Wealth Inequality, Homeownership and Optimality of Wealth Taxes*

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Abstract

We analyze the long-run optimal combination of wealth and labor tax rates in a model with heterogeneous households (wealth-poor and wealth-rich), where wealth derives from business capital and homeownership. We find that the optimal steady state tax structure includes some taxation of labor, zero taxation of financial and business capital, a housing wealth tax on the wealth-rich households and a housing subsidy on the wealth-poor households. Finally, we investigate the consequences on these tax rates of a rising net wage-to-wealth ratio at steady state.

KEYWORDS: housing wealth, wealth inequality, optimal taxation.

JEL CODES: E21, E62, H2, H21, G1.

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1 Introduction

There is mounting consensus, among scholars and commentators, that shifting taxation from labor to wealth may be an optimal response to the increase in wealth–to–income ratios and wealth inequality that has been documented for many advanced economies over the last decades (Piketty, 2014). This policy is mostly motivated by distributional objectives, and the concern with the effects that wealth concentration has on democratic institutions, and it is sometime credited as having small efficiency costs (Piketty et al., 2013; Altman, 2012; Saez and Zucman, 2019). In this paper we investigate the long-run social welfare effects of such tax reform under full commitment by considering a simple model where households are heterogeneous, wealth consists of business capital, housing and financial assets and the government has access to a limited set of tax rates (a flat tax on wages, housing rent and wealth, the latter being possibly contingent on the types of wealth and on the households' net asset position). We show that an optimal tax structure implies heterogeneous tax rates/subsidies on housing wealth and no tax on financial and business capital.

In our model labor supply is inelastic and households can be lenders or borrowers, home-owners or renters. Wealth heterogeneity is based on the assumption that households are endowed with different time discount rates and are subject to borrowing constraints, so that some households end up having zero net wealth. In this set up, the steady state distribution of wealth is perfectly polarized between a set of *wealth-rich* and *wealth-poor* households, although all of them may work and own some housing wealth in different quantities. The only relevant difference between the two sets of households is that the wealth-poor are either renters, with zero home ownership, or home owners with the value of their home being perfectly matched by mortgages. The supply side of the economy includes two produced goods, a perishable consumption good (also called *consumption*), and residential construction. The latter generates an evolving stock of housing subject to physical depreciation. Technologies employ labor and capital, although the housing sector also needs some flow of new land available for construction every period. All the revenues from the sale of land permits are assumed to be appropriated by the government, either because the latter is the owner of land or because, despite land being privately owned, these revenues are taxed away by the government.

Within this set up, we characterize the optimal distortionary tax structure (for a given flow of positive public spending) when the Planner maximizes a weighted average of households' lifetime utilities under full commitment and we simulate the impact of a rise in wealth inequality on the optimal tax structure. Our main finding is that the Chamley-Judd's zero steady state tax on financial and business capital (Chamley, 1986; Judd, 1985) survives, whereas housing wealth is taxed at a non zero rate. In particular, we identify a set of conditions under which it is optimal to impose a positive tax on rich households' housing wealth, and a subsidy on the user cost of housing (or rent) faced by poor households. Somewhat counterintuitively, the optimal taxes and subsidies on housing wealth are zero when the rich households are pure speculators in the housing market, *i.e.*, they are not working and they derive no utility from housing services. In the more general setting, the tax on the rich households' housing wealth is positive for homogeneous utility functions and the housing subsidy is positive for all poor households whose marginal utility of consumption is sufficiently large. In particular, when poor households are all identical, their consumption of housing services must be subsidized. We also show that, when inequality rises (due to a perturbation of the households' preferences), the housing wealth tax on rich households rises, whereas the housing wealth subsidy to poor households and the wage tax decrease. The effects of a rising wealth to income ratio on the structure of housing taxes and subsidies is mainly explained by the change in the housing demand elasticity triggered by the rising wealth-to-income ratio. The negative effect of a rising wealth to income ratio on the wage tax rate is, instead, mainly a consequence of the rising government revenues from the sale of land permits (which increase substantially, as both the housing price and housing stock are increasing). These results are robust to changes in the fraction of rich households, or in the weight of land in the production function of the construction sector.

It must be stressed that the same planning optimum that we derive with the given menu of taxes (labor, rents and wealth) could also be achieved using some alternative set of tax instruments, such as a tax or subsidy on imputed rents. Since we concentrate on steady state equilibria, any distinction between housing wealth taxes and indirect taxes on housing services is somewhat artificial, since the user cost of housing (a proxy for imputed rents) is proportional to the value of housing property. More generally, it is often noted that a tax on imputed rental income could be approximated through an annual recurrent tax on property since imputed rents are typically proportional to property values. However, taxing imputed rents for owner-occupied housing may be difficult in practice, and it is rarely implemented, as it involves some practical difficulties such as evaluating properly depreciation and capital gains. In a recent report, Fatica and Prammer (2017) claim that, "while imputed rents are generally not taxed, all the euro area countries in the HFCS survey - except Malta - levy recurrent taxes on real estate property". The Mirrlees' Review (Mirrlees et al. (2011)) suggests that a tax related to the consumption value of a property bears some resemblance with the British *council tax*, which is essentially a locally collected property tax based on a limited set of brackets (*bands*) for the property values. Similar tax systems for housing wealth are applied in almost all advanced economies. The Mirrlees' Review also claims that the council tax is generally regressive relative to its base and should be replaced by a *housing service tax*, *i.e.*, a flat percentage of the rental value of property, whether it is rented or owner-occupied. However, our findings suggest that, with an inequality averse Planner, this tax should not be flat, but contingent on the size of individuals' net wealth.

Our results depend on some strong assumptions. First, an inelastic labor supply makes the model biased towards the idea that wealth should not be taxed, so that a positive taxation on housing should be fairly robust[1]. Second, deriving the wealth distribution from different subjective discount factors and debt limits has some limitations, although it is a very standard practice in neoclassical growth theory and, in some way, necessary to produce much stronger polarization in wealth than in income (which is observed in the data and not easily reproducible in models with homogeneous preferences). Third, by concentrating the analysis on steady states we miss the analysis of the transition from low to higher tax rates, which is motivated by the need to focus on long-run phenomena.

The literature on wealth taxation is large and controversial. If we abstract from life-cycle considerations, precautionary savings and imperfect information, the optimal tax on capital income is zero even when some households have no wealth or inheritances (Chamley, 1986; Judd, 1985). The reason is that capital taxation implies exponentially growing distortions of investment over time, so that there are large benefits from shifting the tax burden from capital to labor. Note, however, that housing, which constitutes a large fraction of total households' wealth, is at the same time a store of value and a source of housing services providing utility benefits[2]. For this reason, a wealth tax may not be as inefficient as predicted in models where wealth is only held for intertemporal consumption allocation, such as the models with infinitely lived individuals and no frictions in asset markets. In fact, housing taxation has been advocated in several studies, especially as a way to avoid a sub-optimal tax discrimination between factor inputs and sources of wealth, and many authors have highlighted the existence of substantial welfare gains from increased housing taxation, due to the failure to tax implicit rental income and because of mortgage interest deductability characterizing

[1] Saez et al. (2009) argue that the estimated compensated elasticity of labor is small (close to zero for prime-age males).

[2] More generally, Saez and Stantcheva (2017) assume that wealth enters the households utility function directly for various reasons, among which are "social status", "power", "philanthropy", etc.

existing tax codes in most advanced economies (see Poterba (1984), Gahvari (1984), Berkovec and Fullerton (1992), Auerbach and Hines (2002), Gervais (2002) and Mirrlees et al. (2011)). These distortions imply that housing investment crowds out business capital and generates excessive levels of homeownership. Furthermore, a heavier taxation of housing wealth may reduce inequality in economies where, because of capital market imperfections and indivisibilities, rental housing is concentrated among poor households (although Gervais (2002) finds that the distributional effects of eliminating housing tax incentives are quantitatively small). Our contribution differs from this literature because we are specifically interested in (differentiated) wealth taxation and the way it should evolve in response to increasing wealth inequality, instead of examining the welfare gains from reducing fiscal incentives on housing. Whereas the case for housing taxation is usually based on the unavailability of non distorting taxes, in our model housing taxes (and subsidies) survive despite the fact that labor taxes are non distortionary. The papers most related to ours are Bonnet et al. (2019), who consider an economy with heterogeneous wealth composition (business capital and land) and heterogeneous households (capitalists/landlords and workers/tenants). Differently from our model, Bonnet et al. (2019) assume that poor households have no wealth (in particular, no land and housing wealth) and obtain housing services by renting from rich households. In their model, capital should not be taxed and the first best allocation can be implemented by levying a tax on land. The optimality of a land tax follows from the Planner's preference for redistribution and the fact that land is a fixed factor (*i.e.*, a land tax is non-distortionary).

From the empirical perspective, there is a large literature estimating the elasticity of taxable income with respect to marginal tax rates (for example, Saez et al. (2009)), but little evidence regarding the wealth elasticity with respect to wealth taxes. Having a reliable estimate is important, because, if the elasticity of wealth to the tax rate is small, shifting taxation from income to wealth appears to be intuitively reasonable in light of the alleged increasing trend in wealth–to–income ratios. Typically, existing tax codes that contain a wealth tax also set thresholds above which the wealth tax applies. The existence of the threshold creates incentives to reduce, or misreport, wealth to just below the threshold to avoid the tax. Therefore, we should expect bunching in the distribution of wealth around the exemption threshold. Most empirically studies exploit this discontinuity to estimate the elasticity of wealth to wealth taxes. Looking at Danish data, Jacobsen et al. (2017) find some evidence that wealth is relatively inelastic with respect to tax rates, suggesting that, leaving aside efficiency considerations, taxing wealth may be an effective tool for reducing wealth inequality. A relatively low elasticity, in the range of 0.1% and 0.3% is also found by Seim (2017) for Sweden. On the other hand, using variations across Swiss cantons, Brülhart et al. (2019) provides a much larger estimate in the range 20 to 40%. We provide a rough estimate of the elasticity of aggregate net wealth with respect to a wealth tax for our model under a plausible parameter configuration. Specifically, we find that the introduction of a new 1% wealth tax, on financial and housing net wealth, reduces the steady state net wealth by approximately 10% with respect to the case of zero wealth taxes.

The remainder of this paper is organized as follows: section 2 presents the model; section 3 considers the optimal taxation problem and results of our quantitative analysis; section 4 presents our conclusions.

2 The Model

In this section we present a model with two sectors: manufacturing and housing construction; different households, with preferences over consumption of a perishable manufacturing good and a durable good, which we call housing; and a government that uses a set of taxes to finance public spending.

2.1 Framework

We consider an economy with two sectors, *manufacturing* and *(housing) construction*; and a finite set $\mathcal{I}$ of households types indexed by i with preferences over consumption of the manufacturing good and housing services. The manufacturing good is a proxy for all non-construction consumption and the housing stock is a proxy for housing services. Household types have mass $m_i \in (0,1)$ per total population, with $\sum_{i\in\mathcal{I}} m_i = 1$, and belong to infinitely lived dynasties. Life time utilities are represented by

$$\mathcal{U}^i = \sum_{t=0}^{\infty} \beta_i^t U(c_t^i, z_t^i), \tag{1}$$

where $U(.)$ is the per period strictly increasing and strictly concave utility function (identical across types); $\beta_i \in (0,1)$ are the type-specific time discount rates; and c^i, z^i denote, respectively, household i's consumption of manufacturing goods and housing services. All households supply one unit of labor inelastically and have different labor productivities. In particular, we let $\epsilon^i \in (0,1)$ be the household i-specific contribution to production of a unit of labor and assume

$$\sum_{i\in\mathcal{I}} m_i \epsilon^i = 1.$$

Production takes place in the manufacturing (m) and housing (h) sector with heterogeneous neoclassical technologies. While the technology in manufacturing employs labor and capital only, production of new housing requires also land. In particular, technologies in the two sectors are defined by

$$y_t^m = f^m(k_t^m, l_t^m), \qquad y_t^h = f^h(k_t^h, l_t^h, x_t),$$

where k^j is the capital stock and l^j the amount of labor employed in sector j in efficiency units; x_t is the flow of new land available for housing construction. Following the recent literature on housing, we think of the flow of new available land as "land permits" provided by the government on the basis of some physical constraint or environmental concern (Favilukis et al., 2017; Borri and Reichlin, 2018). Both $f^m(.)$ and $f^h(.)$ are assumed to be increasing, strictly concave, to exhibit constant returns to scale, to be continuously differentiable and to verify Inada conditions. For simplicity, we assume that capital fully depreciate in one period and we let

$$c = \sum_{i \in \mathcal{I}} m_i c^i, \qquad z = \sum_{i \in \mathcal{I}} m_i z^i, \qquad k = k^h + k^m.$$

Then, for some given initial allocation of capital, k_0; and housing stock, h_0; a *feasible allocation* of individuals' consumption and sector specific capital and employment is a sequence $\{c_t^i, z_t^i, k_t^j, l_t^j, k_{t+1} h_{t+1}, ;\ i \in \mathcal{I},\ j = h, m\}_{t=0}^{\infty}$, satisfying, for all $t \geq 0$,

$$\begin{aligned} c_t + g_t + k_{t+1} &\leq f^m(k_t^m, l_t^m), & (2)\\ h_{t+1} &\leq f^h(k_t^h, l^h, x_t) + (1-\delta) h_t, & (3)\\ z_t &\leq h_t & (4)\\ l_t^m + l_t^h &\leq 1, & (5)\\ k_t^h + k_t^m &\leq k_t, & (6) \end{aligned}$$

where g_t is the total amount of public spending; $\delta \in (0, 1]$ is the housing depreciation rate; and $\{x_t\}_{t=0}^{\infty}$ is the given sequence of government provided flow of new land permits.

We let manufacturing be the *numeraire* good; q_t the unit price of housing; R_t the real gross interest rate; w_t the average real wage rate, with the i-specific wage rate being set at $\epsilon^i w_t$. Assuming perfect competition in both sectors, profit maximization and perfect labor mobility imply

$$R_t = f_k^m(k_t^m, l_t^m) = q_t f_k^h(k_t^h, l_t^h, x_t), \quad (7)$$

$$w_t = f_l^m(k_t^m, l_t^m) = q_t f_l^h(k_t^h, l_t^h, x_t). \quad (8)$$

where f_k^j, f_l^j, f_x^j, for $j = h, m$, are the marginal products of capital, labor and land. Firms in the construction sector rebate any remaining profits to the government as a compensation for the use of land permits, and the government uses these resources to finance public spending. Then, the government revenue from land permits in units of labor efficiency is

$$\tau_t^L = q_t f_x^h(k_t^h, l_t^h, x_t) x_t. \quad (9)$$

In our model all tax revenues come from (possibly type-specific) income taxes, wealth taxes, and from the sale of land permits. Note that income, wealth, and housing taxes may be differentiated across types of wealth (*i.e.*, financial or housing) and made contingent on the households' net financial wealth position, *i.e.*, on whether a household is a net lender or a net borrower. Any household i, at all time $t \geq 0$, has access to some units, b_{t+1}^i, of a 1-period bond with gross pre-tax interest rate, R_{t+1}, and some units, h_{t+1}^i, of residential property with (before tax) unit price q_t. Housing services enjoyed at time t, z_t^i, come from rental housing, with a before tax unit rental price of s_t, or home ownership. We denote with $z_t^{r,i}$ the housing services from renting and the units of housing rented; and with $z^{o,i}$ the housing services from owner occupied housing as well as the units of housing property occupied by the owner. Hence, one unit of housing capital generates one unit of housing services. These two type of housing services are assumed to be perfect substitutes, so that

$$z_t^i = z_t^{r,i} + z_t^{o,i}.$$

We assume that housing capital is not perfectly divisible (Gervais, 2002). In particular, there exists a minimum size of owner occupied housing, $\bar{z}$, which also represents the smallest amount of housing services a homeowner (but not a renter) can consume. Hence, all households face the constraint:

$$z_t^{o,i} \geq \bar{z}. \tag{10}$$

The government can select tax rates, at all $t \geq 0$, from a menu, $(\tau_t^s, \tau_t^w, \tau_t^{k,i}, \tau^{h,i})$, representing, respectively, a tax rate on housing rent, labor income, financial and housing wealth. The per-period budget constraint of the household is then

$$b_{t+1}^i/R_{t+1} + c_t^i + q_t h_{t+1}^i + s_t z_t^{r,i} = \epsilon^i \hat{w}_t + \hat{s}_t(h_t^i - z_t^{o,i}) + (1 - \tau_t^{k,i})b_t^i + (1-\delta)\hat{q}_t^i h_t^i, \tag{11}$$

where $\hat{s}_t = (1 - \tau_t^s)s_t$ is the after tax housing rent (on landlords); $\hat{w}_t = (1 - \tau_t^w)w_t$ is the after tax wage rate per units of efficiency; and $\hat{q}_t^i = q_t(1 - \tau_t^{h,i})$ is the housing price net of the housing tax. Note that the latter can be considered a tax on housing wealth or, equivalently, a sale tax on housing transactions. Later on, as it is common in most tax codes, we will impose that taxes on financial wealth may differ from zero if and only if the latter is positive, *i.e.*, debt is untaxed ($\tau_t^{k,i} = 0$ if $b_t^i \leq 0$). Now define households' before tax net assets as

$$a_{t+1}^i/R_{t+1} = b_{t+1}^i/R_{t+1} + q_t h_{t+1}^i \tag{12}$$

and the i-specific after tax net assets, $\hat{a}^i = (1 - \tau^{k,i})a^i$, gross interest rates, $\hat{R}^i = R(1 - \tau^{k,i})$, and present value prices, $\{p_t^i\}_{t=0}^{\infty}$, recursively from $p_t^i/p_{t+1}^i = \hat{R}_{t+1}^i$. Then, using (12), the t-period budget constraint becomes

$$p_{t+1}^i \hat{a}_{t+1}^i + p_t^i(c_t^i + s_t z_t^{r,i} + \hat{\pi}_t^i z_t^{o,i} + (\hat{\pi}_t^i - \hat{s}_t)(h_t^i - z_t^{o,i})) = p_t^i(\epsilon^i \hat{w}_t + \hat{a}_t^i), \tag{13}$$

where

$$\hat{\pi}_t^i = \hat{R}_t^i q_{t-1} - (1-\delta)\hat{q}_t^i \tag{14}$$

is the after tax *user cost of housing*. The latter is a measure of the net of tax market price of housing services and it is equivalent to the present value of next period *imputed rents* from owner occupied housing. Finally, we assume that net assets must be non-negative at all periods,

$$\hat{a}_{t+1}^i \geq 0, \tag{15}$$

i.e., households debt must be fully collateralized by the housing wealth.

To close the model, we assume that the government, at all time t, issues one-period bonds in some amount b_{t+1}^g at the market interest rate, R_{t+1}. Then, the government budget constraint is

$$b_{t+1}^g / R_{t+1} \geq g_t + b_t^g - \mathcal{T}_t - \tau_t^w w_t - \tau_t^L, \tag{16}$$

where

$$\mathcal{T}_t = \sum_i m_i \left(\tau_t^{k,i} b_t^i + q_t(1-\delta)\tau_t^{h,i} h_t^i + \tau_t^s s_t (h_t^i - z_t^{o,i}) \right)$$

is the time-t revenue from wealth taxation.

2.2 Equilibrium

The following proposition provides a first order characterization of households' optimal choices at equilibrium. Appendix B contains the proof of the proposition.

Proposition 1. *Any equilibrium where some individuals are homeowners is such that*

$$\begin{aligned}
\textit{either} \quad z_t^{o,i} = 0 \quad \textit{and } z_t^i = z_t^{r,i},\ h_t^i = 0, && (17) \\
\textit{or} \quad z_t^{r,i} = 0 \quad \textit{and } z_t^i = z_t^{o,i} \geq \bar{z},\ h_t^i \geq z_t^i, && (18) \\
p_t^i \quad \geq \quad \beta_i^t U_{1,t}^i p_0^i / U_{1,0}^i, && (19) \\
U_{1,t}^i s_t \quad \geq \quad U_{2,t}^i, && (20) \\
U_{1,t}^i \hat{\pi}_t^i \quad \geq \quad U_{2,t}^i, && (21) \\
s_t \geq \hat{\pi}_t^i \quad \geq \quad \hat{s}_t && (22) \\
(\hat{\pi}_t^i - \hat{s}_t)(h_t^i - z_t^{o,i}) \quad = \quad 0, && (23)
\end{aligned}$$

where $U_{j,t}^i \equiv U_j(c_t^i, z_t^i)$ *and* (20) *and* (21) *hold with inequality only if* $z^{r,i} = 0$ *and* $z_t^{o,i} = 0$*, respectively. Furthermore, letting* $p_t^{z,i} = \hat{\pi}_t^i$ *if household-i is a homeowner and* $p_t^{z,i} = s_t$ *if household-i is a renter, the optimal choices of consumption and housing services,* $\{c_t^i, z_t^i\}_{t=0}^{\infty}$*, are subject to the following life-time present value budget constraint*

$$p_0^i \hat{a}_0^i = \sum_{t=0}^{\infty} p_t^i (c_t^i + p_t^{z,i} z_t^i - \hat{w}_t \epsilon^i), \qquad (24)$$

A *competitive equilibrium* is a sequence of quantities and prices,

$$\{c_t^i, z_t^i, b_{t+1}^i, h_{t+1}^i, k_t^j, l_t^j, k_{t+1}, h_{t+1}, q_t, w_t, R_{t+1}; i \in \mathcal{I},\ j = h, m\}_{t=0}^{\infty},$$

and a policy $\mathcal{P} = \{g_t, b_t^g, x_t, \tau_t^w, \tau_t^{h,i}, \tau_t^{k,i}, \tau_t^s; j = s, w, k, h, i \in \mathcal{I}\}_{t=0}^{\infty}$, satisfying the resource feasibility constraints (equations (2)–(6)); profit maximization, (equations (7)–(8)); utility maximization (equations (17)–(23)); the government budget constraint (equation (16)); and the asset markets equilibrium condition

$$\sum_i m_i b_t^i / R_t = k_t + b_t^g / R_t, \tag{25}$$
$$\sum_i m_i a_t^i / R_t = k_t + b_t^g / R_t + q_{t-1} h_t \tag{26}$$

for all $t \geq 0$ and some given initial stocks of capital, housing and public debt (k_0, h_0, b_0^g).

In the remainder of the paper we assume that households' subjective discount rates may take one of two values. In particular, there exist two time discount rates only, β^H, β^L, with $\beta^H > \beta^L$, and a partition $(\mathcal{R}, \mathcal{P})$ of $\mathcal{I}$, such that $\beta_i = \beta^H$ if $i \in \mathcal{R}$ and $\beta_i = \beta^L$ if $i \in \mathcal{P}$. For convenience, we set

$$\beta^H = 1/(1+r)$$

and we refer to $\mathcal{R}$ as the set of (wealth) *rich* households and $\mathcal{P}$ as the set of (wealth) *poor* households. Hence, rich households are relatively patient and, at any equilibrium around a steady state, they are lenders with respect to the rest of the economy; whereas poor households end up with zero net wealth asymptotically. Motivated by these considerations, in what follows we concentrate only on equilibria such that the debt limits are binding only for poor households, so that $\hat{a}_t^i = 0$ for all $i \in \mathcal{P}$ and $\hat{a}^i \geq 0$ for all $i \in \mathcal{R}$. We also assume that rich households have enough wealth to be homeowners, *i.e.*, to overcome the minimum home size $\bar{z}$ at all existing market prices. On the contrary, poor households can either be homeowners and a borrowers, or renters. Under this simple partition, taxes on financial assets fall on the type $i \in \mathcal{R}$ only, *i.e.*, $\tau_t^{k,i} = 0$ for all $i \in \mathcal{P}$. On the other hand, by allowing $\tau^{h,i}$ to be contingent on types, we consider the possibility of a subsidy on the housing wealth backed by mortgages. Since tax rates can only be contingent on whether a household is a lender or a borrower, there is no ambiguity in setting $\tau^{k,i} = \tau^k$, $\hat{R}^i = \hat{R}$, $p_t^i = p_t$ and, with some abuse of notation, we set

$$(\tau^{h,i}, \hat{\pi}^i) = (\tau^{h,r}, \hat{\pi}^r) \quad \text{for all } i \in \mathcal{R},$$
$$(\tau^{h,i}, \hat{\pi}^i) = (\tau^{h,p}, \hat{\pi}^p) \quad \text{for all } i \in \mathcal{P}.$$

Note that the first order condition from the households' problem is

$$p_t^i/p_{t+1}^i = \hat{R}_{t+1} = (1+r)U_{1,t}^i/U_{1,t+1}^i \qquad \text{for all } i \in \mathcal{R} \tag{27}$$

so that, at a steady state equilibrium, equation (27) provides the following characterization of the gross interest rate and marginal products of capital

$$f_k^m(k^m, l^m) = q f_k^h(k^h, l^h, x) = R = (1+r)/(1-\tau^k), \tag{28}$$

which can only be verified for $\tau^k < 1 - (1+r)\beta^L$. Observe also that, at steady state, a positive financial tax raises the user cost of housing faced by poor households. Specifically, we have

$$\hat{\pi}^r = q\left(r + \delta + (1-\delta)\tau^{h,r}\right), \tag{29}$$
$$\hat{\pi}^p = q\left(r + \delta + (1-\delta)\tau^{h,p} + (1+r)\frac{\tau^k}{1-\tau^k}\right), \tag{30}$$

so that

$$\hat{\pi}^p \geq \hat{\pi}^r \quad \Leftrightarrow \quad \tau^k \geq \frac{(1-\delta)(\tau^{h,r} - \tau^{h,p})}{(1+r) + (1-\delta)(\tau^{h,r} - \tau^{h,p})}.$$

Therefore, with a positive financial tax rate, poor households may end up paying a higher user cost of housing than rich households unless the latter face a high enough housing wealth tax (higher than that faced by poor households). In particular, if taxes on housing wealth cannot be made contingent on types, then equation (22) and $\tau^k \geq 0$

imply

$$s \geq \hat{\pi}^p \geq \hat{\pi}^r \geq \hat{s}.$$

Observe that, by the first order conditions and the complementary slackness condition (23), if $h^i > z^i$, *i.e.*, if rich households are landlords, then it must be that $\hat{\pi}^r = \hat{s}$, so that the existence of poor homeowners, *i.e.*, $s \geq \hat{\pi}^p$, implies

$$\frac{q\hat{R} - (1-\delta)\hat{q}^r}{1-\tau^s} \geq qR - (1-\delta)\hat{q}^p$$

at steady state. By rearranging terms and recalling that, at steady state, $\hat{R} = (1+r)$, the above implies that, if there is a uniform tax on housing property irrespective of wealth, *i.e.*, $\tau^h = \tau^{h,r} = \tau^{h,p}$, the coexistence of rich landlords and poor homeowners requires the taxation of rents, *i.e.*, $\tau^s > 0$. If, on the other hand, $\tau^s = 0$ and there is a uniform wealth tax on the rich ($\tau^{h,r} = \tau^k$), then the above is only verified if the poor households' homeownership is subsidized, *i.e.*, $\tau^{h,p} < 0$.

2.3 Effects of Introducing a General Wealth Tax

Following from the above discussion, in this section we evaluate the quantitative effects of introducing a uniform tax on net wealth by numerically solving our model. We consider two scenarios: a benchmark case where (housing and financial) wealth is untaxed and an alternative scenario characterized by a flat 1% tax rate on total net wealth, which is comparable to the rates we observe in existing tax codes. To reduce the dimensionality of the problem (from the point of view of the effects on the distribution of income and wealth), we only consider the case where the wealth-poor face the same cost of housing services, *i.e.*, we assume that the rent tax is such that $\hat{\pi}^w = s$, so that wealth-poor households are effectively identical. This restriction implies that we are limiting the degree of inequality across households below the level that could be achieved otherwise (*i.e.*, for $s \geq \hat{\pi}^w$).

Here and in the following numerical exercises we use a very parsimonious parametrization of the model based on Cobb-Douglas preferences and technologies

$$U(c,z) = c^{1-\theta} z^{\theta}, \tag{31}$$

$$f^m(k^m, l^m) = (k^m)^{\alpha_k^m} (l^m)^{\alpha_l^m}, \tag{32}$$

$$f^h(k^h, l^h, x) = (k^h)^{\alpha_k^h} (l^h)^{\alpha_l^h} x^{\alpha_x^h} \tag{33}$$

where $\sum_{j=k,l} \alpha_j^m = \sum_{j=k,l,x} \alpha_j^h = 1$. We calibrate the model by borrowing some of the parameter values from existing literature and setting the others in order to match some moments of the data. All the parameters used in the quantitative analysis are reported in Table 1. We set the baseline consumption preference parameter $\theta = 0.2$ in order to match the U.S. households expenditure on housing services (approximately 15% of 2015 GDP according to the BEA NIPA Table 2.3.5), and exploit an exogenous variation in this parameter to generate changes in wealth and wealth inequality. Intuitively, a higher value of θ is associated to a higher demand for housing services and, therefore, higher housing prices, wealth, and inequality. The time discount parameter of patient households (*i.e.*, rich households) is set to $\beta_H = 0.95$, implying a steady state real interest rate of 5%. Impatient households (*i.e.*, poor households) have a lower value for the time discount parameter, which we set to $\beta_L = 0.90$. The annual depreciation of the housing stock is set equal to $\delta = 2\%$ as in Iacoviello and Neri (2010). We use O'Mahony and Timmer (2009)'s KLEMS data to have rough estimates of the capital factor shares in construction and manufacturing in the US over the 1970–2010 period and, accordingly, set $\alpha_k^m = 1/3$ and $\alpha_k^h = 1/5$. These numbers are in line with those in Valentinyi and Herrendorf (2008) who set the capital share in manufacturing and construction respectively to 0.4 and 0.2. We set the weight attached to the land input to $\alpha_x^h = 1/10$, which is in line with the value used by Davis and Heathcote (2005). Finally, we set the government expenditure g to 0.10 to match the U.S. Federal expenditure as fraction of GDP; and the share of patient (*i.e.*, rich) households to 10%, and the government land policy parameter ξ to 1. For simplicity, we also set government debt to zero (*i.e.*, $b^g = 0$)[3].

Table 1: Model Parameters

Preferences		
consumption expenditure share (baseline):	θ	0.80
housing expenditure share (baseline):	θ	0.20
discount rate capitalists:	β_1	0.95
discount rate workers:	β_2	0.90
Technology		
Housing depreciation:	δ	0.02
capital share manufacturing:	α_k^m	0.33
capital share construction:	α_k^h	0.20
housing share construction:	α_x^h	0.10
Economy structure		
Government expenditure:	g	0.10
Government debt:	b^g	0.00
Share rich households:	m_r	0.10
Share poor households:	m_p	0.90

Notes: This table reports all the parameters used to simulate the model. The model is simulated for $\theta = 0.2, \ldots, 0.8$. The utility function u is Cobb-Douglas and described in equation (31). The production functions are Cobb-Douglas.

Although they have zero net wealth, poor households are affected by the wealth tax because of the general equilibrium effect on prices. Specifically, poor and rich households face different net user costs of housing services (equations (29) and (30)). Figure 1 plots the steady state values of the main variables for the benchmark model with zero wealth tax (red dashed lines) and for the model with the 1% wealth tax (black solid line) for different values of θ (the expenditure share for housing). Note that, by incresing θ, we can evaluate how results change as wealth increases. Table 2 presents a summary of our results by reporting the change in the relevant variables in the two scenarios for $\theta = [0.2, 0.5, 0.8]$. First, the introduction of the wealth tax increases the

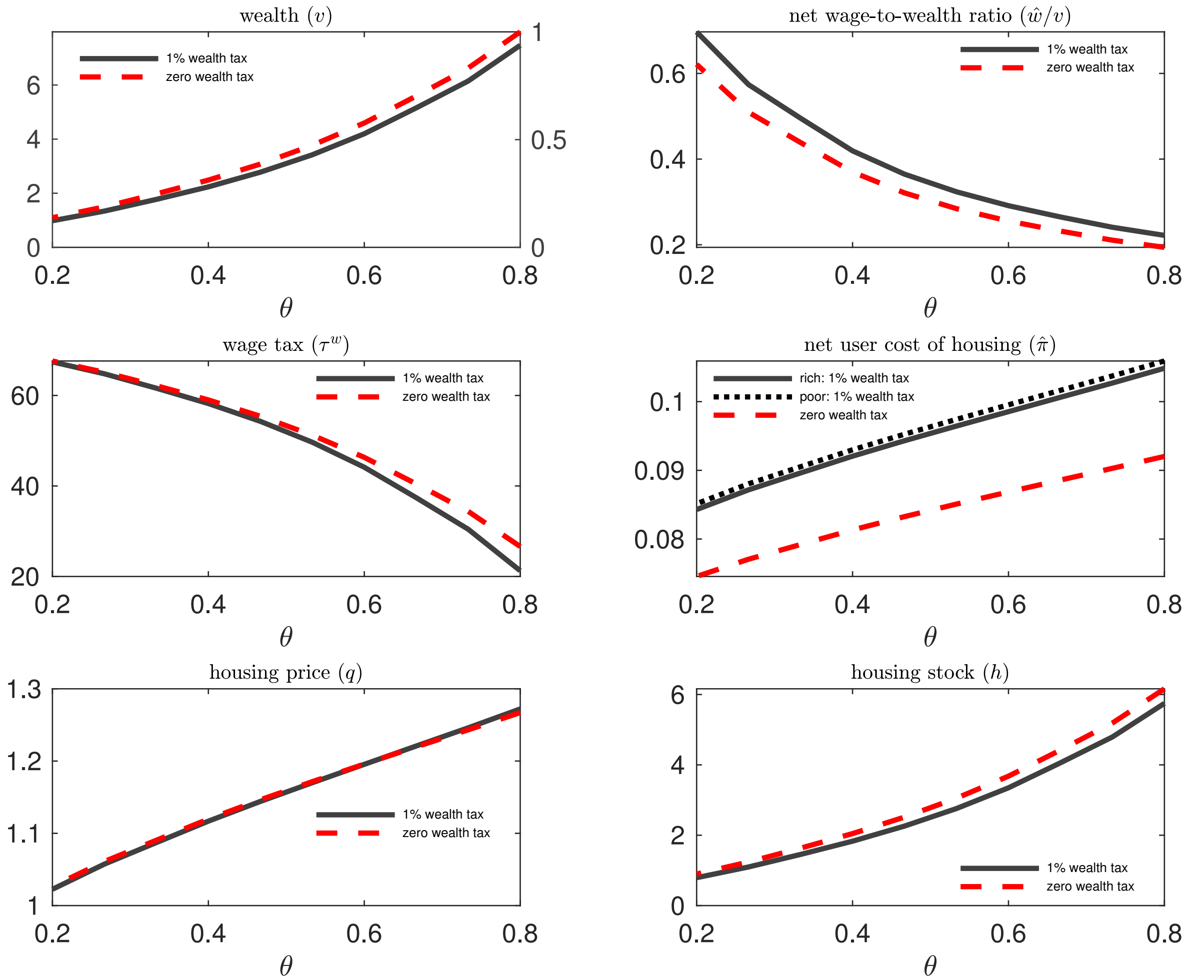
[3] We do not directly calibrate the housing wealth as a fraction of total wealth. In the simulations, when $\theta = 0.2$, this share is approximately equal to 80% and higher then in the data. For example, Iacoviello (2010) reports a value of approximately 50% for the U.S., where a large fraction of housing wealth (80 percent) is made up by the stock of owner-occupied homes).

user costs of housing services for both poor and rich households, and more so for poor households. Specifically, the user cost of housing services increases, respectively, by approximately 14 and 13 per cent for the baseline value of $\theta = 0.2$. Second, the wealth tax reduces the level of wealth by approximately 10% for $\theta = 0.2$. Note that the reduction in wealth depends entirely on the fall in the housing stock (approximately 12%), while housing prices are almost unchanged. Third, the net wage-to-wealth ratio (our measure of inequality) is higher in the model with the wealth tax. In particular, it increases approximately by 7.5% for $\theta = 0.2$, and less so for higher values of θ. The reduction in inequality depends mostly on the decrease in the denominator, as the wealth level drops. On the contrary, the wage tax is mostly unchanged, with a moderate drop of 0.20 percentage points. Note that, although the net wage-to-wealth ratio is higher and the wage tax lower under the scenario with a wealth tax, poor households are worse off. The last row of Table 2 shows that the equivalent income loss for poor households of introducing the wealth tax is equal to approximately 75%. This large loss depends on the large increase in the user cost of housing services after the introduction of the wealth tax. Finally, we find that the wealth elasticity to the wealth tax is equal to approximately 10%, when $\theta = 0.2$. In the next section we consider the optimal tax rates, both from a theoretical and from a quantitative point of view.

3 Optimal Tax Rates

In this section we consider the optimal taxation problem under commitment. We first present the theoretical framework, and then analyze the quantitative effects of a model with optimal tax rates with respect to a model with zero wealth tax.

Figure 1: Steady State with Exogenous Tax Structure



Notes: This figure plots the steady state values for the total wealth (v); the net wage-to-wealth ratio ($\hat{w}/v$); the wage tax (τ^w); the net user costs of housing services ($\hat{\pi}$); the housing price (q); the housing stock (h); for different values of the parameter θ, i.e., the weight of the consumption good in the utility function of households. The wage tax rate is reported in percentage. The black solid line corresponds to a model with a wealth tax of 1% applied to the net financial and housing wealth of rich and poor households; the red dashed line corresponds to a model where the wealth tax is set to zero. For the net user costs of housing services, in the case of a non zero wealth tax, we distinguish between rich (back solid line) and poor households (dotted black line). Parameters are from Table 1, with the exception of θ which is in the range $[0.2, 0.8]$.

3.1 Framework

The Planner maximizes a weighted average of per period utilities across households types at competitive equilibrium allocations by choosing appropriate values of the available tax rates. In order to obtain the steady state allocation as a possible solution to the optimal policy we assume that per period utilities are discounted at the same

Table 2: Effects of Introduction of Flat 1% Wealth Tax

	$\theta = 0.2$	$\theta = 0.5$	$\theta = 0.8$
Δ% wealth (v)	-10.32	-9.62	-6.23
Δ net wage-to-wealth ratio ($\hat{w}/v$)	7.57	3.86	2.80
Δ wage tax (τ^w)	-0.20	-1.43	-5.35
Δ% net user cost of housing poor ($\hat{\pi}^p$)	14.21	14.46	15.10
Δ% net user cost of housing rich ($\hat{\pi}^r$)	13.06	13.32	13.94
Δ% housing price (q)	-0.38	-0.16	0.40
Δ% housing stock (h)	-11.71	-9.91	-6.73
Δ% equivalent income loss	-74.85	-26.76	-7.20

Notes: This table reports the change in wealth; net wage-to-wealth ratio; wage tax; net user cost of housing for poor and rich households; housing price; and housing stock level between the scenario with a flat 1% wealth tax and the scenario with a zero wealth tax. For the net wage-to-wealth ratio and the wage tax we report the difference in percentage points. For all the remaining variables we report percentage changes, expressed in percentage. In addition, the table reports the equivalent income loss of poor households, in percentage, determined by the introduction of the flat 1% wealth tax. The equivalent income loss is equal to $1 - (\hat{\pi}_0^p/\hat{\pi}_1^p)^\theta$, where we denote with "0" the scenario with zero wealth tax and with "1" the scenario with the flat 1% wealth tax. Parameters are from Table 1, with the exception of θ which is in the range $[0.2, 0.8]$.

rate, $(1+r)^{-1}$, *i.e.*, the discount rate of the most patient households. Note that this type of social welfare function implies that the impatient households will be saving more than they would if the Planner was discounting utilities at the (heterogeneous) subjective discount rates. However, if the equilibrium generated by the Planner's policies implies binding debt limits for the impatient households at all $t \geq 0$, replacing their subjective discount rate with the higher value, $(1+r)^{-1}$, has no consequences on these households' net wealth, which is going to be zero in both cases. Social welfare functions with welfare weights reflecting the Planner's (or society's) preferences have been widely used in the literature. For instance, Saez and Stantcheva (2016) propose to evaluate tax reforms using "generalized social marginal welfare weights" to capture society's concerns for fairness without being necessarily tied to individual utilities. We simplify the Planning problem by selecting the poor and the rich households' welfare weights in $\{1, \eta\}$, where $\eta \geq 0$ is the one attached to the rich households' utility, so that the social welfare function is

$$\mathcal{U} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(\eta \sum_{i\in\mathcal{R}} m_i U(c_t^i, z_t^i) + \sum_{i\in\mathcal{P}} U(c_t^i, z_t^i)\right). \tag{34}$$

For simplicity, we restrict our attention to equilibria such that the minimum home size restriction (10) is non binding for all i and such that the renters are sufficiently poor (ϵ^i low enough) and $\bar{z}$ large enough to be better off being renters than homeowners. Finally, we concentrate on equilibria where the households' partition is as specified in section 2, *i.e.*, the patient households' debt limits are non binding at all times and $a_t^i = 0$ for all $t \geq 0$ and $i \in \mathcal{P}$, a condition that is certainly verified if the economy is sufficiently close to the steady state.

To set up the planner's problem we start by exploiting the market clearing conditions in the good, housing and asset markets, as well as profit maximization, to restate the t-period government budget constraint (16) as follows

$$\sum_i m_i \left(\frac{\hat{a}_{t+1}^i}{\hat{R}_{t+1}^i} + c_t^i + \hat{\pi}_t^i h_t^i + (s_t - \hat{s}_t)(h_t^i - z_t^i) - \hat{w}_t - \hat{a}_t^i\right) \geq 0. \tag{35}$$

Equation (35) can be simplified under the assumed household's partition. Specifically, recall that $\hat{a}_t^i = 0$ for all $i \in \mathcal{P}$ and $t \geq 0$. Furthermore, letting $\mathcal{S}$ be the set of poor-renters, note that

$$\sum_{i\in\mathcal{P}\setminus\mathcal{S}} (c_t^i + \hat{\pi}^p h^i - \epsilon^i \hat{w}_t) = \sum_{i\in\mathcal{S}} (c_t^i + s_t z_t^i - \epsilon^i \hat{w}_t) = 0, \tag{36}$$

and

$$\sum_{i\in\mathcal{R}} m_i (h_t^i - z_t^i) = \sum_{i\in\mathcal{S}} m_i z_t^i.$$

Then, using the above into equation (35) and exploiting the no arbitrage condition

(23), the latter is equivalent to

$$\sum_{i\in\mathcal{R}} m_i \left(\frac{\hat{a}^i_{t+1}}{\hat{R}^i_{t+1}} + c^i_t + \hat{\pi}^r z^i_t - \epsilon^i \hat{w}_t - \hat{a}^i_t \right) \geq 0. \tag{37}$$

The available menu of (proportional) tax rates on housing, rents and financial wealth is unrestricted. To find the optimal mix of tax rates, we follow the primal approach (Lucas and Stokey, 1983; Atkeson et al., 1999; Chari and Kehoe, 1999). Since the rich are never financially constrained,

$$p_t = \frac{U^i_{1,t} p_0}{(1+r)^t U^i_{1,0}} \qquad \text{for all } i \in \mathcal{R}.$$

Then, using the first order conditions from utility maximization, (20)-(23); the complementary slackness conditions; and the assumption that the minimum home size restriction is non binding; we can rewrite equations (36), (37) as

$$\sum_{i\in\mathcal{R}} m_i \left(\frac{U^i_{1,t+1}\hat{a}^i_{t+1}}{1+r} + H^i(c^i_t, z^i_t, \hat{w}_t) - U^i_{1,t}\hat{a}^i_t \right) \;\geq\; 0, \tag{38}$$

$$H^i(c^i_t, z^i_t, \hat{w}_t) \;=\; 0 \qquad i \in \mathcal{P} \tag{39}$$

where

$$H^i(c^i, z^i, \hat{w}) \equiv U_1(c^i, z^i)c^i + U_2(c^i, z^i)z^i - U_1(c^i, z^i)\epsilon^i \hat{w}_t.$$

Equations (38), (39) are the *implementability conditions* and define the households' budget constraints in terms of first order conditions, instead of prices. Finally, by (24), equation (38) can be iterated forward from period zero to provide the following present value representation of the government budget constraint

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \sum_{i\in\mathcal{R}} m_i H^i(c^i_t, z^i_t, \hat{w}_t) \geq \sum_{i\in\mathcal{R}} m_i U_1(c^i_0, z^i_0)\hat{a}^i_0, \qquad \text{for all } i \in \mathcal{R}. \tag{40}$$

Any sequence $\{c^i_t, z^i_t, \hat{w}_t; i = l, d, r\}_{t=0}^{\infty}$ satisfying conditions (39), (40) together with the resource feasibility constraints (equations (2)–(6)), for all $t \geq 0$, and for some initial aggregate wealth,

$$\sum_{i \in \mathcal{R}} m_i \hat{a}_0^i = (\hat{R}_0 k_0 + b_0^g + q_{-1} h_0),$$

is a competitive equilibrium implemented by some set of implicit individual specific tax rates.

Now define the *pseudo welfare function*

$$\tilde{U}_t = \eta \sum_{i \in \mathcal{R}} m_i U(c_t^i, z_t^i) + \sum_{i \in \mathcal{P}} U(c_t^i, z_t^i) + \mu \sum_{i \in \mathcal{R}} m_i H^i(c^i, z^i, \hat{w}), \tag{41}$$

where the multiplier μ is positive if the Planner needs distortionary taxation to finance public spending. This multiplier represents a "bonus to date-t allocations that brings in extra government revenues, thereby relieving other periods from distortionary taxation, and the same term imposes a penalty in the opposite situation" (Erosa and Gervais, 2001). Then, for a given policy, $\{g_t, x_t\}_{t=0}^{\infty}$, the Planner's decision variables are defined by the sequence

$$\mathbf{d} = \{c_t^i, z_t^i, k_t^j, h_{t+1}, l_t^j, k_t; i = \mathcal{I},\ j = h, m\}_{t=0}^{\infty},$$

and we define the optimal taxation problem as follows

$$\max_{(\mathbf{d}, \mu) \geq 0} \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t \tilde{U}_t - \mu W_0 \qquad \text{s.t. equations (2)–(6) at all } t \geq 1, \tag{PP}$$

where

$$W_0 = \sum_{i \in \mathcal{R}} m_i U_1(c_0^i, z_0^i) \hat{a}_0^i, \qquad \sum_{i \in \mathcal{R}} m_i a_0^i = R_0 k_0 + b_0^g + q_{-1} h_0.$$

Note that, since we only consider the case of full commitment, the planner is unable to revise the given initial tax rates, so that W_0 is a predetermined initial condition in the planning problem.

3.2 Wealth-Rich as Pure Speculators

To gain intuition about the optimal tax structure (which we will derive in the next section), it is useful to start with a restricted version of the present model which is more easily comparable to the literature on optimal taxation with heterogeneous households (rich and poor). The most celebrated contribution on this matter is due to Judd (1985), who considers an economy with a single good produced by a constant-returns-to-scale production function with capital and labor as inputs, populated by a capitalist (with capital as the only source of income) and a worker. The Planner must select two distortionary tax rates on labor and capital to finance a stream of lamp-sum expenditure. Within this setting, Judd (1985) shows that the Planner would not use tax rates for redistribution (at least asymptotically), *i.e.*, the capital tax rate is zero at steady state, even if the social welfare function is totally biased toward the worker ($\eta = 0$, with our notation). A standard interpretation is that the inefficiency of capital taxation grows extremely large over time due to the infinite elasticity of the supply of capital (Saez and Stantcheva, 2017).

In this section we assume two type of individuals only, rich (with positive net wealth) and poor (with zero net wealth), the former acting as "pure speculators" in the housing market, in the sense that they derive no utility from housing services, and we study the optimal tax structure as their labor income goes to zero. It turns out that Judd's zero capital tax result survives in our model. However, it is optimal to introduce a subsidy on housing services specifically targeted to poor households as long as the rich derive some income from work. Quite clearly, since we allow for homeownership among the poor, this subsidy may be interpreted as a negative capital income tax as well as a negative tax on imputed rents. In the next section, when we consider the unrestricted version of the model, we will show that a positive housing wealth tax on the rich households is optimal under some robust assumptions on rich households' utility function.

Assume that the rich households' preferences are described by a utility function $U^r(c)$, increasing and strictly concave. Rich and poor households' labor productivities are equal to ϵ^r and ϵ^p, respectively, and poor households are indifferent between being homeowners or renters, so that $\pi_t^p = s_t$ for all $t \geq 0$. This is compatible with individual optimality when capital and housing taxes are all zero or, alternatively, when poor households' home ownership is subsidized. Since rich and poor households face the same set of prices and net wages, we can restrict the index set $\mathcal{I}$ to $\{r, p\}$ and use the index "r" to denote a mass m_r of rich households and the index "p" to denote a mass m_p of poor households. We assume that government debt and business capital are zero, so that the consumption good is produced only with labor and the construction good is produced with labor and a fixed flow of land per period. Since $k = 0$ and the manufacturing sector exhibits constant returns to scale, the production functions in manufacturing and construction are

$$f^m(l^m) = wl^m, \qquad f^h(l^h, x) \equiv f(l^h),$$

where $w > 0$ is a productivity parameter and $f_l^h(L^h) > 0$, $f_{ll}^h(L^h) < 0$. These assumptions imply the following profit maximization condition

$$q_t = w/f_l^h(l_t^h), \tag{42}$$

and the first order conditions from utility maximization

$$\begin{aligned} U_1^r(c_t^r)/U_1^r(c_{t+1}^r) &= \hat{R}_{t+1}/(1+r), & (43)\\ U_2^p(c_t^p, h_t)/U_1^p(c_t^p, h_t) &= \hat{\pi}_t^p = s_t. & (44)\\ q_t &= ((1-\delta)\hat{q}_{t+1}^r + \hat{s}_{t+1})/\hat{R}_{t+1}. & (45) \end{aligned}$$

By exploiting the above restrictions; the assumption $\hat{\pi}^p = s$; and recalling that rich households derive no utility from housing services (so that the poor households' consumption of housing services is equal to the stock of housing) we can derive the t-period government budget constraint as

$$q_t h_{t+1} + m_p(c_t^p + s_t h_t - \epsilon^r \hat{w}_t) + m_r(c_t^r - \epsilon^r \hat{w}_t) \geq \hat{R}_t q_{t-1} h_t. \tag{46}$$

The above is the equivalent of equation (37) under this restricted version of our model. It is important to note that, according to the above constraint, any extra unit of net wage reduces the government's revenue by one unit. Therefore, given the initial outstanding net capital income, $\hat{R}_t q_{t-1} h_t$, any extra unit of net wage must be compensated by some extra value of next period wealth or households' consumption. However, note also that, since poor households are financially constrained, *i.e.*,

$$c_t^p + s_t h_t - \epsilon^r \hat{w}_t = 0,$$

we can rewrite (46) as

$$q_t h_{t+1} + m_r(c_t^r - \epsilon^r \hat{w}_t) \geq \hat{R}_t q_{t-1} h_t. \tag{47}$$

Then, an extra unit of the net wage, $\hat{w}_t$, has two separate effects. First, it reduces poor households' total expenditure by $m_p \epsilon^p$, because poor households are financially constrained. Second, it generates a tax revenue shortfall equal to $m_r \epsilon^r$, which can only be compensated by a change in rich households' wealth or consumption. These additional resources have a cost in terms of incremental distortions. To derive the optimal tax rates, we can replace $\hat{R}_t$ using the first order condition (43), and then substitute q_t and q_{t-1} with the no arbitrage condition (45) in equation (47). Then, we can rewrite (47) as

$$\frac{U_1^r(c_{t+1}^r)}{1+r}((1-\delta)\hat{q}_{t+1}^r+\hat{s}_{t+1})+U_1^r(c_t^r)m_r(c_t^r-\epsilon^r\hat{w}_t)\geq U_1^r(c_t^r)((1-\delta)\hat{q}_t^r+\hat{s}_t)h_t. \quad (48)$$

By iterating forward (48) we obtain the *long-run implementability constraint* (equivalent to (40) in the unrestricted model):

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t U_1^r(c_t^r)m_r(c_t^r-\epsilon^r\hat{w}_t)\geq W_0, \quad (49)$$

where $W_0 \equiv U_1^r(c_0^r)((1-\delta)\hat{q}_0^r+\hat{s}_0)h_0$. Since we are looking at the optimal tax problem under commitment, the term W_0 is given exogenously.

In the remaining part of this section, we assume that rich households' utility function, $U^r(c^r)$, exhibits a constant relative degree of risk aversion, $\sigma>0$, and that poor households' utility function, $U^p(c^p,z^p)$ is Cobb-Douglas as in (31). This specification of preferences has the advantage of simplifying the characterization of the optimal tax structure and provides additional intuition. In particular, by the poor households' preference representation, we obtain the constant expenditure shares

$$c_t^p=(1-\theta)\hat{w}_t, \qquad s_tz_t^p=s_th_t/m_p=\theta\hat{w}_t.$$

Then the planning problem (PP) boils down to the maximization of the function

$$\mathcal{L}=\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t[\eta m_rU^r(c_t^r)+m_pU^p\left((1-\theta)\hat{w}_t,h_t/m_r\right)+\mu U_1^r(c_t^r)m_r(c_t^r-\epsilon^r\hat{w}_t)$$

$$+\lambda_t^m(wl_t^m-m_rc_t^r-m_p(1-\theta)\hat{w}_t-g_t)+\lambda_t^h(h_{t+1}-f(l_t^h)-(1-\delta)h_t)+\xi_t(1-l_t^h-l_t^m)],$$

with respect to $\{c_t^r,\hat{w}_t,h_{t+1},l_t^h,\lambda_t^m,\lambda_t^h,\xi_t\}_{t=1}^{\infty}$, where $(\lambda_t^m,\lambda_t^h,\xi_t)$ are the (discounted) Lagrange multipliers associated to the resource feasibility constraints in manufacturing, housing and labor market (*i.e.*, the shadow prices of consumption, housing and labor, respectively).

Now consider an interior solution and observe that optimality requires that the net marginal benefit of increasing the rich households' consumption, c_t^r, and the net wage, $\hat{w}_t$, at time t, must be equal to the shadow price of consumption in manufacturing. In particular, these conditions can be stated as follows

$$\eta U_1^r(c_t^r) = \lambda_t^m + \mathcal{D}_t^r \tag{50}$$

$$U_1^p(c_t^p, z_t^p) = \lambda_t^m + \mathcal{D}_t^p, \tag{51}$$

where

$$\mathcal{D}_t^r \equiv \mu U_1^r(c_t^r) \left(\sigma \left(1 - \epsilon^r \hat{w}_t / c_t^r\right) - 1\right), \qquad \mathcal{D}_t^p = \mu U_1^r(c_t^r) m_r \epsilon^r \hat{w}_t / m_p c_t^p$$

are the net cost of the fiscal distortions generated by a rise in the rich and poor households' consumption, respectively. Hence, the left hand sides of equations (50) and (51) represent the direct benefit on social welfare of increasing the consumption of rich and poor households, and the right hand side is the sum of two costs: the shadow price of consumption, λ_t^m, and the net cost of the fiscal distortions, $\mathcal{D}_t^r$, $\mathcal{D}_t^p$. Note that these are proportional to the multiplier μ, which represents the gain from relaxing the government budget constraint; and they have different size and, possibly, sign. In particular, $\mathcal{D}_t^r$ can be positive or negative depending on the elasticity of the marginal utility, $1/\sigma$, and the rich household's wage-to-consumption ratio. If $\sigma > 1$, *i.e.*, marginal utility is relatively inelastic, the fiscal distortions related to a rise in the rich households' consumption is positive.

Crucially, we note that the fiscal distortion caused by a rising poor households' consumption (or wage), $\mathcal{D}^p$, is always positive for $\epsilon^r > 0$, whereas $\mathcal{D}_t^r = \mathcal{D}_t^p = 0$ if $\epsilon^r = 0$, *i.e.*, if rich households are not working. The intuition is as follows. If rich households are working, a drop in the labor tax, or, equivalently, a rising net wage, is only partly compensated by a rising consumption by poor households. But this is not enough to compensate for the total lost revenue because part of it comes from the labor tax on the rich. Then, some other revenue compensation must be generated by other sources of the rich households' income, implying some additional distortions. Then, the social cost of raising the poor households' consumption is larger than the shadow price of consumption.

Because $\mathcal{D}_t^p > 0$, then the poor households' marginal rate of substitution between housing and consumption at the planning optimum is lower than the shadow relative price of housing services, *i.e.*, housing must be subsidized. In particular, turning to the first order conditions related to housing and labor, we obtain

$$(1+r)\lambda_{t-1}^h = U_2(c_t^p, z_t^p) + (1-\delta)\lambda_t^h, \tag{52}$$

$$\lambda_t^h/\lambda_t^m = w/f'(l_t^h), \tag{53}$$

implying that $\lambda_t^h/\lambda_t^m = q_t$. Now consider a steady state of the optimal allocation. In this case, equations (51) and (52) imply

$$\frac{U_2^p}{U_1^p} = \left(1 - \frac{\mathcal{D}^p}{U_1^p}\right) q(r+\delta). \tag{54}$$

Recalling the definition of $\hat{\pi}^p$ given in (30), the above optimal condition can be implemented in steady state through the assumed menu of tax rates by setting

$$\tau^k = 0, \qquad \tau^{h,p} = -\left(\frac{r+\delta}{1-\delta}\right)\frac{\mathcal{D}^p}{U_1^p} \leq 0.$$

3.3 The Unrestricted Model

We now consider the unrestricted model of section 2. To derive the Planner's problem, we use equation (39) to express the poor households' consumption as a function of housing demand. In particular, note that, for $j = 1, 2$,

$$H_j^i(c^i, z^i, \hat{w}) = U_j(c^i, z^i, \hat{w})\left(1 + g_j(c^i, z^i, \hat{w})\right), \qquad H_3(c^i, z^i, \hat{w}) = -U_1(c^i, z^i)\epsilon^i,$$

where

$$1 + g_j^i = 1 + \frac{U_{1,j}^i c + U_{2,j}^i z}{U_j} - \frac{U_{1,j}^i}{U_j^i}\epsilon^i \hat{w} \tag{55}$$

are the *general equilibrium elasticities* related, respectively, to the tax rates on capital and housing (Chari and Kehoe, 1999). These elasticities capture the extent to which a fall in the corresponding tax rates is reducing distortions. We discuss two properties of these elasticities in the following proposition. Appendix B contains the proof.

Proposition 2. *If $U(c^i, z^i)$ is concave, the following properties hold for all $i \in \mathcal{P}$:*

$$g_1^i = z^i \frac{\partial}{\partial c^i}\left(\frac{U_2^i}{U_1^i}\right) \geq 0, \tag{56}$$

$$g_1^i \geq g_2^i \tag{57}$$

with strict inequalities if $U(c^i, z^i)$ is strictly concave.

By (56) we can use equation (39) to express the poor households' consumption as a function of housing demand and the net wage, *i.e.*,

$$c^i = \psi^i(z_t^i, \hat{w}_t).$$

Under the maintained assumptions, the above is a continuously differentiable function such that

$$\psi_1^i = -\frac{H_2^i}{H_1^i} = -\frac{U_2^i(1+g_2^i)}{U_1^i(1+g_1^i)}, \qquad \psi_2^i = -\frac{H_3^i}{H_1^i} = \frac{\epsilon^i}{(1+g_1^i)}. \tag{58}$$

It follows that the pseudo welfare function $\tilde{U}_t$ in (41) is

$$\tilde{U}_t = \eta \sum_{i\in\mathcal{R}} m_i U(c_t^i, z_t^i) + \sum_{i\in\mathcal{P}} U(\psi^i(z_t^i, \hat{w}_t), z_t^i) + \mu \sum_{i\in\mathcal{R}} m_i H^i(c^i, z^i, \hat{w}).$$

To characterize the planning optimum, let $\{\lambda_t^m, \lambda_t^h\}_{t=0}^{\infty}$ be the non-negative discounted Lagrange multipliers associated to the resource constraint in the manufacturing sector, (2), in the construction sector, (3), and in the labor market, (5), respectively; f_t^j the time-t output per unit of labor efficiency; and $f_{s,t}^j$, for $j = h, m$ and $s = k, l, x$, the time-t marginal products of capital, labor and land in the two sectors. Then, we can split the first order characterization of the optimal taxation problem into two sets of conditions. The first concerns the optimal allocation of capital, labor and land across sectors, consumption of manufacturing, and housing:

$$\lambda_t^h/\lambda_t^m = f_{l,t}^m = (\lambda_t^h/\lambda_t^m) f_{l,t}^h, \tag{59}$$

$$(1+r)\lambda_t^m/\lambda_{t+1}^m = f_{k,t+1}^m = (\lambda_t^h/\lambda_t^m) f_{k,t+1}^h. \tag{60}$$

Note that, by the profit maximization conditions, (7), (8), the above imply

$$\lambda_t^h/\lambda_t^m = q_t, \qquad (1+r)\lambda_{t-1}^m/\lambda_t^m = R_t.$$

The second set of conditions concerns the optimal allocation of consumption, housing and labor across households. Namely, letting

$$\pi_t = q_{t-1}R_t - (1-\delta)q_t,$$

the optimal allocation of consumption and housing services across rich households, *i.e.*, for all $i \in \mathcal{R}$, is defined as

$$\lambda_t^m = U_{1,t}^i(\eta + \mu(1 + g_{1,t}^i)), \tag{61}$$

$$\lambda_t^m \pi_t = U_{2,t}^i(\eta + \mu(1 + g_{2,t}^i)), \tag{62}$$

which provide an interpretation of λ^m and $\lambda^m \pi_t$ as the shadow prices of consumption and housing services, respectively. Note that, by (60), equation (61) implies that, at steady state,

$$f_k^m(k^m) = R \equiv (1+r), \tag{63}$$

which establishes the Chamley-Judd zero capital tax rate result at steady state. Now note that, by (61) and (62), we get the marginal rate of substitution between housing services and consumption

$$\frac{U_{2,t}^i}{U_{1,t}^i} \equiv \hat{\pi}_t^i = \pi_t \left(\frac{\eta + \mu(1+g_{1,t}^i)}{\eta + \mu(1+g_{2,t}^i)} \right) \qquad \text{for all } i \in \mathcal{R}. \tag{64}$$

Hence, the higher is the general equilibrium elasticity g_1^i (g_2^i), the higher (lower) should be the tax rate on housing, *i.e.*, the size of g_1^i relative to g_1^i plays out in favour of a housing tax. For this reason $g_1^i - g_2^i$ is a measure of the social benefit from housing taxation. Turning now to the first order conditions for the optimal allocation of poor households' housing services, using (58) we get

$$\lambda_t^m \pi_t = U_{2,t}^i \left(1 - \left(\frac{1+g_{2,t}^i}{1+g_{1,t}^i} \right) \left(1 - \frac{\lambda_t^m}{U_{1,t}^i} \right) \right) \qquad \text{for all } i \in \mathcal{P}, \tag{65}$$

where the left hand side is the shadow price of housing services and the right hand side is the utility gain from an extra unit of housing services net of the (possible) reduction in consumption that follows from the budget constraint and the feasibility constraint. Finally, the first order condition related to the optimal allocation of the net wages, $\hat{w}_t$, can be stated as follows

$$\underbrace{\left(\sum_{i \in \mathcal{P}} m_i \frac{U_{1,t}^i \epsilon^i}{1+g_{1,t}^i} \right)}_{\text{extra consumption}} = \mu \underbrace{\left(\sum_{i \in \mathcal{R}} m_i U_{1,t}^i \epsilon^i \right)}_{\text{extra distortions}} + \lambda_t^m \underbrace{\left(\sum_{i \in \mathcal{P}} m_i \frac{\epsilon^i}{1+g_{1,t}^i} \right)}_{\text{reduced resources}}. \tag{66}$$

Note that (66) equates the gain from any extra unit of net wage due to poor households' extra consumption to the sum of two different costs: the cost of the additional distortions following from the fall in the tax revenue plus the cost of the fall in the available resources. The last two costs are weighted, respectively, by the Lagrange multiplier μ (representing the gain from a fall in distortionary taxation) and the shadow price of consumption, λ_t^m. Now observe that, by (64),

$$\hat{\pi}_t^i > \pi_t \qquad \Leftrightarrow \qquad g_{1,t}^i > g_{2,t}^i \qquad \forall i \in \mathcal{R}, \tag{67}$$

i.e., the cost of housing services for rich households must be taxed if the gain in efficiency from a fall in the (implicit) tax on consumption exceeds the gain from a fall in the tax on housing. Note that, if $U(c, z)$ is homogeneous of degree $\zeta \geq 0$, we have

$$(U_{1,1}^i c^i + U_{1,2}^i z^i)/U_1^i = (U_{1,2}^i c^i + U_{2,2}^i z^i)/U_2^i = \zeta - 1,$$

so that, for all $i \in \mathcal{I}$,

$$1 + g_1^i = \zeta - \frac{U_{1,1}^i c^i}{U_1^i}\left(\frac{\epsilon^i \hat{w}}{c^i}\right), \qquad 1 + g_2^i = \zeta - \frac{U_{1,2}^i c^i}{U_2^i}\left(\frac{\epsilon^i \hat{w}}{c^i}\right),$$

and, by strict concavity, $g_1^i > g_2^i$ for all $i \in \mathcal{I}$. In this case, by (67), it follows that $U_{2,t}^i/U_{1,t}^i > \pi_t$ for all $i \in \mathcal{R}$. In particular, since $U_{1,1}^i < 0 < U_{1,2}^i$, the difference $(g_1^i - g_2^i)$ is increasing in the ratio between wage income and consumption, $\epsilon^i \hat{w}/c^i$. The latter is constant and equal to $1/(1-\theta)$ for the poor, and it depends on net assets for rich households. Note that, at steady state, $c^i > (1-\theta)\epsilon^i \hat{w}$ and, then, $\epsilon^i \hat{w}/c^i$ is decreasing in the size of wealth for all $i \in \mathcal{R}$. Hence, the difference $(g_1^i - g_2^i)$, *i.e.*, the scope for housing taxation, is decreasing in the size of wealth for all $i \in \mathcal{R}$.

A second important observation is that, by (65), and since $g_{1,t}^i \geq g_{2,t}^i$ for all $i \in \mathcal{P}$,

$$\frac{U_{2,t}^i}{U_{1,t}^i} \equiv \hat{\pi}_t^i = \pi_t \left(\frac{(1+g_{1,t}^i)\lambda_t^m}{U_{1,t}^i(g_{1,t}^i - g_{2,t}^i) + (1+g_{2,t}^i)\lambda_t^m}\right) \qquad \forall i \in \mathcal{P}. \tag{68}$$

implying that, for all $i \in \mathcal{P}$,

$$U^i_{2,t}/U^i_{1,t} < \pi_t \qquad \Leftrightarrow \qquad \lambda^m_t < U^i_{1,t} \qquad \forall i \in \mathcal{P}. \tag{69}$$

In other words, the cost of housing services for the worker must be subsidized if her marginal utility of consumption exceeds the shadow price of consumption. To understand the circumstances under which this condition holds, define the "weights"

$$\xi^i_t = \frac{m_i \epsilon^i/(1+g^i_{1,t})}{\sum_{j\in\mathcal{P}} m_j \epsilon^i/(1+g^j_{1,t})},$$

and notice that, by rearranging the terms in equation (66), we obtain

$$\lambda^m_t = \sum_{i\in\mathcal{P}} \xi^i_t U^i_{1,t} - \mu \left(\frac{\sum_{i\in\mathcal{R}} m_i \epsilon^i U^i_{1,t}}{\sum_{j\in\mathcal{P}} m_j \epsilon^i/(1+g^j_{1,t})} \right). \tag{70}$$

Since the weights, ξ^i_t, are positive and they sum up to one and $\mu > 0$, λ^m_t is strictly smaller than a convex linear combination of the poor households' marginal utilities of consumption. Namely, the shadow price of consumption falls short of an average of the poor households' marginal utility of consumption because of the extra-distortions implied by shifting taxation from labor to housing. This implies that

$$\lambda^m_t < \max_{i\in\mathcal{P}} U^i_{1,t}. \tag{71}$$

Then, by (69), the user cost of housing faced by poor households whose marginal utility of consumption is relatively large must be subsidized. We summarize these findings in the following proposition.

Proposition 3. *Assume that $U(c,z)$ is a homogeneous function and let $\mathcal{P}^* \subset \mathcal{P}$ such that*

$$U_1(c^i_t, z^i_t) \geq U_1(c^j_t, z^j_t) \qquad \textit{for all } i \in \mathcal{P}^* \textit{ and } j \in \mathcal{P}.$$

Then, the optimal tax structure is such that

$$\hat{\pi}_t^i < \pi_t < \hat{\pi}_t^j \qquad \textit{for all } i \in \mathcal{P}^* \textit{ and } j \in \mathcal{R}.$$

By (29) and (30), the implicit tax rates derived in this section can be implemented through the tax instruments considered in section 2. Namely, letting the steady state implicit optimal tax rate be

$$t^{h,i} = \frac{\hat{\pi}^i}{\pi} - 1,$$

we derive

$$\tau^{h,i} = \left(\frac{r+\delta}{1-\delta}\right) t^{h,i}.$$

3.4 Numerical Simulation

To provide a better analytical representation of the optimal tax structure, assume that rich households are all identical (in terms of labor productivities and initial asset holdings) and utility is Cobb-Douglas (31). Assume, also, that $\eta = 0$ and use the index $i = r$ to identify rich households' decisions. In this case, for all $i \in \mathcal{I}$,

$$(1 + g_1^i) = 1 + \theta \epsilon^i \hat{w}/c^i, \qquad (1 + g_2^i) = 1 - (1-\theta)\epsilon^i \hat{w}/c.$$

Note that, for all $t \geq 0$ and $i \in \mathcal{P}$,

$$c_t^i = (1-\theta)\epsilon^i \hat{w}_t. \tag{72}$$

Then,

$$(1 + g_{1,t}^i) = 1/(1-\theta), \qquad (1 + g_{2,t}^i) = 0 \qquad \text{for all } t \geq 0 \text{ and } i \in \mathcal{P}.$$

On the other hand, by exploiting the asset market equilibrium condition, the rich households' general equilibrium elasticities are

$$1+g^r_{1,t}=\frac{(1-\theta)r+(1-\bar{\epsilon}^p)\hat{w}/v}{(1-\theta)(r+(1-\bar{\epsilon}^p)\hat{w}/v)},\qquad 1+g^r_{2,t}=\frac{(1-\theta)r}{r+(1-\bar{\epsilon}^p)\hat{w}/v}.$$

Hence, the rich households' general equilibrium elasticities depend on the net wage-to-wealth ratio, $\hat{w}/v$, and the scope for taxing housing, as measured by the gap $g^r_1-g^r_2$, is increasing in this ratio. Note, also, that, by (65),

$$U^i_{2,t}=\lambda^m_t\pi_t$$

for all $i\in\mathcal{P}$, and, by the linear homogeneity of U, this implies that all marginal utilities are equalized across the set of poor households. Hence, with some abuse of notation, we set

$$U^i_{j,t}=U^p_{j,t}\qquad\text{for all } i\in\mathcal{P}.$$

This implies that (70) becomes

$$\lambda^m_t=U^p_{1,t}-\frac{\mu}{(1-\theta)}\left(\frac{1-\bar{\epsilon}^p}{\bar{\epsilon}^p}\right)U^r_{1,t},\tag{73}$$

where $\bar{\epsilon}^p=\sum_{i\in\mathcal{P}}m_i\epsilon^i$. Now we derive the value of μ by equating (61) to (73) to obtain

$$\mu=\frac{U^p_{1,t}}{U^r_{1,t}}\left(\frac{(1-\theta)\bar{\epsilon}^p}{(1-\theta)\bar{\epsilon}^p(1+\theta\epsilon^r\hat{w}_t/c^r_t)+(1-\bar{\epsilon}^p)}\right).\tag{74}$$

The above is a measure of the cost of the additional distortions following from the fall in the tax revenue due to a higher net wage, $\hat{w}$. Finally, by the above value of μ and the assumption $\eta=0$, the optimal user costs of housing are

$$\hat{\pi}^i_t=\pi_t(1+t^i_t),$$

where

$$t_t^r = \frac{\epsilon^r \hat{w}_t}{c_t^r - (1-\theta)\epsilon_t^r \hat{w}_t}, \qquad t_t^p = -\frac{(1-\bar{\epsilon}^p)}{(1-\bar{\epsilon}^p) + (1-\theta)\bar{\epsilon}^p(1+\theta\epsilon^r \hat{w}/c^r)}. \tag{75}$$

Now consider a steady state, and denote aggregate net wealth as

$$v = k + qh + b^g/(1+r)$$

Then, recalling the asset market equilibrium condition (26), the steady state consumptions of the rich and poor households are

$$c^r = (1-\theta)(\epsilon^r \hat{w} + rv/m_r). \tag{76}$$

The above implies that

$$\frac{\epsilon^r \hat{w}}{c^r} = \frac{(1-\bar{\epsilon}^p)\hat{w}/rv}{(1-\theta)((1-\bar{\epsilon}^p)\hat{w}/rv + 1)}$$

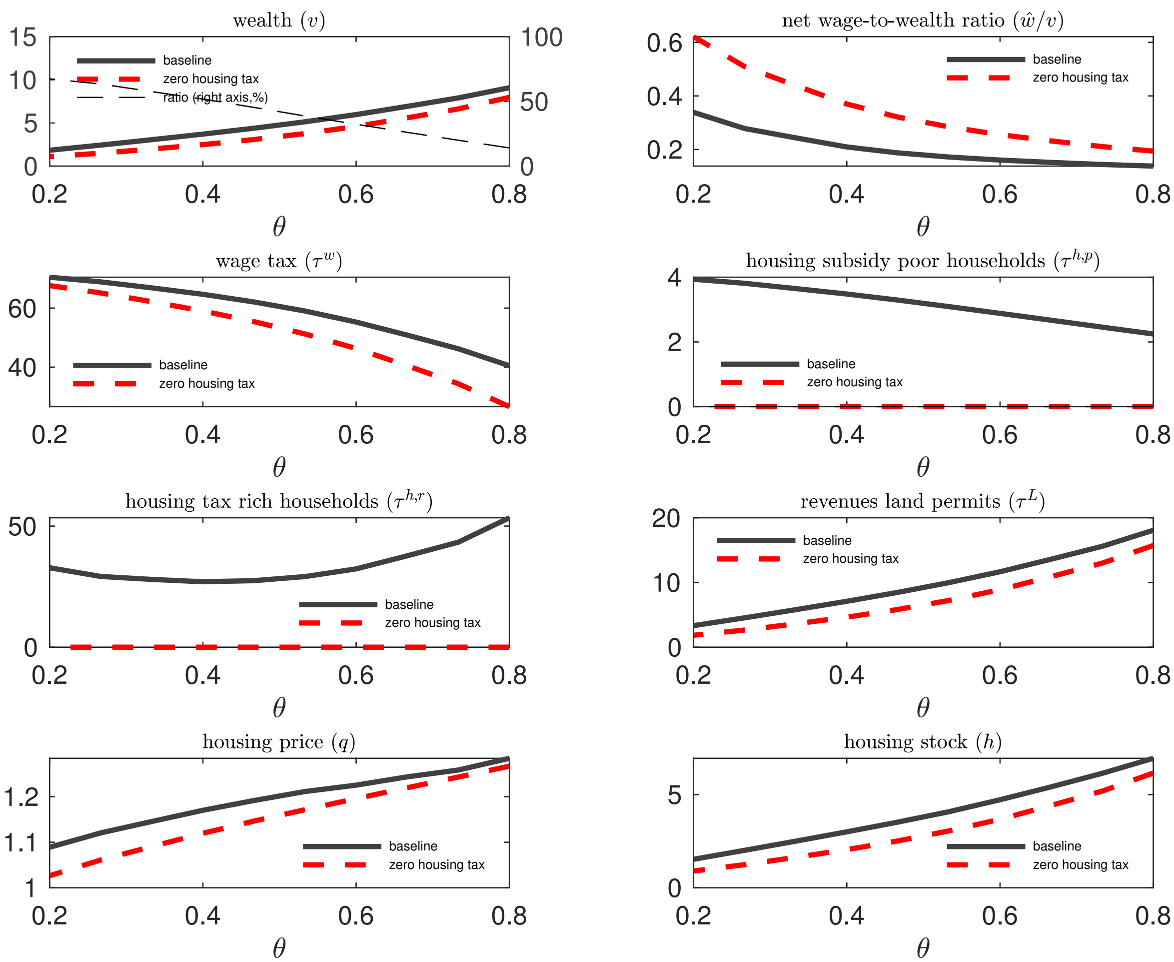
and, then, using the above in (75), we obtain

$$t_t^r = \left(\frac{1-\bar{\epsilon}^p}{1-\theta}\right)\frac{\hat{w}}{rv}, \qquad t_t^p = -\frac{(1-\bar{\epsilon}^p)}{1 - \frac{\theta\bar{\epsilon}^p}{1+(1-\bar{\epsilon}^p)\hat{w}/rv}}. \tag{77}$$

Therefore, the implicit taxation of housing for rich households is decreasing in the net wage-to-wealth ratio, $\hat{w}/v$, and the implicit subsidy on housing for the poor households is increasing in the same ratio. This result follows from the fact that a higher net wage-to-wealth ratio makes the rich households' housing demand more elastic, *i.e.*, it lowers the spread between the general equilibrium elasticities, $g_1^i - g_2^i$.

We now consider the quantitative results of the optimal taxation problem. We generate a path of increasing wealth inequality by exogenously changing the parameter θ, which pins down the households' expenditure for housing. Specifically, we let θ go from $\theta = 0.2$ to $\theta = 0.8$. We solve for the steady state of the model by solving the system described in details in Appendix A and setting $\tau^k = 0$. The model and the parameters are the same as those presented in section 2.3. Figure 2 plots the steady state values for the total wealth (v); the net wage-to-wealth ratio ($\hat{w}/v$); the wage tax (τ^w); the housing subsidy on poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the government revenue from the sale of land permits as a fraction of government expenditure (τ^L); the housing price (q); the housing stock (h); for different values of θ. Figure 2 plots both the steady state values of our baseline model, with the optimal tax rates (solid black line), and the steady state values of a model with zero housing tax rates for rich and poor households (dashed red line). We summarize our results as follows. First, as expected, the level of wealth is increasing in θ. Note how the level of wealth is always higher in our baseline model than in the model with zero housing tax rates, but the difference is decreasing with the wealth level. Second, the net wage-to-wealth ratio is decreasing with wealth and, therefore, inequality is increasing for both model specifications. Note also that the net wage-to-wealth ratio is always lower in the model with zero housing taxes. Therefore, the baseline model with the optimal housing taxes is characterized by less inequality, as the government can curb inequality with additional tax instruments. Third, wage taxes decline with θ, and are always higher in the baseline model then in the model with zero housing tax. Intuitively, in the baseline model the government must raise additional funds to pay a housing subsidy to poor households. The government can, in part, collect these resources by taxing the housing wealth of rich households. However, because rich households are only a small fraction of the population, the government can raise limited resources through this channel. Fourth, it is optimal for the government to set a housing subsidy for poor households of approximately 4% for $\theta = 0.2$, and lower for higher values of θ; and a housing tax for rich households of approximately 40% for $\theta = 0.2$, and lower for lower values of θ. Fifth, government revenues from the sale of land permits, as a fraction of government expenditure, are increasing in θ; always higher in the baseline model then in the model with zero housing tax; and approximately equal to 5% for $\theta = 0.2$. Finally, housing prices and the housing stock are both increasing in θ. Therefore, the increase in the level of wealth is driven by both valuation and quantity effects.

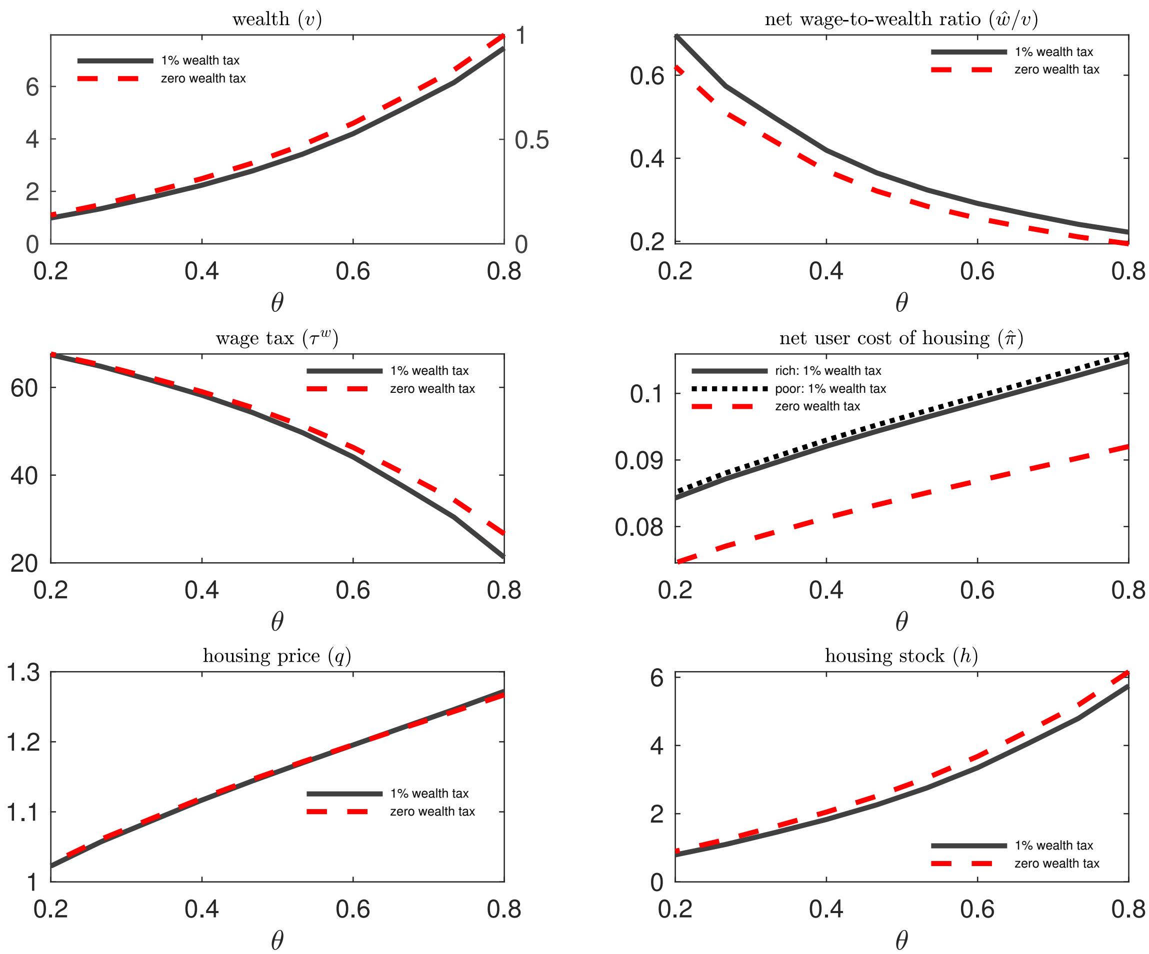
Figure 2: Steady State: Main Variables



Notes: This figure plots the steady state values for the total wealth (v); the net wage-to-wealth ratio ($\hat{w}/v$); the wage tax (τ^w); the housing subsidy on poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the government revenue from the sale of land permits as a fraction of government expenditure (τ^L); the housing price (q); the housing stock (h); for different values of the parameter θ. The wage and housing tax rates, the housing subsidy, and the revenues from the sale of land permits, are reported in percentage. The black solid line corresponds to our baseline model with optimal tax rates; the red dashed line corresponds to a model with zero housing tax on both rich and poor households. For total wealth we also report the ratio between the values under the baseline model and the model with zero housing taxes (black dashed line, right axis). Parameters are from Table 1, with the exception of θ which is in the range $[0.2, 0.8]$.

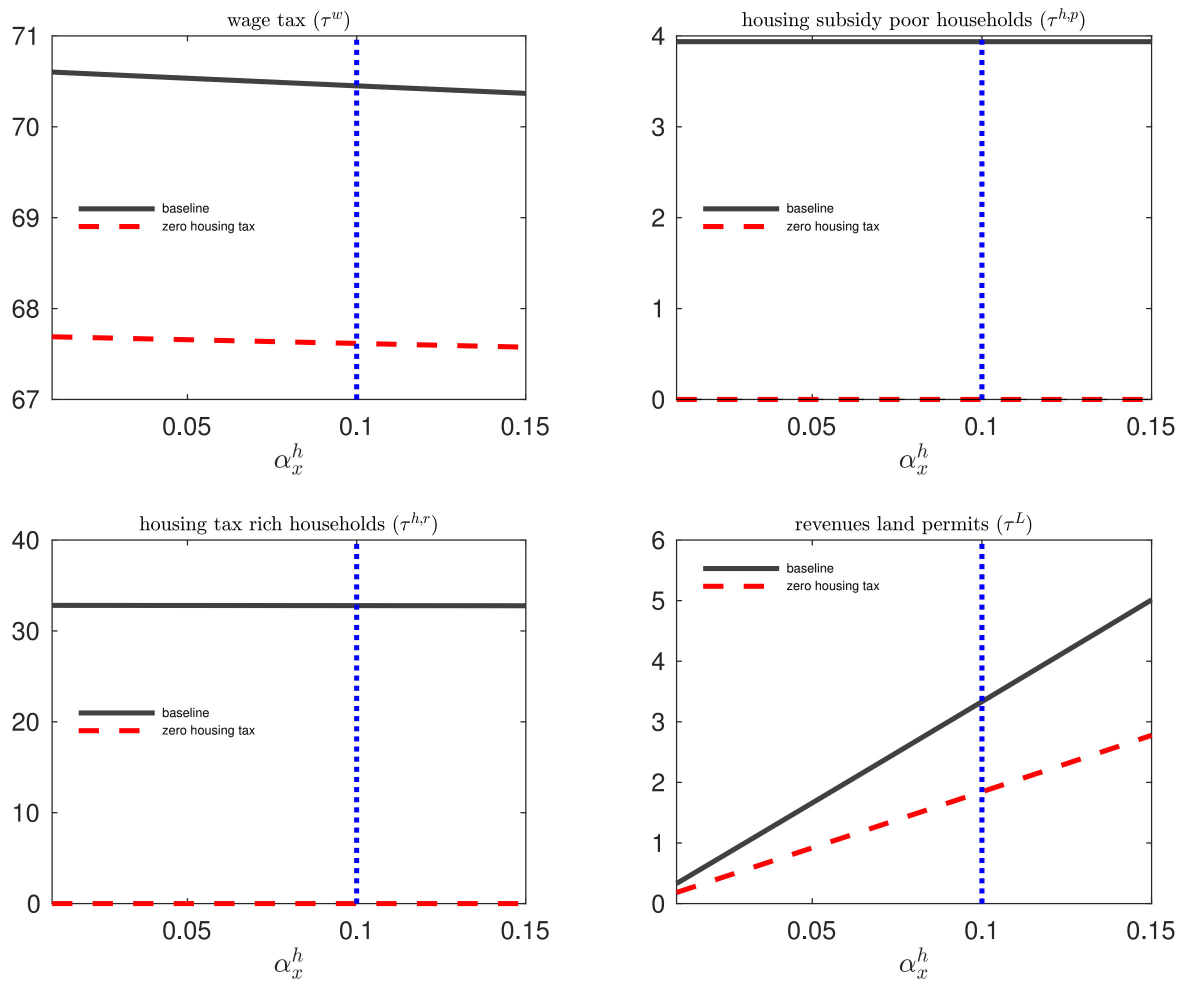
We evaluate the robustness of our results exploring the role of different fractions of rich households (Figure 3) and a different weight of land in the production function of the construction sector (Figure 4). We find that the steady state values of the optimal wage tax; the housing subsidy for poor households; the housing tax for rich households; and the revenues from the same of land permits are robust to changes in both the fraction of rich households and weight of land in the production function of the construction sector.

Figure 3: Robustness: Fraction of Rich Households



Notes: This figure plots the steady state values for the wage tax (τ^w); the housing subsidy on poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the government revenue from the sale of land permits as a fraction of government expenditure (τ^L) for different values of the parameter m_r, i.e., the fraction of rich households. The dotted blue vertical line corresponds to the baseline value of m_r. Parameters are from Table 1 with the exception of m_r (and $m_p = 1 - m_r$) which is in the range $[0.01, 0.15]$.

Figure 4: Robustness: Land Weight in Production



Notes: This figure plots the steady state values for the wage tax (τ^w); the housing subsidy on poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the government revenue from the sale of land permits as a fraction of government expenditure (τ^L) for different values of the parameter α_x^h, i.e., the weight of the land input in the production function for the housing sector. The dotted blue vertical line corresponds to the baseline value of α_x^h. Parameters are from Table 1 with the exception of α_x^h which is in the range $[0.01, 0.15]$. Note that we compute the weight on the labor input, in the production function of the construction sector, as the residual $1 - \alpha_k^h - \alpha_x^h$.

4 Conclusions

Based on the observation that the wealth–to–income ratio and wealth inequality have been generally increasing in advanced economies since 1970, it has been suggested that the tax structure should be rebalanced from labor income to wealth. In this paper we have considered a simple model with rich (lenders) and poor (financially constrained) households, financial and housing wealth, and find that the optimal steady state tax structure includes some taxation of labor, zero taxation of financial wealth, a housing tax on rich households and a housing subsidy on poor households. Therefore, governments can use housing taxation to curb inequality. When wealth inequality increases, it is optimal for a government to maintain a housing subsidy to poor households and reduce the taxation of labor income. The government can raise the required resources by increasing the housing wealth tax on rich households, and with the revenues from the sale of land permits that are increasing in the level of wealth.

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Online Appendix (not for publication)

This online appendix contains additional results and derivations for Borri, N. and P. Reichlin (2019): "Optimal Taxation, Homeownership and Wealth Inequality."

A Quantitative Analysis

In this section, we present details and robustness results for our quantitative analysis presented in section 3

A.I Model

Here we provide a full specification of the model that we use in the numerical simulation for the parameters in Table 1. The utility and production functions are specified as in (31), (32), (33), so that the parameter θ denotes the expenditure share on housing, α_k^j the capital shares in sector $j = h, m$ and α_x^h the land share in sector h. Furthermore, we assume that rich households are all identical and use the index r to denote their decisions, (c^r, z^r), as well as their mass, m_r. The key additional hypothesis is that the cost of housing services faced by the poor households is equal to $\hat{\pi}^p$ irrespective of whether they are home owners or renters. This assumption is arbitrary in the case of exogenous wealth and rent tax rates considered in section 2.3, and, as discussed in section 3.3, it is necessarily verified in the case of optimal taxation (as a consequence of the linear homogeneity of the utility function). Given these assumptions, any demand side solution is such that the ratios c^i/h^i are identical for all $i \in \mathcal{P}$.

The model is solved in two consecutive steps. The first step, which we call the *supply side solution*, consists in using (7), (8) and the steady state versions of (3), (4), (5), (6), to derive the sector-specific capital and labor, (k^m, l^m, k^h, l^h), the gross wage, w, the land price, q, the capital stock, k, and the housing stock, h, as functions of the capital tax rate, τ^k, and the rich households' rate of time preference, r, and the flow of land for construction, x. In particular, $(k^m, k^h, l^m, l^h, w, q, h, q)$ can be derived from

$$
\begin{aligned}
\frac{k^m}{l^m} &= \left(\frac{\alpha_k^m(1-\tau^k)}{1+r}\right)^{1/(1-\alpha_k^m)}, && \text{(A1)}\\
\frac{k^h}{l^h} &= \left(\frac{\alpha_k^h}{\alpha_k^m}\right)\left(\frac{1-\alpha_k^m}{1-\alpha_k^h-\alpha_x^h}\right)\frac{k^m}{l^m}, && \text{(A2)}\\
w &= (1-\alpha_k^m)\left(\frac{k^m}{l^m}\right)^{\alpha_k^m}, && \text{(A3)}\\
q &= \left(\frac{\alpha_k^m}{\alpha_k^h}\right)^{\alpha_k^h}\left(\frac{1-\alpha_k^m}{1-\alpha_k^h-\alpha_x^h}\right)^{1-\alpha_k^h}\left(\frac{k^m}{l^m}\right)^{\alpha_k^m-\alpha_k^h}\left(\frac{x}{l^h}\right)^{-\alpha_x^h}, && \text{(A4)}\\
h\delta &= (k^h)^{\alpha_k^h}x^{\alpha_x^h}(l^h)^{1-\alpha_k^h-\alpha_x^h}, && \text{(A5)}\\
1 &= l^m + l^h && \text{(A6)}\\
k &= k^h + k^m && \text{(A7)}
\end{aligned}
$$

For the give supply side solution, the next step (*demand side solution*) consists in the derivation of the steady state values of the remaining 6 variables, *i.e.*, the net wage, $\hat{w}$, the households' consumption and housing services (c^p, c^r, z^p, z^r), and the aggregate net wealth, $v \equiv qh + k + b^g/R$. The remaining unknowns are determined as follows. First, by the steady state versions of equations (39), (40) we derive

$$
\begin{aligned}
c^i &= (1-\theta)\epsilon^i\hat{w} \qquad \forall i \in \mathcal{P}, && \text{(A8)}\\
c^r &= (1-\theta)((1-\bar{\epsilon}^p)\hat{w} + rv)/m_r, && \text{(A9)}\\
z^i &= \left(\frac{\theta}{1-\theta}\right)c^i/\hat{\pi}^p \qquad \forall i \in \mathcal{P}, && \text{(A10)}\\
z^r &= \left(\frac{\theta}{1-\theta}\right)c^r/\hat{\pi}^r. && \text{(A11)}
\end{aligned}
$$

Then, we use the above in the market clearing conditions in manufacturing and housing,

$$
\begin{aligned}
\sum_{i\in\mathcal{P}} m_i c^i + m_r c^r &= (k^m)^{\alpha_k^m}(l^m)^{1-\alpha_k^m} - k - g, && \text{(A12)}\\
\sum_{i\in\mathcal{P}} m_i z^i + m_r z^r &= h. && \text{(A13)}
\end{aligned}
$$

Note that the homogeneity of the utility function implies that the solution of the above system implies the following two simultaneous conditions:

$$
\begin{aligned}
(1-\theta)(\hat{w} + rv) &= f^m(k^m, l^m) - k - g, && \text{(A14)}\\
\theta\left(\frac{\bar{\epsilon}^p\hat{w}}{\hat{\pi}^p} + \frac{(1-\bar{\epsilon}^p)\hat{w} + rv}{\hat{\pi}^r}\right) &= h. && \text{(A15)}
\end{aligned}
$$

Exogenous Capital Tax. Now suppose that we impose an exogenous uniform wealth tax, τ^k, on the rich households and no wealth tax on the poor households wealth.

Then, the user costs of housing are

$$\hat{\pi}^p = q\left(r + \delta + (1+r)\frac{\tau^k}{1-\tau^k}\right), \qquad \hat{\pi}^r = q\left(r + \delta + (1-\delta)\tau^k\right).$$

Optimal Tax Rates.

$$\hat{\pi}^r = \pi\left(1 + \frac{(1-\bar{\epsilon}^p)\,\hat{w}}{(1-\theta)rv}\right) \tag{A16}$$

$$\hat{\pi}^p = \pi\left(1 - \frac{1-\bar{\epsilon}^p}{1 - \frac{\theta\bar{\epsilon}^p}{1+(1-\bar{\epsilon}^p p)\hat{w}/rv}}\right) \tag{A17}$$

where $\pi = q\,(r+\delta)$.

B Proofs

Proof of proposition 1.

To characterize the households' optimal choices, it is convenient to start by solving an *auxiliary problem*, corresponding to the problem faced by a household that is "forced" to buy an amount of housing at least as large as the minimum amount $\bar{z}$ at all $t \geq 0$. Whether this solution is optimal will be verified *ex-post* by confronting it with the solution of the problem faced by the same household when setting $z_t^o = h_t = 0$ at all $t \geq 0$. The solution to the *auxiliary problem* faced by household i follows from the maximization of the utility function, (1), subject to the the budget constraints (13), the debt limits (15) and

$$z_t^{r,i} \geq 0, \tag{A18}$$

$$z_t^{o,i} \geq \bar{z}, \tag{A19}$$

$$h_t \geq z_t^{o,i}. \tag{A20}$$

Using a standard Lagrange method, we let $\{\eta_t^i, \mu_t^i, \xi_t^i, \lambda_t^i\}_{t=0}^{\infty}$ be a sequence of (non-negative) Lagrange multipliers related to the constraints, (15), (A18), (10), (A20), respectively, and, assuming strict positivity of (c_t^i, z_t^i, l_t^i) for all $t \geq 0$, we state the homeowners's first order conditions as

$$U_1(c_t^i, z_t^i, l_t^i)/\hat{R}_{t+1}^i - \beta_i U_1(c_{t+1}^i, z_{t+1}^i, l_{t+1}^i) = \eta_t^i, \tag{A21}$$

$$U_1(c_t^i, z_t^i, l_t^i)(\hat{\pi}_t^i - \hat{s}_t^i) = \lambda_t^i, \tag{A22}$$

$$U_1(c_t^i, z_t^i, l_t^i)s_t - U_2(c_t^i, z_t^i, l_t^i) = \mu_t^i, \tag{A23}$$

$$U_1(c_t^i, z_t^i, l_t^i)\hat{\pi}_t^i - U_2(c_t^i, z_t^i, l_t^i) = \xi_t^i, \tag{A24}$$

together with the complementary slackness conditions

$$\eta_t^i a_{t+1}^i = \lambda_t^i (h_t^i - z_t^{o,i}) = \mu_t^i z_t^{r,i} = \xi_t^i (z_t^{o,i} - \bar{z}) = 0 \tag{A25}$$

and the transversality condition

$$\lim_{t \to \infty} \beta_i^t U_1(c_t^i, z_t^i) a_{t+1}^i = 0. \tag{A26}$$

If, on the other hand, the household is forced to be a renter, the first order conditions are only those specified in equations (A21), (A23), whereas the values of z_t^o and h_t are zero at all $t \geq 0$ and the transversality condition remains as specified in (A26). It is clear that, a necessary condition for having an equilibrium with home-owners is that conditions (A22) and (A24) are verified for some i. Now note that conditions (A22) and (A25) imply the right hand side inequality in (22) and condition (23) for all $t \geq 0$. Using the transversality condition (A26), we can derive the life-time present value representation of the individuals budget constraints (24). By (A21) and the definition of present value prices, we get (19). Finally, by the form of the budget constraints (24), it is clear that a necessary condition for the existence of some home-owners is the left hand side inequality in (23) for all $t \geq 0$. In fact, suppose that $\hat{\pi}_t^i > s_t$ for all $t \geq 0$. Then, a home-owner can buy the same amount of consumption of manufacturing goods, housing services and leisure at the given prices by spending strictly less than her initial wealth if she was becoming a renter, and this is incompatible with optimality. It follows that (23) must hold in any equilibrium with some home-owner.

Proof of proposition 2.

Since workers are hand-to-mouth, $U_1^i c^i + U_2^i z^i = U_1^i \epsilon^i \hat{w}$, and, then, we have

$$g_1^i = \frac{z^i}{(U_1^i)^2} \left(U_{2,1}^i U_1^i - U_{1,1}^i U_2^i \right) = z^i \frac{\partial}{\partial c^i} \left(\frac{U_2^i}{U_1^i} \right),$$

which, by (strict) concavity, is a (strictly) positive value. Now notice that

$$\begin{aligned} g_1^i - g_2^i &= \left(\frac{U_{1,2}^i}{U_2^i} - \frac{U_{1,1}^i}{U_1^i} \right) (\epsilon^i \hat{w} - c^i) + \left(\frac{U_{1,2}^i}{U_1^i} - \frac{U_{2,2}^i}{U_2^i} \right) z^i \\ &= \left(\frac{U_{1,2}^i}{U_2^i} - \frac{U_{1,1}^i}{U_1^i} \right) \frac{U_2^i z^i}{U_1^i} + \left(\frac{U_{1,2}^i}{U_1^i} - \frac{U_{2,2}^i}{U_2^i} \right) z^i \\ &= \frac{z^i}{(U_1^i)^2 U_2^i} \left(2 U_1^i U_2^i U_{1,2}^i - U_{1,1}^i (U_2^i)^2 - U_{2,2}^i (U_1^i)^2 \right). \end{aligned}$$

By (strict) concavity the above is (strictly) positive.