Public and Private Employment in a Model with Underemployment *

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Abstract

The public sector hires disproportionately more educated workers. Using US data from the CPS, we show that this college bias holds across gender, age, states, level of government, as well as over time. It is also true within industries and in two thirds of 3-digit occupations. To rationalize this finding, we propose a model of private and public employment based on two key features. First, alongside a perfectly competitive private sector, our economy features a cost-minimizing government that acts with a wage schedule that does not equate supply and demand. Second, our economy features heterogeneity across individuals and jobs, and a simple sorting mechanism that generates underemployment. The equilibrium model is parsimonious and can be calibrated to match key moments of the US public and private sectors. We find that, in the US economy, the excess hiring of skilled in the public sector is mainly accounted for by technological consideration, with the public wage differential and excess underemployment in the public sector accounting for 15 percent of the education bias. In addition, in a counterintuitive fashion, we find that more wage compression in the public sector raises inequality in the private sector. A 1 percent increase in unskilled public wages raises skilled private wages by 0.07 percent and lowers unskilled private wages by 0.06 percent.

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1 Introduction

The public sector is a large employer. Behar and Mok (2013) report that, on average for 194 countries, public(-sector) employment accounts for 15 percent of total employment. In developed OECD countries, public sectors account for between 10 and 35 percent of total employment. However, the importance of the public sector is very heterogeneous across different types of workers. As Figure 1 shows for the US, UK, France and Spain, the public sector tends to hire significantly more educated workers. The left panel shows the fraction of public employment out of total employment for workers with and without a college degree. The right panel shows the fraction of college graduates out of total public and private employment. Both graphs reflect the bias of the public sector towards workers with higher education.[1] Using US data from the CPS, we show that the college bias holds across gender, age, states, level of government, as well as over time. It also holds true within industries and in two thirds of 3-digit occupations that are common across the two sectors. The aim of this paper is to understand why this happens.

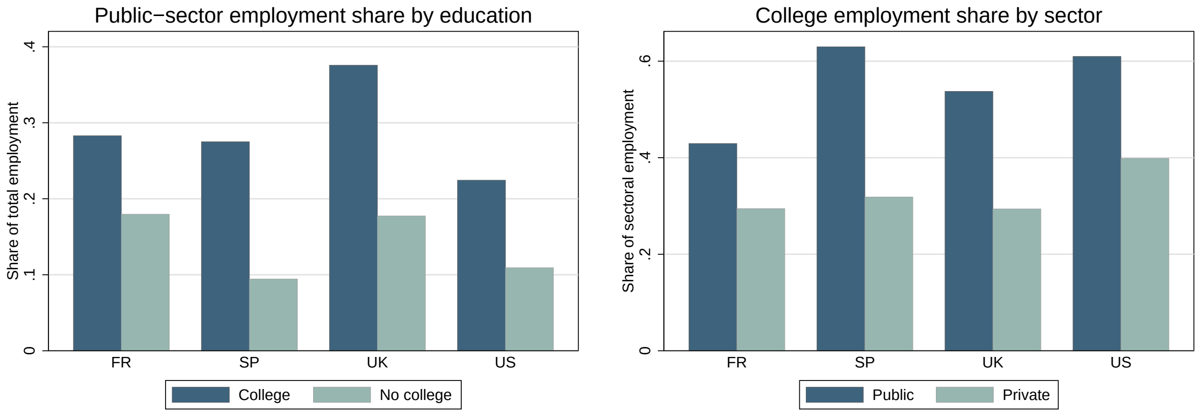
The public sector is very different from the private sector. It does not sell its goods or services, but supplies them directly to the population. The technology used to produce such services is likely to be different from the private sector, and could be more biased towards educated workers. Since the public sector has no revenues from sales, it finances its production through the power of taxation. Further, the public sector does not have shareholders, it does not maximize profits and does not go (often) into bankruptcy. The decisions regarding employment are taken by governments. On the one hand, they partly reflect the preferences of society about the scope of the public sector and whether their services should be produced directly or outsourced to the private sector. On the other hand, it has been documented that they also reflect other government objectives such as: attaining budgetary targets [Gyourko and Tracy (1989)]; implementing a macroeconomic stabilization policy [Keynes (1936)]; redistributing resources [Alesina et al. (2000)]; or satisfying interest groups for electoral gains [Gelb et al. (1991)]. As such, the usual economics mechanisms that drive the private sector adjustments do not map into the public sector. One of the missing adjustment channels is wages. When governments set their wages (or wage growth), there is a discretionary component that can create widely documented wage differentials vis-à-vis the private sector.[2] Hence, public wages do not necessarily equate demand and supply.

Using US CPS data, we confirm the finding in the literature that the public sector provides a wage schedule that is compressed across educational level, with higher (lower) pay for low (high) educated workers vis-a-vis the private sector. This wage compression holds in 50 of the 51 US states. With respect to the educational bias reported in Figure

[1] Although this fact is not necessarily common knowledge, it has been previously documented. See, for instance the Handbook of Labour Economics chapter by Gregory and Borland (1999).

[2] See Katz and Krueger (1991) for the United States, Postel-Vinay and Turon (2007) or Disney and Gosling (1998) for the United Kingdom and Christofides and Michael (2013), Castro et al. (2013) and Giordano et al. (2011) for several European countries.

Figure 1: Public-Sector Employment and Education



Note: The graph on the left shows the fraction of public-sector employment out of total employment for college and not college graduates. The graph of the right, is the share of public-sector workers that have a college degree divided by the share or private sector workers with a college degree. For the United States the data is take from CPS (1996-2018), for the United Kingdom from the UK Labour Force Survey (2003-2018), for France for the French Labour Force Survey (2003-2018) and from Spain from the Spanish Labour Force Survey (2005-2018). For details on the methodology for the European economies see Fontaine et al (2018).

1, the wage compression fact has a demand and a supply implication. On the one hand, a compressed wage schedule might bias labor demand from governments towards relatively inexpensive graduate workers. On the other hand, a compressed wage schedule might shift labor supply of educated workers away from the public sector. A model of public employment should consider both demand and supply effects of wage compression.

A further channel for understanding the public-sector education bias is underemployment. In OECD economies a large share of workers are employed in jobs that require skills and characteristics lower than the ones they have, as reported by McGowan and Andrews (2015) or McGowan and Andrews (2017). We think that underemployment might be have stronger incidence in the public sector as its hiring process for limited positions is largely based on a ranking of candidates. Using PIACC data, we show that in 70 percent of the countries considered, the fraction of workers that appears under-employed (or in other words over qualified) is higher in the public sector. As we argue in the brief literature review below, we lack a benchmark model to evaluate the general equilibrium consequences of public-sector employment and wage policies. The contribution of this paper is to provide such a model.

The private vs public sector model with underemployment developed in Section 3 is based on two key features. First, alongside a perfectly competitive private sector, our economy features a cost minimizing government facing a wage schedule that does not necessarily equate demand and supply. Given a wage schedule, the government decides how many workers for different tasks should be employed for producing a given level of public services. In this sense, our model merges a neoclassical Walrasian private sector with a public sector modeled in the spirit of disequilibrium theories à la Malinvaud (1977) and Barro and Grossman (1971). Second, our economy features heterogeneity across individuals and jobs. Workers can be high- or low-educated while jobs have different skill requirements. Jobs are described through a ladder type mechanism, so that individuals endowed with higher education are able to perform also unskilled jobs, but workers with low education cannot perform skilled jobs.

We assume a variation of the Roy model (Roy (1951) and Borjas (1987)) in which workers attach different "non-pecuniary" value to jobs in different sectors and of different skills. This preference structure generates a non-trivial sorting mechanism that serves two purposes. First, we generate a labor market allocation with endogenous underemployment, that depends on the wage differential between jobs of different skills. Second, it allows for both positive and negative wage premium in the public sector for different workers. On the one hand, when public wages are high enough, there would be more workers interested in having a public-sector job than available jobs, with the excess workers driven to the private sector. In this regime, public employment is demand determined (jobs are rationed). On the other hand, if the public-sector wage premium is negative, the government can only fill all of its jobs if there are enough workers with a strong preference for the public sector. Still, if wages are too low, the government might be constrained in the number of workers of certain type that can hire and forced to substitute to another type of workers to maintain the production of government services. In this regime public employment is determined by supply. It is also possible that if wages decrease below a certain threshold, the public sector can no longer produce the minimum level of services and breaks down. In our baseline equilibrium specification, based on US empirical targets, public jobs appear to be rationed (demand determined).

Our model provides three possible explanations for why public employment is so biased towards educated workers. The first explanation is technological – government hires more educated workers because they are more important inputs in the production of government services. A second explanation is related to the wage schedule. A cost-minimizing government that is constrained to pay a compressed profile of wages (i.e. due to union pressures), shifts the composition from the (relative more expensive) less qualified workers to the (relative less expensive) more qualified workers. The third explanation is underemployment and over-qualification. If wages of unskilled public-sector jobs are very high, they might attract workers with more qualifications. This last channel amplifies the role of the public-sector wage schedule.

In Section 4 we calibrate a variation of the model to match key statistics of the US economy. The model is fairly parsimonious, and seven structural parameters can be obtained by solving a simple algorithm to match seven key moments, including public employment and the public-private wage differential by education, and a conservative estimate of underemployment in the U.S. We use the calibrated model to carry out two quantitative exercises. First, we solve the model under the assumption that wages in the public sector equalize wages in the private sector, which also eliminates excess underemployment in the public sector. We then solve it with the additional assumption that technology is the same in the two sectors. We find that the public-private wage differential and underemployment account for 15 percent of the excess hiring of educated workers in the US public sector.

In our second exercise, we calculate the elasticities of private wages with respect to in public wages. We find that the wage policy is a crucial driver of private wage inequality, but in an counterintuitive fashion – a more compressed wage schedule in the public sector raises inequality in the private sector. A one percent increase in unskilled public wages raises skilled private wages by 0.07 percent and lowers unskilled private wages by 0.06 percent. Given a variation of the public-sector wage premium of 20 percentage points across US states, the variation of this policy alone can determine a variation of 2.6 percentage points in the college premium. More wage compression alters the skill-mix in the public sector from unskilled to skilled jobs and fosters underemployment. As a consequence, the skill-mix in the private sector shifts towards low-educated workers, so their wages fall while wages of high-educated workers go up. While decreasing wage inequality for a sub-set of workers, such policies increase wage inequality for everyone else.

Our paper is related to two strands in the literature. First, the assumption that public wages do not adjust to equate supply and demand is related to the fixed-price equilibrium literature that followed from Barro and Grossman (1971) and Malinvaud (1977). More recent papers in this literature include Benassy (1993) or Michaillat and Saez (2015). Second, it is close to the literature on public employment that mainly uses search and matching models. Examples include Gomes (2015), Gomes (2018), Bradley et al. (2017), Albrecht et al. (2019). These papers study the effects of public employment and wages on unemployment and other labour market outcomes. While search and matching frictions are important to study several aspects of public employment, for instance the role of job security, we think that some of its consequences can be more clearly understood without search unemployment. More precisely, the skill mix chosen by the government is bound to affect the skill mix of the private sector, even in a full employment context. The papers that are most closely related to ours are Domeij and Ljungqvist (2019), Gomes and Kuehn (2017), Gomes (2018) and Michaillat (2014). Domeij and Ljungqvist (2019) build a model where the public sector hires an exogenous number skilled and unskilled workers, to compare the evolution of the skill premium in US and Sweden. They point out that the expansion of the Swedish public sector, that hired more low-skilled workers, can explain the divergence of the skill premium between the two countries. Gomes and Kuehn (2017) study, in a model of occupational choice, the effects of skill-biased hiring in the public sector on the occupational choice of entrepreneurs and on firm size. Relative to these two papers, we endogenise the choice of the type of public-sector workers hired, add underemployment, and allow for different wages across sectors. Our approach to model the choice of workers in the public sector - based on a cost minimization - is similar to Gomes (2018). His model has search and matching frictions and is solved quantitatively. Our model has a simple structure summarized by few equations allowing the study of underemployment. Finally, Michaillat (2014) proposes a general equilibrium model with public spending and endogenous public and private employment. The model uses the matching approach to emphasize the role of stochastic rationing, but features a unique wage in both sectors.

2 Three Key Facts: Education Bias, Wage Compression and Underemployment

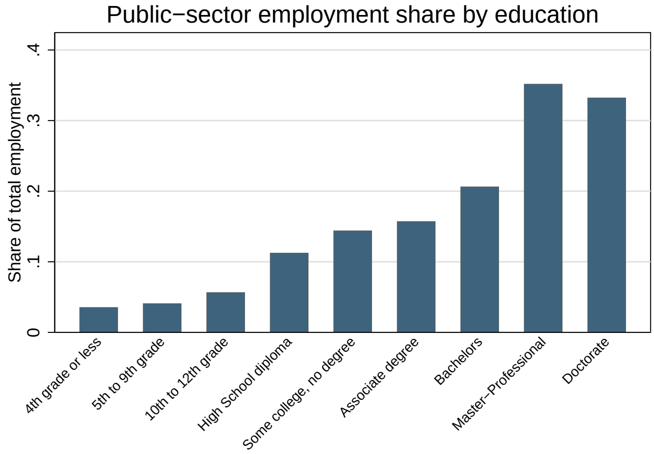
We first report empirical evidence of the three main facts that motivate our research. Section 2.1 reports the evidence on the education bias across various dimensions and across countries. Section 2.2 reports the evidence on wage compression across educational levels in the public sector. Section 2.3 defines and reports estimates of underemployment across countries and across public and private sectors.

We uses a variety of data sources. The main dataset used is the CPS. This survey provides labor force status as well as data on work experience. Comprehensive work experience information is given on the employment status, occupation, industry, weeks worked and hours per week worked. For the calculation of the stocks we use the monthly files from 1996 to 2018. We restrict the sample to individuals aged 16 to 64. When we estimate the public-sector wage premium we use the CPS March Supplement, that has information on total income and income components. The distinction between public and private sector jobs is based on a self-reported variable. Each respondent is asked to classify his/her employer. We define public-sector employment as work for the Government (whether Federal, State or Local government). This method is consistent with the statistics published by the BEA.

We also analyse data from the United Kingdom, France and Spain. We choose these countries with sizable public sectors because their public sectors encompass different industries and they employ distinct hiring processes, and because these large economies are characterized by very different labor market institutions and education policies. This guarantees that common findings across these four countries are likely to be intrinsic characteristics of the public sector and not driven by country specificities. Our analysis is based on microdata and in particular, for each country, we use the representative labor force survey, from which official statistics are drawn: the French *Labour Force Survey* (FLFS), the UK *Labour Force Survey* (UKLFS) and the Spanish *Labour Force Survey* (SLFS). See Fontaine et al. (2018) for details on the definition of the public sector. For the wage regressions, we use microdata from the 2002, 2006, 2010, and 2014 *Structure of Earnings Survey*.

Finally, evidence of underemployment comes from the OECD Survey of Adult Skills, part of the Program for the International Assessment of Adult Competencies (PIAAC). The data were collected between 2011 and 2015. In each country, the survey includes socio-demographic information (gender, education), labor market status and assesses the proficiency of adults aged between 16 and 65 in literacy, numeracy and problem solving.

Figure 2: Public-Sector Employment Share By Educational Levels



Note: CPS data, average between 1996 and 2018. Government workers (Federal, State and Local government), fraction of employment of workers age 16 to 64 with a given level of education. ***College:*** *Associate degree, Bachelors, Master, Doctorate.*

2.1 Education Bias

Figure 1 reports the basic fact on the tendency of the public sector to employ workers with college degree. While workers are divided between two education categories in most of the paper, it is worth emphasizing that education is a quasi-continuous variable. Using CPS data for the US, Figure 2 reports public employment share for 9 educational categories, from few years into primary education until workers with tertiary education. The relationship is almost monotonic. At the very top, the government hires one third of all employed workers with Masters or Professional degree or who hold a PhD. For simplicity, throughout the paper, we summarize education into two categories: college and no-college. We assume that workers with no college include also workers that attended college but did not complete the degree. Still, one should keep in mind that there is further heterogeneity within these groups.

Table 1 reports the accounting definition used in the paper. We normalize the size of the employment pool by 1, and we let n and $1-n$ denote respectively the share of employed workers with a college degree and without college. College workers are indicated with subscript 1 while no-college workers with subscript 2. Superscript g refers to the government/public sector while superscript p refers to the private sector. We thus indicate

Table 1: Basic Accounting With Two Sectors and Two Education Categories

	Public sector	**Private sector**	**Total**
College	l_1^g	l_1^p	n
No-college	l_2^g	l_2^p	$1-n$
Total	l^g	l^p	1

Note: Government (g), private (p), college (1), no-college (2). Total employment normalized to 1. Share of college in total employment (n).

with l_1^g the stock of college workers employed in the public sector (similarly for the other 3 categories). Given the two-by-two matrix described in Table 1, in the paper, we summarize the employment bias in the public sector with one of two indicators. The first indicator is the ratio of public employment shares r^g, simply defined as the ratio of public employment share for college workers over the public employment share for non-college workers (shown on the left of Figure 1). The second statistics is the education intensity ratio ei^g, defined as the ratio of the share of college graduates out of public sector workers over that of the private sector (shown on the right of Figure 1). Formally:

$$r^g = \frac{\frac{l_1^g}{n}}{\frac{l_2^g}{1-n}}, \qquad ei^g = \frac{\frac{l_1^g}{l^g}}{\frac{l_1^p}{l^p}}.$$

These two statistics are complementary (Figure 3). In the unlikely case of perfect symmetry across sectors, both statistics would generate a value of 1. The ratio is above 1.4 for the four countries reported, which points towards a public employment bias for college workers. Throughout the remaining of this section, we focus mainly of the ratio of public-employment shares, but we report in Appendix A all the figures with the education intensity ratio.

While the type of services that governments produce is an important driver of the education intensity, they are not the only explanation. One key empirical finding of this section is that the public-sector education bias holds across industries in the US, France and the UK (Figure 4).[3] On the one hand, even when excluding the Health and Education industries, industries that naturally employs a large share of graduates, the bias remains, although with lower ratio. The US ratio of public employment shares is 1.8 instead of 2. On the other hand, even within the health and education industries, the public sector hires a larger fraction of graduates than the private sector, leading to a ratio larger than 1.

To further support this argument we also analyse the heterogeneity of public-sector jobs, based on occupational classification from 3-digit ISCO-08 in the US.[4] We consider only occupations with non-trivial public sector employment (in each occupation, the share of public-sector employment in total employment is larger than 5%[5]) We find that, in total, two-thirds of the occupations have ratio of public-employment shares larger than 1. Overall, the distribution across industries and occupations appear important, and indeed will play a key role in the theory that we propose, but it does not explain everything.

In Appendix A, focusing on US data, we show the different statistics across gender, age, US states and over time. The ratio of public employment share is constantly around 2

[3]The Spanish LFS does not allow for a disaggregation of public employment by industry.

[4]CPS occupational code is based on 2010 Census 4-digit occupational classification. We use a cross-walk in order to classify occupations based on 4-digit ISCO-08. This occupation classification has several advantages: First, it provides clear guidelines for grouping occupations. Secondly, it provides harmonized classification across countries, which will be helpful when we extend our empirical analysis to other countries.

[5]In doing so, some top-paid occupations are dropped (such as Manufacturing, mining, construction, and distribution managers; Architects, planners, surveyors and designers) as well as some low-paid jobs (such as Domestic, hotel and office cleaners and helpers, Vehicle, window, laundry and other hand cleaning workers, Waiters and bartenders).

Table 2: Regression Of The Log Of Hourly Wages

	Controlling for 2-digit occupation		**Not controlling for occupation**	
	College	No college	College	No college
Public-sector	0.010***	0.077***	-0.0262***	0.095***
	(5.09)	(40.79)	(-14.8)	(51.7)
Controls				
Age and gender	X	X	X	X
Region and year	X	X	X	X
Part-time	X	X	X	X
Occupation	X	X		
Observations	668,287	918,664	668,287	918,664
R-squared	0.294	0.247	0.155	0.167

Note: Estimation by regressing the log of hourly wage on a public-sector dummy and controls (age, gender, region, year and a part-time dummy), separately for workers with and without college graduate. When controlling for occupation we include 2-digit occupation dummies. CPS data between 1996 and 2018.

across gender and age. When we disaggregate by US states, the ratio of public employment shares varies from 1.4 in Washington DC to 3 in Nevada. The ratio is also persistent over time, even though it fell around the great recession, most likely because of large changes in private-sector employment.

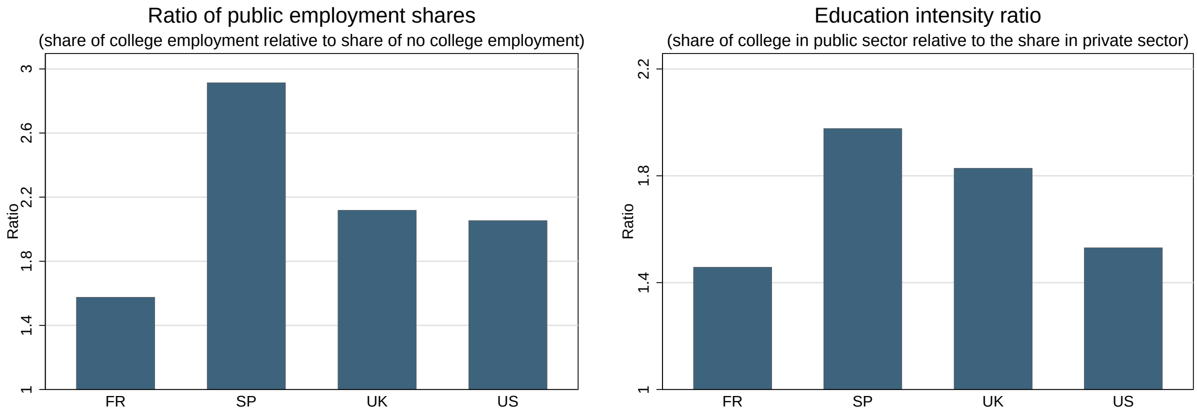
2.2 Wage Compression

The second key fact concerns the wage policy and the tendency in the public sector to compress wage across educational group. Specifically, low-educated public-sector workers tends to be paid more than their private-sector counterparts, while the public-sector wage premium of high-educated workers is lower (and sometimes negative). The basic evidence of wage compression comes from a simple Mincer regression on log hourly wages on a variety of control, including the public-sector dummy. Figure 5 shows the premium for different levels of education. We can see that it is the highest for workers with only the 4th grade, and it decreases with PhD having the largest negative premia.

Table 2 shows the estimations for our two categories: college and no-college workers. Controlling for 2-digit occupations, the estimate of the public sector wage premium is of 1 percent for college graduate and 7.7 percent for workers with no college. We will use these numbers in the quantitative section. If we do not control for occupation, the public-sector wage premium even becomes negative for college workers. There is substantial variation of pay depending on whether the employer is the Federal, State or Local government. The Federal government pays a premium of 0.100 to college graduates and 0.200 to workers without college. State and Local government offer a negative premium for college workers between -0.06 to -0.03 and a positive premium of 0.008 to 0.018 for workers without college.

To highlight the heterogeneity of the public wage policies, even within a country, we look at regional differences across US states, shown in Figure 6. The public-sector wage premium for both college and no-college, as well as the difference between the two, varies across states

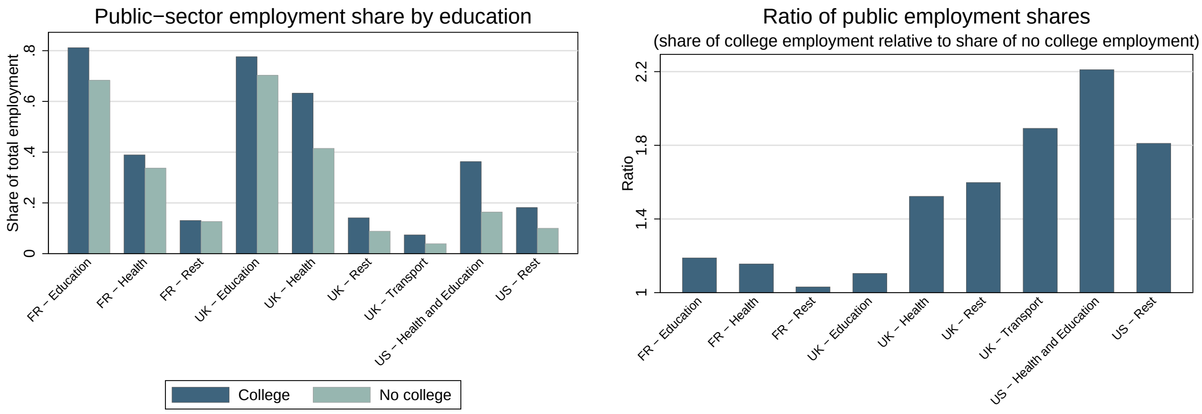
Figure 3: Public-Sector Education Bias: Two Simple Indicators



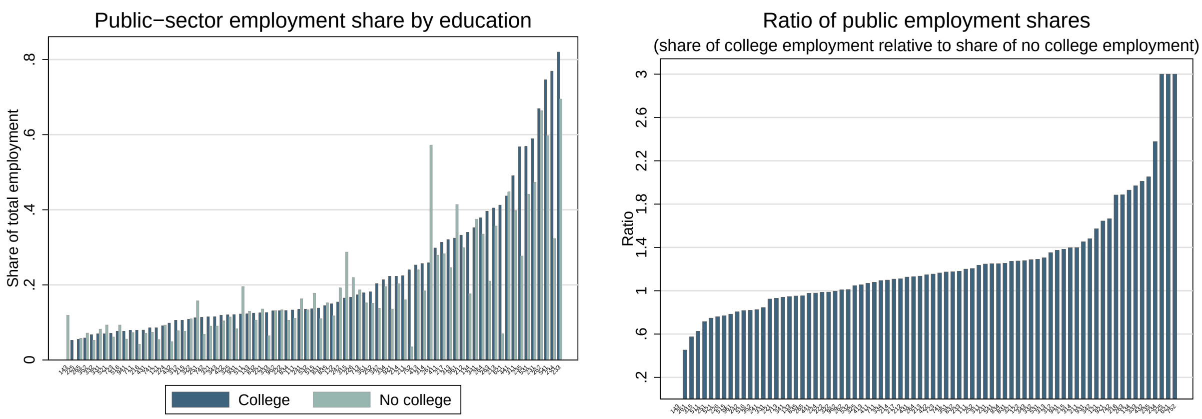
Note: The graph on the left shows the ratio of public employment shares r^g. The graph of the right, is the education intensity ratio ei^g. For the United States the data is take from CPS (1996-2018), for the United Kingdom from the UK Labour Force Survey (2003-2018), for France for the French Labour Force Survey (2003-2018) and from Spain from the Spanish Labour Force Survey (2005-2018). For details on the methodology for the European economies see Fontaine et al (2018).

Figure 4: Public-Sector Employment Share Across Industries and Occupations

Industries



3-digit Occupations



Note: 1st panel uses French, Spanish, UK Labour Force Surveys and the CPS. 2nd panel: CPS data, average between 1996 and 2018. 3-digit occupations that have an overall share of public-sector employment between 0.05 and 0.95. On the right-hand graph, the ratio was capped at 3 for readability.

by close to 20 percentage points. While there is large cross sectional variation in policies, the wage compression holds in 50 out of 51 US states. The state with highest compression is Washington DC and with only one with a negative compression is Kentucky.

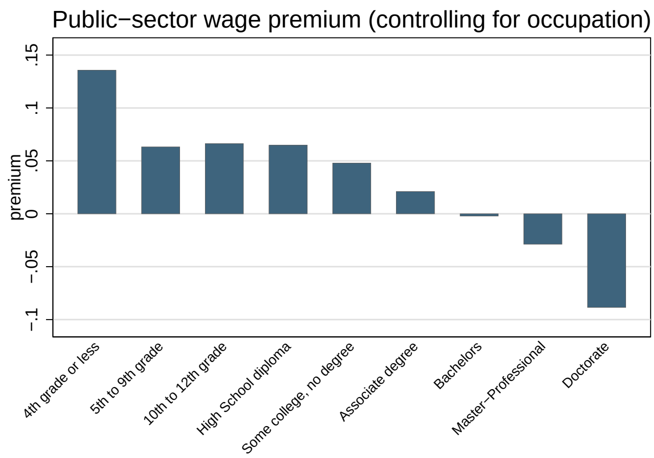
Finally, we show in Appendix A the evolution of the public-sector wage premia over time, in the US, UK, France and Spain. The wage compression across educational group is persistent over time in all countries. The dummy for public sector workers in Mincer regressions is always larger for workers with low education. Remarkably, the policies on wages can vary substantially in a few years. Most striking is the case of France. Between 2006 and 2010 the estimated premium fell by 15 log points for both workers with and without college. In Spain we find that the public-sector premium of college graduates fell from 0.10 in 2006 to 0.03 in 2014, while it remained constant for workers without college.

2.3 Underemployment

Preliminary suggestive evidence of underemployment in the public sector can be obtained using the CPS data. The public sector wage premium for college graduate is lower and negative when we do not control for occupation. This may certainly imply that workers with a college degree in the public-sector are more likely to be in lower paid occupations. To corroborate this suggestion, we correlate the ratio of public-employment shares in 3 digit occupations (shown in the 2nd panel of Figure 4) with the gross public-sector premium for no-college in those occupations.[6] Indeed, Figure 7 indicates a positive and statistically significant relation between the level of public-sector pays for unskilled workers in a given

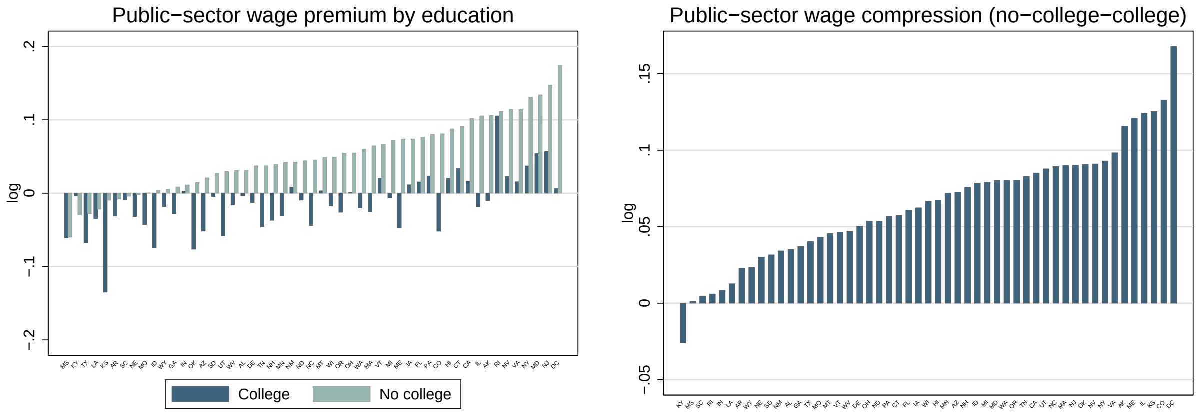
[6]We compute hourly wage as the respondent's total pre-tax wage and salary income for the previous calendar year divided by the product of the number of weeks worked last year times the usual hours worked per week last year. We then consider the mean hourly wage in each occupation for the workers with no-college.

Figure 5: Public-Sector Wage Premia By Education Levels



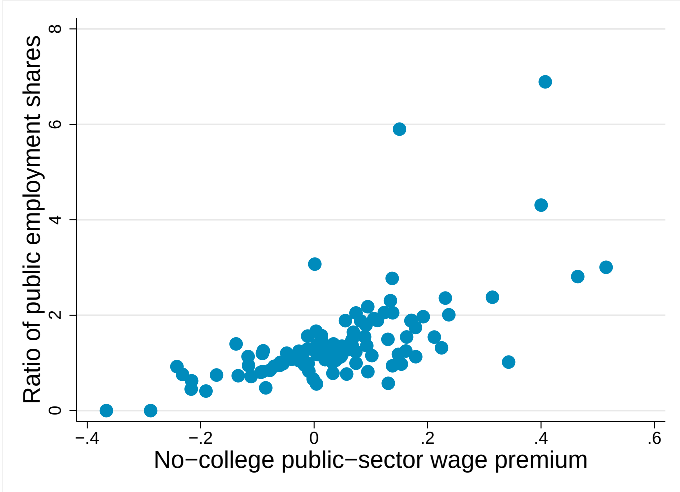
Note: Estimation by regressing the log of hourly wage on a public-sector dummy and controls (age, gender, region, year and a part-time dummy), separately for workers with different education levels. When controlling for occupation we include 2-digit occupation dummies. CPS data between 1996 and 2018.

Figure 6: Public-Sector Wage Compression Across US states



Note: Estimation by regressing, for each state, the log of hourly wage on a public-sector dummy and controls (age, gender, year and a part-time dummy), separately for workers with and without college graduate. When controlling for occupation we include 2-digit occupation dummies. CPS data between 1996 and 2018.

Figure 7: Ratio Of Public-Sector Employment Shares And No-College Public-Sector Premia



Note: Ratio of public employment shares in 3 digit occupations with no-college public sector wage premium in the occupation. CPS data between 1996 and 2018.

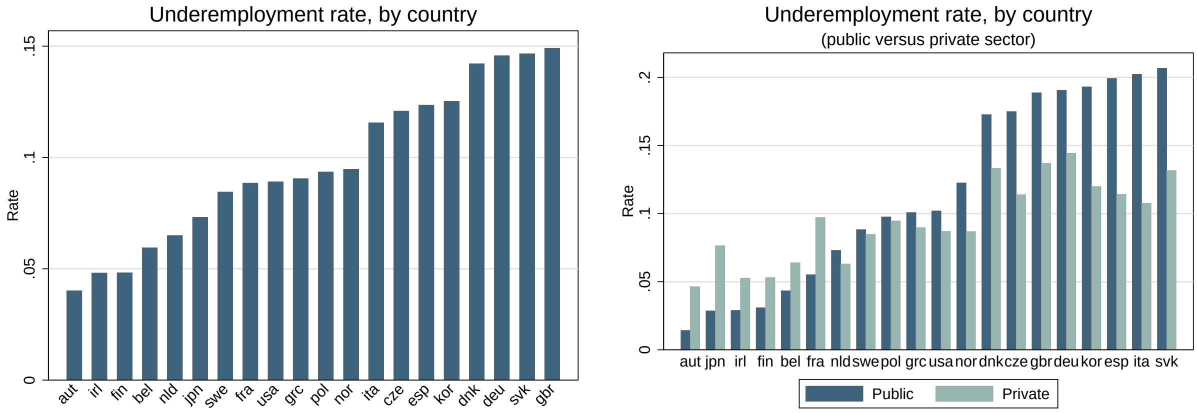
occupation and the education bias within that occupation.

To provide more evidence of underemployment across countries, as well as across public and private sector within countries, we use PIACC database. We need first some accounting. We refer to *underemployment* u, as to the stock of workers with college employed in jobs typically performed by no-college workers. Similarly as above, n is the stock of employed college workers, and $1-n$ is the stock of non college workers. Let j_1 be the stock of skilled related jobs, only filled by graduates, so that

$$j_1 = n - u.$$

Further, j_2 is the stock of unskilled jobs that is filled by workers without college or underemployed college workers, $j_2 = (1-n) + u$. We define the *underemployment rate*, indicated

Figure 8: Underemployment Rate Across Countries



Note: PIAAC (Survey of Adult Skills). By occupation, we calculate the number of years of education of (self-reported) well-matched people. Consider underemployed, college graduates whose years of education are 1.96 s.d. above the mean years of education in an occupation.

with $\tilde{u}$, as the fraction of unskilled jobs performed by college graduates. Formally:

$$\tilde{u} = \frac{u}{j_2}.$$

Similarly, we define the underemployment rate in private and public sector as $\tilde{u^p} = \frac{u^p}{j_2^p}$ and $\tilde{u^g} = \frac{u^g}{j_2^g}$. By occupation, we calculate the number of years of education of (self-reported) well-matched people. Consider underemployed, college graduates whose years of education are 1.96 s.d. above the mean years of education in their occupation.

The left graph in Figure 8 reports underemployment rate across countries. In many countries, more than 10 percent of unskilled jobs are held by people that have years of education much above than most people in that occupation. The minimum level is below 5 percent in countries such as Austria and Ireland. With respect to the topic of the paper, the key empirical evidence is in the graph on the right. In 14 out of 20 countries, including the US, the underemployment rate is indeed larger in the public than in the private sector. While underemployment is more perverse in the public sector, this is not automatic. Countries such as France or Finland have lower underemployment in the public sector.

3 Two-Sector Model With Underemployment

3.1 Technology and Preferences

Individuals are endowed with 1 unit of indivisible labor. There are two types of individuals with high (1) and low (2) education . The supply of educated individuals in the economy is indicated with n, while the supply of the low-educated workers is indicated with $1 - n$.

A representative firm and a government have jobs requiring different skills. The superscript $x = p, g$ refers to the private or public sector and the subscript $e = 1, 2$ refers to both the education of the worker and the skill of the job. The government has j_1^g skilled jobs and

j_2^g unskilled jobs, while the private sector has j_1^p and j_2^p. The representative firm produces a private-sector output y - the numeraire of the economy - with a constant return technology. In what follows we use a Cobb Douglas specification,

$$y = (j_1^p)^{\alpha}(j_2^p)^{1-\alpha}, \tag{1}$$

where α if the skill intensity. The government produces government services g – a different good from the private sector for which there is no market (price) – using,

$$g = (j_1^g)^{\beta}(j_2^g)^{1-\beta}. \tag{2}$$

We allow for technology to be different from the private sector $\beta \neq \alpha$ reflecting the fact that these governments services might require more or less skilled jobs.

A key assumption in our theory concerns the ability of different individuals to perform different jobs. Jobs can be described through a ladder type mechanism, so that individuals endowed with high education are able to perform also unskilled jobs. They can perform at zero effort costs both type of jobs while individuals with low education can perform at no cost only the unskilled job, while we assume that the cost of effort required to perform the skilled job is (infinitely) large.

Individual preferences are linear and the model is static. Each individual worker i has an heterogeneous "non-pecuniary value" over skilled and unskilled jobs in the private and public sector $\epsilon_i^{x,e}$ drawn from a continuous distribution. We assume, for tractability, that they have an extreme type I error distribution. These "non-pecuniary" attributes of the job could reflect preferences, but all other elements such as location of jobs, co-workers, hours, altruism, preference for job stability, etc. For instance, a worker i of type e, working in sector x, has an utility given by sum of the wage net of taxes and "non-pecuniary" shock, $(1-\tau)w_e^x + \nu\epsilon_i^{x,e}$, where τ is the income tax and ν captures the weight of the "non-pecuniary" value in the individual preferences.[7] Our model accommodates the traditional model in the limit where ν tends to zero and workers would select to the highest paying job.

3.2 A Malinvaud Government...

We assume that the government is required to produce a certain level of government services, $\bar{g}$, taken as exogenous. Given a wage schedule, the government chooses its target (ideal) level and composition of employment ($\tilde{j}_1^g$ and $\tilde{j}_2^g$), that minimizes the costs of producing the government services, $\bar{g}$.

For clarity of the model, we assume that the wage schedule is given exogenously for skilled and unskilled jobs (w_1^g and w_2^g). This is not a crucial assumption. The key assumption is

[7]We take the ν as exogenous. A more general model of underemployment could micro-found this parameter, and could potential be the outcome of policies such housing market or transport policies, as well as regulation of specific occupations.

that the government wages do not adjust to equate supply and demand. We think this is a realistic assumption, given that the government does not sell its goods and services and finances the wage bill with (lump-sum or income) taxes, public-sector wages do not have the same impacts as in private sector firms, so they might be influenced by other factors, such as unions, redistribution or elections. Notice that the wages are paid in units of the private-sector good so they are essentially a transfer of resources from private- to public-sector workers.[8]

$$\min_{j_1^g, j_2^g} w_1^g j_1^g + w_2^g j_1^g$$
$$s.t.$$
$$\bar{g} = (j_1^g)^{\beta}(j_2^g)^{1-\beta}.$$

Given the level of public wages, the government employs enough workers to maintain an employment level capable of providing its services. Using the production function and the two first-order conditions, we find the optimal ratio of complex and simple public-sector jobs is:

$$\frac{\tilde{j}_1^g}{\tilde{j}_2^g} = \frac{w_2^g}{w_1^g}\frac{\beta}{(1-\beta)}, \tag{3}$$

Plugging in the production function, the target level of jobs of each type is given by:

$$\tilde{j}_1^g = \bar{g}\left(\frac{w_2^g}{w_1^g}\frac{\beta}{1-\beta}\right)^{1-\beta} \qquad , \qquad \tilde{j}_2^g = \bar{g}\left(\frac{w_1^g}{w_2^g}\frac{1-\beta}{\beta}\right)^{\beta}. \tag{4}$$

Lemma 1 *If the government minimizes costs, the target skilled jobs, $\tilde{j}_1^g$ is increasing in w_2^g and β and decreasing in w_1^g. The target unskilled jobs, $\tilde{j}_2^g$ is increasing in w_1^g and decreasing in w_2^g and β. They are independent of private sector conditions.*

The first dimension of analysis is the government's preferred choice of which workers to hire. Taking the wage schedule and the production function as given, the government chooses how many workers of the two types to minimize the costs of producing $\bar{g}$. Changes in public wages are going to alter the labour demand choice of the government. Higher unskill wages reduce the demand for unskilled jobs and raise demand for skilled jobs.

In a model without frictions, public-private wage differentials can pose some problems. If it is positive, all workers would prefer the public-sector, so one has to assume these jobs are rationed. Perhaps harder to deal is the opposite case, where the differential is negative and no worker would like to work for the government. Our preference structure avoids this problem. It makes the supply of workers to the public sector continuous on the wage, while preserving the different regimes. If the public-sector wages are higher, the government can

[8]For the adamant reader concerned about the assumption of exogenous public wages, we present an extension in subsection 5.1 where the government also chooses wages, but faces an additional union preference constraint. This problem generates an endogenous public-sector premium that depends on an exogenous union power and preference for wage compression, but does not add much to the analysis of the effects of public wages.

attain its target level of jobs, that are rationed. If the public-private wage differential is negative, there would always be workers with high enough preference for the public sector such that the supply of workers is never zero. Still, the supply of workers of a given type might be lower than the target level determined by cost-minimization. In such cases, the government has to hire more workers of the other type in order to maintain the production of services.

The final assumption is that, an educated worker that applies to an unskilled public job always has priority over low-educated workers. The government is financed through a labour income tax, τ. In the baseline model we take it as exogenous, but in subsection 5.3 we discuss one extension in which τ adjusts to satisfy the government budget constraint. In subsection 5.4, we also discuss the differences if we consider the government's dual problem.

3.3 ... And A Walrasian Private Sector...

The representative private sector firm maximizes profits. The labour market is perfectly competitive such that the wages equate demand and supply and jobs are paid their marginal productivity. The labour demand equations are

$$w_1^p = \alpha \left(\frac{j_2^p}{j_1^p}\right)^{1-\alpha} \quad , \quad w_2^p = (1-\alpha)\left(\frac{j_1^p}{j_2^p}\right)^{\alpha}. \tag{5}$$

3.4 ... With Underemployment

The possibility of educated workers to do unskilled jobs creates a dissociation between the number of educated workers and the number of skilled jobs, as well as the number of workers with low education and the number of unskilled jobs. Some of the educated workers might be under-employed in the public or private sector (u^g, u^p) if they choose to. Hence the market clearing condition in high- and low-educated labour markets are given by

$$n = j_1^g + j_1^p + u^g + u^p \quad , \quad 1 - n = j_2^g + j_2^p - u^g - u^p. \tag{6}$$

Sorting

An educated worker i has the possibility of going to private or public sector, in a skilled or unskilled job. Hence they choose between four options:

$$Max\{(1-\tau)w_1^p + \nu\epsilon_i^{p,1}, (1-\tau)w_2^p + \nu\epsilon_i^{p,2}, (1-\tau)w_1^g + \nu\epsilon_i^{g,1}, (1-\tau)w_2^g + \nu\epsilon_i^{g,2}\}. \tag{7}$$

Here, the fact that skilled jobs in the public sector might be rationed is important, given that there might be fewer jobs available than workers wanting to work there at a given public wage. We assume that workers that would wish but could not get a skilled public job, choose the maximum between the three remaining options. Notice that this does not happen for unskilled public jobs because we assume that they have priority over low-educated workers.[9] We include one specific shock for each of the four possible jobs. One alternative would be to consider a preference for public and private sectors and one for complex and simple jobs, but is less tractable.

A worker with low education only has a choice or a private or public unskilled job:

$$Max\{(1-\tau)w_2^p + \nu\epsilon_i^{p,2}, (1-\tau)w_2^g + \nu\epsilon_i^{g,2}\}. \tag{8}$$

It is useful to assume that the non-pecuniary shock is drawn from an extreme type I value distribution. To better understand the different regimes we define, $\tilde{w}_1^g$ and $\tilde{w}_2^g$ - endogenous objects - the minimum skilled and unskilled public wage that allows the government to hire its target level of employment. $\tilde{w}_1^g$ and $\tilde{w}_2^g$ is defined implicitly by

$$\tilde{j}_1^g = n\left[\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g} + e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{9}$$

$$\tilde{j}_2^g - u^g = (1-n)\left[\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_2^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_2^g} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{10}$$

If both public wage are above the thresholds – Regime 1 – the number of interested workers is larger than the number of jobs, so all public jobs are rationed and are determined by demand ($j_1^g = \tilde{j}_1^g$ and $j_2^g = \tilde{j}_2^g$). If one of the wages is below the threshold, in one market there are fewer interested workers than jobs, so the government is constrained and supply determines either j_1^g or j_2^g and the other adjusts to maintain the production of services (Regimes 2 or 3). Finally if both wages are below the threshold, the government is constrained in both jobs, so it is not able to maintain its government services.

Independently of whether the government jobs are determined by supply or demand, underemployment in the two sectors is pinned down by

$$u^p = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^p}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{11}$$

$$u^g = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{12}$$

Notice that when $w_2^g = w_2^p$, $u^p = u^g$ independently of the size of the public and private sectors. This means the underemployment rate in the private sector ($\tilde{u^p} = \frac{u^p}{J_2^p} \neq \tilde{u^g} = \frac{u^g}{J_2^g}$). In one of the extensions we present a slight variation of the sorting problem that, when $w_2^g = w_2^p$ generates $\tilde{u}^p = \tilde{u}^g$.

For the interested reader, we show in Appendix B a version of the two-sector model

[9]It would technically be possible that if the unskilled public wage would be so high that more educated workers would want an unskilled public job than existing jobs that these jobs would be rationed too $u^g = j_2^g$. We find that this case is only a theoretical curiosity with little empirical relevance.

without underemployment and perfect labour mobility, and in Appendix C a 1-sector model of underemployment where we discuss some comparative statics, namely with respect to the tax rate and the supply of educated workers. These two Appendices develop the intuition and isolate the mechanisms present in the model.

3.5 Equilibrium Definition

Definition 1 *A steady-state equilibrium consists of private-sector wages $\{w_1^p, w_2^p\}$, private-sector jobs $\{j_1^p, j_2^p\}$, public-sector jobs $\{j_1^g, j_2^g\}$, and under-employment in the two sectors $\{u^p, u^g\}$, such that, given some exogenous wage policies, technology and composition of the labour force $\{w_1^g, w_2^g, \nu, \bar{g}, \alpha, \beta, n\}$, the following apply.*

1. *Private-sector firms maximizes profits (5).*
2. *Government sets employment either: i) if unconstrained (demand determined), by minimizing the costs of providing government services (4) or ii) if constrained (supply determined), to maintain the production of government services.*
3. *Educated workers sort across labour markets according to (11 and 12).*
4. *Markets clear (6).*

3.6 Solving The Model Under Different Regimes

Regime 1 - Unconstrained Government

This equilibria requires that $w_1^g \geq \tilde{w}_1^g$ and $w_2^g \geq \tilde{w}_2^g$. Given that $j_1^g = \tilde{j}_1^g$ and $j_2^g = \tilde{j}_2^g$ are only function of the exogenous public-sector wages and technology, the solution of the model under Regime 1 can be written in three equations in $u = u^p + u^g$, w_1^p and w_2^p

$$u = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^p} + e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{13}$$

$$w_1^p = \alpha\left(\frac{1 - n - j_2^g + u}{n - j_1^g - u}\right)^{1-\alpha}, \tag{14}$$

$$w_2^p = (1-\alpha)\left(\frac{n - j_1^g - u}{1 - n - j_2^g + u}\right)^{\alpha}, \tag{15}$$

We can further substitute the two wages, and have one equation in one unknown, with the left-hand side is increasing in u and the right-hand side decreasing u. The equilibrium exists and is unique. In Appendix D we show the full system determining the total derivatives of the endogenous variables to the key exogenous variables.

Proposition 1 *Under Regime 1, an increase of w_2^g shifts the composition in the public sector towards skilled jobs and in the private sector to unskilled jobs. It raises skilled wages and* *lowers unskilled wages in the private sector. The effect on underemployment is ambiguous* ($\frac{du}{dw_g^2} \lesseqgtr 0$, $\frac{dw_1^p}{dw_2^g} > 0$, $\frac{dw_2^p}{dw_2^g} < 0$, $\frac{dj_1^g}{dw_2^g} > 0$, $\frac{dj_2^g}{dw_2^g} < 0$*).*

Proposition 2 *Under Regime 1, an increase of w_1^g shifts the composition in the public sector towards unskilled jobs and in the private sector to skilled jobs. It raises unskilled wages and lowers skilled wages in the private sector. It raises underemployment* ($\frac{du}{dw_g^1} > 0$, $\frac{dw_1^p}{dw_1^g} < 0$, $\frac{dw_2^p}{dw_1^g} > 0$, $\frac{dj_1^g}{dw_1^g} < 0$, $\frac{dj_2^g}{dw^g 1} > 0$*).*

The propositions tell us how public wages affect the private sector. The effect of an increase of w_2^g on underemployment is ambiguous. While there is a direct positive effect on underemployment in the public sector, the higher wage inequality in the private sector, has a negative indirect effect on both private and public underemployment. The effect on underemployment of an increase in w_1^g is unambiguously positive. By reducing private-sector wage inequality it fosters underemployment in both sectors.

We can write expressions for the elasticities of private wages with respect to public-sector wages. For instance, private wage elasticities with respect to unskilled public wage are given by:

$$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p} = (1-\alpha)(1-\beta)\frac{j_1^g}{j_1^p} + (1-\alpha)\beta\frac{j_2^g}{j_2^p} + \frac{du}{dw_2^g}\left[\frac{(1-\alpha)}{j_2^p} + \frac{(1-\alpha)}{j_1^p}\right] w_2^g \tag{16}$$

$$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p} = -\alpha(1-\beta)\frac{j_1^g}{j_1^p} - \alpha\beta\frac{j_2^g}{j_2^p} - \frac{du}{dw_2^g}\left[\frac{\alpha}{j_2^p} + \frac{\alpha}{j_1^p}\right] w_2^g \tag{17}$$

These expressions provides a decomposition of the effects of public wages. Higher unskilled public-sector wages induces the government to open more skilled jobs and fewer unskilled jobs. In turn, this means that in the private sector there is a shortage of educated workers (first term) and an excess of low-educated workers (second term), both pushing skilled wages up and the unskilled wages down. Finally, there is an effect on underemployment. If underemployment increases, both the positive effect on skilled wages and the negative effect on the unskilled wages are reinforced. If underemployment decreases, they are mitigated.

Regime 2 - Skilled Public-Sector Wages Too Low

This is a potentially realistic regime. Regime 2 occurs if wages for skilled jobs are too low ($w_1^g < \tilde{w}_1^g$). The government cannot hire its target level of employment so, to maintain the production of government services it has to open more unskilled jobs (provided it still pays high enough wages $w_2^g \geq \tilde{w}_2^g$). Now the level of skilled public employment is constrained and given by

$$j_1^g = n\left[\frac{e^{\frac{(1-\tau)}{\nu}w_1^g}}{e^{\frac{(1-\tau)}{\nu}w_1^g} + e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}w_1^p} + e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{18}$$

while for unskilled jobs is

$$j_2^g = \left[\frac{\bar{g}}{(j_1^g)^\beta}\right]^{\frac{1}{1-\beta}}. \tag{19}$$

The three equations pinning down u, w_1^p and w_2^p are the same as in the previous case, but now they affect both j_1^g and j_2^g that are no longer independent. In Appendix D we show the full system determining the total derivatives of the endogenous variables to the key exogenous variables.

Proposition 3 *Under Regime 2, an increase of w_2^g raises skilled wages and lowers unskilled wages in the private sector. The effect on underemployment and in the skill mix of the public sector is ambiguous ($\frac{du}{dw_g^2} \lesseqgtr 0$, $\frac{dw_1^p}{dw_2^g} > 0$, $\frac{dw_2^p}{dw_2^g} < 0$, $\frac{dj_1^g}{dw_2^g} \lesseqgtr 0$, $\frac{dj_2^g}{dw_2^g} \lesseqgtr 0$).*

Proposition 4 *Under Regime 2, an increase of w_1^g shifts the composition in the public sector towards skilled jobs and in the private sector to unskilled jobs. It raises skilled wages and lowers unskilled wages in the private sector. It lowers underemployment ($\frac{du}{dw_g^1} < 0$, $\frac{dw_1^p}{dw_1^g} > 0$, $\frac{dw_2^p}{dw_1^g} < 0$, $\frac{dj_1^g}{dw_1^g} > 0$, $\frac{dj_2^g}{dw^g 1} < 0$).*

In this case public-sector employment is supply determined so the signs of the effect of public-sector wages on private-sector wages are the opposite of those in Regime 1. Increasing wages at the top, allows the government to attract more educated workers.

Regime 3 - Unskilled Public-Sector Wages Too Low

Regime 3 happens if unskilled public wages are too low ($w_2^g < \tilde{w}_2^g$). The government cannot hire its target level of employment so, to maintain the production of government services, it has to open more skilled jobs (which requires that their wage is high enough $w_1^g \geq \tilde{w}_1^g$). While this case is not realistic, we consider it for completeness. The public employment is given by

$$j_2^g - u^g = (1-n)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g}+e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \quad , \qquad j_1^g = \left[\frac{\bar{g}}{(j_2^g)^{1-\beta}}\right]^{\frac{1}{\beta}} . \tag{20}$$

Regime 4 - Public Sector Breaks Down

Regime 4 occurs if both public wages are too low ($w_1^g < \tilde{w}_1^g$ and $w_2^g < \tilde{w}_2^g$). All government jobs are determined by supply. The government cannot hire enough workers to maintain the production of government services, so they have to be scaled down.

$$j_2^g - u^g = (1-n)\left[\frac{e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}w_2^g}+e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{21}$$

$$j_1^g = n\left[\frac{e^{\frac{(1-\tau)}{\nu}w_1^g}}{e^{\frac{(1-\tau)}{\nu}w_1^g}+e^{\frac{(1-\tau)}{\nu}w_2^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{22}$$

And the government services that are allowed is given by $g = (j_1^g)^{\beta}(j_2^g)^{1-\beta}$.

4 Quantitative analysis

4.1 Baseline model with alternative sorting mechanism and exogenous income tax

For quantitative purposes we solve the baseline model described in section 3 with an alternative sorting mechanism. One of the features of the baseline model is that when public and private wages are equal, the level of underemployment is equal in both sectors. Thus, if the public sector is smaller than the private, the underemployment rate would be larger in the public sector. As such, even in the case of perfect symmetry between the two sectors (in terms of wages and technology), the ratio of public-employment shares is not 1. We set up a variation of the model with an alternative sorting mechanism that avoids this feature, shown in Appendix E.3.

We consider that the underemployment opportunities are proportional to size of sector. Of all the educated workers, a fraction $\frac{j_2^g}{j_2^p+j_2^g}$ has an underemployment opportunity only in the public sector. Those workers choose between three options $Max\{w_1^p+\nu\epsilon_i^{p,1}, w_1^g+\nu\epsilon_i^{g,1}, w_2^g+\nu\epsilon_i^{g,2}\}$. The remaining fraction $\frac{j_2^p}{j_2^p+j_2^g}$ has only an underemployment opportunity in the private sector and chooses between $max\{w_1^p+\nu\epsilon_i^{p,1}, w_1^g+\nu\epsilon_i^{g,1}, w_2^p+\nu\epsilon_i^{p,2}\}$. The mechanism is similar to the baseline model except that equation (13), that pins down underemployment, becomes more complex. As this extension gives a ratio public employment shares of 1, in the symmetric case, we use this variation of the model in this quantitative section.

Furthermore, we take into account an exogenous income tax τ in the baseline model. The tax rate has the same effect as a change in ν, the weight of the non-pecuniary element of preferences. The income tax rate is taken as a parameter assumed constant even in the quantitative experiments carried out in this section. The justification is that we considered that such policies would be financed with government debt or by adjustments in other spending categories. We will take into account the endogenous response of income tax in section 5

4.2 Calibration

We calibrate the variation of model with the alternative sorting mechanism to the United States. The model has seven parameters $\{w_1^g, w_2^g, \frac{\nu}{1-\tau}, \bar{g}, \alpha, \beta, n\}$. As such, we set them to target seven moments of the data, all described in Section 2. Table 3 summarizes the parameter values and target values.

We set n to match 43.2 percent of college graduates. The parameters $\bar{g}$ and β target a public employment of 0.097 and 0.062 of college and non-college, as a proportion of the employed population, taken from the CPS. Notice that the employment of no-college public workers is equal to $j_2^g - u^g$ while the employment of public workers with college is $j_1^g + u^g$. The parameter α targets a college premium of private workers of 58 percent found by regressing

Table 3: Calibration

Parameter	Value	Variable	Description	Model	Data
		Targeted			
α	0.450	$\frac{w_1^p}{w_2^p}$	College premium (private sector)	1.580	1.580
β	0.657	$j_1^g + u^g$	Public employment of college	0.097	0.097
$\bar{g}$	0.082	$j_2^g - u^g$	Public employment of no-college	0.062	0.062
n	0.432	n	Percentage of college workers	0.432	0.432
w_1^g	0.652	$\frac{w_1^g}{w_1^p}$	Public-sector wage premium (college)	1.010	1.010
w_2^g	0.440	$\frac{w_2^g}{w_2^p}$	Public-sector wage premium (college)	1.077	1.077
$\frac{\nu}{1-\tau}$	0.142	$\frac{u}{j_2^g+j_2^p}$	Underemployment rate (economy)	0.089	0.089
		Not Targeted			
		$\frac{u^g}{j_2^g}$	Underemployment rate (public)	0.105	0.102
		$\frac{u^p}{j_2^p}$	Underemployment rate (private)	0.087	0.087

the log of hourly wages of private workers on a college dummy, controlling for age, gender, region, year and a part-time dummy, for a sample between 1996 and 2018.

One important point that our model raises is that the observed public wage premium for college workers might be understated if not controlling for occupation, as it includes underemployed workers. We target the coefficient from Table 2, of the regressions in which we control for two digit occupations, meaning a public-private wage rate for both unskilled jobs of $\frac{w_2^g}{w_2^p} = 1.077$ and for skilled jobs of $\frac{w_1^g}{w_1^p} = 1.010$.

Finally, notice that we cannot dissociate the weight of the preference shock in sorting, ν, from the income tax rate. We set $\frac{\nu}{1-\tau}$, such that the underemployment rate in the economy is 0.089, the number found for the United States using PIAAC data. In the baseline case, the economy is in Regime 1, where wages are high enough such that the government hiring is unrestricted. Despite not being targeted, the model generates an underemployment rate for the private and public sector very close to the ones observed in the data.

4.3 What Drives The Public-Sector Education Bias?

The first exercise shows whether the public-sector education bias is driven by technology or the wage policy and the presence of underemployment. This is shown in Table 4. Column (1) shows the values of variables in the data and Column (2) the values under the baseline calibration. Column (3) shows the counterfactual values when there are no differences across sectors in terms of wages ($w_1^g = w_1^p$ and $w_2^g = w_2^p$). Column (4) equates both wages and technology ($\beta = \alpha$). In that case, the public and private sector have the same skill mix (this would not happen in the baseline model): the government hires 16.6 percent of both types of workers. We then decompose the difference between the baseline and the symmetric case, into contributions of sectorial wage difference (column 3).

In the symmetric case – equating technology and wages – underemployment rates in both sectors are equal and the public employment shares ratio and the education intensity ratios are both be equal to 1. Switching off only the wage differences across sectors, imply cutting

Table 4: Decomposition of public-sector employment education bias

Variable	Data	Baseline	Equating wages	Equating wages and technology
	(1)	(2)	(3)	(4)
Public employment shares				
Skilled	0.224	0.224	0.218	0.166
Unskilled	0.109	0.109	0.115	0.166
Ratio	2.054	2.054	1.892	1.000
Education intensity				
Public	0.610	0.610	0.590	0.432
Private	0.399	0.399	0.402	0.432
Ratio	1.530	1.530	1.468	1.000
Underemployment rate				
Total	0.089	0.089	0.090	0.116
Public*	0.102	0.105	0.090	0.116
Private*	0.087	0.087	0.090	0.116

** not calibrated*

public wages by 1.4 percent for skilled and 6.8 percent for unskilled jobs. In this scenario the underemployment rate is equal in both sectors. This reduces the share of public employment for college graduates by 0.6 percentage point. It would lower the public employment shares ratio from 2.05 to 1.9, roughly 15 percent of the different to 1. It would lower the education intensity ratio from 1.53 to 1.47 - 12 percent of the difference to 1.

In Appendix F we present decomposition for the UK, France and Spain, together with one exercise using the baseline model instead of the model with alternative sorting. In the UK, the wage profile and underemployment account for only 3 percent of the education bias but in France and Spain it accounts for between 13 and 19 percent. Using the baseline model, once we equate both wages and technology, the public employment shares ratio is 1.59 and the education intensity ratio is 1.34. Out of the difference, more than 80 percent is explained by wages and underemployment.

4.4 Elasticities of Private-Sector Wages

We now calculate the elasticities of private wages, with respect to public wages. Following equations 16 and 17, we can decompose them into three components. The first two components relate to the adjustment of the skill-mix in the public sector. Higher unskilled public wages alter the government skill-mix towards skilled jobs, hence employing fewer low-educated workers. The first component measures the impact of the shortage of high-educated worker in the private sector. It is positive for private skilled wages and negative for unskilled wages. Similarly, the excess low-educated workers, has a positive effect on skilled wages and negative effect on unskilled wages, as measured by the second term. These two effects would exist in a model without underemployment.

The contribution of underemployment is measured in the third component, that depends on whether higher unskilled public wages increase or decrease underemployment, which we could not pin down analytically. Hence, we calculate the elasticities and the three

Table 5: Elasticities Of Private-Sector Wages

Variable	**Elasticity**	**Decomposition**		
		Shortage of skilled	**Excess unskilled**	**Underemployment**
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.074	0.059	0.045	-0.029
$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	-0.061	-0.048	-0.037	0.024
Elasticity of private wages w.r.t. skilled public wages				
$\frac{dw_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	-0.046	-0.059	-0.045	0.058
$\frac{dw_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.038	0.048	0.037	-0.047
Elasticity of private wages w.r.t. public wages				
$\frac{dw_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.029	0.000	0.000	0.029
$\frac{dw_2^p}{dw^g}\frac{w^g}{w_2^p}$	-0.023	0.000	0.000	-0.023

Note: the first column is calculated numerically, the decomposition is based on equations 16 and 17.

components numerically, shown in Table 5.

An increase of 1 percent in unskilled public wages translates into an increase of 0.07 percent of skilled private wages and a reduction of 0.06 percent in unskilled private wages, increasing wage inequality in the private sector. We can see that, the presence of underemployment contributes to mitigates the effect. Higher unskilled public wages, raise underemployment in the public sector but reduce it in the private sector. The overall effect in negative. An increase of 1 percent in skilled public wages translates into a reduction of 0.05 percent of skilled private wages and an increase in 0.04 percent of unskilled private wages. Again underemployment mitigates the effect.

The last rows show the elasticity of private wages to an increase of both skilled and unskilled wages. In this case there is no change in the skill-mix of the government, so all the effects come from underemployment. Still, increasing proportionally wages in the public sector has an asymmetric effect. The increase in underemployment in the public sector is larger that the fall in underemployment in the private sector so overall underemployment increases, which raises skilled wages and lowers unskilled wages in the private sector.

In Appendix F we present the same exercise for the UK, France and Spain, as well as using the baseline model instead of the model with alternative sorting. Given the higher share of public employment in the UK and France, their elasticities are up to four times larger than in the US. For instance, an increase of 1 percent in the unskilled public wages raises private skilled wages by 0.21 percent in the UK and 0.14 percent in France. Using the baseline model, the elasticities are also higher. An increase of 1 percent in public wages raises private skilled wages by 0.21 and lowers unskilled wages by 0.2. The decomposition shows that the magnitude of the effect driven by underemployment. Under the baseline case, higher unskilled public sector wages raises underemployment, driven by the direct positive effect on public-sector underemployment, which largely dominates the negative effect on private-sector underemployment. In the model with alternative sorting, because we restrain the set of underemployment opportunities on the public-sector, the positive effect is mitigated and the negative effect is amplified.

4.5 Switching Regimes

Figure 9 shows which regime is in place depending on the wage policy. For the baseline calibration we are in the unconstrained regime where both the skilled and unskilled public wages are high enough. Only cuts larger than 25 percent in skilled wages or larger than 50 percent for unskilled wages would push the economy to one of the three other regimes. Still we perform numerical exercises, varying skilled and unskilled public wages across regimes.

Figure 10 shows the effects of varying skilled public wages. The kink observed for wage cuts above 25 percent, is the switching from regime 1 to regime 2. When government employment switches to become supply determined, the sign of the effects on private wages, underemployment education intensity and public employment shares ratio flips. What is particularly interesting is that even the effect on government spending changes sign. By lowering the skilled public wages in regime 1, the government reduces spending. But when lowering wages implies that fewer educated workers are attracted to public jobs and some are left unfilled, the government has to open more unskilled positions (relative more expensive) to maintain the production of government services. This implies an inefficient skill mix and moving away from the cost-minimizing allocation.

Figure 11 shows the effects of varying unskilled public wages, for the scenario where public skilled wages are at the baseline (regime 1, dark line) and one where they are 35 percent below the baseline (regime 2, light line). The sign of the effects of unskilled wages in most key variables is the same in both regimes. Higher unskilled public wages raises the education intensity and public employment shares ratios. It pushes private skilled wages up and unskilled wages down raising inequality in both regimes, although the slope is larger under regime 1. The one variable that is affected differently by unskilled public wages in the two regimes is underemployment. Higher unskilled wages lower total underemployment in Regime 1 because of the very large quantitative effects on private-sector inequality which reduces the incentive of being underemployed in the private sector. In Regime 2, higher unskilled public-sector wages do not reduce directly the number of unskilled jobs of the government (because the government is not able to substitute away from unskilled labour) so they simply foster underemployment in the public-sector.

5 Extensions

We have analyse several extensions to further understand the model. We now describe the four more relevant extensions. Although we have worked out others, we abstract from discussing the ones that add little to the mechanism (i.e considering a CES production function or introducing capital). We have included all the equations of each model in Appendix E, while here in the main text we describe the main interest and insight.

Figure 9: Regimes as a function of public-sector wage schedule

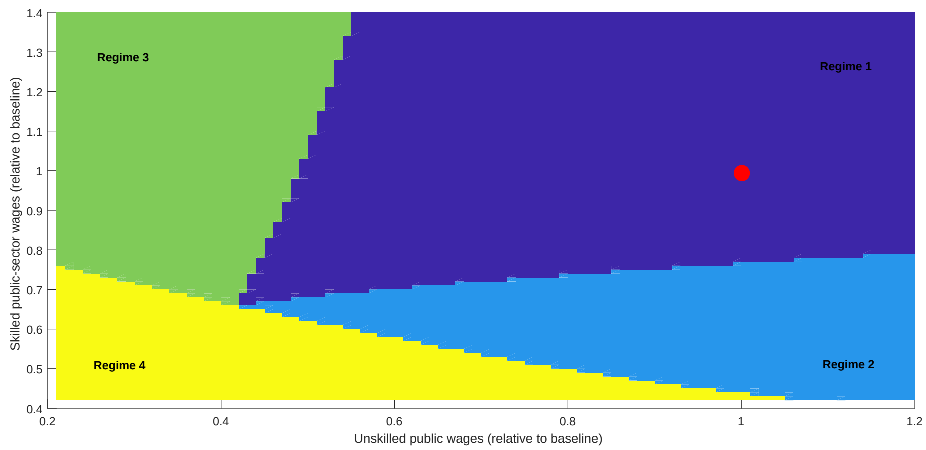


Figure 10: Effects of public-sector wages for skilled jobs

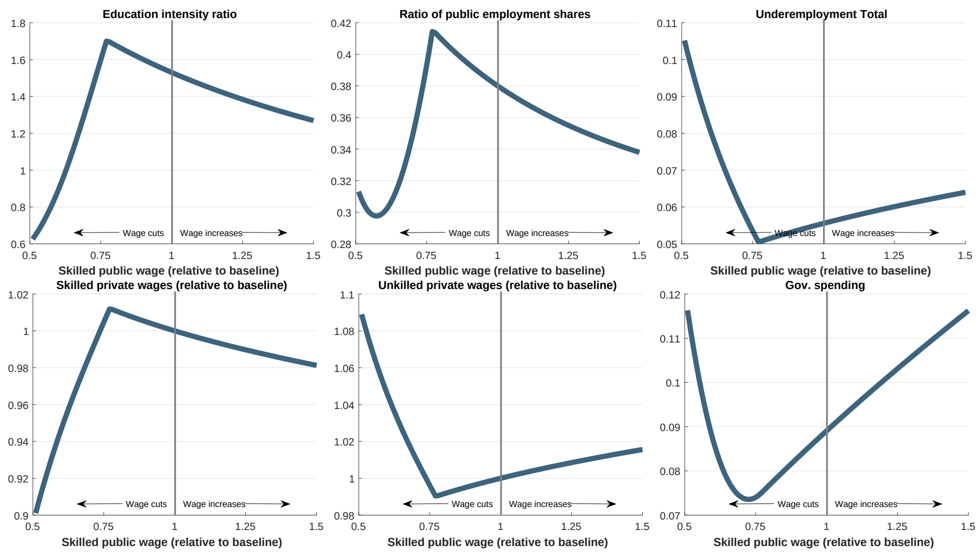
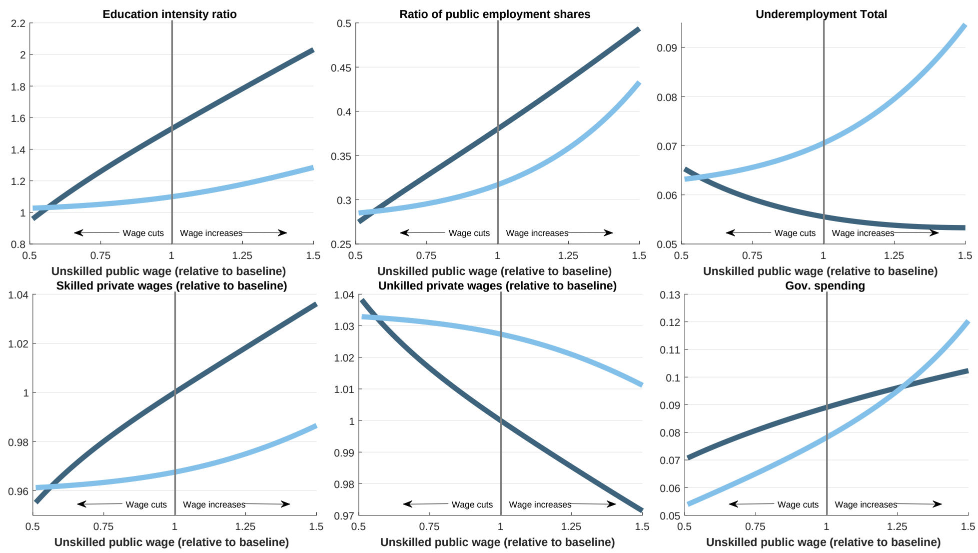


Figure 11: Effects of public-sector wages for unskill jobs



5.1 Endogenous Public-Sector Wages

Our theory for the endogenous determination of public-sector wage is based on a union constraint. We think the higher unionization rates in the public sector might be one of the cause of significant public-private wage differentials and the compression across education levels. However, these could be driven by other political economy factors or simply aversion to inequality. Here we present one possible theory. To the government minimization problem (3.2) that we considered in the baseline model we add a union preference constrained $\bar{U} = \theta \ln(a_1) + (1-\theta)\ln(a_2)$, where a_1 and a_2 are choice variables representing extra payment to public sector workers on top of the minimum threshold level that allows an unconstrained hiring ($w_1^g = \tilde{w}_1^g + a_1$ and $w_2^g = \tilde{w}_2^g + a_2$). θ and $1-\theta$ are the relative importance of each type in the unions function. $\bar{U}$ is the required utility of unions. As $\bar{U}$ tends to zero, $a_1 = a_2 = 0$ and the $w_1^g = \tilde{w}_1^g$ and $w_2^g = \tilde{w}_2^g$: government offers the minimum wage necessary to hire the workers it needs. This would be the case of a benevolent government. In this setting, one would explain a higher premium for unskilled workers with a higher weight of unions on these workers.

5.2 Heterogeneity of educated workers

We think that heterogeneity of ability of educated workers is an important dimension to understand both underemployment and the selection into the public sector. Appendix E.2 describes a variation of the model where high-educated workers are heterogeneous in their effective units of labour. A fraction χ of educated workers have $1+\eta$ efficiency units in skilled jobs, while the remaining only have $1-\eta$. Wages in the private sector reflect perfectly their efficiency units, with the high-ability workers earning $(1+\eta)w_1^p$ and the low-ability workers earning $(1-\eta)w_1^p$. Given that underemployment is a negative function of the wage differential between skilled and unskilled jobs, it is clear that underemployment is concentrated on the low-ability workers.

In the public sector, the payment structure might not reflect entirely the efficiency units of the worker. We assume that the wages of high-ability educated workers is $(1+\eta\delta)w_1^g$ and for the low ability worker $(1-\eta\delta)w_1^g$. δ is the compression parameter of the public-sector. If $\delta < 1$ the are lower wage dispersion in the public sector for skilled workers, fact that has been widely documented.[10] At the limit where $\delta = 0$ the government offers one wage independent of the efficiency units. Our model help understand the implications of the wage compression within education groups. If δ is below 1, the government does not reward fully the efficiency units of high-ability educated workers and rewards too much the efficiency units on low-ability workers. As such, fewer high-ability skilled workers go work for the government, so the government is more likely to be constrained by the supply of high-ability

[10] This has been found running quantile regressions and finding that for the bottom of the earnings distribution the public-sector wage premium is large at the bottom very low or negative. See for instance Christofides and Michael (2013).

workers, and hence it has to employ more of the low-ability skilled workers whose efficiency units are relatively more expensive.

5.3 Endogenous tax rate

One element that we did not developed in the baseline model was the financing side of the government. τ was taken as a parameter in the baseline model. In a third, we endogeneize the tax rate in order to balance the government budget. This implies adding a fourth equation to the model and a fourth endogenous variable.

The tax rate has a same effect as a change in ν, the weight of the non-pecuniary element of preferences. Higher taxes lowers the net income differential between skilled and unskilled jobs, so it raises underemployment. See, for instance, Figure C.2 in Appendix C for the effects of an increase tax rate in the a one sector model. An increase of skilled or unskilled wages, by raising government spending have an additional positive effect on underemployment by raising the income tax.

5.4 Government Dual Problem

We have also done one extension with the dual government problem, where it maximizes services subject to an exogenous wage bill. The first-order conditions are slightly different, with the spending in each type of worker equal to a constant fraction, β of the wage bill. As such, increases in the unskilled wage lowers proportionally the number of unskilled jobs, but do not affect the number of skilled jobs. While the decomposition of the elasticity of private wages is different, the intuition is similar.

6 Conclusion

We present a simple two-sector model with underemployment that highlights the main trade-off regarding public wages, without modeling search frictions. The model highlights three channels to rationalize why public employment is so biased towards educated: technology, the public wage profile and underemployment. We find that in the US economy the excess hiring of educated workers in the public sector is mainly accounted for by technological consideration, while the public wage policy and underemployment account for 15 percent.

We also find that the public wage policy is a crucial driver of private sector inequality: more wage compression in the public sector raises inequality in the private sector. A 1 percent increase in unskilled public wages raises skilled private wages by 0.07 percent and lowers unskilled private wages by 0.06 percent. It has been documented that governments are concerned with inequality when setting their wage policies. For instance, during the Euro Area crisis, many governments implemented wage cuts for their highest paid workers, and spared workers with lower wages, on the grounds that further cuts at the bottom would worsen inequalities. We show that this well intended policy can backfire. Higher wage compression shifts demand from workers with low to workers with high education and worsen underemployment in the public sector. As a consequence, the skill-mix in the private sector shifts towards low-educated workers, so their wages fall while skilled private wages go up. While decreasing wage inequality for a sub-set of workers, such policies increase wage inequality for everyone else.

Our model, despite its simplicity, reveals quite complex mechanisms about the public sector. If public wages do not equate supply and demand of government jobs, different regimes can arise. We have shown that, whether we are in a regime where employment is demand determined or a regime whether employment is supply determined (depending on whether the wage are above or below the market clearing one), the effects of government policies on the private sector are profoundly different. While this switching between regimes did not interfere with the quantitative results on the decomposition, we think it is a defining feature of public-sector labour markets. Given the substantial variation of public wage across US states or across countries, we think it could explain variations in labour market and fiscal outcomes. We leave this question for future research.

As we have shown, the model can be extended in different directions to study questions related to both public employment but also mismatch. In a companion letter, Garibaldi et al. (2019), we generalize a one-sector model, considering both under and over employment, and different efficiency units of educated workers in simple tasks, to measure the output losses of mismatch across OECD economies.

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COMPANION APPENDIX - (almost complete)

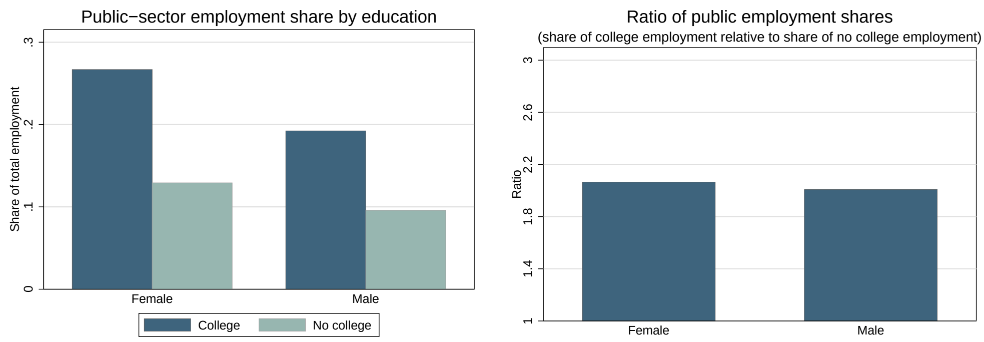
Public and Private Employment in a Model with Underemployment

Pietro Garibaldi, Pedro Gomes and Thepthida Sopraseuth

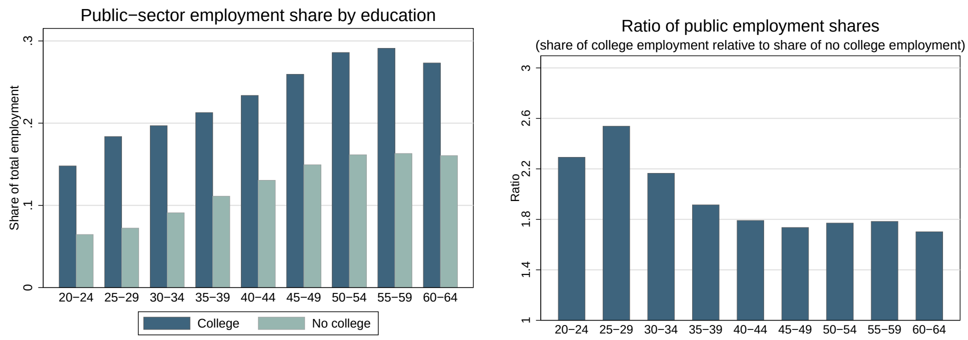
A Additional Statistics

Figure A.1: Public-Sector Employment Share by Education, Different Dimensions

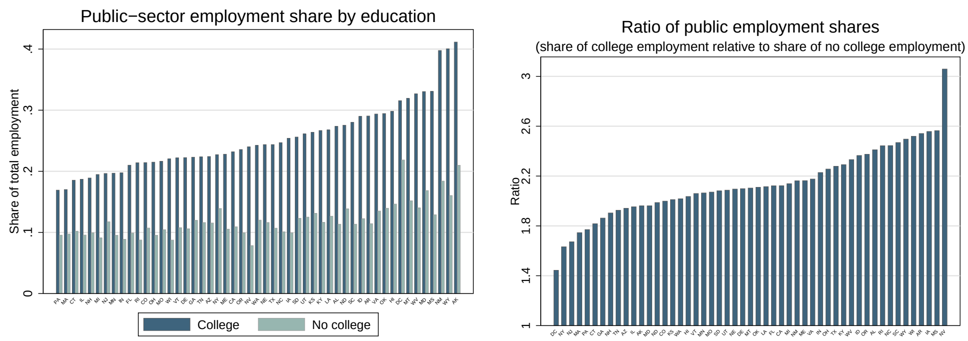
Gender



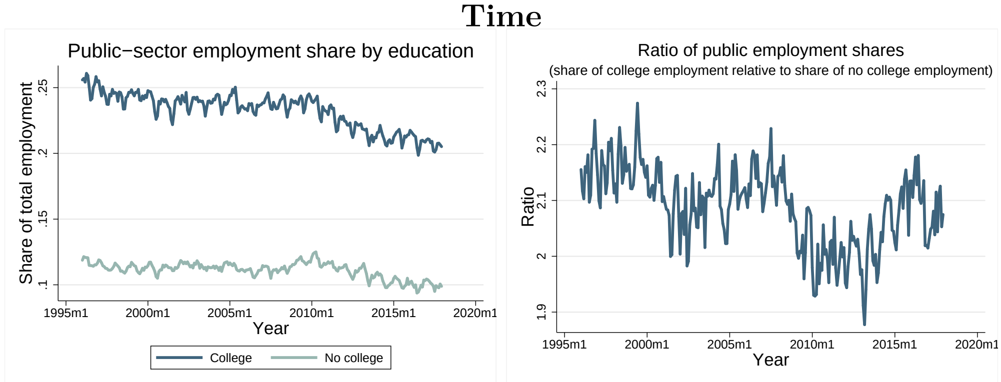
Age



State



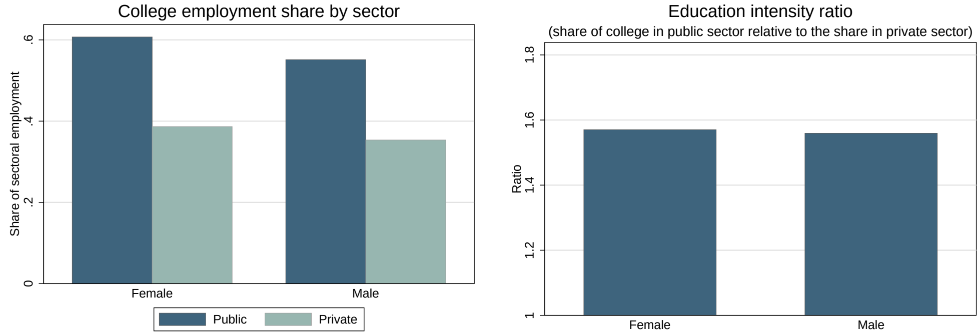
Time



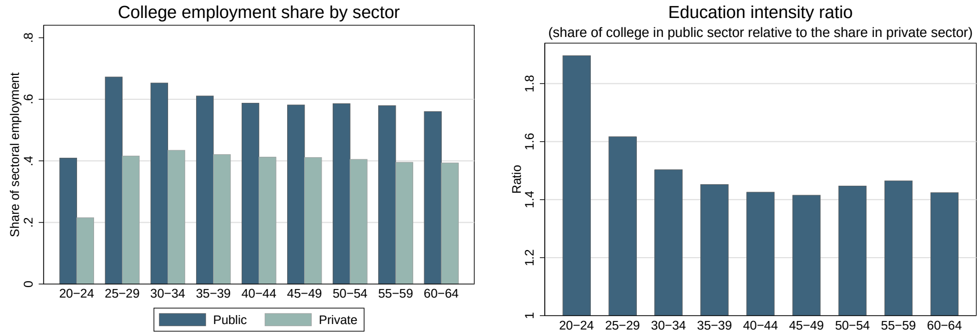
Note: CPS data, average between 1996 and 2018.

Figure A.2: College share by sector, Different Dimensions

Gender



Age



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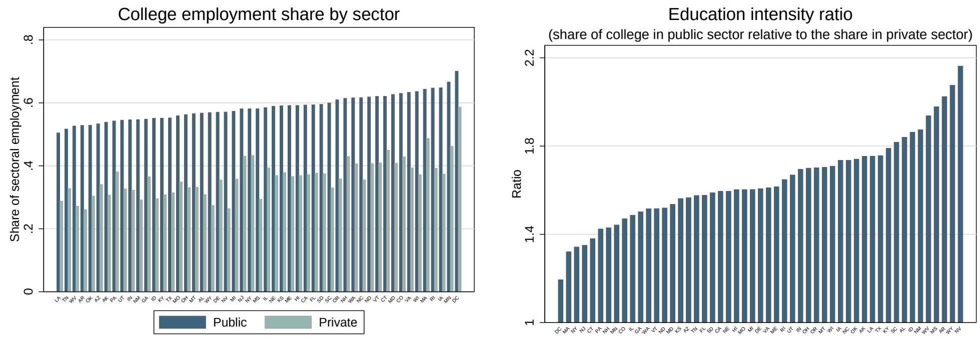
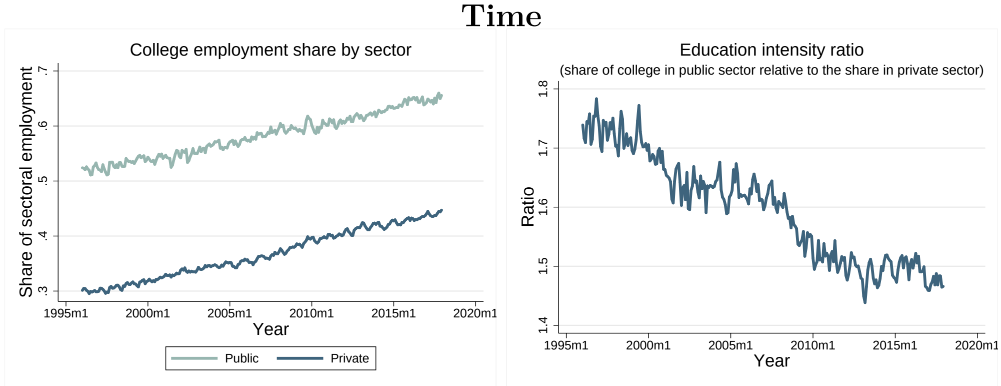


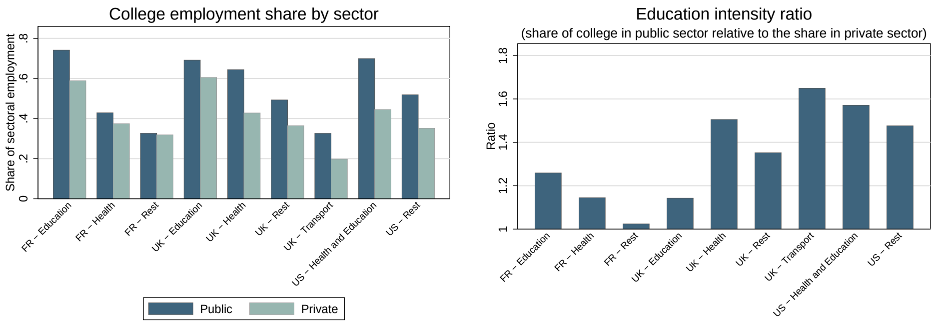
Time



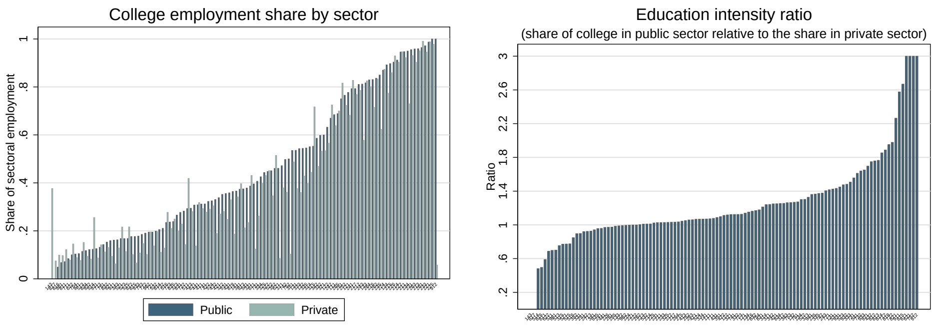
Note: CPS data, average between 1996 and 2018.

Figure A.3: College share by sector, Across Industries and Occupations

Industries

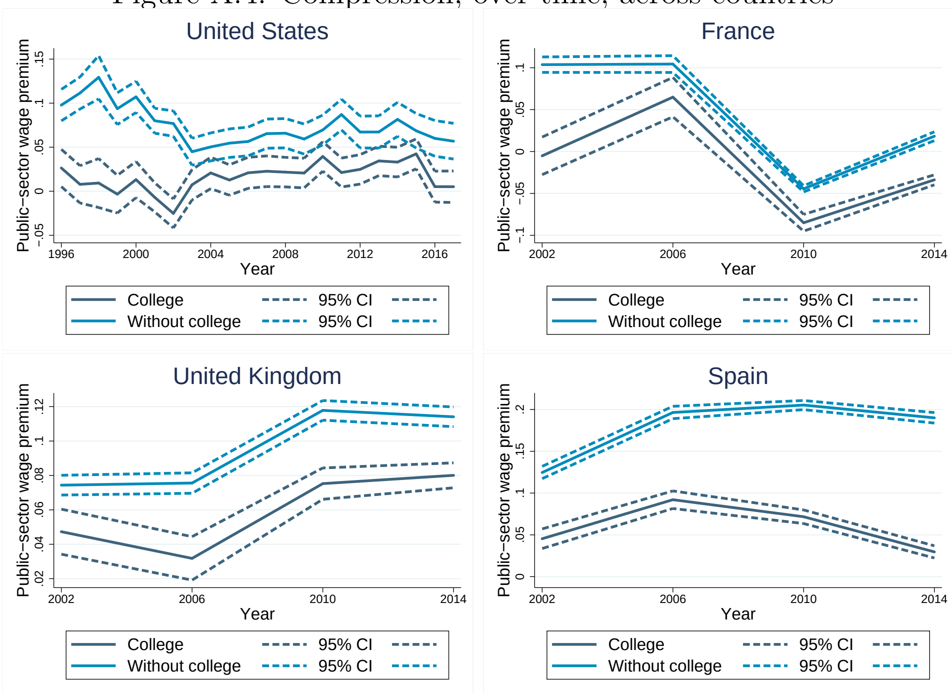


3-digit Occupations



Note: 1st panel uses French, Spanish, UK Labour Force Surveys and the CPS. 2nd panel: CPS data, average between 1996 and 2018. 3-digit occupations that have an overall share of public-sector employment between 0.05 and 0.95.

Figure A.4: Compression, over time, across countries



Note: Estimation by, for each year regressing the log of hourly wage on a public-sector dummy and controls (age, gender, region and a part-time dummy), separately for workers with and without college graduate. Structure of Earning Survey (2002, 2006, 2010, 2014) and CPS data between 2006 and 2018.

B Two-Sector Model Without Under-employment

Technology and Preferences

We present a two-sector model that features a labour market with free mobility. There are two types of individuals with high (1) and low (2) education . The supply of educated individuals in the economy is indicated with n, while the supply of the low-educated workers is indicated with $1-n$. The representative firm produces a private sector output y and the government produces services g with constant return technology:

$$y = (j_1^p)^\alpha (j_2^p)^{1-\alpha} \quad , \quad g = (j_1^g)^\beta (j_2^g)^{1-\beta}. \tag{B.1}$$

Individuals only value wages, so they chose the highest paying job. If the public sector pays a higher wage than the private sector, these jobs would be preferred and would be rationed. If the public sector pays lower wages, no one would work there. As such, the only equilibrium without rationing, implies that the wages in the two sectors have to equate.

Government

We assume the government follows the same minimization problem, determining the target (ideal) level and composition of employment ($\tilde{j}_1^g$ and $\tilde{j}_2^g$), given by.

$$\tilde{j}_1^g = \bar{g}\left(\frac{w_2^g}{w_1^g}\frac{\beta}{1-\beta}\right)^{1-\beta} \quad , \quad \tilde{j}_2^g = \bar{g}\left(\frac{w_1^g}{w_2^g}\frac{1-\beta}{\beta}\right)^{\beta}. \tag{B.2}$$

Private Sector

The representative private sector firm maximizes profits as in the baseline model:

$$w_1^p = \alpha\left(\frac{j_2^p}{j_1^p}\right)^{1-\alpha} \quad , \quad w_2^p = (1-\alpha)\left(\frac{j_1^p}{j_2^p}\right)^{\alpha}. \tag{B.3}$$

And the market clearing conditions are now

$$n = j_1^g + j_1^p \quad , \quad 1-n = j_2^g + j_2^p. \tag{B.4}$$

Equilibrium

Definition 2 *A steady-state equilibrium consists of private-sector wages $\{w_1^p, w_2^p\}$, private-sector jobs $\{j_1^p, j_2^p\}$, public-sector jobs $\{j_1^g, j_2^g\}$, such that, given an exogenous wage policies, technology and composition of the labour force $\{w_1^g, w_2^g, \bar{g}, \alpha, \beta, n\}$, the following apply.*

1. *Private-sector firms maximizes profits.*
2. *Government:*
   (a) *If unconstrained by supply: minimizes costs of providing government services.*
   (b) *If constrained by supply: maintains production of government services.*
3. *Workers sort across labour markets optimally.*
4. *Markets clear.*

The model can be written in two equations in w_1^p and w_2^p, as a function of public-sector employment j_1^g and j_2^g.

$$w_1^p = \alpha \left(\frac{1-n-j_2^g}{n-j_1^g} \right)^{1-\alpha}, \tag{B.5}$$

$$w_2^p = (1-\alpha) \left(\frac{n-j_1^g}{1-n-j_2^g} \right)^{\alpha}, \tag{B.6}$$

Regime 1: wages are high enough in public sector

This is the case where public employment is demand determined. Jobs are rationed so workers who do not get a job in the public sector work in the private.

$$j_1^g = \tilde{j}_1^g = \bar{g} \left(\frac{w_2^g}{w_1^g} \frac{\beta}{1-\beta} \right)^{1-\beta}, \tag{B.7}$$

$$j_2^g = \tilde{j}_2^g = \bar{g} \left(\frac{w_1^g}{w_2^g} \frac{1-\beta}{\beta} \right)^{\beta}. \tag{B.8}$$

For this regime, the wages in the public sector have to be above those in the private.

$$w_1^g > \tilde{w}_1^g = \alpha \left(\frac{1-n-\tilde{j}_2^g}{n-\tilde{j}_1^g} \right)^{1-\alpha} \tag{B.9}$$

$$w_2^g > \tilde{w}_2^g = (1-\alpha) \left(\frac{n-\tilde{j}_1^g}{1-n-\tilde{j}_2^g} \right)^{\alpha} \tag{B.10}$$

The mechanisms here are the same as in the baseline model, except for the absence of underemployment.

Regime 2: Skilled public-sector wages too low

In the case, skilled public wages are below the private wage (when the government hires its target level of workers): $w_1^g < \tilde{w}_1^g$, skilled workers would move away from the public sector. However, not all of them would leave, as doing so would push the private sector wage below the public. Hence, the only equilibrium is that private wages fall until they are equal to public wages ($w_1^p = w_1^g$). This pins down jointly educated private employment, public employment, and unskilled private wages:

$$j_1^p = \left(\frac{\alpha}{w_1^p} \right)^{\frac{1}{1-\alpha}} (1-n-j_2^g), \tag{B.11}$$

$$j_1^g = n - j_1^p, \tag{B.12}$$

$$j_2^g = \left[\frac{\bar{g}}{(j_1^g)^{\beta}} \right]^{\frac{1}{1-\beta}}, \tag{B.13}$$

$$w_2^p = (1-\alpha) \left(\frac{n-j_1^g}{1-n-j_2^g} \right)^{\alpha}, \tag{B.14}$$

provided that $w_2^g \geq \tilde{w}_2^g$. To maintain government services it has to open more low-type jobs. Public employment is supply determined.

Regime 3: Unskilled public-sector wages too low

Again, we show this case for completeness. It requires that unskilled public wages are too low and that skilled wages are high enough $w_1^g \geq \tilde{w}_1^g$ and $w_2^g < \tilde{w}_2^g$ Unskilled workers prefer private sector so private wages fall until they are equal to public wages ($w_2^p = w_2^g$). This pins down jointly educated private employment, public employment, and unskilled private wages

$$j_2^p = \left(\frac{1-\alpha}{w_2^p}\right)^{\frac{1}{\alpha}} (n - j_1^g), \tag{B.15}$$

$$j_2^g = 1 - n - j_2^p \tag{B.16}$$

$$j_1^g = \left[\frac{\bar{g}}{(j_2^g)^{1-\beta}}\right]^{\frac{1}{\beta}}. \tag{B.17}$$

$$w_1^p = \alpha \left(\frac{1 - n - j_2^g}{n - j_1^g}\right)^{1-\alpha}, \tag{B.18}$$

To maintain government services it has to open more high-type jobs. Public employment is supply determined.

Regime 4 - both wages too low - public-sector breakdown

If both wages are two low $w_1^g < \tilde{w}_1^g$ and $w_2^g < \tilde{w}_2^g$ private wages are determined by the public sector: $w_1^p = w_1^g$ and $w_2^p = w_2^g$ Public sector can only hire the remaining

$$j_2^p = \left(\frac{1-\alpha}{w_2^p}\right)^{\frac{1}{\alpha}} (j_1^p), \tag{B.19}$$

$$j_1^p = \left(\frac{\alpha}{w_1^p}\right)^{\frac{1}{1-\alpha}} (j_2^p), \tag{B.20}$$

$$j_2^g = 1 - n - j_2^p \tag{B.21}$$

$$j_1^g = n - j_1^p \tag{B.22}$$

Government services that are allowed is given by $g = (j_1^g)^\beta (j_2^g)^{1-\beta}$. In an extreme case of low wages, all workers would move to the private sector and production would go to zero.

C One-sector Model with Underemployment

Technology and Preferences

Individuals are endowed with 1 unit of indivisible labor and firms have jobs requiring different tasks to produce output. There are two types of individuals with high (1) and low (2) education . The supply of educated individuals in the economy is indicated with n, while the supply of the low educated workers is indicated with $1-n$.

Firms produce with a constant return technology in jobs requiring different skills. There are skilled and unskilled jobs. In what follows we shall use a Cobb Douglas specification.

$$y = (j_1)^{\alpha}(j_2)^{1-\alpha}$$

where $j_1(j_2)$ is the number skilled (unskilled) jobs. Jobs can be described through a ladder type mechanism, so that individuals endowed with higher education are able to perform also unskilled jobs. They can perform at zero effort costs both type of jobs while individuals with low education can only perform the unskilled job.

Individual preferences are linear, and the model is static. The wage paid for the skilled job is indicated with w_1 while the wage paid for unskilled job is indicated with w_2. Each individual worker i has an heterogeneous "non-pecuniary value" over these tasks, ϵ_i^1 and ϵ_i^2, drawn from a continuous distribution with cumulative density Φ and unbounded lower and upper support. For simplicity, we also assume that the expected value of $E[\epsilon^1] = E[\epsilon^2] = 0$. These "non-pecuniary" attributes of the job could reflect preferences, but all other elements such as location of jobs, co-workers, hours, etc. For instance, an educated worker i's utility in the skilled job is given by sum of the wage and "non-pecuniary" shock, $w_1 + \nu\epsilon_i^1$, where ν captures the weight of the "non-pecuniary" shock in the individual preferences. Our model accommodates the traditional model in the limit where ν tends to zero.

Sorting by High-Educated Workers and Underemployment

The key decision rests with the educated workers and concerns the type of sector in which to supply their indivisible unit of labor. An individual i decision is given by

$$U_i^1 = Max\{w_1 + \nu\epsilon_i^1, w_2 + \nu\epsilon_i^2\} \tag{C.1}$$

while type 2 individuals have no choice other than working in the unskilled tasks and their utility is thus $U_i^2 = w_2 + \nu\epsilon_i^2$. Educated individuals join the simple tasks only if $(w_1 + \nu\epsilon_i^1 < w_2 + \nu\epsilon_i^2)+$, or if $\eta_i = \frac{w_2 - w_1}{\nu}$, where $\eta_i = \epsilon_i^1 - \epsilon_i^2$. In what follows, we indicate with Φ_η the probability distribution over the net preference shock $\eta_i = \epsilon_i^1 - \epsilon_i^2$. Educated individuals join the simple job if η is low enough so that $\eta_i < \frac{w_2 - w_1}{\nu}$. This simple sorting condition implies that there is an endogenously determined aggregate number of underemployed defined as

$$u = \Phi_\eta(\frac{w_2 - w_1}{\nu}) \tag{C.2}$$

Labor Demand and Market Clearing

Firms maximise profits taking as given the wage for both tasks. Labor demand is given by

$$w_2 = (1-\alpha)\left(\tfrac{j_1}{j_2}\right)^{\alpha} \quad , \qquad w_1 = \alpha\left(\tfrac{j_2}{j_1}\right)^{1-\alpha}. \tag{C.3}$$

Wages adjust until the demand for jobs requiring a particular task is equal to the supply of workers for that task. Market clearing equilibrium imply

$$j_1 = n - u \quad , \qquad j_2 = (1 - n) + u. \tag{C.4}$$

where labor demand j_1 and j_2 is given by equations (C.3) while underemployment u is derived from equation (C.2)

Equilibrium

Definition 3 *A steady-state equilibrium consists of tasks wages $\{w_1, w_2\}$, jobs in the two tasks $\{j_1, j_2\}$, and under-employment for skilled workers $\{u\}$, such that .*

1. *Private-sector firms maximizes profits (C.3).*

2. *Skilled workers sort across labour markets according to (C.2).*

3. *Markets clear (C.4).*

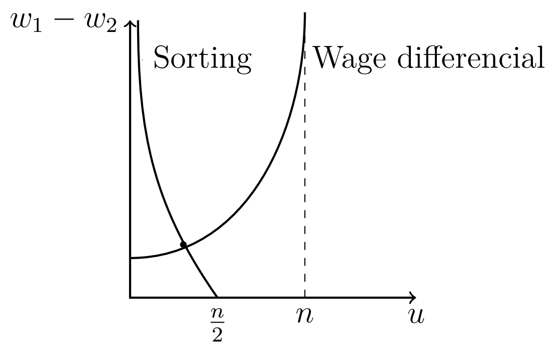
The equilibrium is best summarized in two equations: the sorting condition and a wage differential condition, in u and $w_1 - w_2$:

$$u = n\Phi_\eta\left(\frac{w_2 - w_1}{\nu}\right) \tag{C.5}$$

$$w_1 - w_2 = \alpha\left(\frac{(1-n)+u}{n-u}\right)^{1-\alpha} - (1-\alpha)\left(\frac{n-u}{(1-n)+u}\right)^{\alpha} \tag{C.6}$$

These two conditions are depicted graphically in Figure C.1. The downward sloping line is the sorting condition C.2, that crosses the horizontal axis at $\frac{n}{2}$ underemployment. When the wage differential is zero, workers will split equally between the two types of jobs as none offers a wage advantage. As the wage differential increases, there are fewer educated willing to work in unskilled jobs and as this differential increases to infinity underemployment tends to zero. The upward sloping equation is the wage differential condition, obtained from labor demand (C.3), and the market clearing conditions (equation C.4), is increasing in underemployment. With zero underemployment the intercept represents the wage differential of the typical model where all the educated workers are performing skilled jobs. As underemployment increases, this is reflected on an excess supply of workers to unskilled jobs and a shortage of workers for skilled jobs, increasing the wage differential. As underemployment approaches the total supply of the skilled n, by the Inada conditions the wage differential

Figure C.1: Equilibrium Underemployment



tends to infinity. The equilibrium underemployment is the crossing of the two lines, and is given by a single equation in underemployment:

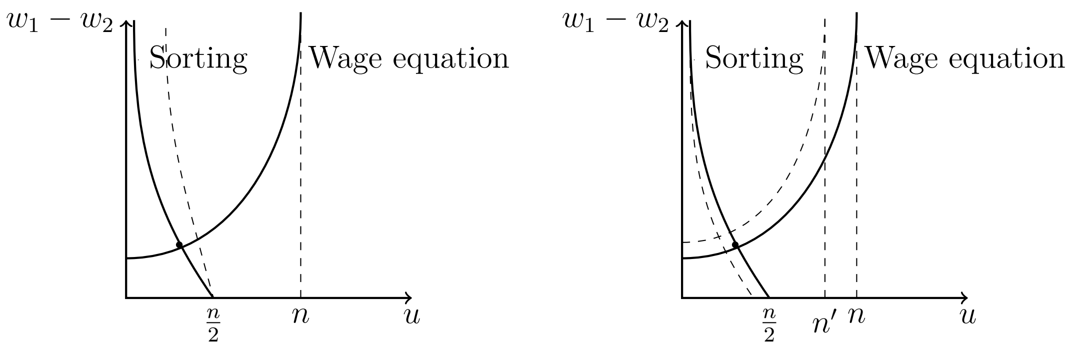
$$u = \Phi_\eta \left(\frac{(1-\alpha)}{\nu} \left(\frac{n-u}{1-n+u} \right)^{1-\alpha} - \frac{\alpha}{\nu} \left(\frac{1-n+u}{n-u} \right)^{\alpha} \right) \tag{C.7}$$

The equilibrium exists and is unique.

Comparative Statics

The simple model can be used to illustrate the effects of two interesting comparative static exercise. Such exercise highlights some features of the public sector that are present in the main model. Suppose first that the government imposes a proportional income tax (Figure C.2, left panel). Other things equal, the net-wage differential is lower and the sorting condition shifts to the right, and equilibrium underemployment rises. Note that despite the fact that the gross wage differential $(w_2 - w_1)$ rises, the take-home differential actually falls. Next, suppose that the supply of skilled workers available shrinks. As shown in the right panel of Figure C.2, both curves shift to the left and equilibrium underemployment falls, but the wage gap is now larger.

Figure C.2: Equilibrium underemployment with an income tax and skill shortage



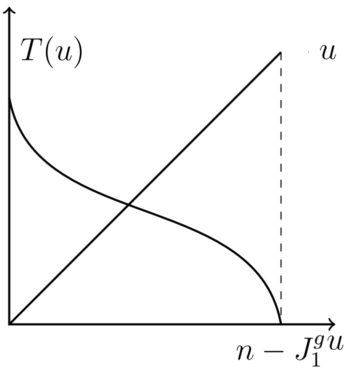
In a companion paper, we generalize this 1-sector model, considering both under and overemployment, and different efficiency units of educated workers in unskilled jobs, to measure the output losses of mismatch (Garibaldi et al. 2019).

D Baseline model

D.1 Regime 1

Substituting the expressions for wages on underemployment, we get one equation that pins down u.

$$u = (n - j_1^g)\left[\frac{e^{\frac{(1-\tau)}{\nu}[(1-\alpha)\left(\frac{n-j_1^g-u}{1-n-j_2^g+u}\right)^{\alpha}]}}{e^{\frac{(1-\tau)}{\nu}w_2^g} + e^{\frac{(1-\tau)}{\nu}[\alpha\left(\frac{1-n-j_2^g+u}{n-j_1^g-u}\right)^{1-\alpha}]} + e^{\frac{(1-\tau)}{\nu}[(1-\alpha)\left(\frac{n-j_1^g-u}{1-n-j_2^g+u}\right)^{\alpha}]}} + e^{\frac{(1-\tau)}{\nu}w_2^g}\right] \equiv T(u) \quad \text{(D.1)}$$



The LHS is the 45 degree line, from 0 to $n-J_1^g$. The RHS evaluated at zero is positive and evaluated at $n-j_1^g$ is zero. We concentrate our analysis on the effects of public-sector wages for both types of workers, the size of the educated population and the level of government services. Under Regime 1, we can write the matrix of marginal effects for the exogenous variables $z \in \{w_g^2, w_g^1, \bar{g}, n\}$ as:

$$\begin{pmatrix} 1 & -\frac{\partial u}{\partial w_p^1} & -\frac{\partial u}{\partial w_p^2} & -\frac{\partial u}{\partial j_g^1} & 0 \\ -\frac{\partial w_p^1}{\partial u} & 1 & 0 & -\frac{\partial w_p^1}{\partial j_1^g} & -\frac{\partial w_p^1}{\partial j_2^g} \\ -\frac{\partial w_p^2}{\partial u} & 0 & 1 & -\frac{\partial w_p^2}{\partial j_1^g} & -\frac{\partial w_p^2}{\partial j_2^g} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \frac{du}{dz} \\ \frac{dw_p^1}{dz} \\ \frac{dw_p^2}{dz} \\ \frac{dj_g^1}{dz} \\ \frac{dj_g^2}{dz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial z} \\ \frac{\partial w_1^p}{\partial z} \\ \frac{\partial w_2^p}{\partial z} \\ \frac{\partial j_1^g}{\partial z} \\ \frac{\partial j_2^g}{\partial z} \end{pmatrix} \quad \text{(D.2)}$$

where:

$\frac{\partial u}{\partial w_p^1} = -\frac{1-\tau}{\nu}u(1-\frac{u}{n-j_1^g}) < 0$	$\frac{\partial u}{\partial w_p^2} = \frac{1-\tau}{\nu}u^p(1-\frac{u}{n-j_1^g}) > 0$	$\frac{\partial u}{\partial j_g^1} = -\frac{u}{n-j_1^g} < 0$
$\frac{\partial w_p^1}{\partial u} = (1-\alpha)w_1^p(\frac{1}{j_1^p}+\frac{1}{j_2^p}) > 0$	$\frac{\partial w_p^1}{\partial j_1^g} = \frac{(1-\alpha)w_1^p}{j_1^p} > 0$	$\frac{\partial w_p^1}{\partial j_2^g} = -\frac{(1-\alpha)w_1^p}{j_2^p} < 0$
$\frac{\partial w_p^2}{\partial u} = -\alpha w_2^p(\frac{1}{j_1^p}+\frac{1}{j_2^p} < 0$	$\frac{\partial w_p^2}{\partial j_1^g} = -\frac{\alpha w_2^p}{j_1^p} < 0$	$\frac{\partial w_p^2}{\partial j_2^g} = \frac{\alpha w_2^p}{j_2^p} > 0$

The right-hand side vector is different depending on which parameter we are doing the comparative statics on

$\frac{\partial u}{\partial w_g^2} = \frac{1-\tau}{\nu}u^g(1-\frac{u}{n-j_1^g}) > 0$	$\frac{\partial u}{\partial w_g^1} = 0$	$\frac{\partial u}{\partial \bar{g}} = 0$	$\frac{\partial u}{\partial n} = \frac{(u)}{n-j_1^g} > 0$
$\frac{\partial w_1^p}{\partial w_2^g} = 0$	$\frac{\partial w_1^p}{\partial w_1^g} = 0$	$\frac{\partial w_1^p}{\partial \bar{g}} = 0$	$\frac{\partial w_1^p}{\partial n} = -(1-\alpha)w_1^p(\frac{1}{j_1^p}+\frac{1}{j_2^p}) < 0$
$\frac{\partial w_2^p}{\partial w_2^g} = 0$	$\frac{\partial w_2^p}{\partial w_1^g} = 0$	$\frac{\partial w_2^p}{\partial \bar{g}} = 0$	$\frac{\partial w_2^p}{\partial n} = \alpha w_2^p(\frac{1}{j_1^p}+\frac{1}{j_2^p}) > 0$
$\frac{\partial j_1^g}{\partial w_2^g} = \frac{(1-\beta)j_1^g}{w_2^g} > 0$	$\frac{\partial j_1^g}{\partial w_1^g} = \frac{-(1-\beta)j_1^g}{w_1^g} < 0$	$\frac{\partial j_1^g}{\partial \bar{g}} = \frac{j_1^g}{\bar{g}} > 0$	$\frac{\partial j_1^g}{\partial n} = 0$
$\frac{\partial j_2^g}{\partial w_2^g} = -\frac{\beta j_2^g}{w_2^g} < 0$	$\frac{\partial j_2^g}{\partial w^g 1} = \frac{\beta j_2^g}{w_1^g} > 0$	$\frac{\partial j_2^g}{\partial \bar{g}} = \frac{j_2^g}{\bar{g}} > 0$	$\frac{\partial j_2^g}{\partial n} = 0$

Solving the matrix system (noticing that $\frac{\partial w_1^p}{\partial j_1^g} \times \frac{\partial w_2^p}{\partial j_2^g} = \frac{\partial w_1^p}{\partial j_2^g} \times \frac{\partial w_2^p}{\partial j_1^g}$, together with $\frac{(u)}{n-j_1^g} < 1$, $-\frac{\partial u}{\partial w_p^1} = \frac{\partial u}{\partial w_g^2} + \frac{\partial u}{\partial w_p^2}$ and that $\frac{\partial j_1^g}{\partial w_2^g} < -\frac{\partial j_2^g}{\partial w_2^g}$ if $w_1^g > w_2^g$. With Matlab Simbolic Toolkit (codes available on request), we show

$\frac{du}{dw_g^2} \lesseqgtr 0$	$\frac{du}{dw_g^1} > 0$	$\frac{du}{d\bar{g}} \lesseqgtr 0$	$\frac{du}{dn} > 0$
$\frac{dw_1^p}{dw_2^g} > 0$	$\frac{dw_1^p}{dw_1^g} < 0$	$\frac{dw_1^p}{d\bar{g}} \lesseqgtr 0$	$\frac{dw_1^p}{dn} \lesseqgtr 0$
$\frac{dw_2^p}{dw_2^g} < 0$	$\frac{dw_2^p}{dw_1^g} > 0$	$\frac{dw_2^p}{d\bar{g}} \lesseqgtr 0$	$\frac{dw_2^p}{dn} \lesseqgtr 0$
$\frac{dj_1^g}{dw_2^g} > 0$	$\frac{dj_1^g}{dw_1^g} < 0$	$\frac{dj_1^g}{d\bar{g}} > 0$	$\frac{dj_1^g}{dn} = 0$
$\frac{dj_2^g}{dw_2^g} < 0$	$\frac{dj_2^g}{dw^g 1} > 0$	$\frac{dj_2^g}{d\bar{g}} > 0$	$\frac{dj_2^g}{dn} = 0$

D.2 Regime 2

Under Regime 2, the last two rows of the matrix of marginal effects for the exogenous variables $z \in \{w_g^2, w_g^1, \bar{g}, n\}$ are different:

$$\begin{pmatrix} 1 & -\frac{\partial u}{\partial w_p^1} & -\frac{\partial u}{\partial w_p^2} & -\frac{\partial u}{\partial j_g^1} & 0 \\ -\frac{\partial w_p^1}{\partial u} & 1 & 0 & -\frac{\partial w_p^1}{\partial j_1^g} & -\frac{\partial w_p^1}{\partial j_2^g} \\ -\frac{\partial w_p^2}{\partial u} & 0 & 1 & -\frac{\partial w_p^2}{\partial j_1^g} & -\frac{\partial w_p^2}{\partial j_2^g} \\ 0 & -\frac{\partial j_1^g}{\partial w_p^1} & -\frac{\partial j_1^g}{\partial w_2^p} & 1 & 0 \\ 0 & 0 & 0 & -\frac{\partial j_2^g}{\partial j_1^g} & 1 \end{pmatrix} \times \begin{pmatrix} \frac{du}{dz} \\ \frac{dw_p^1}{dz} \\ \frac{dw_p^2}{dz} \\ \frac{dj_g^1}{dz} \\ \frac{dj_g^2}{dz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial z} \\ \frac{\partial w_1^p}{\partial z} \\ \frac{\partial w_2^p}{\partial z} \\ \frac{\partial j_1^g}{\partial z} \\ \frac{\partial j_2^g}{\partial z} \end{pmatrix} \tag{D.3}$$

where, in addition

$\frac{\partial j_1^g}{\partial w_1^p} = -\frac{1-\tau}{n\nu} j_1^p j_1^g < 0 \quad \frac{\partial j_1^g}{\partial w_2^p} = -\frac{1-\tau}{n\nu} u^p j_1^g < 0 \quad \frac{\partial j_2^g}{\partial j_1^g} = -\frac{\beta}{1-\beta} \frac{j_2^g}{j_1^g} < 0$

The last two rows of the right-hand side vectors are now

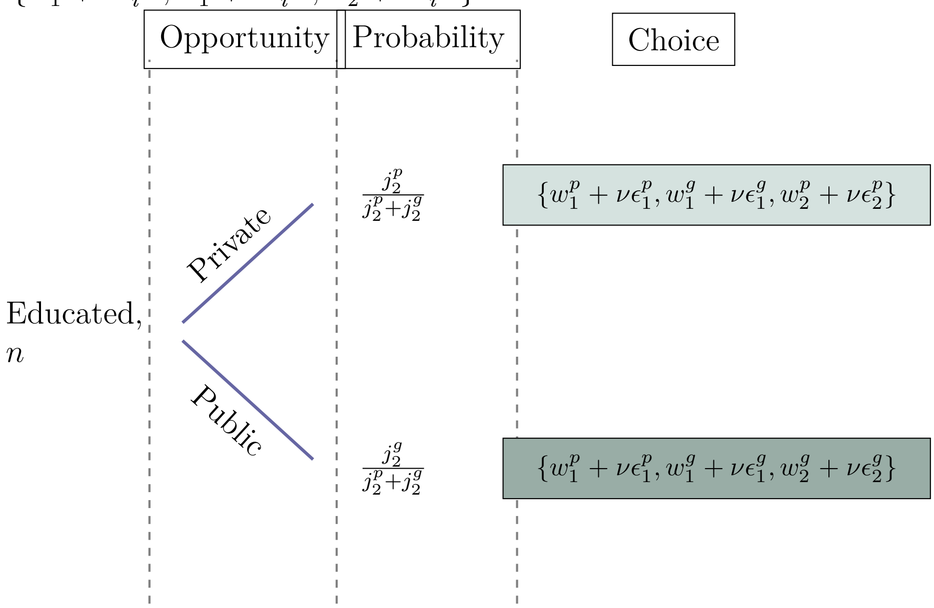
$\frac{\partial j_1^g}{\partial w_2^g} = -\frac{1-\tau}{n\nu} u^g j_1^g < 0$	$\frac{\partial j_1^g}{\partial w_1^g} = \frac{1-\tau}{\nu} \frac{j_1^g}{n} (1 - \frac{j_1^g}{n}) > 0$	$\frac{\partial j_1^g}{\partial \bar{g}} = 0$	$\frac{\partial j_1^g}{\partial n} = \frac{j_1^g}{n} > 0$
$\frac{\partial j_2^g}{\partial w_2^g} = 0$	$\frac{\partial j_2^g}{\partial w^g 1} = 0$	$\frac{\partial j_2^g}{\partial \bar{g}} = \frac{1}{\beta} \frac{j_2^g}{\bar{g}}$	$\frac{\partial j_2^g}{\partial n} = 0$

Solving the matrix system (noticing that $\frac{\partial w_1^p}{\partial j_1^g} \times \frac{\partial w_2^p}{\partial j_2^g} = \frac{\partial w_1^p}{\partial j_2^g} \times \frac{\partial w_2^p}{\partial j_1^g}$, together with $\frac{(u)}{n-j_1^g} < 1$, $-\frac{\partial u}{\partial w_p^1} = \frac{\partial u}{\partial w_g^2} + \frac{\partial u}{\partial w_p^2}$ and that $-\frac{\partial j_2^g}{\partial j_1^g} > 1$ if $w_1^g > w_2^g$. With Matlab Simbolic Toolkit (codes available on request), we show

$\frac{du}{dw_g^2} \lesseqgtr 0$	$\frac{du}{dw_g^1} < 0$	$\frac{du}{d\bar{g}} \lesseqgtr 0$	$\frac{du}{dn} \lesseqgtr 0$
$\frac{dw_1^p}{dw_2^g} > 0$	$\frac{dw_1^p}{dw_1^g} > 0$	$\frac{dw_1^p}{d\bar{g}} < 1$	$\frac{dw_1^p}{dn} \lesseqgtr 0$
$\frac{dw_2^p}{dw_2^g} < 0$	$\frac{dw_2^p}{dw_1^g} < 0$	$\frac{dw_2^p}{d\bar{g}} > 0$	$\frac{dw_2^p}{dn} \lesseqgtr 0$
$\frac{dj_1^g}{dw_2^g} \lesseqgtr 0$	$\frac{dj_1^g}{dw_1^g} > 0$	$\frac{dj_1^g}{d\bar{g}} \lesseqgtr 0$	$\frac{dj_1^g}{dn} \lesseqgtr 0$
$\frac{dj_2^g}{dw_2^g} \lesseqgtr 0$	$\frac{dj_2^g}{dw^g 1} < 0$	$\frac{dj_2^g}{d\bar{g}} \lesseqgtr 0$	$\frac{dj_2^g}{dn} \lesseqgtr 0$

E.3 Baseline model with alternative sorting mechanism

We set up a variation of the model with an alternative sorting mechanism. We consider that the underemployment opportunities are proportional to size of sector. The mechanism is described in the figure below. Of all the educated workers, a fraction $\frac{j_2^g}{j_2^p+j_2^g}$ has an underemployment opportunity only in the public sector. Those workers choose between three options $Max\{w_1^p + \nu\epsilon_i^{p,1}, w_1^g + \nu\epsilon_i^{g,1}, w_2^g + \nu\epsilon_i^{g,2}\}$. The remaining fraction $\frac{j_2^p}{j_2^p+j_2^g}$ has only an underemployment opportunity in the private sector and chooses between $Max\{w_1^p + \nu\epsilon_i^{p,1}, w_1^g + \nu\epsilon_i^{g,1}, w_2^p + \nu\epsilon_i^{p,2}\}$.



The threshold wages $\tilde{w}_1^g$ and $\tilde{w}_2^g$ are defined implicitly by

$$\tilde{j}_1^g = n\left[\frac{j_2^p}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^p}} + \frac{j_2^g}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^g}}\right] \tag{E.1}$$

$$\tilde{j}_2^g - u^g = (1-n)\left[\frac{e^{\frac{(1-\tau)}{\nu}\tilde{w}_2^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_2^g}+e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{E.2}$$

And the different shares in the economy given by:

$$u^g = n\left[\frac{j_2^g}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_2^g}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^g}}\right] \tag{E.3}$$

$$u^p = n\left[\frac{j_2^p}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_2^p}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^p}}\right] \tag{E.4}$$

$$j_1^p = n\left[\frac{j_2^p}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_1^p}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^p}} + \frac{j_2^g}{j_2^p+j_2^g}\frac{e^{\frac{(1-\tau)}{\nu}w_1^p}}{e^{\frac{(1-\tau)}{\nu}\tilde{w}_1^g}+e^{\frac{(1-\tau)}{\nu}w_1^p}+e^{\frac{(1-\tau)}{\nu}w_2^g}}\right] \tag{E.5}$$

The mechanism is similar to the baseline model but with more complicated solution. The advantage of this extension is that it gives a ratio public employment shares of 1, in the symmetric case, which we think is more realistic. Hence, we use this variation of the model in the quantitative section.

E Extensions

E.1 Simple model to endogeneize public policies

We can provide microeconomic foundations for the public-sector employment and wage policies that are taken as exogenous in the baseline model. Consider a government that, similarly wants to minimize cost subject to maitaining the production of government services $\bar{g}$. Additionally, it faces a constrained impose by unions, that arise from political pressure. The the preferences of a union represented by $\theta \ln(a_1) + (1-\theta)\ln(a_2)$. Here θ represents the weight of skilled workers in the union's preferences and a_1 and a_2 are the extra payment to public-sector workers on top of the threshold wage for the unconstrained public sector ($w_1^g = \tilde{w}_1^g + a_1$ and $w_2^g = \tilde{w}_2^g + a_2$). The union knows what this minimum required wage is and tries to push the wages above. For convenience, we assume the function expressing the utility of the extra payment to type i workers,is $\log(a_i)$. The government's problem can be written as:

$$\min_{j_1^g, j_2^g} w_1^g j_1^g + w_2^g j_1^g$$
$$s.t.$$
$$\bar{g} = (j_1^g)^{\beta}(j_2^g)^{1-\beta}.$$
$$\bar{U} = \theta \ln(a_1) + (1-\theta)\ln(a_2).$$
$$w_1^g = \tilde{w}_1^g + a_1.$$
$$w_2^g = \tilde{w}_2^g + a_2.$$

Where $\bar{U}$ is the required utility of unions. The first order conditions of this problem are:

$$j_1^g = \bar{g}\left(\frac{w_2^g}{w_1^g}\frac{\beta}{1-\beta}\right)^{1-\beta}, \tag{E.6}$$

$$j_2^g = \bar{g}\left(\frac{w_1^g}{w_2^g}\frac{1-\beta}{\beta}\right)^{\beta}. \tag{E.7}$$

$$a_1 = \frac{\Omega\theta}{j_1^g} \tag{E.8}$$

$$a_2 = \frac{\Omega(1-\theta)}{j_2^g} \tag{E.9}$$

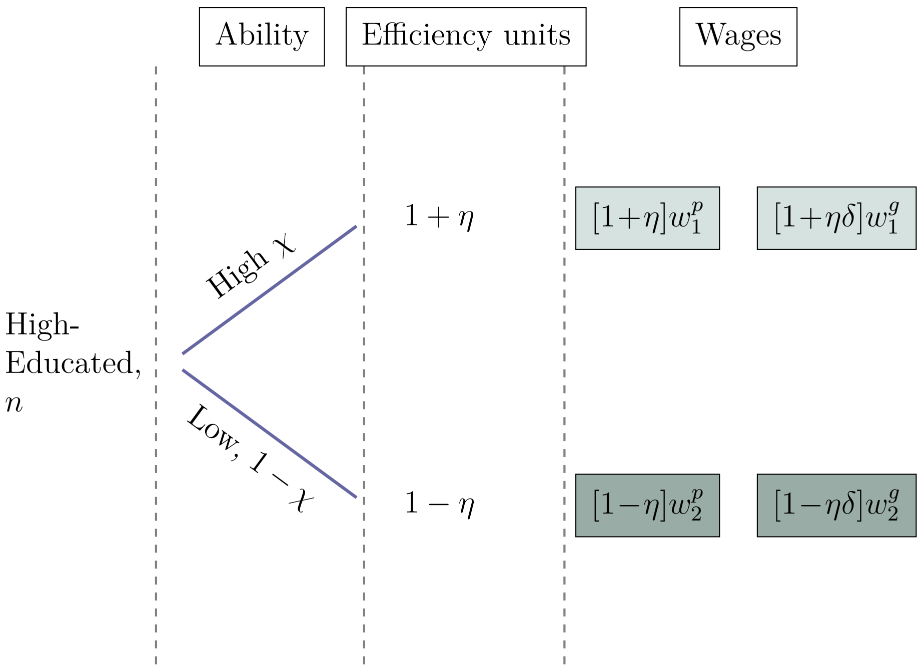
The first two conditions pin down the employment level of the government and are equal to the baseline case. The last two conditions pin down government wages. The additional payment to each type of workers depends on the strength of the union constraint (measure by Ω) and the relative preference of the union over skilled and unskilled workers. Whether it raises more the skilled or unskilled wages, depends on the relative weight on the union preference. This could generate different premia (including negative premia) for different types of workers.

If $\bar{U} = 0$, $a_1 = a_2 = 0$ and the $w_e^g = \tilde{w}_e^g$, the government offers the minimum wage necessary to hire the workers it needs. This would be the outcome of a benevolent government. This is one model of government behaviour, but there could certainly be others.Under this conditions, the economy would be always under Regime 1. To push the government into

Regime 2, we would need to add other elements such as budgetary pressures or preferences for inequality. We think however, when studying the effects of public-sector wages, it is a clearer exercise to take them as exogenous.

E.2 Heterogeneous High-Educated Workers

We then set up a model where skilled workers are heterogeneous in their effective units of labour. A fraction χ of skilled workers have $1+\eta$ efficiency units, while the remaining fraction, $1-\chi$ have $1-\eta$. Workers with high ability are denoted with an additional subscript h while the workers with low efficiency units are denoted with ℓ. Wages in the private sector reflect perfectly their efficiency units, with the high ability earning $(1+\eta)w_1^p$ and the low ability earning $(1-\eta)w_1^p$. On the public sector, the payment structure might not reflect entirely the efficiency units of the worker. We assume that the wages of high ability skilled workers is $(1+\eta\delta)w_1^g$ and for the low ability worker $(1-\eta\delta)w_1^g$. δ is the compression parameter of the public-sector. If $\delta<1$ the are lower wage dispersion in the public sector for skilled workers, fact that has been widely documented. The setting is described in the figure below.



This heterogeneity requires that we distinguish the number of workers in terms of headcount and in efficiency units. Furthermore, we assume that the government always prefers the high-ability workers and restrain the analysis to the case χ is small enough so that, for the government cannot exhaust the high skilled jobs with high-ability educated workers. We can defined the market clearing in headcount:

$$n\chi = l_{1,h}^g + l_{1,h}^p + u_h \tag{E.10}$$
$$n(1-\chi) = l_{1,\ell}^g + l_{1,\ell}^p + u_\ell \tag{E.11}$$
$$1-n = j_2^g + j_2^p - u_h - u_\ell. \tag{E.12}$$

where $l_{1,h}^x$ and $l_{1,\ell}^x$ denote the number of high- and low-ability working in sector x. In terms of efficiency units the market clearing is given by

$$j_1^g = (1+\eta)l_{1,h}^g + (1-\eta)l_{1,\ell}^g \tag{E.13}$$
$$j_1^p = (1+\eta)l_{1,h}^p + (1-\eta)l_{1,\ell}^p \tag{E.14}$$

Regarding the sorting, we assume that the government skilled jobs is always high enough such that high ability workers that want a public-sector job always enter. Hence, for the high ability, the sorting between underemployment, public-sector employment and private-sector employment (remainder) is given by

$$u_h = n\chi \left[\frac{e^{\frac{(1-\tau)}{\nu} w_2^p} + e^{\frac{(1-\tau)}{\nu} w_2^g}}{e^{\frac{(1-\tau)}{\nu} w_2^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta\delta) w_1^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta) w_1^p} + e^{\frac{(1-\tau)}{\nu} w_2^p}} \right] \quad \text{(E.15)}$$

$$l_{1,h}^g = n\chi \left[\frac{e^{\frac{(1-\tau)}{\nu}(1+\eta\delta) w_1^g}}{e^{\frac{(1-\tau)}{\nu} w_2^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta\delta) w_1^g} + e^{\frac{(1-\tau)}{\nu}(1+\eta) w_1^p} + e^{\frac{(1-\tau)}{\nu} w_2^p}} \right] \quad \text{(E.16)}$$

The low-ability workers, take the remaining public-sector jobs and we focus on Regime 1 (public-sector wages are high enough) such that for them, jobs are rationed. Hence, the number of low-ability workers in the public sector and underemployed are given by:

$$l_{2,\ell}^g = \frac{j_1^g - (1+\eta) l_{1,h}^g}{(1-\eta)} \quad \text{(E.17)}$$

$$u_\ell = \left[n(1-\chi) - l_{2,\ell}^g \right] \left[\frac{e^{\frac{(1-\tau)}{\nu} w_2^p} + e^{\frac{(1-\tau)}{\nu} w_2^g}}{e^{\frac{(1-\tau)}{\nu} w_2^g} + e^{\frac{(1-\tau)}{\nu}(1-\eta) w_1^p} + e^{\frac{(1-\tau)}{\nu} w_2^p}} \right] \quad \text{(E.18)}$$

In this version of the model, skilled workers with low efficiency units, have lower wages in skilled jobs, and hence are more likely to be underemployed. Also, if δ is reduced, fewer high ability skilled workers go work for the government, so the government has to employ more of the low ability skilled workers that are relatively more expensive.

E.3 Endogenous income tax

One element that was not developed in the model was the financing side of the government. Although we included τ, reflecting an income tax that finances government spending, this was taken as a parameter assumed constant even when public-sector wages increase. The justification would be that such policies would be financed with government debt or cuts in other spending categories. However, we can easily endogeneize tax rate in the model by introducing an additional budget constraint. The model can be written in four equations in u, w_1^p, w_2^p and τ

$$u = (n - j_1^g) \left[\frac{e^{\frac{(1-\tau)}{\nu} w_2^p} + e^{\frac{(1-\tau)}{\nu} w_2^g}}{e^{\frac{(1-\tau)}{\nu} w_2^g} + e^{\frac{(1-\tau)}{\nu} w_1^p} + e^{\frac{(1-\tau)}{\nu} w_2^p}} \right] \quad \text{(E.19)}$$

$$w_1^p = \alpha \left(\frac{1 - n - j_2^g + u}{n - j_1^g - u} \right)^{1-\alpha}, \quad \text{(E.20)}$$

$$w_2^p = (1-\alpha) \left(\frac{n - j_1^g - u}{1 - n - j_2^g + u} \right)^{\alpha}, \quad \text{(E.21)}$$

$$\tau = \frac{w_1^g j_1^g + w_2^g j_2^g}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} \quad \text{(E.22)}$$

The solution to the system of total derivatives is:

$$
\begin{pmatrix}
1 & -\frac{\partial u}{\partial w_p^1} & -\frac{\partial u}{\partial w_p^2} & -\frac{\partial u}{\partial j_g^1} & 0 & -\frac{\partial u}{\partial \tau} \\
-\frac{\partial w_p^1}{\partial u} & 1 & 0 & -\frac{\partial w_p^1}{\partial j_1^g} & -\frac{\partial w_p^1}{\partial j_2^g} & 0 \\
-\frac{\partial w_p^2}{\partial u} & 0 & 1 & -\frac{\partial w_p^2}{\partial j_1^g} & -\frac{\partial w_p^2}{\partial j_2^g} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-\frac{\partial \tau}{\partial u} & 0 & 0 & -\frac{\partial \tau}{\partial \partial j_g^1} & -\frac{\partial \tau}{\partial \partial j_g^2} & 1
\end{pmatrix}
\times
\begin{pmatrix}
\frac{du}{dz} \\ \frac{dw_p^1}{dz} \\ \frac{dw_p^2}{dz} \\ \frac{dj_g^1}{dz} \\ \frac{dj_g^2}{dz}
\end{pmatrix}
=
\begin{pmatrix}
\frac{\partial u}{\partial z} \\ \frac{\partial w_1^p}{\partial z} \\ \frac{\partial w_2^p}{\partial z} \\ \frac{\partial j_1^g}{\partial z} \\ \frac{\partial j_2^g}{\partial z} \\ \frac{\partial \tau}{\partial z}
\end{pmatrix}
\tag{E.23}
$$

$\frac{\partial u}{\partial \tau} = \frac{j_1^p}{\nu}(w_1^p \frac{u}{n - j_1^g} - w_2^p u^p - w_2^g u^g > 0$ $\quad \frac{\partial \tau}{\partial u} = \frac{(w_1^p - w^p 2)\tau}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$ $\quad \frac{\partial \tau}{\partial \partial j_g^1} = \frac{w_1^g (1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} >$

$\frac{\partial \tau}{\partial \partial j_g^2} = \frac{w_2^g (1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$ $\quad \frac{\partial \tau}{\partial w_2^g} = \frac{j_2^g (1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$ $\quad \frac{\partial \tau}{\partial w_1^g} \frac{j_1^g (1-\tau)}{(j_1^p)^\alpha (j_2^p)^{1-\alpha} + w_1^g j_1^g + w_2^g j_2^g} > 0$

E.4 Dual government problem

We now propose a different government problem. Instead of minimizing costs to produce a certain level of government services, the government maximizes the production of services, having an explicit budget constraint. Consider a government that is limited in its amount of spending to $\bar{\omega}$, exogenous, that arises from budgetary constraints. The objective function is given by:

$$\max_{j_1^g, j_2^g} (j_1^g)^{\beta} (j_2^g)^{1-\beta}$$
$$s.t.$$
$$w_1^g j_1^g + w_2^g j_1^g = \bar{\omega}.$$

The first-order conditions pinning employment are given by:

$$j_1^g = \bar{\omega} \left(\frac{\beta}{w_1^g} \right), \tag{E.24}$$

$$j_2^g = \bar{\omega} \left(\frac{1-\beta}{w_2^g} \right), \tag{E.25}$$

The first two conditions pin down the employment level of the government. Notice that now the number of workers of a given type only depends on their wage. Given technology and a certain wage, the government spends a constant fraction β of its budget on skilled workers and $1-\beta$ on unskilled workers. Differently from the baseline, j_1^g is increasing in β and decreasing in w_1^g and j_2^g is decreasing in w_2^g and β. The derivatives of employment are given by:

$\frac{\partial j_1^g}{\partial w_2^g} = 0$	$\frac{\partial j_1^g}{\partial w_1^g} = \frac{-j_1^g}{w_1^g} < 0$
$\frac{\partial j_2^g}{\partial w_2^g} = -\frac{j_2^g}{w_2^g} < 0$	$\frac{\partial j_2^g}{\partial w^g 1} = 0$

The expressions for the elasticities of private wages with respect to public-sector wages also simplify, with no cross term: For instance, private wage elasticities with respect to unskilled public wage are given by:

$$\frac{dw_1^p}{dw_2^g} \frac{w_2^g}{w_1^p} = (1-\alpha) \frac{j_2^g}{j_2^p} + \frac{du}{dw_2^g} \left[\frac{(1-\alpha)}{j_2^p} + \frac{(1-\alpha)}{j_1^p} \right] w_2^g \tag{E.26}$$

$$\frac{dw_2^p}{dw_2^g} \frac{w_2^g}{w_2^p} = -\alpha \frac{j_2^g}{j_2^p} - \frac{du}{dw_2^g} \left[\frac{\alpha}{j_2^p} + \frac{\alpha}{j_1^p} \right] w_2^g \tag{E.27}$$

F Additional quantitative results

Quantitative results for European countries

Table F.1: Calibration, European Countries

Parameter	**UK**	**France**	**Spain**	**Variable**	**UK**	**France**	**Spain**
				Targeted			
α	0.224	0.302	0.294	$\frac{w_1^p}{w_2^p}$	1.401	1.474	1.434
β	0.530	0.449	0.624	$j_1^g + u^g$	0.133	0.091	0.101
$\bar{g}$	0.123	0.106	0.082	$j_2^g - u^g$	0.115	0.122	0.060
n	0.354	0.323	0.369	n	0.354	0.323	0.369
w_1^g	0.808	0.700	0.744	$\frac{w_1^g}{w_1^p}$	1.059	0.985	1.060
w_2^g	0.597	0.504	0.580	$\frac{w_2^g}{w_2^p}$	1.096	1.045	1.179
$\frac{\nu}{1-\tau}$	1.645	0.224	0.271	$\frac{u}{j_2^g + j_2^p}$	0.149	0.088	0.124
				Not Targeted			
				$\frac{u^g}{j_2^g}$	0.189	0.055	0.199
				$\frac{u^p}{j_2^p}$	0.137	0.097	0.114

Table F.2: Elasticities of Private-Sector Wages, European Countries

Variable	**Data** (1)	**Baseline** (2)	**Equating wages** (3)	**Equating wages and technology** (4)
	Panel A: United Kingdom			
Public employment shares				
Skilled	0.376	0.376	0.374	0.210
Unskilled	0.177	0.177	0.179	0.208
Ratio	2.118	2.118	2.091	1.012
Education intensity				
Public	0.537	0.537	0.534	0.357
Private	0.294	0.294	0.295	0.354
Ratio	1.828	1.828	1.811	1.009
Underemployment rate				
Total	0.149	0.149	0.149	0.190
Public*	0.189	0.152	0.149	0.191
Private*	0.137	0.149	0.149	0.190
	Panel B: France			
Public employment shares				
Skilled	0.283	0.283	0.275	0.197
Unskilled	0.180	0.180	0.185	0.197
Ratio	1.575	1.575	1.491	1.000
Education intensity				
Public	0.429	0.429	0.416	0.323
Private	0.295	0.295	0.298	0.323
Ratio	1.458	1.458	1.395	1.000
Underemployment rate				
Total	0.088	0.089	0.090	0.110
Public*	0.055	0.094	0.090	0.110
Private*	0.097	0.087	0.090	0.110
	Panel C: Spain			
Public employment shares				
Skilled	0.275	0.274	0.262	0.151
Unskilled	0.094	0.094	0.102	0.151
Ratio	2.913	2.925	2.565	1.001
Education intensity				
Public	0.630	0.631	0.600	0.369
Private	0.319	0.319	0.325	0.369
Ratio	1.977	1.978	1.848	1.001
Underemployment rate				
Total	0.124	0.124	0.125	0.164
Public*	0.199	0.154	0.126	0.164
Private*	0.114	0.121	0.125	0.164

** not calibrated*

Table F.3: Elasticities of Private-Sector Wages, European Countries

Variable	**Elasticity**	**Decomposition**		
		Shortage of skilled	*Excess unskilled*	*Underemployment*
		Panel A: United Kingdom		
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.206	0.319	0.089	-0.202
$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	-0.059	-0.092	-0.026	0.058
Elasticity of private wages w.r.t. skilled public wages				
$\frac{dw_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	-0.182	-0.319	-0.089	0.224
$\frac{dw_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.053	0.092	0.026	-0.065
Elasticity of private wages w.r.t. public wages				
$\frac{dw_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.023	0.000	0.000	0.022
$\frac{dw_2^p}{dw^g}\frac{w^g}{w_2^p}$	-0.007	0.000	0.000	-0.006
		Panel B: France		
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.144	0.169	0.069	-0.094
$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	-0.062	-0.073	-0.030	0.041
Elasticity of private wages w.r.t. skilled public wages				
$\frac{dw_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	-0.091	-0.169	-0.069	0.147
$\frac{dw_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.039	0.073	0.030	-0.064
Elasticity of private wages w.r.t. public wages				
$\frac{dw_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.053	0.000	0.000	0.053
$\frac{dw_2^p}{dw^g}\frac{w^g}{w_2^p}$	-0.023	0.000	0.000	-0.023
		Panel C: Spain		
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.094	0.127	0.047	-0.079
$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	-0.039	-0.053	-0.020	0.033
Elasticity of private wages w.r.t. skilled public wages				
$\frac{dw_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	-0.059	-0.127	-0.047	0.114
$\frac{dw_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.025	0.053	0.020	-0.048
Elasticity of private wages w.r.t. public wages				
$\frac{dw_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.036	0.000	0.000	0.035
$\frac{dw_2^p}{dw^g}\frac{w^g}{w_2^p}$	-0.015	0.000	0.000	-0.015

Note: the first column is calculated numerically, the decomposition is based on equations 16 and 17.

Quantitative results baseline model

Table F.4: Calibration, Baseline Model

Parameter	Value	Variable	Description	Model	Data
		Targeted			
α	0.483	$\frac{w_1^p}{w_2^p}$	College premium (private sector)	1.580	1.580
β	0.503	$j_1^g + u^g$	Public employment of college	0.097	0.097
$\bar{g}$	0.078	$j_2^g - u^g$	Public employment of no-college	0.062	0.062
n	0.432	n	Percentage of college workers	0.432	0.432
w_1^g	0.641	$\frac{w_1^g}{w_1^p}$	Public-sector wage premium (college)	1.010	1.010
w_2^g	0.431	$\frac{w_2^g}{w_2^p}$	Public-sector wage premium (college)	1.077	1.077
$\frac{\nu}{1-\tau}$	0.089	$\frac{u}{j_2^g+j_2^p}$	Underemployment rate (economy)	0.089	0.089
		Not Targeted			
		$\frac{u^g}{j_2^g}$	Underemployment rate (public)	0.340	0.102
		$\frac{u^p}{j_2^p}$	Underemployment rate (private)	0.043	0.087

Table F.5: Decomposition of Public-Sector Education Bias, Baseline Model

Variable	**Data** (1)	**Baseline** (2)	**Equating wages** (3)	**Equating wages and technology** (4)
Public employment shares				
Skilled	0.224	0.224	0.207	0.202
Unskilled	0.109	0.110	0.124	0.127
Ratio	2.054	2.034	1.671	1.593
Education intensity				
Public	0.610	0.607	0.560	0.548
Private	0.399	0.399	0.408	0.410
Ratio	1.530	1.523	1.373	1.336
Underemployment rate				
Total	0.089	0.088	0.085	0.087
Public*	0.102	0.340	0.272	0.273
Private*	0.087	0.043	0.050	0.052

* *not calibrated*

Table F.6: Elasticities of Private-Sector Wages, Baseline Model

Variable	**Elasticity**	**Decomposition**		
		Shortage of skilled	**Excess unskilled**	**Underemployment**
Elasticity of private wages w.r.t. unskilled public wages				
$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.211	0.053	0.047	0.111
$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	-0.196	-0.050	-0.043	-0.104
Elasticity of private wages w.r.t. skilled public wages				
$\frac{dw_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	-0.041	-0.053	-0.047	0.058
$\frac{dw_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.039	0.050	0.043	-0.054
Elasticity of private wages w.r.t. public wages				
$\frac{dw_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.169	0.000	0.000	0.169
$\frac{dw_2^p}{dw^g}\frac{w^g}{w_2^p}$	-0.158	0.000	0.000	-0.158

Note: the first column is calculated numerically, the decomposition is based on equations 16 and 17.

G Quantitative results for a more restricted definition of skilled

In our baseline quantitative results, the US economy was always in Regime 1 and far from regime two. However, one should not diminish the importance of modelling the different regimes when studying public employment. To highlight its importance, we do an alternative calibration where the educated workers are defined to have an MSc., Professional or PhD degree. These make up close to 10 percent of the employed population. Out of these, more than one third work in the public sector. These workers have a negative public sector wage premium of about 4 percent. In this particular calibration, we set the same value for $\frac{\nu}{1-\tau}$. Given the education premium for these workers, the model predicts very little underemployment.

In the baseline calibration the economy is in regime 2. This means that the government wage policy actually reduces the number of educated workers, so technology explains more than 100 percent of the education bias. The private-sector wage elasticity with respect to public skilled wages, have the opposite sign of the baseline case in Regime 1.

Table G.1: Calibration

Parameter	**Value**	**Variable**	**Description**	**Model**	**Data**
		Targeted			
α	0.113	$\frac{w_1^p}{w_2^p}$	College premium (private sector)	1.700	1.697
β	0.379	$j_1^g + u^g$	Public employment of PhD-MSc.-Professional	0.033	0.032
$\bar{g}$	0.075	$j_2^g - u^g$	Public employment of non PhD-MSc.-Professional	0.125	0.125
n	0.091	n	Percentage of PhD-MSc.-Professional workers	0.091	0.092
w_1^g	1.049	$\frac{w_1^g}{w_1^p}$	Public-sector wage premium (high-educated)	0.933	0.961
w_2^g	0.700	$\frac{w_2^g}{w_2^p}$	Public-sector wage premium (low-educated)	1.058	1.065
$\frac{\nu}{1-\tau}$	0.142		(kept from main calibration)		
		Not Targeted			
		$\frac{u}{j_2^g+j_2^p}$	Underemployment rate (economy)	0.0023	-
		$\frac{u^g}{j_2^g}$	Underemployment rate (public)	0.0029	-
		$\frac{u^p}{j_2^p}$	Underemployment rate (private)	0.0022	-

Table G.2: Decomposition of Public-Sector Education Bias, at the top

Public employment shares				
Skilled	0.346	0.346	0.360	0.108
Unskilled	0.140	0.140	0.136	0.107
Ratio	2.478	2.478	2.651	1.008
Education intensity				
Public	0.200	0.200	0.211	0.092
Private	0.071	0.071	0.069	0.091
Ratio	2.809	2.809	3.037	1.008
Underemployment rate				
Total		0.0023	0.002	0.010
Public		0.0029	0.002	0.010
Private		0.0022	0.002	0.010

* *not calibrated*

Table G.3: Elasticities of Private-Sector Wages, Alternative definition of educated

Variable	**Elasticity**	
	Baseline model	Alternative definition of educated
Elasticity of private wages w.r.t. unskilled public wages		
$\frac{dw_1^p}{dw_2^g}\frac{w_2^g}{w_1^p}$	0.074	0.005
$\frac{dw_2^p}{dw_2^g}\frac{w_2^g}{w_2^p}$	-0.061	-0.001
Elasticity of private wages w.r.t. skilled public wages		
$\frac{dw_1^p}{dw_1^g}\frac{w_1^g}{w_1^p}$	-0.046	0.657
$\frac{dw_2^p}{dw_1^g}\frac{w_1^g}{w_2^p}$	0.038	-0.084
Elasticity of private wages w.r.t. public wages		
$\frac{dw_1^p}{dw^g}\frac{w^g}{w_1^p}$	0.029	0.662
$\frac{dw_2^p}{dw^g}\frac{w^g}{w_2^p}$	-0.023	-0.084

Note.

Figure G.1: Regimes as a function of public-sector wage schedule

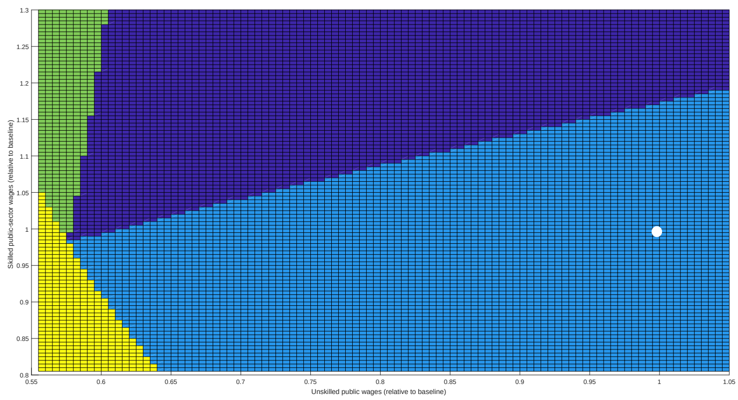


Figure G.2: Effects of public-sector wages for skilled jobs

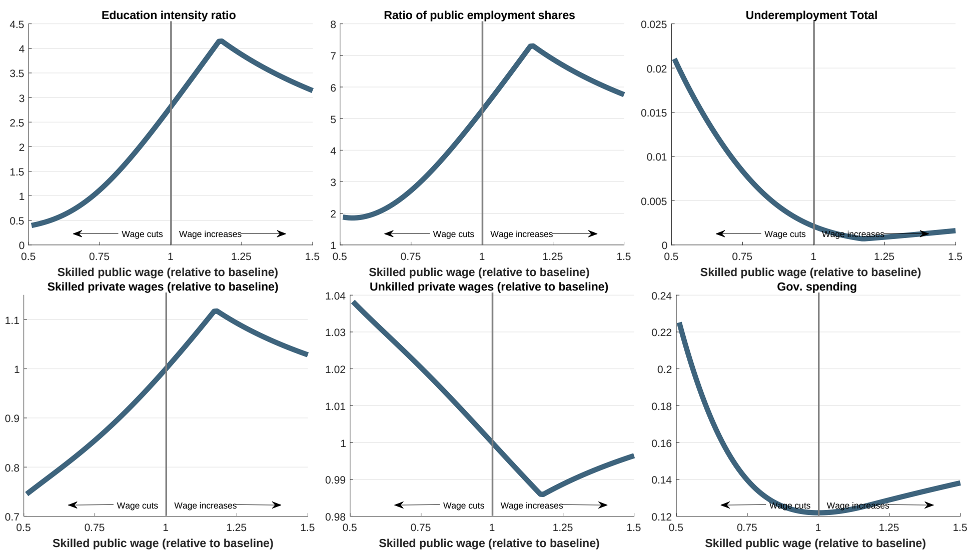


Figure G.3: Effects of public-sector wages for unskill jobs