Monetary Non-neutralities when Credit Constraints Bite

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Abstract

This article studies monetary policy in a model in which credit constraints are the only source of non-neutralities. I show that sizeable real effects can be obtained in a framework that is also able to match the term premium and generate an upward-sloping yield curve. Moreover, monetary policy has the expected effect on real and nominal variables as well as on asset prices. Combining financing frictions with a lower bound on interest rates generates large asymmetries in the transmission mechanism. The state of the economy therefore matters as money becomes close to neutral when credit is ample and easily obtainable. In contrast, monetary policy is particularly effective during periods of credit crunches and high lending spreads. Resorting to higher-order approximations is necessary to detect this asymmetry.

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1 Introduction

Understanding the effects of financial frictions on monetary policy is a central question in macroeconomics. One main lesson from the evidence accumulated over the last 30 years is that financial frictions, in particular frictions affecting access to credit, influence the transmission of monetary policy.[1]

Following the seminal contribution of Bernanke and Gertler (1989) (see also Bernanke et al. 1999), a vast strand of literature has interpreted this line of evidence by developing models in which lending is subject to a "costly state verification" problem (e.g. Townsend, 1979).[2] However, in this literature it is the introduction of nominal rigidities that allows monetary policy to have real effects. Financial frictions amplify the effects of monetary policy shocks but they are not a source of non-neutrality per se.

This paper asks whether the sole presence of credit frictions can break the classical dichotomy. If yes, the objective is then to assess whether the real effects generated via this channel are quantitatively important. I introduce financing constraints by assuming that the representative firm needs to obtain a bank loan to finance the cost of production inputs (e.g. Jermann and Quadrini, 2012). Relative to the financial accelerator literature, a key difference is that this specification creates a wedge that affects labor demand.

This departure from the standard approach can be motivated by a growing literature that has uncovered a link between credit market frictions and employment (e.g. Chodorow-Reich, 2014). In particular, the evidence available for Europe strongly suggests that credit market frictions affect the real economy through their impact on the labor market (e.g. Bentolila et al., 2018; Popov and Rocholl, 2018).

The second main departure from the literature is that monetary policy is studied in a model in which risk matters. Introducing risk considerations into models of monetary policy can be justified by the role played by long-term interest rates in the transmission mechanism (e.g. Den Haan, 1995). Indeed, the term premium is often mentioned as one of the possible channels through which monetary policy is transmitted to the real economy (e.g. Bernanke, 2017). In contrast, in the class of models typically used for policy analysis, the issue of risk is either ignored or circumvented by assuming that term premiums are exogenously determined.

Relative to the literature that uses Epstein-Zin-Weil (e.g. Epstein and Zin, 1989; Weil, 1989; Weil, 1990; Tallarini, 2000; Bansal and Yaron, 2004; Gourio, 2012; Swanson, 2016;

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[1] See among others Kashyap et al. (1994) and Gertler and Gilchrist (1994) for some early evidence on the effect financial frictions on the transmission mechanism.

[2] See Christiano et al. (2014) for a recent example.

Andreasen et al. 2017) or Chew-Dekel preferences (e.g. Campanale et al., 2010) to resolve asset pricing puzzles, I rely on a specification of habit formation in the composite of consumption and leisure (e.g. Jaccard, 2014; Dimitriev, 2017). In order to simultaneously match the low risk-free rate volatility observed in recent years, I also introduce time-variation in the rate at which agents discount the future.

Time-variation in time-discounting can be motivated by the evidence documented in the psychology literature. As shown by Blain et al. (2016) among others, the choice between immediate or delayed rewards is influenced by mental fatigue. In the model, this is captured by introducing a slow-moving state variable that depends on past and current labor effort. Since the time-discount rate in turn varies with work intensity, intertemporal choices depend on fatigue levels. This in turn implies that agents become more impatient to consume when fatigue levels are elevated.

Generating time-variation in stochastic discount factors is a main building block of modern asset pricing theory (e.g. Cochrane, 2011). Combining habits in the composite good with this time-variation in time-discounting allows the model to match the term premium, the yield curve's slope and the risk-free rate volatility by only adding two degrees of freedom.

The third main innovation is to capture the effect of the lower bound on interest rates by introducing cash into the analysis. Cash acts as a constraint on monetary policy because agents will not keep money on their deposit accounts if interest rates become negative. In a model in which the cost of storing cash is negligible, negative rates can always be circumvented by holding money instead of bank deposits. The presence of cash therefore limits the room for manoeuvre of the central bank.

Relative to the New Keynesian literature (e.g. Adam and Billi, 2006, 2007), the key difference is that the zero lower bound emerges as an endogenous outcome. In models in which monetary aggregates are explicitly modelled, the allocation between cash and deposit is dictated by agents' optimality conditions. Avoiding discontinuities in the state space also greatly simplifies the problem as higher-order perturbation methods can be used to solve the model (e.g. Andreasen et al., 2017).

My first main finding is that monetary policy has the expected average effect on real and nominal variables as well as on asset prices. In particular, a one percent standard deviation shock to the quantity of money that is transmitted through this channel increases output, consumption, the price level, hours worked as well as investment. Credit increases in response to a loosening of the monetary policy stance, whereas lending spreads decline. The model also reproduces the liquidity effect as a positive monetary policy shock reduces the real short-term rate (e.g. Christiano and Eichenbaum, 1995). Regarding asset pricing implications, I find that a positive shock steepens the yield curve and reduces the term premium while increasing the price of long-term nominal bonds. This co-movement between nominal and real variables is obtained in a model that reproduces a set of 13 asset pricing and business cycle moments that characterize the Eurozone economy.

In this flexible price environment, I show that this transmission mechanism essentially works through the effect of monetary policy on bank lending rates. Bank deposit rates are determined by the equilibrium between the demand for cash and the quantity of money supplied by the central bank. Since the demand for cash is downward sloping, an expansionary monetary policy shock lowers the deposit rate which in turn leads to a reduction in the cost of credit. Consequently, by reducing the tightness of firms' financing constraints, a lower cost of financing stimulates labor demand, which in turn increases output.

Although nominal interest rates directly affect production costs, prices increase in response to an expansionary monetary policy shock. This illustrates that prices in this economy are determined by the household side of the economy, and not by a Phillips curve linking marginal costs to inflation. This reflects that the benefit of holding real money balances depends on the deposit rate paid by banks. The value of money declines in response to a positive shock and the price level therefore increases because a lower deposit rate reduces the opportunity cost of real money balances.

My second main finding is that the effects of monetary policy strongly depend on financial conditions. During periods of high credit availability and low lending spreads, this transmission channel is insignificant, as monetary policy is close to neutral. By contrast, the real effect of the exact same monetary policy shock is considerably larger during periods of credit stress and high lending spreads. The increase in the real quantity of credit that banks extend to the productive sector only reaches 0.1% in the state of ample credit availability. In contrast, the same shock stimulates credit creation by around 0.7% on impact in the credit stress regime. The effect of an expansionary monetary policy shock on output is also about 7 times greater in the credit stress regime than in the state of high credit availability.

This state-dependence can be explained by the impact of the lower bound on interest rates on the effectiveness of monetary policy. In the ample credit state, the high degree of liquidity available in the economy depresses lending rates, which in turn relaxes credit constraints. The key is that the resulting decline in interest rates also affects the choice between holding cash and lending funds to the real economy. This follows from the fact that the opportunity cost of holding cash depends on the monetary policy rate. As a result, the incentive to channel any further increase in money supply to the productive sector declines when interest rates approach the lower bound. In other words, in such states of the world, any further increase in money supply is hoarded in the form of cash rather than lent to the real economy.

The work of Giovannini and Labadie (1991) is one of the first attempts to study money supply shocks in a model able to generate plausible risk premiums. Relative to their approach, which relies on cash-in-advance constraints, the first main difference is that I study a model where financing constraints are the only source of non-neutrality. The second main departure is that I introduce production into the analysis (e.g. Jermann, 1998).

In recent years, the literature on monetary policy has mostly focussed on models in which the non-neutrality of monetary policy relies on sticky prices (e.g. Woodford 2003; Galí, 2015). However, although they received far less attention, many studies have documented real effects of a sizeable magnitude in models without nominal rigidities. In Fuerst (1992) and Christiano and Eichenbaum (1992), significant non-neutralities are obtained in a model with production in which cash-in-advance constraints are combined with limited participation. At the same time, one limitation of these models is that the non-neutralities that they generate are not sufficiently persistent (e.g. Christiano and Eichenbaum, 1995).[3]

Relative to these studies, Cooley and Quadrini (1999) obtain more persistent real effects of monetary policy in a model with search and matching frictions in the labor market. In Khan and Thomas (2015), persistent real effects are obtained in a model in which market segmentation is endogenously determined. In Gomes et al. (2016), the significant non-neutralities stem from the effect of inflation on long-term nominal debt contracts that are issued by firms. Relative to this strand of literature, the difference is that I study monetary policy in a model that generates an upward-sloping yield curve and reproduces a term premium of the magnitude observed in the data.[4]

Relative to the cost channel of monetary policy, the difference is that in my case prices increase in response to an accommodative monetary policy shock. As initially shown by Barth and Ramey (2002), an expansionary monetary policy shock can lead to a decline in prices if marginal costs depend on the nominal interest rate paid by firms to obtain credit. For instance, in the New Keynesian model developed by Christiano et al. (2005), the role of the working capital channel is to provide an explanation for the "price puzzle" documented is some empirical studies. Indeed, as shown by Ravenna and Walsh (2006), the nominal interest rate enters the Phillips curve with a positive sign once a cost channel is introduced

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[3] This lack of persistence was subsequently addressed by introducing price stickiness into the analysis (e.g. Christiano et al. 2005).

[4] Zervou (2013) studies the volatility of stock prices in a model with limited participation.

into a standard model with nominal rigidities. This contrasts with the flexible price model studied here where the dynamics of prices is determined by the demand for real money balances.

The empirical evidence on the effectiveness of monetary policy over the business cycle is mixed. The lower effectiveness of monetary policy during periods of low interest rates implied by the mechanism under study is consistent with the evidence documented by Borio and Gambacorta (2017). As illustrated in Figure 11, the ineffectiveness of monetary policy at very low rates can be explained by the lower responsiveness of credit supply in those states of the world. This model prediction is also in line with the results documented in Aikman et al. (2019) where monetary policy is less effective during periods of above trend credit growth. The evidence documented in Cariero et al. (2018) also suggests that the effects of monetary policy are strongly asymmetric. Using a regime switching VAR, they find that monetary policy is more effective during periods of credit stress, which is consistent with the mechanism studied here. The empirical facts documented by Santoro et al. (2014) also suggest that monetary policy has a stronger effect on output during recessions and they rationalize their empirical findings by combining loss aversion with nominal rigidities. In contrast, Tenreyro and Thwaites (2016) find that monetary policy is less effective during periods of recessions. In Eickmeier et al. (2016), monetary policy is less effective during periods of high financial market uncertainty, which generally coincide with recessions.

Finally, my approach is also related to a recent strand of literature in which the preference for liquid assets is explicitly modelled. In van den Heuvel (2008), agents' preference for liquidity introduces a wedge between the return on equity and the return on bank deposits. In Begenau (2019), the general equilibrium effects implied by this preference for liquidity can lead to an increase in bank lending in response to higher capital requirements. In Piazzesi et al. (2019), this preference for deposit creates a convenience yield on the policy instrument that significantly alters the transmission mechanism of an otherwise standard New Keynesian model.

2 The Environment

The model is composed of a non-financial or corporate sector, a commercial banking sector, a central bank, a representative household and a government. A role for external financing is introduced by assuming that firms in the non-financial sector need to pay workers and capital owners in advance of production.

Households

The representative agent owns the economy's stock of capital and derives utility from consuming a consumption good, from enjoying leisure and from holding cash. All variables are detrended and the deterministic growth rate along which the economy is growing is denoted by γ (e.g. King and Rebelo, 1999). The period t budget constraint of the representative agent is given as follows:

$$prof_{Tt} + tr_t + r_t k_t + w_t n_t + i_{Dt}\frac{D_t}{P_t} + \frac{M_t}{P_t} + \frac{B_t}{P_t} = c_t + x_t + \gamma\frac{M_{t+1}}{P_t} + \frac{1}{1+i_{Bt}}\gamma\frac{B_{t+1}}{P_t} \tag{1}$$

The left-hand side of equation (1) reports the various sources of income received by agents. Since they own all the sectors of the economy, households receive a dividend income paid by the financial and non-financial sectors denoted by $prof_T$. Each period a lump sum transfer, which is denoted by tr, is received from the government. Households own the economy's capital stock and rent it to the non-financial corporate sector. The capital stock is denoted by k and r is the rental rate of capital. Labor supply being endogenously determined, households divide their total time endowment, which I normalize to 1, between hours worked in the corporate sector and leisure:

$$z_t + n_t = 1 \tag{2}$$

where leisure and hours worked are denoted by z and n, respectively. The total labor income received in period t is therefore wn, where w denote the wage rate. Relative to a real business cycle model, the difference is that households also accumulate real money balances. The nominal stock of money balances carried from the previous period is denoted by M_t, whereas money balances available during period t is denoted by M_{t+1}. Since money is a nominal asset, the real value of money holding is obtained by dividing the nominal stock by the price level, which is denoted by P. Relative to the textbook money-in-the-utility function model, the difference is that households can either keep money in the form of cash or deposit part of it in the banking sector. This portfolio decision is captured by introducing the following constraint:

$$\gamma M_{t+1} = D_t + S_t \tag{3}$$

where γM denotes money holdings available during the period. The fraction that is deposited in the banking sector is denoted by D and earns a within period interest rate i_D.

In real terms, the net income received from bankers is thus given by $i_D \frac{D}{P}$. The remaining fraction that households keep in cash is represented by S. Following the money-in-the utility view, a demand for cash is introduced by assuming that the amount of real money holdings available during the period, which is denoted by S/P, yields utility.

Following the timing adopted in models with money-in-the-utility, it is the amount available at the end of the period that yields utility (e.g. Walsh, 2010). Whereas S represents the fraction of money balances that is liquid and that households can access at any time, the amount deposited in the banking sector is illiquid in the sense that it cannot be converted into cash within the period. The interest rate on deposit therefore represents the opportunity cost of keeping liquid money balances in the form of cash instead of depositing it in the banking sector. Finally, households invest in a short-term risk-free bond issued by the government. The real payoff received by households depends on inflation and is given by the coupon payment B deflated by the price level P.

The right-hand side of equation (1) represents the different expenditures faced by agents in period t. Households firstly choose how to allocate their total income between consumption and investment, which are denoted by c and x, respectively. Real money balances available during period t are denoted by $\gamma \frac{M_{t+1}}{P_t}$, whereas $\gamma \frac{B_{t+1}}{P_t}$ is the stock of government bonds purchased by households during the period. The price at which this one period risk-free bond is purchased is denoted by $\frac{1}{1+i_{Bt}}$.

Capital accumulation is subject to adjustment costs, and following Jermann (1998) and Baxter and Crucini (1993) among others, I use the following specification:

$$\gamma k_{t+1} = (1-\delta)k_t + \left(\frac{\theta_1}{1-\epsilon} \left(\frac{x_t}{k_t} \right)^{1-\epsilon} + \theta_2 \right) k_t \tag{4}$$

where ϵ measures the degree of adjustment costs and can be interpreted as the elasticity of Tobin's Q with respect to changes in the investment to capital ratio. The parameters θ_1 and θ_2 are chosen to ensure that the model with and without adjustment costs have the same deterministic steady state.

Habits are formed over the composite good consisting of the different components of utility (e.g. Jaccard, 2014). The composite good not only depends on consumption and leisure but also on the fraction of real money balances S/P that agents hold in the form of cash. This implies the following law of motion for the habit stock, which is denoted by h :

$$\gamma h_{t+1} = \tau h_t + (1-\tau) c_t^{\kappa} \left(\frac{S_t}{P_t} \right)^{1-\kappa} (\psi + z_t^{\upsilon}) \tag{5}$$

where τ is a memory parameter that affects the rate at which the habit stock depreciates over time. The weight of consumption in the utility function is denoted by the utility parameter κ. υ and ψ are two labor supply parameters that control the Frisch elasticity as well as the steady state allocation of time between hours worked and leisure, respectively. The objective of the representative agent is to maximize lifetime utility, which is given as follows:

$$\max_{c_t, B_{t+1}, S_t, n_t, h_{t+1}, k_{t+1}, x_t, M_{t+1}} E_0 \sum_{t=0}^{\infty} (\beta_t)^t \frac{\left(c_t^{\kappa} \left(\frac{S_t}{P_t} \right)^{1-\kappa} (\psi + z_t^{\upsilon}) - h_t \right)^{1-\sigma}}{1-\sigma}$$

subject to constraints (1) to (5). In an infinite horizon model, the subjective discount factor is affected by the growth rate of the economy along the balance growth path (e.g., Kocherlakota 1990) and I denote the modified discount factor by β_t, where $\beta_t = \widetilde{\beta}_t \gamma^{1-\sigma}$.

Time-varying subjective discount factor

Following the evidence reported in the psychology literature, I introduce a link between agents' degree of impatience to consume and their state of mental fatigue. The facts documented by Blain et al. (2016) suggest that fatigue levels impact intertemporal decisions by affecting the choice between immediate or delayed rewards. A main result from this strand of literature is that the propensity to favor immediate rewards increases with the degree of mental fatigue observed among participants. Intense cognitive work could therefore increase impulsivity in economic decisions by reducing agents' willingness to accept to delay an immediate reward in anticipation of a larger future one. In this model, the intertemporal choice between immediate and delayed gratification is captured by the subjective discount factor β, a parameter which is typically held constant in the vast majority of models used in macro-finance.

In order to formalize the link uncovered in the psychology literature, fatigue is captured by introducing a state variables e that depends of agents' accumulated labor effort n. This implies the following law of motion for mental fatigue:

$$e_{t+1} = (1-\xi) e_t + n_t \tag{6}$$

where ξ is a parameter that measures the rate at which the stock of fatigue depreciates over time. A link between the subjective discount factor and fatigue levels is then introduced by assuming a negative relationship between the subjective discount factor and the state variable e:

$$\beta_t = \overline{\beta} - (e_t - \overline{e}) \tag{7}$$

where $\overline{\beta}$ and $\overline{e}$ represent the steady state values for these two variables.

In line with the evidence reported in the literature, this negative relationship between β and e therefore captures the idea that agents' willingness to delay immediate rewards by accepting to consume more tomorrow rather than today will decline with their fatigue level. According to this formulation, what matters is the difference between the actual state of mental fatigue of the representative agent relative to its steady state value $\overline{e}$. If e_t is kept constant at its steady state value, the model reduces to the standard case with a constant subjective discount factor.

The non-financial or corporate sector

Firms in the non-financial sector produce a composite output good using labor n and capital k as production inputs. The final output good produced by the corporate sector is denoted by y and the production function takes a standard Cobb-Douglas form:

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha} \tag{8}$$

where α is the capital share parameter. The random technology shock a follows an autoregressive process of order one,

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{at}$$

where the random disturbance ε_a is normally distributed with mean zero and standard deviation σ_a. The autoregressive parameter is denoted by ρ_a, where $0 \leq \rho_a \leq 1$.

Profits at time t, which are denoted by $prof_F$, are given as follows:

$$prof_{Ft} = a_t k_t^{\alpha} n_t^{1-\alpha} - r_{Kt} k_t - w_t n_t - i_{Lt} \frac{L_t}{P_t} \tag{9}$$

Relative to the neoclassical growth model, a cost channel of monetary policy is introduced by assuming that firms need to obtain credit in order to operate. The lending decision is intratemporal in the sense that the loan is received at the beginning of the period and needs to be reimbursed with an interest payment before the end of period t. The amount of bank-based financing received at the beginning of the period is denoted by L/P and i_L is the interest rate that is paid to bankers. The amount of external financing needed at the beginning of period t is determined by the following loan-in-advance constraint:

$$\frac{L_t}{P_t} \geq \mu \left(w_t n_t + r_{Kt} k_t \right) \tag{10}$$

where the parameter μ represents the fraction of total labor and capital costs, *i.e.* $wN + rk$, that needs to be paid in advance and which therefore requires financing. The objective of managers in the corporate sector is to maximize the value of the firm, which is given by the infinite discounted sum of future profits:

$$\max_{n_t, k_t, L_t} \ E_0 \sum_{t=0}^{\infty} \left(\widehat{\beta}_t \right)^t \frac{\lambda_t}{\lambda_0} prof_{Ft}$$

where $\widehat{\beta}_t^{\,t} \lambda_t / \lambda_0$ is the stochastic discount factor of the representative agent who owns firms in the non-financial sector, subject to equations (8), (9) and (10).

The central bank

The central bank provides money to the private sector and any profit or loss made by the monetary authorities, which is denoted by rcp_t, is directly transferred to the government. In period t, this implies the following budget constraint:

$$rcp_t = \gamma \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} \tag{11}$$

Since money is explicitly modelled, and given that central banks use quantities to influence interest rates, the quantity of money M is the policy tool. The deposit rate i_D is therefore treated as an endogenous variable. This choice can also be motivated by the environment in which monetary policy was conducted in recent years. Since the onset of the financial crisis, balance sheet policies have become the main policy instrument. In addition, in a model in which the quantity of money is explicitly introduced, the extent to which monetary policy is constrained by the zero lower bound is a priori not clear. The empirical results documented in Belongia and Ireland (2018) for instance suggest that this constraint could be alleviated by using money supply rather than interest rate rules. Monetary policy is determined by the following money supply rule:

$$M_{t+1} = \overline{M} + \rho_M \left(M_t - \overline{M} \right) - \phi_\pi (P_t - \overline{P}) + \varepsilon_{Mt} \tag{12}$$

where $\overline{M}$ denotes the quantity of money when the economy reaches its steady state. The monetary policy shock is denoted by the random disturbance ε_M, which is normally distributed with mean zero and standard deviation σ_M.

Relative to steady state, the quantity of money set by the central bank firstly depends on the quantity available in the previous period, where ρ_M is a monetary policy smoothing parameter. The objective of monetary policy is to stabilize inflation and the central bank adjust its monetary policy stance each time the price level P deviates from its steady state value, which is denoted by $\overline{P}$. Monetary policy therefore becomes more restrictive when P rises above $\overline{P}$ and more accommodative when the price level falls below its steady state value. The sensitivity of monetary policy to deviations of the price level from its target is captured by the parameter ϕ_π. The model could also be closed by assuming that the deposit rate i_D is the main policy instrument, which would imply that the stock of money M is an endogenous variable.

The commercial banking sector

Following van den Heuvel (2008, 2016), the commercial banking sector intermediates funds between households and the non-financial sector. Banks collect deposits at the beginning of the period, which are then lent to the corporate sector. I simplify the analysis by assuming that the lending and deposit decisions occur within the period.

As in Goodfriend and McCallum (2007), I assume that banks are endowed with a technology that can be used to produce credit using deposits as an input. The production function is given by a linear technology that links the quantity of loans extended to the non-financial sector to the quantity of deposits raised at the beginning of the period:

$$L_t = \eta D_t \tag{13}$$

where η is a technology parameter measuring the efficiency of the intermediation technology operated by banks. Each period bankers optimally choose the amount of deposits to collect from households D and the quantity of credit to extend to firms L to maximize profits, which are given as follows:

$$\max_{L_t, D_t} prof_{Bt} = i_{Lt}\frac{L_t}{P_t} - i_{Dt}\frac{D_t}{P_t}$$

subject to constraint (13).

The government

Government only plays a passive role in this environment. The lump sum transfer made to the representative agent is financed by issuing a short-term risk free bond and by the receipts received from the central bank. In period t, this implies the following budget constraint:

$$tr_t = rcp_t + \frac{1}{1+i_{Bt}}\gamma\frac{B_{t+1}}{P_t} - \frac{B_t}{P_t} \tag{14}$$

Market clearing condition

The aggregate budget constraint can be derived by combining the budget constraint of the different sectors. Since the representative agent owns the non-financial and banking sectors, the total dividend income that is received is given by $prof_{Tt} = prof_{Ft} + prof_{Bt}$. Any loss or profit made by the central bank is transferred to the government and since the government in turns makes a lump sum transfer to the representative agent, the economy's consolidated budget constraint is given as follows:

$$y_t = c_t + x_t \tag{15}$$

Equilibrium definition

A competitive equilibrium in the economy is a sequence of prices:

$$\varpi, q, \lambda, \varphi, w, r_K, i_D, i_L, i_B, P$$

where ϖ denotes the Lagrange multipliers associated with the loan-in-advance constraint, q is Tobin's Q, λ is marginal utility, φ is the Lagrange multiplier associated with the law of motion of the habit stock, and quantities:

$$L, D, M, S, y, c, x, n, k, h$$

that satisfy households and firms efficiency conditions as well as the resource constraint (15) for all states, for t=1...∞, and given initial values for the two endogenous state variables k and h.

The transmission of monetary policy to the real economy

The household optimality conditions can be analyzed to gain intuition into how a monetary policy shock is transmitted to the real economy. The optimality conditions with respect to S and c can firstly be combined to derive the following relationship between the deposit rate i_D, consumption c, the price level P, and the amount of cash held by agents

S:[5]

$$i_{Dt} = \frac{1-\kappa}{\kappa}\frac{P_t c_t}{S_t} \tag{16}$$

The deposit rate represents the opportunity cost foregone by the agent when cash balances are held liquid instead of being deposited into a bank account. This creates a downward demand schedule that implies a negative relationship between interest rates and cash holdings. Given the choice to use the quantity of money as the policy instrument, equilibrium in the money market can be represented by a vertical money supply curve. An exogenous increase in money supply, which shifts this supply curve to the right, therefore has a direct impact on this opportunity cost by creating a surplus of liquidity, which in turn puts downward pressure on the deposit rate i_D.

A change in the deposit rate is then passed through to firms via the banking sector. Profit maximization in the banking sector implies the following relationship between deposit and bank lending rates:

$$i_{Lt} = \frac{1}{\eta} i_{Dt} \tag{17}$$

where the degree of pass-through depends on the efficiency of the financial intermediation technology. A change in lending rates then impacts firms in the non-financial corporate sector by modifying the cost of obtaining funds from the banking sector. The effects of a change in the cost of funds on the behaviour of firms can be illustrated by deriving their optimal demand for inputs:

$$w_t = (1-\alpha)\frac{y_t}{N_t}\frac{1}{1+\mu i_{Lt}} \tag{18}$$

$$r_t = \alpha\frac{y_t}{k_t}\frac{1}{1+\mu i_{Lt}} \tag{19}$$

A change in funding costs i_L therefore firstly affects the real economy through a labor wedge by moving firms' demand for labor. Secondly, since lending rates affect the marginal productivity of capital, credit frictions also distort the investment decision through an investment wedge. This latter effect can be illustrated by deriving the Euler condition associated with capital accumulation, which with adjustment costs, is given by the following

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[5] See technical appendix in section 10.

equation:

$$\begin{aligned}\lambda_t q_t &= \left(\beta_t/\gamma\right) E_t \lambda_{t+1} q_{t+1} \left((1-\delta_K) + \frac{\theta_1}{1-\epsilon}\left(\frac{x_{t+1}}{k_{t+1}}\right)^{1-\epsilon} + \theta_2 - \theta_1 \left(\frac{x_{t+1}}{k_{t+1}}\right)^{1-\epsilon} \right) \\ &\quad + \left(\beta_t/\gamma\right) E_t \lambda_{t+1} r_{t+1}\end{aligned}$$

A change in borrowing costs therefore modifies the marginal productivity of capital, which in turn affects Tobin's Q and thus investment.

Money and the lower bound on interest rates

For realistic calibrations, the components of utility and the price level are always strictly positive in this model. Since in equilibrium, the deposit rate is equal to the marginal utility of holding cash (see equation 16), this ensures that the deposit rate i_D will take values that are always strictly positive. Introducing an endogenous choice between between S and D therefore captures the restriction on interest rates created by the existence of cash. If the return on bank deposits is too low, agents can always choose to hold money in cash. As will be discussed in section 6, this lower bound on i_D is a significant source of asymmetry in the transmission mechanism of monetary policy. It is however necessary to solve the model using a non-linear solution method to capture this effect. Notice also that the occurrence of slightly negative rates, which was recently observed in some jurisdictions, could be rationalized by introducing a cost of storing cash.

In the absence of such costs however, the deposit rate will always stay in positive territory, which also implies strictly positive values for the bank lending rate i_L (see equation 17). The second advantage of introducing a bound on deposit rates is that strictly positive values for bank lending rates also rule out the case of occasionally binding constraints. Indeed, the optimality condition in the non-financial sector with respect to L implies that the Lagrange multiplier associated to constraint (10), which is denoted by ϖ, is proportional to the bank lending rate i_L:

$$i_{Lt} = \frac{\varpi_t}{\lambda_t} \tag{20}$$

Since marginal utility λ is always strictly positive and given the lower bound on the deposit rate implied by equation (16), i_L and thus ϖ are always strictly positive. If present, the loan-in-advance constraint is therefore always satisfied with strict equality.

Equilibrium in the market for loanable funds

Combining the demand for production factors with the liquidity constraint faced by firms, and since this constraint is always binding, the following credit demand condition can be derived:

$$L_t = P_t y_t \frac{\mu}{1 + \mu i_{Lt}} \tag{21}$$

which implies a negative relationship between the amount of credit needed by firms L and the lending rate i_L. The supply of loanable funds can be derived by firstly combining the portfolio allocation constraint, *i.e.* $\gamma M_{t+1} = D_t + S_t$, with the demand for cash given by equation (16). Next, using the financial intermediation technology in the banking sector given by equation (13), the following supply curve implying a positive relationship between credit and lending rates can be derived:

$$L_t = \eta \gamma M_{t+1} - \frac{1-\kappa}{\kappa} \frac{P_t c_t}{i_{Lt}} \tag{22}$$

The dynamics of prices

The link between prices and monetary policy can be illustrated by deriving the optimality condition with respect to the demand for money M :

$$\frac{\lambda_t}{P_t} = (\beta_t/\gamma)\, E_t \lambda_{t+1} \frac{1}{P_{t+1}} + \frac{\lambda_t}{P_t} i_{Dt} \tag{23}$$

where λ is marginal utility. Optimality in the accumulation of real money balances implies that the cost of sacrificing one unit of consumption today has to equate the current and expected marginal benefit of doing so. Since agents can choose to deposit money in the banking sector, the marginal benefit from holding money also includes the interest rate paid on deposit i_D. To illustrate how fluctuations in the money market rate affects inflation, I log-linearize equations (23) around the deterministic steady state of the model. After a few manipulations, the dynamics of prices can be characterized by the following formula:

$$\widehat{P}_t = -\left(\widehat{\beta}_t + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t\right) - \frac{1-\beta/\gamma}{\beta/\gamma} \widehat{i_{Dt}} + E_t \widehat{P}_{t+1} \tag{24}$$

where variables with a hat are expressed in deviation from steady state. On the right-hand side, the first term denotes the stochastic discount factor, $\left(\widehat{\beta}_t + E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t\right)$, that agents use to evaluate future payoffs. The negative relationship between current prices and the stochastic discount factor illustrates that attitudes towards risk have an impact on inflation, a channel that is typically overlooked in the literature (e.g. Jaccard, 2018b). Everything else equal, an increase in stochastic discounting, which implies a stronger willingness to postpone current consumption in favor of future consumption, reduces the price level. In other words, when agents become more willing to save, they reduce consumption and accumulate assets, such as real money balances, that they can use to transfer wealth across time. Since $\widehat{P}_t$ is the price of consumption, an increase in the degree of patience puts downwards pressure on prices.

The steady state value for the subjective discount factor β/γ being smaller than one, and keeping all other variables constant, this condition also shows that an increase in the short-term rate reduces the price level. Keeping everything else constant, the effect stemming from the second term in equation (24) illustrates that an increase in the deposit rate increases the benefit of holding money. As the accumulation of real money balances comes at the expense of other components of aggregate demand, such as consumption expenditures, the price level declines when $\widehat{i_D}$ goes up. Finally the price level is a forward-looking variable in this model and current prices also depend on future expected values, *i.e.* $E_t \widehat{P}_{t+1}$.

Asset pricing implications

Optimality in the household sector implies the following Euler condition that relates the price of purchasing a risk-free nominal government bond to its expected payoff:

$$\lambda_t \frac{1}{1+i_{Bt}} = \left(\beta_t/\gamma\right) E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{25}$$

where $\frac{1}{1+i_B}$ is the price of the short-term bond, and where the expected payoff from holding a one period risk-free bond also depends on expected inflation.

The stochastic discount factor of the representative agent can also be used to derive an asset pricing formula that characterizes the dynamics of a risk-free long-term bond with infinite maturity and that pays a constant coupon normalized to 1:

$$p_{Bt} = \left(\beta_t/\gamma\right) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left(1 + \varrho p_{Bt+1}\right) \tag{26}$$

where ϱ is the rate of decay of the asset measuring the duration of the asset. The realized return in period t is defined as follows:

$$r_{Bt}^{LT} = \frac{1 + \varrho p_{Bt}^{LT}}{p_{Bt-1}^{LT}}$$

The average yield to maturity of this perpetual bond can be computed by solving the following equation:

$$p_{Bt}^{LT} = \sum_{k=1}^{\infty} \frac{\delta_C^{k-1}}{(1 + yield_t)^k}$$

Given the simplifying assumption of an infinite maturity, the yield is given by the inverse of the price of the long-term bond and also depends on the longevity of the asset:

$$yield_t = \frac{1}{p_{Bt}^{LT}} + \varrho - 1$$

To compute the corresponding term premium, we first need to derive the price of this long-term bond under the assumption that investors are risk neutral. Denoting the risk neutral price $\widetilde{p}_{Bt}^{LT}$, this price can be obtained by replacing the stochastic discount factor of the agent by the risk-free nominal rate:

$$\widetilde{p}_{Bt}^{LT} = \frac{1}{1 + i_{Bt}} E_t \left(1 + \varrho \widetilde{p}_{Bt+1}^{LT}\right)$$

The yield to maturity under risk neutral probabilities $yield_{RNt}$ is then given by the inverse of the risk neutral price:

$$yield_{RNt} = \frac{1}{\widetilde{p}_{Bt}^{LT}} + \varrho - 1$$

Up to a first-order approximation or in the deterministic version of the model, $\widetilde{p}_{Bt}^{LT}$ and p_{Bt}^{LT} are equivalent since certainty equivalent holds in these two particular cases. However, once the model is solved using higher-order approximations, the effect of uncertainty on the valuation drives a wedge between the two concepts, because risk averse investors will require a compensation for holding an asset whose price declines during recessions, when marginal utility is high. In a model in which risk and stochastic discounting both matter, the price of a long-term bond computed under the assumption of risk neutrality is therefore higher than the price obtained using the stochastic discount factor of a risk averse agent. Since risk adjustments reduce asset prices, risk aversion increases the yield of a long-term bond. The average term premium $E(tp)$ can then be computed as follows:

$$E(tp_t) = E(yield_t) - E(yield_{RNt}) \tag{27}$$

and provides a measure of the effect of risk adjustments on bond yields.

The stochastic discount factor of the representative agent can also be used to derive a long-term interest rate corresponding to an investment made today and that promises a fixed nominal future payoff. The quarterly interest rate i^{10Y} corresponding to a risk-free payoff that will be received in k quarters is determined by the following formula:

$$\frac{1}{1+i_t^{10Y}} = (\beta_t/\gamma)^k \, E_{t+k} \frac{\lambda_{t+k}}{\lambda_t} \frac{P_t}{P_{t+k}} \tag{28}$$

The yield curve's slope is then the difference between the long-term rate corresponding to an investment of 40 quarters, which is equivalent to 10 years in this quarterly model, and the 3-month short-term rate, which can be obtained by setting k to 1 in the above formula. Once expressed in annualized terms, on average, the difference between the short and long-term rate is equal to zero in the deterministic version of the model or if the model is solved using a first-order linear approximation. The expected return of holding a long-term asset being affected by attitudes towards risk, a non-zero average yield curve's slope can only be obtained if the model is solved using higher-order approximations.

3 Parameter selection and results

Given that the objective of this paper is evaluate the importance of credit constraints in the transmission of monetary policy, the model is calibrated using euro area data. This choice can be explained by the importance played by small firms in the European economy. Indeed, small and medium-sized enterprises (SMEs) represent about 99% of all euro area firms and account for around two-thirds of the jurisdiction's workforce (e.g. ECB, 2014). Since SMEs are more reliant on access to credit than large firms, which typically have access to market-based financing, the mechanisms studied here should be particularly relevant in economies with a bank-based financial system such as the euro area, China or Japan.

The data used to calibrate the model is described in the data appendix. Whereas data availability for the eurozone economy can be an issue, in most cases it is possible to find series that start in the late 1990's. Since one objective is to understand the effect of balance sheet policies on lending conditions, the quantity of money rather than the short-term rate is used as the monetary policy instrument. For the sake of parsimony, I only introduce two exogenous shocks and technology shocks are the only real source of business cycle fluctuations.

Deterministic growth rate, capital share and labor supply

The deterministic growth rate of the economy is calibrated using data on population growth, which is available on an annual basis since 1960. Between 1960 and 2018, the average rate of population growth for the country group that is currently forming the eurozone is 0.45% per year, which implies a value for the quarterly growth rate γ of 1.00112. The capital share parameter in the production function of the final output good α is set to 1/3, which implies a labor share of 2/3. With internal habits, long-term risk aversion increases with the curvature coefficient σ but is independent from the habit parameter (e.g., Constantinides 1990; Jermann (1998); Boldrin, Christiano, and Fisher (1997); Swanson 2012). I therefore set the curvature parameter σ to 1. As shown in Jaccard (2018), increasing this parameter does not help to resolve asset pricing puzzles in a model with labor and where consumption is endogenously determined.

Frisch elasticity of labor supply and steady state time allocation

The first labor supply parameter ψ is calibrated to ensure that in the steady state, agents spend about 20 percent of their time on work related activities, which corresponds to a value for N of 0.2. The curvature parameter υ is chosen to imply a value for the Frisch elasticity of labor supply of about 1 (e.g., Hall 2009; Chetty et al. 2011).

Steady state money supply and McCaulay duration

The steady state quantity of money available in the economy, which is denoted by $\overline{M}$, affects the steady state price level $\overline{P}$. If the steady state money supply is doubled, the steady state price level and the steady state quantity of cash holding $\overline{S}$ doubles as well but a change in money supply has no effect on the real side of the economy. Given this invariance of the real economy to the value of $\overline{M}$, I set the steady state money supply to 1.

The rate of decay of long-term bonds is determined by the parameter ϱ. If ϱ is set to 0, the asset reduces to a one period risk-free security. Setting this parameter to 0.9865 implies a McCaulay duration of about 10 years (e.g., Rudebusch and Swanson 2012).

Matching moment procedure

The remaining 13 parameters are calibrated to match a set of 13 moments that characterize the Eurozone business cycle. Since with uncertainty higher-order terms in the Taylor expansion drive a wedge between the deterministic and the stochastic versions of the model, it is necessary to simulate the model and find the combination of parameter values that minimizes the distance between the estimated and simulated moments. Table 1 below reports the combination of parameter values that minimizes this distance and the comparison between the model and the data is shown in Table 2.

Table 1: Moment Matching Procedure, Structural Parameters

$\overline{\beta}$	κ	τ	ξ	μ	ϕ_π	η	ϵ	δ	σ_A	ρ_A	σ_M	ρ_M
0.9925	0.999	0.91	0.36	0.92	2.3	0.48	1.1	0.01	0.007	0.98	0.023	0.6

In Table 2, g_y, g_c, g_x, g_M, g_D, g_P denote the growth rate of output, consumption, investment, money, deposits and prices expressed in year-over-year growth rate, respectively, where for output the growth rate is computed as $(\log(y_t) - \log(y_{t-4}))$. $\sigma(i_D)$ is the standard deviation of the short-term money market rate, $E(i_{10Y})$ is the mean long-term rate, $E(tp)$ is the mean term-premium, $E(i_{10Y} - i_{3M})$ is the mean yield curve's slope, and $E(i_L - i_D)$ the mean intermediation spread. Finally, $E(l/y)$ and $E(x/y)$ denote the average loan to output and investment to output ratios.

Table 2: Model vs. Data

	Data		Model
	95% confidence interval	Estimated empirical moments	Theoretical moments
$std(g_y)$	[1.6, 2.1]	1.8	1.8
$std(g_c)$	[0.9, 1.2]	1.0	0.9
$std(g_x)$	[5.0, 6.6]	5.7	5.7
$std(g_M)$	[2.8, 3.8]	3.3	3.3
$std(g_D)$	[1.8, 2.4]	2.0	2.0
$std(g_P)$	[0.8, 1.1]	0.9	0.9
$std(i_D)$	[1.8, 2.4]	2.0	2.0
$E(i_{10Y})$	[3.6, 4.4]	4.0	3.9
$E(tp)$	[0.9, 1.1]	1.0	1.0
$E(i_{10Y} - i_{3M})$	[1.4, 1.8]	1.6	1.7
$E(i_L - i_D)$	[2.2, 2.4]	2.3	2.3
$E(l/y)$	[0.88, 0.95]	0.91	0.91
$E(x/y)$	[0.21, 0.22]	0.22	0.21

Table 3: Variance Decomposition

	g_y	g_c	g_x	i_{10Y}	i_D	g_P	g_M
Technology	98	89	99.9	85	73	60	31
Monetary	2	11	0.1	15	27	40	69

Relation between structural parameters and model implied first and second-

order moments

It is difficult to associate each structural parameter with only one moment as most parameters have a significant impact on the entire system through general equilibrium effects. Some parameters do however have larger effects on a subset of model implications. As illustrated by the variance decomposition shown in Table 3, technology shocks are the main drivers of business cycle aggregates and account for nearly all the variation in output. The technology shock standard deviation parameter σ_A can therefore be associated to the volatility of output and business cycle aggregates in general.

The capital adjustment cost parameter ϵ controls the supply elasticity of capital and has a first-order impact on the volatility of investment. Since capital is the main asset available to transfer wealth across time, this parameter also affects the ease at which the economy's storage technology can be used to achieve consumption smoothing. When combined with habit persistence, the degree of which is captured by the parameter τ, the capital adjustment cost parameter also impacts the volatility of marginal utility which in turn affects how agents discount future payoffs. The model's ability to match the average term premium therefore critically depends on these two parameters. Agents' propensity to save also depends on whether technology shocks, which are the main source of business cycle fluctuations, are perceived as temporary or permanent. The mean term premium and the volatility of consumption and investment, denoted by $E(tp), \sigma(g_x)$ and $\sigma(g_c)$, can thus be associated with the capital adjustment costs, habit and shock persistence parameters ϵ, τ and ρ_A, respectively.

The last column of Table 3 shows that fluctuations in monetary aggregate M are mostly driven by the monetary policy shock, which illustrates that this moment helps to identify the monetary policy shock standard deviation σ_M. The preference parameter κ determines the weight of real cash balances in the utility function. The volatility of deposits $std(g_D)$ is therefore particularly sensitive to this parameter value. In a model in which supply shocks are the main drivers of business cycle fluctuations, inflation can be a main source of risk for bondholders. As a result, whereas the inflation coefficient in the monetary rule ϕ_π has a first-order impact on the volatility of prices g_P, this parameter also affects the model's asset pricing implications.

The steady state value for the subjective discount factor $\overline{\beta}$ determines agents' intertemporal choices and has a significant effect on the mean term premium and on the volatility of consumption and investment. Since this parameter also has a direct impact on interest rates, it can be used to pin down the mean long-term rate $E(i_{10Y})$. Whereas combining habits with adjustment costs allows this class of models to reproduce the volatility of stock returns, one shortcoming of this mechanism is that it tends to generate excessive risk-free rate variations. As will be discussed shortly, introducing endogenous fluctuations in subjective discounting, that are counter-cyclical, helps to alleviate this problem. The fatigue parameter ξ can therefore be associated to the risk-free rate standard deviation $std(g_D)$. Without this parameter, it would not be possible to simultaneously match the yield curve's slope as well as the term premium without generating excessive risk-free rate variations.

As illustrated by equation (17), the magnitude of the intermediation spread $E(i_L - i_D)$ is pinned down by the technology parameter in the production function of loans η. The parameter μ measures the fraction of total costs that firms need to pay in advance. This parameter therefore determines the quantity of credit needed to operate firms and mainly affects the steady state importance of bank-based financing in the economy. This parameter can thus be associated to the loan-to-output ratio $E(l/y)$. The amount of output that is invested critically depends on the rate at which capital depreciates and δ is therefore identified by including the mean investment to output ratio in the loss function $E(x/y)$.

The last remaining parameter, the monetary policy smoothing parameter ρ_M, is poorly identified. In particular, it is difficult to distinguish its effect on the model dynamics from that of the shock standard deviation σ_M. This parameter however affects the volatility of inflation, interest rates and monetary aggregates.

4 Results

Its ability to simultaneously reproduce the average term premium and yield curve's slope, $E(tp)$ and $E(i_{10Y} - i_{3M})$, is the main distinguishing feature of this model. Relative to the class of models with habits used in the asset pricing literature, the difference is that these moments can be reproduced without generating excessive risk-free rate variations, since the standard deviation of money market rates $\sigma(i_D)$ can also be matched. I use the shadow-rate term structure model developed and estimated using European data by Wu and Xia (2017) as an empirical counterpart for the model term premium. Using data from 2005 to 2017, they find an average value for the term premium of 1%, a value which is also close to those used for models calibrated to the U.S. economy (e.g. Rudebusch and Swanson 2008, 2012). The yield curve's slope is computed as the average difference between the yield of a 10 year AAA government bond and the 3-month money market rate.

These asset pricing facts can be matched in a model also able to reproduce the volatility of output, consumption and investment. Relative to a real business cycle model (e.g., King and Rebelo 1999), the monetary aggregate M, the price level P, and household deposits

D are the new variables that are introduced. It is thus encouraging to see that the model can also match the volatility of these nominal variables. Noteworthy is the fact that the moment matching procedure described in section 3 only assigns a very small value to the utility share of money, *i.e.* $1 - \kappa = 0.01$. This small value implies that agents hold on average 5 percent of their total liquid wealth in cash and 95 percent in the form of bank deposit. As will be discussed in section 6, even a small utility share of real cash balances is sufficient to ensure that the money market rate will always remain in positive territory.

One of the key model parameters that determines the effectiveness of monetary policy is denoted by μ. Given that this parameter measures the economy's credit dependence, it can be identified using data on credit to GDP ratios made available by the Bank for International Settlements. In line with the decentralized equilibrium described in section 2, the empirical value for the loan-to-output ratio $E(l/y)$ only includes credit to non-financial corporations obtained by banks, and excludes credit to households or other forms of non-bank's source of funding.

It is also possible to match the average spread between bank lending rates and the money market rate $E(i_L - i_D)$. Due to a lack of data availability, it is only possible to compute this spread for a sample period that starts from the first quarter of 2000 onwards. The lending rate data used to compute this spread is the cost paid by non-financial corporations to obtain a new loan corresponding to amounts smaller than 1 mio and for a period of less than a year. Since the representative firm is a SME, using a measure of interest rate on loans smaller than 1 mio is a better proxy of the cost of funding than the rate paid on larger loans. Finally, the fact that the investment share of output $E(x/y)$ can be matched also ensures that the high volatility of investment that is obtained is not due to a steady state effect.

Whereas the model is able to match the stylized facts reported in Table 2, it is important to keep in mind that it fails on some other dimensions. For instance, although the volatility of deposits can be reproduced, it is not possible to simultaneously account for the large fluctuations in credit observed in the data. Moreover, the model understates the volatility of hours worked, a limitation which suggests that incorporating both an extensive and an intensive margin of labor supply would be necessary to match the data. Since the model abstracts from unemployment, it also cannot be used to study the link between different measures of slack and inflation (e.g., Den Haan et al. 2017, Stock and Watson 2018).

What determines the term premium?

Recall that the model's ability to generate a sizeable term premium as well as an upward sloping average yield curve is entirely due to the higher-order terms in the Taylor expansion.

Up to a first-order linear approximation or in the deterministic version of the model, $E(tp)$ and $E(i_{10Y} - i_{3M})$, are therefore both equal to zero in this model. The term premium being a compensation for risk, what are the main drivers of this covariance between the return on a long-term bond and the stochastic discount factor?

In order to isolate the contribution of inflation risk to the term premium, let us start by deriving the pricing equation that corresponds to an inflation indexed long-term bond, the price of which is denoted by p_R. Relative to the formula shown in equation (27), expected inflation no longer affects the valuation of an inflation indexed long-term bonds, the price of which is given by the following expression:

$$p_{Rt} = (\beta_t/\gamma)\, E_t \frac{\lambda_{t+1}}{\lambda_t} (1 + \varrho p_{Rt+1})$$

Following the definition of term premium given by equation (28), a risk neutral measure can be obtained by firstly deriving a formula for the real risk-free rate, which I denote by i_R:

$$\lambda_t \frac{1}{1 + i_{Rt}} = (\beta_t/\gamma)\, E_t \lambda_{t+1}$$

The term premium on the inflation indexed bond, which is denoted by tp^R, is then determined by the difference between the yield computed using the stochastic discount factor of the risk averse agent minus the risk neutral measure. Since tp^R abstracts from inflation risk, the inflation risk premium π_{RP} can be defined as the difference between the term premium on a nominal and an inflation indexed long-term bond:

$$E(RP_\pi) = E(tp) - E(tp^R)$$

This measure therefore captures the impact of expected inflation on the compensation required by investors for holding a nominal asset subject to inflation risk.

Since the inflation risk premium critically depends on the cyclical properties of expected inflation, it is important to first understand how this measure varies over the business cycle. Table 4 uses the measure of expected inflation from the ECB survey of Professional Forecasters and reports its correlation with output growth. Since the measure reported in this survey corresponds to inflation expectation three quarters ahead, the model-based measure of inflation expectation is computed as follows: $E_t(\pi_{t+3}) = E_t((P_{t+3}/P_{t+2} - 1) \cdot 400)$. Both in the model and in the data, the correlation between output growth and inflation expectations is shown in the first row of Table 4, whereas the second row reports the standard deviation of inflation expectations, *i.e.* $std(E_t(\pi_{t+3}))$.

Table 4: Inflation expectations and inflation risk premium

	Data	Model
$corr(g_y, E_t(\pi_{t+3}))$	0.19	0.19
$std\,(E_t(\pi_{t+3}))$	0.30	0.22
$E(RP_\pi)$	-	-0.13

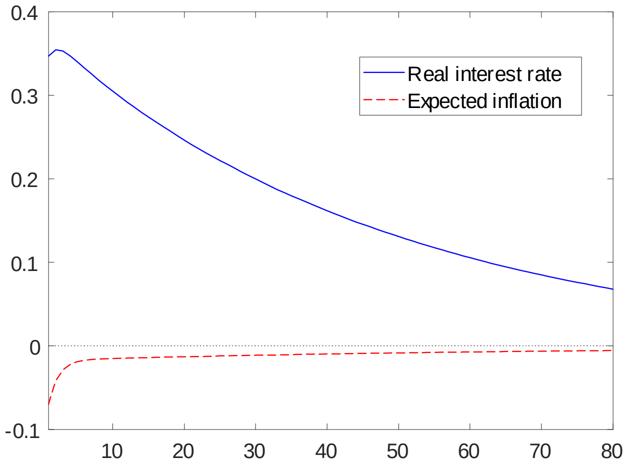

Figure 1. Response of the real interest rate and expected inflation to a negative technology shock. y axis: annualized percent. x axis: quarters after the shock.

Although this moment was not targeted, the first main result that stands out is that the model is able to match the correlation between inflation expectations and output growth observed in the data. Moreover, as illustrated by the second row of Table 3, it is possible to generate fluctuations in the three quarter ahead expected inflation rate of a magnitude that is broadly in line with the one observed in the data. The average inflation risk premium predicted by the model is shown in the last row of Table 3. Interestingly, the co-movement between expected inflation and output growth observed in the data and reproduced by the model gives rise to a negative inflation risk premium of 13 basis points. In other words, inflation contributes to reduce, rather than increase, the risk associated with an investment in long-term bonds.

Seen through the lens of this model, the fact that in the euro area inflation expectations are slightly procyclical is therefore a source of risk reduction. To gain intuition into this result, consider the case of a recession, a period in which marginal utility is higher than usual. This positive correlation lowers the risk of investing in long-term bonds because it implies a decline in expected inflation during periods of recession. Since a decline in expected inflation increases the real value of a nominal asset, this negative co-movement between marginal utility and expected inflation acts as a hedge against consumption risk.

If inflation is a source of risk reduction, what explains the model's ability to reproduce a sizeable term premium? To answer this question, it is important to first note that, in contrast to expected inflation, real interest rates are countercyclical in this environment. For the calibration summarized in Table 1, the model-generated correlation between output growth and the real interest rate i_R is -0.27. Since higher real rates depress prices by reducing the discounted value of future payoffs, holders of long-term bonds can thus expect capital losses to occur during periods of recession, precisely when marginal utility to consume is high. In spite of the negative inflation risk premium implied by the dynamics of expected inflation, the behaviour of real interest rates therefore explains why long-term bonds are risky in this model.

This difference in co-movement is illustrated in Figure 1, which compares the response of expected inflation $E_t(\pi_{t+3})$ with that of real interest rates i_R in the case of a recessionary shock that raises marginal utility. On impact, a negative technology shock increases the real interest rate by 0.3 percent and lowers expected inflation by a bit less than 0.1 percent.

What determines yield curve's slope?

The sensitivity analysis performed in Table 4 shows that it is the introduction of time-variation in the subjective discount factor β that allows the model to generate a yield curve that is sufficiently upward sloping on average. To deconstruct the mechanism, the first and second rows of Table 4 compare the result obtained using the calibration shown in Table 2 with the case in which the subjective discount factor is kept constant. Without time-variation in β, the average yield curve's slope decreases from 1.7% to 0.3%. The term premium also declines, whereas the risk-free rate volatility increases.

Without the additional degree of freedom provided by the fatigue parameter ξ, it would be necessary to rely more intensively on adjustment costs in order to generate an upward sloping yield curve of a plausible magnitude. Relative to the calibration shown in Table 2, and without time-variation in β, increasing the capital adjustment costs parameter from 1.1 to 3.0 allows the model with constant β to reproduce a 1.6% average yield curve's slope. However, in this case, the term premium reaches 3.2%, and the volatility of the short-term nominal rate increases from 2.0 to 5.1%. This illustrates that it would be more difficult to simultaneously reproduce these three moments without this additional source of time-variation in the stochastic discount factor.

Table 5: Sensitivity to decision fatigue

	$E(i_{10Y}-i_D)$	$E(tp)$	$std(i_D)$
Time-varying β, benchmark model	1.7	1.0	2.0
Constant β	0.3	0.7	2.4
Constant β, high adjustment costs	1.6	3.2	5.1

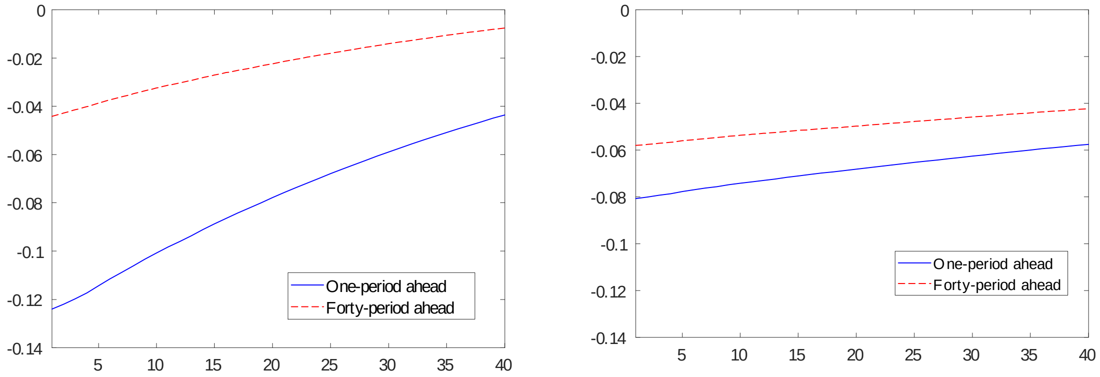

Figure 2: Response of marginal utility to a positive shock. y axis: percentage deviation from steady state. x axis: quarters after the shock.

To illustrate why introducing time-variation in β allows this model to generate lower fluctuations in $\log \lambda_{t+40}$ than in $\log \lambda_{t+1}$, the left panel of Figure 2 plots the response of expected marginal utility to a negative shock using the calibration that matches the moments shown in Table 2. The case in which β is kept constant is depicted in the right panel. In the model featuring a constant subjective discount factor, the response of expected marginal utility in $t+1$ and in $t+40$ to a positive shock is too similar, which explains the small yield curve's slope obtained in this case. By contrast, as illustrated by the much larger difference between the red dotted and blue continuous lines shown in the left panel, this difference in volatility is much more pronounced when the subjective discount factor is allowed to vary over time.

Since labor effort increases during periods of expansion, the introduction of fatigue effects leads to variations in β that are countercyclical. On impact, this effect induces a higher degree of impatience in intertemporal choices that reduces investment and increases consumption, which in turn accentuates the decline in marginal utility. Relative to the model with constant discounting, the key is that these fluctuations in marginal utility not only become more volatile on impact but also less persistent. Indeed, as can be seen by comparing the red dotted lines across the two panels, long-term expected marginal utility converges faster to the steady state in the model with time-variation in β. Intuitively, in good times, the key is that the decline in β that occurs in the benchmark model reduces the consumption smoothing motive induced by habit formation, by increasing the preference for the present. This intertemporal reallocation of consumption away from the future and in favor of the present reduces the persistence of consumption, which is what is needed to reduce the persistence of marginal utility. As a consequence, fluctuations in expected marginal utility in 40 periods decline when the consumption smoothing of the agent is attenuated by a change in the subjective discount factor.

The risk-free rate volatility

These counteryclical fluctuations in β also have a direct effect on the short-term risk-free rate volatility. In response to a positive shock, agents realize that these favorable conditions will not last and therefore that marginal utility of consumption tomorrow has to exceed its current value. Since expectation of higher expected marginal utility relative to today's value increases agents' willingness to save, the short-term real rate diminishes to reflect that the price of present relative to future consumption declines. Without time-variation in β, models with habits and adjustment costs overstate the importance of this intertemporal smoothing effect, which is responsible for the excessive reduction in real rates occurring in boom times. In the model with fatigue effects, this well-known issue is addressed by the countercyclical fluctuations in β that helps to attenuate the strength of the intertemporal smoothing motive.

The cyclicality of the intermediation spread

As can be seen in Figure A.1 in the appendix, in the eurozone, the intermediation spread is strongly negatively correlated with real credit growth. Periods of high credit growth are therefore associated with low levels of the intermediation spread, whereas these spreads increased during the Subprime and Sovereign Debt Crises. Over the period from the first quarter of 2000 to the first quarter of 2014, this negative correlation reaches -0.76. Although this correlation increased slightly after 2014 and stands at -0.58 for the entire sample, this negative co-movement is nevertheless a robust empirical regularity.

An important test for the model is therefore to check whether this negative co-movement can be reproduced. Figure A.2 below shows the response to a positive technology shock of real credit and of the intermediation spread, which is the main driver of business cycle fluctuations in this model. The negative co-movement between spreads and credit aggregates that is obtained confirms that this fact can also be reproduced. A period of low credit availability therefore coincides with high intermediation spreads, as observed during the Sovereign Debt Crisis for instance.

5 The real effects of monetary policy

Although prices and wages are fully flexible, monetary policy can have real effects in this model. The strength of the transmission mechanism depends on firms' dependence on credit, which is captured by the liquidity parameter μ, and monetary policy is neutral when μ is set to zero. A monetary policy shock takes the form of an exogenous increase in the innovation ε_M to the policy rule shown in equation (12). A positive innovation to the policy rule increases the quantity of money M in circulation and, although a fraction of this increase is absorbed by households who like to hold money in cash, the shock increases the supply of funds deposited in the banking sector D. This increase in deposits is then funneled to firms in the non-financial sector in the form of short-term credit by financial intermediaries. The resulting increase in the supply of loanable funds in turn reduces the rate at which banks lend funds to firms. A decline in the cost of funding, which is denoted by i_L, then affects the demand for factors through its effects on equations (19) and (20).

In order to deconstruct the transmission mechanism of monetary policy shocks, Figure 3 shows the response of the monetary aggregate M, the price level P, the short-term money market rate i_D and credit L to a one standard deviation positive monetary policy shock. Since the model's state space is non-linear, the response to shocks can be influenced by the particular point from which the impulse response is computed. To capture this potential state dependence, the impulse responses shown in this section are computed using a higher-order approximation to the policy function. The results reported in Figure 3 to 6 represent the average effect of a monetary policy shock. In each case, it is obtained by calculating an impulse response starting from many different points in the state space and by computing the average response (e.g., Adjemian et al. 2014). These responses correspond to the case in which monetary policy shocks are the only source of fluctuations. The response to a monetary policy shock conditional on also having technology shocks is studied in section 6.

As illustrated by the top left panel of Figure 3, a positive innovation to the money supply rule increases M by 1.8% on impact. Since it shifts the money supply curve to the right, equilibrium in the money market implies that an expansionary monetary policy shock reduces the money market rate i_D, which, in annualized terms, declines by 0.8%. As illustrated by the negative relationship between prices and the short-term deposit rate shown in equation (25), a decline in the nominal short-term rate has a positive impact on the price level P, which increases by about 0.3% on impact. The increase in credit L, which is shown in the bottom right panel, is less than proportional than the increase in money supply, which reflects that part of the effect of the shock is absorbed by households who increase their cash holdings S. Since the response of credit also depends on the efficiency of the financial intermediation technology, banks are only able to channel a fraction of the funds they receive to firms. This explains why a positive shock that raises money supply by 1.8% on impact only leads to an increase in credit by 0.8%.

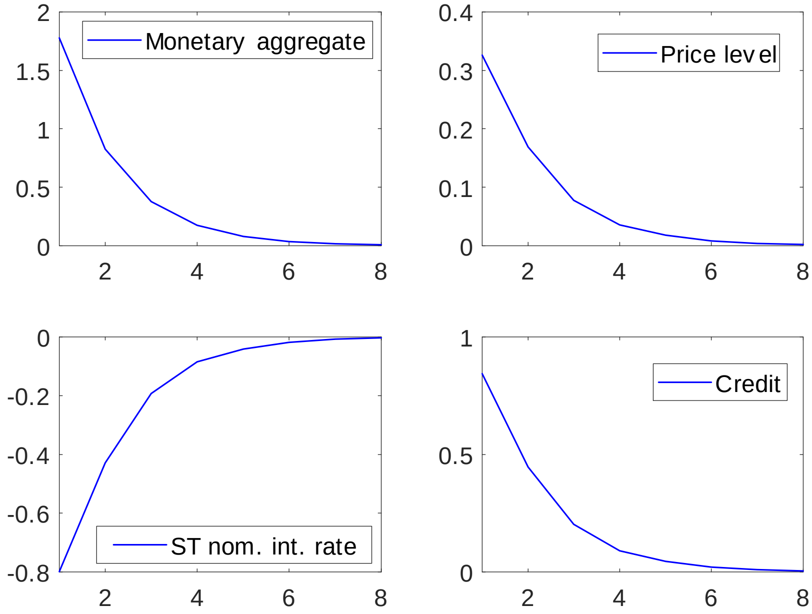

Figure 3: Response of the monetary aggregate, the price level, the short-term deposit rate and credit to one standard deviation monetary policy shock. y axis: percentage deviation from steady state and annualized percent for short-term rate. x axis: quarters after the shock.

The real effects of monetary policy are illustrated in Figure 4, which shows the response of output y, investment x, hours worked n and consumption c to the monetary policy shock. As can be seen on the top left panel, a monetary policy shock that increases money supply by 1.8% and reduces the nominal short-term interest rate by 0.8% raises output on impact by 0.14%. The shock has a somewhat stronger effect on hours worked, the increase of which reaches about 0.2% on impact. This stronger effect on hours worked contrasts with the considerably more muted increase in investment. The expansionary monetary policy stance also stimulates aggregate consumption, the increase of which reaches about 0.15% on impact.

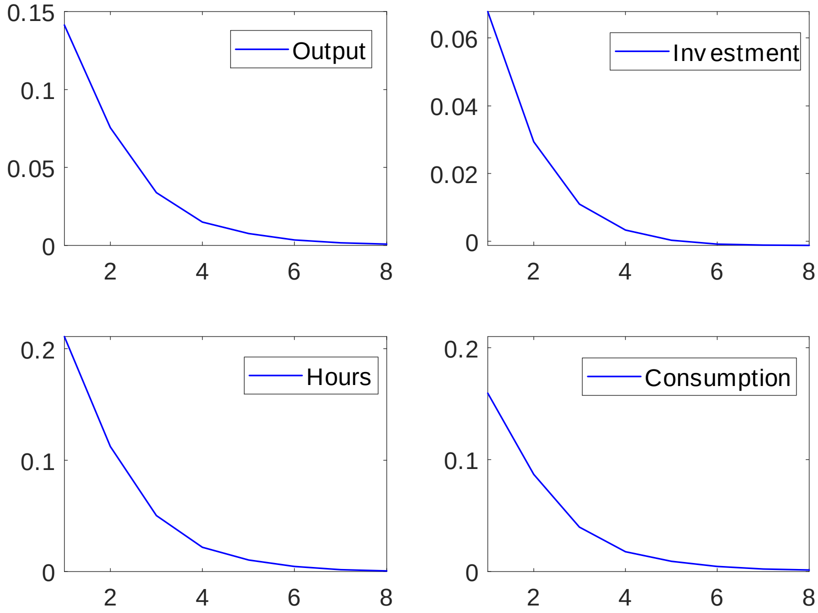

Figure 4: Response of output, investment, hours and consumption to a one standard deviation monetary policy shock. y axis: percentage deviation from steady state. x axis: quarters after the shock.

Along with the increase in investment and hours, the increase in the real wage and marginal productivity of capital shown in the two upper panel of Figure 5 confirms that an expansionary monetary policy shock mainly affects the economy by increasing firms' demand for production factors, as the demand for capital and labor both shift to the right. Combined with the weak impact of monetary policy on investment, the stronger increase in the marginal productivity of capital, which reaches about 0.5% compared to 0.3% for real wages, can be attributed to a discounting effect that simultaneously reduces the supply of capital. Indeed, as can be seen by comparing the two lower panels of Figure 5, whereas the short-term real rate declines on impact, a positive monetary policy increases long-term real rates. The lower increase in Tobin's Q caused by the response of long-term rates in turn explains the muted response of investment shown in the upper right panel of Figure 4. Without habits, a case which in obtained by setting τ to 1, the response of investment would even turn negative after a few quarters, as the decline in the subjective discount factor β reduces agents' incentives to postpone consumption by accumulating capital. For the calibration that reproduces the moments shown in Table 2, however, the decline in β is partially offset by the lower elasticity of intertemporal substitution implied by habits formation and, although small in magnitude, the response of investment remains positive.

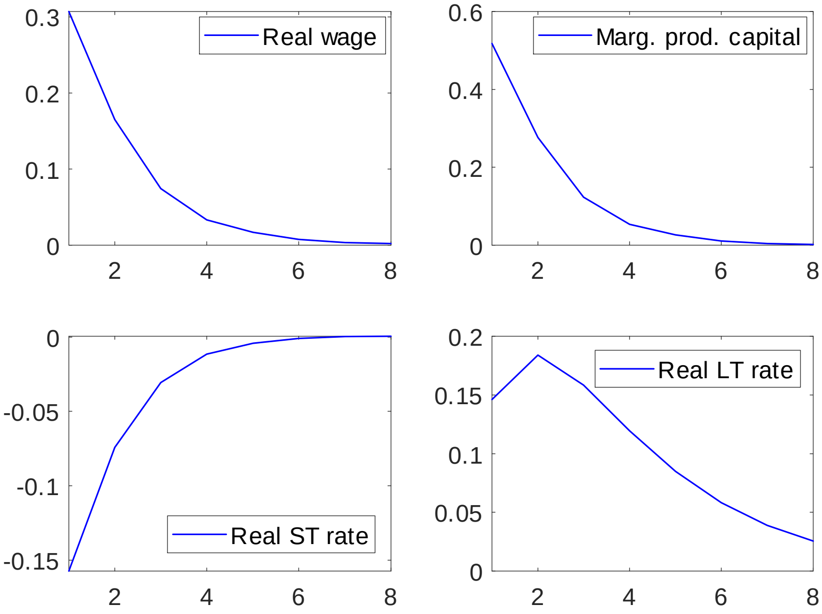

Figure 5: Response of real wage, marginal productivity of capital, the real short-term rate and the real long-term rate to a one standard deviation monetary policy shock. y axis: percentage deviation from steady state for upper panels and annualized percent for lower panels. x axis: quarters after the shock.

The positive impact on consumption can firstly be explained by the substitution effect induced by the decline in the short-term real rate depicted in the lower left panel of Figure 5. Moreover, since wages and the income from renting capital to firms also increase, this substitution effect is reinforced by a positive income effect that contributes to stimulate consumption on impact. As the subjective discount factor declines when labor effort increases, the time-variation in β is another factor that explains the larger response of consumption relative to investment.

The response of the term premium to the positive monetary shock is shown in the upper left panel of Figure 6. It is necessary to resort to a third-order approximation in order to generate time-variation in the term premium. This reflects that the term premium is determined by the covariance between the stochastic discount factor and the return on a long-term bond. The very small effect of monetary policy on the term premium illustrates that it is generally difficult to generate large variations in conditional second moments within this class of models. At the same time, the decline in the term premium obtained in response to an expansionary monetary policy shock is consistent with view that monetary policy affects the real economy by lowering risk premiums.

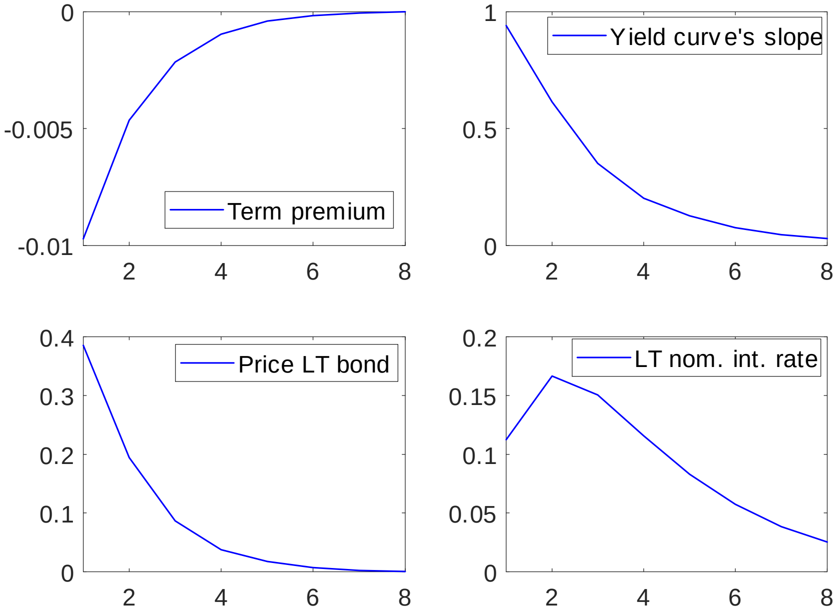

Figure 6: Response of the term premium, the yield curve's slope, the price of long-term bonds, and the long-term nominal rate to a one standard deviation monetary policy shock. y axis: annualized percent and percentage deviation from steady state for long-term bonds. x axis: quarters after the shock.

Although monetary policy only has a small effect on the term premium, it has significant effect on the yield curve. Indeed, as shown by the upper right panel of Figure 6, the positive shock raises the yield curve's slope by almost one percent on impact. Whereas this increase is mainly driven by the decline in the short-term rate, as illustrated by the lower right panel of that chart, an expansionary monetary policy shock also raises the nominal long-term rate by about 0.1 percent on impact. Finally, as shown by the lower left panel, the price of a long-term nominal bond is sensitive to a change in monetary policy and increases by almost 0.4 percent in response to an expansionary shock.

The importance of credit frictions

In this environment, the loan-in-advance constraint given by equation (10) is the only source of monetary policy non-neutrality. Whether a change in M has real effect therefore critically depends on the credit friction parameter μ. If μ is set to zero, the demand for loanable funds given by equation (22), falls to zero and a monetary policy shock has no effect on the real quantity of credit demanded by firms. To illustrate this point, Figure 7, firstly shows how the level of μ affects the steady state of the economy. The sensitivity analysis performed in this chart shows how the loan-to-output and cash holding ratios, *i.e.* $E(l/y)$ and $E(S/M)$ respectively, vary with the financial frictions parameter μ.

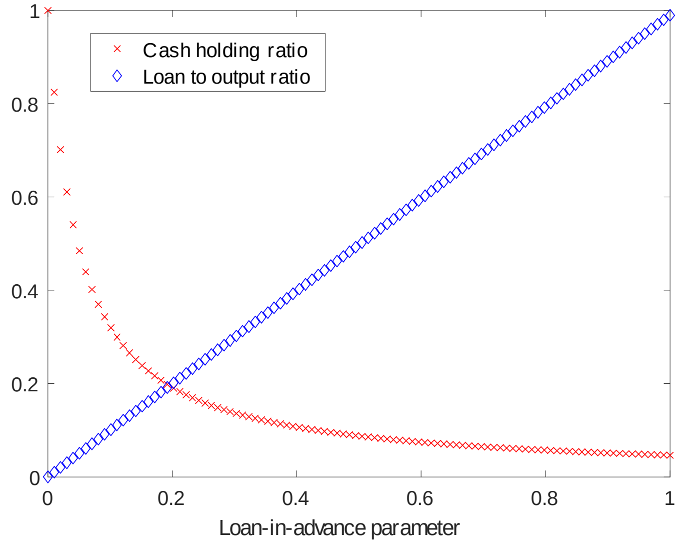

Figure 7. Sensitivity to credit friction parameter μ

As μ approaches zero, the loan-to-output ratio, which is depicted by the blue diamonded line, tends towards zero, the model reducing to a creditless economy in this limiting case. Since the demand for credit becomes negligible as μ approaches zero, the degree of credit friction also affects the allocation of money between cash S and deposits D. In Figure 7, this is illustrated by the red crossed line, which shows how a change in μ affects the average share of liquid wealth that households keep in their pocket in the form of cash. A lower dependence on credit implies a lower share of bank deposit. In the limit, the ratios D/M and S/M tend towards zero and 1, respectively, and all the money available in the economy is held in cash. As a result, in the limit, financial intermediation completely disappears and the model reduces to a version of the neoclassical growth model with money in the utility function.

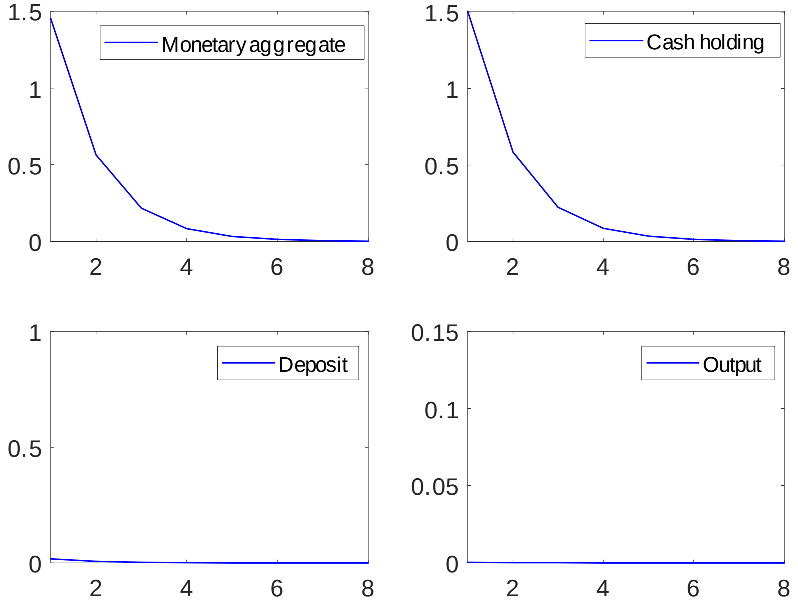

Figure 8. Response to a one standard monetary policy shock when μ is set to 0.001.

To illustrate that the value of μ also has a crucial impact on the model's dynamics, Figure 8 shows the response to a positive monetary policy shock in the case in which the financial friction parameter μ is set to 0.001. As shown by the two upper panels of Figure 8, in this limiting case, the change in cash holding S is almost exactly proportional to the change in M. In terms of the portfolio allocation constraint (3), this implies that the shock has a negligible impact on quantity deposited in the banking sector and thus on credit creation. As shown by the response of output in the lower right panel of Figure 8, the monetary policy shock is almost neutral in this case, which reflects that a change in lending conditions has virtually no real effects when the fraction of costs that needs to be paid in advance becomes negligible.

The importance of the labor wedge

Whereas a variation in the cost of lending directly affects the demand for both capital and labor, the transmission mechanism operates primarily through the labor market. The importance of the labor market can be illustrated by considering the case in which the constraint only depends on capital. If wages do not need to be paid in advance, the loan-in-advance constraint takes the following form:

$$\frac{L_t}{P_t} \geq \mu r_{Kt} k_t$$

and a monetary policy shock no longer affects the labor demand equation.

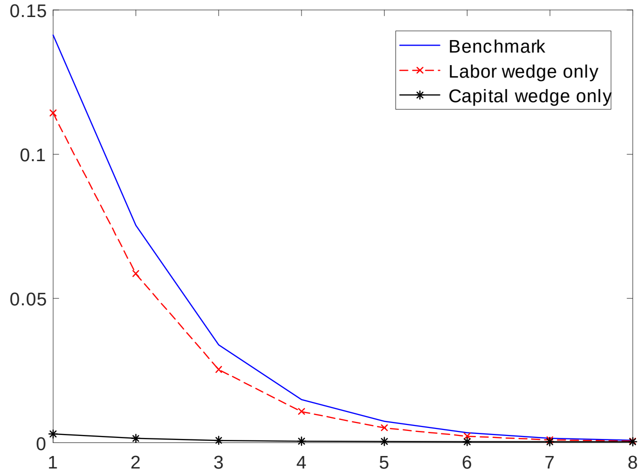

Figure 9. Response of output to a one standard monetary policy shock in the three different models. y axis: percentage deviation from steady state. x axis: quarter after the shock.

The difference between the black dotted and blue continuous lines in Figure 9 shows how removing the wage component from the loan-in-advance constraint would affect the response of output to a monetary policy shock. This comparison demonstrates that, without a labor wedge, the model loses most of its ability to generate monetary non-neutralities. This dramatic reduction in the effectiveness of monetary policy is mainly due to the lower credit share of output in the version of the model in which labor costs do not need to be paid in advance, which in Figure 9 is referred to as the capital wedge model. Since the capital share represents one third of output, compared to two thirds for the labor share, a much lower quantity of credit is needed in the capital wedge model.

The red dashed line shows the case in which the loan-in-advance constraint only depends on labor costs:

$$\frac{L_t}{P_t} \geq \mu w_t n_t$$

Since a lower quantity of credit is needed to operate firms when capital costs are not paid in advance, relative to the benchmark model that reproduces the facts shown in Table 2, the model loses some of its ability to generate monetary non-neutralities. Whereas the effect of monetary policy is stronger when both margins are included, this comparison nevertheless illustrates that the labor wedge remains the main channel through which monetary policy is transmitted to the real economy.

6 The state dependence of monetary policy transmission

Another interesting result implied by this mechanism is that the effects of monetary policy depend on the state of the economy. Since monetary policy is transmitted to the real economy by affecting firms' borrowing costs, the transmission mechanism critically depends on the central bank's ability to control lending rates. By varying the quantity of liquidity it supplies, the central bank has a direct impact of the deposit rate, which in turn affects banks' lending rates. This ability to control market rates however depends on how agents react to a change in the monetary policy stance. In particular, the fact that households can choose between depositing money in the banking sector or keeping money in cash interferes with the conduct of monetary policy. This can be explained by showing the equilibrium condition in the money market, which implies the following non-linear relationship between the deposit rate i_D and the quantity of money held by households in the form of cash balances:

$$i_{Dt} = \frac{1-\kappa}{\kappa}\frac{P_t c_t}{S_t}$$

Keeping P and c constant at their average value, and using the calibration summarized in Table 1, this relationship is plotted in Figure 10, which shows i_D on the vertical axis as a function of cash balances S. For the calibration that reproduces the moments shown in Table 2, the average short-term nominal money market rate i_D, in annualized terms, stands at 2.1%. As can be inferred from the blue diamonded line an average interest rate of 2.1% implies a quantity of money held in cash of about 0.05. In terms of the portfolio allocation, this calibration implies that households hold on average 5% percent of their liquid wealth in cash and the remaining 95% in the banking sector in the form of bank deposits.

An interesting consequence of this inverse relationship between cash holdings and the money market rate is the potential asymmetry that it entails. Indeed, for very low levels of the money market rate, the demand for cash flattens and converges towards one. What this means is that households can always choose to withdraw money from their deposit account and keep money in cash, if the remuneration on their deposit account is too low. In the limit, the share of money invested in cash reaches 100%, as the deposit rate i_D approaches zero.

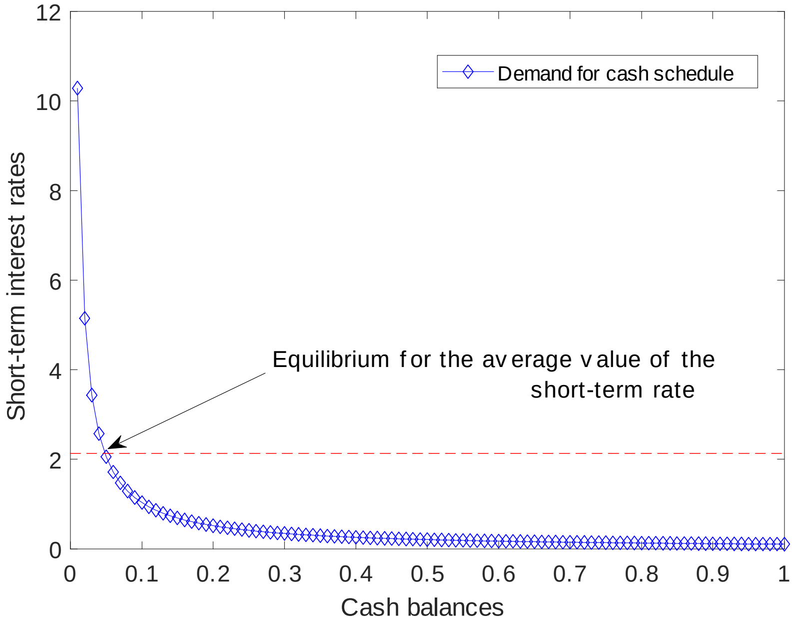

Figure 10. Demand for cash schedule.

This non-linear relationship formalizes the idea that the existence of cash creates a zero lower bound on deposit rates that constrains the transmission of monetary policy. To gain intuition into this result, recall that a proportional decline in the amount deposited in the banking sector D is a necessary by-product of any increase in cash holdings. In other words, the possibility to hold cash instead of bank deposits limits the room for manoeuvre of the central bank by preventing deposit rates from turning negative. Indeed, households will always strictly prefer to hold cash rather than incur a cost for depositing their money in the banking sector.

Consider for instance the case of a very aggressive injection of liquidity occurring during a period of low interest rates. Clearly, as illustrated by the non-linear relationship between i_D and S depicted in Figure 10, once the flatter portion of the cash demand curve is reached, any further increase in liquidity only has a small effect on deposit rates. In terms of portfolio allocation, this reflects that the supply of deposits becomes insensitive to changes in the policy instrument M, when the flat portion of the demand curve is reached. In this case, any further increase in M is absorbed by a proportional increase in cash holdings. And if the amount of deposits remains broadly unaffected, a change in monetary policy only has a muted effect on the real economy. In this case, since the opportunity cost of holding cash is so low, any further increase in M is absorbed by a proportional increase in cash holdings S.

Deconstructing the mechanism

To better understand how the demand for cash affects the transmission mechanism of monetary policy, it is useful to illustrate how initial credit conditions affects the effectiveness of a monetary policy shock. The equilibrium in the credit market is represented in Figure 11 where the credit supply and credit demand curves given by equations (20) and (21) are depicted. In both panels, the red dotted lines represent the demand curves corresponding to equation (20). These two demand curves are obtained by using the value for the parameter μ taken from Table 1 and by fixing P, c and y at their average values.

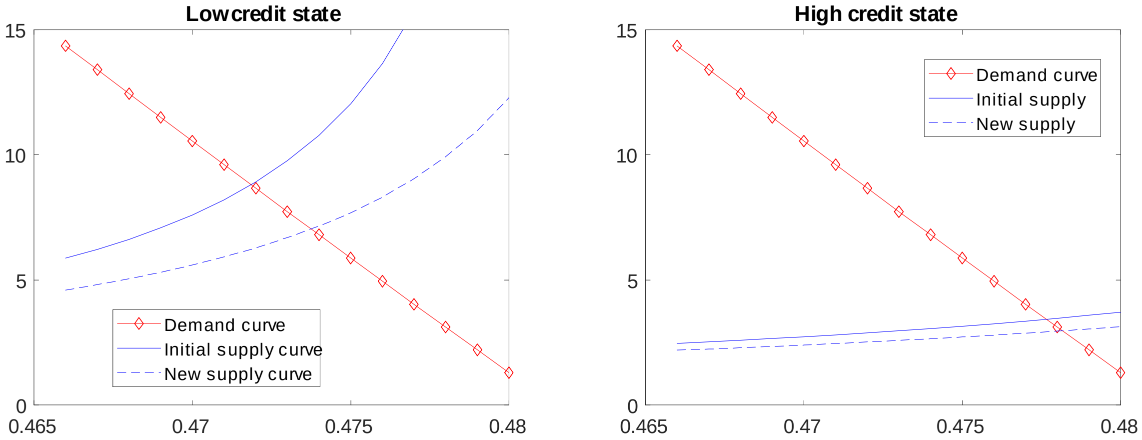

Figure 11. Equilibrium in the credit market in response to a 1% change in money supply. x axis: credit. y axis: bank lending rate in annual terms.

In the left panel, the blue continuous line shows the credit supply curve in the case in which the quantity of money available in the economy M stands below its average value. When monetary conditions are tighter than usual, the credit supply curve is also steeper. This in turn implies a higher average lending rate and a lower equilibrium quantity of credit. The blue dashed line shows how a 1% increase in money supply shifts the credit supply curve when the economy is in this state of below average equilibrium quantity of credit.

In the right hand-side panel, the blue continuous line represents an initial credit supply curve that corresponds to a case in which the quantity of money available in the economy is higher than average. The initial state of the economy is therefore one in which the equilibrium quantity of credit available in the economy is above average. The blue dashed line illustrates how a 1% increase in the money stock shifts the credit supply curve in this case.

As can be seen by comparing the blue continuous lines across the two panels of Figure 11, the first difference is that the initial supply curve is flatter in the high credit state. Second, a 1 percent shock to money supply, which in each panel is determined by the

distance between the blue continuous and blue dashed lines, has a much smaller effect on equilibrium quantities and prices in the high money, high credit state. The much lower effect obtained in the high credit state illustrates the limits of monetary policy in a world in which agents can choose to hold cash. When the flat portion of the cash demand curve shown in Figure 10 is reached, any further increase in money supply is hoarded instead of being deposited in the banking sector. The small effect of monetary policy on amounts deposited in the banking sector can be explained by the low opportunity cost of holding cash when the deposit rate paid to households becomes sufficiently low. Since without additional funding the banking sector is in turn unable to increase lending to the productive sector, the credit supply curve becomes insensitive to changes in monetary policy. As a result, and as can be seen by comparing the blue continuous and blue dashed lines shown in the right panel, a 1% change in money supply only has tiny effect on the credit supply curve in this case.

The low effectiveness of monetary policy obtained in the high credit state contrasts with the much stronger effect depicted in the left panel, which corresponds to the low credit state. In terms of the cash demand curve depicted in Figure 10, the difference is that the demand for cash schedule becomes steeper when monetary conditions are tighter than average. As illustrated in Figure 10, the key element is that the demand for cash S becomes less sensitive to variations in interest rates when the opportunity cost of hoarding cash is higher. In other words, any change in money supply M has a stronger effect on deposits D in this case because the demand for cash S becomes less sensitive to monetary policy. Since a change in the deposit base in turn shifts the credit supply curve, the effectiveness of monetary policy increases when the incentive to hoard cash rather than deposit money in the banking sector declines.

Impulse response analysis

Given that the model studied so far can match a relatively large set of moments, the next step is to provide a quantitative evaluation of the non-linearity implied by this mechanism. Using the outcome of the calibration summarized in Table 1, this section studies the response of output and real credit to a one standard deviation monetary policy shock by differentiating between periods of low and high credit availability. This is achieved by firstly simulating 10'000 different trajectories for real credit L/P by drawing a corresponding number of realizations for the two exogenous shocks. Since each trajectory depends on a sequence of random shocks, the result of this simulation can be used to generate a distribution of realized values for the short-term rate in a given period, say period 100. Following Adjemian et al. (2014) an impulse response is then computed as the difference between the series to which a one standard deviation monetary policy shock is added and the initial simulated path. Periods of low and high interest rates can then be identified by selecting the set of impulse response that are computed from points in the state space where the equilibrium quantity of credit is higher or lower than average.[6]

In the left panel of Figure 12, the red dotted line shows the impulse response of the short-term rate to a positive money supply shock in a low interest rate environment. The credit stress state corresponds to impulse responses that were computed in states of the economy in which credit lies within the lower quartile of the distribution. In other words, the blue continuous line represents the average impulse response obtained in states of the economy in which the quantity of credit in the economy is less than or equal to 25% of all realized observations. Similarly, the red dotted line shows the average impulse response to a positive one standard deviation shock in the ample credit state. I define the high credit state as a case in which the realized value for credit is included in the last quartile of the distribution, which corresponds to values greater than or equal to 75% of all realizations.

When looking at Figure 12, what immediately stands out is the asymmetry in the response to a monetary policy shock between the high and low credit states. On impact, the exact same monetary policy shock generates an increase in the real quantity of credit of around 0.1 percent in the high credit state, as compared to an increase that reaches 0.7% in the credit stress regime. Interestingly, and as illustrated by the right panel, this asymmetric response of credit market variables also has a significant impact on the real economy. Indeed, during periods of ample credit availability, a one standard deviation positive monetary policy shock only has a negligible effect on output, which increases by around 0.025% on impact. By contrast, monetary policy is much more effective during periods of credit crunches. As depicted by the blue continuous line in the right panel of Figure 12, the exact same monetary policy shock increases output by almost 0.2% in this

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[6] The impulse response analysis shown in this section has greatly benefitted from comments and suggestions from M. Juillard whose help is gratefully acknowledged. Any remaining error is my own responsibility.

case. We can therefore conclude that the non-linear relationship between money, interest rates and credit illustrated in Figures 10 and 11 is a significant source of state dependence in the transmission mechanism of monetary policy.

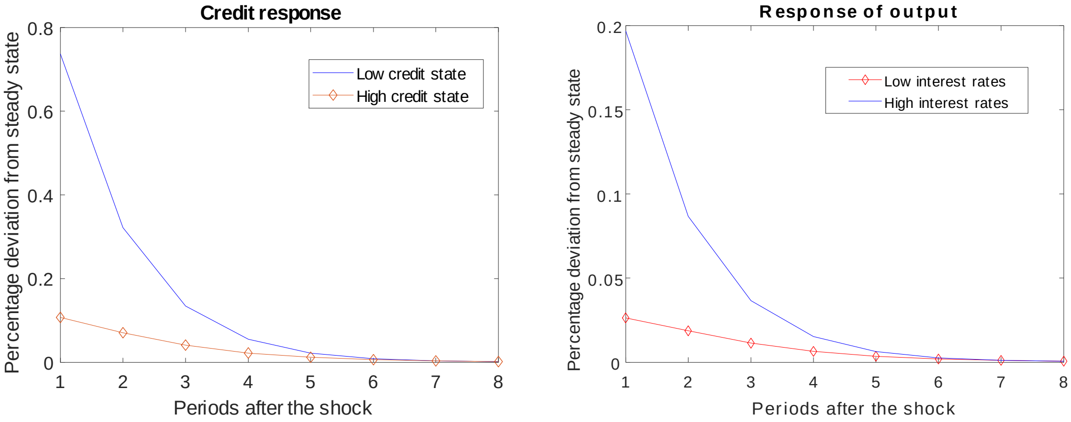

Figure 12. Response of the short-term rate and output in a low and high interest rate environment.

Notice that this non-linearity can also be interpreted from the perspective of the representative firm. In the non-financial sector, firms' ability to operate crucially depends on access to credit, which in turn depends on the tightness of the loan-in-advance constraint given by equation (10). In this model, the Lagrange multiplier associated to this constraint, which is denoted by ϖ, therefore provides a measure of ease of access to credit.

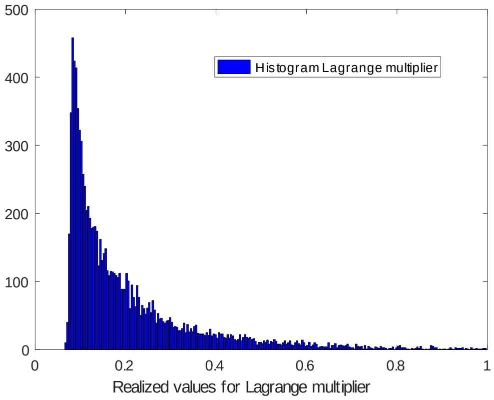

Figure 13. Lagrange multiplier associated to loan-in-advance constraint.

Another way to interpret the non-linearity illustrated in Figure 11 is to analyze how this Lagrange multiplier behaves over the business cycle. As shown in Figure 13, which plots the histogram of realized values for ϖ using a sample of 10'000 simulated observations, this multiplier always takes values that are strictly positive. The strong asymmetry in the distribution of ϖ reflects that there is a limit to which monetary policy can be effective in alleviating the effects of credit constraints. By contrast monetary policy is particularly effective during periods of credit stress because the increase in liquidity occurs precisely when firms encounter difficulties in obtaining financing. The relief provided by an expansionary monetary policy shock is therefore much stronger when the loan-in-advance constraint is tighter, which in turn implies larger real effects in periods of credit stress than in periods of ample credit availability.

7 Conclusion

This paper studies monetary non-neutralities stemming from credit frictions in a model also able to generate an upward-sloping yield curve and that reproduces a term premium of the magnitude observed in the data. In terms of policy implications, one main takeaway is that this channel is more likely to matter during periods of credit stress and high lending spreads. Although I calibrate the model using euro area data, one limitation of the analysis is that the cross-country heterogeneity observed in the Eurozone is not taken into account. Indeed, there are good reasons to think that the strength of this channel is very likely to differ across Eurozone economies. The deterioration in credit standards observed in the euro area during the Subprime and Sovereign Debt crises was for instance much more severe is Southern European economies. This heterogeneity in credit conditions was also reflected by the large dispersion in lending rates observed across European regions at the height of the Sovereign debt crisis (e.g. Neri, 2013).

As for the transmission mechanism, the main takeaway is that credit frictions can create a channel of monetary policy that is independent from price stickiness. Since the effectiveness of this channel critically depends on the tightness of credit constraints, the effects of monetary policy are strongly state-dependent. It is however necessary to use a nonlinear solution method to detect this asymmetry. My results therefore suggest that higher-order terms in the Taylor expansion could play a more important role in monetary economics than previously thought (e.g. Fernández-Villaverde et al., 2015).

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9 Data Appendix

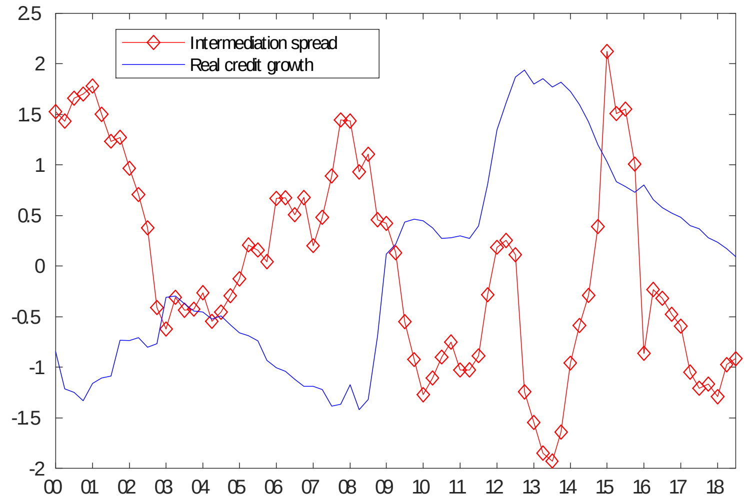

Figure A.1. Real credit growth and intermediation spreads in the Eurozone.

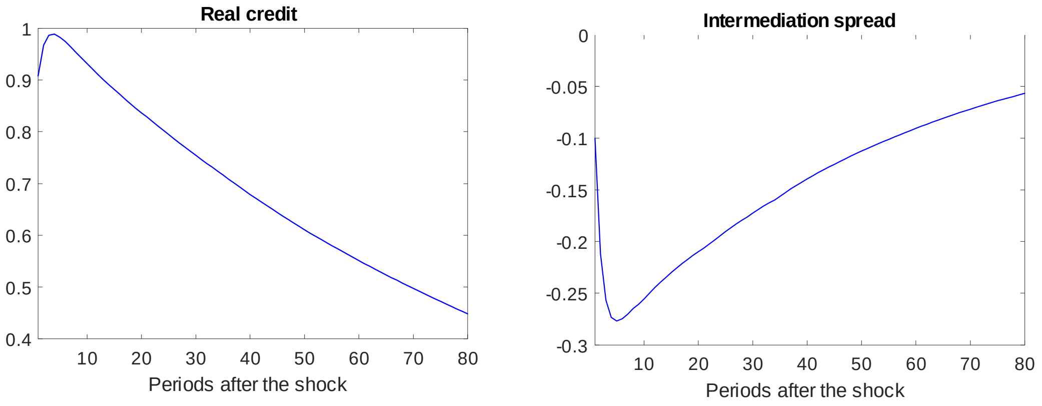

Figure A.1. Response of real credit and the intermediations spread to a positive technology shock.

Data appendix to section 3

Variables	Description	Source
Output, y	Real GDP	Stat. Office of the EC
	EA 19, mio of chained 2010 Euros	1995q1-2018q4
Consumption, c	Final consumption expenditures	Stat. Office of the EC
	EA 19, mio of chained 2010 Euros	1995q1-2018q4
Investment, x	Gross capital formation	Stat. Office of the EC
	EA 19, mio of chained 2010 Euros	1995q1-2011q4
Money supply, M	M1 Money supply adjusted for	ECB
	reclassifications, EA 11-19	1997q3-2018q4
Deposit, D	Total deposits of Euro area households	ECB
	amount outstanding	1997q3-2018q4
Price level, P	Harmonized index of consumer prices	ECB
	2015=100, EA 11-19	1997q1-2018q4
Monetary policy rate, i_D	3-month deposit (EURIBOR)	ECB
	EA 11-19	1995q1-2018q4
Long-term rate, i_{10Y}	10-year government benchmark	ECB
	bond yield, EA 11-19	1995q1-2018q4
Term premium, $E(tp)$	Based on a shadow-rate term structure	Wu and Xia (2017)
	model for Europe	2005m7-2017m6
Slope yield curve, $i_{10Y} - i_D$	EURIBOR 3-month deposit minus	ECB
	10-year government yield	1995q1-2018q4
Intermediation spread, $i_L - i_D$	Rate on new loans to non-financial	ECB
	corporations (less than 1 mio, less	2000q1-2018q4
	than a year) minus 3-month deposit	
Loan-to-output ratio, l/y	Credit to non-financial corporations	BIS
	% of GDP, market value	1997q3-2018q4

10 Technical Appendix

10.1 The competitive equilibrium

Households

$$\max_{c_t,S_t,B_{t+1},n_t,M_{t+1},x_t,h_{t+1},k_{t+1}} E_0 \sum_{t=0}^{\infty} \left(\beta_t\right)^t \frac{\left(c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}\left(\psi+z_t^{\upsilon}\right)-h_t\right)^{1-\sigma}}{1-\sigma}$$

where:

$$\beta_t = \widetilde{\beta}_t \gamma^{1-\sigma}$$

such that:

$$\gamma \frac{M_{t+1}}{P_t} = \frac{S_t}{P_t} + \frac{D_t}{P_t}$$

$$1 = n_t + z_t$$

$$tr_t + prof_{Tt} + w_t N_t + i_{Dt}\frac{D_t}{P_t} + r_{Kt}k_t + \frac{B_t}{P_t} = c_t + x_t + \gamma\frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} + \frac{1}{1+i_{Bt}}\gamma\frac{B_{t+1}}{P_t}$$

$$\gamma k_{t+1} = (1-\delta)k_t + \left(\frac{\theta_1}{1-\epsilon}\left(\frac{x_t}{k_t}\right)^{1-\epsilon} + \theta_2\right)k_t$$

$$\gamma h_{t+1} = \tau h_t + (1-\tau)c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}\left(\psi+z_t^{\upsilon}\right)$$

The Lagrangian:

$$L = E_0 \left\{ \sum_{t=0}^{\infty} \left(\beta_t\right)^t \frac{\left(c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}\left(\psi+z_t^{\upsilon}\right)-h_t\right)^{1-\sigma}}{1-\sigma} \right.$$

$$+\sum_{t=0}^{\infty}\left(\beta_t\right)^t\lambda_t\left[\begin{array}{c} tr_t+prof_{Tt}+w_tN_t+i_{Dt}\left(\gamma\frac{M_{t+1}}{P_t}-\frac{S_t}{P_t}\right)+r_{Kt}k_t+\frac{B_t}{P_t}-c_t-x_t \\ -\left(\gamma\frac{M_{t+1}}{P_t}-\frac{M_t}{P_t}\right)-\frac{1}{1+i_{Bt}}\gamma\frac{B_{t+1}}{P_t} \end{array}\right]$$

$$+\sum_{t=0}^{\infty}\left(\beta_t\right)^t q_t\lambda_t\left[(1-\delta)k_t+\left(\frac{\theta_1}{1-\epsilon}\left(\frac{x_t}{k_t}\right)^{1-\epsilon}+\theta_2\right)k_t-\gamma k_{t+1}\right]$$

$$+\sum_{t=0}^{\infty}\left(\beta_t\right)^t\varphi_t\left[\tau h_t+(1-\tau)c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}\left(\psi+z_t^{\upsilon}\right)-\gamma h_{t+1}\right]\right\}$$

Banks

Banks maximize profits, which are given by:

$$\pi_t^F=i_{Lt}\frac{L_t}{P_t}-i_{Dt}\frac{D_t}{P_t}$$

such that:

$$L_t=\vartheta D_t$$

Firms

$$\max_{k_t,N_t,L_t} prof_{Ft}=A_tk_t^{\alpha}n_t^{1-\alpha}-w_tn-r_{Kt}k_t-i_{Lt}\frac{L_t}{P_t}$$

such that:

$$\frac{L_t}{P_t}\geq\mu\left(w_tn_t+r_{Kt}k_t\right)$$

The Lagrangian:

$$\mathcal{L}=E_0\sum_{t=0}^{\infty}\left(\beta_t\right)^t\frac{\lambda_t}{\lambda_0}\left\{\begin{array}{c} A_tk_t^{\alpha}n_t^{1-\alpha}-w_tn_t-r_{Kt}k_t-i_{Lt}\frac{L_t}{P_t} \\ +\frac{\varpi_t}{\lambda_t}\left[\frac{L_t}{P_t}-\mu\left(w_tn_t+r_{Kt}k_t\right)\right] \end{array}\right\}$$

Market clearing condition

$$A_tk_t^{\alpha}n_t^{1-\alpha}=c_t+x_t$$

10.2 The dynamic system

$$\left\{\left[c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}(\psi+z_t^{\upsilon})-h_t\right]^{-\sigma}+(1-\tau)\varphi_t\right\}\kappa c_t^{\kappa-1}\left(\frac{S_t}{P_t}\right)^{1-\kappa}(\psi+z_t^{\upsilon})=\lambda_t$$

$$\left\{\left[c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}(\psi+z_t^{\upsilon})-h_t\right]^{-\sigma}+(1-\tau)\varphi_t\right\}c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}\upsilon z_t^{\upsilon-1}=w_t\lambda_t$$

$$i_{Dt}\frac{S_t}{P_t}=\frac{1-\kappa}{\kappa}c_t$$

$$\gamma\lambda_t q_t=\beta_t E_t\lambda_{t+1}q_{t+1}\left[(1-\delta_K)+\frac{\theta_1}{1-\epsilon}\left(\frac{x_{t+1}}{k_{t+1}}\right)^{1-\epsilon}+\theta_2-\theta_1\left(\frac{x_{t+1}}{k_{t+1}}\right)^{1-\epsilon}\right]+\beta_t E_t\lambda_{t+1}r_{t+1}$$

$$1=q_t\theta_1\left(\frac{x_t}{k_t}\right)^{-\epsilon}$$

$$\gamma\frac{\lambda_t}{P_t}=\beta_t E_t\lambda_{t+1}\frac{1}{P_{t+1}}+\gamma\frac{\lambda_t}{P_t}i_{Dt}$$

$$\gamma\frac{\lambda_t}{P_t}\frac{1}{1+i_{Bt}}=\beta_t E_t\lambda_{t+1}\frac{1}{P_{t+1}}$$

$$\gamma M_t=S_t+D_t$$

$$\varphi_t=\tau\beta_t E_t\varphi_{t+1}-\beta_t E_t\left(c_{t+1}^{\kappa}\left(\frac{S_{t+1}}{P_{t+1}}\right)^{1-\kappa}(\psi+z_{t+1}^{\upsilon})-h_{t+1}\right)^{-\sigma}$$

$$(1-\delta)k_t+\left(\frac{\theta_1}{1-\epsilon}\left(\frac{x_t}{k_t}\right)^{1-\epsilon}+\theta_2\right)k_t-\gamma k_{t+1}=0$$

$$\tau h_t+(1-\tau)c_t^{\kappa}\left(\frac{S_t}{P_t}\right)^{1-\kappa}(\psi+z_t^{\upsilon})-\gamma h_{t+1}=0$$

$$\beta_t=\beta-(e_{t+1}-e)$$

$$e_{t+1}=\varsigma e_t+n_t$$

$$L_t = \vartheta D_t$$

$$i_{Lt} = \frac{1}{\vartheta} i_{Dt}$$

$$y_t = a_t k_t^{\alpha} N_t^{1-\alpha}$$

$$\frac{L_t}{P_t} = y_t \frac{\mu}{1 + \mu \frac{\varpi_t}{\lambda_t}}$$

$$w_t = (1-\alpha) \frac{y_t}{n_t} \frac{1}{1 + \mu \frac{\varpi_t}{\lambda_t}}$$

$$r_t = \alpha \frac{y_t}{k_t} \frac{1}{1 + \mu \frac{\varpi_t}{\lambda_t}}$$

$$i_{Lt} = \frac{\varpi_t}{\lambda_t}$$

$$y_t = c_t + x_t$$

$$M_t = \overline{M} + \rho_M \left(M_{t-1} - \overline{M} \right) - \phi_\pi (P_t - P) + \varepsilon_M$$

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_a$$