Aggregate Dynamics and Microeconomic Heterogeneity: The Role of Vintage Technology*

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Abstract

We study the role of capital accumulation for the aggregate dynamics of total factor productivity in a general equilibrium model with rich firm heterogeneity. Using data on the census of incorporated Italian firms, we document that firms that have more recently exhibited large investment episodes, or *spikes*, are more productive than firms that exhibited comparable spikes in a more distant past. Our estimates show that the timing of capital expenditures at the firm level explains about 15 percent of the productivity heterogeneity observed in the sample. Building on Khan and Thomas (2008), we formulate a state-of-the-art model of firm heterogeneity to assess the aggregate relevance of this microeconomic behavior. A non-convex adoption cost prevents firms from adopting the latest technology. In equilibrium, as new and old vintages coexist, the non-degenerate distribution of capital stocks and technologies across firms determines aggregate total factor productivity. After fitting the model to reproduce the distribution of investment rates observed in the data, we show that vintage effects constitute a microeconomic-based amplification mechanism of aggregate shocks relative to a benchmark real business cycle model. A deterioration in financial conditions or a slowdown in the growth rate of technological progress induce firms to postpone investment expenditures leading to aggregate productivity losses. Through this channel, the shift in the microeconomic distribution contributes to deepening the recession and amplifying the effect of the initial shock. In an application to Italian data, we show that the model accounts for about one-third of the stagnant productivity growth observed following the 2012 recession.

JEL Codes: E24; E32; E62; J11; J63; J64.
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1 Introduction

One of the classic questions in macroeconomics is to study how the role of capital accumulation contributes to the dynamics of aggregate productivity. Since the work of Johansen (1959) and Solow (1960), a large body of the literature has emphasized the role of investment for productivity dynamics, as the newer vintages are of better quality. In the aftermath of the Great Recession, this literature has received renewed attention as many advanced economies experienced a slow recovery in investment and gross domestic product (GDP) together with stagnant productivity growth.

In this paper, we make two contributions to the existing literature. First, we bring firm-level evidence to bear on the role of capital accumulation for productivity dynamics. Using data on the census of incorporated Italian firms, we document that firms that have more recently experienced large capital expenditures, or *spikes*, have higher productivity than firms that exhibited spikes in a more distant past. Controlling for a plethora of confounding factors, we estimate this gap at about one percentage point per year for labor productivity and about one-half a percentage point for total factor productivity (TFP). Our results show that about 15 percent of the productivity heterogeneity in the sample depends on the timing of firms' investment behavior. Second, we formulate a state-of-the-art model of firm heterogeneity that explicitly accounts for the empirical regularity that technology adoption is a byproduct of capital accumulation. In the model, a non-convex adoption cost prevents firms from adopting the most recent vintage in every period. In equilibrium, new and old vintages coexist yielding a non-degenerate distribution of capital stocks and technologies across firms. Following aggregate shocks, shifts in this distribution result in endogenous movements in economy-wide productivity that amplify macroeconomic fluctuations beyond the effect of the initial shock.

In the empirical analysis, we employ firm-level data that cover roughly 80 percent of the value added in the Italian economy and span 30 years. The dimension of our panel, both in the cross section and in the time series, allows us to quantify technology vintage effects controlling for many potential confounding factors: Firms' age and size, aggregate-, industry-, and firm-specific effects.

We start by documenting the nature of capital accumulation at the firm-level and its importance for aggregate investment dynamics. As in other advanced economies, investment at the firm-level is a large and infrequent, or *lumpy* episode. On average, only 18 percent of firms exhibit investment spikes or an investment rate above 20 percent, but they account for about two-thirds of total investment in our data. Then, we establish that the probability of firms exhibiting spikes is increasing with the time elapsed since the last spike. This finding shows that the lumpiness of investment at the firm-level is consistent with the presence of non-convex adjustment cost, see Haltiwanger, Cooper and Power (1999) for a discussion.

The timing of investment spikes at the firm level constitutes the basis of our strategy to identify vintage effects in the data. We follow the approach in Power (1998) and use investment age, the time elapsed between the firm's investment spikes, to capture the vintage technology available to firms. Our results highlight that investment age is a pivotal contributor to firms' productivity dynamics: firms with a lower investment age are, other things being equal, more productive than firms with a higher investment age. This result is robust to alternative empirical definitions of spikes, the sample period considered, accounting for firms' propensity to innovate, and the age composition of the firms in our sample. Also, we rule out alternative explanations related to news about current or future profitability as a driver of our result. One significant implication of the empirical analysis is that the timing of firms' investment decisions leads to productivity heterogeneity across firms and, thus, contributes to determining aggregate productivity.

Measuring the macroeconomic relevance of this microeconomic behavior requires a theoretical framework that (i) agrees with the micro evidence on capital accumulation and the timing of investment spikes, (ii) reproduces the link between investment and productivity, and (iii) takes into account general equilibrium effects. As detailed in Section 1.1, despite the advancements in the investment literature, little to none theoretical work is aimed at quantifying the implications of technology adoption for aggregate productivity. We aim to fill this gap in the existing literature.

Our model builds on Khan and Thomas (2008). They formulate a general equilibrium framework with rich firm heterogeneity that reproduces the pattern of capital accumulation at the firm level. We amend their model to include a vintage technology structure. In the model, the firm's productivity includes *(i)* the *permanent* vintage component, that is endogenous to the timing of technology adoption, and *(ii)* a temporary but persistent idiosyncratic component, that is fully exogenous. The vintage component evolves as in a quality ladder model. Firms optimally decide if, and when to adopt the latest vintage, i.e., the latest technology. As this choice is subject to a non-convex adoption cost, the firm's policy functions are of the (S,s) type: some firms adopt the latest technology, while others postpone it. Conditional on the adoption decision, firms optimally choose the capital stock. In equilibrium, vintages of different quality coexist. This microeconomic heterogeneity determines aggregate productivity. Instead, when the adoption cost is set to zero, our framework boils down to the real business cycle (RBC) model where firms' and aggregate productivity coincide and are exogenous as all the firms find optimal to adopt the latest technology in every period.

To study the role of microeconomic heterogeneity in shaping aggregate fluctuations, we parameterize the model to reproduce the cross-sectional distribution of investment rates and investment age in the data, as well as the results of the empirical analysis.[1] In response to aggregate shocks, microeconomic heterogeneity *amplifies* macroeconomic dynamics. The presence of vintage effects at the *firm level* is key for this result: shifts in the distribution of capital stocks and vintage technologies across firms induced by the aggregate shock, also lead to fluctuations in the economy-wide TFP. In turn, the endogenous response of productivity constitutes an additional amplification mechanism for macroeconomic dynamics beyond the effect of the initial shock.

We turn to the quantitative implications of the model to consider the aggregate dynamics following a shock that leads to a temporary deterioration in financial conditions, making it more expensive for firms to undertake capital expenditures. The ensuing drop in investment also results in stagnant productivity, leading to output losses 40 percent larger than losses predicted by a standard *RBC* that abstracts from vintage effects. Also, the vintage technology model accounts for about one-third of the drop in aggregate pro-

[1] Vintages in the data are identified using investment spikes, in the model, firms update their vintage by paying a non-convex cost. Thus, technology adoption does not necessarily require large capital expenditures. Nonetheless, in Section 5.2, we show that the model closely reproduces the estimates in the data.

ductivity observed in the data (productivity is constant in the *RBC* model). This result supports the view that an investment slump, such as the one observed in the aftermath of the Great Recession, contributed to stagnant productivity growth.

We also consider permanent technology shocks. In the context of our model, this amounts to a shock to the efficiency of newer vintages relative to the previous one.[2] When the growth rate of productivity of newer vintages slows down, firms postpone adopting the latest technology and reduce capital expenditures, as the productivity gap with the technology frontier increases less than expected. In the presence of vintage effects, pure technology shocks increase the volatility of the growth rates of aggregate series (such as output and investment) by about one-half relative to the standard model.

Our paper is organized as follows. In Section 1.1, we describe our contribution relative to the existing literature. We document the nature of capital accumulation at the firm level in Section 2 and the relationship between capital accumulation and productivity in Section 3. In Sections 4 and 5, we outline the model and its quantitative performance relative to the data. In Section 6, we perform some experiments to quantify the role of vintage capital for aggregate dynamics. Section 7 concludes.

1.1 Literature Review

Our work contributes to different strands of the existing literature. Our empirical analysis investigates the link between capital accumulation and productivity using firm-level data. After Gordon (1990) and Cummins and Violante (2002), who uses product-level and sectoral data, most of the existing literature on vintage capital has focused on aggregate data, see for instance Hulten (1992), Wolff (1996), Greenwood, Hercowitz and Krusell (1997), and Greenwood, Hercowitz and Krusell (2000). The central insight of these papers is that the growth rate of the price of investment goods (relative to the one of consumption) can be interpreted as a measure of investment-specific technological progress. There is, instead, little systematic evidence on the role of capital accumulation for productivity dynamics at the firm level partly because a rigorous analysis at the firm level requires a

[2] When the cost of adoption is zero; this is equivalent to productivity shocks in the *RBC* model.

set of data not commonly available to researchers. The exceptions Licandro, Maroto Illera and Puch (2005), Power (1998) and Sakellaris and Wilson (2004) provide mixed evidence on vintage effects. Using U.S. manufacturing data Power (1998) finds no evidence that investment spikes contribute to increasing a firm's productivity. Using similar data Sakellaris (2004) and Sakellaris and Wilson (2004), and more recently Licandro, Maroto Illera and Puch (2005) using Spanish manufacturing data find the opposite result. Relative to these studies, our data consists of a cross-section 10 times as large and a sample 2 times as long. Moreover, our data cover all the sectors in the economy except for the financial and the banking sectors.

From a theoretical standpoint, our paper relates to the literature that studies investment dynamics in models with rich firm heterogeneity, see for instance Cooper and Haltiwanger (1993), Caballero and Engel (1999), Khan and Thomas (2008) and Bachmann, Caballero and Engel (2013), to name a few. We retain several elements that have determined the quantitative success of this class of models in accounting for the pattern of capital accumulation at the firm-level. In our model, firms face idiosyncratic shocks and decide if and when to adopt the latest vintage and, conditional on this choice, the next-period stock of capital. The quantitative focus of our analysis distinguishes our work from the abundant literature on vintage capital. As in Solow (1960), in general, the dynamics of capital vintage models cannot be captured through a representative firm unless knife-edge conditions are met. As a result, the number of studies that have confronted vintage models with microeconomic data has been limited to non-existent.[3] For a complete list of references and a historical perspective on the evolution of the literature on vintage capital see the excellent surveys of Boucekkine, de la Croix and Licandro (2011) and Boucekkine and de Oliveira Cruz (2015).

[3] Cooley, Greenwood and Yorukoglu (1997) study the balanced growth path and the transitional dynamics of a deterministic model with two sectors and vintage capital and compare it with the neoclassical growth model. Samaniego (2006) formulates a model that emphasises the role of organizational capital as friction that prevents firms from adopting newer technology.

2 Microeconomic Evidence on Capital Adjustment

In this section, we describe the data set employed to document the role of investment for productivity dynamics. We proceed in steps. First, we provide details about the source of our data; then we report descriptive statistics on the age composition of the census of incorporated Italian firms. Finally, we document the pattern of capital accumulation at the firm level.

2.1 Data Set

We obtained our data set combining different sources. To construct the variables of interest, firm-level investment rates and measures of productivity, we require information on payroll, gross value-added, and employment. See Appendixes A and C for detailed information on data sources and variables construction. The sample spans a period of 30 years, from 1986 to 2015. The data set includes 5,004,894 firm-year observations from 395,169 different firms. On average, the number of firms in the cross section of any given year is 169,223. The time series and the cross-sectional dimensions of our panel make these data ideal to study the role of investment for productivity dynamics. Our data match the size and the distribution of Italian firms accounting for more than 75 percent of the value added produced in the Italian economy. In Table $A.1$ we report the composition of the data set by sector. Sectors are identified following the statistical classification of economic activities in the European Community, abbreviated as NACE. Consistent with their share of the economy, the manufacturing and the trade sectors constitute more than one-half of the observations in the data.

2.2 Age Distribution of Firms

We now turn to study the age composition of the firms in our sample. The ability to distinguish between firm age and investment age will be essential in Section 3 where we measure how the investment pattern at the firm-level accounts for differences in productivity across firms.

Table 1: Descriptive Statistics - Age

Firm Age	Share in Data Set (A)	Share of Output (B)	Share of Investment (C)	Share of Employment (D)
$0-5$ years-old	29.90%	13.90%	16.69%	15.74%
$5-10$ years-old	23.05%	17.20%	17.81%	16.60%
$10-20$ years-old	24.97%	25.80%	24.87%	25.04%
$20+$ years-old	22.08%	43.10%	40.63%	42.62%
Total	100.00%	100.00%	100.00%	100.00%

Notes: The sample period is 1998-2015. Statistics are computed as averages over the sample period considered.

Table 1 reports aggregate statistics conditioning on firms' age. Table entries are averages over the sample period from 1998 to 2015 but are representative even if the sample ends in 2009.[4] We follow Fort et al. (2013) and denote as young firms that have an age below five years. Also, we consider age groups for mature and old firms: 5-10, 10-20 and over 20 years old. The average (median) firm is 11 years old (10 years old). As shown in column A, our data set is constituted by young, mature and old firms alike. Young firms account for 30 percent of the firms in the sample. As expected, mature and old firms represent more than 80 percent of output, investment and labor, with firms over 20 years old that account for about one-half of output, investment, and employment. These shares are stable over the sample period we consider. We now turn to study the nature of capital accumulation at the firm level.

2.3 The Lumpy Nature of Capital Accumulation

In this subsection, we document that the nature of capital accumulation at the firm-level is lumpy: at the firm level capital adjustment is large and infrequent. We compute the distribution of investment rates for the sample 1998-2015. As is customary in the litera-

[4] The sample period is consistent with the analysis in Section 3. There, data over the period 1986-1997 are used to initialize the distribution of investment age that constitutes the variable of interest in our empirical analysis.

ture, we calculate real capital stocks applying a perpetual inventory method from balance sheets data, see Appendix C for details. Following Bloom (2009) we define the investment rate for a given firm f at time t as $ik_{f,t} = \frac{I_{f,t}}{0.5(K_{f,t-1}+K_{f,t})}$, where $I_{f,t}$ is real investment net of disinvestment.[5] Table 2 reports the empirical distribution of $ik_{f,t}$ in our sample. As in Bachmann and Bayer (2014) we define lumpy adjusters as those firms that exhibit a spike, i.e., an investment rate above 20 percent. These investors account for 60 percent of total investment. Instead, firms that experience small capital adjustments (defined as in Nilsen and Schiantarelli (2003) as experiencing $ik_{f,t}$ between -5 and 5 percent) account for only 6 percent of total investment.[6] While the share of investment accounted for by these two groups of firms differs substantially, the share of output and employment are instead equivalent.

Table 2: Cross-Sectional Distribution of Investment Rates

Investment Rate	Share in Data Set (A)	Share of Output (B)	Share of Investment (C)	Share of Employment (D)
$ik \geq 0.20\%$	18.81%	26.77%	61.04%	27.52%
$-0.05\% \leq ik \leq 0.05\%$	34.19%	25.67%	5.76%	27.01%
$ik \leq -0.20\%$	3.11%	1.98%	-6.65%	2.14%

Notes: ik denotes the investment rate. See the main text for the definition. The distribution of investment rates is computed over the sample period 1998-2015.

Higher moments of the cross-sectional distribution of investment rates exhibit positive skewness (0.88). The existing literature interprets this evidence as suggesting the presence of investment lumpiness at the firm level, see Caballero, Engel and Haltiwanger (1995).[7]

[5] Doms and Dunne (1998) define investment rates net of capital depreciation while Cooper and Haltiwanger (2006) and Gourio and Kashyap (2007) follow the convention that $ik_{f,t} = I_{f,t}/K_{f,t}$. The lumpy nature of the capital accumulation process in our data does not depend on the specific formula used to compute investment rates.

[6] The lumpy nature of the capital accumulation process is a feature of the data also in other countries. Doms and Dunne (1998) report evidence for the United States; Bachmann and Bayer (2014) for Germany; Licandro, Maroto Illera and Puch (2005) for Spain; Nilsen and Schiantarelli (2003) for Norway; and Gourio and Kashyap (2007) for Chile.

[7] The cross-sectional dispersion of firm-level investment rates is pro-cyclical with a correlation of 0.34

To shed light on the source of the type of capital adjustment cost at the firm level, we estimate the probability of the firm experiencing an investment spike given the time elapsed since the last investment spike. As is customary in the existing literature, we refer to this probability as the hazard rate. As discussed in Haltiwanger, Cooper and Power (1999) and Nilsen and Schiantarelli (2003), a positively sloped hazard function is consistent with non-convex capital adjustment costs. Instead, convex capital adjustment costs will not, in general, imply upward-sloping hazard rates.

To estimate hazard rates from the distribution of spells between spikes, we use the semi-parametric heterogeneity model proposed by Heckman and Singer (1984) and employed in Haltiwanger, Cooper and Power (1999).

This approach consists of fitting a particular functional form for the hazard rate: the proportional hazard model. This class of models is flexible because it allows (i) to estimate the hazard using the distribution of investment spells defined over a fixed number of discrete intervals and (ii) to control for unobserved heterogeneity. As shown in Figure $A.1$, the hazard rates are upward sloping. This result is consistent with the presence of non-convexities at the firm level. (Refer to Appendix B for details about the estimation.)

3 Investment Age and Productivity

In this section, we document the link between investment and productivity using firm-level data. Consistent with the idea that technological progress is embodied in new equipment, our main result is that firms that have recently experienced a large investment episode (spike) are, other things being equal, more productive than firms that have had such a spike in a more distant past. The richness of the data set in terms of firm size, industry and coverage makes it suited to study the relationship between investment and (several measures of) productivity. A key advantage of our study is the possibility to control for potential confounding factors at the firm, industry and aggregate levels.

We start by describing our empirical strategy; then, we discuss the main results. Fi-

with the growth rate of GDP. This feature of the data echoes the finding of Bachmann and Bayer (2014) for the German economy.

nally, using industry-level data, we document the relevance of the extensive margin of investment for productivity dynamics.

3.1 Empirical Specification and Variables Construction

To quantify the link between productivity and investment, we follow the approach in Power (1998) and estimate the following equation:

$$log(Productivity)_{f,t} = \alpha + \sum_{j=1}^{J} \beta_j Inv.Age_{j,f,t} + Controls_{f,t} + \epsilon_{f,t}. \quad (1)$$

The dependent variable is either the log of labor productivity or that of TFP. Labor productivity is obtained, in any given year, as the ratio between real value-added over the number of workers. TFP is measured through the Solow residual. As the stock of capital is not quality-adjusted, our TFP measure does not distinguish between neutral or investment-specific technological progress.[8] Appendix C reports additional details on the construction of the variables. The regressors of interest are the set of dummies $Inv.Age_j$ that capture the j vintage of capital. The latter is measured using the time elapsed since the last investment spike ($ik_{f,t} \geq 0.20$) at the firm level. When a firm experiences an investment spike, the dummy $Inv.Age_0$ equals one while the other dummies $Inv.Age_j$ from 1 to J take the value of zero. If the same firm does not experience an investment spike in the next period, then $Inv.Age_0$ is equal to zero and $Inv.Age_1$ to one. The investment age increases as firms delay large capital adjustment. We denote with J the maximum investment age — in our case the category 12+ years. This category includes firms that have never experienced an investment spike as well as firms that have not experienced one for over 12 years. Given the long time-series dimension in our data, we split the sample and use the first part (1985-1997) to initialize the distribution of $Inv.Age_j$. We estimate equation 1 using the second part of the sample (1998-2015). This operation reduces the number of firm observations to 2,973,696, with an average cross section in a given year of

[8] The Solow residual is computed assuming a Cobb-Douglas production function. Following Bachmann and Bayer (2014) we estimate the output elasticities of the production function as median factor expenditures share over gross value-added within each industry.

174,859. We exclude from the regressors the dummy $Inv.Age_{0,f,t}$, i.e., investment age for firms that have just invested are not part of the equation. As a result, the β_j coefficients measure the productivity gap of older capital vintages relative to the newest vintage. As discussed in Section 2.3, we follow Bachmann and Bayer (2014) and define an investment spike using a threshold of 20 percent. In Section 3.4, we show that our results are not sensitive to this choice.

Equation 1 is estimated using Ordinary Least Squares (OLS). The set of controls includes firm- and industry-specific effects. Also, dummy years are included to capture aggregate shocks, such as fiscal or monetary policy. Finally, to avoid confounding vintage capital effects with firm's age-specific effects, we include five dummies for the age and six for the size of the firm.[9]

3.2 Measuring Vintage Effects in the Data

To facilitate the interpretation of our estimates, we report in Figure 1 the β_j and confine to Appendix D the full set of coefficients and description of the estimation details. The dependent variable is the log of labor productivity in Panel A and the log of TFP in Panel B. The horizontal axis measures $Inv.Age_j$, the years elapsed since the last investment spike. The vertical axis reports the point estimates of the gap between the latest and previous vintages of capital expressed in percent. Dashed lines indicate 95 percent confidence bands. The coefficients are precisely estimated and document a negative relationship between firms' productivity and investment age. Controlling for a full set of possible confounding factors, we find that the labor productivity gap between firms that have experienced spikes in different years is, on average, about 0.8 percent per year. The year-to-year gap is about 1 percent up to an investment age of eight years, and it then flattens up to about one-half percent. Results in Panel B indicate that about half of the labor productivity gap can be attributed to differences in TFP, suggesting a quantitatively important role for capital deepening. When we consider as a dependent variable the growth

[9] Age dummies define categories for the age of the firm at any given point in time given intervals of 0-5, 5-10, 10-20, 20-30, and above 30 years of age. The range for firm size dummies is 1-5, 5-20, 20,-50, 50-100, 100-300, and above 300 employees.

rate of productivity, we do not find any discernible effect of investment age (not shown). We find that vintage effects account for about 15 percent of the heterogeneity in labor and TFP measured in the data.[10]

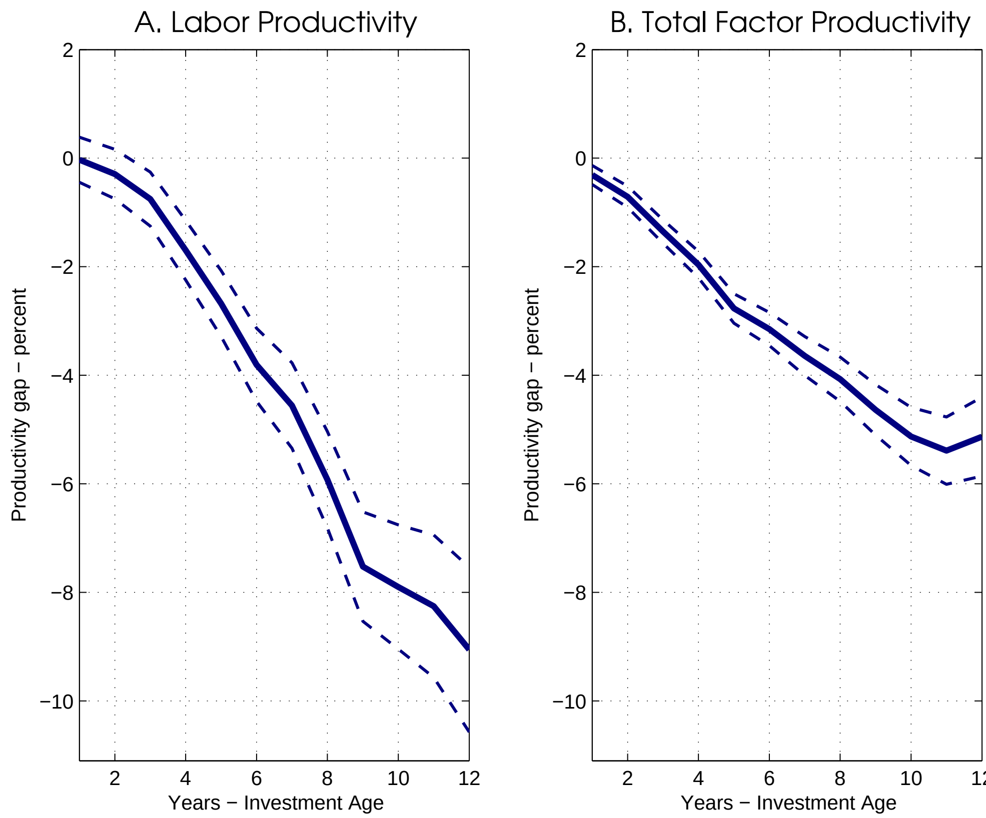

Figure 1: Investment Age and Productivity

Notes: The figure reports estimates of the β_j coefficients in equation 1. Dashed lines denote 95 percent confidence bands. Each equation is estimated with Ordinary Least Squares, and it includes fixed-, industry-, year-effects and a series of dummies for firm's age and size.

3.3 Ruling Out Alternative Hypotheses

To interpret the estimated relationship between investment and productivity in a causal sense, we rule out alternative hypotheses that can explain our results. More specifically, we verify that our results are not driven by potentially time-varying firm-specific factors

[10]To obtain this number, we take the ratio between the average contribution investment age of productivity and the standard deviation of the productivity residual of the estimated regression. Using the interquartile range of the residual, yields an equivalent number.

that are not accounted for by the firm fixed-effects already included in the empirical specification. For instance, news about current profitability or the persistence of current shocks could induce firms to undertake a large capital adjustment. Also, higher productivity for firms with more recent investment spikes could be the byproduct of the propensity of the firm to innovate. To address these concerns, we proceed as follows. First, we fit an autoregressive process of order one to our measures of productivity, controlling for firm and industry fixed effects together with firm age and size dummies and year effects. The estimated coefficient on the lag of productivity is 0.39 for labor productivity and 0.37 for TFP. Such a low persistence, consistent with the results in Cooper and Haltiwanger (2006) for the U.S. economy, makes it hard to rationalize the productivity gap observed over 12 years with the idiosyncratic component of productivity. Second, to control for news about future profitability that could potentially drive current investment decisions, we augment the baseline specification with a proxy for news. We construct the latter as the log difference between revenues at time $t+1$ and $t-1$ (or t). This strategy amounts to assume that the firm at time t (or $t-1$) has knowledge and can correctly predict future revenues, or, equivalently, profitability. The estimated β_j are not affected and are virtually unchanged. Third, we split the sample between innovative and non-innovative firms and re-estimate our baseline regression. To do so, we follow the insight of Corrado, Hulten and Sichel (2006) and use the share of intangible capital (over total capital) as a proxy for innovation. The rationale for this choice is that innovation results in intangible capital, for instance, patents, research and development expenditures, and software. We do not find differences in the estimates of the vintage effects (Appendix E reports the results). Fourth, as discussed in more detail in Section 5.2, we fit the same empirical specification in equation 1 to data simulated from the model with rich firm heterogeneity and vintage capital, described in Section 4. Using a parametrized version of the model consistent with the cross-sectional and time-series evidence on investment behavior, we find that the estimated coefficients on simulated data reproduce the results in Figure 1. In light of this empirical and theoretical evidence, we conclude that the estimated coefficients on $Inv.Age_j$ measure the contribution of new capital injections to productivity. All in all, this evidence rules out alternative explanations for our results.

3.4 Sensitivity Analysis

In this section, we show that our results are robust to alternative assumptions regarding the definition of an investment spike, firm age, and sample composition. As the identification of a spike is an empirical convention, we show that our results are robust to an alternative definition of spikes. In the spirit of Power (1998), we employ a measure that does not rely on capital. A firm exhibits an *absolute* investment spike if its real investment exceeds the 80th percentile of the firm-specific distribution of investment. Figure *A*.2 reports the estimated coefficients. Independently of the definition adopted, the data point to the presence of vintage effects. The estimated coefficients obtained with a different definition of spikes are very close in magnitude relative to the baseline case. To verify firm entry does not drive our results, we follow Bachmann and Bayer (2014) and consider only firms five years after their birth. We also assess the robustness of our results concerning the sample composition by including only firms that are observed for, at least, 10 consecutive periods. Results are confirmed and reported in column 2, 3, 5 and 6 in Table *A*.2. Finally, we consider a sectoral analysis that confirms that the lumpiness of capital adjustment at the firm level is a characteristic of all the sectors We find evidence of vintage effects in almost every sector, except for the financial sector and hotel and food services. Equipment goods do not constitute a significant component of the production process in these sectors. Appendix G reports the findings. These results confirm the presence of vintage effects associated with large investment episodes.

3.5 The Extensive and the Intensive Margin

We now turn to evaluate the aggregate relevance of the documented vintage effects at the firm level. Using industry variables, we test whether an increase in investment at time $t-1$, due to the extensive margin (number of firms experiencing spikes) or the intensive margin (the average size of a spike) leads to higher productivity between time $t-1$ and t. To do so, we perform an industry-based analysis in which labor and TFP are regressed on the fraction of spikes adjusters and the average adjustment undertaken by those investors. The purpose of this approach is two-fold. First, estimating such empirical specification allows us to measure if the firm-level evidence contributes to determining productivity at the industry level. Second, we can disentangle whether this occurs through the extensive or the intensive margin. Notice that, despite the evidence of vintage effects at the firm-level, a priori there is no reason why this firm-level characteristic should be a quantitatively-relevant determinant of productivity also at the industry level.

Table 3: The Extensive Margin of Investment and Productivity

	LP (A)	TFP (B)	LP (C)	TFP (D)
Extensive Margin (-1)	0.43^{**}	0.36^{**}		
Lumpy-Adjusters Count	(2.28)	(2.13)		
Extensive Margin (-1)			0.09^{*}	0.15^{***}
Lumpy-Adjusters K-weighted			(1.95)	(3.67)
Intensive Margin (-1)	−0.07	0.03	−0.05	0.05
	(−0.50)	(0.41)	(−0.61)	(0.62)
No. obs.	2073	2073	2073	2073
R^2	0.72	0.76	0.78	0.79

Notes: *p<0.10, **p<0.05, ***p<0.01, where p is the marginal probability level; t-statistics in parenthesis. Each equation is estimated using Ordinary Least Squares, and it includes industry-specific fixed effects and year dummies. The dependent variable is the labor productivity in columns A and C, and total factor productivity in columns B and D. Both are measured at the two-digit NACE classification. The sample period is 1987-2015.

The sample includes 55 industries and runs from 1987 to 2015, see Appendix A for details. The number of years is larger than the sample employed in Section 3.2 and G because there is no need to initialize the distribution of investment age. As in Bachmann and Bayer (2014), the extensive margin is measured as the fraction of firms that experience spikes (or equivalently lumpy adjusters) in a given year. In columns A and B we use a count of firms, and, in columns C and D we weight firms by their capital. The intensive margin consists of the average adjustment of those same firms. Table 3 reports the estimated coefficients. The extensive margin of capital accumulation is a quantitatively relevant determinant of productivity at the industry level. The coefficients are both statistically and economically significant. To get a sense of the magnitude implied by our estimates, a one-time decrease in the fraction of lumpy adjusters equal to a standard deviation leads to lower labor productivity by about 1 percent and 0.7 percent for TFP. The same experiment yields 0.8 and 1.1 percent when we measure the extensive margin as the weight of capital accounted for by lumpy adjusters. The magnitude of these effects is economically significant but must be interpreted with care since, neglecting general equilibrium effects, it likely constitutes an upper bound. Indeed, a sizable shift in the number of lumpy adjusters may affect factor prices and hence, firms' investment decisions. We take into account these considerations in the next section, where we perform a full quantitative evaluation to assess the role of microeconomic heterogeneity for the dynamics of aggregate productivity in the Italian economy.

4 Model

In this section, we describe the theoretical framework that we employ to study how microeconomic heterogeneity contributes to the dynamics of aggregate productivity in response to macroeconomic shocks. Our starting point is the neoclassical growth model of Khan and Thomas (2008), the benchmark for quantitative analysis involving firm dynamics. This framework allows us to capture the features of capital accumulation at the firm-level discussed in Section 2 in a general equilibrium framework. Our model innovation consists of introducing technology vintage structure through capital accumulation. The firm's productivity not only depends on idiosyncratic factors but also on the technological vintage available to the firm. In other words, vintage technology is a defining characteristic of the firm. The firm's problem consists of deciding the optimal timing to obtain the latest vintage and, if so, how much to invest. This option is subject to a non-convex adjustment cost that, in equilibrium, leads to the coexistence of vintages of different quality and contributes to reproducing the firm-level evidence on capital accumulation. In this respect, we follow the existing literature since Cooper and Haltiwanger (1993), Haltiwanger, Cooper and Power (1999), and Caballero and Engel (1999), as well as the empirical evidence in Section 2.3 that supports for the presence of non-convex capital adjustment cost. In our vintage model, this cost captures any friction, whether real or financial, that prevents large capital adjustment and delays the adoption of new technology.

Our framework contributes to the existing literature that studies the role of investment indivisibility in general equilibrium; recent contributions include Thomas (2002); Khan and Thomas (2008); Bachmann, Caballero and Engel (2013); and Fiori (2012). In Sections 4.1 and 4.2, we outline the tradeoffs that determine the production and investment decision of each firm. Sections 4.4 and 4.3 describe the households' problem and the recursive equilibrium of the economy 4.5. In Sections 4.6 and 4.7 we discuss the implications of the model for aggregate productivity and the mapping between the model and the evidence presented in Section 3.2.

4.1 Production

The economy consists of a continuum of firms that is normalized to one.[11] One commodity that can be consumed or invested. Each firm has access to an increasing and concave production function that combines predetermined capital stock k with its available technology to produce output y:

$$y = \varepsilon z k^{\theta}, \tag{2}$$

where $0 < \theta < 1$.[12] The efficiency of production depends upon two variables, ε and z. ε denotes the idiosyncratic productivity that is exogenous to the firm. z identifies the current vintage of technology and is optimally chosen by the firm. Every period, firms decide whether to pay the cost ξ and adopt the latest vintage or to postpone it. The technological frontier grows deterministically at the gross rate of $\gamma_A > 1$. Along the balanced growth path, z_0 indicates the latest vintage or the technological frontier that, along the balanced growth path deflated by its trend, is equal to 1. A firm that chooses not to obtain the latest vintage keeps its current technology that becomes more obsolete

[11] As discussed in the sensitivity analysis in Section 3.4, our results are not driven by entry and exit. In light of this evidence, we abstract from entry and exit dynamics.

[12] Variables reported are deflated by their respective trends. Along the balanced growth path, γ_A denote the gross trend growth rate of the technology frontier. Consumption and capital grow at a gross rate $\gamma_A^{1/(1-\theta)}$.

relative to the frontier at a per-period rate γ_A, so that $z^{'} = z/\gamma_A$.[13] We describe in the next section the economic tradeoff associated with the investment choice. As in Khan and Thomas (2008), $\varepsilon \in \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_{N_\varepsilon}\}$ where $Pr(\varepsilon = \varepsilon_m | \varepsilon = \varepsilon_l) \equiv \pi^{\varepsilon}_{lm} \geq 0$, and $\sum_{m=1}^{N_\varepsilon} \pi^{\varepsilon}_{lm} = 1$ for each $l = 1, ..., N_\varepsilon$. In each period, a firm is defined by its vintage productivity z, its idiosyncratic productivity level $\varepsilon \in \mathcal{E} \equiv \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_{N_\varepsilon}\}$, its predetermined stock of capital $k \in \mathbf{R}_+$, and its cost associated with vintage adoption $\xi \in [0, \bar{\xi}]$, which is denominated in units of output.

4.2 Firm's Adoption and Investment Decision

In every period, each firm faces the choice between keeping its current vintage or adopting the latest available technology. This choice consists of choosing whether it pays or avoids its current adoption cost ξ. By paying ξ, the firm obtains the latest vintage z_0 and optimally chooses the stock of capital $k^{'}$, where primes denote next-period variables. The firm's capital stock evolves according to $k = (1 - \delta - \delta_S)/\gamma k + i$, where i is its current investment and $\delta \in (0, 1)$ is the rate of physical capital depreciation. The parameter δ_S captures potential incompatibility of existing capital with the old and new vintage in a parsimonious way.[14] When δ_S is equal to zero, this amounts to assume a full retrofitting, which is that the productivity associated with the new vintage applies to the capital stock of the firm already installed. When δ_S is positive, adopting the new technology comes at the cost of scrapping δ_S units of its current k. Also, a positive δ_S amounts to a rescaling of the fixed cost of adoption that makes it firm specific, as it depends on the current capital stock available to the firm. Specifically, by forfeiting ξ, units of current output, the firm can invest in any future capital $k \in \mathbf{R}_+$ and upgrade technology z to the latest vintage of technology z_0. Firms that postpone paying the adjustment cost keep their current vintage and can undertake a constrained investment i^C. In this case, the firm's distance

[13] Given that the frontier of z_0 evolves deterministically at a rate of γ_A, z indicates the time elapsed since the firm has adopted the latest vintage.

[14] As in Solow (1960), the dynamics of vintage models cannot be represented through a representative firm unless knife-edge conditions are met. This case is not valid in models that feature a vintage structure and non-convex adjustment costs. Motivated by our empirical evidence in Section 3, our identifying assumption is that the firm vintage, linked to the capital accumulation decision of the firm, is a defining characteristic of the firm. Khan and Thomas (2003) employ a different identifying assumption. They assume that the latest vintage technology applies only to the most recent investment.

from the technological frontier or the degree of technological obsolescence increases so that $z' = z/\gamma_A$, and $k' \in \Omega \subseteq \mathbf{R}_+$, where

$$\Omega(k) \equiv \left[\frac{1-\delta+a}{\gamma}k, \frac{1-\delta+b}{\gamma}k\right]. \tag{3}$$

Introducing the possibility of small capital adjustment captures the empirical regularity in Table 2 that a large fraction of investors experiences small investment rates.

Table 4 summarizes the decision set available to the firm regarding its capital stock and vintage technology between two consecutive periods, from k to k' and z to z'.

Table 4: Firm's Adoption and Investment Decision

	Fixed Cost Paid	Future Technology z'	Future Capital k'	Total Investment
$i \neq 0$	ξ	z_0	$k' > 0 \in \mathbf{R}_+$	$\gamma k' - (1-\delta-\delta_S)k$
$i = i^C$	0	z/γ_A	$k' > 0 \in \Omega(k)$	$\gamma k' - (1-\delta)k$

As in Khan and Ravikumar (2002), the adoption adjustment cost ξ is non-convex and its modeling strategy follows Caballero and Engel (1999) and the subsequent literature on lumpy investment. Thus, the decision to adopt the latest vintage involves a non convexity; conditional on adjusting capital and upgrading technology, the cost ξ incurred is independent of the scale of adjustment. As in Thomas (2002), we assume that ξ is independently and identically distributed across firms and across time. Every period, each firm draws its current cost of vintage adoption $\xi \geq 0$ (denominated in units of output) from the time-invariant distribution G common to all production units. As the firm's current adjustment cost has no implication for its future adjustment, the distribution of firms is summarized by (ε, z, k) : the idiosyncratic productivity ε, vintage technology z, and capital stock k. To characterize the distribution of firms over (ε, z, k), we use the probability measure μ defined on the Borel algebra S for the product space $S = \mathcal{E} \times \mathbf{R}_+ \times \mathbf{R}_+$.

The distribution of firms evolves over time according to a mapping (defined below) Γ : $\mu^{'} = \Gamma(\mu)$.

4.3 Firm's Dynamic Programming Problem

To describe the adoption and the investment decision of the firm, as is customary in the literature we adopt the approach in Khan and Thomas (2008) and state the problem in terms of utils of the representative households (rather than physical units) and denote the marginal utility of consumption by $p = p(\mu)$. This variable denotes the pricing kernel used by firms to price output streams. Given the i.i.d. nature of the adjustment cost ξ continuation values can be integrated out of future continuation values.

Let $v^1(\varepsilon_l, z, k, \xi; \mu)$ denote the expected discounted value of a firm entering the period with (ε_l, z, k) and drawing an adjustment cost ξ when the aggregate state of the economy is μ. The dynamic optimization problem for the typical firm is described using a functional equation defined by (4)–(6). First, we define the beginning-of-period expected value of a firm before the realization of its fixed cost draw, but after the determination of (ε_l, z, k):

$$V^0(\varepsilon_l, z, k; \mu) = \int_0^{\bar{\xi}} V^1(\varepsilon_l, z, k, \xi; \mu) dG(\xi). \tag{4}$$

The firm's profit maximization problem, which takes as given the evolution of the firm distribution, $\mu = \Gamma(\mu)$, is then described by

$$V^1(\varepsilon, z, k, \xi; \mu) = \max_{k^*,\, k^C} \left\{ \begin{array}{c} [F(\varepsilon, z, k) + (1-\delta)k]\, p(\mu) + \\ \max \left[\begin{array}{c} -\xi p(\mu) - \delta_S k + R(\varepsilon, z_0, k^*; \mu^{'}), \\ R(\varepsilon, z/\gamma_A, k^C; \mu^{'}) \end{array} \right] \end{array} \right\} \tag{5}$$

$s.t.\ k^* \in \mathbf{R}_+$ and $k^C \in \Omega(k)$,

where $R(\varepsilon, z^{'}, k^{'}; \mu^{'})$ represents the continuation value associated with a given combination of the idiosyncratic shock, the vintage, and the stock of capital:

$$R(\varepsilon, z, k^{'}; \mu^{'}) \equiv -\gamma k^{'} p(\mu) + \beta \sum_{m=1}^{N_{\varepsilon}} \pi^{\varepsilon}_{lm} V^{0}(\varepsilon_{m}, z^{'}, k^{'}; \mu^{'}) \tag{6}$$

Every period, the firm decides whether to pay the fixed cost (ξ) to obtain its current vintage and then invest. Otherwise, the firm keeps its current vintage. For notational convenience, as in Khan and Thomas (2008), rather than subtracting investment from current profits, the value of undepreciated capital augments current profits, and the firm is seen to repurchase its capital stock each period.[15] Because of the perfect mapping between technology adoption and the capital stock decision, we find it more transparent to focus on the capital decision. Thus, we let $K(\varepsilon, z, k, \xi; \mu)$ represent the choice of capital for the next period by firms of type (ε, z, k) with adjustment cost ξ, as this choice subsumes the adoption decision.

4.4 Households

The economy features a continuum of identical households that have access to a complete set of state-contingent claims. As there is no heterogeneity across households, these assets are in zero net supply in equilibrium. Moreover, they own shares in the production units, denoted by the measure $\lambda(\varepsilon, z, k; \mu)$ and value $\rho_0(\varepsilon, z, k; \mu)$. Given the value for their current shares, $\rho_0(\varepsilon, z, k; \mu)$, households maximize their lifetime expected utility choosing current consumption c as well as the numbers of $\lambda(\varepsilon, z, k)$ to purchase at prices $\rho_1(\varepsilon, z, k; \mu)$:

$$W(\lambda; \mu) = \max_{C, \lambda^{'}} \left[U(c) + \beta W(\lambda^{'}; \mu^{'}) \right] \tag{7}$$

subject to

$$\begin{aligned} & c + \int_{S} \rho_1(\varepsilon, z, k; \mu) \lambda^{'} \left(d \left[\varepsilon^{'} \times z^{'} \times k^{'} \right] \right) \\ \leq & \int \rho_0(\varepsilon, z, k; \mu) \lambda \left(d \left[\varepsilon \times z \times k \right] \right). \end{aligned} \tag{8}$$

[15] This approach is equivalent but notationally more convenient.

The household optimality condition yields:

$$p(\mu) = U_C(c) \tag{9}$$

Let us denote the optimal choices for the household as $C(\lambda;\mu)$ and $\Lambda^h(\varepsilon', z', \lambda, k'; \mu)$.

4.5 Recursive Equilibrium

A recursive competitive equilibrium is a set of functions $(p,\ v^1,\ K,\ W,\ C,\ \Lambda^h,\ \Gamma)$ that satisfy firms' and households' problem and clear the markets for assets, labor, and output:

(i) Firm's optimality: Taking p as given, $V^1(\varepsilon, z, k, \xi; p)$ solves (4)-(6) and the corresponding policy functions $K = K(\varepsilon, z, k, \xi; p)$.

(ii) Household's optimality: Taking p as given, the household's consumption satisfies equation (9) and (C, Λ^h).

(iii) $\Lambda^h(\varepsilon_m, z, k; \mu) = \mu(\varepsilon_m, z, k)$ for each $(\varepsilon_m, z, k,) \in S$.

(iv) Commodity market clearing: $C = \int y d\mu - \int \int_0^{\bar{\xi}} [\gamma K(\varepsilon, z, k, \xi; p) - (1-\delta) K] \, dG d\mu$ $- \int \xi dG d\mu$.

(v) Model-consistent dynamics: The evolution of the cross-sectional distribution that characterizes the economy, $\mu' = \Gamma(\mu)$, is induced by the adjustment decision and the exogenous processes for ε. Conditions (i), (ii), (iii), and (iv) define an equilibrium given Γ, while conditions (v) determines the equilibrium condition for Γ. We confine to Appendix H the discussion about the (S,s) decision rule for the firm upgrading and investing decision and the details on the evolution of the cross-sectional distribution of firms' productivity and capital stocks.

4.6 Firm-Level Vintage Technology and Aggregate Productivity

In this section, we discuss the role of the vintage structure for the aggregate economy. The presence of non-convex adoption cost implies that the firm's technology adoption decision follows an (S,s) rule: Some firms adopt the latest vintage while others postpone it.[16] Conditioning on the adoption decision, firms decide next-period capital stock. The degree of obsolescence of the vintage currently available to the firm (the productivity gap from the technological frontier) as well as the realization of the idiosyncratic, and potentially aggregate shocks affect the timing of technology adoption at the firm level. Thus, in equilibrium, vintages of different quality coexist implying that the distribution of productivity across firms determines the economy-wide production efficiency. Shifts in the cross-sectional distribution, determined by variations in the firms' adoption decision, result in fluctuation in aggregate productivity. We highlight that at the firm level, there is no one-to-one mapping between the size of the investment adjustment and the adoption of the latest vintage. As a result, the response of aggregate investment (or measuring vintage effects at the firm level) is not a sufficient statistic to characterize the evolution of aggregate productivity and its role for aggregate dynamics in response to shocks.

4.7 Mapping Between the Model and the Evidence on Vintage Effects

Before discussing the calibration strategy, we emphasize one crucial aspect related to the mapping between the model and the data. The empirical evidence in Section 3 highlights the relationship between recent investment spikes and productivity: Firms with lower investment age (measured as the time elapsed between investment spikes) are, other things being equal, more productive than firms with higher investment age. This link is not hard-wired into the model. Adopting the latest vintage entails a non-convex adoption cost independently of the size of the investment necessary to reach the target capital k^*. A priori, despite the vintage structure embedded in the model, there is no reason to expect that the model reproduces the empirical evidence in Section 3.2. We show in Section 5.3 that this is indeed the case.

[16] See Appendix H for additional details.

5 Taking the Model to the Data

In this section, we take the model to the data. We start by describing the parameterization of the model in Section 5.1. Given our focus on the role of microeconomic heterogeneity for aggregate dynamics, one critical aspect in evaluating the empirical performance of the model is its ability to fit the cross-sectional distribution of investment rates and the timing of investment spikes across firms, i.e., the empirical proxy for vintage technology. We discuss these issues in Sections 5.2 and 5.3.[17]

5.1 Parameterization

Following the business cycle literature, we calibrate the model to fit key first-order moments of the Italian economy. Table 5 summarizes parameter values, targeted moments, and data sources. We are to assign values to 11 parameters related to the growth rate of aggregate variables (γ and γ_A), the production process (δ, δ_S and θ), individual preferences (β), the adjustment cost function and boundaries to the investment process ($\bar{\xi}$, a and b), and the idiosyncratic productivity process (ρ_ε, and σ_ε). We first describe the set of parameters that are externally calibrated, i.e., using independent evidence. Then, we focus on those estimated within the model.

Externally Calibrated. One period in the model represents one year, which corresponds to the frequency of the data employed in Section 2. The depreciation rate is taken from the Italian National Statistical Institute and is equal to 9 percent. The average growth rate of output is set to 0.7 percent, consistent with its sample average computed using data until 2012. To calibrate the productivity increase of the latest vintage relative to the previous one (γ_A), we use our estimates of the vintage effects in Section 2, so that $\gamma_A = 1.005$. We set δ_S to zero because firms that experience a negative gross investment rate are, on average, less than 3 percent. Also, more than 95 percent of all the firms in the sample report a disinvestment rate of less than 1.5 percent.

Internally Calibrated. The discount factor β is set to 0.975 to reproduce the real annual interest rate in the data. The elasticity of output to capital is set to 0.4 to satisfy

[17] Appendix I reports details about the computation of the stationary equilibrium of the model.

the balanced growth path restriction that links the growth rate of output and the one on technology.

Table 5: Benchmark Calibration

Parameter		Value	Target
Depreciation rate	δ	0.091	Data
Scrapping rate	δ_S	0	Data
Growth rate of productivity	γ_A	1.005	Estimated vintage effects
Growth rate of output	γ	1.010	Data
Discount factor	β	0.975	Annual real interest rate = 2.3%
Elasticity of output w.r.t. capital	θ	0.4	Balanced growth path restriction
Upper support adj. cost distribution	$\bar{\xi}$	0.007	ik distribution
Persistence idiosyncratic productivity	ρ_ε	0.87	ik distribution
St. dev. idiosyncratic productivity	σ_ε	0.065	ik distribution
Lower bound of Ω	a	-0.050	ik distribution
Upper bound of Ω	b	0.050	ik distribution

To select the remaining parameters we follow Khan and Thomas (2008). We set the upper support of the adjustment cost function ($\bar{\xi}$), the boundaries that define constrained investment (a and b), the persistence (ρ_ε), and standard deviation of the idiosyncratic productivity process (σ_ε) to reproduce selected moments of the cross-sectional distribution of investment rates (ik) in the data. Results are reported in Section 5.2. Before reviewing the empirical performance of the model, it is worth examining the role of each parameter separately. The upper support of the adjustment cost distribution ($\bar{\xi}$) determines the magnitude of the adjustment costs. The higher $\bar{\xi}$, the higher the potential cost of adopting the latest vintage. Increasing this parameter leads to a higher average investment age. The persistence and the standard deviation of the idiosyncratic process interact with the vintage effect in shaping the economic incentives that make the firm adopt the latest vintage and choose capital. Finally, the parameter a and b are set to imply symmetric investment rates around zero and determine the extent of the frictions investment without adopting the latest vintage of productivity.

5.2 Cross-Sectional Distribution of Investment Rates

We now turn to examine the model performance in accounting for the cross-sectional distribution of investment rates. Given our focus, the model must reproduce the firm level pattern of capital accumulation observed in the data. As in Cooper and Haltiwanger (2006) and Khan and Thomas (2008), the cross-sectional distribution is summarized using five groups: inaction, positive and negative investment, and positive and negative spikes. A specific threshold for the investment rate identifies each group.[18] Before examining the results in Table 6, we note that our definition of the inaction region is broader than the definition employed in the existing literature. This choice allows us to capture the small investment rates occurring in about one-third of the firms in the sample.

Table 6: Distribution of Firm Investment Rates

	Inaction	Positive Spikes	Negative Spikes	Positive Investment	Negative Investment
	(A)	(B)	(C)	(D)	(E)
Data	34.19%	18.81%	3.11%	59.81%	6.00%
Model	36.25%	18.32%	0.15%	60.58%	3.17%

Notes: Each entry reports the fraction of firms that, on average, exhibit investment rates that fall in each category. See the text for the definition of inaction, positive and negative spikes, and positive and negative investment.

The model provides an excellent account of the cross-sectional distribution of investment rates. Of course, this result is the byproduct of the calibration strategy discussed in Section 5.1. While the model accurately reproduces the fraction of firms in the inaction region and those exhibiting positive spikes, it does slightly less well in accounting for the behavior of downsizing investors. This feature obtains because the realizations of the idiosyncratic process and the gap from the vintage frontier have conflicting effects on the adoption decision of the firm. Unfavorable realizations of the idiosyncratic process tend

[18] The Inaction region is identified as the fraction of investors with an investment rate less than or equal to 5 percent in absolute terms. Positive (negative) investors are defined as firms experiencing an investment rate above to (or below negative) 5 percent. An investment rate above (below) 20 percent identifies positive (negative) spikes.

to make firms postpone the adoption decision. However, postponing the adoption of the latest vintage increases the distance from the productivity frontier and reduces the capital available to the firm. This second effect is quantitatively stronger and implies that firms adopting the latest vintage do so with positive investment.

The model is also successful in accounting for the share of aggregate investment accounted for by each group. Investors exhibiting positive spikes account for 64 percent of total investment; in the data, this number is 61 percent. Moreover, firms in the inaction group are responsible for 3 percent of the total investment. Its data counterpart is about 6 percent.

5.3 Investment Age Distribution

We now turn to validate the model across dimensions that are left untargeted in the calibration. We proceed in two ways to connect the model with the data. First, we compute the model-based distribution of investment age, calculated as the time elapsed since the last time the firm experienced an investment spike. Reproducing this dimension of the data is essential because investment age is the variable at the core of the empirical strategy in Section 3 to identify vintage effects in the data.

Figure 2 reports the result. Firms that experience an investment spike have an investment age of zero. They move along the distribution until they exhibit another spike. As shown by the table, the model-based distribution displays less mass on the right tail relative to the data. It is not surprising then that the average investment age in the model is three years while it is five in the data. Overall, we consider the performance of the model quite satisfactory. We notice that the ability of the framework to reproduce the timing of investment spikes across firms is distinct from the model success to account for the cross section of investment rates. While the fraction of firms with investment age zero coincides by construction to the fraction of firms exhibiting spikes, the fraction of firms with age one and above depends on the investment behavior of firms, determined by the realization of individual states.

Second, we use our model for interpreting the empirical evidence in Section 3. To-

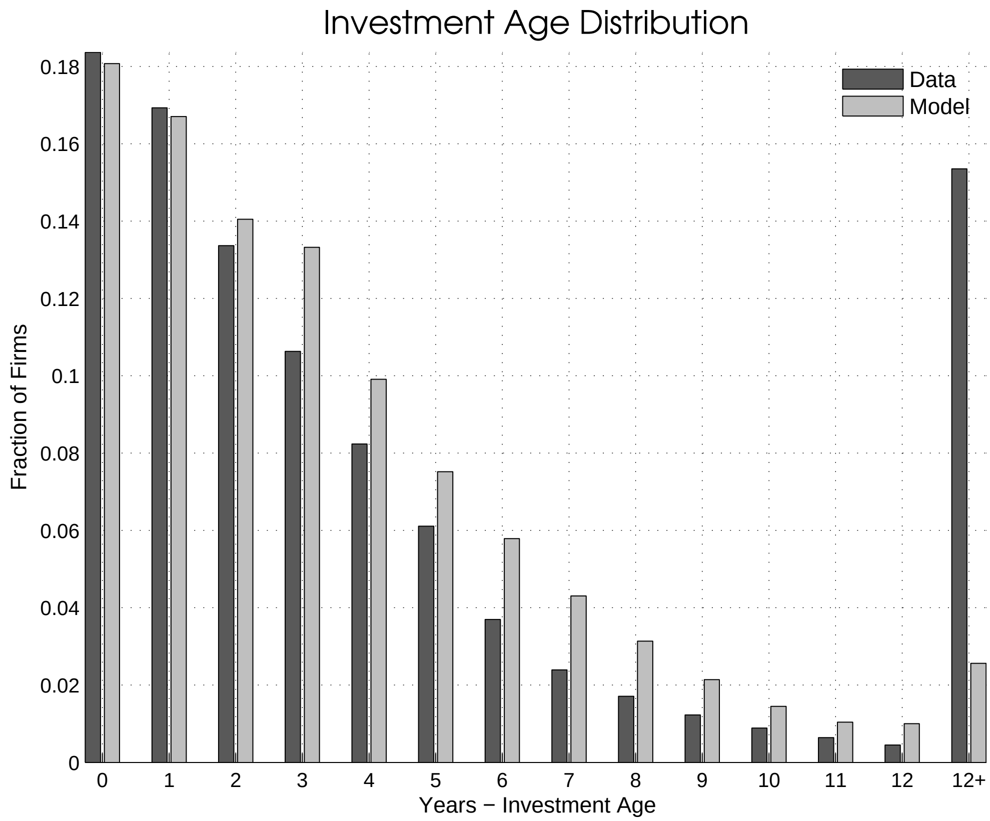

Figure 2: Comparison between the Empirical and the Model-Based Investment Age Distribution

ward this goal, we fit the empirical specifications in equation 1 using data simulated from the model. While there is a deterministic relationship between adoption and the vintage effect, two factors affect the ability of the model in reproducing the magnitude of the empirical estimates. Our parameterization implies that in the model (as in the data) the variance of firm's TFP is dominated by its idiosyncratic component. As a result, detecting vintage effects may be harder. Moreover, the empirical proxy for vintage technology is an investment spike in the data. In the model, firms can adopt the latest vintage even without exhibiting an investment spike. Figure 3 compares the empirical estimates in the data with those obtained in the model.[19] We find reassuring that the model matches the empirical evidence. The larger discrepancy occurs on the effects estimated right after

[19]We simulate data for a sample of 170000 firms, the average size of the cross section in the data, for 100 periods. The first 70 observations are discarded to reduce the dependence of our results on initial conditions. Of the remaining 30 observations, we use the first 15 observations to train the distribution of investment age and the last 15 to regress the log of TFP of each firm on its the investment age.

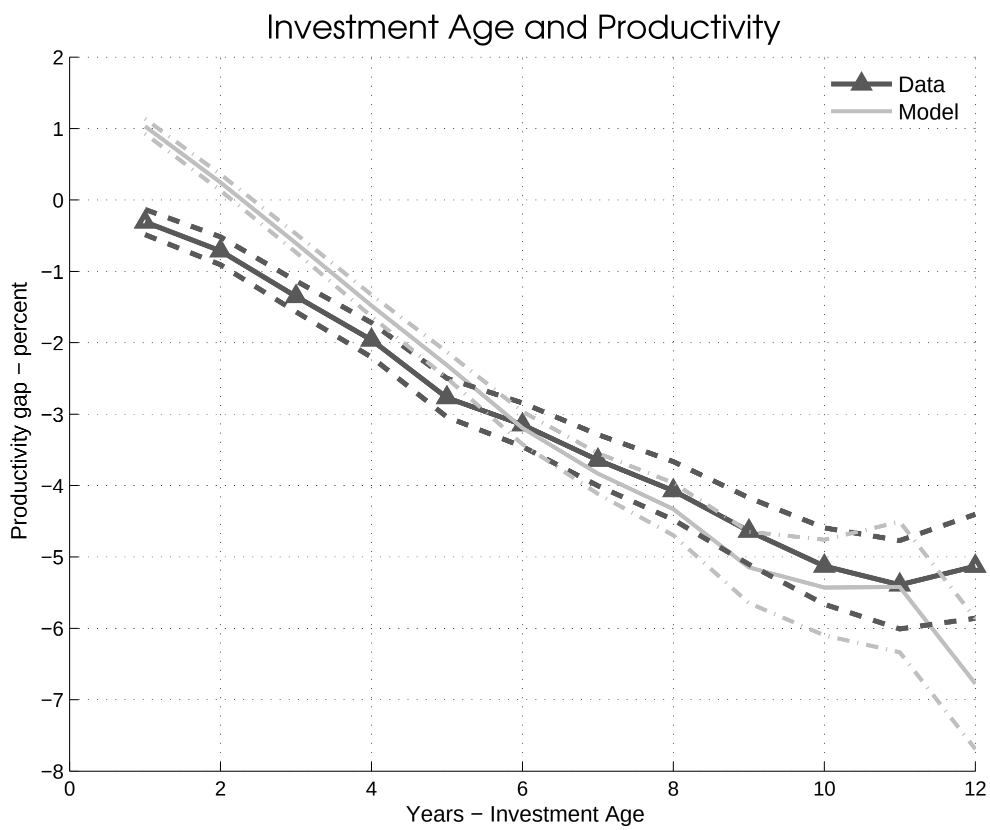

Figure 3: Regressions with Actual versus Simulated Data.

firms experience an investment spike. This effect is related to the timing assumption in the model. Firms that obtain a favorable idiosyncratic shock are more likely to adopt the latest vintage, and the effect of the new vintage gets compounded by the persistence of the idiosyncratic productivity. Moreover, in the data spikes may occur over multiple periods, while in the model capital adjustment takes place in one period. Before we move to the next section, note that the discipline imposed by the calibration is essential for the success of the model in reproducing Figure 3. More specifically, we build a counterfactual scenario in which we increase the boundaries of constrained capital adjustment (lowering a and increasing b) to allow firms a large capital adjustment without paying the adjustment cost. The purpose of this exercise is to show that investment spikes are not *per se* an indicator of vintage adoption. The model fails to reproduce the cross-sectional distribution in Table 6. Simulated data from this model indicate no relationship between investment age and productivity as investment spikes cease to be an indicator of vintage adoption. Overall, our findings corroborate the evidence that large investment episodes lead to higher productivity at the firm-level. After establishing the empirical success of the model in accounting for the pattern of capital accumulation at the firm level as well as the result of the empirical analysis in Section 3 we turn to assess the relevance of vintage effects for aggregate dynamics.

6 Investment, Productivity, and Macroeconomic Shocks

In this section, we use our framework to quantify the relevance of microeconomic heterogeneity induced by vintage technology in the propagation of macroeconomic shocks. Toward this goal, we compare the properties of the aggregate series obtained in our baseline model with a positive adoption cost and a benchmark with zero adoption cost, i.e., the standard *RBC* with idiosyncratic shocks. In Section 6.1, we consider a deterioration in financial conditions to study the role of vintage effects in accounting for the stagnant productivity observed in developed economies following the Great Recession. Then, in Section 6.2, we characterize business cycle dynamics in the presence of technology shocks.

Unlike the RBC model, when the technology available to each firm also depends on investment age, shocks that alter the timing of capital expenditures at the individual firms also affect the production possibility frontier of each firm. As a result, the joint distribution of capital stocks and technology across firms determines the *aggregate* efficiency of the economy. As firms postpone capital expenditures, they also postpone introducing the latest technology vintage. In a parameterized version of the model that closely reproduces the cross-sectional distribution of investment rates and investment age, this mechanism amplifies macroeconomic dynamics in response to standard aggregate shocks. We now discuss in detail our main exercises.

6.1 Financial Shock

Our first experiment characterizes the model dynamics in response to an aggregate deterioration in financial conditions. Following the approach in Gavazza, Mongey and Violante (2018), we study the *perfect foresight* transitional dynamics of the model in response to a one-time, unexpected temporary aggregate shock. This scenario is obtained assuming a temporary increase in the real cost of investment goods for firms that experience an investment rate above the threshold level of 5 percent. The economy starts in steady state, and the path of the shock always reverts to its initial value, so the economy also returns to its initial steady state.[20]

Rather than providing a microfounded characterization of financial frictions in the model, we assess how shocks that increase the cost of undertaking large capital expenditure to individual firms affect the endogenous dispersion of productivity and capital across firms and contribute to aggregate outcomes.[21] In the spirit of Gomes (2001), firms are subject to an additional real cost equal to $\lambda_1 i_{f,t}$, when they choose an investment rate $ik_{f,t}$ larger than 5 percent.

To discipline our exercise, we rely on firm-level Italian data. We parameterize λ_1 so that the model reproduces the fluctuations in the distribution of investment age consistent with the data, i.e., a reduction in the fraction of firms experiencing spikes observed in the data.[22] Thus, the share of firms with investment age equal to zero, matches exactly its data counterpart. To quantify the role of vintage effects, we then compare the dynamics implied of our baseline model with the nested RBC, the typical benchmark of quantitative macroeconomic analysis.

Table 7 reports the impulse response function of the two models. In the RBC model, a persistent increase in the cost of investment makes capital expenditures more expensive, leading to a drop in aggregate investment, that affects the economy for a few periods. As capital is predetermined and is the only input of production, output is unaffected on im-

[20]Details about the computation of the transitional dynamics are reported in Appendix I.

[21]The literature on financial frictions is large. For an explicit characterization of financial frictions in macroeconomic models see the seminal contribution of Bernanke, Gertler and Gilchrist (1999) and, in model featuring production heterogeneity, Khan and Thomas (2013).

[22]The fraction of firms experiencing spikes drops by 4 percentage points in 2012, relative to its value in 2011.

Table 7: Financial Shock - Aggregate Responses

	GDP RBC (A)	GDP VINTAGE (B)	INVESTMENT RBC (C)	INVESTMENT VINTAGE (D)
Impact	0.00%	0.00%	-4.87%	-4.48%
Period 1	-0.19%	-0.60%	-3.60%	-3.09%
Period 2	-0.31%	-0.85%	-2.66%	-2.83%
Period 3	-0.42%	-0.77%	-1.95%	-2.31%
Period 4	-0.38%	-0.68%	-1.43%	-1.57%

Notes: Each entry is in percent relative from trend values. RBC refers to the model with zero adoption cost, while VINTAGE to the baseline model.

pact but declines over time due to the lack of investment. As financial conditions revert to the steady state, the economy recovers and returns to the initial stationary equilibrium. While the qualitative pattern of the aggregate series in the *RBC* and the baseline model are similar, the *quantitative* predictions of the two models are starkly different. In the baseline model, vintage effects amplify aggregate dynamics. The distribution of productivity and capital stocks is responsible for this difference. As capital expenditures become more expensive, the average capital expenditure (intensive margin) and the number of investors (extensive margin) shrink. Despite the increase in the cost of investment, a significant fraction of firms finds optimal paying the non-convex adjustment cost and adopting the latest vintage. (By virtue of the calibration strategy, the model matches the fraction of spikes adjusters.) Nonetheless, such investors choose a lower target capital relative to the stationary equilibrium. Current financial conditions reduce the number of firms that adopt the latest technology. On impact, the drop in investment is virtually the same in both economies. Over time, in the baseline model, the response is amplified. As a byproduct of the drop in capital accumulation, the productivity of non-adjusting firms stagnates. As the distance of non-adjusting firms from the technological frontier increases, the economy-wide productivity dispersion increases. Lower average productivity and capital deepen the recession. Over time, the two economies recover at a similar speed.

In Table 7, we report the response of aggregate productivity across models and in the data, following the 2012 recession. Model-based measures of TFP are computed imposing a Cobb-Douglas production function. Through the lens of our model, parameterized to reproduce the micro and the macro pattern of capital accumulation in the data, vintage effects account for 35 percent of the average response of aggregate productivity.[23] Increasing the size of the shock increases the decline in aggregate investment and aggregate productivity. In the nested RBC, the response of aggregate productivity is exogenous and constant over time.

Table 8: Financial Shock - TFP Response

	TFP DATA (A)	TFP VINTAGE (B)	TFP RBC (C)
2012	-1.27%	-0.42%	0.00%
2013	-1.08%	-0.57%	0.00%
2014	-1.15%	-0.31%	0.00%
2015	-0.89%	-0.26%	0.00%

Notes: Each entry is in percent relative from trend values. TFP is computed using an aggregate production function.

6.2 Technology Shock

In this section, we extend the baseline model in two dimensions. First, we introduce aggregate uncertainty by assuming that the rate at which technology evolves is stochastic rather than deterministic. Thus, newer vintages are more productive than the previous

[23] Results are equivalent if TFP is computed as a weighted average of individual TFP across firms. Moreover, the measure of TFP in the data is adjusted for the utilization rate in the economy; this ensures comparability between the model and the data.

one at a gross rate of $\gamma_{A,t}$ that is now time varying according to an autoregressive process of order one. Second, we include a labor supply decision on the household side. Both modifications allow us to study how microeconomic heterogeneity contributes to the propagation of technology shocks. While productivity and technology are often used interchangeably to label stochastic disturbances to production efficiency, the same is not true in the context of our baseline model as shocks affect only the *current* vintage. Adding aggregate uncertainty complicates the solution of the model as the distribution (and its evolution) enters the space of the model. To solve the model, we adapt the solution method in Khan and Thomas (2008), described in Appendix J.2. The calibration strategy follows the same targets outlined in Section 5.1 and discussed in Appendix J.1.

We perform the conventional business cycle exercise by simulating the model in response to technology shocks. We restrict the aggregate technological process so that the realizations of the shock are such that there is no technological regress, i.e., the growth rate of technological efficiency, $\gamma_{A,t}$, is always non-negative.

Table 9: RBC Moments - Technology Shock

	ΔGDP	ΔC	ΔI	LABOR
	(A)	(B)	(C)	(D)
RBC Model				
σ_X	0.28%	0.27%	3.57%	0.38%
VINTAGE MODEL				
σ_X	0.42%	0.22%	4.37%	0.41%

Notes: Each entry represents the volatility of the respective variable. Δ we indicate the growth rate. C, I and L refer to consumption, investment, and labor, respectively.

As shown in Table 9, vintage effects amplify aggregate dynamics relative to the nested *RBC* model. The standard deviation of GDP increases by about 50 percent. This result obtains because the sensitivity of aggregate investment to technology shocks is higher in the vintage model. On average, large capital expenditures also entail productivity gains. The gap from the technological frontier is now stochastic and depends upon the realization of the aggregate technology shock. When the efficiency of the latest vintage is higher than expected, more firms pay the adoption cost and, on average, incur more significant capital expenditures. The ensuing shift in the distribution magnifies the response of GDP and investment.

The vintage model is also consistent with the cyclicality of the dispersion of TFP and investment rates across firms, discussed in Bloom et al. (2018) and Bachmann and Bayer (2014). This property is not induced by a countercyclical dispersion in idiosyncratic shocks faced by firms, but it endogenously arises in equilibrium in response to first-moment shocks. In good times, as more firms adopt the latest technology, the dispersion of TFP decreases. Moreover, as adopting the latest technology, on average, involves larger capital expenditure, the dispersion of investment rates increases as well.

7 Concluding Remarks

As shown in Table 9, vintage effects amplify aggregate dynamics relative to the nested *RBC* model. Using firm-level data, we find that the timing of investment adjustment is a source of TFP heterogeneity across firms: Firms that have undertaken large capital adjustment in more recent periods are more productive than firms that have done so in a more distant past. To study the aggregate relevance of this micro-based mechanism, we formulate a general equilibrium model of rich firm heterogeneity where, as in the data, individual firms' TFP depends upon persistent idiosyncratic component, but, as in the data, is also endogenous to the timing of investment. A non-convex cost of technology adoption reconciles the lumpiness of capital accumulation with the TFP heterogeneity observed in the data. As firms pursue (S,s) decision rule, old and new technologies coexist in the data.

In response to aggregate shocks, the shift in the distribution of productivity and capital stocks constitutes an independent and additional mechanism that amplifies macroeconomic dynamics beyond the effect of the initial primitive shock. A negative shock that induces firms to postpone adoption and, therefore, investment also leads to lower productivity. We show that this mechanism is quantitatively significant in response to a financial shock and a slowdown in the growth rate of technology efficiency.

In an application to the Italian recession of 2012, through the lens of the model, parameterized to reproduce the microeconomic pattern of capital accumulation, we find that the shift in the distribution of investors accounts for about one-third of the missing productivity growth of the Italian economy. Thus, a prolonged investment slump, as the one experienced by advanced economies in the decade after the Great Recession, has significantly contributed to the observed stagnant productivity growth.

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APPENDIX

A Data Sources

Detailed information on yearly balance sheets comes from Cerved Group S.P.A. (Cerved Database) while data on employment and wages are obtained from the Italian National Social Security Institute (INPS). Industry-specific price deflators and depreciation rates are obtained from the Italian National Statistical Institute (ISTAT). Sectors are constructed aggregating available data from two-digit industries, according to the 2007 NACE classification. The agriculture sector includes industries 1, 2, 3, and 8. The manufacturing sector comprises industries 10, 11, and 13 to 33.

Table $A.1$: Sectoral Data

Sector	No. of Obs.
Agriculture, forestry and fishing	96,087
Manufacturing	1,487,826
Electricity and gas supply	12,324
Water supply	40,249
Construction	614,258
Wholesale and retail trade	1,324,078
Transportation and storage activities	189,789
Accommodation and food service	267,581
Information and communication	223,826
Financial and insurance activities	25,160
Real estate activities	60,759
Professional, scientific and technical activities	224,766
Administrative and support service activities	172,656
Public administration and defense	31,138
Education	121,044
Human health and social work	66,950
Other activities	46,403

The electricity and gas supply includes industry 35. The water supply sector includes industries 36-39. The construction sector includes industries 41-43. The wholesale and retail trade sector includes industries 45-47. The transportation and storage activities sector includes industries 49-53. The accommodation and food service sector includes industries 55 and 56. The information and communication sector includes industries 58-63. The financial and insurance activities sector includes industry 66. The real estate activities sector includes industry 68. The professional, scientific and technical activities sector includes industries 69-75. The administrative and support service activities sector includes industries 77-82. The public administration and defense sector includes industry 85. The education sector includes industries 86-88. The human health and social work sector includes industries 90-93. The other activities sector includes industries 95 and 96. The composition of the data set by sector is reported in Table $A.1$.

B Hazard Rates - Estimation Details

In this section, we describe the procedure employed to estimate investment hazard rates. Our discussion follows Haltiwanger, Cooper and Power (1999) and Cameron and Trivedi (2005).

Let us denote by $h_f(t)$ the hazard for firm f at time t. The hazard is parameterized using the popular proportional hazard form:

$$h_f(t) = h_0(t) exp\{z_f(t)\beta\} \tag{A.1}$$

where $h_0(t)$ is the baseline hazard rate at time t (which is unknown), z is a vector of covariates, and β is the vector of unknown parameters. The probability that a spell lasts until $t+1$ given it has lasted for t periods is given by:

$$P[T_f \geq t+1 | T_f \geq t] = exp[-exp(z_f(t)'\beta + \gamma(t) + \mu)] \tag{A.2}$$

where

$$\gamma(t) = ln\left[\int_t^{t+1} h_0(u)du\right] \tag{A.3}$$

The likelihood function for a sample of N firms can then be written in terms of $A.2$ as:

$$L = \prod_{f=1}^{N} \sum_{j=1}^{J} \alpha_j L_f \qquad (A.4)$$

where

$$L_f = \left[1 - exp\{-exp[\gamma k_f + z_f(k_f)'\beta]^{d_f}\}\right] \times \prod_{t=1}^{k_f - 1} \left[exp\{-exp[\gamma t + z_f(t)'\beta + \mu_j]\}\right] \qquad (A.5)$$

where C_f is the censoring time and d_f if $T_f \leq C_f$ and 0 otherwise, and k_f is the minimum between the observed length of the spell and the C_f. The first term in $A.5$ represents the probability that the firm exhibits an investment spike in the interval $[k_f, k_f + 1]$ given that the spell has lasted until k_f. The second term in $A.5$ represents the probability that a spell lasts until k_f. The unobserved heterogeneity across firms is approximated by a discrete distribution with a finite number of points. The μ_j are the J points of support of the distribution where each of them has an associated probability α_j. μ_j and α_j are estimated jointly with the other parameters by maximizing the log of the likelihood in $A.4$.[24] As discussed in Cameron and Trivedi (2005), one can interpret the j as a a discrete number of unobserved types from which the data are assumed to be drawn. We consider investment spells in the 1987 and 2015 period, and we then track the firm's investment age until the next spike. All firms that have an investment age over 12 years are censored. For computational reasons, we consider only firms for which we have at least 15 consecutive observations. Figure $A.1$ reports the results from estimating $A.4$ with our data. We report results considering three groups, i.e., J is equal to 3. The estimation yields three groups.[25] In all cases, the hazard rates are upward-sloping, although the slopes are different across groups. Each group is to be interpreted as a type. The weight associated with each group is the probability that the sample is drawn from one of the groups. Group 1 has a weight

[24] In the estimation μ_0 is normalized to zero and the sum of the α_j is constrained to one.

[25] The estimation requires specifying the number of groups. As there is no guiding theory on the choice of J, we follow standard practice in the existing literature. We start with $J = 2$ and increase the number of groups until the log-likelihood does not change significantly. When we consider four groups, the log-likelihood is similar to the case with three groups, and the estimates associated with the fourth group virtually reproduce the ones for group 1.

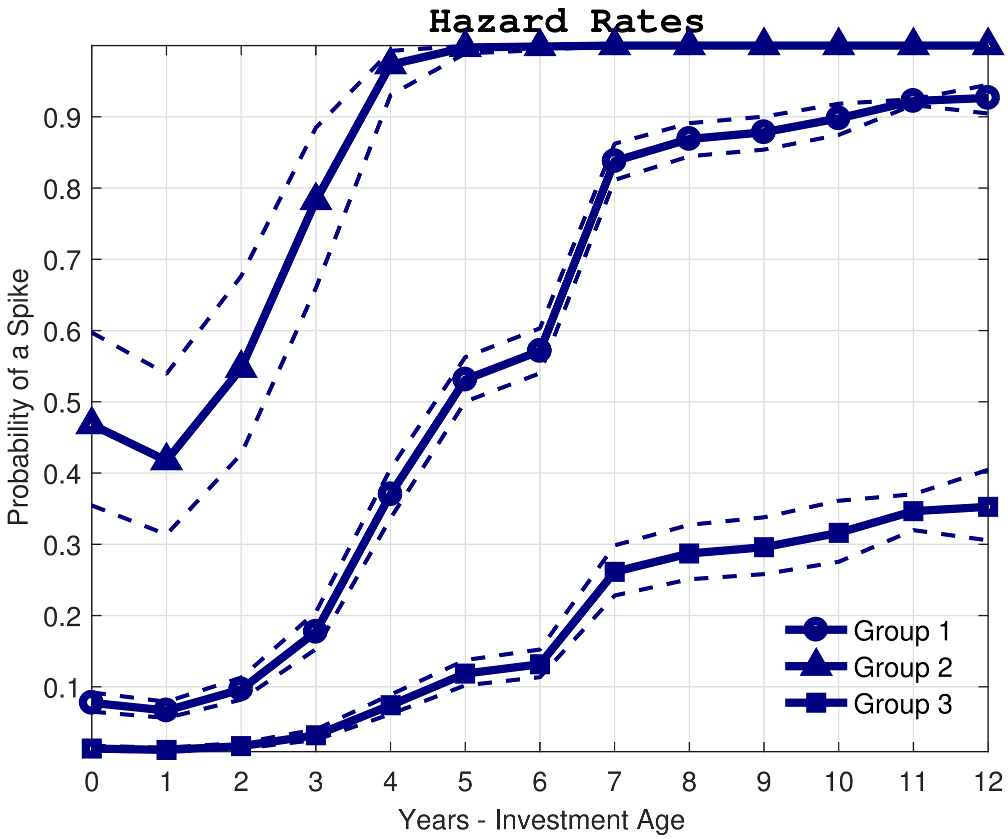

Figure A.1: Semi-Parametric Estimates

Notes: Appendix B reports details about the estimation procedure. The weight to the first group is 0.42, the weight to the second group is 0.06, and to the third 0.52.

of 0.42, group 2 of 0.06, and group 3 of 0.52. For low levels of investment age up to five years, the hazard rates in groups 1 and 3 are considerably smaller than the ones for group 2. Over time, while the probability of a spike increases dramatically for group 1, the one for group 3 increases at a slower pace. Overall, the probability that a firm exhibits an investment spike is rising with the time since the last spike.

C Investment Rates and Productivity Measures

Our measure of interest is labor and TFP together with investment age. Next, we discuss the construction of intermediate variables. Our computations follow the prevalent practice in the existing literature.

C.1 Labor Productivity

Labor productivity is obtained as the ratio between real value-added and the number of employees in a given firm. As in Bloom et al. (2018) we measure value-added $v_{f,t}$ for each firm f at year t as

$$v_{f,t} = Q_{f,t} - M_{f,t}, \tag{A.6}$$

where $Q_{f,t}$ is nominal output and $M_{f,t}$ is cost of materials. Nominal quantities are deflated by the corresponding sectoral deflators to obtain a measure of real value-added. Concerning labor input, we directly observe the wage bill and the number of employees for the firm at a given time t.

C.2 Total Factor Productivity

We follow Bloom et al. (2018) and define value-added based total factor productivity as

$$log(\hat{z}_{f,t}) = log(v_{f,t}) - \theta_f log(k_{f,t}) - \nu_f log(N_{f,t}), \tag{A.7}$$

where $v_{f,t}$ denotes real value added, $k_{f,t}$ the real capital stock, and $N_{f,t}$ labor input, and θ and ν are the cost shares for capital and labor. We follow Bachmann and Bayer (2014) and estimate θ and ν by the median of the firm average share of factor expenditure in total value-added, as defined by

$$\begin{aligned} \hat{\theta}_f &= T^{-1} \sum_t \frac{wn_{f,t}}{v_{f,t}} \text{ and} \\ \hat{\nu}_f &= T^{-1} \sum_t \frac{(r_{f,t} + \delta_{f,t}) k_{f,t}}{v_{f,t}}, \end{aligned} \tag{A.8}$$

where $wn_{f,t}$ is the real wage bill and $r_{f,t}$ the real cost of funds for the corporate sector and is estimated using the average real interest rate on banking loans for the corporate sector. As in Becker et al. (2006) and most of the existing literature, we construct the real capital stock series using the perpetual inventory method so that

$$k_{f,t} = (1 - \delta_{f,t}) k_{f,t-1} + i_{f,t}, \tag{A.9}$$

where $i_{f,t}$ is real net investment (deflated using sectoral deflators for capital expenditures) on tangible and intangible assets. To initialize the recursion, we estimate the real stock of capital using the book value of fixed assets net of funds amortization. The depreciation rate δ is common within sectors.

D Empirical Analysis - Estimates

Table A.2 reports estimates of equation 1 under different sample choices. In columns 1 and 4, we use our baseline sample; in columns 2 and 5, we include firms three years after their birth; and in column 3 and 6, we keep only firms for which we have at least 10 consecutive observations.

Table $A.2$: Investment Age and Vintage Productivity

	(1) LP	(2) LP	(3) LP	(4) TFP	(5) TFP	(6) TFP
Inv. Age 1	-0.006^{***}	-0.007^{***}	-0.002	-0.015^{***}	-0.010^{***}	-0.003^{**}
	(-4.33)	(-5.05)	(-0.75)	(-21.27)	(-13.62)	(-2.70)
Inv. Age 2	-0.010^{***}	-0.012^{***}	-0.005	-0.024^{***}	-0.018^{***}	-0.006^{***}
	(-6.39)	(-7.37)	(-1.90)	(-28.84)	(-20.75)	(-6.18)
Inv. Age 3	-0.015^{***}	-0.016^{***}	-0.009^{**}	-0.031^{***}	-0.025^{***}	-0.013^{***}
	(-8.42)	(-8.93)	(-3.17)	(-32.26)	(-25.84)	(-11.56)
Inv. Age 4	-0.025^{***}	-0.025^{***}	-0.016^{***}	-0.038^{***}	-0.032^{***}	-0.019^{***}
	(-12.42)	(-12.09)	(-5.54)	(-33.93)	(-28.67)	(-14.93)
Inv. Age 5	-0.036^{***}	-0.036^{***}	-0.026^{***}	-0.048^{***}	-0.042^{***}	-0.028^{***}
	(-15.41)	(-15.01)	(-8.12)	(-36.47)	(-32.22)	(-19.46)
Inv. Age 6	-0.044^{***}	-0.043^{***}	-0.036^{***}	-0.053^{***}	-0.047^{***}	-0.031^{***}
	(-16.22)	(-15.84)	(-10.02)	(-34.39)	(-30.87)	(-19.54)
Inv. Age 7	-0.053^{***}	-0.051^{***}	-0.043^{***}	-0.058^{***}	-0.052^{***}	-0.038^{***}
	(-16.23)	(-15.73)	(-10.23)	(-31.26)	(-28.44)	(-20.17)
Inv. Age 8	-0.072^{***}	-0.069^{***}	-0.058^{***}	-0.063^{***}	-0.057^{***}	-0.043^{***}
	(-18.98)	(-18.39)	(-12.31)	(-29.20)	(-26.64)	(-20.17)
Inv. Age 9	-0.081^{***}	-0.078^{***}	-0.073^{***}	-0.064^{***}	-0.058^{***}	-0.049^{***}
	(-18.66)	(-18.01)	(-13.76)	(-25.42)	(-23.18)	(-20.26)
Inv. Age 10	-0.084^{***}	-0.081^{***}	-0.077^{***}	-0.069^{***}	-0.062^{***}	-0.053^{***}
	(-16.71)	(-16.16)	(-12.92)	(-23.55)	(-21.58)	(-19.09)
Inv. Age 11	-0.085^{***}	-0.082^{***}	-0.082^{***}	-0.072^{***}	-0.064^{***}	-0.057^{***}
	(-14.62)	(-14.12)	(-12.08)	(-20.91)	(-19.16)	(-17.91)
Inv. Age 12	-0.095^{***}	-0.091^{***}	-0.091^{***}	-0.064^{***}	-0.057^{***}	-0.054^{***}
	(-13.93)	(-13.43)	(-11.53)	(-15.74)	(-14.25)	(-14.59)
Inv. Age 12+	-0.028^{***}	-0.022^{***}	-0.033^{***}	-0.012^{***}	-0.007^{***}	-0.017^{***}
	(-13.64)	(-8.81)	(-8.97)	(-11.48)	(-5.92)	(-11.29)
No. Observations	2,952,117	2,493,655	895,239	4,008,046	3,354,342	1,690,604
R^2	0.321	0.327	0.457	0.050	0.048	0.053

Notes: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$, where p is the marginal probability level; t-statistics in parentheses. Each equation is estimated using Ordinary Least Squares. All the equations include firm-specific constants, industry and year-effects. The dependent variable is the log of labor productivity (LP) or total factor productivity (TFP). The sample period is 1998-2015. Column 1 and 4 report our baseline case. Column 2 and 5 estimates the baseline specification a sample that includes firms 3 years after their birth. In Column 3 and 6, the sample includes firms for which we have at least 10 consecutive observations.

E Sensitivity Analysis - Intangible Capital

Table $A.3$ reports estimates of equation 1 splitting the sample based on the intensity of intangible capital. The latter is measured as the ratio between investment in intangibles and total capital.

Table $A.3$: Investment Age and Vintage Productivity - Robustness

	(1) LP	(2) LP	(3) LP	(4) TFP	(5) TFP	(6) TFP
Inv. Age 1	-0.006^{***}	-0.002	-0.010^{***}	-0.015^{***}	-0.019^{***}	-0.011^{***}
	(-4.33)	(-1.31)	(-5.40)	(-21.27)	(-18.81)	(-11.48)
Inv. Age 2	-0.010^{***}	-0.005^{*}	-0.016^{***}	-0.024^{***}	-0.029^{***}	-0.020^{***}
	(-6.39)	(-2.30)	(-7.53)	(-28.84)	(-23.45)	(-17.70)
Inv. Age 3	-0.015^{***}	-0.009^{***}	-0.022^{***}	-0.031^{***}	-0.037^{***}	-0.027^{***}
	(-8.42)	(-3.32)	(-9.25)	(-32.26)	(-25.28)	(-20.56)
Inv. Age 4	-0.025^{***}	-0.022^{***}	-0.030^{***}	-0.038^{***}	-0.043^{***}	-0.035^{***}
	(-12.42)	(-7.21)	(-10.93)	(-33.93)	(-24.08)	(-23.72)
Inv. Age 5	-0.036^{***}	-0.033^{***}	-0.041^{***}	-0.048^{***}	-0.056^{***}	-0.042^{***}
	(-15.41)	(-8.90)	(-13.35)	(-36.47)	(-26.50)	(-24.84)
Inv. Age 6	-0.044^{***}	-0.040^{***}	-0.048^{***}	-0.053^{***}	-0.066^{***}	-0.043^{***}
	(-16.22)	(-9.33)	(-13.82)	(-34.39)	(-25.85)	(-22.49)
Inv. Age 7	-0.053^{***}	-0.048^{***}	-0.059^{***}	-0.058^{***}	-0.067^{***}	-0.051^{***}
	(-16.23)	(-9.04)	(-14.10)	(-31.26)	(-21.33)	(-22.35)
Inv. Age 8	-0.072^{***}	-0.064^{***}	-0.078^{***}	-0.063^{***}	-0.076^{***}	-0.054^{***}
	(-18.98)	(-10.14)	(-16.54)	(-29.20)	(-20.24)	(-20.47)
Inv. Age 9	-0.081^{***}	-0.078^{***}	-0.085^{***}	-0.064^{***}	-0.077^{***}	-0.055^{***}
	(-18.66)	(-10.32)	(-15.82)	(-25.42)	(-16.96)	(-18.10)
Inv. Age 10	-0.084^{***}	-0.082^{***}	-0.087^{***}	-0.069^{***}	-0.088^{***}	-0.057^{***}
	(-16.71)	(-9.07)	(-14.25)	(-23.55)	(-16.24)	(-16.37)
Inv. Age 11	-0.085^{***}	-0.079^{***}	-0.090^{***}	-0.072^{***}	-0.084^{***}	-0.062^{***}
	(-14.62)	(-7.33)	(-12.88)	(-20.91)	(-12.82)	(-15.50)
Inv. Age 12	-0.095^{***}	-0.098^{***}	-0.095^{***}	-0.064^{***}	-0.089^{***}	-0.049^{***}
	(-13.93)	(-7.41)	(-11.88)	(-15.74)	(-10.97)	(-10.53)
Inv. Age 12+	-0.028^{***}	-0.020^{***}	-0.036^{***}	-0.012^{***}	-0.012^{***}	-0.011^{***}
	(-13.64)	(-6.49)	(-13.22)	(-11.48)	(-7.39)	(-8.26)
No. Observations	2,952,117	1,353,358	1,598,759	4,008,046	1,830,779	2,177,267
R^2	0.321	0.313	0.329	0.050	0.047	0.054

Notes: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$, where p is the marginal probability level; t-statistics in parentheses. Each equation is estimated using Ordinary Least Squares. All the equations include firm-specific constants, industry and year-effects. The dependent variable is the log of labor productivity (LP) or total factor productivity (TFP). The sample period is 1998-2015. Column 1 and 4 report our baseline case. Column 2 and 5 estimates the baseline specification a sample that includes firms with the intensity of intangible capital above the 75th percentile. In Column 3 and 6, the sample includes firms with the intensity of intangible capital below the 75th percentile.

F Sensitivity Analysis - Alternative Definition of Spikes

Figure *A*.2 reports the estimate of $Inv.Age_j$ coefficients in 1 employing an alternative definition of investment spikes.

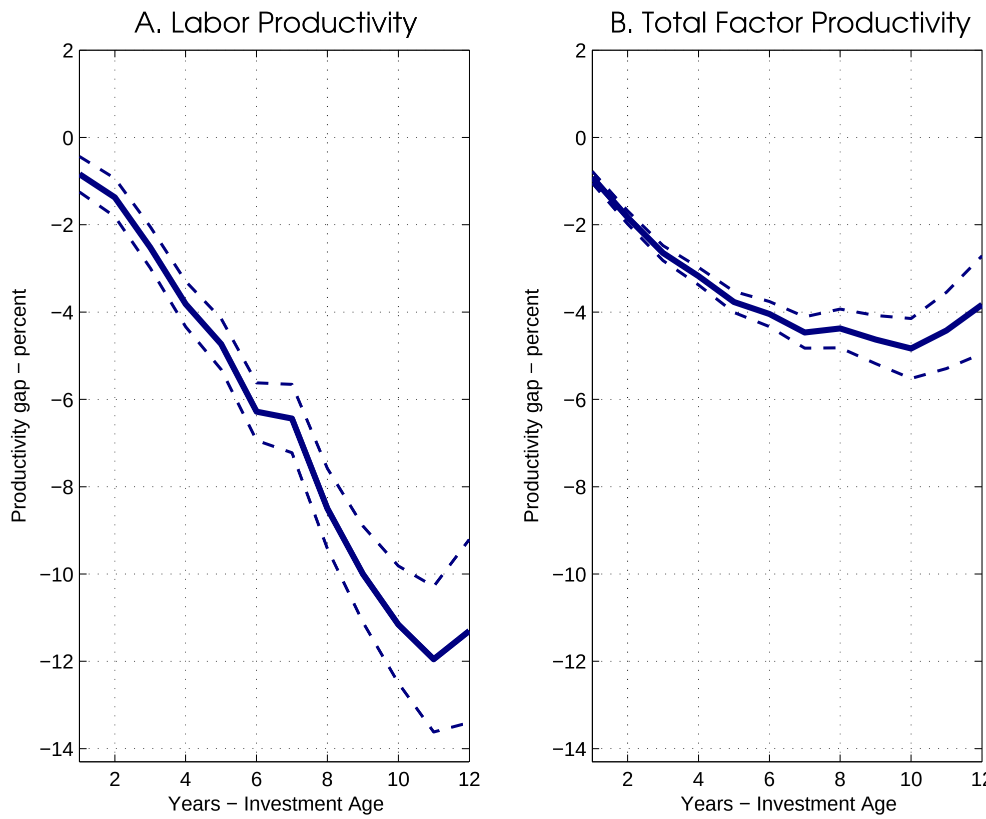

Figure $A.2$: Investment Age and Total Factor Productivity - Absolute Spikes

Notes: The figure reports the estimated β_j coefficients in equation 1. See the text for the definition of *absolute* spike. Dashed lines are 95 percent confidence bands. Each equation is estimated with Ordinary Least Squares, and it includes fixed-, industry-, year-effects and a series of dummies for a firm's age and size.

G Sectoral Analysis - Estimates

Results in the previous section provide empirical evidence of the link between investment and productivity. Here, instead of pooling all firms together, we fit our empirical specification to firms in each sector separately. This exercise sheds light on which sectors are driving our results and whether vintage effects are a defining characteristic of firm level productivity in all sectors. Before discussing the estimated coefficients, we notice that the lumpy nature of capital accumulation is a characteristic of all sectors.

Figures A.3 and A.4 plot estimated coefficients of $Inv.Age_j$ obtained by fitting equation 1 to each sector.

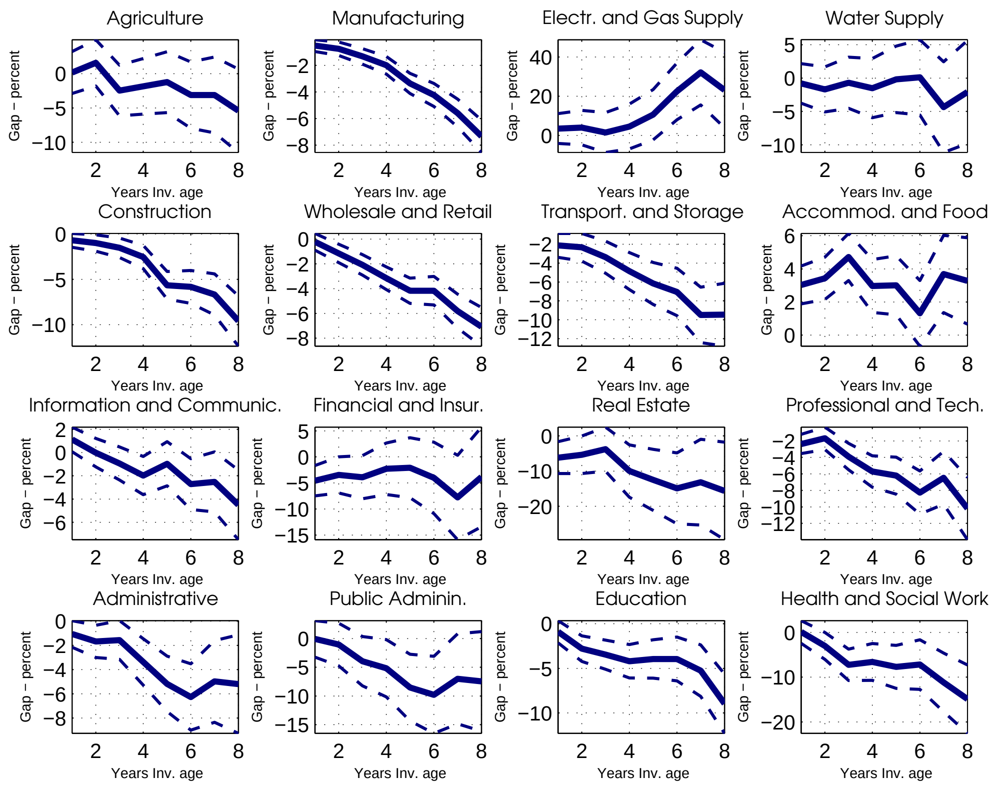

Figure A.3: Investment Age and Labor Productivity - Sectoral Analysis

Notes: The figure reports the estimated β_j coefficients in equation 1 using the baseline definition of investment spike. Dashed lines are 95 percent confidence bands. Each equation is estimated with Ordinary Least Squares, and it includes fixed-, year-effects and a series of dummies for a firm's age and size.

The fraction of firms experiencing spikes (measured as exhibiting an investment rate over 20 percent) ranges from 13 percent in the water supply sector to 21 percent in construction. Given the significant differences in the number of observations in each sector, to estimate a consistent specification we combine in a single variable investment age dummies over eight years. Figures A.3 and A.4 report the results. Because of the smaller number of observations, estimates are less precise, and confidence intervals are wider than the ones obtained with the pooled sample. Nonetheless, we find evidence of vintage effects in almost every sector, except for the financial sector and hotel and food services.

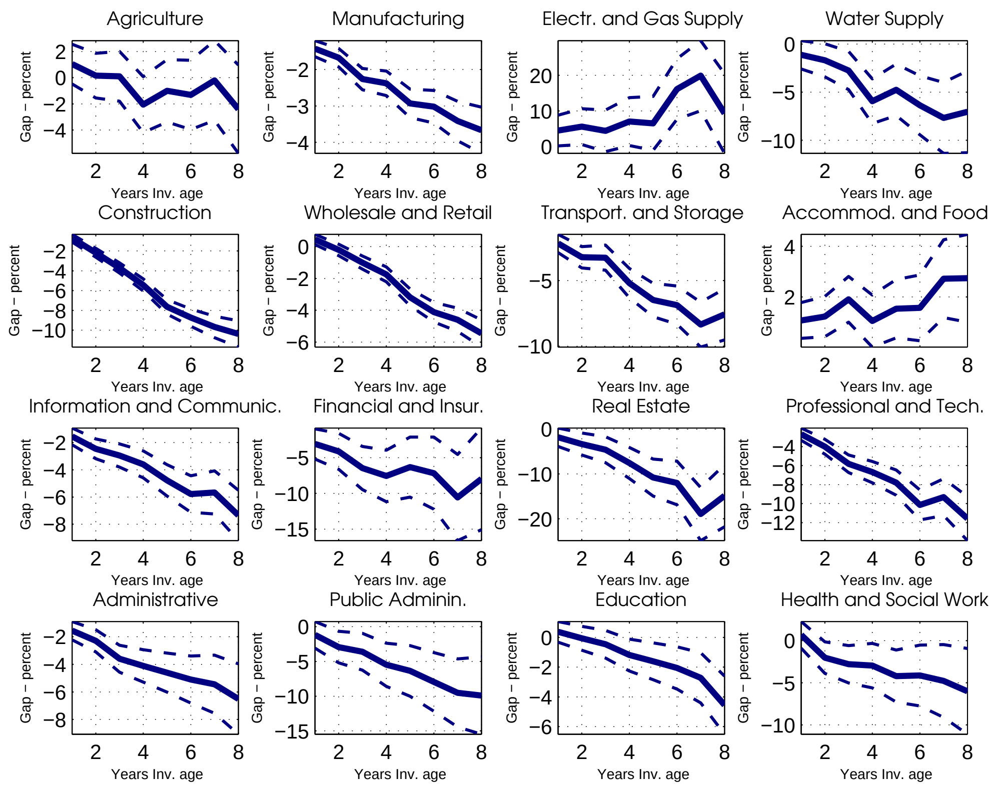

Figure $A.4$: Investment Age and Total Factor Productivity - Sectoral Analysis

Notes: The figure reports the estimated β_j coefficients in equation 1 for each sector. Dashed lines are 95 percent confidence bands. Each equation is estimated with Ordinary Least Squares, and it includes fixed-, year-effects and a series of dummies for a firm's age and size.

Equipment goods do not constitute a significant component of the production process in these sectors. Two points are worth emphasizing. First, when detected in the data, the magnitude of the estimated vintage effects is similar across sectors. Second, the confidence intervals for the sectoral estimates include the estimates obtained with the pooled sample.

H Equilibrium and (S,s) Decision Rules

Given the presence of fixed cost, the adoption and investment decision is akin to exercising an option. Consider a firm of a type (ε, z, k) drawing adjustment cost ξ. Define the value associated with the value of action $V^A(\varepsilon, z, k; \mu)$ and the one with the inaction choice $V^I(\varepsilon, z, k; \mu)$ as

$$V^A(\varepsilon, z_0, k; \mu) \equiv \max_{k' \in \mathbf{R}_+} R(\varepsilon, z_0, k'; \mu), \quad (A.10)$$

$$V^I(\varepsilon, k; \mu) \equiv \max_{k' \in \Omega(k)} R(\varepsilon, z_0, k'; \mu), \quad (A.11)$$

Next, define the firm's target capital k^* as the optimal choice of k — when the firm obtains the latest vintage — that solves the right-hand side of (A.10). The solution to the problem in (A.10) is independent of the current stock of capital k and ξ, but not ε (and of course z_0), given persistence in firm-specific productivity. As a result, all firms with current productivity ε and pay their fixed costs to upgrade to their latest vintage will choose a common target capital for the next period, $k^* = k(\varepsilon, \mu)$, and achieve a common gross value $V^A(\varepsilon, z_0, k; \mu)$. By contrast, firms that do not pay adjustment costs have value $V^I(\varepsilon, z, k; \mu)$. In this case, firms keep its current vintage z that becomes more obsolete (i.e., more distant from the technological frontier) at a rate γ_A. The firm gets to adjust its stock of capital, that is constrained to be included in $\Omega(k)$.

A firm will pay the fixed cost if $V^A(\varepsilon, k; \mu) - p(\xi + \delta_S k)$ – the value of adjusting — is at least as great as $V^I(\varepsilon, z, k; \mu)$ — the value of inaction. Given continuity in the adjustment cost ξ, it is possible to identify threshold value such that a type (ε, z, k) firm indifferent between action and inaction:

$$-p(\mu)(\hat{\xi}(\varepsilon, z, k; \mu) + \delta_S k) + V^A(\varepsilon, k; \mu) = V^I(\varepsilon, z, k; \mu). \quad (A.12)$$

To summarize the adoption and investment decision define $\xi^T(\varepsilon, z, k; \mu) \equiv \min [\ \bar{\xi}, \hat{\xi}(\varepsilon,z,k;\mu)\ ]$ so that $0 \leq \xi^T(\varepsilon, z, k; \mu) \leq \bar{\xi}$. Any firm (ε, z, k) that draws an adjustment cost at or below its type-specific threshold, $\xi^T(\varepsilon, z, k; \mu)$ will pay the fixed cost and adjust k and

z.

Thus, for a given group of firms of type $(\varepsilon, z, k; \mu)$, a fraction $G\left[\xi^T(\varepsilon, z, k; \mu)\right]$ pay their fixed cost to adopt the latest vintage and optimally choose capital. Thus, the market-clearing levels of consumption required to determine p using (9) is given by

$$\begin{aligned} C &= \int_S F(\varepsilon, z, k) \qquad (A.13) \\ &\quad - G\left[\xi^T(\varepsilon, z, k; \mu)\right] J(\xi \le \xi^T(\varepsilon, z, k; \mu))(i - \xi) \\ &\quad - \left[1 - G\left[\xi^T(\varepsilon, z, k; \mu)\right]\right] J(\xi > \xi^T(\varepsilon, z, k; \mu)) \left[i^C \, \mu\left(d\left[\varepsilon \times z \times k\right]\right)\right], \qquad (A.14) \end{aligned}$$

where it is understood that i and i^C depend upon the firm's current state. Finally, the evolution of the firm distribution, $\mu' = \Gamma(\mu)$. It is useful to define the indicator function $J(x) = 1$ if x is true, and 0 otherwise. For each $(\varepsilon_m, z, k) \in S$

$$\begin{aligned} &\mu'(\varepsilon_m, z, k) \\ &= \sum_{l=1}^{N_\varepsilon} \pi^{\varepsilon}_{lm} \left[\begin{array}{c} \int J(\xi \le \xi^T(\varepsilon, z, k; \mu)) G\left[\xi^T(\varepsilon_l, z, k; \mu)\right] \mu(\varepsilon_l, z, dk) \\ + \int \left[1 - \int G\left[\xi^T(\varepsilon_l, z, k; \mu)\right] \mu(\varepsilon_l, z, dk)\right] J(\xi > \xi^T(\varepsilon, z, k; \mu)) \end{array} \right] \end{aligned}$$

I Computational Details - Stationary Equilibrium

I.1 Value Function and Steady State

The value function to solve the firm's problem defined in equation (4)-(6) will be the basis of our numerical solution of the economy. The solution algorithm involves repeated application of the contraction mapping implied by (4)-(6) to solve for firms' value function, given the price functions $p(\mu)$. More specifically, the firm's problem amounts to find the next-period value of capital k'. To do so, we resort on a golden section search to allow for continuous control. We discretize the state space using a fine grid between 0.1 and 8.5 for capital k and between 1 and 0.2 for z. The process for the idiosyncratic process ε is approximated using the procedure in Tauchen (1986) over 13 possible values. We compute the value function exactly at the grid points above and interpolate for in-between values.

This procedure is implemented using a multidimensional cubic splines procedure, with a so-called "'not-a-knot"'-condition to address the large number of degrees of freedom problem, when using splines, see Judd (1998). With the firm's policy function at hand, we compute the stationary distribution and verify that the guessed price is consistent with market clearing. We update the guessed price function $p(\mu)$ until convergence, i.e., until the guessed and the market-clearing price converges.

I.2 Transitional Dynamics

We solve for the transitional dynamics as follows. We specify a path for $\{X_t\}_{t=0}^T$ with $X_0 = X_T = \bar{X}$, where $\bar{X}$ indicate the steady-state value of X. Following the approach in Ríos-Rúll (1999), we conjecture a path for the marginal utility of consumption $\{p_t\}_{t=1}^T$. Assuming that at time T the economy is in steady-state we can solve backward the expected value function at all dates T-1, T-2,... till 1. Using the expected value function, the value of the shocks, and the conjecture path, we explicitly solve for the market clearing in the goods market in every time period and obtain new values for $\{p_t^{MktClearing}\}_{t=1}^T$. We iterate until the proposed path $\{p_t\}_{t=1}^T$ and equilibrium path $\{p_t^{MktClearing}\}_{t=1}^T$ for the marginal utility of consumption converge.

I.3 Aggregate Process for the Financial Shock

We study the aggregate and cross-sectional dynamics in response to a macroeconomic shock under two alternative scenarios. First, we consider a deterioration in financial conditions. In the spirit of Gomes (2001), firms (that adopt the latest vintage) are subject to an additional financial cost equal to $\lambda_0 + \lambda_1 i_{f,t}$. The processes for lambdas are parameterized so that the model reproduces the micro and macro behavior of investment during the 2012 recession. More specifically, the shock processes result on a drop in the fraction of firms experiencing spikes and in aggregate investment observed in the data.[26]

Let X indicate either shock, depending on the experiment. The path for $\{X_t\}_{t=0}^T$ is

[26] The fraction of firms experiencing spikes drops by 5 percentage points in 2012, relative to its value in 2011.

Table $A.4$: Calibration of the Financial Shock

Shock	$\bar{X}$	X_1	ΔX	ρ_X
λ_0	0	0.0045	0.0045	0.75
λ_1	0	1.20	1.20	0.75

such that the economy is initialized at the steady state, and once the shock is absorbed, the economy reverts to the steady state. We assume that X is an autoregressive process of order one. More formally, this results in $X_0 = X_T = \bar{X}$ and $(X_t - \bar{X}) = \rho_X(X_{t-1} - \bar{X})$ for $t \in \{1, ...T-1\}$, where $\bar{X}$ is the value taken in the steady state. Table $A.4$ reports the details of the calibration exercise.

J Computational Details - Aggregate Uncertainty

J.1 Calibration

We retain the parameters in Table 5, with few exceptions. We introduce labor supply considerations by assuming a perfectly elastic labor supply, see Hansen (1985) and Thomas (2002). A denotes the parameter that governs the disutility of labor in the utility function and is set to 0.79 to ensure that aggregate labor is, on average, equal to one. We estimate the elasticity of output with respect to capital (θ) and the one to labor (ν) from the data using the procedure detailed in Appendix C.1. In the sample, θ is equal to 0.18 and ν to 0.64. To calibrate the process of idiosyncratic shocks and the upper support of the adjustment cost distribution, we follow the same strategy in Section 5.1 and choose these parameters to reproduce the cross-sectional distribution of investment rates. This yields ρ_ε and σ_ε equal to 0.86 and 0.0163, respectively. The upper support of the distribution (ξ) is set to 0.026. Finally, the log of $\gamma_{A,t}$ follows an autoregressive process centered around its mean value $\bar{\gamma_A}$: $log(\gamma_{A,t}) = (1-\rho_{\gamma_A})\, log(\bar{\gamma_A}) + \rho_{\gamma_A} log(\gamma_{A,t-1}) + \sigma_{\gamma A}\varepsilon_{\gamma_A}$, where ε_{γ_A} is a normally distributed i.i.d. process with standard deviation $\sigma_{\gamma A}$. In absence of empirical guidance for the evolution of the technological frontier, we choose ρ_{γ_A} and $\sigma_{\gamma A}$ so that average TFP in the model matches the persistence and the volatility of its data counterpart for the sample 1992-2007. We then set ρ_{γ_A} equal to 0.15 and $\sigma_{\gamma A}$ to 0.0026.

J.2 Solution Algorithm

When the growth rate of technology is stochastic, the endogenous distribution of capital stocks and productivity enters the state space of the model. To solve the model, we follow the approach in Khan and Thomas (2008). This strategy replaces the aggregate law of motion for the distribution with a forecast rule. Typically, to predict prices and the future proxy aggregate state, agents use the mean capital stock. In our framework, two endogenous distributions for the capital stocks and the vintage technologies are available to the firms. In theory, this could complicate the solution algorithm by requiring agents to forecast the behavior of two distributions rather than one. In practice, when the persistence of the shock is relatively low, the standard rule that uses the mean of the capital stocks as a regressor works very well, yielding an accurate forecast of prices and future proxy aggregate state. We forecast the mean capital K' and the marginal utility of consumption p using $log(K')=\beta_0 + \beta_1 log(K) + \varepsilon_K$ and $log(p)=\beta_0 + \beta_1 log(K) + \varepsilon_p$. As we approximate $\gamma_{A,t}$ using the discretization procedure in Tauchen (1986) using a grid with 7 points, we estimate a regression conditional on each realization of the aggregate process, $\gamma_{A,t}$.

In Table *A*.5 and *A*.6 we assess the accuracy of the forecasting rule for both models. We find that the algorithm yields a very accurate solution as testified by the high R^2 and small standard errors. As discussed by Den Haan (2010), R-squares are averages and scaled by the variance of the dependent variable. To provide a robust statistic, we report the maximum forecast error for each regression. For the vintage model, the maximum percentage error for p is 0.002% and for K' is 0.095%. For the *RBC* model, the maximum percentage error for p is 0.076% and for K' is 0.013%. We conclude that the forecasting rules are extremely precise.

Table $A.5$: Forecasting Rules - VINTAGE MODEL

	Technology	β_0	β_1	$S.E.$	Adj. R^2
Forecasting K'					
	$\gamma_{A,1}$	0.01461	0.73473	2.84e-05	0.99998
	$\gamma_{A,2}$	0.01122	0.72941	7.15e-05	0.99989
	$\gamma_{A,3}$	0.00776	0.72151	0.00018	0.99923
	$\gamma_{A,4}$	0.00410	0.71584	0.00030	0.99818
	$\gamma_{A,5}$	0.00041	0.70716	0.00040	0.99668
	$\gamma_{A,6}$	-0.00329	0.69926	0.00053	0.99248
	$\gamma_{A,7}$	-0.00697	0.69627	0.00051	0.99311
Forecasting p					
	$\gamma_{A,1}$	0.52492	-0.28237	1.96e-05	0.99995
	$\gamma_{A,2}$	0.52446	-0.27822	5.38e-05	0.99957
	$\gamma_{A,3}$	0.52395	-0.27482	7.23e-05	0.99917
	$\gamma_{A,4}$	0.52342	-0.27425	8.34e-05	0.99907
	$\gamma_{A,5}$	0.52280	-0.27157	7.88e-05	0.99915
	$\gamma_{A,6}$	0.52216	-0.26935	7.94e-05	0.99888
	$\gamma_{A,7}$	0.52149	-0.26924	6.21e-05	0.99932

Table $A.6$: Forecasting Rules - RBC MODEL

	Technology	β_0	β_1	$S.E.$	Adj. R^2
Forecasting K'					
	$\gamma_{A,1}$	0.01575	0.70978	1.94e-05	0.99999
	$\gamma_{A,2}$	0.01223	0.70868	1.96e-05	0.99999
	$\gamma_{A,3}$	0.00867	0.70901	2.12e-05	0.99998
	$\gamma_{A,4}$	0.00513	0.70891	2.42e-05	0.99998
	$\gamma_{A,5}$	0.00159	0.70895	2.31e-05	0.99998
	$\gamma_{A,6}$	-0.00195	0.70860	1.98e-05	0.99998
	$\gamma_{A,7}$	-0.00549	0.70893	1.82e-05	0.99999
Forecasting p					
	$\gamma_{A,1}$	0.52557	-0.29789	1.06e-05	0.99998
	$\gamma_{A,2}$	0.52478	-0.29789	6.43e-06	0.99999
	$\gamma_{A,3}$	0.52400	-0.29834	7.51e-06	0.99999
	$\gamma_{A,4}$	0.52322	-0.29867	9.19e-06	0.99998
	$\gamma_{A,5}$	0.52244	-0.29902	9.22e-06	0.99998
	$\gamma_{A,6}$	0.52166	-0.29934	1.05e-05	0.99998
	$\gamma_{A,7}$	0.52089	-0.29975	7.69e-06	0.99999