

Macro Uncertainty and Unemployment Risk*

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Abstract

This paper illustrates how households' heterogeneity is crucial for the propagation of uncertainty shocks. We empirically show that an uncertainty shock generates a drop in aggregate consumption, job finding rate, and inflation: the aggregate consumption response is mainly driven by the consumption response of the bottom 60% of the income distribution. A New Keynesian model with uninsurable unemployment risk rationalizes our findings. Uncertainty shocks induce households' precautionary saving and firms' precautionary pricing behaviors, triggering a fall in aggregate demand and supply. The two precautionary behaviors increase the unemployment risk of the imperfectly insured, who strengthen their precautionary saving behavior. When the feedback loop between unemployment risk and precautionary saving is strong enough, a rise in uncertainty leads to a decrease in inflation. Contrary to standard representative agent New Keynesian models, our model qualitatively and quantitatively matches the empirical evidence on uncertainty shock propagation.

Keywords: uncertainty, inflation, unemployment risk, precautionary savings

JEL Classification: E12, E31, E32, J64

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1 Introduction

The Great Recession has sparked a wide debate on the impact of uncertainty on the macroeconomy. After the seminal paper of Bloom (2009), close attention has been devoted to study the consequences of uncertainty shocks over the business cycle. An increase in uncertainty has been shown to cause a contraction of output and its subcomponents.¹

While the existing literature has focused on the transmission of uncertainty shocks to the macroeconomy, it has not considered how households' heterogeneity affects their propagation. This paper illustrates how heterogeneity is key to the transmission of uncertainty to the macroeconomy and, in particular, to inflation. Empirical work has shown that an increase in uncertainty leads to a drop in output and its main components, as well as a drop in inflation, and an increase in unemployment. The theoretical literature, on the other hand, while being able to explain how a rise in uncertainty propagates to output, consumption, and unemployment, has not been successful in robustly explaining why inflation drops.² Our paper shows that households respond heterogeneously to increases in uncertainty and this heterogeneity is able to explain why inflation decreases following an uncertainty shock.

To corroborate the already existing empirical evidence on the propagation of macro uncertainty shocks, we start by estimating a vector autoregression (VAR) of macro variables, labor market variables, and the macro uncertainty index of Jurado et al. (2015). We use a recursive identification where macro uncertainty is ordered first. We show that a rise in macro uncertainty leads to a drop in output, the job finding rate, consumption, and inflation, and an increase in the unemployment rate and the separation rate. To gain a deeper understanding of the mechanism driving the macro dynamics, we estimate a VAR by using consumption and income micro data from the Consumer Expenditure Surveys (CEX). This allows us to study the heterogeneous response of consumption across the households' income distribution. We show that the response of aggregate consumption is driven by the response of households belonging to the bottom 60% of the income distribution. Instead, the consumption response of households in the top 40% of the income distribution is not significant.

To rationalize these findings, our paper proposes a theoretical mechanism whereby an increase in macro uncertainty results in a drop in inflation and generates responses of output, consumption, unemployment rate, job finding rate, and separation rate, which are quantitatively, as well as qualitatively in line with the empirical evidence. We develop a dynamic stochastic general equilibrium model with the following

¹Following the macro literature, we use the word 'uncertainty' to refer to 'objective uncertainty' or 'risk', in which the probabilities are well understood by all agents. There could be an alternative source of uncertainty, that is ambiguity, in which the probabilities are not well understood.

²While Leduc and Liu (2016) show that an uncertainty shock resembles an aggregate demand shock as it increases unemployment, while decreasing inflation, Fasani and Rossi (2018) argue that their result hinges upon the Taylor rule specification and it can actually be flipped by using different Taylor rules.

features: household heterogeneity induced by unemployment risk and imperfect risk sharing à la [Challe et al. \(2017\)](#), labor market search and matching (SaM) frictions à la [Mortensen and Pissarides \(1994\)](#), and [Calvo \(1983\)](#)-type price rigidities. We model uncertainty as a second moment shock to technology.

Within this framework, we study how a positive uncertainty shock propagates throughout the economy. In representative agent New Keynesian models such as [Born and Pfeifer \(2014\)](#), [Fernández-Villaverde et al. \(2015\)](#), and [Mumtaz and Theodoridis \(2015\)](#), uncertainty shocks have two effects. The first effect is on aggregate demand and works through the precautionary saving behavior of risk-averse households. Due to the convexity of the marginal rate of substitution between present and future consumption, higher uncertainty induces households to increase their savings. The second effect is on aggregate supply and works through the precautionary pricing behavior of firms. When uncertainty increases, firms which are allowed to reset their price, increase it to self-insure against the risk of being stuck with low prices in the future. Since the increase in prices induced by the precautionary pricing behavior of firms is stronger than the drop in prices induced by the precautionary saving behavior of risk-averse households, inflation increases after a positive uncertainty shock. Enhancing this framework with households' heterogeneity adds an indirect channel of precautionary savings, which has powerful implications on the propagation of uncertainty shocks. This channel works as follows. The drop in aggregate demand and aggregate supply induces firms to lower their vacancy posting. This reduces households' job finding rate and increases unemployment risk. Since some households are borrowing constrained and subject to only partial risk sharing, an increase in unemployment risk pushes them to further strengthen their precautionary saving behavior. When the feedback loop between precautionary savings and unemployment risk sufficiently amplifies the negative demand effects of uncertainty shocks, the latter have deflationary effects. Moreover, this feedback effect is able to reinforce the responses of output, consumption, and unemployment rate so as to be quantitatively in line with the empirical evidence.

Related Literature Our paper is related to the fast growing literature of heterogeneous agent New Keynesian (HANK) models, such as those developed by [McKay and Reis \(2016\)](#) and [Kaplan et al. \(2018\)](#). More specifically, it is part of the novel literature of HANK models with SaM frictions, which studies how labor market frictions interact with households' precautionary saving behavior. Within this literature, [Gornemann et al. \(2016\)](#) show how unemployment risk is endogenous to monetary policy, [McKay and Reis \(2017\)](#) investigate optimal social insurance against uninsurable risks to income and unemployment, [Ravn and Sterk \(2017\)](#) study how nominal and labor market rigidities along with household heterogeneity produce amplification and account for key features of the Great Recession, [Ravn and Sterk \(2018\)](#) revisit the qualitative results of the New Keynesian literature in light of the interaction between HANK and SaM, [Cho \(2018\)](#) assesses the importance of unemployment risk for aggregate business cycle dynamics, and [Dolado et](#)

al. (2018) analyze the distributional effects of monetary policy in the presence of SaM frictions and capital-skill complementarity. More precisely, our paper is related to a specific stream of the HANK literature, which introduces households' heterogeneity in a simplified, but effective framework. This setup allows us to gain tractability, which is essential to study the propagation of uncertainty shocks, while at the same time retaining the main feature of introducing households' heterogeneity, which is the precautionary saving motive. This framework is presented by Challe et al. (2017), who construct and estimate a tractable HANK model with SaM frictions, where the cross-sectional heterogeneity of households remains finite dimensional. A similar framework where households' heterogeneity is kept to the minimum to retain model tractability is the one of Challe (2019), who studies optimal monetary policy in the presence of uninsured unemployment risk and nominal rigidities. To our knowledge, our paper is the first to study uncertainty shocks in the context of a HANK model with SaM frictions and highlight how these features are crucial to explain the propagation of uncertainty throughout the economy.

The second stream of literature this paper is related to is the one on uncertainty. Since the seminal work of Bloom (2009), many papers have studied how uncertainty affects economic activity. The literature has focused on different types of uncertainty: financial uncertainty (Ludvigson et al., 2019), stock market volatility (Bloom, 2009 and Basu and Bundick, 2017), uncertainty as risk or ambiguity (Backus et al., 2015), consumers' perceived uncertainty (Leduc and Liu, 2016), firm-specific uncertainty (Bachmann et al., 2013), economic policy uncertainty (Baker et al., 2016), and fiscal policy uncertainty (Born and Pfeifer, 2014 and Fernández-Villaverde et al., 2015). This paper focuses specifically on macro uncertainty as estimated by Jurado et al. (2015). Based on a series of recent studies showing that macro uncertainty is exogenous to the business cycle (see e.g. Carriero et al., 2018a, Piffer and Podstawski, 2018, Angelini et al., 2019, and Angelini and Fanelli, 2019), this paper investigates how an exogenous shock to macro uncertainty affects the macroeconomy. Our main contribution to the literature on uncertainty is to highlight the importance of the interaction between households' heterogeneity and labor market SaM frictions in the transmission of uncertainty shocks to the macroeconomy.

Our contribution to the literature on uncertainty is both empirical and theoretical. On the empirical side, this paper studies the propagation of macro uncertainty shocks across different levels of households' income by using CEX Surveys data. This data has been collected by Heathcote et al. (2010), and then used by Anderson et al. (2016) and Ma (2019) to study government spending shocks, by De Giorgi and Gambetti (2017) to analyze the interaction between business cycles and the consumption distribution, and by Wong (2019) to show the effects of demographic changes on the transmission of monetary policy to consumption. On the theoretical side, this paper adds to the literature on uncertainty shock propagation along two dimensions. First, it is able to match quantitatively the empirical responses of consumption and

inflation to an identified uncertainty shock. Second, and most importantly, it is able to generate a decrease in inflation in response to a rise in uncertainty and to uncover the underlying mechanism explaining this decrease. Papers like [Born and Pfeifer \(2014\)](#) and [Mumtaz and Theodoridis \(2015\)](#) obtain an increase in prices as a consequence of higher uncertainty. This is due to price rigidities à la Calvo, which trigger a precautionary pricing behavior of firms. On the other hand, papers like [Leduc and Liu \(2016\)](#), [Basu and Bundick \(2017\)](#), and [Cesa-Bianchi and Fernandez-Corugedo \(2018\)](#) find that an increase in uncertainty leads to a decrease in prices. This is mainly due to their assumption of price rigidities à la [Rotemberg \(1982\)](#). There are multiple reasons why Calvo-type rigidities are preferable to Rotemberg-type rigidities, especially when solving a model at higher order approximation. First, it is quite difficult to attach a structural interpretation to the Rotemberg adjustment cost parameter, as there is no natural equivalent in the data. In contrast, for the Calvo approach various papers have computed average price durations, e.g. [Bils and Klenow \(2004\)](#) and [Nakamura and Steinsson \(2008\)](#). The literature on price rigidities has therefore regularly made use of the first-order equivalence of Rotemberg- and Calvo-type adjustment frictions by translating the Rotemberg adjustment costs to an implied Calvo price duration via the slope of the New Keynesian price Phillips Curve. Second, despite being equivalent to Calvo-type rigidities at first order approximation, Rotemberg-type rigidities generate opposite responses of prices to uncertainty shocks as shown in [Oh \(2019\)](#). In particular, Rotemberg-type rigidities lack the precautionary pricing channel, which has been shown to be at play by micro-founded menu cost models ([Vavra, 2014](#) and [Bachmann et al., 2018](#)). To the contrary, Calvo-type rigidities allow for this channel and are therefore preferable. Moreover, [Fasani and Rossi \(2018\)](#) show that the responses of inflation to uncertainty shocks in the presence of Rotemberg-type rigidities are very much dependant on the Taylor rule specification and could become positive once empirically plausible degree of interest rate smoothing is considered.

Another paper focusing on uncertainty and heterogeneity is [Bayer et al. \(2019\)](#). Our paper differs from it along several dimensions. While [Bayer et al. \(2019\)](#) study individual households' income volatility, we focus on the propagation of aggregate macro uncertainty. In addition, when solving for aggregate dynamics, [Bayer et al. \(2019\)](#) use a first-order perturbation. Instead, we solve the model at third order, which allows us to obtain a precautionary pricing motive for firms, which would not be present at a first order approximation. Third, we have a frictional labor market, which is necessary to explain the feedback effect between unemployment risk and precautionary saving, which is the one driving our main results.

Last but not least, on the methodological side, we contribute to the literature by studying the propagation of uncertainty shocks in a heterogeneous agent framework that is tractable. Studying uncertainty shocks requires to solve the model to a third order approximation. This gets extremely complicated in fully fledged heterogeneous models, which are solved by [Krusell and Smith \(1998\)](#) projection method and [Reiter \(2009\)](#).

However, [Challe et al. \(2017\)](#)'s assumptions on unemployment spells and binding borrowing constraints allow us to simplify the heterogeneity of households, thus being able to study uncertainty shocks in a tractable framework.

The rest of the paper is structured as follows. Section 2 shows empirical evidence on the responses of macroeconomic variables to an increase in uncertainty. Section 3 describes the New Keynesian model with uninsured unemployment risk. Section 4 displays the quantitative results. Section 5 illustrates how much each precautionary saving and pricing channel contributes to our quantitative results. Section 6 concludes.

2 Empirical Evidence

2.1 Macro Data

Recent papers such as [Carriero et al. \(2018a\)](#), [Piffer and Podstawski \(2018\)](#), [Angelini et al. \(2019\)](#), and [Angelini and Fanelli \(2019\)](#) argue that macroeconomic uncertainty is exogenous when evaluating its effects on the US macroeconomy.³ Based on this extensive evidence, we consider macro uncertainty as exogenous to the business cycle. To show how the US economy reacts to an exogenous increase in uncertainty, we estimate a quarterly frequency VAR with a constant and two lags suggested by the Hannan-Quinn information criterion. The variables included in our VAR are: macroeconomic uncertainty, log of per capita real GDP, the job finding rate, the separation rate, the unemployment rate, log of per capita real consumption (including nondurable goods and services), inflation (first-differenced logged consumer price index), and the policy rate. To measure macroeconomic uncertainty we use the macro uncertainty index estimated by [Jurado et al. \(2015\)](#).⁴ For the job finding rate and the separation rate we use the series computed by [Shimer \(2012\)](#) and updated by [Pizzinelli et al. \(2018\)](#).⁵ As for the policy rate, we use the quarterly average of the effective Federal funds rate. However, since the sample includes a period during which the Federal funds rate hits the zero lower bound (ZLB), from 2009Q1 to 2015Q3 we use the shadow Federal funds rate constructed by [Wu and Xia \(2016\)](#).⁶ This shadow rate is not bounded below by zero and better summarizes the stance of monetary policy. The remaining series are retrieved from the FRED of St. Louis Fed.⁷

³For a thorough review on macro uncertainty and its exogeneity to the business cycle, see [Castelnuovo \(2019\)](#), Section 2.

⁴The updated version of the macro uncertainty series is obtained from the author's website, <https://www.sydneyludvigson.com/data-and-appendixes>. We use the quarterly average of their monthly series with $h = 3$ (i.e., 3-month-ahead uncertainty).

⁵We are grateful to Carlo Pizzinelli for sharing with us the updated version of Shimer's series as can be found at <https://sites.google.com/site/robertshimer/research/flows>.

⁶The shadow Federal funds rate is obtained from the author's website, <https://sites.google.com/view/jingcynthiawu/shadow-rates>.

⁷The retrieved series are the following (FRED series IDs are in parentheses): Gross Domestic Product (GDP), Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL), Civilian Unemployment Rate (UNRATE), Personal Consumption Expenditures: Nondurable Goods (PCND), Personal consumption expenditures: Nondurable goods (chain-type price index) (DNDGRG3M086SBEA), Personal Consumption Expenditures: Services (PCESV), Personal consumption expenditures: Services (chain-type price index) (DSERRG3M086SBEA), and Effective Federal Funds Rate (FEDFUNDS). Then, we obtain the quantity indices by deflating the expenditures. Per capita variables are divided by Civilian Noninstitutional Population

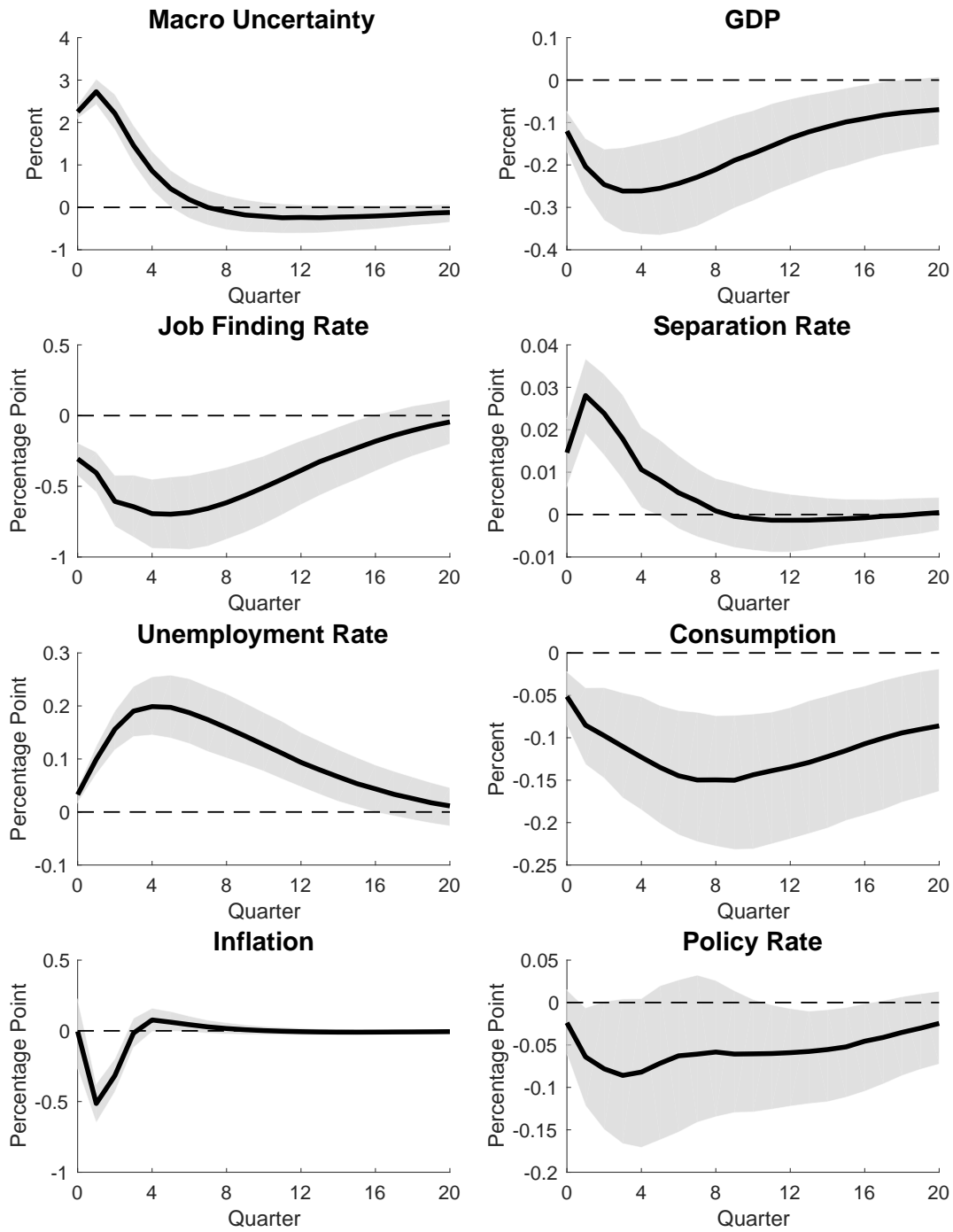


Figure 1: Empirical Responses to One-Standard Deviation Macro Uncertainty Shocks

Note: Grey areas indicate 68 percent bootstrap confidence bands.

We identify uncertainty shocks by using a Cholesky decomposition where macro uncertainty is ordered (CNP16OV).

first. This ordering implies that uncertainty does not react contemporaneously to the other variables included in the VAR. We use US quarterly data over the sample period 1982Q1-2015Q3. As it is common practice in this literature, to avoid parameter instability we start our sample only after the beginning of Paul Volcker’s mandate as the Federal Reserve Chairman.⁸

Figure 1 shows the impulse responses to a one standard deviation shock in the macro uncertainty index. GDP and the job finding rate drop significantly and persistently for sixteen quarters, while the separation rate rises significantly for four quarters. The response of the unemployment rate is positive and persistent and reaches a 0.2 percentage point increase at its peak. The unemployment rate response is in line with the linear specification results of [Caggiano et al. \(2014\)](#), who examine the impact of uncertainty on unemployment dynamics. Moving to the consumption response, we find that it declines at its minimum by more than 0.15 percent after seven quarters. The policy rate drops, but is only mildly significant. Importantly, inflation falls by 0.5 percentage points after one quarter. The response of inflation is in line with what other papers studying uncertainty shocks find - see [Fernández-Villaverde et al. \(2015\)](#), [Bonciani and van Roye \(2016\)](#), [Leduc and Liu \(2016\)](#), [Basu and Bundick \(2017\)](#), and [Oh \(2019\)](#).⁹

To make sure that our results are robust to different Cholesky ordering, sample periods, data series, and VAR specifications, we conduct several robustness checks, which are shown by Figure 2. The first row displays responses of a VAR where we put macro uncertainty as last in the recursive ordering of the variables. The second row reports the impulse responses when we exclude the ZLB period. The third row replaces the CPI inflation with the GDP deflator inflation. The last row shows responses of a VAR with one suggested by the Bayesian information criterion, instead of two lags. In all cases, following a positive uncertainty shock we get: a drop in the finding rate, an increase in the separation rate and the unemployment rate, and decrease in consumption and inflation.

Given this empirical evidence, Section 3 is going to build a model, which is able to replicate our empirical findings. In particular, our goal is to obtain a drop in inflation and a significant amplification in the response of macro and labor market variables following a positive uncertainty shock.

2.2 Micro Data

To gain a deeper understanding of the mechanism driving the macroeconomic dynamics, we carry out a similar VAR exercise to Section 2.1, but we now use consumption micro data instead of aggregate consumption.

This allows us to disentangle the responses of households’ consumption across their income distribution. We

⁸Paul Volcker started his mandate on August 6, 1979.

⁹The few exceptions are [Mumtaz and Theodoridis \(2015\)](#), [Katayama and Kim \(2018\)](#), and [Carriero et al. \(2018b\)](#). The former finds an inflationary effect of uncertainty shocks, while the last two find a non-significant response of inflation to uncertainty shocks. However, they start their sample in 1975Q1, 1960Q3, and 1961M1 respectively, thus including the pre-Volcker period.

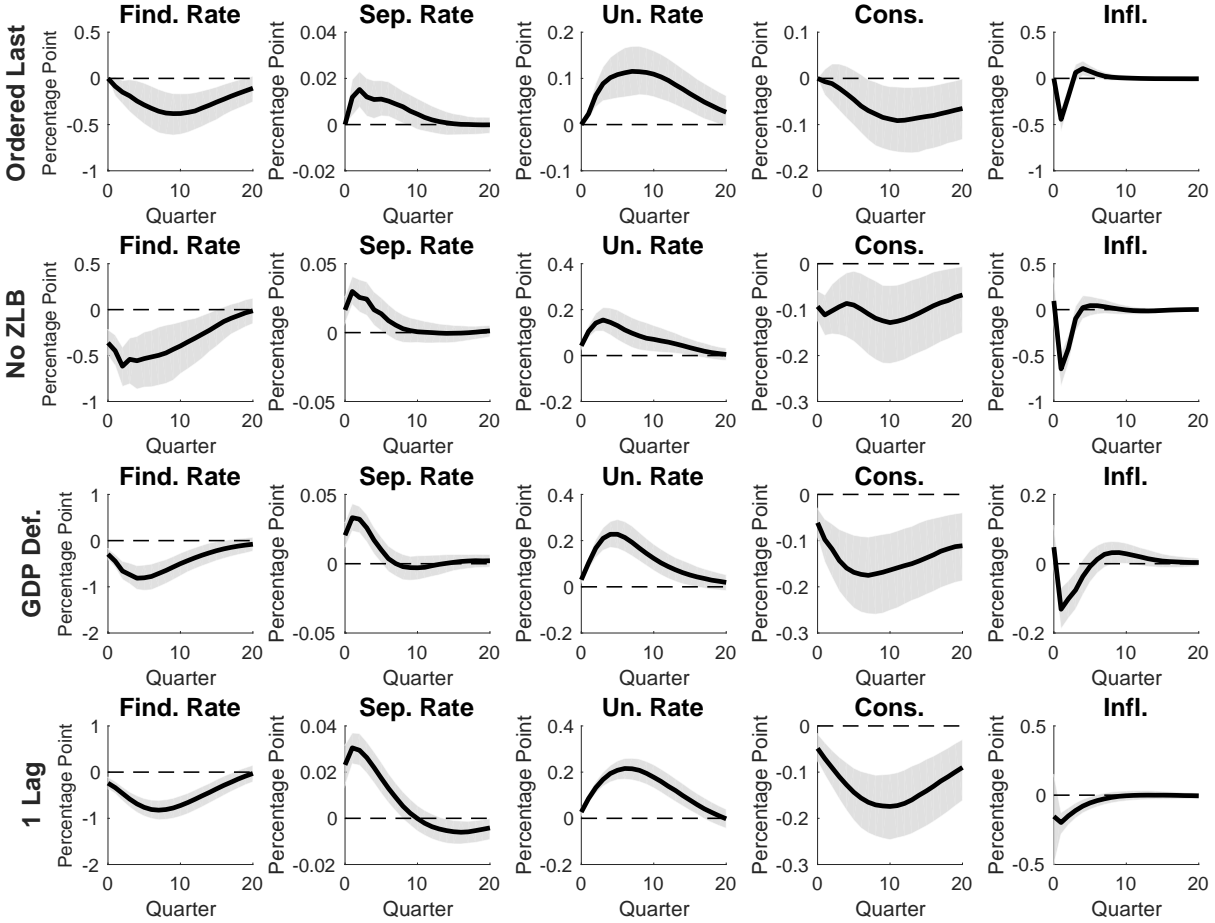


Figure 2: Robustness Checks for Empirical Responses to One-Standard Deviation Macro Uncertainty Shocks

Note: Grey areas indicate 68 percent bootstrap confidence bands.

use the Consumer Expenditure Survey (CEX) data on consumption and income over the period 1982Q1-2015Q3. We follow [Heathcote et al. \(2010\)](#), [Anderson et al. \(2016\)](#), and [Ma \(2019\)](#) in defining nondurable consumption. This comprises food and beverages, tobacco, apparel and services, personal care, gasoline, public transportation, household operation, medical care, entertainment, reading material, and education. As in [Ma \(2019\)](#), income is defined as before-tax income, which is the sum of wages, salaries, business and farm income, financial income, and transfers. To get income and nondurable consumption for households in real per capita values, we divide them by family size (the number of family members), deflate by CPI-U series, and seasonally adjust by X-12-ARIMA.¹⁰

Figure 3 exhibits the consumption responses to macro uncertainty shocks for the bottom 60% and the top 40% of the households' income distribution.¹¹ The response of consumption is heterogeneous between

¹⁰We are grateful to Eunseong Ma for sharing with us his CEX data on consumption.

¹¹We chose the breakdown between the bottom 60% and the top 40% of the income distribution to match the calibration of

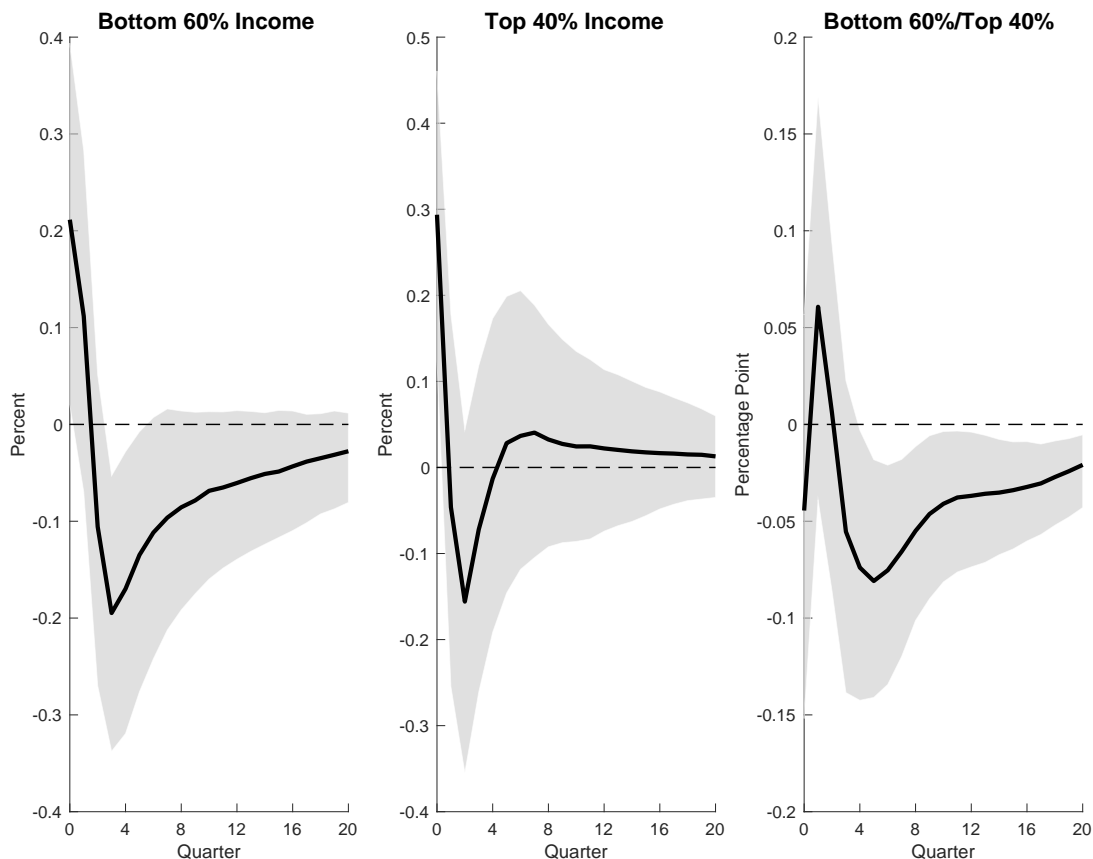


Figure 3: Empirical Responses of Consumption across Income Distribution to One-Standard Deviation Macro Uncertainty Shocks

Note: “Bottom 60% Income” and “Top 40% Income” denote the consumption response of households respectively in the lowest 60% and the highest 40% of the income distribution. Grey areas indicate 68 percent bootstrap confidence bands.

these two groups. In particular, what Figure 3 illustrates is that the drop in aggregate consumption is mainly driven by the consumption response of the bottom 60%. Instead, the consumption response of households in the top 40% is not significant. To show that the heterogeneity in the consumption responses is significant, the third plot of Figure 3 displays the response of the ratio between the consumption of the bottom 60% and the consumption of the top 40%. This response is negative and significant from the fourth quarter onward and remains persistently negative until the twentieth quarter. This indicates that the consumption response of households is heterogeneous: the most responsive to uncertainty are those who are at the bottom of the income distribution.

We check the robustness of our results to the recursive ordering, the sample period, and the VAR specification. Results are shown by Figure 4. The first row reports responses to an uncertainty shock when our model in Section 3, as in Challe et al. (2017) and Cho (2018). However, we have also run the VAR across the five quintiles of the income distribution and we have found that the aggregate response is driven by the response of households in the three lowest quintiles. The response of households in the fourth quintile is only mildly significant, while the response of households in the fifth quintile is not significant.

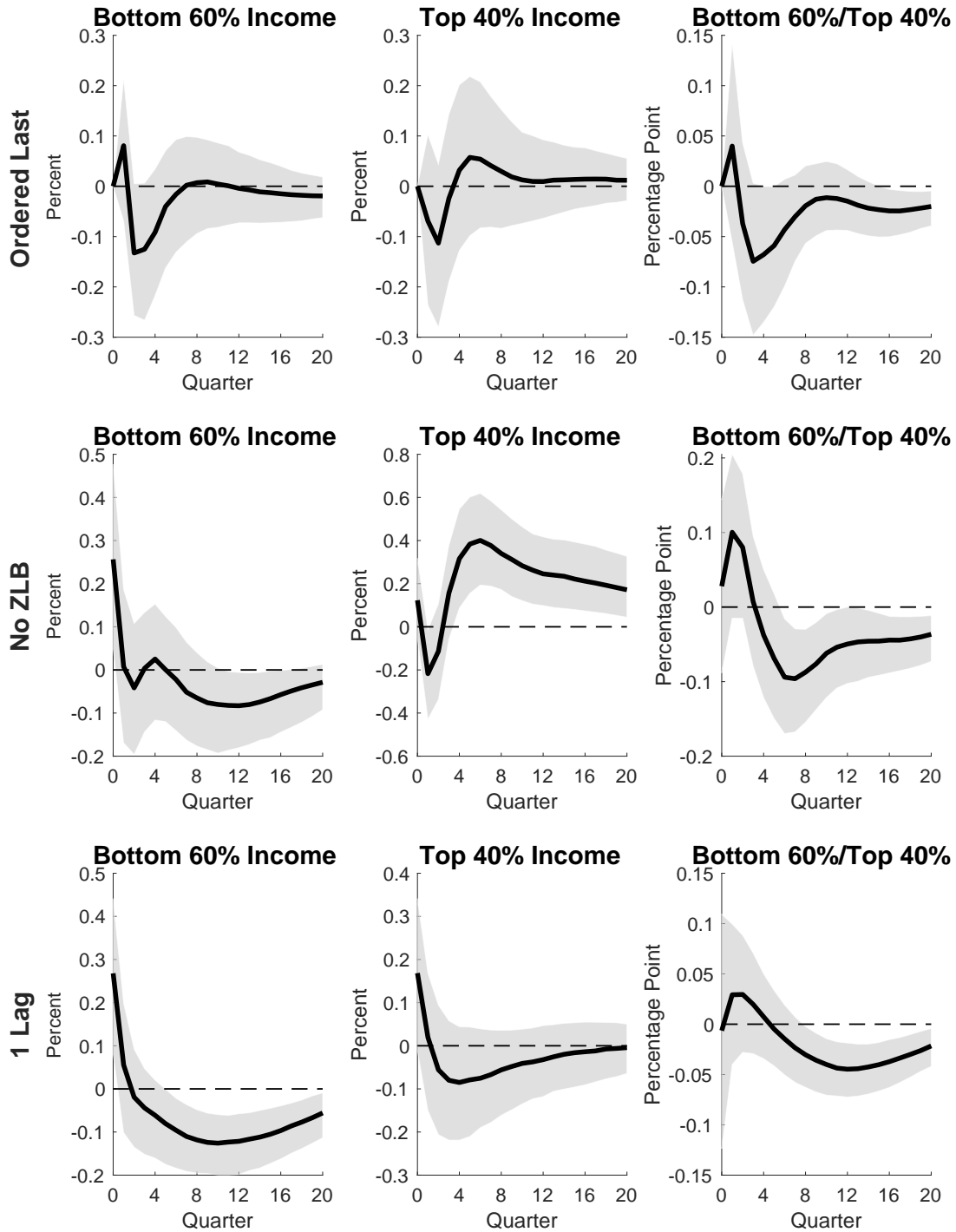


Figure 4: Robustness Checks for Empirical Responses of Consumption across Income Distribution to One-Standard Deviation Macro Uncertainty Shocks

Note: “Bottom 60% Income” and “Top 40% Income” denote the consumption response of households respectively in the lowest 60% and the highest 40% of the income distribution. Grey areas indicate 68 percent bootstrap confidence bands.

the macro uncertainty is ordered last in the Cholesky recursion. The second row exhibits responses of the two income groups when we exclude the ZLB period. The last row displays responses when we run a VAR with only one lag. All robustness checks indicate that the aggregate response of consumption is driven by the response of households in the bottom 60%.

This micro data evidence suggests that households respond in a heterogeneous way across their income distribution. Therefore, households' heterogeneity is an important feature of the data that should not be overlooked when studying the propagation of uncertainty shocks. Hence, in Section 3 we build a tractable model with heterogeneous agents subject to uninsurable unemployment risk to study the propagation of uncertainty shocks throughout the economy.

3 The Model

To reproduce our empirical findings, we build a tractable New Keynesian model with imperfectly insured unemployment risk à la Challe et al. (2017) and Challe (2019), where we introduce a technology process with stochastic volatility. We then simulate a temporary increase in the stochastic volatility of technology and study how the economy reacts. The reduced-form analysis conducted in Section 2 studies the impact of macro uncertainty. This is a comprehensive measure, which aims to capture ‘uncertainty that may be observed in many economic indicators at the same time, across firms, sectors, markets, and geographic regions’, Jurado et al. (2015). As Oh (2019) shows that the theoretical responses to an uncertainty shock do not depend on the source of uncertainty,¹² in our theoretical analysis we capture macro uncertainty by focusing on a technology uncertainty shock.

The model features *imperfect* insurance against idiosyncratic unemployment risk in a New Keynesian framework with labor market frictions à la Mortensen and Pissarides (1994). There are two types of households, a perfectly and an imperfectly insured one. Only perfectly insured households can own firms. Both perfectly and imperfectly insured households participate in the labor and bond market and are subject to idiosyncratic unemployment risk. However, while perfectly insured households fully share risk among each other, imperfectly insured households cannot fully insure themselves against unemployment risk and face a borrowing constraint. The two latter features generate precautionary saving motives for employed households who are not perfectly insured.

To simplify the introduction of both labor market frictions and nominal rigidities, the production side is made of four types of firms as in Gertler et al. (2008). First, labor market intermediaries hire labor from both perfectly and imperfectly insured households, subject to search and matching frictions, and transform it into

¹²More precisely, responses are qualitatively similar when a second moment shock hits technology, preferences, markup, or the interest rate.

labor services. Second, wholesale goods firms buy labor services in a competitive market to produce wholesale goods used by intermediate goods firms. Third, intermediate goods firms buy wholesale goods, differentiate it, and sell it monopolistically while facing price stickiness à la [Calvo \(1983\)](#). Fourth, a competitive final good sector aggregates the intermediate good into a final good used for consumption and vacancy posting costs. The nominal interest rate is set by a central bank which follows a standard Taylor rule.

To specify the timing of events within a period, every period can be divided into three sub-periods: a labor market transition stage, a production stage and a consumption-saving stage. In the first stage, the exogenous state is revealed, workers are separated from firms, firms open vacancies and new matches are created. In the second stage, production takes place and the income components are paid out to the economy agents as wages, unemployment benefits, and profits. In the third stage, asset holding choices are made and the family heads redistribute assets across household members.

[Challe et al. \(2017\)](#)'s assumptions on imperfect risk sharing and a tight borrowing constraint faced by imperfectly insured households allow us to reduce the state space to a finite dimensional object. If we also assume that the borrowing constraint becomes binding after one period of unemployment spell, we can further reduce the heterogeneity of imperfectly insured households to three types. In [Section 3.1 - 3.6](#), we are going to describe the model in detail by focusing on the specific case in which imperfectly insured households are reduced to three types. For notation purposes, aggregate variables are in bold characters. In addition, variables corresponding to the beginning of the labor transition stage are denoted with a tilde.

3.1 Households

There is a unit mass of households in the economy. Each household is endowed with one unit of labor. If at the beginning of the production stage the household is employed, she supplies her unit of labor inelastically. All households are subject to idiosyncratic changes to their employment status. A share $f \in [0, 1]$ of the unemployed households at the beginning of the labor market transition stage finds a job by the beginning of the production stage, while a share $s \in [0, 1]$ loses her job over the same period. There are two types of households: a measure $\Omega \in [0, 1]$ of imperfectly insured ones and a measure $1 - \Omega$ of perfectly insured ones. They have different subjective discount factors. In particular, the discount factor β^P of perfectly insured households is higher than the discount factor β^I of imperfectly insured ones. They all share the same period utility function $u(c) = \frac{(c-h\mathbf{c})^{1-\sigma}}{1-\sigma}$, where c is consumption, \mathbf{c} is the level of consumption habits, and h is a constant habit parameter. Consumption habits are external. We define \mathbf{c}^P as the common consumption habits of the perfectly insured households in the current period. These habits are assumed to be the average of the perfectly insured households' consumption in the previous period. Consumption habits

of the imperfectly insured, instead, depend on their unemployment spell $N \geq 0$. Namely, we assume that imperfectly insured households with unemployment spell N are going to have consumption habits $\mathbf{c}^I(N)$. These habits are equal to the average consumption of the imperfectly insured households with unemployment spell N in the previous period.

3.1.1 Imperfectly Insured Households

Imperfectly insured households face idiosyncratic shocks to their employment state and are subject to a borrowing limit that prevents them from borrowing beyond a given threshold \underline{a} .

Employed households earn a wage w that gets taxed by a rate τ to pay for the unemployment benefit b^u that unemployed households receive. Since the unemployment insurance scheme is balanced every period, the following equation has to hold:

$$\tau w \mathbf{n}^I = b^u (1 - \mathbf{n}^I), \quad (1)$$

where \mathbf{n}^I is the imperfectly insured households' employment rate at the end of the labor market transition stage. Following the literature, we adopt the family structure according to which every imperfectly insured household belongs to a representative family, whose head makes consumption and saving decisions to maximize the family current and expected utility.

There are two crucial assumptions that [Challe et al. \(2017\)](#) make to keep the model tractable, while still preserving the heterogeneity across imperfectly insured households: *i*) the borrowing limit is tighter than the natural debt limit; *ii*) there is only partial risk sharing across members of the imperfectly insured households. In particular, only employed members can fully insure each other by transferring assets. Instead, no transfer is admitted between employed and unemployed members or across unemployed members.

Because of idiosyncratic shocks and imperfect risk sharing, there is heterogeneity across imperfectly insured households. This heterogeneity implies a distribution $\mu(a^I, N)$ of imperfectly insured households over assets a^I and unemployment spells $N \geq 0$. Thanks to the two aforementioned assumptions, for every N the cross-sectional distribution $\mu(a^I, N)$ of imperfectly insured households can be summarized by the unique mass point $a^I(N)$ and the associated number of imperfectly insured households $n^I(N)$.

Given X the vector of aggregate states,¹³ the head of a representative family of imperfectly insured households maximizes the family current and future utility with respect to assets $a^I(N)$ and consumption

¹³See Section 3.6 for the aggregate state definition.

$c(N)$:

$$V^I(a^I(N), n^I(N), X) = \max_{\{a^{I'}(N), c^I(N)\}_{N \in \mathbb{Z}_+}} \left\{ \sum_{N \geq 0} n^I(N) u(c^I(N) - hc^I(N)) + \beta^I \mathbb{E}_{\mu, X} [V^I(a^{I'}(N), n^{I'}(N), X')] \right\}, \quad (2)$$

subject to:

$$a^{I'}(N) \geq \underline{a}, \quad (3)$$

$$a^{I'}(0) + c^I(0) = (1 - \tau)w + (1 + r)A, \quad N = 0, \quad (4)$$

$$a^{I'}(N) + c^I(N) = b^u + (1 + r)a, \quad N \geq 1. \quad (5)$$

Equation (3) is the borrowing constraint, where \underline{a} is higher than the natural borrowing limit. Equation (4) is the budget constraint of an employed household (the unemployment spell N is zero). An employed household consumes $c^I(0)$ and buys assets $a^I(0)$, while receiving after tax income $(1 - \tau)w$ and return from previously held assets $(1 + r)A$. Equation (5) is the budget constraint of a household, who has been unemployed for N periods. This household consumes $c^I(N)$, buys assets $a^I(N)$, gets the unemployment benefit b^u and the return $(1 + r)a$ from previously held assets (of course, if these are negative assets, i.e. debt, r is the interest paid on debt).

If $N = 0$, the value of assets and the employed households' law of motion are given by:

$$A' = \frac{1}{n^{I'}(0)} \left[(1 - s')a^{I'}(0) + f' \sum_{N \geq 1} a^{I'}(N) n^I(N) \right], \quad (6)$$

$$n^{I'}(0) = (1 - s')n^I(0) + f'(1 - n^I(0)). \quad (7)$$

Equation (6) says that the next period value of assets that each employed imperfectly insured household gets is the total of assets that next period employed imperfectly insured households bring divided by the total number of employed imperfectly insured households $n^{I'}(0)$, who belong to the family. The total of assets that next period employed imperfectly insured households bring is given by the fraction of assets that households who *remain* employed bring to the family $(1 - s')a^{I'}(0)$, plus the fraction of assets that households, who *become* employed bring to the family $f' \sum_{N \geq 1} a^{I'}(N) n^I(N)$. Equation (7) says that next period employed imperfectly insured households are given by the fraction of this period employed imperfectly insured households who *remain* employed $(1 - s')n^I(0)$, plus the fraction of this period unemployed imperfectly insured households who *become* employed $f'(1 - n^I(0))$.

If $N \geq 1$, the value of next period assets and next period unemployed households' law of motion are

given by:

$$a^I(N) = a^{I'}(N-1), \quad (8)$$

$$n^{I'}(1) = s'n^I(0) \text{ and } n^{I'}(N) = (1-f')n^I(N-1) \text{ if } N \geq 2. \quad (9)$$

Equation (8) says that the value of next period assets of an imperfectly insured household, who has been unemployed for $N-1$ periods is equal to the value of this period assets of an imperfectly insured household, who has been unemployed for N periods. Equation (9) says that next period unemployed people with one period unemployment spell are the fraction of this period employed households, who become unemployed, while next period unemployed with more than one period unemployment spell are the fraction of this period unemployed households, who stay unemployed.

Imperfectly insured households face a binding borrowing limit after \hat{N} consecutive periods of unemployment. This problem has a particularly easy solution for the case of $\hat{N} = 1$, which, following [Challe et al. \(2017\)](#), is supported by empirical evidence (liquid wealth is fully liquidated after one period). When $\hat{N} = 1$, in every period there are *three* types of imperfectly insured households: $N = 0$, $N = 1$, and $N \geq 2$. To these three types, there are the three following associated consumption levels $c^I(0)$, $c^I(1)$, and $c^I(2)$ for all $N \geq 2$, and the two following assets levels $a^I(0)$, and \underline{a} . $a^I(0)$ is the asset level of employed households, while \underline{a} is the asset level of unemployed households. Since all unemployed households face a binding borrowing constraint, their asset level is the same regardless of their unemployment spell. These three types of imperfectly insured households are in number $\Omega \mathbf{n}^I$, $\Omega s \tilde{\mathbf{n}}^I$, and $\Omega (1 - \mathbf{n}^I - s \tilde{\mathbf{n}}^I)$. In equilibrium, for any $N \geq 0$ the Euler condition for imperfectly insured households is:

$$\mathbb{E}_{\mu, X} [M^{I'}(N)(1+r')] = 1 - \frac{\Gamma(N)}{u_c(c^I(N) - \mathbf{c}^I(N))n(N)}, \quad (10)$$

where $M^I(N)$ is the intertemporal marginal rate of substitution (IMRS) and $\Gamma(N)$ is the Lagrange multiplier associated to the borrowing limit. When the household is employed ($N = 0$), the borrowing limit is not binding. Therefore, $\Gamma(N) = 0$ and the Euler condition holds with equality:

$$\mathbb{E}_{\mu, X} [M^{I'}(0)(1+r')] = 1. \quad (11)$$

Instead, when the household is unemployed ($N \geq 1$), the borrowing limit is binding, $\Gamma(N) > 0$, and $\mathbb{E}_{\mu, X} [M^{I'}(N)(1+r')] < 1$. The IMRS is the ratio of the next-period and the current period marginal utility:

$$M^{I'}(0) = \beta^I \frac{(1-s')u_c^{I'}(0) + s'u_c^{I'}(1)}{u_c^I(0)}, \quad N = 0, \quad (12)$$

$$M^{I'}(N) = \beta^I \frac{(1 - f') u_c^{I'}(N + 1) + f' u_c^{I'}(0)}{u_c^I(N)}, \quad N \geq 1. \quad (13)$$

Equation (12) is the IMRS of an employed household. The denominator is the current period marginal utility. The numerator is the next period marginal utility, which is a weighted average of the household's marginal utility if she remains employed $u_c^{I'}(0)$ times the probability of remaining employed $1 - s'$, and her marginal utility if she becomes unemployed $u_c^{I'}(1)$ times the probability of becoming unemployed s' . Similarly, Equation (13) is the IMRS of an unemployed household. In this case, the numerator is the weighted average of the household's marginal utility if she remains unemployed $u_c^{I'}(N + 1)$ times the probability of remaining unemployed while already being unemployed $1 - f'$, and her marginal utility if she becomes employed $u_c^{I'}(0)$ times the probability of becoming employed f' .

3.1.2 Perfectly Insured Households

The fraction of employed members within every family of perfectly insured households before and after the labor-market transitions stage are denoted by \tilde{n}^P and n^P , respectively. We thus have:

$$n^{P'} = (1 - s') n^P + f' (1 - n^P), \quad (14)$$

$$n^P = \tilde{n}^{P'}. \quad (15)$$

As before, these are family-level variables. The corresponding aggregate variables are denoted by $\tilde{\mathbf{n}}^P$ and \mathbf{n}^P . Employed perfectly insured households earn after tax wage $(1 - \tau)w^P$, while unemployed perfectly insured households get unemployment benefit b^{uP} . Also the unemployment insurance scheme of perfectly insured households is balanced every period, thus the following equation holds:

$$\tau w^P \tilde{\mathbf{n}}^P = b^{uP} (1 - \mathbf{n}^P). \quad (16)$$

Besides having a higher discount factor, what differentiates perfectly insured households from imperfectly insured ones is that there is full risk sharing among their family members, regardless of their employment status. This implies that all family members are symmetric, consume c^P and save $a^{P'}$. The family head of perfectly insured households solves:

$$V^P(a^P, n^P, X) = \max_{a^{P'}, c^P} \{u(c^P - hc^P) + \beta^P \mathbb{E}_{n^P, X} [V^P(a^{P'}, n^{P'}, X')]\}, \quad (17)$$

subject to:

$$c^P + a^{P'} = w^P n^P + (1 + r) a^P + \Pi, \quad (18)$$

where w^P is the real wage that perfectly insured households get and Π is the profit from intermediate goods firms and labor intermediaries, which are owned by perfectly insured households.

Since all perfectly insured households are homogeneous, they have the same Euler equation:

$$\mathbb{E}_X [M^{P'} (1 + r')] = 1, \quad (19)$$

where the IMRS $M^{P'}$ is given by:

$$M^{P'} = \beta^P \frac{u_c^{P'}}{u_c^P}. \quad (20)$$

3.2 Firms

There are four types of firms in the economy. Labor intermediaries hire labor in a frictional labor market and sell labor services to wholesale goods firms. Wholesale goods firms buy labor to produce wholesale goods in a competitive market. Intermediate goods firms buy wholesale goods and sell them to the final goods firms while facing [Calvo \(1983\)](#) price rigidities. Final goods firms aggregate intermediate goods into a final good.

3.2.1 Final Goods Firms

A continuum of perfectly competitive final goods firms combine intermediate goods, which are uniformly distributed on the interval $[0, 1]$, according to the production function:

$$y = \left(\int_0^1 y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (21)$$

where ε is the elasticity of substitution between two intermediate goods. Let p_i denote the real price of intermediate good variety i in terms of final good price. The final goods firm solves:

$$\max_y y - \int_0^1 p_i y_i di, \quad (22)$$

subject to Equation (21). The solution of the maximization gives the final firm's demand of intermediate good:

$$y_i(p_i) = p_i^{-\varepsilon} y, \quad (23)$$

while the zero-profit condition for final goods firms gives:

$$\left(\int_0^1 p_i^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} = 1. \quad (24)$$

3.2.2 Intermediate Goods Firms

Intermediate goods firm i produces x_i with a linear technology $y_i = x_i - \Phi$, where Φ is a fixed cost of production. Firm i 's profit is then given by $\Xi = (p_i - p_m)y_i - p_m\Phi$, where p_m is the real price of intermediate goods in terms of final goods. Intermediate goods firms choose p_i to maximize the present discounted value of future profits subject to the demand curve (23). They face pricing frictions à la Calvo (1983). Therefore, every period only a share $1 - \theta \in [0, 1]$ of firms is allowed to reoptimize over the price. The value of an intermediate goods firm $V^R(X)$ that is allowed to reoptimize is:

$$V^R(X) = \max_{p_i} \{ \Xi + \theta \mathbb{E}_X [M^{P'} V^N(p_i, X')] + (1 - \theta) \mathbb{E}_X [M^{P'} V^R(X')] \}. \quad (25)$$

The value of an intermediate goods firm $V^N(p_{i,-1}, X)$ that is not allowed to reoptimize is:

$$V^N(p_{i,-1}, X) = \Xi + \theta \mathbb{E}_X [M^{P'} V^N(p_i, X')] + (1 - \theta) \mathbb{E}_X [M^{P'} V^R(X')]. \quad (26)$$

Intermediate goods firms which do not reoptimize set their price by fully indexing it to steady state inflation $\bar{\pi}$:

$$p_i = \frac{1 + \bar{\pi}}{1 + \pi} p_{i,-1}. \quad (27)$$

Instead, optimizing firms set their price as:

$$p^* = \frac{\varepsilon}{\varepsilon - 1} \frac{p^A}{p^B}, \quad (28)$$

where

$$p^A = p_m y + \theta \mathbb{E}_X \left[M^{P'} \left(\frac{1 + \pi'}{1 + \bar{\pi}} \right)^\varepsilon p^{A'} \right], \quad (29)$$

$$p^B = y + \theta \mathbb{E}_X \left[M^{P'} \left(\frac{1 + \pi'}{1 + \bar{\pi}} \right)^{\varepsilon-1} p^{B'} \right]. \quad (30)$$

The inflation law of motion associated with the optimal price p^* , the indexation rule (27) and the zero profit condition (24) is

$$\pi = \frac{\theta(1 + \bar{\pi})}{(1 - (1 - \theta)p^{*1-\varepsilon})^{\frac{1}{1-\varepsilon}}} - 1. \quad (31)$$

This pricing generates price dispersion. The price dispersion index $\Delta = \int_0^1 p_i^{-\varepsilon} di$ evolves according to the following law of motion:

$$\Delta = (1 - \theta) p^{*- \varepsilon} + \theta \left(\frac{1 + \pi}{1 + \bar{\pi}} \right)^\varepsilon \Delta_{-1}. \quad (32)$$

3.2.3 Wholesale Goods Firms

The wholesale good y_m is produced by a continuum of perfectly competitive identical firms, which use a linear technology in labor $y_m = z\check{n}$, where \check{n} is labor demand and z is technology. These firms solve:

$$\max_{\check{n}^d} \{p_m z \check{n} - Q \check{n}\}. \quad (33)$$

The real unit price Q of labor services n is given by the first order condition:

$$Q = p_m z. \quad (34)$$

3.2.4 Labor Intermediaries

Labor intermediaries hire labor from both perfectly and imperfectly insured households in a frictional labor market and sell labor services to wholesale goods firms. Every period there is exogenous separation rate ρ between employers and workers. At the same time, labor intermediaries post vacancies at the unit cost κ . There is a skill premium for perfectly insured households over imperfectly insured ones.¹⁴ In particular, while an employed imperfectly insured household provides one unit of labor services and earns a wage w , an employed perfectly insured household provides $\psi > 1$ units of labor services and earns $w^P = \psi w$. Hence, the values for a labor intermediary of a match with imperfectly and perfectly insured households are:

$$J^I = Q - w + \mathbb{E}_X [(1 - \rho') M^{I'} J^{I'}], \quad (35)$$

$$J^P = \psi Q - \psi w + \mathbb{E}_X [(1 - \rho') M^{P'} J^{P'}], \quad (36)$$

which implies that $J^I = \psi J^P$. Moreover, given the vacancy filling rate λ , the free entry condition of labor intermediaries implies that the value of opening a vacancy has to equalize its cost:

$$\lambda (\Omega J^I + (1 - \Omega) J^P) = \kappa. \quad (37)$$

¹⁴We follow [Challe et al. \(2017\)](#) in introducing a skill premium for the perfectly insured. As a matter of fact, consumption heterogeneity in the U.S. cannot be fully imputed to the heterogeneity in asset income. Some heterogeneity in labor income is needed to match the heterogeneity in consumption. We test the sensitivity of our results to the skill premium in Section 4.3.

The aggregate employment rate at the beginning and at the end of the labor market transition stage are given respectively by

$$\tilde{\mathbf{n}} = \Omega \tilde{\mathbf{n}}^I + (1 - \Omega) \psi \tilde{\mathbf{n}}^P, \quad (38)$$

$$\mathbf{n} = \Omega \mathbf{n}^I + (1 - \Omega) \psi \mathbf{n}^P, \quad (39)$$

which implies that $\tilde{\mathbf{n}}' = \mathbf{n}$.

The aggregate unemployment rate \mathbf{u} is given by the unemployed households $\mathbf{1} - \tilde{\mathbf{n}}$ at the beginning of the labor market transition stage plus the fraction ρ of employed households, who loose their job over the period:

$$\mathbf{u} = \mathbf{1} - \tilde{\mathbf{n}} + \rho \tilde{\mathbf{n}}. \quad (40)$$

Firm-worker matches are created through the following matching technology

$$m = \mu \mathbf{u}^\chi v^{1-\chi}, \quad (41)$$

where v are the posted vacancies, μ is the matching efficiency parameter, and χ is the elasticity of matches with respect to unemployed households. The aggregate job finding and job filling rates are given by:

$$f = \frac{m}{\mathbf{u}}, \quad (42)$$

$$\lambda = \frac{m}{v}. \quad (43)$$

Since the workers who loose their job at the beginning of the labor market transition period can be rematched within the same period, the period-to-period separation rate is:

$$s = \rho(1 - f). \quad (44)$$

Given the job finding rate f and the job separation rate s , the law of motion of aggregate labor is:

$$\mathbf{n} = f \tilde{\mathbf{n}} + (1 - s) \tilde{\mathbf{n}}. \quad (45)$$

We assume that wages are set according to the following wage rule:

$$w = \bar{w} \left(\frac{\mathbf{n}}{\tilde{\mathbf{n}}} \right)^{\phi_w}, \quad (46)$$

where ϕ_w indicates the elasticity of wages to deviations of employment from its steady-state value \bar{n} and \bar{w} is the steady state wage.

3.3 Monetary Authority

The monetary authority follows a standard Taylor rule, where the nominal interest rate R reacts to inflation and output growth. The rule is:

$$\frac{1+R}{1+\bar{R}} = \left(\frac{1+\pi}{1+\bar{\pi}}\right)^{\phi_\pi} \left(\frac{y}{\mathbf{y}_{-1}}\right)^{\phi_y}, \quad (47)$$

where \bar{R} is the steady-state nominal interest rate, and ϕ_π and ϕ_y are the reaction coefficients to inflation and output growth.

The real interest rate is determined as follows:

$$1+r = \frac{1+\mathbf{R}_{-1}}{1+\pi}. \quad (48)$$

3.4 Exogenous Processes

The technology z used by wholesale goods firms is subject to first and second moment shocks according to the following stochastic processes:

$$\log z = \rho_z \log z_{-1} + \sigma^z \varepsilon^z, \quad (49)$$

$$\log \sigma^z = (1 - \rho_{\sigma^z}) \log \bar{\sigma}^z + \rho_{\sigma^z} \log \sigma_{-1}^z + \sigma^{\sigma^z} \varepsilon^{\sigma^z}. \quad (50)$$

In particular, $\varepsilon^z \sim N(0,1)$ is a first-moment shock capturing innovations to the level of technology, while $\varepsilon^{\sigma^z} \sim N(0,1)$ is a second moment shock capturing innovations to the standard deviation σ^z of technology. ρ_z and ρ_{σ^z} indicate the persistence of the two processes and σ^{σ^z} is the standard deviation of σ^z . The second moment shock is how we introduce uncertainty into the model.¹⁵ We interpret a positive second moment shock as an increase in uncertainty in the economy.

3.5 Market Clearing

3.5.1 Labor Market

All households face the same job finding rate f and job separation rate s . Since we assume that employment is symmetric between perfectly and imperfectly insured households at the beginning of period zero, for the law of large numbers it remains symmetric at every point in time. Hence, the share of perfectly and

¹⁵Oh (2019) shows that responses of macro variables do not qualitatively depend on the source of uncertainty.

imperfectly insured agents which is employed is the same, and family-level variables are equal to aggregate variables:

$$\tilde{n}^P = \tilde{n}^I = \tilde{\mathbf{n}}^P = \tilde{\mathbf{n}}^I \equiv \tilde{\mathbf{n}}, \quad (51)$$

$$n^P = n^I = \mathbf{n}^P = \mathbf{n}^I \equiv \mathbf{n}. \quad (52)$$

Moreover, the aggregate labor supply is:

$$\Omega \mathbf{n}^I + (1 - \Omega) \psi \mathbf{n}^P = (\Omega + (1 - \Omega) \psi) \mathbf{n}, \quad (53)$$

and the labor market clearing condition is:

$$(\Omega + (1 - \Omega) \psi) \mathbf{n} = \tilde{n}. \quad (54)$$

3.5.2 Assets Market

All households participate in the assets market, which is in zero net supply:

$$\Omega (A + (1 - \mathbf{n}) \underline{a}) + (1 - \Omega) a^P = 0. \quad (55)$$

There are Ω imperfectly insured households and $1 - \Omega$ perfectly insured households. Imperfectly insured households own either A if their budget constraint is not binding or \underline{a} if it is binding.¹⁶ Perfectly insured households own assets a^P .

3.5.3 Goods Market

The final good production y has to be equal to the final good aggregate consumption c plus the cost of posting vacancies:

$$c + \kappa v = y. \quad (56)$$

Aggregate consumption is the share Ω of imperfectly insured households' consumption plus the share $1 - \Omega$ of perfectly insured households' consumption c^P . The former is made of the consumption of imperfectly insured households who are employed $n^I(0) c^I(0)$, who have been unemployed for one period $n^I(1) c^I(1)$,

¹⁶Since we have assumed that the borrowing constraint of unemployed imperfectly insured households becomes binding after one period of unemployment spell, the assets that they own is equal to the borrowing limit \underline{a} regardless of the length of their unemployment spell N . This would not be the case if the borrowing limit became binding after more than one period of unemployment spell.

and who have been unemployed for at least two periods $n^I(2) c^I(2)$:

$$c \equiv \Omega (n^I(0) c^I(0) + n^I(1) c^I(1) + n^I(2) c^I(2)) + (1 - \Omega) c^P. \quad (57)$$

Intermediate goods market is in equilibrium when the intermediate goods demand Δy is equal to its supply $y_i - \Phi$:

$$\Delta y = y_m - \Phi. \quad (58)$$

Finally, the market clearing condition for the wholesale goods is:

$$\int_0^1 x_i di = y_m = z\check{n}. \quad (59)$$

3.6 Aggregate State and Equilibrium

We focus on symmetric equilibrium, where variables at family-level are identical. The aggregate state X is then given by:

$$X = \{\tilde{\mu}(\cdot), a^P, a^I(0), \mathbf{c}^P, \mathbf{c}^I(N)_{N \geq 0}, \mathbf{R}_{-1}, \mathbf{y}_{-1}, \mathbf{\Delta}_{-1}, \tilde{\mathbf{n}}, z, \sigma^z\}. \quad (60)$$

When $\hat{N} = 1$, i.e. when the borrowing constraint becomes binding after one period of unemployment spell, the heterogeneity of the imperfectly insured households can be reduced to three types: the employed type $N = 0$, the unemployed type for one period $N = 1$, and the unemployed type for more than one period $N \geq 2$. These types are in shares of respectively: $\Omega \mathbf{n}$, $\Omega s\tilde{\mathbf{n}}$, and $\Omega(1 - \mathbf{n} - s\tilde{\mathbf{n}})$. In this specific case, a symmetric equilibrium is given by the following conditions:

1. the Euler condition (19) and the IMRS (20) for the perfectly insured households hold, and the Euler condition (11) and the IMRS (12) for the imperfectly insured households hold;
2. the budget constraint for the perfectly insured households (18) and the budget constraints for the three types of imperfectly insured households (4) and (5) with assets determined by (6) and (7);
3. the price set by optimizing firms, the inflation rate and the price dispersion are determined by (28) to (32), and the real unit price of labor services by (34);
4. the aggregate employment and unemployment rates are given by (38), (39), and (40), the job finding rate, the job filling rate, the period-to-period separation rate, and the matching function technology by (42), (43), (44) and (41), the aggregate labor law of motion by (45), the value of a match and the value of opening a vacancy are given by (35) to (37);

5. wages are determined according to (46), social contributions to (1) and (16), and nominal and real interest rates to (47) and (48);
6. the market clearing conditions (51) to (59) hold;
7. consumption habits are as follows: $\mathbf{c}^{P'} = \mathbf{c}^P$, $\mathbf{c}^{I'}(0) = \mathbf{c}^I(0)$, $\mathbf{c}^{I'}(1) = \mathbf{c}^I(1)$, and $\mathbf{c}^{I'}(2) = \mathbf{c}^I(2)$.

3.7 Precautionary Savings

The model features precautionary savings induced by positive uncertainty shocks through two different channels, a direct and an indirect one. The direct channel is due to households' risk aversion. Since all households are risk-averse, they behave in a precautionary manner when uncertainty increases. The indirect channel is due to uninsured unemployment risk. While both perfectly and imperfectly insured households bear unemployment risk, perfectly insured households fully share this risk, while imperfectly insured households face partial risk sharing. Partial insurance further strengthens the precautionary saving behavior of imperfectly insured households. Subsection 3.7.1 and 3.7.2 closely explain the two motives driving the precautionary saving behavior of perfectly and imperfectly insured households.

3.7.1 Direct Precautionary Savings: Household Risk Aversion

Increased uncertainty directly triggers a precautionary saving behavior of risk-averse households. Let's assume that households have the following IMRS:

$$M = \beta \mathbb{E} \left(\frac{c'}{c} \right)^{-\sigma}. \quad (61)$$

Without loss of generality, we can shed light on the precautionary saving behavior by using the steady-state IMRS and our baseline parametrization of $\sigma = 1$. If we assume that under certainty, relative consumption is $cc = 1$,

$$\bar{M}^c = \beta cc^{-\sigma} = \beta. \quad (62)$$

If we assume that, under uncertainty, relative consumption can take either the low value of $cc_l = 0.9$, or the high value of $cc_h = 1.1$, both with probability $q = \frac{1}{2}$, then the IMRS is

$$\bar{M}^u = q \times \beta cc_l^{-\sigma} + (1 - q) \times \beta cc_h^{-\sigma} = 1.01 \times \beta, \quad (63)$$

$$\bar{M}^c < \bar{M}^u. \quad (64)$$

Due to convexity, the IMRS under uncertainty is larger than that under certainty. A higher IMRS induces households to substitute out of consumption towards savings.

3.7.2 Indirect Precautionary Savings: Uninsured Unemployment Risk

Increased uncertainty further strengthens the precautionary behavior of imperfectly insured households through an indirect channel. In particular, higher uncertainty triggers a drop in aggregate demand. This, in turn, generates a fall in production and a decrease in posted vacancies. Less vacancies lead to a drop in the finding rate f , which increases the endogenous separation rate $s = \rho(1 - f)$. A lower finding rate and a higher separation rate increase the imperfectly insured households' propensity to save. This last implication can be derived from the IMRS of imperfectly insured households. In particular, if imperfectly insured households are employed ($N = 0$), their IMRS is as follows:

$$M^{I'}(0) = \beta^I \frac{(1 - s') u_c^{I'}(0) + s' u_c^{I'}(1)}{u_c^I(0)}, \quad N = 0. \quad (65)$$

Their marginal utility of consumption when becoming unemployed $u_c^{I'}(1)$ is higher than their marginal utility of consumption when remaining employed $u_c^{I'}(0)$, as falling into unemployment generates a drop in consumption and marginal utility is decreasing in consumption. Therefore, whenever the separation rate s' rises, the IMRS increases, thus pushing imperfectly insured households to save more. A similar reasoning applies to the IMRS of imperfectly insured households who are unemployed ($N \geq 1$):

$$M^{I'}(N) = \beta^I \frac{(1 - f') u_c^{I'}(N + 1) + f' u_c^{I'}(0)}{u_c^I(N)}, \quad N \geq 1. \quad (66)$$

Whenever the finding rate f' drops, the IMRS increases as the marginal utility of consumption when remaining unemployed $u_c^{I'}(N + 1)$ is higher than the marginal utility of consumption when becoming employed.

Notice that since throughout the paper we assume that the borrowing limit becomes binding after one period of unemployment spell, only the Euler condition for $N = 0$ will hold with equality, while the Euler condition for $N > 0$ will be slack. This implies that the precautionary saving motive will only concern *employed* imperfectly insured households, who are the only type of imperfectly insured households allowed to save. To the contrary, *unemployed* imperfectly insured households will be at their borrowing limit, so their asset position will simply be \underline{a} .

4 Quantitative Results

4.1 Calibration and Solution Method

Table 1 reports the parameter values for a quarterly calibration to the US economy over the period 1982Q1-2015Q3. We mainly follow Challe et al. (2017) and Cho (2018). The share of imperfectly insured households Ω is calibrated to 0.60. Risk aversion σ is set to the standard value of 1.00 to have log utility, while the habit persistence is in the range estimated by Challe et al. (2017). The discount factor of perfectly insured households β^P is set to match an annual interest rate of 3%, while the discount factor of imperfectly insured households β^I is set to target a 21% consumption drop when falling into unemployment. The unemployment benefits are calibrated to target a replacement rate of 33%. As for parameters related to firms, we set the elasticity of substitution between goods to get a 20% markup. The fixed cost of production Φ is set to have a zero steady-state profit, while the price stickiness θ is calibrated to have a price resetting spell of four quarters. Moving to labor market parameters, the matching efficiency μ is set to target a job filling rate of 71%, which follows den Haan et al. (2000). The job separation rate ρ targets a job loss rate of 6.1% and a job finding rate of 73%. The former follows Challe et al. (2017). The latter is computed following Shimer (2005) by using unemployment and short-term unemployment data from the Current Population Survey. The matching function elasticity χ is set according to Petrongolo and Pissarides (2001). The vacancy posting cost κ is calibrated to being 1% of output. The skill premium ψ is set to 42% so as to match the consumption share of the poorest 60% of the households. The wage elasticity with respect to employment ϕ_w is in the range estimated by Challe et al. (2017). As far as monetary policy parameters are concerned, we set the steady-state inflation $\bar{\pi}$ to target a 2% annual inflation, the interest rate responsiveness to inflation ϕ_π to 1.50 and the interest rate responsiveness to output growth ϕ_y to 0.25. Moving to the shock processes, we set the persistence ρ_z and the steady-state volatility $\bar{\sigma}^z$ of the technology shock to the standard values of 0.95 and 0.007. As for the uncertainty shock process, we follow Katayama and Kim (2018) in modelling our counterpart to the macro uncertainty used in Section 2 as stochastic volatility to technology. We set the persistence ρ_{σ^z} and the volatility σ^{σ^z} to 0.85 and 0.37, values which are also in line with Leduc and Liu (2016).

To study the effects of uncertainty shocks, we solve the model using a third-order perturbation method, as suggested by Fernández-Villaverde et al. (2011). The third-order perturbation moves the ergodic means of the endogenous variables of the model away from their deterministic steady-state values. Hence, we compute the impulse responses in percent deviation from the stochastic steady state of each endogenous variable. For that, we use the Dynare software package developed by Adjemian et al. (2011) and the pruning algorithm designed by Andreasen et al. (2018).

Table 1: Quarterly Calibration

| Parameter | Description | Value | Target/Source |
|---------------------|---------------------------------------|-------|--|
| Households | | | |
| Ω | Share of imp. insured HHs | 0.60 | Challe et al. (2017) |
| \underline{a} | Borrowing limit | 0 | Challe et al. (2017) |
| σ | Risk aversion | 1.00 | Log utility |
| h | Habit persistence | 0.60 | Challe et al. (2017) |
| β^I | Discount factor of imp. insured HHs | 0.961 | 21% consumption loss |
| β^P | Discount factor of perf. insured HHs | 0.993 | 3% annual real interest rate |
| b^u | Unemployment benefits | 0.27 | 33% replacement rate |
| Firms | | | |
| ε | Elasticity of substitution btw goods | 6.00 | 20% markup |
| Φ | Production fixed cost | 0.22 | Zero steady-state profit |
| θ | Price stickiness | 0.75 | 4-quarter stickiness |
| Labor Market | | | |
| μ | Matching efficiency | 0.72 | 71% job filling rate |
| χ | Matching function elasticity | 0.50 | Petrongolo and Pissarides (2001) |
| ρ | Job separation rate | 0.23 | 73% job finding & 6.1% job loss rates |
| κ | Vacancy posting cost | 0.037 | 1% of output |
| ψ | Skill premium | 2.04 | Bottom 60% consumption share (42%) |
| ϕ_w | Wage elasticity wrt employment | 1.50 | Challe et al. (2017) |
| Monetary Authority | | | |
| $\bar{\pi}$ | Steady-state inflation | 1.005 | 2% annual inflation rate |
| ϕ_π | Taylor rule coefficient for inflation | 1.50 | Standard |
| ϕ_y | Taylor rule coefficient for output | 0.25 | Standard |
| Exogenous Processes | | | |
| ρ_z | Persistence of technology shock | 0.95 | Standard |
| $\bar{\sigma}^z$ | Volatility of technology shock | 0.007 | Standard |
| ρ_{σ^z} | Persistence of uncertainty shock | 0.85 | Katayama and Kim (2018) |
| σ^{σ^z} | Volatility of uncertainty shock | 0.37 | Katayama and Kim (2018) |

4.2 Baseline Results

Figure 5 shows the impulse responses of the variables of interest to a one standard deviation shock in technology uncertainty. The solid blue line shows the responses of the model with imperfectly insured unemployment risk as described in Section 3. The dashed red line shows the responses of the corresponding representative agent New Keynesian model where unemployment risk is fully insured. This model is identical to the former model except that there are no imperfectly insured households, that is $\Omega = 0$. In this case, there is only one type of households, the perfectly insured ones, who fully share risk. As a benchmark, we first describe the responses of the model with perfect insurance (PI), before illustrating the responses

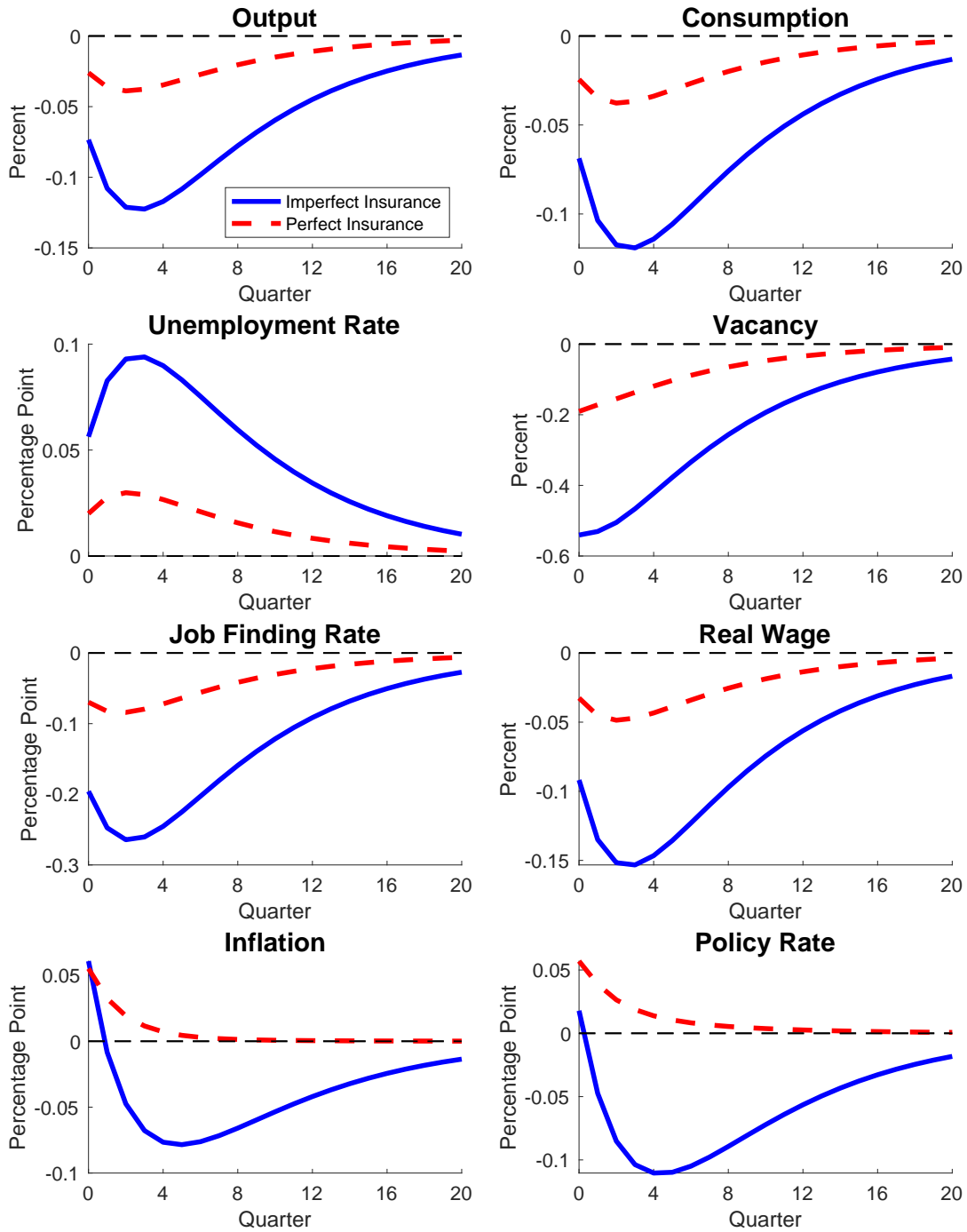


Figure 5: Impulse Responses to One-Standard Deviation Technology Uncertainty Shocks

Note: Impulse responses of output, consumption, vacancy, and real wage are in percent deviation from their stochastic steady state, impulse responses of unemployment rate and job finding rate are in percentage point deviations from their stochastic steady state, while inflation and policy rate are in annualized percentage point deviations from their stochastic steady state.

generated by the model with imperfect insurance (II).

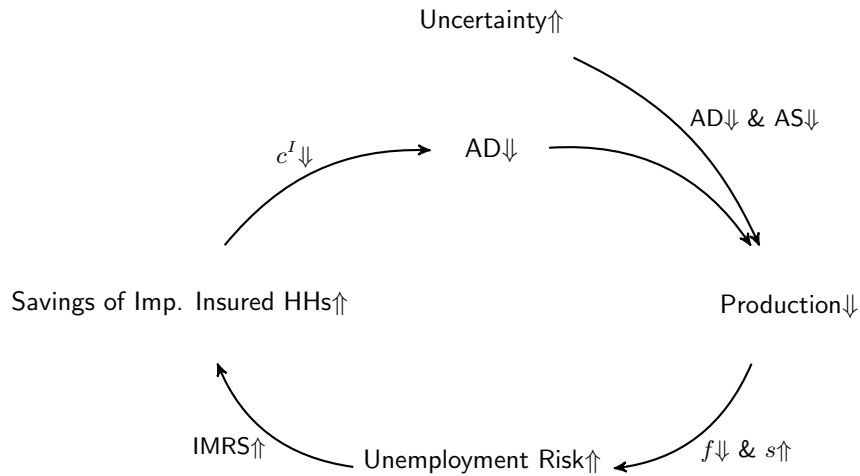


Figure 6: Propagation Mechanism of a Positive Uncertainty Shock

4.2.1 Responses of the Model with Perfect Insurance

In the PI model, a positive uncertainty shock in technology has both an aggregate demand effect through households' saving decisions and an aggregate supply effect through firms' pricing decisions. On the one hand, higher uncertainty induces a negative wealth effect on risk-averse households, who increase savings and decrease consumption (see [Fernández-Villaverde et al., 2015](#), [Leduc and Liu, 2016](#), [Basu and Bundick, 2017](#), and [Oh, 2019](#) for this precautionary saving channel). This causes a drop in aggregate demand. The decrease in aggregate demand reduces the marginal cost that firms are facing and pushes them to lower prices to stimulate demand. On the other hand, an increase in uncertainty triggers a precautionary pricing behavior of firms, which are subject to Calvo pricing. When uncertainty increases, optimizing firms increase their prices to self-insure against the risk of being stuck with low prices in the future (see [Born and Pfeifer, 2014](#), [Fernández-Villaverde et al., 2015](#), and [Oh, 2019](#) for this precautionary pricing channel). Since the increase in prices induced by the precautionary pricing behavior of firms is stronger than the drop in prices induced by the precautionary saving behavior of households, inflation increases after a positive uncertainty shock.

4.2.2 Responses of the Model with Imperfect Insurance

The II model adds a new channel of transmission and amplification of the uncertainty shock to the precautionary saving and pricing behavior described above for the PI model. This is graphically illustrated by [Figure 6](#).

As explained for the PI model, an uncertainty shock causes a drop in aggregate demand triggered

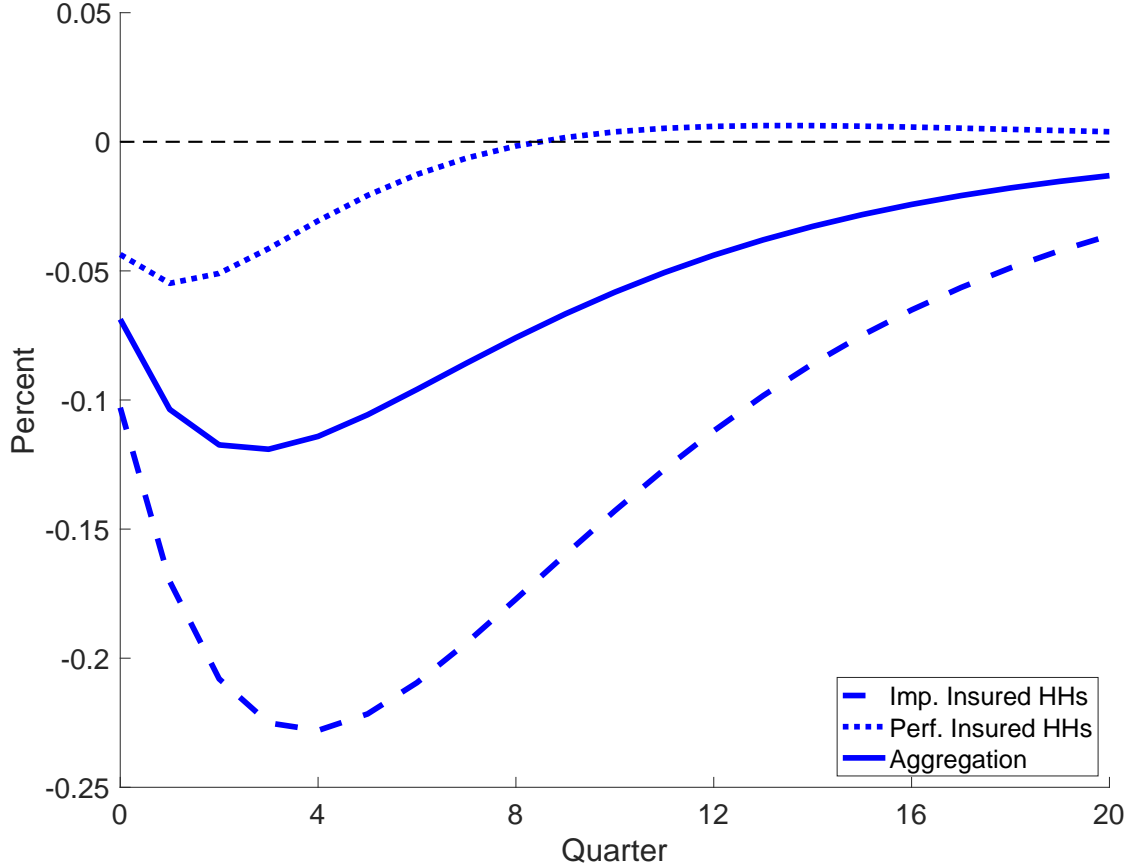


Figure 7: Consumption Heterogeneity

Note: Impulse responses of consumption are in percent deviation from their stochastic steady state.

by the precautionary saving behavior of households. The drop in demand induces firms to lower their vacancy posting, thus reducing the job finding rate and increasing the unemployment rate. At this point the presence of imperfectly insured households becomes key to explain the dynamics of the model. Since imperfectly insured households cannot fully insure against unemployment as they are subject to *imperfect* risk sharing, a higher unemployment risk induces them to further increase savings and decrease consumption. The imperfectly insured households' precautionary saving behavior triggers a feedback loop, which reinforces the drop in aggregate demand. At the same time, firms precautionary pricing behavior generates a reduction in vacancy posting and an increase in unemployment. This further reinforces the precautionary saving behavior of imperfectly insured households and strengthen the feedback loop. Figure 7 illustrates the responses of consumption for both imperfectly (dashed line) and perfectly (dotted line) insured households. Because of the precautionary saving behavior that partial risk sharing induces on imperfectly insured households, their consumption response is much stronger than the one of perfectly insured households.

The presence of heterogeneous agents bears two consequences on the propagation mechanism of un-

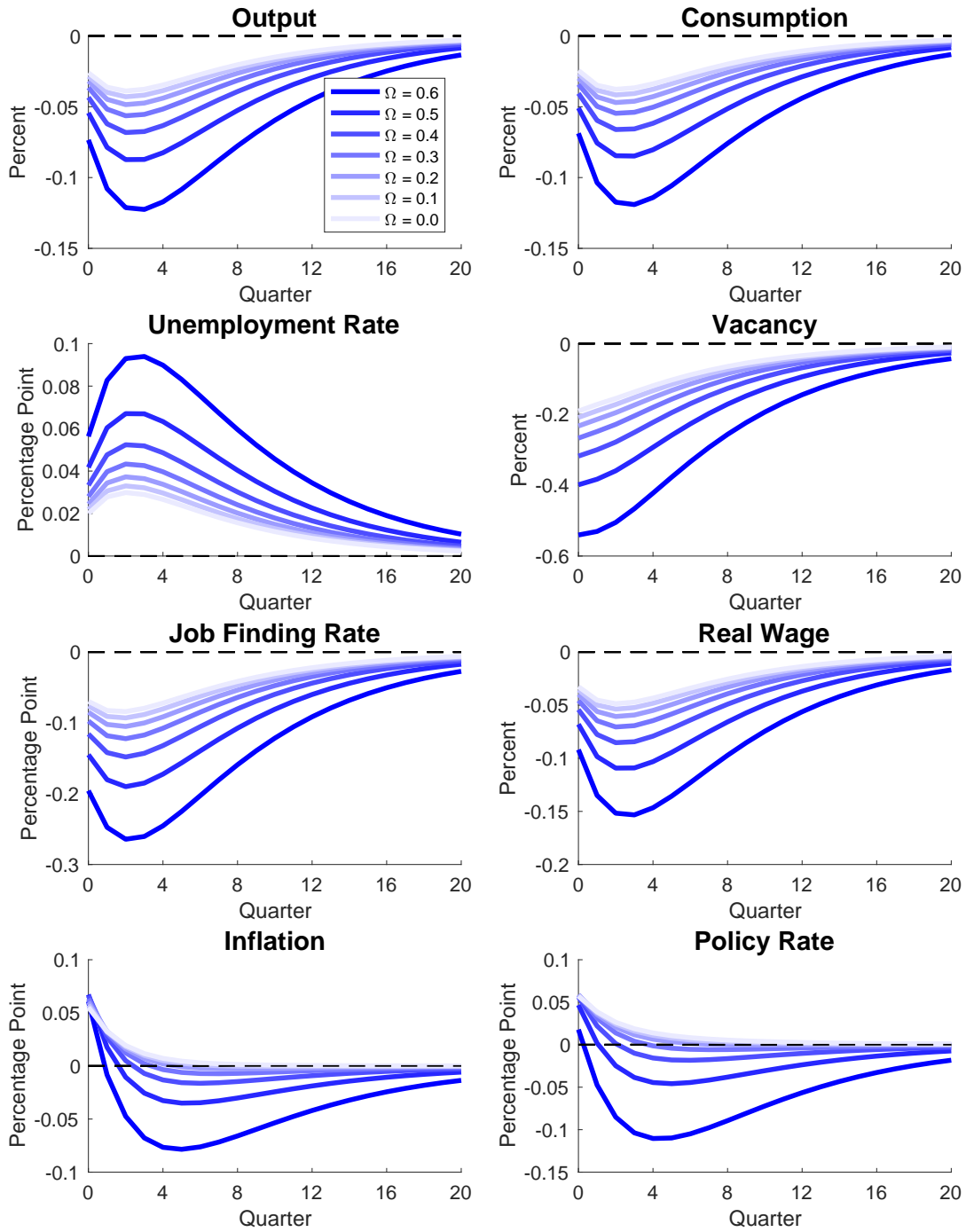


Figure 8: Different Degrees of Heterogeneity

Note: Impulse responses of output, consumption, vacancy, and real wage are in percent deviation from their stochastic steady state, impulse responses of unemployment rate and job finding rate are in percentage point deviations from their stochastic steady state, while inflation and policy rate are in annualized percentage point deviations from their stochastic steady state.

certainty shocks. First, the feedback loop triggered by the precautionary saving behavior of imperfectly insured households is strong enough to induce a drop in prices that outweighs the increase in prices due to the precautionary pricing behavior of optimizing firms. This is the reason why, after two quarters, inflation response becomes negative, which is in line with our empirical results as shown by Figure 1. Second, the feedback loop amplifies all the responses. The precautionary behavior of imperfectly insured households triggers a drop in aggregate demand, which is much stronger than in the PI model. In parallel, the decrease in vacancy posting and the increase in unemployment rate are sharper.

It is worth noticing that our results hinge upon the interaction between the precautionary saving behavior of agents induced by imperfect risk sharing and the precautionary pricing behavior of firms induced by price rigidities à la Calvo (1983). It is the interaction between these two features that allows us to obtain a drop in inflation and an amplification of responses, which quantitatively match the empirical evidence. Absent these features, this would have been possible only by relying on unusual Taylor rules. An example of these is the ‘Alternative Taylor rule’ of Fernández-Villaverde et al. (2015), where the central bank responds to fiscal volatility shocks. This Taylor rule specification is hard to reconcile with central bank independence. Besides, it is still unable to generate a nominal interest rate response to an increase in uncertainty that is consistent with their empirical evidence.

Since the presence of imperfectly insured households is crucial both to determine the response of inflation and to amplify the responses of the other variables, Figure 8 shows how the impulse responses vary when varying the share of imperfectly insured households. On impact, inflation increases regardless of the share of imperfectly insured households. As soon as the negative feedback loop on aggregate demand induced by the precautionary saving behavior of imperfectly insured households kicks in, inflation decreases. Indeed, the higher is the share of imperfectly insured households, the stronger the feedback effect becomes and the more inflation drops. Figure 8 also shows that a bigger share of imperfectly insured households amplifies the responses of the other variables. In particular, output, consumption, vacancies, job finding rate, and wages drop more, while unemployment rate increases more, the higher is the share of imperfectly insured households.

4.3 Sensitivity Analyses

This section illustrates sensitivity exercises on various parameters, which affect the strength of the precautionary saving motive for imperfectly insured households.

The first row of Figure 9 shows how consumption and inflation respond when we vary households’ risk aversion σ . A higher risk aversion generates a stronger precautionary response of imperfectly insured

households, who cannot fully insure against risk. Hence, the more risk-averse imperfectly insured households are, the bigger the shift of their response out of consumption and towards savings. At the same time, inflation, which increases on impact, drops faster the higher the risk aversion is. This is due to the feedback effect that the precautionary saving behavior of households has on aggregate demand.

The second row of Figure 9 shows sensitivity of consumption and inflation response to various consumption differences between employed and unemployed households. Indeed, the bigger the consumption differential is between the two employment states, the stronger the precautionary saving motive that leads employed imperfectly insured households to save more, thus triggering a sharper drop in consumption and inflation.

The third sensitivity exercise that we carry out is on imperfectly insured households' consumption share ($C60/C$). This share is important as it negatively affects the skill premium ψ of perfectly insured households over imperfectly insured ones (as shown in Table 1, we calibrate the skill premium by targeting the share of imperfectly insured households' consumption). The bigger the imperfectly insured households' consumption share, the more the precautionary saving behavior of imperfectly insured households affects aggregate consumption, thus amplifying the drop in consumption and inflation caused by an uncertainty shock.

The next sensitivity exercise is on the elasticity of substitution between two intermediate goods ε . As shown in Oh (2019), a higher elasticity makes the marginal profit curve of intermediate firms more convex, thus strengthening the precautionary pricing behavior of firms. This is why, on impact, a higher elasticity causes a sharper increase in inflation. On the contrary, as soon as the higher prices set by intermediate firms trigger an increase in unemployment, the amplification effect of imperfectly insured households' precautionary saving behavior on aggregate demand kicks in, thus counteracting the price increase and leading to a sharper fall in inflation.

In our baseline model, we have assumed that there is no wage rigidity. Nevertheless, some degree of wage inertia may affect the consumption response of the households as well as the pricing behavior of firms. We therefore check what happens when we modify Equation (46) to introduce some wage rigidity:

$$w = \mathbf{w}_{-1}^{\gamma_w} \left(\bar{w} \left(\frac{\mathbf{n}}{\bar{n}} \right)^{\phi_w} \right)^{1-\gamma_w}, \quad (67)$$

where γ_w indicates the indexation to previous period wage. The first row of Figure 10 shows the sensitivity of consumption and inflation responses to different levels of wage rigidity. The effect of more rigid wages on consumption is not particularly strong. In response to stickier wages, firms tend to increase their prices on impact, thus generating higher inflation.

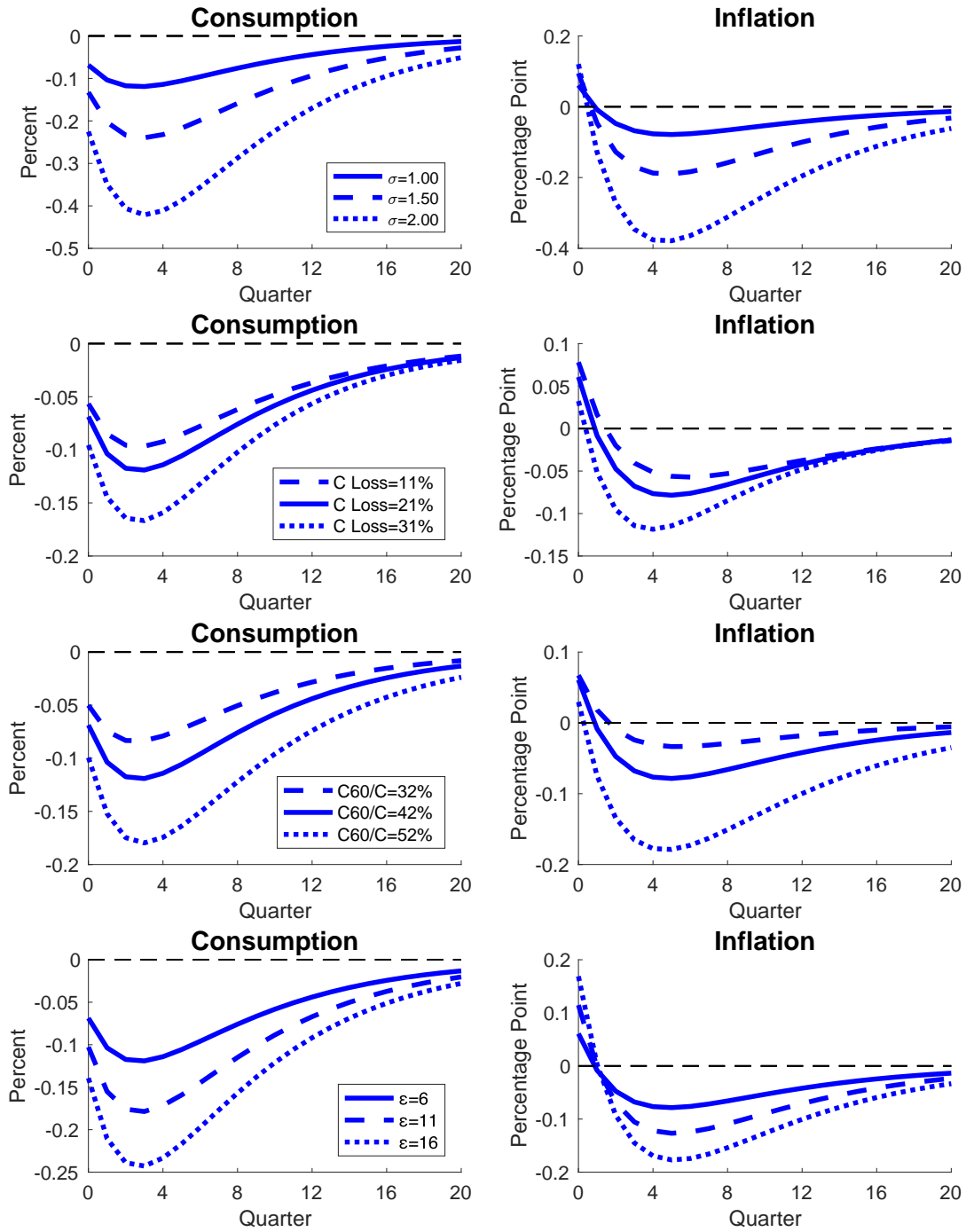


Figure 9: Sensitivity Analyses 1

Note: Impulse responses of consumption are in percent deviation from their stochastic steady state, while impulse responses of inflation are in annualized percentage point deviations from their stochastic steady state.

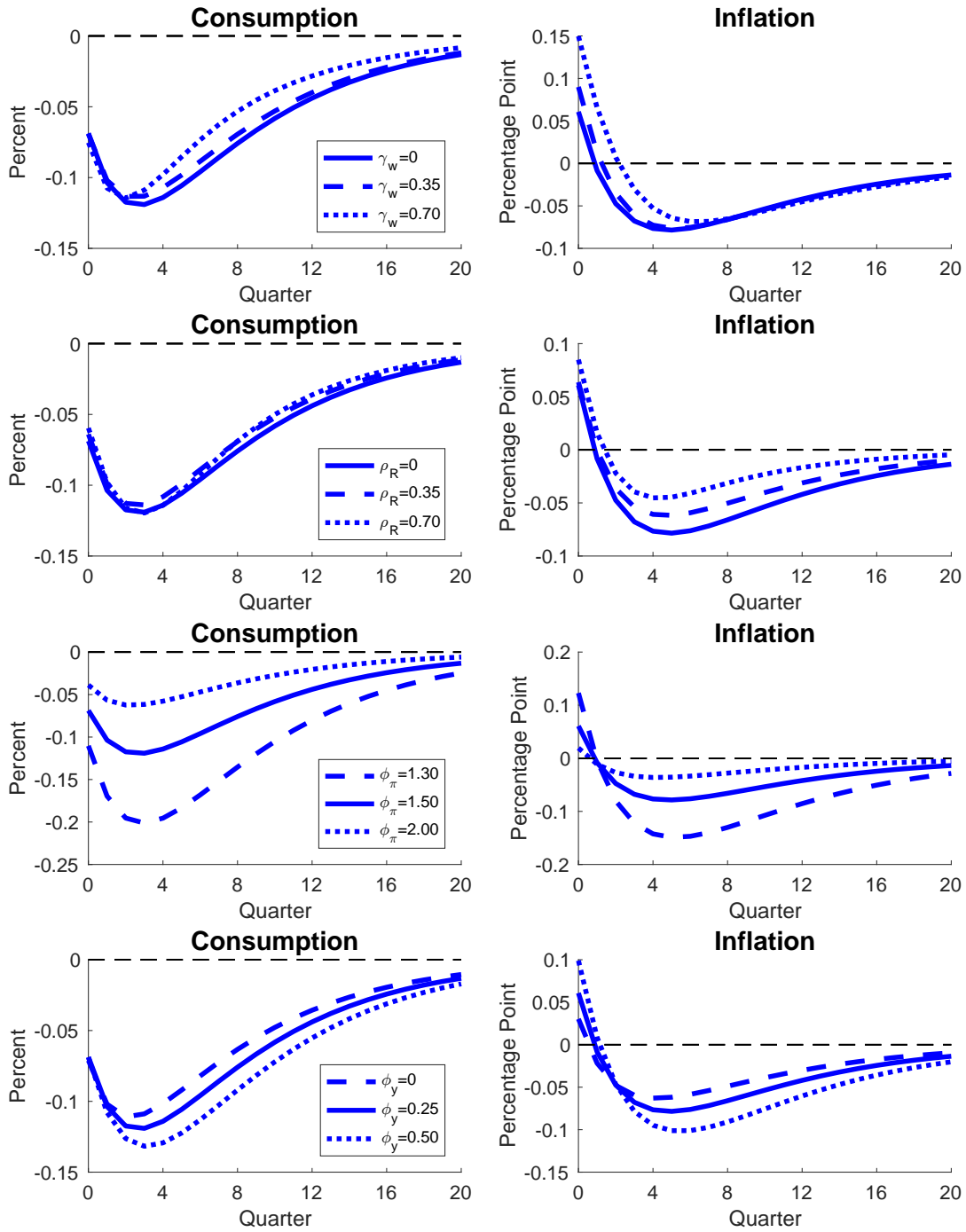


Figure 10: Sensitivity Analyses 2

Note: Impulse responses of consumption are in percent deviation from their stochastic steady state, while impulse responses of inflation are in annualized percentage point deviations from their stochastic steady state.

The next sensitivity exercises concern the parameters of the Taylor rule. In the baseline model we have assumed no persistence in the interest rate. We now check what happens when there is some persistence. We therefore modify Equation (47) as follows:

$$\frac{1+R}{1+\bar{R}} = \left(\frac{1+\mathbf{R}_{-1}}{1+\bar{R}} \right)^{\rho_R} \left(\left(\frac{1+\pi}{1+\bar{\pi}} \right)^{\phi_\pi} \left(\frac{y}{y_{-1}} \right)^{\phi_y} \right)^{1-\rho_R}, \quad (68)$$

where ρ_R is the parameter controlling the degree of persistence. The second row of Figure 10 shows consumption and inflation responses when we vary the persistence ρ_R of the interest rate in the Taylor rule. While interest rate persistence barely affects the consumption response, inflation drops by less the higher the persistence is.

The third and fourth rows of Figure 10 show consumption and inflation responses to an uncertainty shock for different levels of monetary policy responsiveness. In particular, the more responsive monetary policy is to inflation (the higher ϕ_π), the smoother the real interest rate. A smoother real interest rate path reduces the inter-temporal substitution of imperfectly insured households, thus dampening the drop in consumption induced by an uncertainty shock. Indeed, the more responsive monetary policy is to inflation, the less inflation responds to an uncertainty shock. To the contrary, when monetary policy is more responsive to output growth, we get more volatility in consumption and inflation response.

5 Disentangling the Precautionary Channels

To decompose how much of our results is driven by the direct and the indirect precautionary saving channel as well as by the precautionary pricing channel, this section compares the PI and the II models studied in the previous sections to identical models where we substitute the Calvo (1983)-type price rigidity with the Rotemberg (1982)-type price rigidity. As the Rotemberg pricing assumption does not feature any precautionary pricing effect, comparing the responses of models with the two different pricing assumptions allows us to quantify how much of the uncertainty shock propagation is due to the precautionary pricing effect. Before exploring in detail how comparing II and PI models with Calvo and Rotemberg pricing is helpful in disentangling the three precautionary channels, let us discuss what changes need to be made to the model when we substitute Rotemberg pricing to Calvo pricing.

As before, an intermediate good firm chooses price p_i to maximize the present discounted value of future profits subject to the demand curve (23). Now, its value is given by:

$$V^{Rotem}(p_{i,-1}, X) = \max_{p_i} \left\{ \Xi - \frac{\eta}{2} \left(\frac{(1+\pi)p_i}{(1+\bar{\pi})p_{i,-1}} - 1 \right)^2 y + \mathbb{E}_X [M^{P'} V^{Rotem}(p_i, X')] \right\}, \quad (69)$$

where $\frac{\eta}{2} \left(\frac{(1+\pi)p_i}{(1+\bar{\pi})p_{i,-1}} - 1 \right)^2 y$ is a quadratic price adjustment cost. Imposing a symmetric equilibrium across firms implies that $p_i = 1$ and $y_i = y$. The optimal Calvo price equilibrium conditions (28), (29), and (30) are now replaced with the following equation:

$$\eta \left(\frac{1+\pi}{1+\bar{\pi}} - 1 \right) \frac{1+\pi}{1+\bar{\pi}} = \eta \mathbb{E}_X M^{P'} \left(\frac{1+\pi'}{1+\bar{\pi}} - 1 \right) \frac{1+\pi'}{1+\bar{\pi}} \frac{y'}{y} + 1 - \varepsilon + \varepsilon p_m. \quad (70)$$

Moreover, the intermediate goods market clearing condition (58) is replaced with

$$y = y_m - \Phi, \quad (71)$$

as Rotemberg-type frictions do not generate price dispersion. On the other hand, they generate price adjustment costs, which appear in the final good market clearing condition. Hence, condition (56) is replaced with

$$c + \kappa v + \frac{\eta}{2} \left(\frac{1+\pi}{1+\bar{\pi}} - 1 \right)^2 y = y. \quad (72)$$

Except for the equations mentioned above, all the other equilibrium conditions stay the same.

Figure 11 plots impulse responses to a positive uncertainty shock for the II ($\Omega = 0.6$) and the PI ($\Omega = 0$) model with Calvo and Rotemberg pricing. By comparing the four models we can precisely isolate the three precautionary channels: the direct precautionary saving channel, the indirect precautionary saving channel, and the precautionary pricing channel.

Let's first focus on the PI models. The PI model with Rotemberg pricing only features the direct precautionary saving channel explained in Section 3.7.1. Through this channel, a positive uncertainty shock generates a negative wealth effect on risk-averse households, who decrease their consumption and increase their savings, thus lowering aggregate demand. While the only precautionary channel at play in the PI model with Rotemberg pricing is the direct precautionary saving one, the PI model with Calvo pricing adds the precautionary pricing channel. Hence, the difference between the responses of the PI model with Calvo pricing and the PI model with Rotemberg pricing helps us gauging the strength of the precautionary pricing channel. As explained in Section 4.2.1, with Calvo-type frictions firms engage in a precautionary pricing behavior. This behavior leads them to increase prices to such an extent to overcompensate the downward pressure that the aggregate demand drop exerts on prices. That is the reason why the inflation response is positive on impact in the Calvo PI model. On the contrary, the precautionary pricing motive is absent in the Rotemberg pricing model, where all firms are symmetric and are allowed to reset their price every period, even though subject to an adjustment cost - see Oh (2019) for a thorough comparison between the Calvo and Rotemberg pricing models in response to uncertainty shocks. The absence of the precautionary

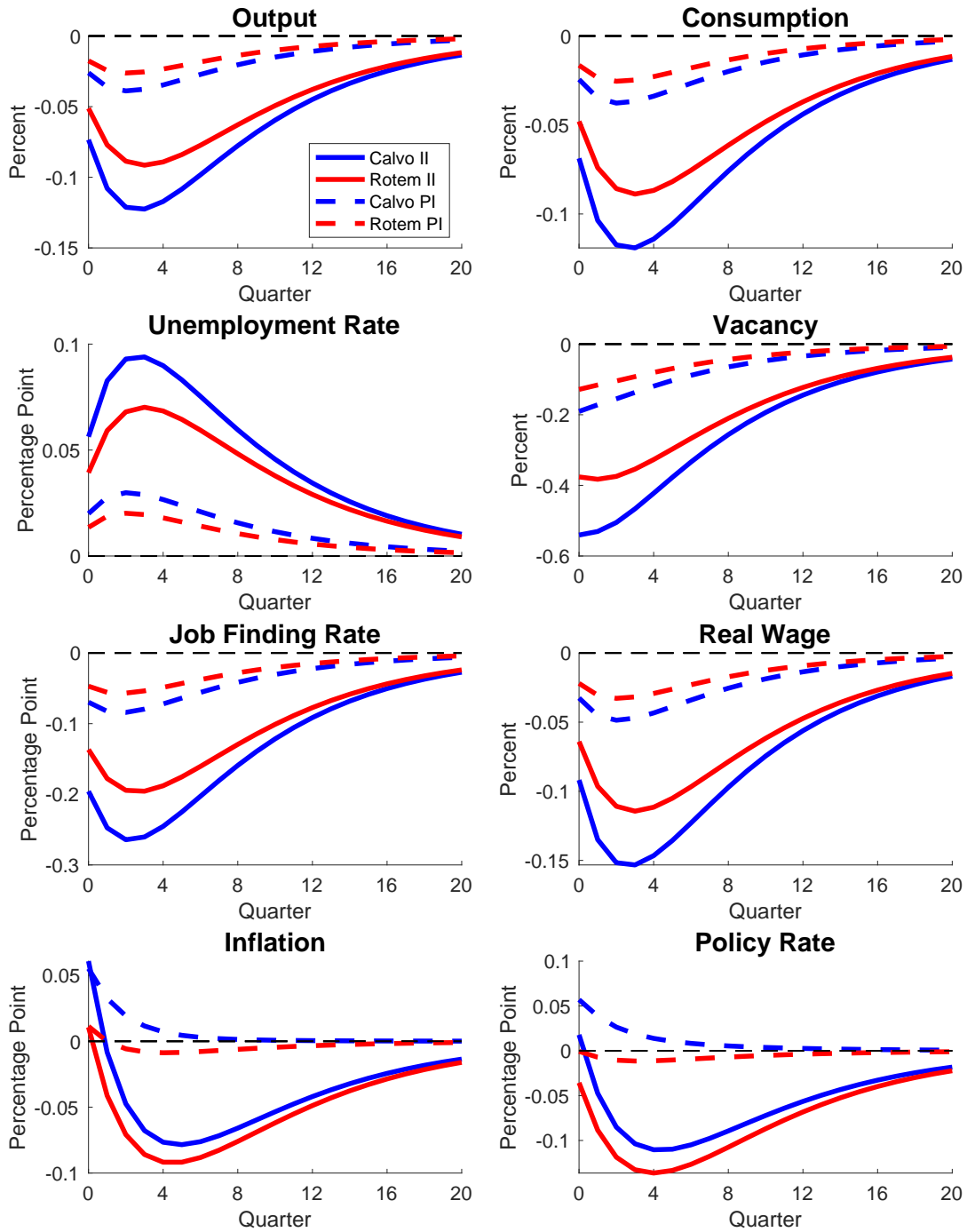


Figure 11: Comparison to Rotemberg Pricing

Note: Impulse responses of output, consumption, vacancy, and real wage are in percent deviation from their stochastic steady state, impulse responses of unemployment rate and job finding rate are in percentage point deviations from their stochastic steady state, while inflation and policy rate are in annualized percentage point deviations from their stochastic steady state.

pricing motive results in a drop in the inflation response to an increase in uncertainty. In addition to the opposite response of inflation, a further difference between the two PI models is that the Calvo pricing model generates more amplified responses. This difference is again induced by the precautionary pricing behavior of firms. Higher prices reduce consumption and push firms to cut their vacancy posting, thus decreasing the job finding rate and increasing the unemployment rate more than in the Rotemberg model. To generate even more amplification and a response of inflation fully in line with the data, a II model with Calvo pricing is necessary. This model features all three precautionary channels: the direct precautionary saving, the indirect precautionary saving and the precautionary pricing channel. Comparing the responses of the II model with Calvo pricing to the PI model with Calvo pricing allows us to isolate the effect of the indirect precautionary saving channel, which is the only precautionary channel that differentiates the two models. The heterogeneity of households in the II model enriches the dynamics of the PI model with the precautionary saving behavior of imperfectly insured households, who reduce their consumption more when unemployment risk rises. This depresses aggregate demand more than in the PI model. This indirect precautionary saving channel is necessary to contemporaneously obtain a drop in inflation as well as an amplification in the responses of the other variables that is quantitatively in line with the empirical evidence.

6 Conclusion

This paper has shown how households' heterogeneity helps explaining the propagation of uncertainty shocks to the macroeconomy. First, it has estimated a VAR of macro variables and the macro uncertainty index of [Jurado et al. \(2015\)](#) to provide empirical evidence that an increase in uncertainty generates a drop in output, consumption, inflation and the job finding rate rate, while it triggers a rise in the unemployment and the separation rate. Second, it has estimated a VAR by using CEX Surveys data instead of aggregate consumption data to show that households respond heterogeneously across the income distribution and that the households belonging to the bottom 60% of the income distribution are more responsive than those belonging to the top 40%. Third, it has built a model with imperfectly insured unemployment risk, SaM frictions and Calvo-type price rigidities to rationalize our empirical findings. In response to a positive uncertainty shock, the interaction between the precautionary saving behavior of partially insured households, the labor market SaM frictions, and the precautionary pricing behavior of firms is able to generate: i) a drop in inflation, and ii) responses of output, consumption, and the policy rate, which are quantitatively as well as qualitatively in line with the empirical evidence.

The goal of our model has been to study the propagation of uncertainty shocks in a model with unemployment risk and imperfect insurance. This has been possible thanks to the tractability of our framework.

Our setup has allowed us to introduce a minimal heterogeneity across households, while at the same time retaining the main precautionary saving motive implied by heterogeneity. As our heterogeneity is kept to the minimum, it does not make our model particularly suitable to study distributional issues.

Lastly, our model abstracts from capital and investment. Introducing capital would provide households with an illiquid asset through which to precautionarily save when uncertainty increases. The option to accumulate capital would dampen the decrease in aggregate demand following a rise in uncertainty. This would somewhat weaken the feedback loop triggered by the precautionary saving behavior of uninsured households, which would nevertheless still be present. To get a response in aggregate demand similar to the model without capital, we would need to give households the possibility to also save through a liquid bond. Further, this addition would allow us to match a more realistic distribution of marginal propensities to consume. We leave the inclusion of capital and a liquid bond as well as a more thorough analysis of their implications to future studies.

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