

Foreign Shocks as Granular Fluctuations*

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Abstract

This paper uses a dataset covering the universe of French firm-level sales, imports, and exports over the period 1993-2007, and a quantitative multi-country model to study the international transmission of business cycle shocks at both the micro and the macro levels. The largest firms are both important enough to generate aggregate fluctuations (Gabaix, 2011), and most likely to be internationally connected. This implies that the largest firms are the key channel through which foreign shocks are transmitted to France. We first document a novel stylized fact: larger French firms are significantly more sensitive to foreign GDP growth. Our quantitative framework is calibrated to the observed firm- and country-level trade data, capturing the full extent of firm-level heterogeneity in firm size, exporting, and importing. We simulate the propagation of foreign shocks to the French economy. “Granular” firms are quantitatively important in transmitting the foreign shocks, due to the combination of their import and export linkages with foreign countries and their large size relative to the overall French economy. As much as two-thirds of the impact of a foreign shock on French GDP is accounted for by the covariance between firm size and firm sensitivity to the foreign shock.

JEL Classifications: E32; F15; F23; F44; F62; L14

Keywords: Shock transmission; Input linkages; International trade; Aggregate fluctuations

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1 Introduction

This paper studies the international transmission of business cycle shocks at the firm and the aggregate levels. After decades of globalization, the structure of production is increasingly international, with supply chains overlapping with country borders. An important feature of this internationalization of production is granularity: the largest firms are the ones responsible for the bulk of international trade linkages in a typical economy (e.g., Freund and Pierola, 2015). As a result, while only a minority of firms have direct trade linkages with foreign countries, those firms tend to account for a large share of aggregate economic activity (di Giovanni et al., 2017, 2018).

We study the consequences of this phenomenon for international shock transmission. Our main hypothesis is that foreign shocks, even if they are purely aggregate, affect firms differentially depending on the extent and nature of their international linkages. In that sense, an aggregate shock to a country's trading partners manifests itself as a set of idiosyncratic shocks to individual firms. The following simple expression conveys the role of this heterogeneity in aggregate outcomes. Let ϵ^Y be the elasticity of GDP to a foreign shock, and let the economy be composed of a number of firms indexed by f . Then, ϵ^Y can be written as:

$$\epsilon^Y = \bar{\epsilon} + Cov\left(\frac{\omega_f}{\bar{\omega}}, \epsilon^f\right), \quad (1)$$

where ϵ^f is the elasticity of firm f 's value added with respect to that same foreign shock, $\bar{\epsilon}$ is the unweighted average of ϵ^f across firms, and $\omega_f/\bar{\omega}$ is the share of firm f in aggregate value added relative to its unweighted average.

The response of GDP to a foreign shock is the sum of the average response of all firms to that shock, and the covariance across firms between sensitivity to that shock and relative size. In a model environment with a representative firm, the entire impact is captured by the first term, $\bar{\epsilon}$. When firms are heterogeneous in both size (ω_f) and sensitivity to foreign shocks (ϵ^f), then part of the impact of a foreign disturbance on GDP is due to the covariance term. We would expect this term to be positive, as large firms are more internationally connected, and thus disproportionately more affected by foreign shocks. Because the foreign shocks affect predominantly the largest firms in France, they lead to aggregate – granular – fluctuations.

Our analysis combines a dataset covering the universe of French firm sales and country-specific imports and exports over the period 1993-2007 with a quantitative multi-country multi-sector model with heterogeneous firms. We begin by documenting a novel stylized fact: larger French firms are significantly more sensitive to foreign GDP growth. This pattern is not driven by differences in the overall procyclicality, as larger firms are not differentially more sensitive to the domestic GDP growth. Though the regression is heuristic, it is *prima facie* evidence that larger firms are more correlated with foreign GDP, supporting the conjecture that the covariance term $Cov(\omega_f/\bar{\omega}, \epsilon^f)$ is likely positive. We also document that in the data (i) there is a great deal of heterogeneity in both

import and export participation among French firms; and (ii) larger firms are systematically more likely to trade internationally, consistent with much of the previous literature.

The econometric estimates do not lend themselves well to aggregation, as they yield the relative impact of foreign GDP growth across firms, but not the overall impact. That is, the regression evidence relates the variation in ϵ^f to firm size, but does not pin down either the level of individual ϵ^f 's, nor their average $\bar{\epsilon}$. Thus, we employ the quantitative framework to simulate the effects of foreign shocks on the French economy. The model is calibrated to the observed firm-level information for France, and to the sector-level information for France's trading partners from the World Input-Output Database (WIOD). The model is general-equilibrium, and thus takes into account all the changes in wages, prices, and market shares in France and the rest of the world. As a result, this quantitative framework not only allows us to simulate the impact of a foreign shock on French GDP, but also to compute all the components of (1) and thus assess the role of granularity in the transmission of foreign shocks. Most importantly, since it is implemented on the complete data on firm imports, exports, and size, the model captures the full extent of heterogeneity across French firms in international linkages, as well as any relationship between those linkages and overall firm size. Thus, it is the right environment to quantify the impact of the $Cov(\omega_f/\bar{\omega}, \epsilon^f)$ term on aggregate outcomes.

The transmission mechanisms in the model are standard. Following a positive foreign productivity shock, firms importing foreign inputs experience a fall in the prices of those inputs, and thus expand production. Following an increase in foreign demand (which could be due to a foreign productivity shock or a foreign demand shock), exporting firms increase their foreign sales. External shocks are transmitted inside the French economy via domestic input-output linkages and general equilibrium effects on the domestic goods and factor prices. Thus, even purely domestic firms in France are in principle affected by foreign shocks.

We simulate 2 types of foreign shocks: a 10% productivity shock, and a 10% foreign demand shock for French goods. We examine both a global shock to all the countries other than France, and a shock to Germany, one of France's most important partners. We express results directly in terms of elasticities. The elasticity of French real GDP with respect to the global productivity shock is 0.44 in our baseline calibration: a 10% improvement in global productivity increases French GDP by 4.4%. The impact of a German shock is predictably smaller, with an aggregate elasticity of 0.06. The elasticities of French GDP to a foreign demand shock are an order of magnitude smaller, which is expected since unlike foreign productivity, the foreign demand shock does not lower the costs of production in France.

Most importantly, the $Cov(\omega_f/\bar{\omega}, \epsilon^f)$ term accounts for 63% of the overall aggregate elasticity for the productivity counterfactual, and 39% in the demand shock counterfactual. We perform two alternative exercises to establish that this quantitatively important covariance term is a consequence

of firm heterogeneity in international linkages. In the first, we simulate the economy’s response to the same shocks in a model with homogeneous firms in each sector. That is, we assign to each firm within a sector the exact same export and import linkages. The share accounted for by the covariance term falls to zero for both shocks. We also compute the covariance term at the sector instead of the firm level in the baseline model. Following a productivity shock, the covariance term across sectors accounts for only 14% of the total effect, whereas for the foreign demand shock the covariance term is negative, implying that the largest sectors actually have lower elasticities with respect to foreign demand shocks. Both of these alternative exercises illustrate that it is the firm, rather than sectoral, heterogeneity in the international linkages that matters.

To judge the quantitative importance of the granular transmission channel for French GDP fluctuations, we simulate the response of the economy to actual foreign productivity shocks, sourced from the Penn World Table. Foreign TFP shocks can account for one-quarter to one-third of the actual GDP fluctuations in France. More importantly for us, nearly 70% of the fluctuations in French GDP due to foreign shocks are due to the granular component. All in all, both of our quantitative exercises show that foreign shocks manifest themselves as largely granular fluctuations.

The paper draws on the active closed-economy literature on the propagation of shocks in production networks (Carvalho, 2010; Acemoglu et al., 2012; Barrot and Sauvagnat, 2016; Baqaee, 2016; Carvalho et al., 2016; Atalay, 2017; Tintelnot et al., 2017), and the importance of large firms in aggregate fluctuations (Gabaix, 2011; di Giovanni et al., 2014; Carvalho and Grassi, 2018). We apply the insights and tools from this literature to the international transmission of shocks. The international business cycle literature is vast, but by and large has not used firm-level data in empirical and quantitative assessments of international comovement. The few recent exceptions include Kleinert et al. (2015), Boehm et al. (2017), Cravino and Levchenko (2017), Blaum et al. (2016), Blaum (2018), and di Giovanni et al. (2018). Ghironi and Melitz (2005) and Alessandria and Choi (2007) provide quantitative assessments of the transmission of aggregate shocks using international real business cycle models with heterogeneous firms. These papers explore the role of the extensive margin whereas we focus on the intensive margin in the context of heterogeneous export and import participation.¹

The rest of the paper is organized as follows. After describing the data, Section 2 presents the basic facts that relate firm size to the sensitivity to the foreign business cycle and foreign market participation. Section 3 presents a multi-country general equilibrium model of trade, featuring firm heterogeneity and input-output linkages, as well as details of parameter estimation and calibration. Section 4 quantifies the importance of the cross-border transmission of shocks at the micro and macro levels. Section 5 concludes.

¹The intensive margin is quantitatively more important for aggregate fluctuations and cross-border business cycle comovement in a granular world (di Giovanni et al., 2014, 2018).

2 Data and Basic Facts

We combine administrative data on the universe of French firms’ value added, imports, and exports with standard multi-country sector-level databases of production, trade, and producer prices. The use of micro data for one country allows us to capture the heterogeneous exposure of individual firms to foreign shocks. While such heterogeneity obviously exists in all countries, firm-level information at level of detail is not available for multiple countries at once. As a consequence, we will study the impact of firm heterogeneity using the French firm-level data, suppressing heterogeneity within sectors in the rest of the country sample.

2.1 Firm-Level Variables

We make use of an administrative dataset that contains balance sheet information collected from individual firms’ tax forms, and includes sales, value added, total exports, the cost structure, as well as its sector of activity for the universe of French firms over 1993-2007. This dataset is complemented with customs data on bilateral export and import flows at the firm level. The resulting dataset is described in greater detail in [di Giovanni et al. \(2018\)](#). [Table A1](#) reports the distribution of firms across sectors in 2005. Interestingly, the largest sector in terms of its contribution to aggregate value added is the one providing “Business Activities” to the rest of the economy. This underscores how important input-output relationships are to the functioning of modern economies. More generally, non-traded good sectors are a large share of the French economy, accounting for more than 80% of firms and 72% of the value added in our sample. The comparison of these two numbers indicates that non-traded sector firms tend to be relatively small. There are some exceptions, however. For instance, firms in the “Post and Telecommunications” or the “Air Transport” sectors are relatively large.

The customs data for imports and exports do not include trade in services. However, goods trade by the service sector firms is observed. Export data can be used to refine the definition of sales to the level of destination market ($X_{mn,j,t}(f)$ for $m = France$ in the notation below). Following [di Giovanni et al. \(2014\)](#) this is done by first allocating sales to the domestic or foreign market using the information available in the tax files on domestic and export sales. The foreign component of demand is then further decomposed by destination using the customs data.

2.2 Aggregate and Sectoral Variables

The main source of data at the multilateral, sectoral level is the World Input Output Database (WIOD) ([Timmer et al., 2015](#)). This dataset combines national input-output tables and data on bilateral trade flows to build the matrix of all intra- and international flows of goods and services between sectors and final consumers. We use the 2013 release of the dataset which covers 40

countries plus a rest of the world aggregate and 35 sectors classified according to the ISIC Revision 3 nomenclature. These data are available over 1995 to 2011 and the benchmark year for the calibration below is 2005.

When describing the variables in this section, we anticipate the notation used in the quantitative framework (Section 3) throughout. The WIOD dataset can be used to recover: i) final consumption spending ($P_{n,t}C_{n,t}$); ii) the value of bilateral sales by sector ($X_{mn,j,t}$); and iii) the sectoral production function parameters, which are used whenever more disaggregated data are not available. We use these data to measure the share of labor in country n sector j 's total costs ($\pi_{n,j}^l$) as well as the components of the input-output matrix, as measured by the share of inputs sourced from country m sector j by firms operating in country n sector i ($\pi_{mn,ji}^x$). The IO coefficients are readily available from the WIOD. Labor shares are measured by the ratio of value added over output, to be consistent with the interpretation of L as “equipped labor.”

The French administrative data and the WIOD data must be made consistent with each other, as the final dataset must feature firm-level trade flows that aggregate up to the sector-level bilateral trade flows reported in WIOD. In addition, shares of value added in total output implied by the French data must match those implied by WIOD for France. Appendix A.1 describes in detail the harmonization procedure.

2.3 Basic Facts

Fact 1: Larger firms are more sensitive to foreign GDP growth We establish this stylized fact by means the following heuristic regression:

$$d \ln Y_{m,j,t+1}(f) = \alpha d \ln Y_{W,t} + \beta \ln Y_{m,j,t}(f) \times d \ln Y_{W,t} + \gamma \ln Y_{m,j,t}(f) + \delta_{jt} + \epsilon_{ft}, \quad (2)$$

where $d \ln Y_{m,j,t+1}(f)$ is the log change in firm value added, $\ln Y_{m,j,t}(f)$ is its initial log level, $d \ln Y_{W,t}$ is the GDP growth in the world outside of France, and δ_{jt} is the sector-time effect. The coefficient of interest β captures whether firms of different sizes have differential elasticity of value added growth with respect to foreign GDP.

Table 1 reports the results. The first column presents estimates of (2) without any fixed effects. Column 2 adds year effects, which implies that we can no longer estimate the main effect of foreign GDP growth. Columns 3-4 include interacted sector-year effects, implying that the coefficient of interest is estimated from the variation across firms within a sector along the size dimension. The coefficient of interest is strongly positive and significant: larger firms are more sensitive to foreign growth. The coefficient is stable across specifications, falling only modestly when sector-year effects are added. It is sizeable in magnitude, implying that a doubling of firm size increases the elasticity of firm growth to world GDP growth by about 0.12.

Next, we check whether larger firms are more sensitive to the foreign business cycle specifically,

Table 1. Sensitivity to Foreign GDP Growth by Firm Size

	(1)	(2)	(3)	(4)	(5)	(6)
					Model	
Dep. Var.: $d \ln Y_{m,j,t+1}(f)$					World	World
					Prod.	Prof.
					Shock	Shock
	Data					
$\ln Y_{m,j,t}(f) \times d \ln Y_{W,t}$	0.175 ^a (0.017)	0.173 ^a (0.017)	0.105 ^a (0.018)	0.118 ^a (0.019)	0.020 ^a (0.0001)	0.333 ^a (0.001)
$\ln Y_{m,j,t}(f)$	-0.024 ^a (0.001)	-0.024 ^a (0.001)	-0.025 ^a (0.001)	-0.025 ^a (0.001)		
$d \ln Y_{W,t}$	-1.025 ^a (0.105)					
$\ln Y_{m,j,t}(f) \times d \ln Y_{FRA,t}$				-0.030 ^b (0.014)		
Observations	3,632,281	3,632,281	3,632,281	3,632,281	385,926	385,926
# years	11	11	11	11	1	1
# firms	655,596	655,596	655,596	655,596	385,926	385,926
Adjusted R^2	0.009	0.013	0.020	0.020	0.444	0.432
Fixed Effects	–	Year	Sector \times Year	Sector \times Year	Sector	Sector

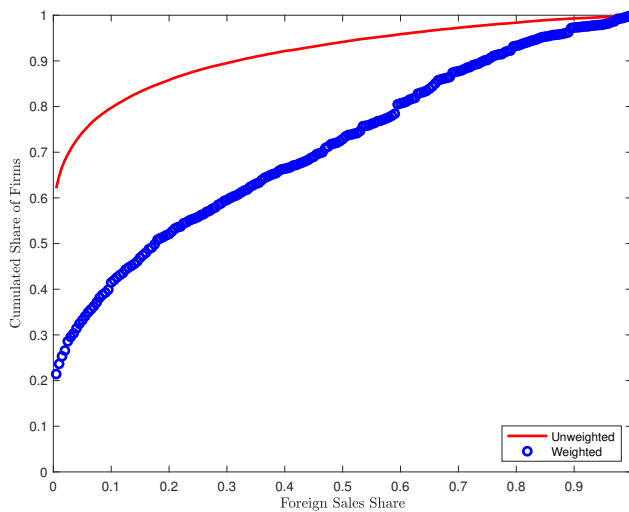
Notes: This table reports the estimates of [Equation \(2\)](#). Standard errors clustered at the firm level in parentheses with ^a, ^b and ^c denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively. $d \ln Y_{FRA,t}$ denotes French GDP growth.

or just more procyclical. Column 4 adds an interaction between firm size and French GDP growth. It is clear that larger firms are more sensitive to foreign growth specifically: the interaction term of firm size with respect to the domestic GDP growth is actually negative, but close to zero. The elasticity with respect to foreign growth is almost the same whether we control for the domestic growth interaction term or not.

Fact 2: Larger firms are more likely to both export and import [Figure 1](#) plots the cumulative distribution function of firm-level share of exports in total sales. Similarly, [Figure 2](#) plots the distribution of the intensity of imported input use, summarized by the share of foreign inputs in firms' total input expenditure ($\sum_{n \neq m} \sum_{i \in T} \pi_{mn,ij}^x(f)$). In both figures, the solid (red) line depicts the unweighted distribution and the (blue) circles the distribution weighted by the firms' share in overall value added.

We stress two features of these figures, both of which are known in the trade literature and are confirmed in our data. First, there is a great deal of heterogeneity across firms in both export intensity and imported input use. Overall, 58% of the firms producing tradable goods do not export in our data. Among the firms that do export, many have sales that are strongly biased towards

Figure 1. Distribution of Export Intensity Across French Firms



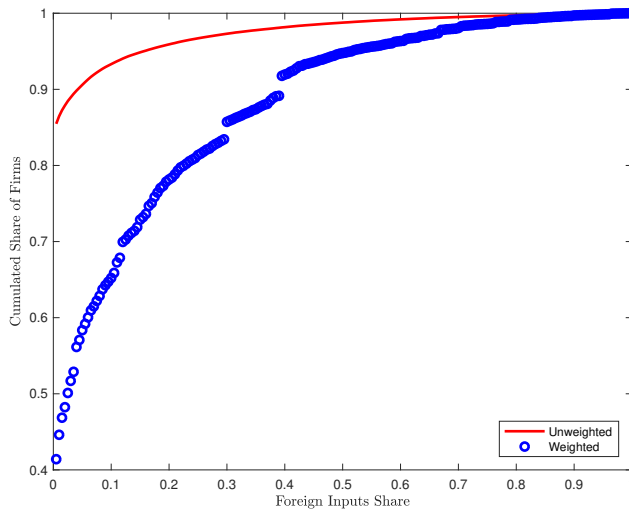
Notes: This figure plots the cumulative distribution of firms according to their degree of openness, defined by the share of their sales coming from a foreign market. The solid (red) line corresponds to the unweighted distribution and the (blue) circles to the weighted distribution, where firms' weights are defined according to their share in aggregate value added. The figure is restricted to firms in traded good sectors. Source: French customs and balance-sheet data, for 2005.

the domestic market. Still, about 6% of firms have export/total sales shares above 50%, and thus quite exposed to foreign demand shocks. Similarly, more than 85% of firms source the entirety of their inputs locally, thus isolating themselves from (direct) foreign input price shocks. At the other end of the spectrum, about 2% of firms source more than 40% of their inputs from abroad.

Second, participation in foreign markets is heavily tilted towards larger firms. This is illustrated in Figures 1 and 2 by the comparison between the weighted and unweighted distributions. In both cases, the cdfs of the weighted distributions are substantially below the unweighted ones, meaning that on average larger firms have higher export and import intensities. For instance, the 6% of firms making more than 50% of their turnover abroad represent as much as 30% of the overall value added in tradable sectors. On the import side, the 15% of firms that source some inputs from abroad account for nearly 60% of aggregate value added, and firms sourcing more than 40% of their inputs abroad account for 10% of aggregate value added. In unreported results, we checked that the heterogeneity is not driven by cross-sector differences in overall exposure. While non-traded good sectors tend to be relatively less dependent on foreign inputs, most of the heterogeneity is actually driven by the within-sector variation.

The patterns illustrated in Facts 1 and 2 have a natural connection: the import and export linkages to foreign countries make the larger firms respond more to foreign shocks. Our earlier work provides reduced-form evidence linking firm-level trade directly to greater responsiveness to foreign

Figure 2. Distribution in Imported Input Use Intensity Across French Firms



Notes: This figure plots the cumulative distribution of firms according to their degree of exposure to foreign input price shocks, as defined by the share of inputs coming from other countries. The solid (red) line corresponds to the unweighted distribution and the (blue) circles to the weighted distribution, where firms’ weights are defined according to their share in aggregate value added. Source: French customs and balance-sheet data, for 2005.

shocks. [Di Giovanni et al. \(2014\)](#) shows that firms exporting to foreign countries are subject to demand shocks from those countries. [Di Giovanni et al. \(2018\)](#) provides econometric evidence that firms importing from, and exporting to, a foreign country are more correlated with GDP growth in that country. The quantitative framework in the following section models these linkages formally and simulates the economy’s response to foreign shocks in an environment with firms heterogeneous in both size and trade linkages.

3 Quantitative Framework

This section builds a heterogeneous-firm, multi-country, multi-sector model of trade. Crucially, we allow for heterogeneity of input linkages at the firm level, as well as heterogeneity across export markets.² The model features endogenous factor supply so that we can analyze how domestic and foreign shocks are transmitted to aggregate fluctuations.

The world is comprised of M countries and J sectors. Time is denoted by t , countries by m , n , and k , sectors by i and j , and firms by f and g . The notation follows the convention that the first subscript always denotes exporting (source) country, and the second subscript the importing (destination) country.

²We only have firm-level data for France, and thus for the other countries the model collapses to an international trade model with sector-level input-output linkages that is standard in the literature (see, e.g. the Handbook chapter by [Costinot and Rodríguez-Clare, 2014](#)).

Households There are \bar{L}_n households in country n . Each one consumes goods and supplies labor. Their income includes profits of domestically-owned firms. Preferences over consumption and leisure are GHH (Greenwood et al., 1988):

$$U(\{c_{n,t}, l_{n,t}\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \delta^t \nu \left(c_{n,t} - \frac{\psi_0}{\psi} l_{n,t}^{\bar{\psi}} \right),$$

where $c_{n,t}$ is per-capita consumption, $l_{n,t}$ the per-capita labor supply, and the function ν is increasing and concave. Note that the $l_{n,t}$ should be thought of as “equipped labor” (Alvarez and Lucas, 2007), and thus captures the supply of all the primary factors.

The final consumption aggregate is Cobb-Douglas in the j sectors, with expenditure shares ϑ_j :

$$c_{n,t} = \prod_j c_{n,j,t}^{\vartheta_j},$$

where $c_{n,j,t}$ is the per capita final consumption of sector j . Therefore, the ideal consumption price index is:

$$P_{n,t} = \prod_j \left(\frac{P_{n,j,t}}{\vartheta_j} \right)^{\vartheta_j},$$

where $P_{n,j,t}$ is the price index of sector j goods in country n at time t .

Denote by $\Pi_{n,t}$ the aggregate profits of firms owned by households in n , and by $D_{n,t}$ any trade imbalance in period t . Assume that both $\Pi_{n,t}$ and $D_{n,t}$ are divided equally among households in n (as will become clear below, under the GHH and homothetic preferences, this assumption affects neither the labor supply decision nor the allocation of consumption expenditure across j). Straightforward steps lead to the following labor supply:

$$L_{n,t} = \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\bar{\psi}-1}} \bar{L}_n,$$

where $w_{n,t}$ is the wage in country n at time t .

Denote by $C_{n,t} \equiv c_{n,t} \bar{L}_n$ the aggregate final consumption in country n , and let $C_{n,j,t} \equiv c_{n,j,t} \bar{L}_n$ be the aggregate final consumption of sector j . Countries m sell (export) to country n . Origin-specific output is apportioned to consumption and intermediate input usage. Let each sector’s consumption be aggregated from origin-specific components:

$$C_{n,j,t} = \left[\sum_m \mu_{mn,j}^{\frac{1}{\sigma_j}} C_{mn,j,t}^{\frac{\sigma_j-1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j-1}},$$

where $C_{mn,j,t}$ is final consumption of imports from country m in sector j , country n . Then the price index for consumption in sector j , country n is:

$$P_{n,j,t} = \left[\sum_m \mu_{mn,j} P_{mn,j,t}^{1-\sigma_j} \right]^{\frac{1}{1-\sigma_j}},$$

where $P_{mn,j,t}$ is the price index for exports from m to n in sector j , defined below. Final demand for goods from m is:

$$P_{mn,j,t}C_{mn,j,t} = \frac{\mu_{mn,j}P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}}P_{n,j,t}C_{n,j,t} = \frac{\mu_{mn,j}P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}}\vartheta_j P_{n,t}C_{n,t}.$$

Then:

$$P_{mn,j,t}C_{mn,j,t} = \frac{\mu_{mn,j}P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}}\vartheta_j \left[w_{n,t} \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_{n,t} + D_{n,t} \right].$$

Note that we use the French customs data for imports at the firm level, and thus every import transaction is associated with a French firm (which may be wholesaler or a retailer). Thus, French final consumers are never observed to import final consumption goods directly, and as a result French final consumption is composed only of domestically-supplied final goods. Formally, when $n = France$, $\mu_{mn,j} = 0 \forall m \neq n$, and:

$$P_{n,j,t} = P_{nn,j,t},$$

$$P_{nn,j,t}C_{nn,j,t} = P_{n,j,t}C_{n,j,t} = \vartheta_j \left[w_{n,t} \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_{n,t} + D_{n,t} \right],$$

where $P_{nn,j,t}$ is the ideal price index of output produced by French firms in France. For all the other countries, we do not have firm-level data on imports, but instead have final consumption data by source country from WIOD. Thus, we assume that foreign consumers import final goods directly.

Sectors Sectors are populated by heterogeneous, monopolistically-competitive firms. Not all firms sell to all destinations. Denote by $\Omega_{mn,j}$ the set of firms from country m , sector j that sell to country n . The CES aggregate of output in sector j of firms from m selling in country n is:

$$Q_{mn,j,t} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{mn,j,t}(f)^{\frac{1}{\rho_j}} Q_{mn,j,t}(f)^{\frac{\rho_j-1}{\rho_j}} \right]^{\frac{\rho_j}{\rho_j-1}}, \quad (3)$$

where $Q_{mn,j,t}(f)$ is the quantity of firm f 's good from country m and sector j selling to country n .³ The taste shock to a firm's destination-specific sales $\xi_{mn,j,t}(f)$ is at this point left unrestricted. It could be allowed to have a firm-specific global component, and/or a source-destination-sector common component across firms. The latter would be isomorphic to $\mu_{mn,j}$ in the cross-section. The price level of the aggregate of sellers from m in n, j, t is:

$$P_{mn,j,t} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{mn,j,t}(f) p_{mn,j,t}(f)^{1-\rho_j} \right]^{\frac{1}{1-\rho_j}},$$

³In the counterfactual experiments below, we assume that following a foreign shock, the sets of firms serving each market $\Omega_{mn,j}$ are unchanged. See di Giovanni et al. (2014, 2018) for evidence that the extensive margin adjustments are not quantitatively important at the business cycle frequency.

where $p_{mn,j,t}(f)$ is the price charged by firm f in country n .

Let X denote expenditure (at each level of aggregation). Then demand faced by firm f in country n is:

$$X_{mn,j,t}(f) = \xi_{mn,j,t}(f) \frac{p_{mn,j,t}(f)^{1-\rho_j}}{P_{mn,j,t}^{1-\rho_j}} X_{mn,j,t}.$$

Thus, $X_{mn,j,t}$ is the total value exports from m to n in sector j at t , and $X_{mn,j,t}(f)$ is the value of exports by firm f .

Firms Firms face downward-sloping demand and set price equal to a constant markup $\frac{\rho_j}{\rho_j-1}$ over the marginal cost. Firms located in m face an iceberg cost of $\tau_{mn,j}$ to export to n . They have a unit input requirement $a_t(f)$, and the cost of the input bundle

$$b_{m,j,t}(f) = \left[\alpha_{m,j}(f) w_{m,t}^{1-\lambda} + (1 - \alpha_{m,j}(f)) (P_{m,j,t}^M(f))^{1-\lambda} \right]^{\frac{1}{1-\lambda}},$$

where $\alpha_{m,j}(f)$ is a firm-specific parameter governing the firm's labor share. The cost of intermediate inputs $P_{m,j,t}^M(f)$ is firm-specific, and given by:

$$P_{m,j,t}^M(f) = \left[\sum_i \sum_k \gamma_{km,ij}(f) P_{km,it}^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

where $\gamma_{km,ij}(f)$ is the parameter governing the use of inputs sourced from country k sector i by firm f operating in country m , sector j . That is, firms in m use inputs from potentially all countries k in each sector i , with firm-specific taste shifters $\gamma_{km,ij}(f)$. Some of these will be zero, i.e. the firm does not use inputs in a particular sector from a particular country. For French firms, $\gamma_{km,ij}(f)$ will be disciplined by the data for imported inputs while the domestic input-output linkages are inferred using firm-level data on input usage and sector-level information on domestic IO linkages – see [Section 2](#) for details. Sales by firm f from country m in destination n are then

$$X_{mn,j,t}(f) = \xi_{mn,j,t}(f) \frac{\left(\frac{\rho_j}{\rho_j-1} \tau_{mn,j} b_{m,j,t}(f) a_t(f) \right)^{1-\rho_j}}{P_{mn,j,t}^{1-\rho_j}} X_{mn,j,t}.$$

Heterogeneity in firm size is thus driven by productivity, taste/quality, and differences in input sourcing across firms. To illustrate, the share of firm f 's sales in total sales by domestic firms to the home market in sector j is:

$$\pi_{mm,j,t}(f) = \frac{\xi_{mm,j,t}(f) a_t(f)^{1-\rho_j} \left[\alpha_{m,j}(f) w_{m,t}^{1-\lambda} + (1 - \alpha_{m,j}(f)) (P_{m,j,t}^M(f))^{1-\lambda} \right]^{\frac{1-\rho_j}{1-\lambda}}}{\sum_{g \in \Omega_{mm,j}} \xi_{mm,j,t}(g) a_t(g)^{1-\rho_j} \left[\alpha_{m,j}(g) w_{m,t}^{1-\lambda} + (1 - \alpha_{m,j}(g)) (P_{m,j,t}^M(g))^{1-\lambda} \right]^{\frac{1-\rho_j}{1-\lambda}}}.$$

Sales dispersion across firms in the same market is generated by differences in productivity $a_t(f)$, the taste shifter $\xi_{mn,j,t}(f)$, and the fact that sourcing shares $\gamma_{km,ij}(f)$ differ across firms (even though we assume that all firms face the same input prices $P_{km,i,t}$). As will become clear below, we will not need to take a stand on the levels of $a_t(f)$ and $\xi_{mn,j,t}(f)$. Instead the counterfactual exercises will use the observed shares such as $\pi_{mn,j,t}(f)$ directly to calibrate the model at the baseline period and then use the equilibrium conditions to compute the changes in those $\pi_{mn,j,t}(f)$'s between the baseline and the counterfactual equilibrium.

Equilibrium Market clearing for exports from m to n in sector j is:

$$X_{mn,j,t} = \frac{\mu_{mn,j} P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}} \vartheta_j \left[w_{n,t} \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_{n,t} + D_{n,t} \right] \quad (4)$$

$$+ \sum_i \sum_{f \in i} \frac{\rho_i - 1}{\rho_i} (1 - \pi_{n,i,t}^l(f)) \pi_{mn,ji,t}^M(f) \sum_k \frac{\xi_{nk,i,t}(f) \left(\frac{\rho_i}{\rho_i - 1} \tau_{nk,i} b_{n,i,t}(f) a_t(f) \right)^{1-\rho_i}}{P_{nk,i,t}^{1-\rho_i}} X_{nk,i,t},$$

where $\pi_{m,j,t}^l(f)$ and $\pi_{km,ij,t}^M(f)$ are firm f 's expenditure shares on labor and input from sector i , country k , respectively:

$$\pi_{m,j,t}^l(f) = \frac{\alpha_{m,j}(f) w_{m,t}^{1-\lambda}}{\alpha_{m,j}(f) w_{m,t}^{1-\lambda} + (1 - \alpha_{m,j}(f)) \left(P_{m,j,t}^M(f) \right)^{1-\lambda}}$$

$$\pi_{km,ij,t}^M(f) = \frac{\gamma_{km,ij}(f) P_{km,i,t}^{1-\eta}}{\sum_i \sum_n \gamma_{nm,ij}(f) P_{nm,i,t}^{1-\eta}}.$$

In Equation (4), the first line is the final demand, and the second is the intermediate demand. Note that the intermediate demand is a summation of firm-level intermediate demands, and thus captures the notion that not all firms, even within the same sector, will import inputs from a particular foreign sector-country with the same intensity. The price indices are:

$$P_{mn,j,t} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{mn,j,t}(f) \left(\frac{\rho_j}{\rho_j - 1} \tau_{mn,j} b_{m,j,t}(f) a_t(f) \right)^{1-\rho_j} \right]^{\frac{1}{1-\rho_j}}. \quad (5)$$

Total labor compensation in the sector is the sum of firm-level expenditures on labor:

$$w_{n,t} L_{n,j,t} = \frac{\rho_j - 1}{\rho_j} \sum_{f \in j} \alpha_{n,j}(f) \sum_k X_{nk,j,t}(f)$$

$$= \frac{\rho_j - 1}{\rho_j} \sum_{f \in j} \alpha_{n,j}(f) \sum_k \frac{\xi_{nk,j,t}(f) \left(\frac{\rho_j}{\rho_j - 1} \tau_{nk,j} b_{n,j,t}(f) a_t(f) \right)^{1-\rho_j}}{P_{nk,j,t}^{1-\rho_j}} X_{nk,j,t}$$

Labor market clearing ensures that real wages adjust to equate the aggregate labor demand (right-hand side) with labor supply:

$$\begin{aligned} \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}}\right)^{\frac{1}{\psi-1}} \bar{L}_n &= \sum_j L_{n,j,t} \\ &= \frac{1}{w_{n,t}} \sum_j \frac{\rho_j - 1}{\rho_j} \sum_{f \in j} \pi_{n,i,t}^l(f) \sum_k \frac{\xi_{nk,j,t}(f) \left(\frac{\rho_j}{\rho_j - 1} \tau_{nk,j} b_{n,j,t}(f) a_t(f)\right)^{1-\rho_j}}{P_{nk,j,t}^{1-\rho_j}} X_{nk,j,t}. \end{aligned} \quad (6)$$

Equations (4), (5), and (6) are a system of equations that define equilibrium wages, prices, and expenditures.

3.1 The Role of Heterogeneity

Let $Y_{m,t}$ denote aggregate GDP in country m , and let $Y_{m,t}(f)$ denote the value added of firm f . We are interested in evaluating the elasticity of GDP with respect to a foreign shock χ . Define $\epsilon^Y \equiv \frac{d \ln Y_{m,t}}{d \ln \chi}$ to be the elasticity of m 's GDP with respect to the shock, $\epsilon^f \equiv \frac{d \ln Y_{m,t}(f)}{d \ln \chi}$ the elasticity of the value added of firm f with respect to the shock, and $\omega_{m,t}(f) \equiv \frac{Y_{m,t}(f)}{Y_{m,t}}$ the share of firm f in total value added. GDP is just the sum of firm-level value added:

$$Y_{m,t} = \sum_f Y_{m,t}(f).$$

Therefore, the aggregate elasticity with respect to the foreign shock is a weighted sum of firm-level elasticities:

$$\epsilon^Y = \sum_f \omega_{m,t}(f) \epsilon^f.$$

The aggregate elasticity can then be written as:

$$\epsilon^Y = \bar{\epsilon} + Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}}, \epsilon^f\right),$$

where $\bar{\epsilon} \equiv \frac{1}{N} \sum_f \epsilon^f$ is the unweighted average elasticity to the shock across firms, $\bar{\omega} \equiv \frac{1}{N} \sum_f \omega_{m,t}(f) = \frac{1}{N}$ is the average share of a firm in the total GDP, and N the total number of firms. Thus, the responsiveness of GDP to a shock is determined by the average responsiveness of all firms in the economy to this shock, and the covariance between firm size with its responsiveness to the shock. Writing the aggregate elasticity this way helps illustrate the role of granularity in international shock transmission. Since the largest firms are more likely to be internationally connected, we would expect them to have higher ϵ^f , and thus $Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}}, \epsilon^f\right) > 0$.

What are the the reasons that firms will differ in their ϵ^f ? With some manipulation, we can write the approximate log change in value added of firm f as:

$$d \ln Y_{m,j,t}(f) \approx (1 - \rho_j) \left[d \ln a(f) + \pi_{m,j}^l(f) d \ln w_{m,t} + \sum_i \sum_k (1 - \pi_{m,j}^l(f)) \pi_{km,ij}^x(f) d \ln P_{km,i,t} \right] + \sum_n \tilde{s}_{mn,j,t}(f) d \ln \left[\xi_{mn,j,t}(f) \left(\frac{\tau_{mn,j}}{P_{mn,j,t}} \right)^{1-\rho_j} X_{mn,j,t} \right], \quad (7)$$

where the summation over n is a summation over all the markets firm f actually serves, and $\tilde{s}_{mn,j,t}(f)$ is the share of market n in the total gross sales of firm f . Thus, a firm that only serves the domestic market has $\tilde{s}_{mm,j,t}(f) = 1$ and $\tilde{s}_{mn,j,t}(f) = 0 \forall n \neq m$.

The first term in (7) captures the change in the firm's costs, and the second term the change in the firm's demand following any external shock. Equation (7) highlights the sources of heterogeneity. On the cost side, following a shock in country k , only firms that import from k – $\pi_{km,ij}^x(f) \neq 0$ – directly experience a change in input costs. At the same time, the change in foreign demand – be it from the price-adjusted foreign expenditure $X_{mn,j,t}/P_{mn,j,t}^{1-\rho_j}$, or from a taste ($\xi_{mn,j,t}(f)$) or trade cost shock – will to first order affect only firms that export to country n , and even among those firms will vary with the sales share to that market.

At the same time, this expression underscores the general-equilibrium channels that will in principle operate and thus should be accounted for. To the extent that the foreign shock changes domestic wages ($d \ln w_{m,t}$), all firms in m will be affected. Also, all firms sell domestically. Thus, if the foreign shock affects domestic demand $d \ln \left(X_{mm,j,t}/P_{mm,j,t}^{1-\rho_j} \right)$, it will reach all firms in m . Finally, it could be that through second-order input linkages, even the non-importing firms' input prices $d \ln P_{mm,i,t}$ change.

It is ultimately an empirical and quantitative question how much ϵ^f varies across firms, and how it covaries with firm size. In particular, the relative importance of the direct effects on the connected firms and the general equilibrium effects on all firms in the economy has not been established. This is the main question addressed in the empirical and quantitative analysis below.

3.2 A Shock Formulation of the Model

To perform counterfactuals that simulate the impact of foreign shocks on domestic firms and the aggregate economy, we follow the approach of Dekle et al. (2008) and express the equilibrium conditions in terms of gross changes $\hat{x}_{t+1} = x_{t+1}/x_t$ in endogenous variables, to be solved for as a function of shocks expressed in gross changes, and initial (time- t) observables. Starting with (4),

we write it as a function of observed initial expenditure shares:

$$X_{mn,j,t} = \pi_{mn,j,t}^c \pi_{n,j,t}^c \left[w_{n,t} \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_{n,t} + D_{n,t} \right] + \sum_i \frac{\rho_i - 1}{\rho_i} \sum_{f \in i} (1 - \pi_{n,i,t}^l(f)) \pi_{mn,ji,t}^M(f) \sum_k \pi_{nk,i,t}(f) X_{nk,i,t}, \quad (8)$$

where $\pi_{mn,j,t}^c$ is the share of final consumption spending on goods from m in the total consumption spending on goods in sector j , country n , $\pi_{n,j,t}^c = \vartheta_j$ is simply the share of sector j in total final consumption spending, and $\pi_{nk,i,t}(f)$ is the share of firm f in the total exports from country n to country k in sector i . All of these π 's are observable when $n = \text{France}$. $\pi_{mn,j,t}^c$ and $\pi_{n,j,t}^c$ are observable in WIOD. $\pi_{nk,i,t}(f)$ when neither n nor k are France is not observable, so would require an assumption on which firms use imported intermediates. Since we do not have firm-level information on other countries, we assume that in those countries there is a representative firm in each sector. Writing out the shares:

$$\begin{aligned} \pi_{n,j,t}^c &= \vartheta_j, \\ \pi_{mn,j,t}^c &= \frac{\mu_{mn,j} P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}} = \frac{\mu_{mn,j} P_{mn,j,t}^{1-\sigma_j}}{\sum_k \mu_{kn,j} P_{kn,j,t}^{1-\sigma_j}}, \\ \pi_{nk,i,t}(f) &= \frac{\xi_{nk,i,t}(f) \left(\frac{\rho_i}{\rho_i-1} \tau_{nk,i} b_{n,i,t}(f) a_t(f) \right)^{1-\rho_i}}{P_{nk,i,t}^{1-\rho_i}}. \end{aligned}$$

Then, in proportional changes, (8) can be written as:

$$\begin{aligned} \hat{X}_{mn,j,t+1} X_{mn,j,t} &= \pi_{mn,j,t+1}^c \pi_{n,j,t+1}^c \left[\hat{w}_{n,t+1} \left(\frac{\hat{w}_{n,t+1}}{\hat{P}_{n,t+1}} \right)^{\frac{1}{\psi-1}} s_{n,t}^L + \hat{\Pi}_{n,t+1} s_{n,t}^\Pi + \hat{D}_{n,t+1} s_{n,t}^D \right] P_{n,t} C_{n,t} \\ &+ \sum_i \frac{\rho_i - 1}{\rho_i} \sum_{f \in i} (1 - \pi_{n,i,t+1}^l(f)) \pi_{mn,ji,t+1}^M(f) \sum_k \pi_{nk,i,t+1}(f) \hat{X}_{nk,i,t+1} X_{nk,i,t}. \end{aligned} \quad (9)$$

where $s_{n,t}^L$ is the share of labor (more generally factor payments) in the total final consumption expenditure at time t , and same for $s_{n,t}^\Pi$ and $s_{n,t}^D$.

Equation (6) is expressed in changes as:

$$\sum_j \sum_{f \in j} \sum_k \frac{\rho_j - 1}{\rho_j} \pi_{n,j,t}^l(f) \pi_{nk,j,t}(f) X_{nk,j,t} \left[\hat{\pi}_{n,j,t+1}^l(f) \hat{\pi}_{nk,j,t+1}(f) \hat{X}_{nk,j,t+1} - \hat{w}_{n,t+1}^{\frac{\psi}{\psi-1}} \hat{P}_{n,t+1}^{\frac{1}{1-\psi}} \right] = 0. \quad (10)$$

The prices (5) are expressed in changes as:

$$\widehat{P}_{mn,j,t+1} = \left[\sum_{f \in \Omega_{mn,j}} \pi_{mn,j,t}(f) \widehat{\xi}_{mn,j,t+1}(f) \left(\widehat{b}_{m,j,t+1}(f) \widehat{a}_{t+1}(f) \right)^{1-\rho_j} \right]^{\frac{1}{1-\rho_j}}, \quad (11)$$

$$\widehat{P}_{n,j,t+1} = \left[\sum_m \widehat{P}_{mn,j,t+1}^{1-\sigma_j} \pi_{mn,j,t}^c \right]^{\frac{1}{1-\sigma_j}}, \quad (12)$$

$$\widehat{P}_{n,t+1} = \prod_j \widehat{P}_{n,j,t+1}^{\vartheta_j}. \quad (13)$$

Finally, the expressions above require knowing next period's π 's. These can be expressed as:

$$\pi_{mn,j,t+1}^c = \frac{\widehat{P}_{mn,j,t+1}^{1-\sigma_j} \pi_{mn,j,t}^c}{\sum_k \widehat{P}_{kn,j,t+1}^{1-\sigma_j} \pi_{kn,j,t}^c} \quad (14)$$

$$\pi_{nk,j,t+1}(f) = \frac{\widehat{\xi}_{nk,j,t+1}(f) \left(\widehat{b}_{n,j,t+1}(f) \widehat{a}_{t+1}(f) \right)^{1-\rho_j} \pi_{nk,j,t}(f)}{\sum_{g \in \Omega_{nk,j}} \widehat{\xi}_{nk,j,t+1}(g) \left(\widehat{b}_{n,j,t+1}(g) \widehat{a}_{t+1}(g) \right)^{1-\rho_j} \pi_{nk,j,t}(g)} \quad (15)$$

$$\widehat{b}_{m,j,t+1}(f) = \left[\pi_{m,j,t}^l \widehat{w}_{m,t+1}^{1-\lambda} + (1 - \pi_{m,j,t}^l) \left(\widehat{P}_{m,j,t+1}^M \right)^{1-\lambda} \right]^{\frac{1}{1-\lambda}} \quad (16)$$

$$\widehat{P}_{m,j,t+1}^M = \left[\sum_i \sum_k \pi_{km,ij,t}^M \widehat{P}_{km,i,t+1}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (17)$$

$$\pi_{m,j,t+1}^l = \frac{\pi_{m,j,t}^l \widehat{w}_{m,t+1}^{1-\lambda}}{\pi_{m,j,t}^l \widehat{w}_{m,t+1}^{1-\lambda} + (1 - \pi_{m,j,t}^l) \left(\widehat{P}_{m,j,t+1}^M \right)^{1-\lambda}} \quad (18)$$

$$\pi_{km,ij,t+1}^M = \frac{\pi_{km,ij,t}^M \widehat{P}_{km,i,t+1}^{1-\eta}}{\sum_i \sum_n \pi_{nm,ij,t}^M \widehat{P}_{nm,i,t+1}^{1-\eta}}. \quad (19)$$

3.3 GDP Accounting in the Model

GDP is real value added. We follow the national accounting practices and deflate nominal value added by the GDP deflator, defined implicitly as the ratio between nominal GDP and the aggregate value added evaluated at the base period prices.⁴ In our framework, nominal value added associated with firm f 's sales to market n is a constant fraction of its sales there:

$$Y_{mn,j,t}^{NOM}(f) = \frac{1 + \alpha_{m,j}(f)(\rho_j - 1)}{\rho_j} X_{mn,j,t}(f),$$

⁴An alternative would be to deflate nominal value added by the CPI, which is to be precise the ideal consumption price index $P_{m,t}$. The two differences between the GDP deflator and the CPI are that (i) the CPI includes the prices of foreign final consumption imports and (ii) the CPI does not include the prices of domestically-produced intermediates. The results when deflating by the CPI are similar and available upon request.

and thus total firm value added is given by:

$$Y_{m,j,t}^{NOM}(f) = \frac{1 + \alpha_{m,j}(f)(\rho_j - 1)}{\rho_j} \sum_n X_{mn,j,t}(f),$$

where the summation is over the markets the firm actually serves.

GDP is simply the sum over all firm-level value added, as in (7). Expressed in gross changes it becomes:

$$\hat{Y}_{m,t+1}^{NOM} = \sum_f \sum_n \omega_{m,j,t}(f) \tilde{s}_{mn,j,t}(f) \hat{X}_{mn,j,t+1}(f),$$

where, as in [Section 3.1](#), $\omega_{m,j,t}(f)$ is the share of firm f 's value added in total GDP, and $\tilde{s}_{mn,j,t}(f)$ is the share of sales to n in firm f 's total gross sales. Finally, the aggregate outcome of interest is the real GDP change:

$$\hat{Y}_{m,t+1} = \frac{\hat{Y}_{m,t+1}^{NOM}}{\hat{P}_{m,t+1}^G}, \quad (20)$$

where $\hat{P}_{m,t+1}^G$ is the GDP deflator. [Appendix B.1](#) presents the formulas underlying the construction of the GDP deflator. When implementing the decomposition (1), we deflate each firm's value added growth with the GDP deflator, since doing this way ensures that aggregate real GDP is the sum of all firms' real value added.

3.4 Calibration

The model implementation involves solving equations (9)-(19).⁵ Implementing these equations requires a small number of structural parameters, and a set of initial-period values taken from the data. [Table 2](#) summarizes the calibration. We set the elasticity of substitution between firms in the same sector selling to the same destination to $\rho = 3$. A value of elasticity of substitution across firms of 3 is a common value according to standard methodologies (see e.g. [Broda and Weinstein, 2006](#)). In the baseline analysis we do not assign different values to ρ across sectors. We set the Armington elasticity of substitution between goods coming from different source countries to $\sigma = 1.5$. This is the value favored by the international business cycle literature following [Backus et al. \(1995\)](#), and is supported by the recent estimates by [Feenstra et al. \(2018\)](#). We set the labor supply parameter to $\bar{\psi} = 3$, implying the Frisch labor supply elasticity of 0.5, as advocated by [Chetty et al. \(2013\)](#). In the baseline analysis, we set the production function elasticities $\eta = \lambda = 1$, as is standard in the literature. In the robustness analysis we implement both higher and lower values. For the firm-specific production parameters and trade shares, we use our combined French and WIOD data, described in detail in [Appendix A](#).

Our model does not feature endogenous deficits. In all our experiments, we thus assume that the change in deficits is zero: $\hat{D}_{n,t+1} = 0$. We adopt a similar approach to profits: $\hat{\Pi}_{n,t+1} = 0$.

⁵See [Appendix B.2](#) for details.

In the absence of a model of multinational production, in an open economy like France changes in profits are not pinned down in our framework. This is because the aggregate profits in equation (9) refer to those used by French residents for domestic consumption spending. These are not the same as profits of firms operating in France, both because French residents own French multinationals operating abroad and thus have claims on those foreign-generated profits, and because not all firms operating in France are domestically-owned, and the profits of foreign multinational affiliates operating in France are not available to French residents for consumption spending. Since the profit share of GDP is under 10%, and for our counterfactuals what matters is not the level of profit share but the change, as an approximation we abstract from the impact of changes in profits on final consumption in our counterfactuals.

Table 2. Parameter values

Param.	Value	Source	Related to
ρ	3	Broda and Weinstein (2006)	subst. elasticity btw. firms
σ	1.5	Feenstra et al. (2018)	Armington elasticity
η	1	standard	subst. elasticity btw. inputs
λ	1	standard	subst. elasticity btw. inputs and labor
$\bar{\psi}$	3	Chetty et al. (2013)	Frisch elasticity
$\pi_{n,i}^l(f), \pi_{mn,ji}^x(f)$	}	Our calculations based on French data and WIOD	labor and intermediate shares
ϑ_j			final consumption shares
$\pi_{mn,j,t}^c$			final trade shares
$\pi_{nk,j,t}(f)$			intermediate use trade shares

Notes: This table summarizes the parameter values used in the calibration.

4 Quantitative Results

4.1 Productivity Shocks

We start by simulating the impact of two shocks on the French economy: a 10% productivity improvement in every other country in the sample other than France, and a 10% productivity improvement in Germany, one of France’s most important trading partners. We report the results directly in terms of elasticities, as those lend themselves to the decomposition (1). The baseline results are reported in the top of the two panels of Table 3. French GDP increases by 4.37% when world productivity grows by 10%. This is a sizeable elasticity, considering that France itself does not experience the productivity shock, and thus the entire change is due to it being transmitted to France via goods trade linkages. The response to a German shock (bottom panel) is understandably much smaller at 0.55%, since that shock affects only one of France’s trading partners.

Table 3. Responses of French Real GDP to Foreign Productivity Shocks
(10% Productivity Shocks)

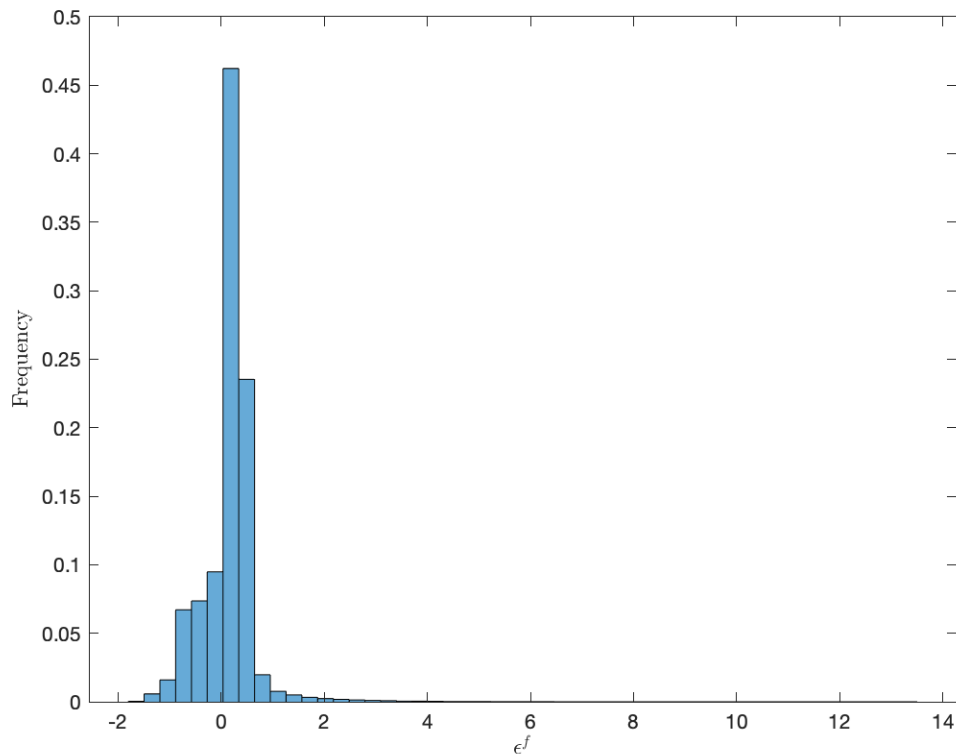
	ϵ^Y	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}_t}, \epsilon^f\right)$
World Productivity Shock			
Baseline	0.374	0.145	0.229
<i>Share:</i>		<i>0.39</i>	<i>0.61</i>
Homogeneous firms	0.430	0.424	0.006
<i>Share:</i>		<i>0.99</i>	<i>0.01</i>
German Productivity Shock			
Baseline	0.055	0.011	0.044
<i>Share:</i>		<i>0.20</i>	<i>0.80</i>
Homogeneous firms	0.065	0.066	-0.001
<i>Share:</i>		<i>1.02</i>	<i>-0.02</i>
Sector-Level Decomposition			
	ϵ^Y	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}}{\bar{\omega}_t}, \epsilon^j\right)$
World Productivity Shock			
Baseline	0.374	0.313	0.061
<i>Share:</i>		<i>0.84</i>	<i>0.16</i>
German Productivity Shock			
Baseline	0.055	0.055	0.000
<i>Share:</i>		<i>1.01</i>	<i>-0.01</i>

Notes: This table reports the elasticity of French GDP with respect to a 10% productivity shock in every other country in the world and with respect to a 10% productivity shock to Germany, both the baseline model and the alternative model that suppresses firm heterogeneity. The table reports the decomposition of the aggregate elasticity into the mean and the covariance terms as in (1).

Our central result concerns not so much the overall magnitude, but the role of heterogeneity. Decomposing the aggregate elasticity into the mean and the covariance term, we find that the covariance term is positive as expected, and quite large. It is responsible for 63% of the overall effect of a world shock, and 80% of the German shock. Thus, our results reveal a quantitatively large role of the heterogeneity in firm-level international linkages in business cycle transmission across countries.

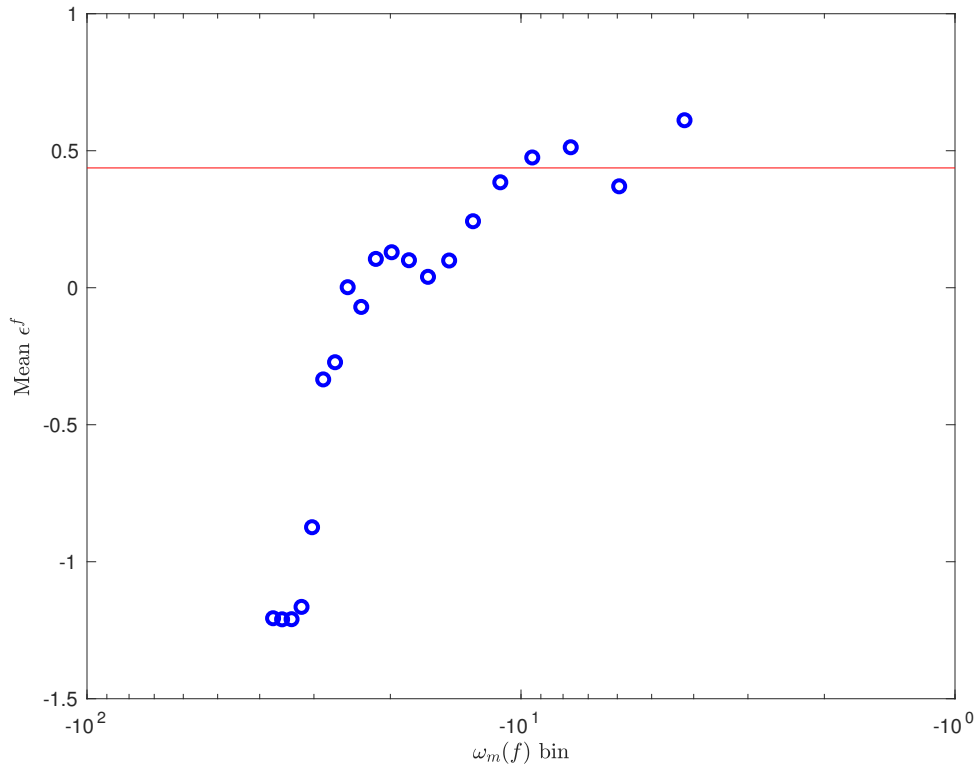
To provide a graphic illustration of this result, [Figure 3](#) plots the histogram of ϵ^f across firms in the baseline model for the world shock. It is evident that firm-level elasticities have a non-trivial distribution. While most of them are positive, there is substantial density below zero as well – some firms shrink in response to a positive shock in the rest of the world. At the same time, there is an upper tail as well, as the density of ϵ^f above 1 is visible. Next, [Figure 4](#) presents the average ϵ^f for firms of different sizes $\omega_{m,t}(f)$. We break firm shares in aggregate value added into size bins, and plot the mean ϵ^f in each size bin. This figure provides a graphical illustration of the positive $Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}}, \epsilon^f\right)$ term. The horizontal line plots the aggregate elasticity ϵ^Y . It is notable that it is towards the top of the plot, coinciding with the ϵ^f of the largest firms.

Figure 3. Histogram of ϵ^f Following a 10% World Productivity Shock



Notes: This figure displays the histogram of ϵ^f following a 10% world productivity shock in the baseline model.

Figure 4. Average ϵ^f for Different Values of $\omega_{m,t}(f)$
(10% World Productivity Shock)

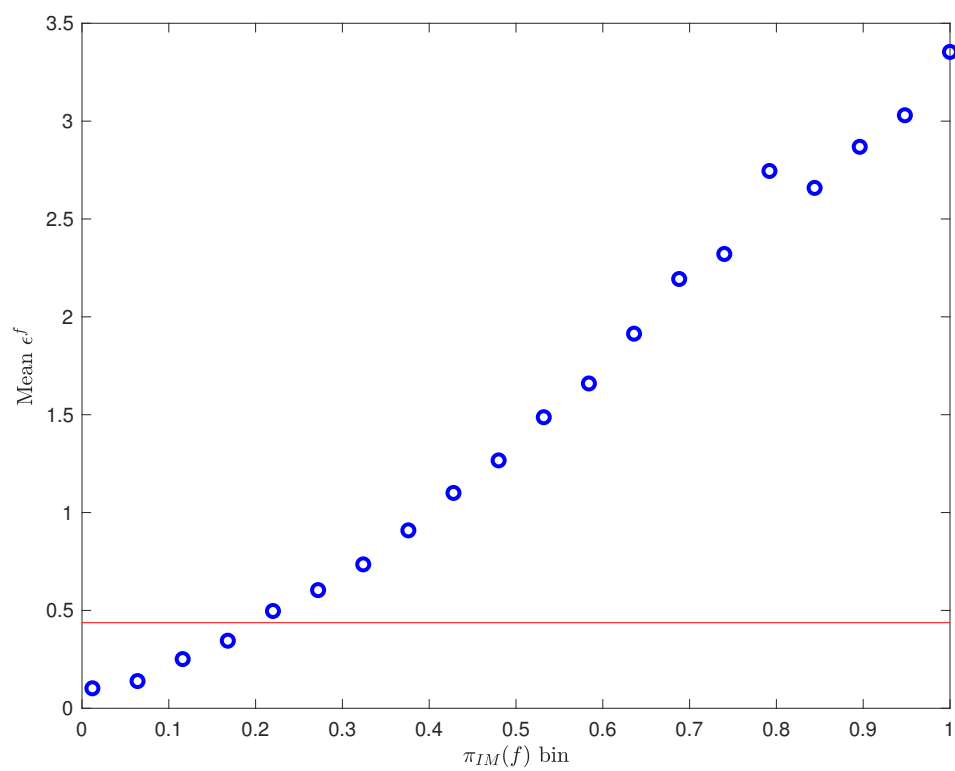


Notes: This figure displays the mean ϵ^f for each size bin following a 10% world productivity shock in the baseline model.

The variation in firm-specific elasticities with respect to foreign shocks has the expected relationship to the intensity of intermediate input purchases from abroad and to export intensity. **Figure 5** plots the average ϵ^f for each value of total imported input share, $\pi_{IM}^x(f) \equiv \sum_{n \neq m} \sum_i \pi_{mn,ji}^x(f)$. There is a pronounced positive relationship. **Figure 6** plots the average ϵ^f against the total export intensity of each firm, defined as the ratio of total firm exports to total firm sales, $\pi_{EX}(f) \equiv \sum_{n \neq m} \tilde{s}_{mn,j,t}(f)$. Once again there is a pronounced positive relationship, with more export-oriented firms having higher elasticities to the foreign shock. Note that unlike the relationship with $\gamma_{IM}(f)$, the unweighted mean ϵ^f in each export intensity bin is actually below the aggregate country-level ϵ^Y . This outcome can serve as an illustration of the granularity in the data, and the mechanisms behind the variation in *epsilon*^f. It indicates that the export intensity by itself is heterogeneous enough that the average firm in essentially all the export intensity bins is still somewhat small. Also, at the same time it shows that following a foreign productivity shock, exporting in and of itself does not necessarily lead to expansion, since substitution effects more than cancel out income effects.

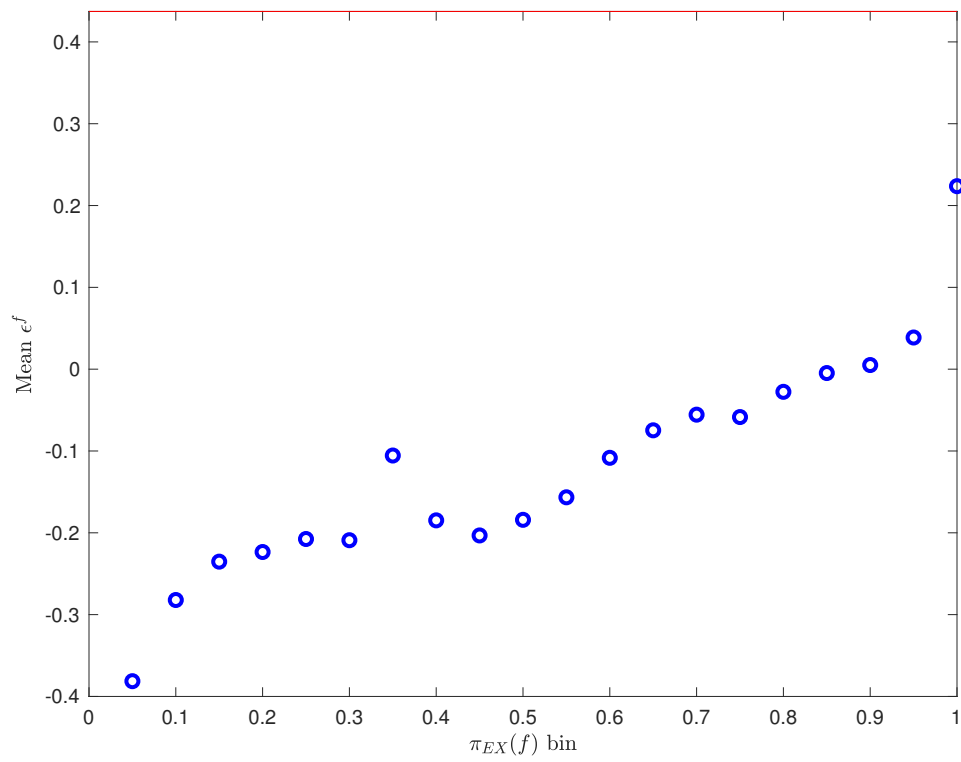
We compare the baseline model to an alternative implementations that suppresses heterogeneity

Figure 5. Average ϵ^f for Firms with different Intermediate Import Intensities (10% World Productivity Shock)



Notes: This figure displays the averages of ϵ^f for each value of total imported intermediate input intensity following a 10% world productivity shock in the baseline model.

Figure 6. Average ϵ^f for Firms with Different Export Intensities
(10% World Productivity Shock)



Notes: This figure displays the mean ϵ^f for each value of overall export intensity following a 10% world productivity shock in the baseline model.

along the importing and exporting dimensions. The line labeled “Homogeneous firms” of [Table 3](#) reports the elasticities in an alternative model in which firm export participation (the trade shares $\pi_{nk,j,t+1}(f)$) and firm-level intermediate import usage ($\pi_{mn,ji}^x(f)$) are made homogeneous within each sector. This scenario is thus a model with a sector-specific representative firm. Importantly, to preserve the overall levels of trade in this scenario, the $\pi_{mn,ji}^x(f)$ ’s are set to match the sector-level imported input coefficients, and the export shares $\pi_{nk,j,t+1}(f)$ are set to match aggregate export shares in each sector. This implies that in this homogeneous firm scenario, the imported input coefficients are lower for the firms that in the data actually import inputs, but higher for firms that in the data do not. Similarly, firms that in the data export nothing in this scenario export to all countries.

The overall elasticity to foreign shocks is comparable to the baseline one. However, the micro patterns are very different. The entirety of the aggregate elasticity is explained by the average $\bar{\epsilon}$ component, with no role for the covariance term.

Next, we evaluate whether in the baseline model, the heterogeneity that drives the high covariance term is within or across sectors. To that end, we take the results from the baseline model, and instead of writing the decomposition [\(1\)](#) at the firm level, write it at the sector level instead:

$$\epsilon^Y = \bar{\epsilon}_j + Cov\left(\frac{\omega_{j,t}}{\bar{\omega}_j}, \epsilon^j\right), \quad (21)$$

where ϵ^j is the elasticity of total value added in sector j to the foreign shock, $\bar{\epsilon}_j$ is its unweighted average, and $\omega_{j,t}$ is sector j share in aggregate value added. Importantly, we implement this decomposition on the baseline model featuring the full heterogeneity across firms, but we compute the sector-level shares and elasticities. The results are presented in the panel labeled “Sector-Level Decomposition” of [Table 3](#). By construction, the overall elasticity ϵ^Y is exactly the same as in the top panel of the table. The sector-level covariance term is 14%, dramatically smaller than the firm-level covariance term of 63%, indicating that the impact of heterogeneity is to a large extent captured by the sectoral dimension.

4.2 Foreign Demand Shocks

Next, we evaluate the propagation of a foreign demand shock to France. To that end, we simulate an increase in the taste shock $\xi_{mn,j,t}(f)$ to all firms in $m = \text{France}$ in all foreign markets $n \neq m$, as well as only in Germany. Examining equation [\(3\)](#), it is clear that an increase in the taste for all French firms abroad amounts to a $\widehat{\xi_{mn,j,t}^{\frac{1}{\rho-1}}}$ productivity increase for French exports abroad, and thus an increase in demand for French goods by foreign firms and consumers. (We assume that this is a purely external shock, such that the French domestic demand shifter $\xi_{mm,j,t}(f)$ is unchanged.) We thus simulate a 10% shift in demand for French goods, namely $\widehat{\xi_{mn,j,t}^{\frac{1}{\rho-1}}} = 0.1$.

Table 4. Responses of French Real GDP to Foreign Demand Shocks
(10% Demand Shocks)

	ϵ^Y	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}_t}, \epsilon^f\right)$
World Demand Shock			
Baseline	0.039	0.024	0.015
<i>Share:</i>		<i>0.61</i>	<i>0.39</i>
Homogeneous firms	0.042	0.043	-0.001
<i>Share:</i>		<i>1.02</i>	<i>-0.02</i>
German Demand Shock			
Baseline	0.006	0.003	0.003
<i>Share:</i>		<i>0.54</i>	<i>0.46</i>
Homogeneous firms	0.007	0.006	0.000
<i>Share:</i>		<i>0.94</i>	<i>0.06</i>
Sector-Level Decomposition			
	ϵ^Y	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}}{\bar{\omega}_t}, \epsilon^j\right)$
World Demand Shock			
Baseline	0.039	0.064	-0.025
<i>Share:</i>		<i>1.63</i>	<i>-0.63</i>
German Demand Shock			
Baseline	0.006	0.008	-0.002
<i>Share:</i>		<i>1.35</i>	<i>-0.35</i>

Notes: This table reports the elasticity of French GDP with respect to a 10% demand shock for French goods in every other country in the world and with respect to a 10% demand shock for French goods in Germany, both the baseline model and the alternative models that suppress sources of firm heterogeneity. The table reports the decomposition of the aggregate elasticity into the mean and the covariance terms as in (1).

Table 4 reports the results. It is structured in exactly the same way as **Table 3**. In the baseline, a 10% demand shocks for French goods abroad raises French real GDP by 0.39%. This is a smaller elasticity than that of a foreign productivity shock, but that is because the overall shock is much smaller, as it affects only the French tradeable sector. The relative importance of the covariance term accounts for 39% of the overall impact for the global demand shock, and 46% for the German demand shock. Once again, when export and import shares are homogeneous, the covariance term drops to zero.

Finally, the bottom panel reports the $\bar{\epsilon}_j - Cov\left(\frac{\omega_{j,t}}{\bar{\omega}_j}, \epsilon^j\right)$ decomposition at the sector level for the foreign demand shock. Not only is the covariance term not positive, it is actually strongly negative, accounting for -63% of the overall effect for the world demand shock, and -35% for the German demand shock. Evidently, sectors with the highest positive elasticities with respect to foreign demand shocks tend to actually be relatively smaller in size. This is sensible, as some of the largest sectors in our data are non-tradeable, and thus by construction not experiencing the positive foreign demand shock.

Finally, we run the heuristic regression (2) from Section 2 inside the model. The results are reported in **Table 1**, columns 5 (for the world productivity shock) and 6 (world demand shock). Since the model simulation is of a single year’s growth rate, there are fewer firms in this regression, and sector-time fixed effects become sector fixed effects. The model reproduces the pattern in the data qualitatively. Larger firms are more sensitive to both the world productivity and world demand shocks. Interestingly, the coefficient of interest is much smaller than in the data in the productivity shock simulation, but much larger than in the data in the demand shock simulation. Given that actual world GDP is a mix of productivity and demand shocks, we should not expect a single shock inside the model to replicate the data coefficient. The fact that the data coefficient is between those for productivity and demand shocks is perhaps telling that foreign shocks experienced by France are a mixture of the two.

4.3 Contribution of Foreign Shocks to the Granular Residual

Using different approaches, [Gabaix \(2011\)](#), [di Giovanni et al. \(2014\)](#), and [Carvalho and Grassi \(2018\)](#) document that a significant fraction of GDP fluctuations is driven by idiosyncratic shocks to individual firms. The contribution of firm idiosyncratic shocks to aggregate fluctuations is captured by the granular residual. Beyond accounting for aggregate fluctuations, the granular residual is an object of interest in other contexts, see for instance its use as an instrument ([Gabaix and Koijen, 2019](#)). Because of the systematically heterogeneous cross-border linkages across firms, foreign shocks are a quantitatively important contributor to the granular residual, and are thus one of the sources of granular fluctuations.

Let $\epsilon_n^f \equiv d \ln Y_{m,t}(f) / d \ln a_{n,t}$ denote the elasticity of output of firm f to a productivity shock

in country n . Then firm f 's real value added growth rate due to the foreign shocks as

$$d \ln Y_{m,t}(f) = \sum_n \epsilon_n^f d \ln a_{n,t}. \quad (22)$$

Then the change in French GDP due to foreign shocks is simply:

$$d \ln Y_{m,t}^F = \sum_f \omega_{m,t-1}(f) d \ln Y_{m,t}^F(f), \quad (23)$$

where the superscript F denotes the fact that this is the change in value added and GDP exclusively due to foreign shocks.

Define the *foreign granular residual* as the size-weighted firm deviation from the unweighted average, in a manner similar to [Gabaix \(2011\)](#):

$$\Gamma_{m,t}^F = \sum_f \omega_{m,t-1}(f) d \ln Y_{m,t}^F(f) - \frac{1}{N} \sum_f d \ln Y_{m,t}^F(f). \quad (24)$$

The object $\Gamma_{m,t}^F$ answers the question: how large would the granular residual be if France experienced only foreign shocks?

The overall change in the French GDP can be written as the sum of the foreign granular residual Γ_t^F and the equal-weighted change $E_{m,t}^F$:

$$d \ln Y_{m,t}^F = \Gamma_{m,t}^F + E_{m,t}^F, \quad (25)$$

where

$$E_{m,t}^F = \frac{1}{N} \sum_f d \ln Y_{m,t}^F(f) \quad (26)$$

is the simple mean of the value added change across all French firms due to foreign shocks.

We implement [\(24\)-\(26\)](#) for a realistic size of foreign shocks in two ways. First, we obtain the aggregate TFP shocks for our sample of countries from the Penn World Tables, and feed them directly into [\(22\)](#) to compute each firm's response to those foreign TFP shocks. In our second approach, we obtain actual GDP growth for all the countries in our sample from the World Development Indicators. To compute the propagation of foreign GDP growth rates into France, we re-express [\(22\)](#) directly in terms of elasticities of French firms to foreign GDP:

$$d \ln Y_{m,t}(f) = \sum_n \tilde{\epsilon}_n^f d \ln Y_{n,t}, \quad (27)$$

where $\tilde{\epsilon}_n^f \equiv \frac{d \ln Y_{m,t}(f)}{d \ln Y_{n,t}}$ is the elasticity of firm f 's value added growth to country n 's GDP, rather than the TFP shock directly. The advantage of this approach is that it in principle accounts for all of GDP movements abroad, not just the movements in measured TFP. The disadvantage is that it implicitly attributes all of the foreign GDP changes to TFP, which may not be accurate.

Table 5. Standard Deviations of Actual and Foreign-Induced GDP Growth and Its Components, Percentage Points

Period	Data		Foreign TFP			Foreign GDP		
	$d \ln Y_{m,t}$	$\Gamma_{m,t}$	$d \ln Y_{m,t}^F$	$\Gamma_{m,t}^F$	$E_{m,t}^F$	$d \ln Y_{m,t}^F$	$\Gamma_{m,t}^F$	$E_{m,t}^F$
1975-2014	1.54		0.38	0.26	0.12	0.16	0.11	0.05
1991-2007	1.11	0.96	0.37	0.25	0.13	0.11	0.07	0.04

Notes: This table reports the standard deviations of actual French GDP growth ($d \ln Y_{m,t}$), the actual French granular residual ($\Gamma_{m,t}$) and each component of (25).

Table 5 reports the results for two time periods: 1975-2014, and 1991-2007. There are two reasons to focus on the shorter time period. The first is that for this time period we can report the standard deviation of the overall French granular residual ($\Gamma_{m,t}$), sourced from our earlier work (di Giovanni et al., 2014). Second, our model is implemented on the trade and production data from this period, and it is not clear that the cross-border trade linkages we assume are realistic prior to the 1990s. The first two columns report the standard deviations of actual French GDP growth and the granular residual. The middle panel reports the standard deviations $d \ln Y_{m,t}^F$, $\Gamma_{m,t}^F$, and $E_{m,t}^F$ generated purely by foreign TFP shocks. Foreign shocks by themselves can generate one-quarter to one-third of the observed GDP fluctuations of France, depending on the time period. More importantly for us, nearly 70% of the foreign shock-induced GDP fluctuations are due to the foreign granular residual $\Gamma_{m,t}^F$. Thus, foreign shocks are indeed predominantly granular fluctuations. The right-most panel reports the results of feeding in GDP growth. Here, the overall fluctuations generated by foreign shocks are about 50% less volatile. Nonetheless, the relative contribution of the foreign granular residual to the overall foreign impact is similarly close to 70%.

5 Conclusion

Large firms are more likely to import and export. A natural conjecture is that this greater participation in international markets also makes the large firms more sensitive to foreign shocks. In this paper, we explored both the micro and the macro implications of this joint heterogeneity in size and international linkages. We first provided firm-level econometric evidence that firms importing intermediate inputs are significantly more responsive to foreign input price shocks. We then implemented a quantitative multi-country model in which French firms exhibit the observed joint distribution of size, importing, and exporting. The covariance between firm size and sensitivity to

foreign shocks accounts for as much as 60% of the aggregate impact of foreign productivity and demand shocks. We conclude that capturing the positive association between size and international linkages is essential for understanding the firm-level and aggregate international transmission of business cycle shocks.

References

- Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, “The Network Origins of Aggregate Fluctuations,” *Econometrica*, September 2012, 80 (5), 1977–2016.
- Alessandria, George and Horag Choi, “Do Sunk Costs of Exporting Matter for Net Export Dynamics?,” *Quarterly Journal of Economics*, 2007, 122 (1), 289–336.
- Alvarez, Fernando and Robert E., Jr. Lucas, “General Equilibrium Analysis of the Eaton-Kortum Model of International Trade,” *Journal of Monetary Economics*, September 2007, 54 (6), 1726–1768.
- Atalay, Enghin, “How Important Are Sectoral Shocks?,” *American Economic Journal: Macroeconomics*, October 2017, 9 (4), 254–280.
- Backus, David K., Patrick J. Kehoe, and Finn E. Kydland, “International Business Cycles: Theory and Evidence,” in Thomas Cooley, ed., *Frontiers of business cycle research*, Princeton: Princeton University Press, 1995, pp. 331–356.
- Baqae, David Rezza, “Cascading Failures in Production Networks,” September 2016. Mimeo, LSE.
- Barrot, Jean-Noël and Julien Sauvagnat, “Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks,” *Quarterly Journal of Economics*, 2016, 131 (3), 1543–1592.
- Blaum, Joaquin, “Importing, Exporting and Aggregate Productivity in Large Devaluations,” March 2018. Mimeo, Brown University.
- , Claire Lelarge, and Michael Peters, “The Gains from Input Trade with Heterogeneous Importers,” December 2016. Forthcoming, *American Economic Journal: Macroeconomics*.
- Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai Nayar, “Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tohoku Earthquake,” December 2017. Forthcoming, *Review of Economics and Statistics*.
- Broda, Christian and David Weinstein, “Globalization and the Gains from Variety,” *Quarterly Journal of Economics*, May 2006, 121 (2), 541–85.
- Bureau of Economic Analysis, “Updated Summary of NIPA Methodologies,” November 2017. <https://www.bea.gov/scb/pdf/2017/11-November/1117-updated-summary-of-nipa-methodologies.pdf>.
- Carvalho, Vasco M., “Aggregate Fluctuations and the Network Structure of Intersectoral Trade,” October 2010. Mimeo, CREi and Universitat Pompeu Fabra.
- and Basile Grassi, “Large Firm Dynamics and the Business Cycle,” September 2018. Forthcoming, *American Economic Review*.
- , Makoto Nirei, Yukiko U. Saito, and Alireza Tahbaz-Salehi, “Supply Chain Disruptions: Evidence from the Great East Japan Earthquake,” December 2016. mimeo, University of Cambridge, Policy Research Institute, Ministry of Finance of Japan, RIETI, and Columbia GSB.
- Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber, “Does Indivisible Labor Explain the Difference between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities,” *NBER Macroeconomics Annual*, 2013, 27 (1), 1 – 56.
- Costinot, Arnaud and Andrés Rodríguez-Clare, “Trade Theory with Numbers: Quantifying the Consequences of Globalization,” in Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4, Elsevier, 2014, chapter 4, pp. 197–261.
- Cravino, Javier and Andrei A. Levchenko, “Multinational Firms and International Business Cycle Transmission,” *Quarterly Journal of Economics*, May 2017, 132 (2), 921–962.

- Dekle, Robert, Jonathan Eaton, and Samuel Kortum, “Global Rebalancing with Gravity: Measuring the Burden of Adjustment,” *IMF Staff Papers*, July 2008, 55 (3), 511–540.
- di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Mejean, “Firms, Destinations, and Aggregate Fluctuations,” *Econometrica*, July 2014, 82 (4), 1303–1340.
- , —, and —, “Large Firms and International Business Cycle Comovement,” *American Economic Review: Papers & Proceedings*, May 2017, 107 (5), 598–602.
- , —, and —, “The Micro Origins of International Business Cycle Comovement,” *American Economic Review*, January 2018, 108 (1), 82–108.
- Feenstra, Robert C., Philip Luck, Maurice Obstfeld, and Katheryn N. Russ, “In Search of the Armington Elasticity,” *The Review of Economics and Statistics*, 2018, 100 (1), 135–150.
- Freund, Caroline and Martha Denisse Pierola, “Export Superstars,” *Review of Economics and Statistics*, 2015, 97 (5), 1023–1032.
- Gabaix, Xavier, “The Granular Origins of Aggregate Fluctuations,” *Econometrica*, May 2011, 79 (3), 733–772.
- and Ralph S. J. Koijen, “Granular Instrumental Variables,” April 2019. mimeo, Harvard and Chicago Booth.
- Ghironi, Fabio and Marc J. Melitz, “International Trade and Macroeconomic Dynamics with Heterogeneous Firms,” *Quarterly Journal of Economics*, August 2005, 120 (3), 865–915.
- Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman, “Investment, Capacity Utilization, and the Real Business Cycle,” *American Economic Review*, June 1988, 78 (3), 402–17.
- Kleinert, Jörn, Julien Martin, and Farid Toubal, “The Few Leading the Many: Foreign Affiliates and Business Cycle Comovement,” *American Economic Journal: Macroeconomics*, October 2015, 7 (4), 134–159.
- Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries, “An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production,” *Review of International Economics*, August 2015, 23 (3), 575–605.
- Tintelnot, Felix, Ken Kikkawa, Magne Mogstad, and Emmanuel Dhyne, “Trade and Domestic Production Networks,” 2017. Mimeo, University of Chicago.

Appendix A Data

A.1 Harmonizing French Firm-Level Data with Global Sectoral Data

The firm’s sector in the French data is reported in the *Nomenclature d’Activités Françaises* classification, which we convert into the 35 sectors of the WIOD nomenclature. Note that the balance-sheet data do not cover Financial Activities and Private Households with Employed Persons (sectors J and P in WIOD), and thus those sectors are dropped from the analysis. We also dropped the “Public Administration” sector (sector L) which represents 23 firms and less than 0.1% of overall value added in our data.

Data on individual bilateral imports, together with information on each firm’s cost structure, are used to recover the technical coefficients of each firm’s production function. Firm-specific labor shares $\alpha_{n,j}(f)$ are defined as the ratio of value added over sales, both available in the balance-sheet data. In order to ensure comparability with the rest of the sample, in which labor shares are calibrated using WIOD for each country and sector, the distribution of firm-level labor shares is rescaled sector-by-sector in a way that preserves the heterogeneity but ensures that the average across firms matches the corresponding information in the WIOD. Namely:

$$\alpha_{n,j}(f) = \tilde{\alpha}_{n,j}(f) \frac{\alpha_{n,j}}{\tilde{\alpha}_{n,j}}.$$

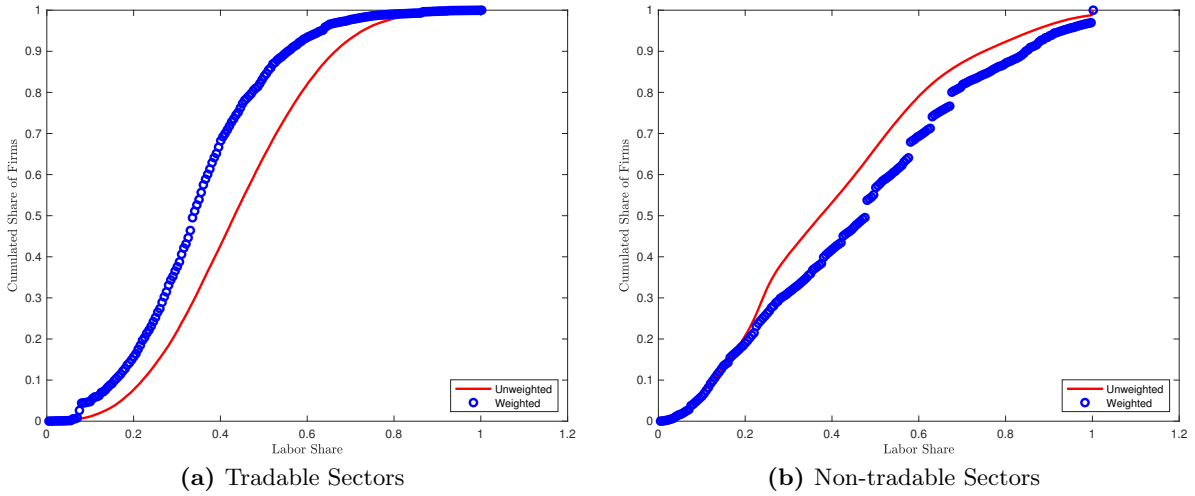
In this equation, $\alpha_{n,j}(f)$ and $\tilde{\alpha}_{n,j}(f)$ are the rescaled and original firm-level coefficients, respectively, and $\alpha_{n,j}$ is the sectoral counterpart recovered from the WIOD data. Finally, $\tilde{\alpha}_{n,j}$ is a weighted average of the original firm-level coefficients, where each firm is weighted according to its share in sectoral sales: $\tilde{\alpha}_{n,j} = \sum_{f \in (n,j)} w_{n,j}^S(f) \tilde{\alpha}_{n,j}(f)$.⁶

Figure 7 displays the cumulative distribution of labor shares, distinguishing between tradable and non-tradable sectors. The solid (red) line correspond to the unweighted distributions and the (blue) circles to the weighted ones. These distributions show a high degree of heterogeneity across firms, within and across broad sectors. In traded good sectors, large firms tend to be less labor intensive, although the pattern is not systematic in all individual sectors and is not very strong. On the contrary, large firms in non-traded good sectors are often more labor-intensive than smaller ones.⁷

⁶The rescaling strategy implies that some rescaled firm-level coefficients end up lying outside of the range of possible values ($[0, 1]$). The corresponding coefficients are winsorized at the maximum and minimum values. This affects less than 0.02% of the firms in the total sample. The rescaling strategy is applied to all sectors but three, namely Wholesale and Retail, including Motor Vehicles and Fuel. For these three sectors, the average labor share is low in the French data compared to the WIOD. This comes from the treatment of merchandise which we categorize as intermediates while WIOD does not. Our approach is consistent with the model in the case of France, when it is assumed that consumers never interact directly with foreign firms. From that point of view, all merchandise imported from abroad is used as inputs by a French firm which ultimately sells to the final consumer. Because this is all the more important for retailing and wholesaling activities, we decided to keep the distribution of measured $\alpha_{n,j}(f)$ unchanged in these sectors.

⁷In tradable sectors, the correlation between the firm’s labor share and its size varies between 0 and -0.09 (Wood

Figure 7. Distribution of Labor Shares Across French Firms



Notes: This figure plots the cumulative distribution of firm-level labor shares ($\alpha_{n,j}(f)$), in tradable and in non-tradable sectors. The solid (red) lines correspond to the unweighted distribution and the (blue) circles to the weighted distribution, where firms’ weights are defined according to their share in aggregate value added. Calculated from French balance-sheet data together with the WIOD information on sectoral labor shares, for 2005.

Total input usage at the firm level equals one minus the labor share (in our setting “labor” stands for the composite of primary factors). We further disaggregate total input usage across sectors and source countries using the information on imports, by product. This allows us to recover the $\gamma_{mn,ij}(f)$ coefficients for $n = France$. While in principle straightforward, calibrating these parameters entails two key difficulties: i) it requires the use of two sources of firm-level data, which raises concerns regarding comparability; and ii) not all of these coefficients can be recovered from the firm-level data. In particular, we don’t have detailed information on inputs purchased domestically and thus need to infer their sectoral breakdown using (more aggregated) information from WIOD. We proceed as follows.

For each sector i among the subset of tradable sectors and each source country $m \neq n$, we first compute a technical coefficient as the ratio of bilateral imports of goods produced by country m , sector i over the firm’s input expenses.⁸ Since this ratio uses data collected from two databases, the overall import share obtained from the summation of these $\gamma_{mn,ij}(f)$ coefficients over all tradable sectors and foreign countries is larger than one in some cases (for less than 1% of firms). Whenever

products) and is often significant. In non-tradable sectors, it is positive and significant in 10 sectors out of 18 and is as high as 0.13 for Post and Telecommunication Services.

⁸This requires the conversion of product-level import data expressed in the highly disaggregated *Harmonized System* into broader sectoral categories. Since the customs data do not allow us to distinguish between the import of intermediates and merchandise (goods that are not further processed before being sold by the firm), we measure the firm’s input expenses accordingly as the sum of raw materials *and* merchandise purchases (taking into account changes in inventories). See [Blaum et al. \(2016\)](#) for a similar treatment of the data.

this happens, the import share is winsorized to one and the bilateral sectoral coefficients rescaled accordingly.

Beyond comparability issues between the two firm-level sources, the introduction of these firm-level technical coefficients into the broader multi-country model also means we must ensure consistency with the sectoral coefficients in the global data. As we did with the labor shares, this implies rescaling the overall distribution of firm-level coefficients to the mean observed in the WIOD data:

$$\gamma_{mn,ij}(f) = \tilde{\gamma}_{mn,ij}(f) \frac{\gamma_{mn,ij}}{\tilde{\gamma}_{mn,ij}},$$

where $\gamma_{mn,ij}(f)$ and $\tilde{\gamma}_{mn,ij}(f)$ denote the rescaled and original firm-level coefficients, respectively, $\gamma_{mn,ij}$ is the sectoral counterpart measured with the WIOD data, and $\tilde{\gamma}_{mn,ij}$ is the weighted average of the firm-level original coefficients, where each firm is weighted according to its share in sectoral input purchases: $\tilde{\gamma}_{mn,ij} = \sum_{f \in (n,j)} \omega_{n,j}^I(f) \tilde{\gamma}_{mn,ij}(f)$. The normalization preserves as much heterogeneity across firms as possible, while avoiding overestimates of the international transmission of shocks through foreign input purchases via an exaggeration of the degree to which French firms actually rely on foreign inputs. From that point of view, our calibration is conservative.

By definition, the remaining input purchases, those not sourced abroad, include tradable goods purchased in France and all expenses on non-tradable inputs. While we do not have any information on how these domestic expenses are spread across sectors, we can recover the firm-level share of individual input purchases as $\sum_i \gamma_{nn,ij}(f) = 1 - \sum_{m \neq n} \sum_{i \in T} \gamma_{mn,ij}(f)$. This domestic input share is then assigned to domestic input sectors using information in the WIOD:⁹

$$\gamma_{nn,ij}(f) = \frac{\gamma_{nn,ij}}{\sum_i \gamma_{nn,ij}} \times \sum_i \gamma_{nn,ij}(f).$$

We have tested an alternative calibration strategy in which the input coefficients for non-traded sectors are all set exactly to their values in the WIOD. The remaining (homogeneous) share in input purchases is then spread across tradable sectors and countries using the bilateral import shares available at the firm level. The residual which corresponds to tradable inputs purchased domestically is spread across sectors using the WIOD coefficients. Note that this strategy tends to underestimate the share of tradable goods that are purchased domestically, i.e., it overestimates the participation of French firms to foreign input markets. For this reason, we have chosen to use the more conservative strategy described above as our benchmark.

⁹Our definition of non-tradable sectors is somewhat unconventional since we de facto exclude from the tradable sector all services that are potentially traded but that we do not observe in the customs data. As a consequence, some of our NT sectors might display strictly positive foreign input shares in WIOD, i.e. $\gamma_{mn,ij} \neq 0$ for $j \in NT$. We adjust the WIOD data to make them consistent with our definition of non-tradable sectors by allocating all purchases from a NT sector to the same French sector, i.e.: $\gamma_{nn,ij} = \sum_m \gamma_{mn,ij}$ and $\gamma_{mn,ij} = 0, \forall i \in NT$. We apply the same adjustment to the other countries in the sample, to ensure comparability.

Appendix B Theory and Quantitative Implementation

B.1 The GDP Deflator Construction in the Model

The GDP deflator measures the change in prices of output produced, rather than consumed. Let the base period be year t throughout. At time $t + 1$, the real GDP (i.e., expressed in base period prices) is defined as the time $t + 1$ quantities produced evaluated at year t prices. Denote the real GDP by $Y_{m,t+1}$. Then the gross change in the GDP deflator is defined implicitly by:

$$\widehat{P}_{m,t+1}^G = \frac{\widehat{Y}_{m,t+1}^{NOM}}{\widehat{Y}_{m,t+1}}.$$

In turn, the real GDP level is:

$$Y_{m,t+1} = \sum_f \sum_n \frac{1 + \alpha_{m,j}(f)(\rho_j - 1)}{\rho_j} Q_{mn,j,t+1}(f) p_{mn,j,t}(f),$$

and thus its gross change is:

$$\widehat{Y}_{m,t+1} = \sum_f \sum_n \omega_{m,j,t}(f) \widetilde{s}_{mn,j,t}(f) \widehat{Q}_{mn,j,t+1}(f),$$

which is, intuitively, the value-added weighted change in quantities. In practice, national statistical agencies compute the real GDP change $\widehat{Y}_{m,t+1}$ by using sectoral quantity changes ([Bureau of Economic Analysis, 2017](#)). The sectoral quantity changes are obtained by deflating the nominal growth rates of sectoral output by either CPI or PPI price indices for the sectors. In our implementation, we stick closely to this procedure, but compute quantity changes at the firm-destination rather than sector-level.

B.2 Model Solution and Calibration

The model is solved for the changes for all variables numerically by relying on the equilibrium equations outlined in the main text. In particular, we solve for the following equilibrium variables:

1. Changes in trade values $\widehat{X}_{mn,j,t+1} \forall m, n, j$
2. Changes in wages $\widehat{w}_{n,t+1} \forall n$
3. Changes in the price indices $\widehat{P}_{n,t+1} \forall n, \widehat{P}_{n,j,t+1} \forall n, j, \widehat{P}_{mn,j,t+1} \forall m, n, j$
4. Next period's trade shares $\pi_{mn,j,t+1}^c \forall m, n, j, \pi_{nk,j,t+1}(f) \forall k, n, j, f, \pi_{n,j,t+1}^l(f) \forall n, j, f, \pi_{mn,ij,t+1}^x(f) \forall n, m, i, j, f$.

The solution of the model further requires setting parameter values for ρ_j , and ϑ_j . We base the parameter values either on those in the previous literature, or use firm-level data (for France) or sector-level information from WIOD to calculate them (see [Section 2](#) for more details).

We further require several base period data series, either at the firm or sector level. Specifically, we require information on:

1. Gross sales $X_{mn,j,t} \forall m, n, j$
2. Final consumption shares within a sector across sources $\pi_{mn,j,t}^c \forall m, n, j$
3. Firm-level within sector, within-destination trade shares $\pi_{nk,j,t}(f) \forall k, n, j, f$
4. Final consumption spending $P_{n,t}C_{n,t}$
5. Shares of labor (factor) income, pure profits, and deficits in final consumption spending $s_{n,t}^L$, $s_{n,t}^\Pi$ and $s_{n,t}^D \forall n$
6. Initial input shares $\pi_{n,j,t}^l(f) \forall n, j, f$, $\pi_{mn,ij,t}^x(f) \forall m, n, i, j, f$.

The construction of these variables and the relevant data sources are described in [A](#).

B.2.1 Satisfying market clearing

In order to proceed correctly with the hat algebra in each sector/country pair, in the pre-period the market clearing condition in levels must be satisfied:

$$X_{mn,j,t} = \pi_{mn,j,t}^c \pi_{n,j,t}^c P_{n,t} C_{n,t} + \sum_i \frac{\rho_i - 1}{\rho_i} \sum_{f \in i} (1 - \pi_{n,i}^l(f)) \pi_{mn,ji}^x(f) \sum_k \pi_{nk,i,t}(f) X_{nk,i,t}. \quad (\text{B.1})$$

In the data, this is unlikely to be the case. We therefore adopt the proposed solution: in each mn, j, t , trivially we can find a wedge $\zeta_{mn,j,t}$ such that conditional on all the other data, [\(B.1\)](#) does hold with equality:

$$X_{mn,j,t} = \pi_{mn,j,t}^c \pi_{n,j,t}^c P_{n,t} C_{n,t} + \sum_i \frac{\rho_i - 1}{\rho_i} \sum_{f \in i} (1 - \pi_{n,i}^l(f)) \pi_{mn,ji}^x(f) \sum_k \pi_{nk,i,t}(f) X_{nk,i,t} + \zeta_{mn,j,t}.$$

Then applying the hat algebra to this equation:

$$\begin{aligned} \widehat{X}_{mn,j,t+1} X_{mn,j,t} &= \pi_{mn,j,t+1}^c \pi_{n,j,t+1}^c \left[\left(\frac{\widehat{w}_{n,t+1}}{\widehat{P}_{n,t+1}} \right)^{\frac{1}{\psi-1}} s_{n,t}^L + \widehat{\Pi}_{n,t+1} s_{n,t}^\Pi + \widehat{D}_{n,t+1} s_{n,t}^D \right] P_{n,t} C_{n,t} \\ &+ \sum_i \frac{\rho_i - 1}{\rho_i} \sum_{f \in i} (1 - \pi_{n,i}^l(f)) \pi_{mn,ji}^x(f) \sum_k \pi_{nk,i,t+1}(f) \widehat{X}_{nk,i,t+1} X_{nk,i,t} \\ &+ \widehat{\zeta}_{mn,j,t+1} \zeta_{mn,j,t}. \end{aligned} \quad (\text{B.2})$$

Next, we solve the entire model while feeding in a “shock” that eliminates this wedge, namely: $\widehat{\zeta}_{mn,j,t+1} = 0$. Finding the model solution will give the a set of $\widehat{X}_{mn,j,t+1}$ ’s that are required to arrive at a set of levels of $X_{mn,j,t+1}$ for which the market clearing condition is satisfied with equality for every mn, j . Then use these $X_{mn,j,t+1}$ as the starting values for all the real counterfactuals we want to run. The antecedent of this approach is in [Costinot and Rodríguez-Clare \(2014\)](#), where they use a similar device to eliminate the trade deficits.

Table A1. Summary Statistics of Firms, by Sector

WIOT sector	# firms	Share VA	Traded/ non-traded
Agriculture, Hunting, Forestry, Fishing	7,718	.0067	T
Mining, Quarrying	1,022	.0041	T
Food, Beverages, Tobacco	10,883	.0354	T
Textile Products	1,684	.0039	T
Leather, Footwear	2,501	.0058	T
Wood Products	3,045	.0044	T
Pulp, Paper, Publishing	7,721	.0202	T
Coke, Refined Petroleum, Nuclear Fuel	50	.0056	T
Chemical Products	2,051	.0358	T
Rubber and Plastics	2,992	.0155	T
Other Non-Metallic Minerals	2,607	.0127	T
Basic and Fabricated Metals	14,561	.0373	T
Machinery n.e.c.	6,442	.0243	T
Electrical, Optical Equipment	6,599	.0288	T
Transport Equipment	1,804	.0315	T
Manufacturing n.e.c.	4,946	.0086	T
Electricity, Gas, Water Supply	321	.0364	NT
Construction	54,428	.0664	NT
Wholesale and Retail Motor Vehicles and Fuel	25,975	.0218	NT
Wholesale Trade	49,166	.0867	NT
Retail Trade	76,069	.0739	NT
Hotels and restaurants	29,135	.0259	NT
Inland Transport	9,244	.0401	NT
Water Transport	171	.0017	NT
Air Transport	66	.0085	NT
Other Transport Activities	2,068	.0256	NT
Post and Telecommunications	276	.0488	NT
Real Estate	7,726	.0425	NT
Business Activities	31,605	.1849	NT
Education	1,569	.0037	NT
Health and Social Work	6,200	.0200	NT
Other Personal Services	15,283	.0324	NT
Total	385,928	1.000	

Notes: This table reports summary statistics on the number and cumulated value added of firms, by WIOT sector. The data are from INSEE-Ficus/Fare and correspond to year 2005.

Table A2. Estimated Sector-Level Elasticities of Substitution

Sector	Coefficient	Std.Err.	R ²	Observations
Agriculture, Hunting, Forestry, Fishing	0.257 ^a	0.219	0.038	93,577
Mining and Quarrying	2.838 ^b	0.715	0.059	15,482
Food, Beverages and Tobacco	2.146 ^a	0.268	0.041	230,056
Textiles Products	2.866 ^a	0.535	0.035	82,846
Leather and Footwear	1.345	0.439	0.028	119,625
Wood Products	1.765	0.465	0.044	56,825
Pulp, Paper, Publishing	1.850 ^b	0.374	0.030	146,748
Coke, Refined Petroleum, Nuclear Fuel	5.393 ^c	2.367	0.051	3,420
Chemicals Products	1.021	0.605	0.029	151,370
Rubber and Plastics	0.959	0.501	0.032	121,515
Other Non-Metallic Minerals	2.818 ^a	0.551	0.037	64,336
Basic and Fabricated Metal	1.978 ^a	0.254	0.037	291,907
Machinery n.e.c.	2.074 ^b	0.340	0.026	219,840
Electrical, Optical Equipment	1.678 ^c	0.359	0.028	211,029
Transport Equipment	1.261	0.685	0.034	50,812
Manufacturing n.e.c.	3.972 ^a	0.421	0.034	111,364
Electricity, Gas, Water Supply	2.994 ^b	0.920	0.110	2,714
Construction	3.240 ^a	0.107	0.014	434,568
Wholesale and Retail Motor Vehicles and Fuel	2.566 ^a	0.145	0.044	262,160
Wholesale Trade	1.044 ^a	0.163	0.030	938,636
Retail Trade	2.430 ^a	0.118	0.030	679,539
Hotels and Restaurants	2.043 ^a	0.117	0.014	213,275
Inland Transport	3.148 ^a	0.224	0.020	81,017
Water Transport	2.817	1.227	0.089	1,305
Air Transport	2.198	3.474	0.121	961
Other Transport Activities	2.578 ^a	0.463	0.050	16,847
Post and Telecommunications	3.323	1.532	0.126	1,559
Real Estate	-1.753 ^a	0.298	0.013	36,727
Business Activities	1.648 ^a	0.128	0.024	253,941
Education	2.612	0.582	0.045	10,543
Health and Social Work	2.529 ^a	0.232	0.019	45,407
Other Personal Services	3.823 ^a	0.145	0.032	117,678

Notes: This table reports the sector-level estimates of substitution elasticities obtained from equation ???. Standard errors reported under parentheses with ^a, ^b and ^c denoting coefficients significantly different from one at the 1, 5 and 10% levels, respectively.