DEFAULT DRIVEN SKEWED BUSINESS CYCLES

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ABSTRACT. We augment a simple Real Business Cycle model with financial intermediaries that may default on their liabilities and a financial friction generating social costs of default. We provide a closed-form solution for the general equilibrium of the economy under specific assumptions, allowing for analytical results and straightforward intuitions. Endogenous default generates a negative skew for GDP and a positive skew for credit spreads, as found in US data. Stronger financial frictions cause a rise in asymmetry, which amplifies the welfare costs of fluctuations. A Pigouvian tax on financial intermediation alleviates the costs of fluctuations at the expense of a steady-state distortion. In most cases, the welfare-maximizing policy also attenuates asymmetry.

JEL Codes: E32, E44, G21

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1. Introduction

Business cycles are asymmetric in the United States: expansions are characterized by long-lasting but moderate increases in aggregate variables such as GDP and employment, whereas recessions correspond to sudden but substantial drops in activity.[1] This pattern, which generates negative asymmetry, or negative skewness, has been documented by a number of authors, including Neftci (1984), Hamilton (1989), and Morley and Piger (2012). It also appears to be strengthening: recent work by Jensen, Petrella, Ravn, and Santoro (2019) finds that the skewness of US business cycles has become increasingly negative since the mid-1980s. These authors suggest that financial factors, in the form of rising private-sector leverage associated with occasionally binding borrowing constraints, can account for this surge in asymmetry. In addition, Adrian, Boyarchenko, and Giannone (2019) find that the lower tail in the distribution of GDP growth is associated with periods of deteriorating financial conditions, confirming the role of financial forces in driving macroeconomic skewness.

Building on this literature, our paper studies cyclical asymmetry in a Real Business Cycle (RBC) model augmented with financial intermediaries that may default on their liabilities, as in Gertler and Karadi (2011). We use this setup to yield insights about the relationship between the strength of financial frictions and asymmetry in general equilibrium. In doing so, we substantiate the idea that financial forces have the potential to explain cyclical asymmetry. We also study the relationship between asymmetry and welfare. In particular, the financial friction generating cyclical asymmetry in our model also causes welfare losses, and we study the effectiveness of a simple regulation in overcoming these properties.

More precisely, our framework builds on a RBC model with standard households and firms and a single technology shock, which we augment with a financial sector channeling funds from saving households to borrowing firms. We adopt a particular overlapping-generations structure for the financial sector: financial intermediaries live for two periods, with an old cohort exiting the market at each period and a new one entering. This setup generates an endogenous default decision in the financial sector: old intermediaries receive state-contingent earnings but face predetermined payments, so that they choose to default in bad states of the world. To propagate financial stress to the economy, we

[1] Morley and Panovska (2019) document that business cycles are asymmetric in other industrialized economies as well.

add a financial friction in the form of sunk accounting costs paid by new intermediaries entering in such a default state. This mechanism results in an endogenous amplification of bad technology shocks, which generates business cycle asymmetry in our model. We also introduce a tax on financial intermediation, which has a Pigouvian interpretation since it helps correct the externality arising from the financial sector. Bianchi (2011), Di Tella (2019), and Jeanne and Korinek (2019) use very similar instruments as shortcuts for macro-prudential regulation.

We solve the model under the specific assumptions of log utility, a Cobb-Douglas production function, and full capital depreciation. These restrictions allow us to provide an exact non-linear representation of the general equilibrium of our economy with occasional defaults and to obtain analytical results characterizing the behavior of the model. We obtain the following four results.

First, we show that the financial friction in our model penalizes capital accumulation in a way that mirrors the effects in DSGE models of negative shocks to investment efficiency (Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Moura, 2018) or capital quality (Gertler and Karadi, 2011; Gourio, 2012; Brunnermeier and Sannikov, 2014). This property echoes Justiniano, Primiceri, and Tambalotti's (2011) view that investment shocks proxy for time-varying frictions to financial intermediation in general-equilibrium models. A novelty of our setup is to provide a transparent mechanism through which standard technology shocks are turned into investment wedges through financial frictions.

Second, the analytical solution highlights that asymmetry in our economy originates from a non-linearity due to the default decision: the distribution of default is truncated from above at zero (no-default periods) and features a right tail of positive realizations (default periods). Through the financial friction, this positive skew of default translates into negative skew for capital, output, and consumption. This is the mechanism through which our framework reproduces the stylized fact that US business cycles are negatively skewed.

Third, we obtain a closed-form expression for the equilibrium lending-deposit spread as a function of regulation, the financial friction, and current-period default. The spread inherits the positive skew of default, which makes our model qualitatively consistent with the empirical evidence provided in Ordonez (2013). Our framework also features the positive relationship between the size of financial frictions and the interest-rate asymmetry documented by Ordonez.

Fourth, we use an analytical approximation to characterize the welfare loss, with respect to a central planner economy, caused in our economy by financial frictions and regulation.[2] We decompose the loss in two terms: the first captures the steady-state costs of regulation, which acts as a wedge on capital accumulation, and the second corresponds to cyclical costs linked to default events. We show that tighter regulation worsens steady-state costs but yields cyclical gains, and we prove that these cyclical gains increase with the size of the financial friction propagating default. This demonstrates that stronger financial frictions call for tighter regulation in our model.

Based on these analytical results, we then propose various quantitative illustrations. Although very stylized, the model is able to reproduce the skewness measured in US data, and an additional advantage of the closed-form solution is that the simulations involve no approximation error. We report impulse-response functions (IRFs) showing that positive and negative technology shocks trigger asymmetric responses: the economy behaves like a RBC model after a positive shock, but negative shocks push intermediaries into default and generate financial stress, which amplifies the recession. We also analyze the relationship between the structural parameters and cyclical skewness in our model. In particular, we show that asymmetry in quantities increases with the strength of the financial friction, in line with the findings of Jensen, Ravn, and Santoro (2018) and Jensen, Petrella, Ravn, and Santoro (2019). Finally, we numerically compute the welfare-maximizing regulation scheme and show that for most parametrization, maximizing welfare reduces macroeconomic asymmetry. The model simplicity yields straightforward intuitions for all these numerical results.

Our paper lies at the intersection of several strands of literature. First, it belongs to the large collection of work focusing on business cycle asymmetry. In terms of documenting the skewness of US business cycles, we can mention Potter (1995) and Bloom, Guvenen, and Salgado (2016) in addition to the papers cited above. Many authors have also proposed theoretical explanations for this asymmetry, based on increasing returns (Acemoglu and Scott, 1997), capacity constraints (Hansen and Prescott, 2005), information constraints (Jovanovic, 2006; Van Nieuwerburgh and Veldkamp, 2006), or

[2]Numerical simulations report very small approximation errors for a large range of parameter values.

non-linear adjustment costs related to the labor market (Abbritti and Fahr, 2013). Compared to these papers, our main novelty is to focus on financial default as a source of non-linearity. Second, we contribute to the more recent literature linking business cycle asymmetry with financial factors. In particular, our model reproduces the association between negative skew and financial stress documented by Adrian, Boyarchenko, and Giannone (2019). Compared to Jensen, Petrella, Ravn, and Santoro (2019), we offer a simpler analytical framework and we emphasize default rather than leverage constraints as the potential source of asymmetry. Third and finally, our work relates to papers studying the stabilization properties of macro-prudential regulation in general equilibrium, for instance De Walque, Pierrard, and Rouabah (2010), Angeloni and Faia (2013), and Farhi and Werning (2016). However, we adopt a slightly different perspective: while most papers focus on volatility and its interplay with financial frictions and regulation, we put more weight on the welfare gains of reducing cyclical asymmetry by limiting the size and occurrence of default tail events. In that spirit, our work also echoes Mendoza and Yue (2012), who study the welfare consequences of default in small open economies.

The paper is organized as follows. Section 2 describes the model and provides the closed-form solution. Section 3 illustrates the asymmetric behavior of the economy and the role of financial frictions. Finally, Section 4 turns to welfare and regulation. Section 5 concludes. To increase readability, we relegate most mathematical proofs to Appendices.

2. Model

This section introduces our model of a real economy, which includes a representative household, a representative firm, and a financial sector channeling funds between the household and the firm. There is also a government raising taxes from the financial sector. The household owns all assets in the economy. The model has three key elements: (i) financial intermediaries bear all risk and may default on their liabilities, (ii) there is an externality arising from the social cost of default that intermediaries do not internalize, and (iii) there is a tax on financial intermediation akin to a regulation instrument. We impose conditions that guarantee an exact analytical solution and show how to solve the model.

2.1. Setup.

Except for a small twist related to default, the household side of the model is fairly standard. At each period, the representative household consumes an amount c_t of the final good and saves d_t in deposits issued by financial intermediaries. Thus, the problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t),$$

subject to the following budget constraint

$$c_t + d_t = \left(r^d_{t-1} - \Delta_t\right) d_{t-1} + \pi^c_t + \pi^f_t + t_t.$$

Here, E_0 is the expectation operator conditional on date-0 information, $\beta \in [0,1[$ is the subjective discount factor, r^d_{t-1} is the (predetermined) gross return on deposits, π^c_t is corporate profits, π^f_t is financial profit, and t_t is a lump-sum transfer from the government. The only unusual term in the budget constraint is Δ_t, with $\Delta_t d_{t-1}$ capturing the financial loss incurred by the household when intermediaries default on their liabilities. We define the default rate on financial liabilities as the missed payment $\Delta_t d_{t-1}$ over the schedule payment $r^d_{t-1} d_{t-1}$, which simplifies into Δ_t / r^d_{t-1}. We provide an equilibrium expression for Δ_t in equation (3) below. The consumption-saving plan is characterized by the Euler equation

$$1 = \beta E_t \frac{\left(r^d_t - \Delta_{t+1}\right) c_t}{c_{t+1}}.$$

The production side is also standard. The representative firm uses the k_{t-1} units of capital available at date t to produce the final good in quantity

$$y_t = \epsilon_t k^{\alpha}_{t-1},$$

where $\alpha \in]0,1[$. Total factor productivity evolves according to

$$\epsilon_t = \epsilon^{\rho}_{t-1} \exp(u_t),$$

with $\rho \in [0,1[$, $u_t \sim N(\mu, \sigma^2)$, and $\sigma \geq 0$. Corporate profits are given by

$$\pi^c_t = \epsilon_t k^{\alpha}_{t-1} - r^k_t k_{t-1}$$

and therefore the optimal production plan verifies

$$\alpha \epsilon_t k^{\alpha-1}_{t-1} = r^k_t.$$

To engineer endogenous default events with macroeconomic effects while preserving an exact analytical solution, our modeling of financial intermediation is more involved. We postulate an overlapping-generations structure: intermediaries live for two periods so that, at each date, an old generation exits the market and a new cohort enters. Within this 2-period framework, we rely on three mechanisms:

- First, financial intermediaries bear all risk in the economy. Formally, we assume that intermediaries pay a predetermined return on their deposit liabilities and earn a return linked to the current state of technology on their assets.[3] As a result, bad technology shocks translate into unexpected lower profits in the financial sector, given predetermined costs.
- Second, it must be possible for financial intermediaries to default endogenously in bad states of the world. Our 2-period structure makes this straightforward: old intermediaries leave the economy at the end of each period and do not internalize future costs, so that they choose to default when bad shocks generate negative profits in the financial sector. We also assume that the government seizes any positive profit in case of default, which ensures that old intermediaries default only when their profits are negative.
- Third, new intermediaries must pay a cost when entering an economy in a state with default. This additional friction, which resembles sunk auditing or accounting costs, ensures that financial stress generates social costs for the economy. Similar mechanisms can be found in, among others, Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Malherbe (2019). Without it, default would simply reallocate household income away from deposit earnings and toward profit earnings in a lump-sum fashion, with no effect on equilibrium allocations.

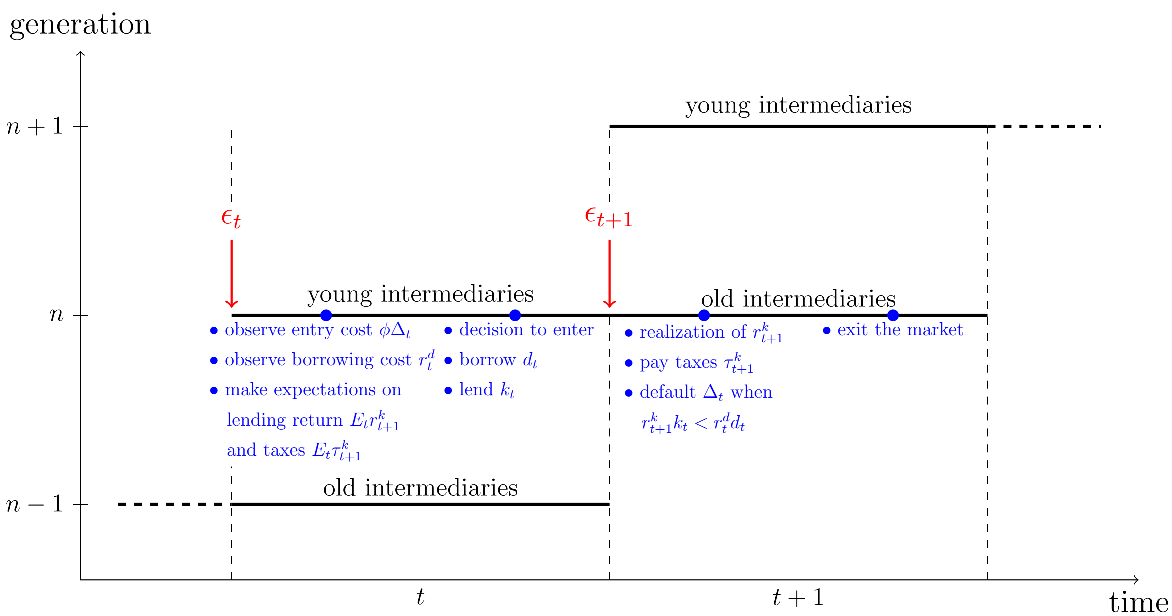
Figure 1 summarizes the timeline of the 2-period OLG financial intermediaries. More precisely, young financial intermediaries entering the market at date t raise an amount d_t of deposits from the household, purchase k_t units of capital, and lend these to the firm. When the economy is in a default state, that is when $\Delta_t > 0$, young intermediaries must also pay an auditing cost equal to a fraction $\phi\Delta_t$ of their balance sheet, with $\phi \geq 0$. As a result, the aggregate balance sheet of financial intermediaries at the end of period t verifies

$$(1 + \phi\Delta_t)k_t = d_t. \tag{1}$$

In the following, we call ϕ the financial friction because it determines the size of the economic costs associated with default in our model. This friction generates an externality

[3]Models with financial frictions typically postulate predetermined deposit rates; see for instance Bernanke, Gertler, and Gilchrist (1999), Iacoviello (2005), or Gertler and Karadi (2011).

FIGURE 1. Financial intermediaries: Timeline of the 2-period OLG structure



because default is decided by old intermediaries, which do not to take into account the feedback effects on other agents, in particular on the financing cost of new intermediaries.

In addition, new intermediaries have to pay a tax τ_{t+1}^k to the government in the next period. This tax is the policy instrument in our model and it has a direct Pigouvian interpretation, since it helps correct the externality arising from the financial sector. It has also implications similar to standard capital requirements: we show below that a higher tax leads to a higher equilibrium spread between the lending and deposit rates and to a lower probability of default, so that the tax makes it possible to limit the riskiness of the financial sector. Bianchi (2011), Di Tella (2019), and Jeanne and Korinek (2019) use a similar shortcut to represent macro-prudential regulation. We assume that the tax is rebated lump sum to households within the period, so that the government budget constraint verifies

$$t_t = \tau_t^k.$$

At date $t+1$, old intermediaries earn $r_{t+1}^k k_t$ from their assets (we assume full capital depreciation) and have to pay $r_t^d d_t$ to the household and τ_{t+1}^k to the government. Old intermediaries may choose to default on their deposit liabilities, in which case they instead transfer their pre-tax income to the household. In contrast, they cannot default *vis-à-vis* the government: this assumption is consistent with the state being a senior creditor and ensures that the policy instrument remains effective in the model.[4] As a result, the profit of old intermediaries at date $t+1$ is given by

$$\pi^f_{t+1} = \max\left(r^k_{t+1}k_t - r^d_t d_t - \tau^k_{t+1},\, -\tau^k_{t+1}\right), \tag{2}$$

where the first argument of the max operator corresponds to the no-default case and the second argument to the default case. It is immediate that intermediaries default only when asset income is below debt servicing costs:

$$r^k_{t+1}k_t < r^d_t d_t,$$

a situation we interpret as insolvency in the financial sector. Shifting time indexes backward, the size of default at date t verifies

$$\max\left(0,\, r^d_{t-1}d_{t-1} - r^k_t k_{t-1}\right).$$

From the household budget constraint, we know that default is also equal to $\Delta_t d_{t-1}$, which implies

$$\Delta_t = \max\left(0,\, r^d_{t-1} - r^k_t \frac{k_{t-1}}{d_{t-1}}\right). \tag{3}$$

The representative household owns financial intermediaries, so that free entry in the market for intermediaries translates into the expected zero-profit condition:

$$\beta E_t\left(\frac{c_t}{c_{t+1}}\pi^f_{t+1}\right) = 0. \tag{4}$$

Finally, we note that our default model is reminiscent of the sovereign default literature (e.g. Aguiar and Gopinath, 2006; Arellano, 2008; Mendoza and Yue, 2012). In these papers, a country chooses to default when the short-term gains of not reimbursing debt are higher than the long-term costs, typically linked to exclusion from world financial markets for a number of periods. In our model, intermediaries default when there is an immediate advantage, since they exit immediately and do not internalize future costs. In addition, in the sovereign default literature, a country going into default experiences productivity losses reflecting inefficiencies linked to financial stress. In our economy, the financial friction ϕ creates a similar mechanism and governs the general equilibrium effects of default.

[4]Usually, the highest priority claim in liquidation goes to fees and outstanding wages, which do not appear here. The state and tax collectors come next. Remaining creditors are then ranked in a descending order of seniority.

2.2. **Solution.** Gathering and rearranging the equations, the equilibrium of our model is characterized by the following system:

$$
(6)\quad
\begin{cases}
\Delta_t = \max\left(r^d_{t-1} - \dfrac{r^k_t}{1+\phi\Delta_{t-1}},\, 0\right), & (5a)\\
c_t + (1+\phi\Delta_t)k_t = \epsilon_t k^{\alpha}_{t-1}, & (5b)\\
\beta E_t\left[\dfrac{\left(r^d_t - \Delta_{t+1}\right)c_t}{c_{t+1}}\right] = 1, & (5c)\\
r^k_t = \alpha\epsilon_t k^{\alpha-1}_{t-1}, & (5d)\\
E_t \max\left[\beta\dfrac{c_t}{c_{t+1}}\left(r^k_{t+1} - [1+\phi\Delta_t]r^d_t\right),\, 0\right] = E_t\, \beta\dfrac{c_t}{c_{t+1}}\,\dfrac{\tau^k_{t+1}}{k_t}, & (5e)\\
\epsilon_t = \epsilon^{\rho}_{t-1}\exp(u_t),\;\; u_t \sim N(\mu,\sigma^2). & (5f)
\end{cases}
$$

These equations highlight the three mechanisms we use to engineer default. First, the expression for Δ_t in (5a) makes it clear how financial intermediaries bear all aggregate risk: at each period, their cost is given in the form of predetermined deposit rates, but their earnings respond to current productivity developments *via* the return to capital. Second, the same equation shows that intermediates default when the return to capital, appropriately weighted, is not sufficient to cover their liability cost. Third, the financial friction $\phi\Delta_t k_t$ in the resource constraint (5b) propagates default to aggregate variables.

Equation (5e), which combines equations (2) and (4), corresponds to the zero-profit condition in the market for intermediation and defines the equilibrium deposit rate r^d_t. It shows why the tax τ^k_{t+1} can be interpreted as a regulatory instrument, since an increase in τ^k_{t+1} implies, *ceteris paribus*, a higher lending-deposit spread and a lower probability of default. Moreover, the left-hand side of equation (5e) is the expectation of a random variable with support over positive values, so that it has to be strictly positive. As a result, system (5) is well defined only when $\tau^k_{t+1} > 0$ ensures that the right-hand side is also above zero. From an economic perspective, the lending-deposit spread becomes irrelevant for financial intermediaries as $\tau^k_{t+1} \to 0$, since in that case they may propose an infinitely high deposit rate, default at each period, and still earn a non-negative profit. However, this generates huge social costs and both capital and consumption converge to zero, so that the economy collapses. Below, we assume that $\tau^k_{t+1} > 0$ to avoid this pathological equilibrium.[5]

[5]This mechanism, which allows unregulated financial intermediaries to offer unsustainable returns on their liabilities (excessive risk taking) and end up defaulting (financial collapse) with negative spillovers

Below, we provide an analytical solution to system (5) that preserves this non-linearity. To build that solution, we impose a specific form on the policy instrument τ_t^k that provides a factorization of the free-entry condition in the market for financial intermediation:

Assumption 1. *The policy instrument is given by*

$$\tau_t^k = \tau \; r_t^k \; k_{t-1},$$

with

$$\tau \equiv \tau(A,\sigma) = \Phi\left(\frac{A}{\sigma} - \frac{\sigma}{2}\right) - \Phi\left(\frac{A}{\sigma} - \frac{3\sigma}{2}\right)\exp(\sigma^2 - A) > 0,$$

where $A \in]0,\infty[$ *if* $\sigma = 0$ *and* $A \in]-\infty,\infty[$ *if* $\sigma > 0$ *and* $\Phi(.)$ *is the cdf of the standard normal distribution.*

Assumption 1 is sufficient to obtain an exact solution. It requires the policy instrument τ_t^k to be a tax on capital income, with constant rate $\tau > 0$. To keep an exact solution, τ has to depend on the volatility parameter σ, so we introduce an additional coefficient A to index the extent of regulation: given a value of σ, the policymaker can choose a value for τ by varying A. In light of this correspondence, from now on we refer to either τ and A as the policy instrument, depending on the context. The mapping from the desired τ to the implied A has no closed form. However, we prove in Appendix B that $\tau(A,0) = 1 - 1/\exp(A)$, $\lim_{A\to-\infty}\tau(A,\sigma) = 0$, $\lim_{A\to\infty}\tau(A,\sigma) = 1$, and $\partial\tau(A,\sigma)/\partial A > 0$ when $\sigma > 0$. For all positive σ, τ is therefore a continuous and monotonically increasing function of A. Finally, Assumption 1 ensures that $\tau_t^k > 0$, so that the free-entry condition in system (5) is well defined.

We are now in position to state:

Proposition 1. *Under Assumption 1, system (5) has the closed-form solution*

$$(DEF)\quad \begin{cases} c_t = [1 - \alpha\beta(1-\tau)]\,\epsilon_t k_{t-1}^{\alpha}, & \text{(6a)} \\ k_t = \dfrac{\alpha\beta}{1+\phi\Delta_t}(1-\tau)\,\epsilon_t k_{t-1}^{\alpha}, & \text{(6b)} \\ \Delta_t = \dfrac{\alpha\epsilon_t k_{t-1}^{\alpha-1}}{1+\phi\Delta_{t-1}} \max\left[\exp\left(\mu + \dfrac{\sigma^2}{2} - u_t - A\right) - 1,\, 0\right], & \text{(6c)} \\ \epsilon_t = \epsilon_{t-1}^{\rho}\exp(u_t),\ \ u_t \sim N(\mu,\sigma^2). & \text{(6d)} \end{cases}$$

Proof. See Appendix C. □

to the whole economy, is close to the usual narrative of the 2008 financial crisis (see, among others, Hanson, Kashyap, and Stein, 2011).

We now use our closed-form solution (DEF) to highlight analytically the properties of the saving rate and capital. We then present the optimal allocation chosen by a benevolent central planner. It will serve as a benchmark when exploring equilibrium properties related to asymmetry and regulation in Sections 3 and 4.

2.3. **Saving rate and capital law of motion.** Equation (6a) indicates that neither default nor the financial friction ϕ affect the equilibrium saving rate $(y_t - c_t)/y_t$, given by $\alpha\beta(1-\tau)$. This saving rate is decreasing in τ, reflecting that tighter regulation weighs on capital accumulation. Moreover, the law of motion of capital (6b) is changed in a way that makes our economy observationally equivalent to a RBC model with shocks to investment efficiency (Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Moura, 2018) or capital quality (Gertler and Karadi, 2011; Gourio, 2012; Brunnermeier and Sannikov, 2014). Indeed, the financial friction arising from default lowers the amount of productive capital obtained from each unit of savings, which exactly mirrors the effect of negative investment efficiency shocks. We formally prove this equivalence in Appendix D, in which we also demonstrate the correspondence with capital quality shocks in our economy with full capital depreciation. Thus, our framework provides a potential micro-foundation for both investment efficiency and capital quality shocks in DSGE models. In particular, it rationalizes why these shocks proxy well for financial factors: in our setup, a negative productivity shock triggering default induces at the same time a fall in aggregate quantities, a rise in credit spreads, and a wedge that resembles investment efficiency and capital quality shocks.

2.4. **Central planner benchmark.** The benevolent central planner maximizes $E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t)$ subject to the resource constraint $c_t + k_t = \epsilon_t k_{t-1}^{\alpha}$. It is straightforward to show:

Proposition 2. *The central planner allocation verifies (McCallum, 1988)*

$$(CP) \quad \begin{cases} k_t = \alpha\beta\epsilon_t k_{t-1}^{\alpha}, \\ c_t = (1-\alpha\beta)\epsilon_t k_{t-1}^{\alpha}, \\ \epsilon_t = \epsilon_{t-1}^{\rho}\exp(u_t), \;\; u_t \sim N(\mu, \sigma^2). \end{cases}$$

Since we impose $\tau_t^k > 0$, Model (DEF) always features an inefficiency distorting capital accumulation. As a result, it is not possible to obtain the efficient allocation as an equilibrium outcome. However, the equilibrium allocation in Model (DEF) becomes arbitrarily close to the efficient outcome when there is no financial friction ($\phi = 0$) and when the regulation distortion vanishes ($\tau \to 0$).

3. Financial Frictions and Business Cycle Asymmetry

In this section, we show how default generates asymmetry and we provide a numerical illustration.

3.1. **Analytical properties.** We directly infer from Proposition 1 some skewness properties. First, equation (6c) shows that our analytical solution preserves the asymmetry of the model. The max operator truncates the equilibrium distribution of the default variable Δ_t to non-negative values, which implies a right tail and a positive skew. In that case, a mass of the distribution lies at zero while the tail corresponds to positive values. In turn, through equation (6b), the positive skew for Δ_t translates into a negative skew for capital: bad shocks cause financial intermediaries to default and the associated social costs generate abnormally low capital realizations in the left tail of the distribution. Finally, given the log-linear decision rule for consumption and production function, both consumption, through equation (6a), and output inherit the negative skew of capital. It follows that our model is able to reproduce the negative skewness of aggregate macroeconomic time series documented by the literature initiated by Neftci (1984) and Hamilton (1989), while maintaining the assumption of symmetric Gaussian productivity shocks.[6] Importantly, the transmission from right-skewed default to left-skewed capital, consumption and output increases with financial frictions ϕ. In particular, without financial frictions ($\phi = 0$), default exists but affects neither capital accumulation nor welfare. Capital is then normally distributed with a zero skew.

Second, the model also generates asymmetry in the spread between the lending and deposit rates. We show in Appendix C that the equilibrium spread between the expected return on credit and on deposits between t and $t+1$ verifies

$$s_t \equiv \frac{E_t r^k_{t+1}}{r^d_t} = \exp(A)(1 + \phi\Delta_t). \tag{7}$$

The spread increases in the default variable Δ_t when there are financial frictions ($\phi > 0$), reflecting higher entry costs facing new intermediaries in bad states of the world. The relationship is linear, so that s_t inherits the asymmetry of Δ_t: credit spreads have a

[6]Altug, Ashley, and Patterson (1999) find no evidence of non-linearity in the Solow residual in the US economy.

positive skew when defaults are rare events. Accordingly, our model is consistent with the positive skewness of spreads in both advanced and emerging economies found by Ordonez (2013). Since the strength of the link between s_t and Δ_t depends on ϕ, our framework is also consistent with the positive relationship between the extent of financial frictions and interest rate asymmetry that Ordonez finds in the data.

Third, we show how the regulation parameter A shapes the level of right skewness of Δ_t, and hence – through the transmission mechanism underlined above – the left skewness of capital, consumption and output, and the right skewness of spread. To do this, we compute the probability that default occurs at any given date as

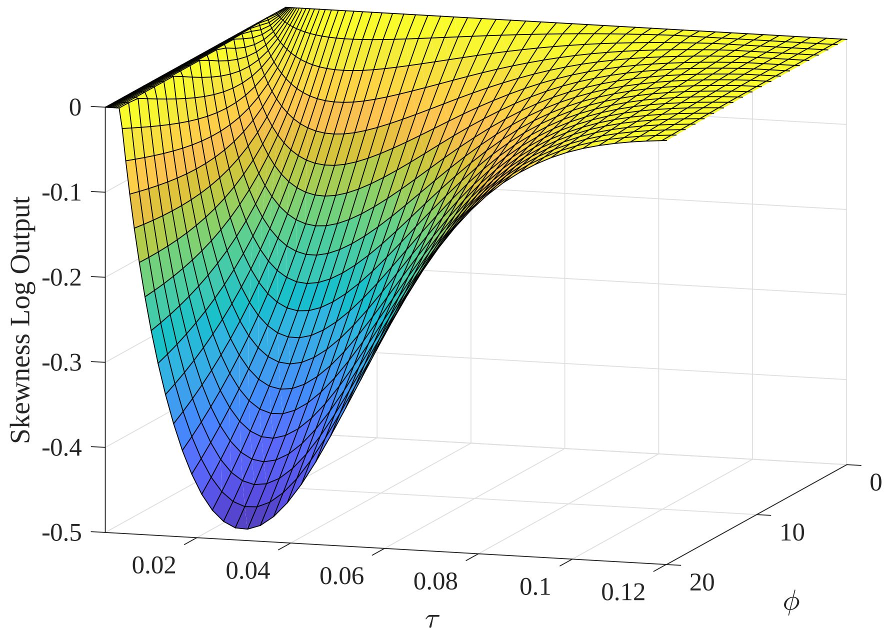
$$\begin{aligned} \Pr[default] &= \Pr\left[r^k_{t+1}k_t - r^d_t d_t < 0\right] = \Pr\left[\exp(u_t) < \exp(\mu + \sigma^2/2 - A)\right] \\ &= \Pr\left[\exp(u_t) < E\exp(u_t - A)\right] = \Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right). \end{aligned} \tag{8}$$

This expression follows from the definition of the equilibrium spread (see Appendix C). Thanks to our 2-period overlapping-generations structure, the probability of default depends on neither current nor past economic conditions, which is key for analytical tractability. We observe that the probability of default decreases with A, that is with τ according to Assumption 1: tighter regulation lowers the occurrence of default events in our model. More precisely, when $\sigma > 0$ and $A \to -\infty$, then $\tau \to 0$ and $\Pr[default] \to 1$, meaning that intermediaries always default when they are not regulated. As a result, all time series are normally distributed, i.e. we obtain a zero-skew economy. When $A \to \infty$, $\tau \to 1$ and default never happens. Again, time series are normally distributed in such an economy. For all intermediate A (between the infinitely small and the infinitely large), the skewness of default is positive.

3.2. **Numerical illustration.** We learned from the closed-form solution (DEF) that (i) Δ_t has a positive skewness unless regulation τ is very small or very large, in which case the skewness tends to zero; (ii) the positive skewness in Δ_t translates into a negative skewness in (log-) output; (iii) the level of financial frictions ϕ amplifies this transmission. To illustrate numerically these properties, we first need to parametrize the model.

We target an annual frequency and pick reference values from the literature: these include the Cobb-Douglas exponent, the subjective discount factor, the persistence and the standard deviation of the technology process, which we set at $\alpha = 0.33$, $\beta = 0.97$, $\rho = 0.90$ and $\sigma = 0.04$. We then compute the skewness of the log output for all pairs (ϕ, τ) with $0 \leq \phi \leq 20$ and $0 < \tau \leq 0.12$. Figure 2 shows that skewness is zero when

Figure 2. Relationship between regulation τ, financial friction ϕ and asymmetry.



Notes. The figure shows the skewness of log output as a function of the financial friction ϕ and regulation τ, with other model parameters fixed at the values $\alpha = 0.33$, $\beta = 0.97$, $\rho = 0.90$ and $\sigma = 0.04$. Statistics computed on samples with $500,000$ observations.

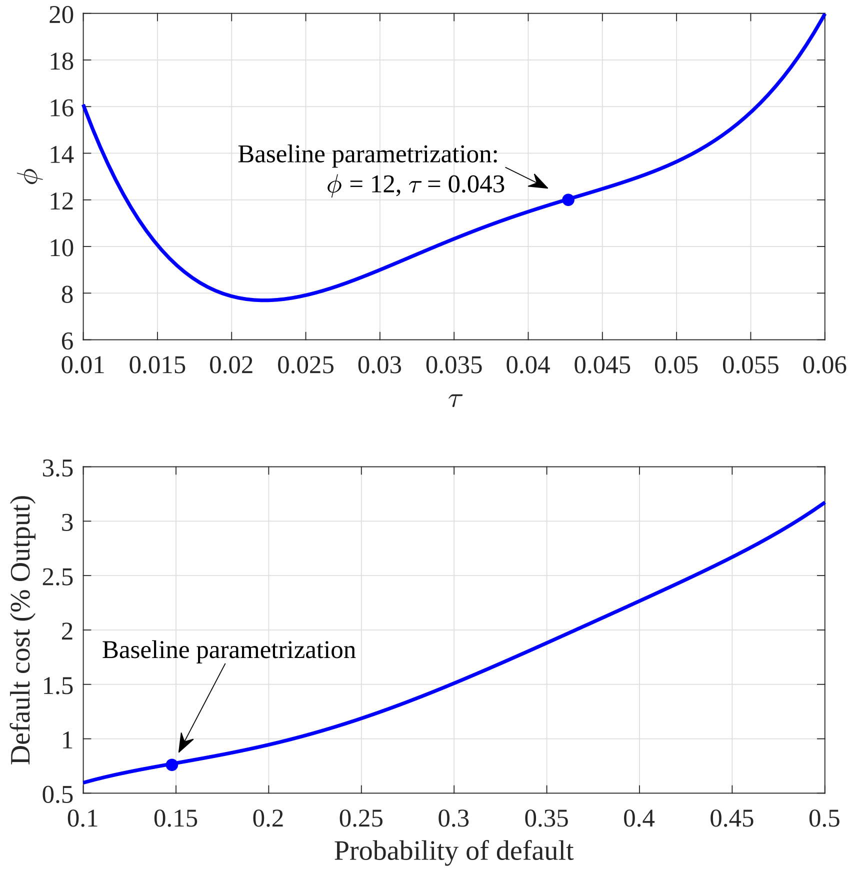
regulation is either very lax (default is the norm and does not generate asymmetry) or very stringent (default never happens). Also, the size of financial frictions affects the transmission of default to real variables and amplifies the asymmetry of output: for a given τ, skewness is zero when $\phi = 0$ and reaches more negative values as ϕ increases.

Although the model is very stylized, we want to know how far it can go in reproducing the asymmetry observed in the data.[7] For instance, the skewness of log GDP was -0.24 between 1953 and 2018.[8] The top panel in Figure 3 shows there are other possible (ϕ, τ) parametrizations such that log output skewness is -0.24. When regulation τ is small or large, the skewness of default is close to zero and we need high financial frictions ϕ to

[7]We use time series extracted from the FRED database. Output is annual real GDP in chained 2012 dollars (GDPC1), investment is annual real gross private domestic investment in chained 2012 dollars (GDPIC1), while the credit spread is the yearly average of Moody's seasoned Baa corporate bond yield relative to the yield on 10-year treasury bonds (BAA10YM). We remove the long-run trend of GDP using the HP filter with smoothing parameter 100, the standard value for annual series.

[8]Note that asymmetries in US data have increased in recent years, with a skewness of log GDP of -0.44 between 1980 and 2018. See also Jensen, Petrella, Ravn, and Santoro (2019) for a discussion.

FIGURE 3. Possible (ϕ, τ) parametrizations such that log output skewness is -0.24.

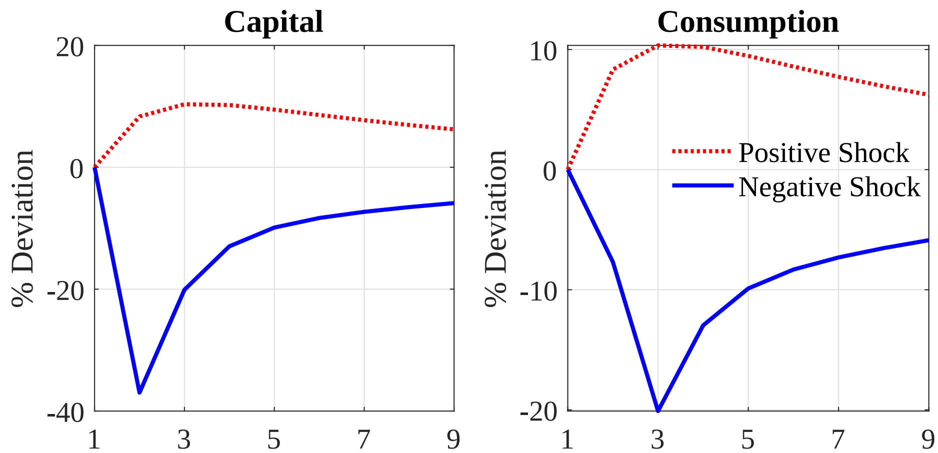


Notes. The figure shows possible parametrizations such that the skewness of log output is -0.24, with other model parameters fixed at the values $\alpha = 0.33$, $\beta = 0.97$, $\rho = 0.90$ and $\sigma = 0.04$. Statistics computed on samples with $500,000$ observations.

generate enough asymmetry. The bottom panel transforms these (ϕ, τ) combinations into more explicit default cost, computed as $E(\phi\Delta_t k_t/y_t)$, and probability of default, as defined in equation (8). For instance, assuming an average cost of default of about 0.8% of output (as a comparison Laeven and Valencia, 2018, estimate at 4.5% the annual GDP cost of the 2007-2011 US financial crisis) and an average probability of default of 15% (meaning that default occurs on average every 7 years) implies choosing the pair $(\phi, \tau) = (12, 4.3\%)$. We call these values our baseline parametrization.

Interestingly, the baseline parametrization makes our model quantitatively consistent with the asymmetry in investment and credit spreads measured in US data. For instance,

Figure 4. Asymmetric effects of technology shocks.



Notes. IRFs to two-standard-deviation positive and negative technology shocks, starting from the deterministic steady state of the model. Baseline parametrization $(\phi, \tau) = (12, 4.3\%)$ with other model parameters fixed at the values $\alpha = 0.33$, $\beta = 0.97$, $\rho = 0.90$, $\sigma = 0.04$.

the skewness of log investment, which is equivalent to log capital in our model, measured in US data was -0.81 between 1953 and 2018. Our baseline parametrization implies a slightly higher value of -1.0. As for the credit spread, it had a positive skewness of 0.38 between 1953 and 2018. Our baseline parametrization overshoots this value, by generating a skew of 3.7 for the spread. Still, the model correctly implies that credit spreads have a positive skew.

Finally, based on this baseline parametrization, we compute the equilibrium path of the model following positive and negative technology shocks. We choose the deterministic steady state as the initial condition, so that the level of default was always zero in the past, and we hit the economy with a one-time, two-standard-deviation technology shock, either positive or negative.[9] We report the resulting IRFs in Figure 4. Equation (6a) demonstrates that consumption and output have identical dynamics in our model, and we report the response of consumption only.

The dashed red lines represent the dynamic effects induced by the positive shock. These are pretty standard. Productivity increases by 8% on impact (two-standard-deviation shock) and then gradually returns to its long-run level. Higher productivity leads to an immediate rise in production of similar magnitude, which is absorbed by

[9]We need a sufficiently negative shock to trigger default and hence asymmetry. This is the case with a two-standard-deviation shock.

increased levels of consumption and investment. The additional units of capital available for future production, together with the persistence of the technology shock, slightly amplify the economy's response in the short run and generate hump-shaped dynamics in capital and consumption. There is no default and and our model displays exactly the same movements as the central planner benchmark.

The solid blue lines represent the economy's response to the negative technology shock. There is a clear asymmetry compared to the effects of the positive shock, as well as significant amplification. The unexpected fall in productivity lowers the return to capital and the income of financial intermediaries. As a result, intermediaries go into default. Although default occurs only when the shock hits the economy, since there is no surprise afterward, the effects are long lasting. Through the financial friction, default entails a large social cost that weighs on investment, which drops by 35% on impact. Consumption also falls immediately, though this only reflects lower productivity (recall that the equilibrium saving rate is constant). At future dates, there are negative spillovers due to the lower capital levels and consumption reaches a trough of -20% two periods after the shock. The economy then gradually returns to its steady state.

These IRFs highlight two key properties of our model. First, positive and negative technology shocks generate asymmetric responses from the endogenous variables because of the non-linearity of default. Second, the financial friction provides an amplification mechanism for negative shocks, both on impact and in later periods. All amplification originates from the social cost associated with the financial friction: setting $\phi = 0$ would eliminate both the asymmetry transmission and the amplification related to default.

4. Welfare and Regulation

So far, we have proposed a positive analysis of how occasional defaults give rise to business cycle asymmetry in our model, through financial frictions. We have also seen the ambiguous role of regulation for skewness. In this final section, we take a different perspective: frictions generate inefficiencies and yield a suboptimal equilibrium allocation. How could regulation restore – or not – an optimal equilibrium allocation? In this normative analysis, we build on our analytical solution and we provide an exact expression for welfare, which we use to study the costs of default. To push the computations further, we introduce an approximation that preserves the non-linearity of the max operator. Finally, we resort to numerical simulations to illustrate the welfare-maximizing regulation policy and to show the relationship between optimal regulation and asymmetry.[10]

4.1. **Analytical results.** As in Lester, Pries, and Sims (2014), we focus on unconditional welfare as measured by the average value function of the representative household. Using the results from Propositions 1 and 2, straightforward algebra demonstrates that, in Models (CP) and (DEF), welfare verifies

$$\begin{aligned}(1-\alpha)(1-\beta)\mathcal{W}_{CP} &= (1-\alpha)\ln(1-\alpha\beta)+\alpha\ln(\alpha\beta)+\frac{\mu}{1-\rho}\\ (1-\alpha)(1-\beta)\mathcal{W}_{DEF} &= (1-\alpha)(1-\beta)\mathcal{W}_{CP}\\ &\quad+(1-\alpha)\ln\left[\frac{1-\alpha\beta(1-\tau)}{1-\alpha\beta}\right]+\alpha\ln(1-\tau)-\alpha E\ln(1+\phi\Delta_t),\end{aligned}$$

where E is the unconditional expectation operator. It follows that the welfare difference between Models (DEF) and (CP) is given by

$$(1-\alpha)(1-\beta)\left(\mathcal{W}_{DEF}-\mathcal{W}_{CP}\right) = -\Big[\underbrace{g(\tau)}_{\text{steady-state cost}} + \underbrace{\alpha E\ln(1+\phi\Delta_t)}_{\text{cyclical cost}}\Big], \tag{9}$$

where

$$g(\tau)=(1-\alpha)\ln\left[\frac{1-\alpha\beta}{1-\alpha\beta(1-\tau)}\right]-\alpha\ln(1-\tau).$$

We show in Appendix E that $g(\tau)>0$ for $\tau\in\ [0,1[$.

The next proposition summarizes the welfare ranking between the central planner benchmark and the economy with default:

Proposition 3. *For all $\sigma\geq 0$ and $\tau(A,\sigma)\in\,]0,1[$, we have*

$$\mathcal{W}_{DEF}<\mathcal{W}_{CP}.$$

In addition, $\mathcal{W}_{DEF}\to\mathcal{W}_{CP}$ when $\sigma=0$ and $\tau\to 0$, and $\mathcal{W}_{DEF}\to\mathcal{W}_{CP}$ when $\phi=0$ and $\tau\to 0$.

Proof. See Appendix E. □

Proposition 3 shows that welfare is always lower in the model with default. Equation (9) decomposes this welfare loss into two sources. First, there is a cost $g(\tau)$ representing the distortion to capital accumulation induced by regulation. We refer to it as

[10] In the following, we refer to this welfare-maximizing policy as the optimal policy, keeping implicit that it is only constrained-optimal and does not restore the efficient central-planner allocation.

the steady-state cost because tighter regulation reduces the average levels of GDP, consumption, and capital even in a deterministic economy without default. Second, there is a cost $\alpha E \ln(1+\phi\Delta_t)$ linked to default events. It is strictly positive when uncertainty generates occasional default events ($\sigma > 0$) and when default is amplified by financial frictions ($\phi > 0$). This cost encapsulates the negative welfare consequences induced by uncertainty, which explains why we refer to it as the cyclical cost. In the limiting cases in which default does not occur ($\sigma = 0$) or does not propagate ($\phi = 0$), only the regulation cost matters and the economy converges to the central planner allocation when $\tau \to 0$. Obviously, this discussion neglects the key point that tighter regulation also lowers the probability and size of default, as well as the related social costs. We discuss this point below.

It is difficult to obtain further analytical results about welfare because the cost $E\ln(1+\phi\Delta_t)$ in equation (9) cannot be explicitly written in terms of structural parameters. Still, we can make some progress using the analytical approximation:

Assumption 2. *The equilibrium is such that*

(A1) $\ln(1+\phi\Delta_t) \approx \phi\Delta_t$*;*

(A2) $\Delta_t\Delta_{t-1} \approx 0$*;*

(A3) $\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1}$ *is accurately approximated by a first-order Taylor expansion around the deterministic steady state;*

(A4) $\phi < 1/\theta$*, where*

$$\theta = \frac{\exp\left(\sigma^2/2\right)\left[\exp(-A)\Phi\left(\sigma/2 - A/\sigma\right) - \Phi\left(-\sigma/2 - A/\sigma\right)\right]}{\beta(1-\tau)} > 0.$$

Assumption 2 essentially requires that the economy is not too volatile around its deterministic steady state, so that the linearizations involved in $(A1)$ and $(A3)$ remain accurate; that defaults are not too large, so that $(A2)$ holds; and that the financial friction is not too strong, so that $(A4)$ is verified. At the same time, the assumption preserves the non-linearity of the max operator and thus the asymmetry due to default. We show numerically in Appendix F that the resulting approximation error is small for a wide range of parameter values.

Still in Appendix F, we demonstrate that under Assumption 2 the cyclical cost in equation (9) simplifies to

$$\alpha E\ln(1+\phi\Delta_t) \approx \frac{\alpha\phi\theta}{1-\phi\theta},$$

where $(A4)$ from Assumption 2 ensures that the right-hand side is positive. We can then show:

Proposition 4. *Under Assumptions 1 and 2,*

$$\frac{\partial E \ln(1+\phi\Delta_t)}{\partial \phi} > 0, \quad \frac{\partial E \ln(1+\phi\Delta_t)}{\partial \tau} < 0, \quad \frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial \tau \partial \phi} < 0.$$

Proof. See Appendix G. □

Given equation (9), Proposition 4 makes three statements valid in the vicinity of the deterministic steady state of our model.

First, higher financial frictions ϕ deteriorate welfare in Model (DEF), since they amplify the cyclical cost. This is not surprising, as stronger frictions make default events more costly to the economy.

Second, there is a clear tradeoff related to regulation. On the one hand, tighter regulation impairs welfare through $g(\tau)$, since $\partial g(\tau)/\partial \tau > 0$. This is a steady-state cost that reflects the distortion to capital accumulation induced by regulation. On the other hand, tighter regulation improves welfare by mitigating the social costs of default, as can be seen from the negative response of $E \ln(1+\phi\Delta_t)$ to an increase in τ. This is a cyclical effect that captures the lower frequency and smaller size of default events in a regulated economy.

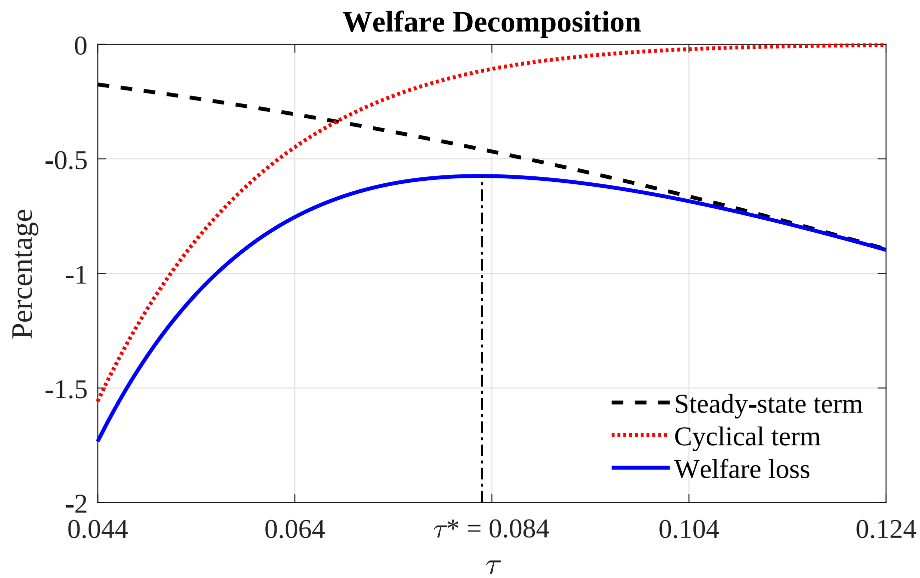
Third, the cyclical effect is more important when financial frictions are high, as shown by the cross-partial derivative of $E \ln(1+\phi\Delta_t)$ with respect to ϕ and τ. Defining the optimal policy as the one balancing positive and negative effects on welfare, our analytical argument makes it clear that higher financial frictions justify tighter regulation: the cyclical cost reduction from increasing τ is larger, while the steady-state cost is unchanged.

4.2. **Numerical illustrations.** Finally, we document the properties of optimal regulation in our model using numerical analyses.

In our baseline parametrization, we find that the welfare function is concave in the regulation instrument τ.[11] This is clear from Figure 5 (solid blue line), which reports the welfare losses resulting from varying τ while keeping other model parameters constant at the baseline values. Concavity of welfare essentially follows from the two costs apparent

[11]We checked numerically that welfare being concave in τ is a robust implication of our model by varying the parameters ϕ and σ.

FIGURE 5. Forces shaping optimal regulation.

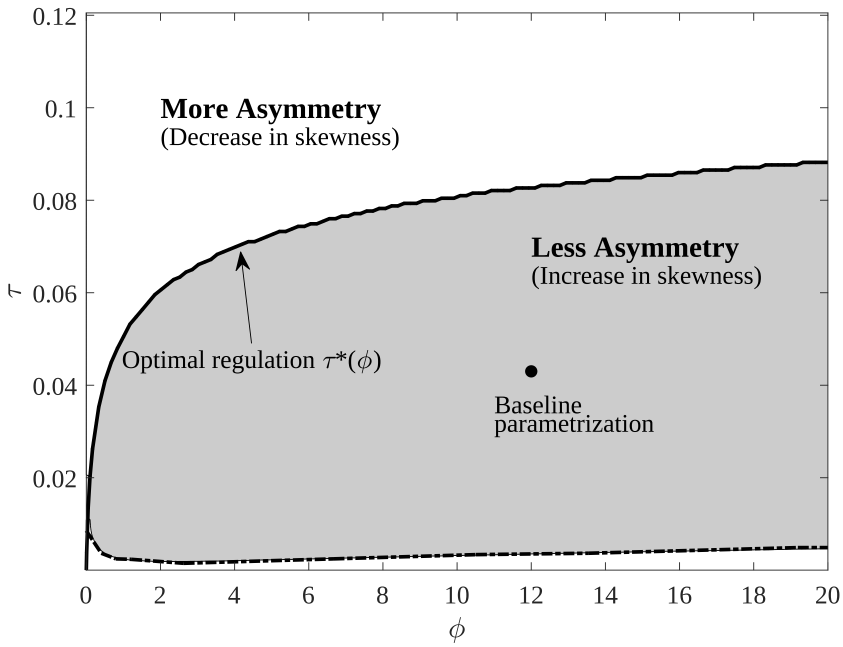


Notes. The figure shows the welfare loss resulting from varying the regulation instrument τ around its optimal value $\tau^{\star}$, with other model parameters fixed at the baseline parametrization $\phi = 12$, $\alpha = 0.33$, $\beta = 0.97$, $\rho = 0.90$, $\sigma = 0.04$. The dashed and dotted lines provide the decomposition of welfare into the steady-state and cyclical costs defined in equation (9). Welfare losses are expressed as percent deviations from the central planner benchmark W^{CP}. Statistics computed on samples with $500,000$ observations.

in equation (9). For low values of τ, regulation is lenient: as a result, the steady-state distortion is small while the cyclical costs associated with default are important, so that there are welfare gains from tightening regulation. In contrast, when τ is high regulation is tight so default occurs rarely: in this case, the distortions on capital accumulation are stronger and deregulation improves welfare.

Because of concavity, there exists an interior value of τ that maximizes welfare. In our baseline parametrization, this optimal regulation instrument corresponds to a tax rate of $\tau^{\star} = 8.4\%$. In comparison, regulation is $\tau = 4.3\%$ in the baseline parametrization. More generally, optimal regulation balances the welfare effects of marginally raising or lowering the tax rate: at the optimal level $\tau^{\star}$, the cyclical welfare benefit of limiting the negative consequences of default by raising the tax rate is equal to the steady-state welfare cost of larger capital distortions. This indifference condition is apparent in Figure 5, which decomposes the welfare function around $\tau^{\star}$: the slopes of the steady-state and cyclical terms are equal at the optimal regulation. The chart also confirms the

FIGURE 6. Change in skewness implied by optimal policy for all pair (ϕ, τ).



Notes. The figure shows how the optimal regulation policy $\tau^*(\phi)$ changes the skewness of log output. More precisely, for all pair (ϕ, τ), we compute $skew(\phi, \tau^*(\phi)) - skew(\phi, \tau)$. A positive change means a less negative skewness, i.e. optimal regulation reduces asymmetry. A negative change means a more negative skewness, i.e. optimal regulation increases asymmetry. The other model parameters are fixed at the baseline parametrization $\alpha = 0.33$, $\beta = 0.97$, $\rho = 0.90$ and $\sigma = 0.04$. Statistics computed on samples with $500,000$ observations.

analytical insight that optimal regulation is tighter when financial frictions are larger: a rise in ϕ amplifies cyclical costs and leaves steady-state costs unchanged, shifting the dotted red line downward and calling for a higher tax rate to restore the slope equality between the two cost functions.

Finally, we assess how the welfare-maximizing regulation affects skewness. Figure 6 plots, for all pair (ϕ, τ), the change in the skewness of log output implied by the implementation of the optimal regulation. This chart highlights four important properties of our economy.

First, the upper solid line, which corresponds to the optimal regulation $\tau^*(\phi)$, reveals an increasing relationship between the optimal tax τ^* and the financial friction ϕ: optimal regulation is close to 0 when the friction is weak, and then increases gradually with ϕ. This is in line with our analytical argument from Section 4.1.

Second, the gray area represents all parametrizations such that implementing the optimal policy implies a rise in τ and a fall in asymmetry. Hence, in this area welfare improvements are associated with a fall in asymmetry: the optimal policy lowers the occurrence of default and reduces the left tail of the equilibrium distribution of log output. Our baseline parametrization $(\phi, \tau) = (12, 4.3\%)$ lies inside this gray area, as do the other possible parametrizations matching the skew of log output shown Figure 3. This indicates that, for all sensible parametrizations of our model, implementing the optimal regulation will limit business-cycle asymmetry.

Third, for all parametrizations above the optimal regulation schedule, the original tax level is very high. As a result, implementing the optimal policy requires a fall in τ, which generates a rise in asymmetry. Still, this region verifies the usual negative relationship between regulation and equilibrium asymmetry.

Fourth, parametrizations below the gray area are associated with a different logic since moving toward the optimal policy is associated with both tighter regulation (increase in τ) and more asymmetry (decrease in the skew of log output). To understand this positive relationship between regulation and asymmetry, remark that taxation is originally very low in this area, which results in extreme frequencies of default: in other words, default is the norm rather than an occasional event, so that the asymmetry related to the occasionally binding constraint vanishes. By consequence, the equilibrium skewness of log output is roughly zero in this area. Implementing the optimal policy implies a rise in τ, a large fall in the frequency of default, and as a result more asymmetry.

5. Conclusion

This paper develops a Real Business Cycle model with endogenous default and financial regulation. We prove analytically that: (i) financial frictions mirror the effect of a negative shock to capital accumulation; (ii) endogenous default generates asymmetric business cycles; (iii) tighter regulation decreases steady-state consumption but lowers the probability of default, which may generate welfare gains. We illustrate these theoretical results through various quantitative experiments. In particular, we show that the size of financial frictions amplifies business-cycle asymmetry and that skewed business cycles are usually associated with welfare losses.

We see at least four interesting extensions of our stylized framework. First, considering partial capital depreciation would allow the equilibrium saving rate to vary over time and potentially depend on the level of financial frictions. Second, and in the same vein, an endogenous labor supply would highlight interactions between financial frictions and the labor market, especially during crises. Third, we could take an extended version of the model with partial depreciation and endogenous labor to the data to check whether it is able to reproduce observed asymmetries. Fourth and finally, the variance of productivity shocks deserves more attention, since volatility affects the decision rule for capital accumulation through the cost of default.

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Appendix A. Equivalence between Tax and Capital Requirements

In this appendix, we show that regulatory policies implemented through taxes or capital requirements on financial intermediaries produce equivalent effects. Our argument largely mirrors that of Bianchi (2011).

Consider a partial-equilibrium version of the model from Section 2.1. Furthermore, assume that there is no financial friction ($\phi = 0$). The financial intermediary borrows d and lends k, subject to the balance-sheet constraint $k = d$. It earns $r^k k$ from assets and pays $r^d d$ on liabilities. Default happens when $r^k k < r^d d$, that is with probability

$$\Pr[default] = \Pr\left[\frac{r^k}{r^d} < 1\right],$$

where we used the balance-sheet constraint to simplify quantities. Independently of default, the financial intermediary must pay a tax $\tau r^k k$ proportional to capital income, where $\tau > 0$ is a tax rate. The free-entry condition for financial intermediation is therefore

$$E \max[r^k k - r^d d,\, 0] = \tau r^k k,$$

which can be simplified to

$$E \max\left[1 - \frac{r^d}{r^k},\, 0\right] = \tau.$$

It follows immediately that raising the tax rate τ increases the equilibrium lending-deposit spread r^k/r^d, which in turn reduces the probability of default $\Pr[r^k/r^d < 1]$.

Now, consider a similar economy in which capital requirements replace taxes: the financial intermediary must finance at least a fraction $\gamma \in [0, 1[$ of the loans it issues with equity e, i.e. $e \geq \gamma k$. Assume that raising equity is more costly than raising deposits, for instance because deposits yield a liquidity service to the household. Other possible justifications include the outcome of moral-hazard problems or tax disadvantages on equity (Bianchi, 2011). Since the return on equity is higher than the return on deposits, the equity constraint is always binding and $e = \gamma k$. The intermediary's balance-sheet becomes $k = d + \gamma k$, equivalently $(1 - \gamma)k = d$. Default still occurs whenever $r^k k < r^d d$, that is with probability

$$\Pr[default] = \Pr\left[\frac{r^k}{(1-\gamma) r^d} < 1\right].$$

For simplicity, we take the cost of equity to be given by $(1 + \eta) r^d \gamma k$, with $\eta > 0$ being the additional cost of equity relative to deposits. The free-entry condition in the market for intermediation is then

$$E\max[r^k k - r^d d,\, 0] = (1+\eta) r^d \gamma k,$$

which can be simplified to

$$E\max\left[\frac{r^k}{(1-\gamma)r^d} - 1,\, 0\right] = (1+\eta)\frac{\gamma}{1-\gamma}.$$

Hence, raising the capital adequacy ratio γ increases the credit spread r^k/r^d, which ends up reducing the probability of default $\Pr[r^k/((1-\gamma)r^d) < 1]$. It follows that a regulator could use either of a tax policy or a capital requirement to reduce the probability of financial default in this model.

Appendix B. Properties of τ

Consider first the case of $\sigma = 0$. Then $A \leq 0$ implies $\tau(A, 0) = 0$, which we exclude since we need $\tau_t^k > 0$. We therefore impose $A \in]0, \infty[$ when $\sigma = 0$. Under this constraint, we have $\tau(A, 0) = 1 - 1/\exp(A)$, which is increasing in A. Moreover, $\lim_{A\to 0}\tau(A, 0) = 0$ and $\lim_{A\to\infty}\tau(A, 0) = 1$.

In the more general case of $\sigma > 0$, we do not need to restrict the support of A, which belongs to $]-\infty, \infty[$. The derivative of the tax rate with respect to A is

$$\frac{\partial\tau(A,\sigma)}{\partial A} = \exp(\sigma^2 - A)\Phi\left(\frac{A}{\sigma} - \frac{3\sigma}{2}\right) > 0.$$

Using l'Hospital rule to deal with an indeterminate form, we find that $\lim_{A\to-\infty}\tau(A,\sigma) = 0$. Furthermore, $\lim_{A\to\infty}\tau(A,\sigma) = 1$ is evident.

Appendix C. Proof of Proposition 1

We use a guess-and-verify approach. Suppose that the policy function for consumption verifies

$$c_t = \Gamma\epsilon_t k_{t-1}^{\alpha},$$

where $\Gamma \geq 0$ is an unknown coefficient. Using this guess, equation (5d) and Assumption 1, the free-entry equation (5e) can be written as

$$E_t \max\left[\alpha\epsilon_t^{\rho} k_t^{\alpha-1} - \frac{r_t^d(1+\phi\Delta_t)}{\exp(u_{t+1})},\, 0\right] = \tau\alpha\epsilon_t^{\rho} k_t^{\alpha-1}.$$

To simplify the notation, define $\mu_{1,t} = \alpha\epsilon_t^{\rho} k_t^{\alpha-1}$ and $\mu_{2,t} = r_t^d(1+\phi\Delta_t)$. Remark that both $\mu_{1,t}$ and $\mu_{2,t}$ are known as of date t, so that the only source of uncertainty is u_{t+1}.

Knowing that $u \sim N(\mu, \sigma^2)$ and using $f(\cdot)$ to denote its pdf., the above equation is equivalent to

$$\int_{\ln \frac{\mu_{2,t}}{\mu_{1,t}}}^{\infty} f(u)du - \frac{\mu_{2,t}}{\mu_{1,t}} \int_{\ln \frac{\mu_{2,t}}{\mu_{1,t}}}^{\infty} \exp(-u) f(u) du = \tau.$$

After some algebra, this can be expressed as

$$\Phi\left(\frac{\mu - \ln \frac{\mu_{2,t}}{\mu_{1,t}}}{\sigma}\right) - \frac{\mu_{2,t}}{\mu_{1,t}} \exp\left(-\mu + \frac{\sigma^2}{2}\right) \Phi\left(\frac{\mu - \sigma^2 - \ln \frac{\mu_{2,t}}{\mu_{1,t}}}{\sigma}\right) = \tau.$$

A solution to this equation is

$$\mu_{1,t} = \kappa\, \mu_{2,t}, \tag{10}$$

where $\kappa > 0$ must verify

$$\Phi\left(\frac{\mu + \ln \kappa}{\sigma}\right) - \frac{1}{\kappa} \exp\left(-\mu + \frac{\sigma^2}{2}\right) \Phi\left(\frac{\mu - \sigma^2 + \ln \kappa}{\sigma}\right) = \tau.$$

It is impossible to find a closed-form expression $\kappa = \kappa(\mu, \sigma, \tau)$ in the general case. However, we can impose Assumption 1 requiring that

$$\tau = \Phi\left(\frac{A}{\sigma} - \frac{\sigma}{2}\right) - \Phi\left(\frac{A}{\sigma} - \frac{3\sigma}{2}\right) \exp(\sigma^2 - A),$$

with $A \in]0, \infty[$ if $\sigma = 0$ and $A \in]-\infty, \infty[$ if $\sigma > 0$. Then it turns out that

$$\kappa = \exp\left(-\mu - \frac{\sigma^2}{2} + A\right),$$

solves the equation.

Plugging κ, $\mu_{1,t}$ and $\mu_{2,t}$ into equation (10), we obtain

$$E_t r_{t+1}^k = \exp\left(\mu + \frac{\sigma^2}{2}\right) \alpha \epsilon_t^\rho k_t^{\alpha-1} = \exp(A) r_t^d (1 + \phi \Delta_t).$$

We use this relationship between the marginal product of capital and the deposit rate to simplify equations (5a) and (5c) into

$$\Delta_t = \frac{\alpha \epsilon_{t-1}^\rho k_{t-1}^{\alpha-1}}{1 + \phi \Delta_{t-1}} \max\left[0,\, \exp\left(\mu + \frac{\sigma^2}{2} - u_t - A\right) - 1\right],$$

$$\frac{1}{\epsilon_t k_{t-1}^\alpha} = \frac{\alpha\beta}{k_t(1 + \phi\Delta_t)} E_t \min\left[\exp\left(\mu + \frac{\sigma^2}{2} - u_{t+1} - A\right), 1\right]. \tag{11}$$

Define $l_t = E_t \min[\exp(\mu + \sigma^2/2 - u_{t+1} - A), 1]$. Then,

$$\begin{aligned} l_t &= \Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right) + \exp\left(\mu + \frac{\sigma^2}{2} - A\right) \int_{\mu+\frac{\sigma^2}{2}-A}^{\infty} \exp(-u) f(u) du \\ &= \Phi\left(\frac{\sigma}{2} - \frac{A}{\sigma}\right) + \exp\left(\sigma^2 - A\right) \left[1 - \Phi\left(\frac{3\sigma}{2} - \frac{A}{\sigma}\right)\right] \\ &= 1 - \tau. \end{aligned}$$

Inserting this expression into the Euler equation (11) yields

$$k_t = \frac{\alpha\beta}{1 + \phi\Delta_t}(1 - \tau)\epsilon_t k_{t-1}^{\alpha}.$$

Merging this equation with the resource constraint (5b), we obtain

$$c_t = [1 - \alpha\beta(1 - \tau)]\, \epsilon_t k_{t-1}^{\alpha}.$$

This validates our initial guess for the consumption policy function, whose unknown coefficient verifies

$$\Gamma = [1 - \alpha\beta(1 - \tau)]\,.$$

Appendix D. Equivalence with Investment Efficiency Shocks and Capital Quality Shocks

This appendix shows the observational equivalence between our set-up with default and financial frictions and a model with shocks to the efficiency of investment. We also demonstrate the correspondence with capital quality shock when capital fully depreciates. Throughout, we abstract from the tax rate τ without loss of generality.

An influential strand of the literature argues that investment shocks, which affect the transformation of private savings into productive capital, play a prominent role in US business cycles (see, among others, Greenwood, Hercowitz, and Krusell, 2000; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Moura, 2018). In addition, Justiniano, Primiceri, and Tambalotti (2011) show that shocks to investment efficiency proxy for financial disturbances in DSGE models, an insight that our framework corroborate.

We introduce an investment efficiency shock into the central planner model from Section 2.4; see Fernandez-Villaverde and Rubio-Ramirez (2007) for a very similar set-up. Household preferences and firm technology remain unchanged, but the aggregate resource constraint becomes

$$k_t = z_t(y_t - c_t), \tag{12}$$

where $z_t \in]0,1]$ is the investment shock. If $z_t = 1$ at all periods, we recover the central planner economy in which savings are fully transformed into productive capital. Here, we instead assume that, although z_t equals 1 at the deterministic steady state, it may occasionally be below than 1. In that case, a contraction in the efficiency of investment lowers the amount of productive capital obtained out of savings, with negative consequences on aggregate production.

The model has a simple solution. At each period, the consumption-saving plan is characterized by the Euler equation

$$\frac{1}{z_t c_t} = \alpha\beta E_t \frac{y_{t+1}}{k_t c_{t+1}}.$$

Using the aggregate resource constraint, this is also

$$\frac{y_t}{c_t} = 1 + \alpha\beta E_t \frac{y_{t+1}}{c_{t+1}}.$$

Since $\alpha\beta < 1$, substituting forward and imposing the transversality condition yields

$$c_t = (1-\alpha\beta) y_t, \tag{13}$$

so that the equilibrium saving rate does not depend on the investment shock z_t. This is not the case of capital accumulation, which is given by

$$k_t = z_t \alpha\beta \epsilon_t k_{t-1}^{\alpha}. \tag{14}$$

It is immediate that equations (13) and (14) are equivalent to equations (6a) and (6b) from Proposition 1 when

$$z_t = \frac{1}{1+\phi\Delta_t} \in]0,1].$$

It follows that our model with default and financial frictions provides a micro-foundation for investment efficiency shocks. More precisely, a negative investment shock in the above model is observationally equivalent to the negative externality arising from endogenous default in Model (DEF).

Several papers mimic the aggregate effects of financial crisis using disturbances to capital quality (see for instance Gertler and Karadi, 2011; Gertler, Kiyotaki, and Queralto, 2012, and many others). To model these shocks, we slightly modify the central planner economy from Section 2.4 to introduce incomplete capital depreciation at rate $\delta \in]0,1]$ and a capital quality shock $\psi_t \in]0,1]$. Define in-process capital s_t as the sum of after-depreciation productive capital $(1-\delta)k_{t-1}$ and investment i_t:

$$s_t = (1-\delta)k_{t-1} + i_t,$$

and assume that in-process capital is transformed into next-period productive capital after the realization of a multiplicative capital quality shock:

$$k_t = \psi_t s_t.$$

Merging these equations, the aggregate resource constraint becomes

$$y_t = c_t + i_t = c_t + s_t - (1-\delta)k_{t-1} = c_t + \frac{k_t}{\psi_t} - (1-\delta)k_{t-1}.$$

In the special case of $\delta = 1$, this simplifies into

$$k_t = \psi_t(y_t - c_t),$$

which is equivalent to the resource constraint (12) from the model with investment efficiency shocks. It follows that the model solution is given by equations (13) and (14), in which the capital quality shock ψ_t simply replaces the investment efficiency shock z_t. Hence, in an economy with full capital depreciation a capital quality shock is also observationally equivalent to the negative externality arising from endogenous default in Model (DEF).

Appendix E. Proof of Proposition 3

Define

$$g(\tau) = (1-\alpha)\ln\left[\frac{1-\alpha\beta}{1-\alpha\beta(1-\tau)}\right] - \alpha\ln(1-\tau),$$

with $\tau \in]0,1[$ according to Assumption 1. Since $\lim_{\tau\to 0} g(\tau) = 0$, $\lim_{\tau\to 1} g(\tau) = \infty$, and $\partial g(\tau)/\partial\tau > 0$, we have $g(\tau) > 0$. Moreover, $\Delta_t \geq 0$ by definition, so that $\phi \geq 0$ implies $E\ln(1+\phi\Delta_t) \geq 0$. Together, these restrictions prove the first part of the proposition.

When $\sigma = 0$, $\Delta_t = 0$ from equation (6c) and $\mathcal{W}_{DEF} \to \mathcal{W}_{CP}$ when $\tau \to 0$. This proves the second part of the proposition.

When $\phi = 0$, $\ln(1+\phi\Delta_t) = 0$ and $\mathcal{W}_{DEF} \to \mathcal{W}_{CP}$ when $\tau \to 0$. This proves the last part of the proposition.

Appendix F. Welfare Approximation

This appendix proves the welfare approximations from Section 4.1.

Replacing ϵ_t by its expression (6d) in equation (6c) gives

$$\Delta_t(1+\phi\Delta_{t-1}) = \alpha\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1} \max\left[\exp\left(\mu + \frac{\sigma^2}{2} - A\right) - \exp(u_t),\, 0\right].$$

Under simplification ($A2$) from Assumption 2, taking the unconditional expectation of both sides of the equality and applying the Law of Iterated Expectations gives

$$E\Delta_t \approx E\left\{\alpha\epsilon_{t-1}^{\rho}k_{t-1}^{\alpha-1}E_{t-1}\max\left[\exp\left(\mu+\frac{\sigma^2}{2}-A\right)-\exp(u_t),\ 0\right]\right\}.$$

The conditional expectation is

$$\int_{-\infty}^{\mu+\sigma^2/2-A}\left[\exp\left(\mu+\frac{\sigma^2}{2}-A\right)-\exp(u)\right]f(u)du=\exp\left(\mu+\frac{\sigma^2}{2}\right)h(\sigma,A),$$

where we define

$$h(\sigma,A)=\exp(-A)\Phi\left(\frac{\sigma}{2}-\frac{A}{\sigma}\right)-\Phi\left(-\frac{\sigma}{2}-\frac{A}{\sigma}\right).$$

It easy to show that $\lim_{A\to-\infty}h(\sigma,A)=\infty$ and $\lim_{A\to\infty}h(\sigma,A)=0$. Moreover, the partial derivative verifies

$$\frac{\partial h(\sigma,A)}{\partial A}=-\exp(-A)\Phi\left(\frac{\sigma}{2}-\frac{A}{\sigma}\right)<0.$$

Together with the limits as $A\to\pm\infty$, this implies $h(\sigma,A)>0$. Overall, the expected value of the default term is thus

$$E\Delta_t=\alpha\exp\left(\mu+\frac{\sigma^2}{2}\right)h(\sigma,A)E\left(\epsilon_{t-1}^{\rho}k_{t-1}^{\alpha-1}\right). \tag{15}$$

To obtain an analytical expression for the last term, we use simplification ($A3$) from Assumption 2 and take a log-linear approximation around the non-stochastic steady state of the model. This gives

$$E\left(\epsilon_{t-1}^{\rho}k_{t-1}^{\alpha-1}\right)\approx\bar{\epsilon}^{\rho}\bar{k}^{\alpha-1}E\left[1+\rho\left(\ln\epsilon_{t-1}-\ln\bar{\epsilon}\right)+(\alpha-1)\left(\ln k_{t-1}-\ln\bar{k}\right)\right],$$

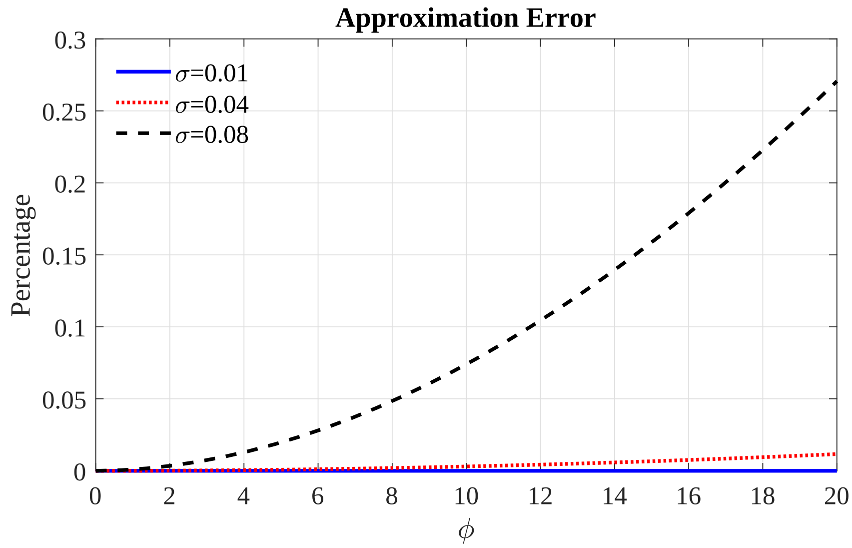
where upper bars denote non-stochastic steady-state levels. From equations (6b) and (6d), we obtain

$$\begin{aligned}\bar{\epsilon}&=\exp(\mu)^{\frac{1}{1-\rho}},\\ \bar{k}&=\left[\alpha\beta(1-\tau)\bar{\epsilon}\right]^{\frac{1}{1-\alpha}},\\ E\ln\epsilon_t&=\frac{\mu}{1-\rho},\\ (1-\alpha)E\ln k_t&=\ln[\alpha\beta(1-\tau)]+\frac{\mu}{1-\rho}-E\ln(1+\phi\Delta_t).\end{aligned}$$

Finally, simplification ($A1$) from Assumption 2 allows to write the last equation as

$$(1-\alpha)E\ln k_t\approx\ln[\alpha\beta(1-\tau)]+\frac{\mu}{1-\rho}-\phi E\Delta_t.$$

FIGURE 7. Approximation error due to Assumption 2.



Notes. The figure shows the approximation error in welfare computations induced by Assumption 2. It reports, for different (ϕ, σ) combinations, the absolute value of the ratio $(\widetilde{\mathcal{W}}_{DEF} - \mathcal{W}_{DEF})/\mathcal{W}_{DEF}$, where $\widetilde{\mathcal{W}}_{DEF}$ is the analytical welfare approximation and $\mathcal{W}_{DEF}$ is the exact model welfare. Statistics computed on samples with $500,000$ observations. Baseline parametrization $\tau = 4.3\%$ with other model parameters fixed at the values $\alpha = 0.33$, $\beta = 0.97$, $\rho = 0.90$.

It follows that

$$E\left[\epsilon_{t-1}^{\rho} k_{t-1}^{\alpha-1}\right] = \frac{1 + \phi E\Delta_t}{\alpha\beta(1-\tau)\exp(\mu)}. \tag{16}$$

Consolidating equations (16) and (15) then yields

$$E\Delta_t = \frac{\theta}{1 - \phi\theta}, \quad \text{with } \theta = \exp\left(\frac{\sigma^2}{2}\right)\frac{h(\sigma, A)}{\beta(1-\tau)}.$$

It is clear that $h(\sigma, A) > 0$ implies $\theta > 0$. Equation $(A4)$ from Assumption 2 then implies that $E\Delta_t > 0$, which is consistent with default having a non-negative support.

Finally, these computations yield an analytical expression for the last term in welfare $\mathcal{W}_{DEF}$: relying once more on simplification $(A1)$ from Assumption 2, we have

$$E\ln(1 + \phi\Delta_t) \approx \phi E\Delta_t = \frac{\phi\theta}{1 - \phi\theta} \geq 0.$$

Figure 7 shows that the approximation error resulting from Assumption 2 is small for a wide range of parameter. We have also verified that $(A4)$ holds for all parameter configurations used in this figure.

Appendix G. Proof of Proposition 4

The sign of the first partial derivative is immediate since $\partial E \ln(1+\phi\Delta_t)/\partial\phi = \theta/(1-\phi\theta)^2 > 0$.

To prove the sign of the second partial derivative, we know from Appendix B that $\partial\tau/\partial A > 0$. Therefore, it is equivalent to prove $\partial E \ln(1+\phi\Delta_t)/\partial\tau < 0$ or $\partial E \ln(1+\phi\Delta_t)/\partial A < 0$. Using the properties of τ and $h(\sigma, A)$ derived in Appendices B and F, we obtain

$$\frac{\partial\theta}{\partial A} = -\exp\left(\frac{\sigma^2}{2}\right) \frac{(1-\tau)\Phi\left(-\sigma/2 - A/\sigma\right) + h(A,\sigma)\Phi\left(\sigma/2 - A/\sigma\right)}{\beta(1-\tau)^2} < 0,$$

which implies in turn

$$\frac{\partial E \ln(1+\phi\Delta_t)}{\partial A} = \frac{\partial\theta/\partial A}{(1-\phi\theta)^2} < 0.$$

This proves the second result.

Finally, it is equivalent to prove the sign of the last partial derivative with respect to τ or A:

$$\operatorname{sign}\left(\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial\tau\partial\phi}\right) = \operatorname{sign}\left(\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial A\partial\phi}\right).$$

Then, the above results as well as (A4) in Assumption 2 imply

$$\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial A\partial\phi} = \frac{\partial}{\partial A}\left[\frac{\theta}{(1-\phi\theta)^2}\right] = \frac{(1-\phi\theta)(1+\phi\theta)}{(1-\phi\theta)^4}\ \frac{\partial\theta}{\partial A} < 0.$$

Since all partial derivatives are themselves differentiable, Schwarz's theorem (see, e.g., Rudin, 1976, for details) implies

$$\frac{\partial^2 E \ln(1+\phi\Delta_t)}{\partial\phi\partial\tau} < 0.$$