Potential Output Pessimism and Austerity in the European Union*

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Abstract

EU fiscal policymakers' estimates of potential output appear to be pro-cyclical and too sensitive to latest economic data. The Great Recession led to over-pessimism about potential output which amplified adverse shocks and underestimation of structural balance triggering fiscal austerity. A business cycle model capturing this feature is developed with incorporating the policymaker's learning about potential output. It replicates the pattern of revisions to estimates of potential output, output gap and structural balance made by EU fiscal policymakers as well as several new evidence on fiscal consolidation and its relation to economic performance we document. The mutual reinforcement between pessimism and fiscal austerity may have contributed markedly to the prolonged recession. Placing less weight to new data (compared to actual EU policy makers) may have reduced real-time mis-measurement of output gap and led to a less severe recession.

1 Introduction

In the wake of the 2008 Global Financial Crisis, the euro area experienced a long economic recession. Figure 1 plots real GDP per capita for the euro area, United States, and Japan.[1] From 2011, real GDP per capita in the euro area declined further for a few more years after the initial slump. It took nearly three more years for the euro area real GDP per capita to rebound to its 2007 pre-crisis

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[1] The data is from IMF World Economic Outlook Database, April 2018.

level than the two other economies. The real GDP per capita for the euro area in 2017 is only 3.4% higher than its 2007 pre-crisis level, while the corresponding number for the United States and Japan are 6.5% and 6.1% respectively.

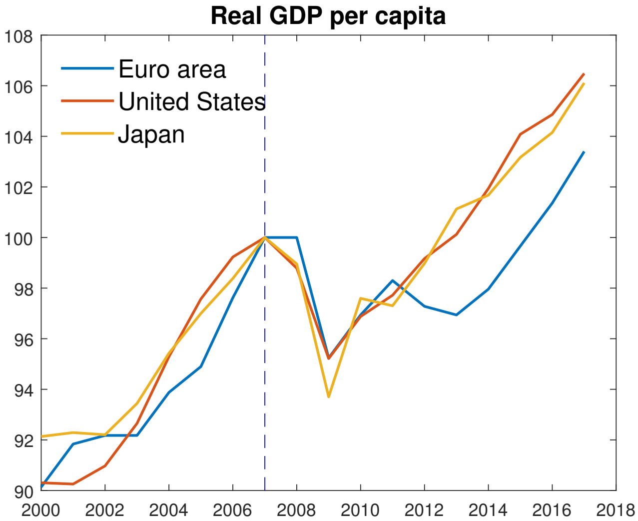

Figure 1: Real GDP per capita. Index 2007 =100.

Why has the euro area experienced a more prolonged recession and slow recovery? The paper develops a formal economic model and offers an explanation. We argue in the wake of the Great Recession, estimates of potential output and structural fiscal balance made by EU fiscal policymakers were overly pessimistic. Mis-measurement in fiscal stance subsequently led to undue austerity which in turn reinforced the pessimism and contributed markedly to the long recession. Placing less weight to new data (compared to actual EU policymakers) to form potential output estimates would have reduced real-time mis-measurement of structural balance and led to a less severe recession.

Since the 2005 reform of the Stability and Growth Pact, structural budget balance – the budget balance-to-GDP ratio that would prevail if the economy was at potential – has become the main indicator of the fiscal stance of member states and plays a central role in EU fiscal policy surveillance. Fiscal policymakers try to target certain level of structural balance. In practice, the European Commission (EC) computes structural balance and makes recommendations on if and how much fiscal austerity is needed for member states primarily based on structural balance estimates. Estimates of structural balance depend crucially on real-time estimates of the output gap which are usually unreliable.

The paper provides new evidence showing the estimates of potential output and structural balance made by the EC appear to be pro-cyclical and too sensitive to latest economic data; see Section 2.2 and 3 for details. Estimates of potential output and structural balance were revised massively downward in the aftermath of the Great Recession and subsequently revised upward during the recovery phase, particularly for crisis countries, such as Spain, Portugal, Ireland and Greece. There have been large and corresponding revisions to estimates of output gaps and structural balance made by the EC.

The over-pessimistic estimate of structural balance following the financial crisis is not innocuous as it translates into excessive requirement on member states to reduce structural deficits. First, we find a country with a higher level of the estimate of structural deficits made by the EC in the aftermath of the Great Recession tends to associate with fiscal consolidation of a bigger size and faster pace subsequently. Second, across these EU countries, fiscal consolidation is strongly negatively associated with changes in real-time estimates of potential output growth rates. A country with a larger fiscal consolidation tends to exprience larger declines in estimates of its potential output growth rates. Third, we show a strong negative cross-country correlation between fiscal consolidation and subsequent growth forecast errors during 2010 − 2012, following Blanchard and Leigh (2013) (but using EC data). A country which plans a larger fiscal consolidation tends to have lower growth than expected subsequently. These empirical findings are robust to using either actual changes in structural balance or forecasts of changes in structural balance as the measure of fiscal consolidation.

Despite the central role for structural balance targeting in the EU fiscal policy framework, academic literature rarely studies this major policy in the context of macroeconomic models. Especially, structural balance is rarely analyzed in macroeconomic models. The paper contributes to the literature by developing a business cycle model to analyze the implications of mis-measuring potential output and structural balance for fiscal policy decisions and the EU recession. In our model, the policymaker faces real-time uncertainty about potential output and forms subjective estimates of potential output and structural balance via learning. Fiscal decisions are made based on these estimates, which in turn influences the macro economy. Using new data observations, the policymaker continually revises these estimates.

In the model, the policymaker and households over-react to sustained declines in aggregate output following the Great Recession with too aggressive downward revisions to potential output. The learning model generates secular-stagnation-like slowdown according to the policymaker's subjective estimates of potential output. Pessimistic beliefs feed back to the economy and amplify the decline in aggregate activities. Crucially, pessimistic estimates of potential output made by the policymaker leads to underestimation of the size of the output gap and structural fiscal balance which in turn triggers undue fiscal austerity due to targeting structural balance in the EU fiscal policy framework.[2] According to our benchmark calculation, the size of the underestimation of structural balance for the eurozone as a whole reaches 3.8% in the aftermath of the Great Recession. Fiscal austerity leads to further downward revision to estimates of potential output and magnifies the underestimation of structural balance. This mutual reinforcement between potential output pessimism and undue fiscal austerity contributes significantly to the recession. Our benchmark calculation suggests a output loss of 3.1% per year for the Eurozone during the fiscal consolidation

[2] By undue fiscal austerity we do not mean relative to an optimal benchmark but rather relative to a situation when there is no mis-measurement of the output gap.

period.

The model features a negative feedback loop created by the dynamic interaction between pessimism about potential output and structural balance, fiscal austerity and the macroeconomy. It generates a realistic magnitude of government spending multiplier of roughtly 2 in line with e.g. House, Proebsting and Tesar (2019). Importantly, the model captures the pattern of the EC's revisions to estimates of potential output, output gap and structural balance. Moreover, it helps to replicate the cross-country correlation between fiscal consolidation and changes in potential output estimates as well as the correlation between fiscal consolidation and growth forecast errors.

Since policymakers have imperfect knowledge, mis-measurement of the output gap (and structural balance) in real time is inevitable. However, policymakers have strong motivation to minimize these mis-measurements since they may otherwise magnify the adverse consequences for the economy. The question arises as to how policymakers may minimize these real-time mis-measurements. In the learning model, the size of mis-measurement of output gap and structural balances depend on the extent to which the policymaker discounts past data when forming estimates of the trend output (this is captured by what is called *gain* parameter in the learning literature).

The policymaker's gain parameter is calibrated to match the (large) revisions to estimates of potential output following the Great Recession and hence is relatively large (i.e., 0.034). In practical terms, the magnitude of the gain parameter indicates the weight attached to historical data so a large gain means more weight is attached to recent data. For example, the gain parameter of 0.034 means the weight received by output growth data 20 years ago, is $(1-0.034)^{80} \simeq 0.063$. In contrast, the model suggests the policymaker should place a more modest weight on recent data points: this gain parameter is approximately 0.0045 and the weight received by output growth data 20 years ago is 0.70 (much larger number). If policymakers had placed less weight to the data during the Great Recession period, this would have led to higher estimates of structural balances, less fiscal austerity and a less severe recession.

Kuang and Mitra (2016) provided evidence on pro-cyclical estimates of US potential (or long-run) output growth using different sources of data. Some of the analysis in Section 3 below extends our earlier results. Kuang and Mitra (2016) also developed a business cycle model with learning which replicates this evidence and suggests a crucial role for pro-cyclical long-run growth forecasts in business cycle fluctuations.

Several recent papers provide evidence supporting the view that real-time estimates of potential output after the crisis were overly pessimistic. Jarocinski and Lenza (2018) estimate and compare various versions of dynamic factor models, i.e., Philips curve-type relationship linking the output gap with inflation. They find that the model which best predicts euro area inflation suggests no slowdown in trend output growth rate after 2011 and large under-estimation of output gaps in real time by policymaking institutions. For instance, their estimate of 2014 eurozone output gap reaches -6% which is twice as negative as the official estimates of international institutions.[3] Coibion,

[3] Some policymakers, such as Benoît Coeuré, Member of the Executive Board of the ECB, shared the same view that real-time estimates of potential output after the crisis were overly pessimistic; see Coeuré (2018).

Gorodnichenko, Ulate (2018) show that estimates of potential output in the US and other OECD countries made by a wide range of public institutions and private forecasters respond sensitively to shocks which have only transitory effects on GDP.[4] They propose a new method to estimate potential output which suggests a more limited decline in potential output following the Great Recession. Again, this has implications for estimates of output gaps. For example, their estimate of 2017 output gap in the US is close to -10%, while the corresponding estimate made by the US Congressional Budget Office is close to zero.[5]

The rest of the paper is organized as follows. Section 2 discusses the role for structural balance in EU fiscal policymaking, its measurement and associated problem; it uses Spain as an illustrating example. Section 3 discusses the pattern of revisions to policymakers' estimates of potential output, output gaps and structural balances. Section 4 presents new evidence on fiscal consolidation and its relation to macroeconomic performance in EU countries. The model setup is presented in Section 5. Agents' beliefs and learning are introduced in Section 6. Section 7 simulates the Great Recession scenario and illustrates the consequences of undue fiscal consolidation. Section 8 provides country-specific simulations which quantitatively replicate a large number of evidence. Section 9 examines a way to minimize real-time mis-measurement of output gaps and conducts a counterfactual analysis. Section 10 concludes.

2 Structural balance in EU fiscal policy framework

Structural budget balance has become the main indicator of the fiscal stance of EU member states and plays a central role in EU fiscal policy surveillance since 2005. Policymakers try to target certain level of structural balance. Following the European Sovereign Debt Crisis, the effort of fiscal discipline has been strengthened. Twenty-five EU member states signed the Treaty on Stability, Coordination and Governance in the Economic and Monetary Union to incorporate a 'debt brake' into their national law or constitution.[6] The fiscal component of the treaty is also called "Fiscal Compact". It entered into force on 1 January 2013 and requires the structural balance ratio for EU member states to be at least -1% (or -0.5%) if the public debt-to-GDP ratio is at most (or above) 60%; see European Parliament (2017). This rule is the main element of the preventive arm of the EU fiscal policy framework.

2.1 Structural balance: measurement and concern

In practice, the EC computes structural balance in real time and makes recommendations on if and how much fiscal austerity is needed for member states based on the estimate. Government budget

[4] Similar observations are made in Dovern and Zuber (2019).

[5] Fatas (2019) argue policymakers were overly pessimistic about potential output in the aftermath of the Great Recession and discusses the feedback loop between mis-measuring potential output and fiscal austerity.

[6] The debt brake applies not only to eurozone countries but also non-eurozone countries. The UK and Czech Republic are not signatories of this treaty. They are nevertheless required by Stability and Growth Pact to achieve similar targets for structural balance.

balance GDP ratio contains a structural and cyclical component. The structural component is obtained by adjusting the actual balance using real-time estimates of the output gap (OG_t). Output gap is defined as deviations of output from its trend. As elaborated in Mourre et al (2013), the EC computes the cyclically-adjusted budget balance (CAB) as

$$CAB_t = BB_t/Y_t - \varepsilon * OG_t.^7 \tag{1}$$

BB_t/Y_t is general government budget balance (BB_t) to GDP ratio. ε is a cyclical sensitivity parameter which measures the sensitivity of CAB with respect to the business cycle. The budgetary semi-elasticity, ε, varies somewhat across EU countries (ranging from 0.3 to 0.61) and has an average value of 0.53; see Mourre et al (2013). Note CAB is lower (higher) than actual balance during expansions (recessions).

The EC employs a uniform methodology – the production function approach – as the main method to estimate potential output for member states, see Havik et al (2014) for a detailed explanation of the methodology.[8] This method uses a Cobb-Douglas aggregate production function assuming constant returns to scale and a factor price elasticity equal to one. To obtain the estimate of their trend component, statistical filtering methods (e.g., the HP filter) are applied to historical data on factor inputs such as total factor productivity and labor participation rate. The Kalman filter is used to obtain the level of employment consistent with non-accelerating (wage) inflation (NAWRU). The trend level of the factor inputs are then substituted into the production function to obtain the estimate of potential output.[9] Each year, the EC produces two vintages of estimates of potential output, output gap and structural balance. So the EC revises these estimates when there are new information or data observation about the macroeconomy.[10] A main problem of the method is that changes in aggregate output due to cyclical factors will automatically and partly be attributed to shifts in trend output.[11] Later the paper provides evidence illustrating that potential output estimates are pro-cyclical and too sensitive to new data.

There have been concerns about the implementation of the debt brake as real-time estimates of the output gap can be unreliable. Orphanides and van Norden (2002) illustrate real-time output gap estimates are unreliable and depend on the filtering methods used. Orphanides (2001, 2003, 2004) argues that the Federal Reserve's mis-measurement of the output gap in the 1970s was one

[7] In practice, structural budget balance ratio is measured as cyclically-adjusted budget balance ratio net of one-off and temporary measures. The latter is usually small and hence we omit it in our analysis and use structural balance and cyclically-adjusted budget balance interchangeably in our model later.

[8] Other institutions, such as IMF, OECD and the US Congressional Budget Office, also use a production function approach.

[9] An alternative method of the EC to estimate potential output is applying the HP filter directly to aggregate output. The EC publishes the estimation results produced by both methods. The former method provides more information about the sources of deviations of output from the trend.

[10] From 2016, the EC started to produce three vintages of estimates each year.

[11] Some observers express skepticism about the standard methodology for estimating potential output and suspect that estimates of potential output have over-reacted to the declines in output. For instance, Krugman (2013) notes "*the standard methods for estimating economic potential are working very badly in this slump; they are, all too often, causing officials to interpret the slump as 'structural'...*"

of the primary reasons for the big inflation in 1970s. In a European Central Bank occasional paper, Anderton et al (2014) find that the Great Recession led to a sign change to estimates of 2007 output gaps for a large number of countries: "*For all countries for which the 2007 output gap was estimated at the time to be negative – Portugal, Cyprus, Italy, Ireland, Malta, Spain, the Netherlands, France and Belgium – it was subsequently, in 2014, estimated to have been positive.*" Bundesbank (2014) warned of the high uncertainty of output gap estimates and expressed doubts on the suitability of such estimates in economic policy.

In the wake of the Great Recession, the EC's estimates suggest a large increase in structural deficits and violation of the fiscal rule for almost all EU countries. This required these countries to implement fiscal consolidation programmes to reduce structural deficits. Many observers expressed concerns about the consequence of under-estimating potential output for fiscal decisions. According to equation (1), mis-measurement in CAB is proportional to mis-measurement in the output gap. If the policymaker is too pessimistic about potential output (during a recession), she will underestimate the size of the output gap and structural balance, which may in turn lead to undue fiscal austerity for the member country concerned, i.e., excessive requirement on reducing structural deficits. The CPB Netherlands Bureau for Economic Policy Analysis (in Hers and Suyker (2014)) noted that "*The volatility of the structural balance is especially problematic because the indicator is used in the EU as a basis for the recommendations for a country. Volatile estimations could, for example, cause a government to be obliged to undertake significant additional fiscal consolidation in order to meet the structural balance requirement – only to be confronted with a structural balance that does not change at all, because of revisions in potential growth.*" In a March 2016 letter to the EC, the ministers of finance of eight EU member states expressed similar concerns and called for intensification of the technical work on this issue, see Letter (2016). Thus, EU fiscal policymakers have strong incentive to minimize real time mis-measurement of structural balance.

2.2 Revisions to potential output, output gap and structural balance: Spain as an example

Using Spain as an example, this section illustrates the evolution of the EC's estimates of potential output, the output gap and CAB. The next section provides more evidence. Figure 2 plots three vintages of estimates for four variables. Denote by X_t^i vintage i estimate of variable X in year t, where $i \in \{$Spring 2008, Spring 2012, Spring 2016$\}$ and $X \in \{$actual output growth rate, potential output growth rate, output gap, CAB$\}$. Note the figure plots estimates for both the past and future. For instance, $X_{2009}^{Spring\ 2008}$ is estimate (or forecast) of X in year 2009 made at Spring 2008.

Following the recession, both historical estimates and forecasts of Spanish potential output growth rates were revised downward, comparing the 2008 Spring vintage with the 2012 Spring vintage. Estimates of pre-crisis output gap (or CAB) are revised upward (downward) for almost all years. During the recovery phase, there have been big downward revisions to output gap and upward revision to CAB, associated with upward revisions to potential output growth rates. From 2012 Spring to 2016 Spring, the estimate of Spanish output gap in year 2011, 2012 and 2013 are revised from −3.76%, −4.44%, −3.59% to −5.53%, −7.46% and −8.28%, respectively; the estimate of Spanish CAB in year 2009 − 2013 are revised from −8.9%, −7%, −6.6%, −4.3%, −4.6% to −4.1%, 2.1%, 0.5%, 0.2%, 4%.

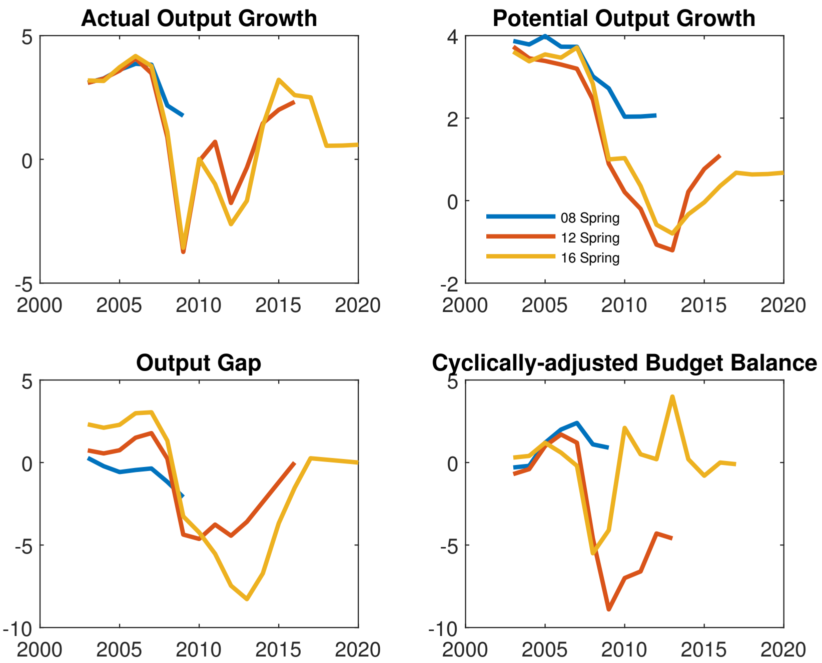

Figure 2: Spain actual output growth, potential output growth, output gap and CAB: three vintages

3 Evidence I: revision pattern

In the wake of the 2008 Global Financial Crisis, the observed persistent and large declines in aggregate output led to large downward revisions to the estimates of potential output in the Euro Area, the UK and the US by national and international institutions.[12] This section presents properties of policymakers' real-time estimates which we seek to replicate. The data is taken from the annual macroeconomic database published in *European Economic Forecast* by the EC and covers data vintages from 2004 Autumn to 2016 Autumn. This dateset is used for policy making purposes in the context of the EU fiscal compact and hence is particularly relevant. The dataset has been computed consistently across countries by the EC using a uniform estimation methodology for potential output.

[12]The large downward revisions to potential output for OECD countries is documented, e.g., in Ball (2014).

3.1 Evidence 1: how did the Great Recession change policymakers' view?

We document how the Great Recession changed policymakers' view on the evolution of three unobserved variables (potential output, ouput gap, and structural balance) for 12 EU countries. Figure 3 plots the revision to estimate of the growth rate of real potential GDP in $2003-2012$ from 2008 Spring to 2011 Spring. Estimates of potential growth rates were revised downward for almost all countries and all years. These estimates are produced by the same procedure (i.e., production function approach) for all countries and appear very sensitive to new data. Taking Ireland as an example, the estimates of annual potential output growth rate were revised downward by 3.1%, 5.6%, 4.7%, 4.4%, 3.7% for the year $2008-2012$ and by 0.14%, 0.69%, 1.15%, 1.56%, 2.03% for the years $2003-2007$.[13] Figure 3 also reveals heterogeneity in the size of downward revision to the growth rate of real potential output across countries for the period of $2008-2010$. For example, the downward revision to average annual growth rate of real potential output in Ireland over $2008-10$ is 4.5%, while the corresponding revision for Germany is 0.6%.

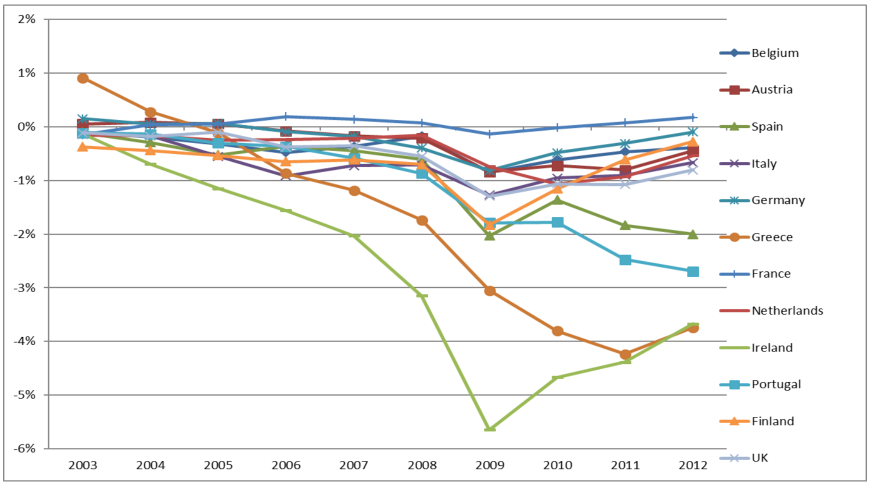

Figure 3: Revision to estimates of potential GDP growth rates: from 2008 Spring to 2011 Spring

Figure 4 shows that the size of the revision to the average growth rate of real potential output of a country during $2008-2010$ is strongly positively correlated with the severity of the recession during $2008-10$ measured by the average growth rate of real output over $2008-10$ minus that over a previous period (i.e., $2003-07$); the correlation coefficient is 0.86 and statistically significant at 1% level. A country which has a larger recession during $2008-10$ tends to have a larger downward revision to average growth rate of real potential output over $2008-10$.

[13]In the dataset, from 2008 Spring to 2011 Spring, potential growth rates are often revised without any revision to actual GDP growth data. For some years, there is data revision in actual growth but the size of the revision is much smaller than the revision in potential growth rates. This suggests only a small role for data revision in explaining the revision in potential growth rates in this sample. We, therefore, focus on the role for policy makers' uncertainty in estimating potential output in explaining the pattern of revisions in the three unobserved variables. Orphanides and Van Norden (2002) also find data revision is not the primary source of revision in measured output gap.

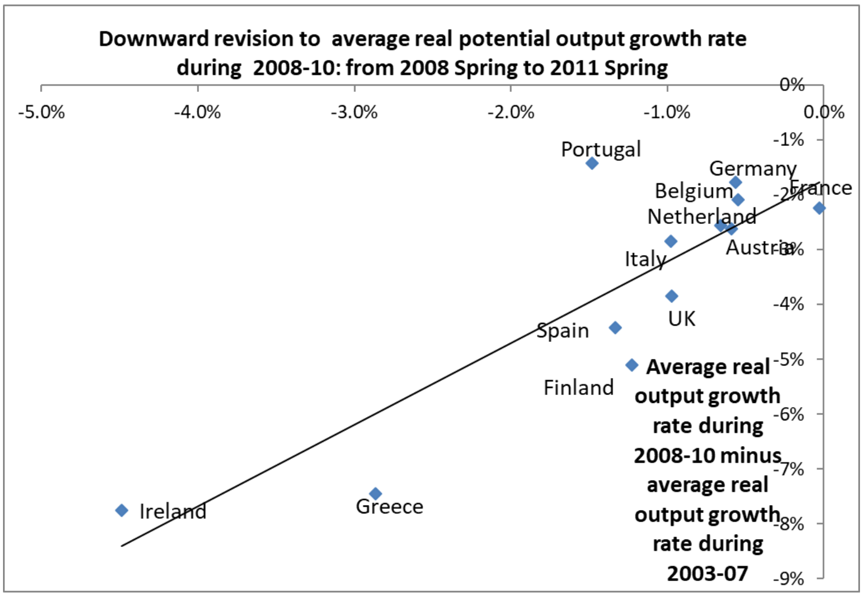

Figure 4: Relation between severity of the recession and revisions to the growth rate of real potential output during 2008 – 2010 across countries

Figure A1 in the Appendix shows that a country which has a larger recession during 2008 – 10 tends to have a larger downward revision to average growth rate of real potential output over a *pre-crisis* period 2005 – 07. Figure 5 plots the revision to 2003 – 07 output gaps from 2008 Spring to 2011 Spring. For almost all countries and years, the downward revision to 2003 – 07 potential growth is associated with an upward revision to 2003 – 07 output gaps. For example, the estimate of 2006 and 2007 Italian output gaps made in 2008 Spring are -0.3% and -0.3%, which suggests policymakers thought the Italian economy had been growing slightly under potential. They were revised up to 2.2% and 2.9% respectively in 2011 Spring; policymakers then perceive that Italian economy during 2006 – 2007 was much above the potential level.

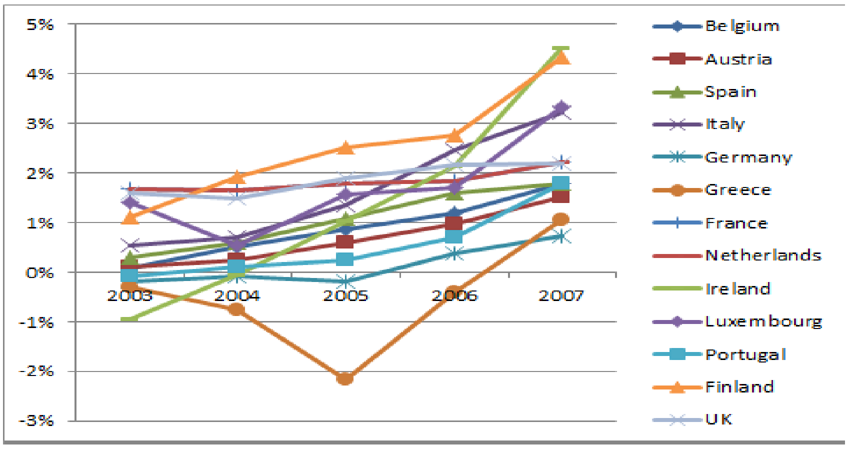

Figure 5: Revision to estimates of pre-crisis (2003-2007) output gap: from 2008 Spring to 2011 Spring

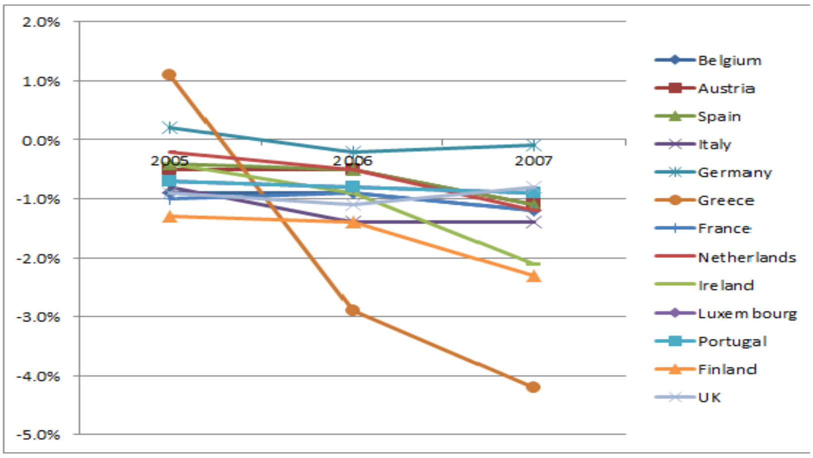

Figure 6: Revision to estimates of pre-crisis (2005-2007) structural budget balance: from 2008 Spring to 2011 Spring

Figure 6 shows the revision to 2005 − 2007 estimates of structural balance to GDP ratio. For almost all countries and years, associated with downward revisions to the estimate of potential output and upward revisions to output gaps, the Great Recession has led to downward revision to pre-crisis estimates of structural balance ratio. For example, the estimate of 2005 − 07 structural balance ratio for Italy made in 2008 Spring are −3.9%, −3.2% and −1.7%. In 2011 Spring, they were revised to −4.7%, −4.5% and −3.0%.

Evidence 1: The Great Recession has led the EC to revise massively downward estimates of potential output growth rates in the eurozone and the UK. Moreover, across these countries, the size of the downward revisions are heterogeneous and there is a strong positive correlation between the size of downward revisions and the severity of recession with a correlation coefficient of 0.86.

The downward revisions to estimates of potential output growth rates have led to upward revisions to estimates of output gaps and downward revisions to estimates of structural balance for almost all countries during a pre-crisis period.

3.2 Evidence 2: how did policymakers' view evolve during the recovery phase?

In the wake of the financial crisis, the eurozone economy was hit by two waves of recessions and started to recover around 2013. This section examines how policymakers' beliefs about potential output growth rate of different countries in the aftermath of the recessions (i.e., beliefs about 2012 potential output growth rate) evolve during the recovery phase. Figure 7 displays a heterogeneity in the pattern of revisions to 2012 potential growth rates made in 2013 Autumn, comparing to a more recent data vintage (2016 Autumn). For seven countries (Ireland, Greece, Spain, Portugal, Netherlands, UK and Belgium), the growth rate of 2012 potential output had been revised upward. For Germany, Italy, France, Finland and Austria, the growth rate of potential output had been revised downward.

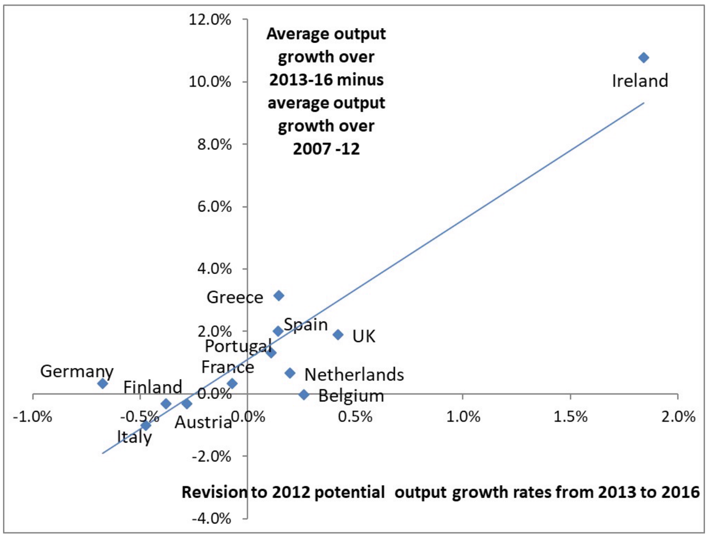

Figure 7: Relation between revision to 2012 potential output growth rates and the pace of recovery from 2013 to 2016

Figure 7 reveals a strong positive cross-country correlation (i.e., correlation coefficient of 0.91) between revision to 2012 growth rate of potential output and the pace of recovery. The pace of recovery is measured by average GDP growth rate during $2013-16$ minus that during $2007-11$. When the average growth rate of real GDP of an economy during $2013-16$ is higher (or lower) than its average growth rate during $2007-12$, this economy tends to have an upward (or downward) revision to 2012 potential output growth rate from 2013 to 2016. Moreover, a country which has a larger increase (or decline) in the average growth rate from the period $2007-12$ to the period $2013-16$ tends to have a larger upward (or downward) revision to the growth rate of 2012 potential output.

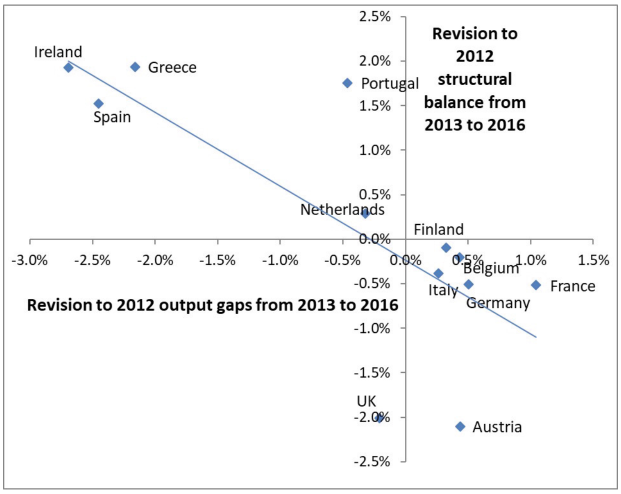

Figure 8: Revision to estimate of 2012 output gaps and structural balance during the recovery: from 2013 Autumn to 2016 Autumn

Figure A.2 in the Appendix displays the revision to the estimate of 2012 potential growth rates and output gaps from 2013 Autumn and 2016 Autumn. A heterogeneity in the pattern of revision appears. Associated with upward (or downward) revision to potential growth rates is downward (upward) revision to output gaps (with Belgium as an exception). For Ireland, Spain and Greece, the revision to 2012 output gaps is sizable and more than 2%.

Figure 8 plots the revision to 2012 structural balance ratio and output gap from 2013 Autumn to 2016 Autumn. There appears a strong cross-country negative relation between output gaps revisions and structural balance ratio revisions (correlation coefficient of -0.75). Associated with upward (or downward) revision to output gaps is downward (or upward) revision to structural balance ratio (with the UK as an exception). For five countries (Ireland, Spain, Portugal, Greece and Netherlands), the 2012 structural budget balance ratio is revised upward from 2013 to 2016. For Ireland, Spain, Portugal and Greece, the revision is sizable and ranges from 1.5% to 2.2%.

Evidence 2: Using a recent data vintage (i.e., Autumn 2016 data vintage), we find that for most countries in the sample, the 2012 potential output growth rates formed in 2013 Autumn have been revised upward during the recovery phase, while for the remaining countries they have been revised downward. The heterogeneity in belief revision is strongly positively correlated with heterogeneous economic performance of EU economies during the recovery phase. A country which has higher output growth during $2013-2016$ (relative to a previous period) tends to have a larger upward revision to 2012 potential output growth rates from 2013 to 2016.

3.3 Evidence 3: how were surprises to output growth translated to revisions to potential output growth forecasts?

Table 1: Correlation coefficient between surprises to estimates of output growth rates and revisions to potential output growth forecasts

	Belgium	Germany	Greece	Spain	France	Ireland
corr.	0.52	0.61	0.75	0.51	0.77	0.62
p-value	0.013	0.003	0.000	0.016	0.000	0.002
	Italy	Netherlands	Austria	Portugal	Finland	UK
corr.	0.58	0.63	0.46	0.44	0.65	0.52
p-value	0.005	0.003	0.049	0.042	0.001	0.013

From Spring of year t to Autumn of year t (or from Autumn of year t to Spring of year $t+1$), policymakers receive new information about real output growth in year t and revise their estimate of year t real output growth rate. We investigate to what extent the surprises to actual growth rate of real output (or the arrival of new information) is translated into the revision to their forecast of growth rate of real potential output in year $t+1$. Table 1 shows that for all countries, there is a strong positive correlation between revisions to the forecast of the growth rate of real potential output and revisions to the estimate of the growth rate of real output using all data vintages covering $2004-2015$. An upward (or downward) revision to the forecast of the potential output growth rate tends to associate with an upward (or downward) revision to the estimate of real output growth rate. All correlation coefficients are significant at 5% level and most are significant at 1% level.

Evidence 3: For all countries, surprises to the estimate of real output growth rates are strongly positively correlated with revisions to the forecast of the growth rate of real potential output.

4 Evidence II: fiscal consolidation, changes in potential output estimates, and growth forecast errors

The Great Recession led to deterioration in estimates of structural balance for EU member states.[14] Column 1 to 4 of Table 2 reports projections of structural balance for twelve EU countries taken from $2009-2010$ updates of *Stability and Convergence Programmes* (SCP) submitted by member states by March 2010 and published in the EC's 2010 *Report on Public Finances in EMU*; see Table 1.3.3 (p.54) of this report. For instance, estimates of structural deficits in 2009 for Spain, Greece, Ireland, Portugal, the UK are 9.9%, 11.4%, 9.3%, 8.3% and 10.7%.

[14] By definition, structural balance is not expected to disppear over the business cycle.

Table 2: Projections and real-time estimates of structural balance (%)

		Panel A: Projection			Panel B: real-time estimate		
	2009	2010	2011	2012	2010	2011	2012
Belgium	−3.8	−3.4	−2.9	−2.2	−2.9	−3.4	−3.0
Germany	−1.8	−4.4	−3.9	−3.0	−1.9	−0.8	0.3
Greece	−11.4	−7.7	−4.4	−1.9	−8.6	−5.7	−1
Spain	−9.9	−7.9	−6.1	−4.6	−7.0	−7.3	−5.5
France	−6.5	−6.8	−4.9	−4.0	−4.9	−4.1	−3.6
Ireland	−9.3	−9.2	−8.2	−6.3	−10.5	−8.4	−7.4
Italy	−3.8	−3.3	−2.7	−1.9	−3.1	−3.6	−1.4
Netherlands	−3.8	−4.8	−3.9	−3.5	−3.7	−3.5	−2.6
Austria	−2.7	−3.9	−3.3	−2.7	−3.7	−2.4	−1.5
Portugal	−8.3	−7.5	−5.9	−4.1	−9.2	−6.2	−4.2
Finland	0.3	−0.9	−1.0	−1.2	0.3	0.6	−0.7
UK	−10.7	−10.0	−7.7	−6.6	−8.2	−6.9	−7

Coordinated fiscal consolidations were implemented to reduce structural deficits. Both actual and planned fiscal consolidations are considered, as in e.g., Blanchard and Leigh (2013). Actual fiscal consolidation is measured by changes in the real-time estimate of structural balances computed using Table 2. Real-time estimates of structural balance ratios are estimates of year t structural balance ratio reported in the European Economic Spring Forecast in year $t+1$ for $t = 2010$, 2011, 2012. Planned fiscal consolidation is measured by the forecast of the change in general government structural fiscal balance in percent of potential GDP. We focus on three-year intervals 2010 -2012 to allow for delayed effects of fiscal policy.

4.1 Relation between structural deficits in 2009 and the size of fiscal consolidation during 2010 - 2012

This section studies the relation between the policymakers' beliefs about structural deficits in 2009 (the opposite of the first column of Table 2) and the size of subsequent (planned and actual) fiscal consolidation. The size of planned fiscal consolidation is calculated as the forecast of the change in structural balance as a percentage of potential output during a three-year period 2010 – 2012 (column 2 minus column 1 in Table 2). Figure 9 plots planned fiscal consolidation during 2010 - 2012 against the estimate of structural deficits in 2009. This gives a cross-country correlation coefficient of 0.90 which is significant at 1% level. Using actual changes in real-time estimates of structural balance as an alternative measure of consolidation yields a correlation coefficient of 0.77 (which is significant at 1% level too).

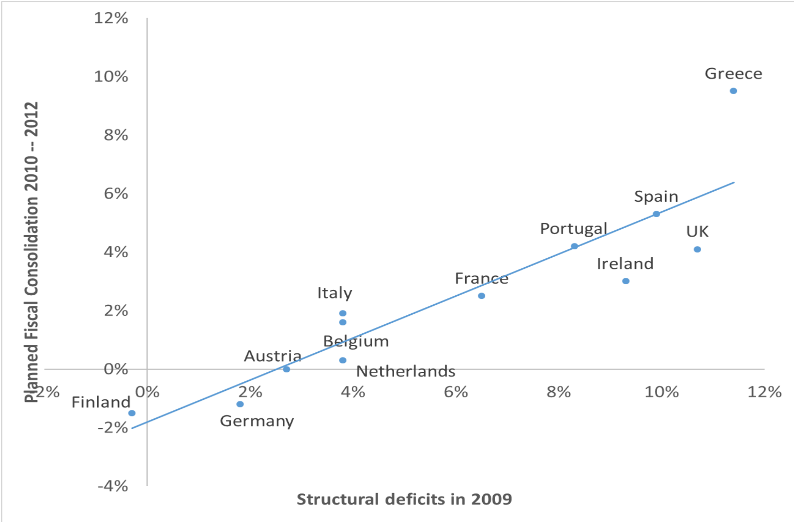

Figure 9: Planned fiscal consolidation vs structural deficits in 2009

Note: figure plots the planned fiscal consolidation during 2010 - 2012 against the estimate of structural deficits in 2009.

Evidence 4: A country with a higher level of estimate of structural deficits in year 2009 tends to associate with a larger actual and planned fiscal consolidation subsequently.

4.2 Relation between fiscal consolidation and changes in real-time estimate of potential GDP growth

We examine the relationship between fiscal consolidation and economic performance in two ways. This section studies the relationship between fiscal consolidation and the change in the real-time estimate of real potential GDP growth rates during the consolidation period $2010-2012$. Real-time estimates of potential GDP growth rates are measured by Spring year $t+1$ estimate of year t potential GDP growth rates. Using data in Table 2, Figure 10 plots the change in real-time estimate of real potential GDP growth rates during the consolidation period $2010-2012$ against the change in the real-time estimate of structural balance during the same period.

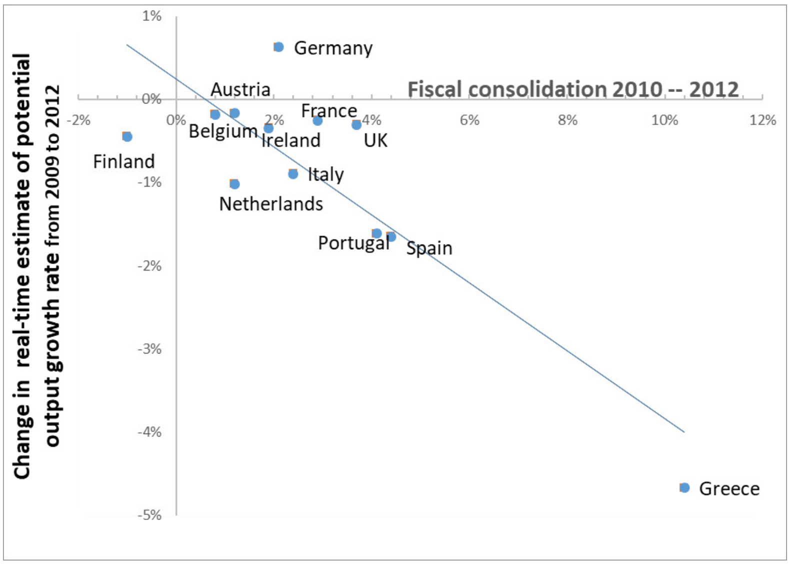

Figure 10: Fiscal consolidation during 2010 – 2012 and changes in real-time estimate of real potential GDP growth rates

Evidence 5: For the 2010 − 2012 period, there is a strong negative cross-country correlation (−0.86 and significant at 1% level) between fiscal consolidation and the change in real-time estimate of the growth rates of real potential GDP during the same period. A country which implements a larger fiscal consolidaton during 2010 – 2012 tends to associate with a bigger downward revision to the real-time estimate of potential GDP growth rates from 2009 to 2012.

Figure A.3 in the Appendix shows that a similar result is obtained when we use planned fiscal consolidation as an alternative measure. The correlation coefficient between planned fiscal consolidation and changes in real-time estimate of real potential GDP growth rates is −0.85 and significant at 1% level.

4.3 Relation between fiscal consolidation and growth forecast errors

This section studies the relation between fiscal consolidation and growth forecast errors, following Blanchard and Leigh (2013) but using the EC data. They find a country which plans a larger fiscal consolidation tends to experience lower economic growth than expected, using data from IMF World Economic Outlook for 26 (EU and other advanced) economies.[15] They interpret the result as substantial underestimation of fiscal multiplier by forecasters. House, Proebsting, and Tesar (2019) find fiscal austerity, defined as government purchases below forecast, account for roughly three quarters of the cross-sectional variation in GDP during 2010 − 2014 period.

Figure 10 illustrates a strong negative correlation between fiscal consolidation and subsequent

[15] Under RE, fiscal consolidation forecasts should be unrelated to subsequent growth forecast errors. Blanchard and Leigh (2013) shows this implication is rejected by the data.

growth forecast errors for our sample of countries.[16] Growth forecast errors measure the difference between actual cumulative real GDP growth during $2010 - 2012$, minus the forecast made in Spring 2010.

Evidence 6: During the consolidation period $2010 - 2012$, there is a strong negative cross-country correlation (-0.89) between fiscal consolidation and growth forecast errors. A country which implements a larger fiscal consolidation tends to have lower growth than expected.

Figure A.4 in the Appendix shows during $2010 - 2012$, the cross-country correlation between planned fiscal consolidation and growth forecast errors is -0.89 too.

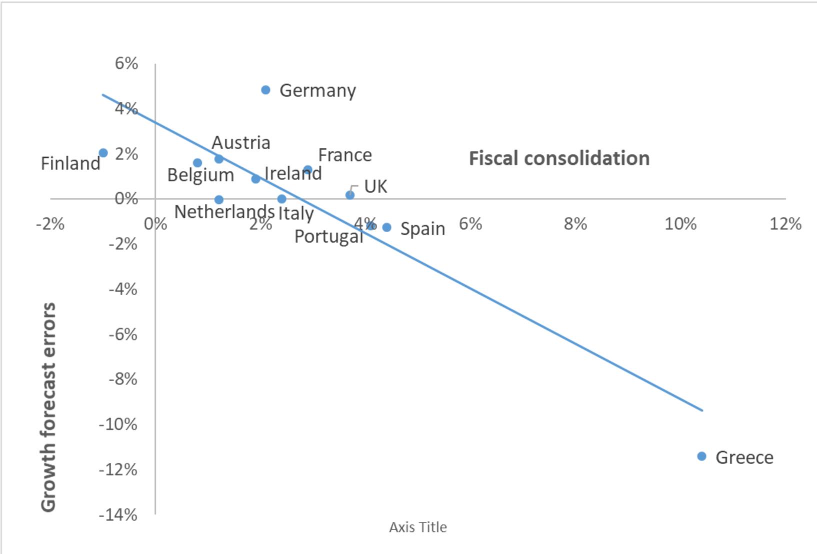

Figure 11: Growth forecast errors versus fiscal consolidation

Notes: Figure plots forecast error for real GDP growth in 2010, 2011 and 2012 relative to the fiscal consolidation for 2010, 2011 and 2012 made in Spring of year 2010. Fiscal consolidation is measured by actual changes in real-time estimates of structural balance during 2010 – 2012.

5 Model setup

This section presents the setup of our model which extends Kuang and Mitra (2016, henceforth KM) with a fiscal policymaker and distortionary taxes. KM incorporates households' uncertainty about the trend and cycle of endogenous variables into a standard Real Business Cycle (RBC) model. Households learn continually about the long-run growth rate.[17] Four main features of the KM model are King-Plosser-Rebelo (KPR) non-separable preferences between consumption and

[16] The forecast of fiscal consolidation is again the forecast of the change in the structural balance ratio during 2010-2011 made in Spring 2010.

[17] The KM model replicates pro-cyclical forecasts of the growth rate of potential output observed in the data and suggests a crucial role for optimism and pessimism about the growth rate of potential (or long-run) output in business cycle fluctuations.

leisure, constant returns to scale technology, variable capital utilization, and a random walk with drift productivity process.

Real business cycle models usually have small government spending multipliers, see e.g., Woodford (2011). Zero lower bound on interest rates has been emphasized as an important feature in determining the size of the multiplier. Nevertheless, our RBC model with learning can generate a realistic magnitude of the government spending multiplier in Europe data (e.g., House, Proebsting, Tesar (2019)) because households' learning and pessimism following the recession amplify the impact of fiscal austerity in the model. Moreover, households' learning also helps to generate state-dependence in multipliers or a larger government spending multiplier during recessions (e.g. as in Auerbach and Gorodnichenko (2012)) due to the dependence of the multipliers on agents' beliefs. Also, the learning model generates positive comovement between government expenditures and private investment, as opposed to typical RBC models with RE.

The representative household maximizes

$$\widehat{E}_t \sum_{t=0}^{\infty} \beta^t u(C_t, L_t);\ u(C_t, L_t) = \frac{C_t^{1-\sigma} v(1-L_t)}{1-\sigma}$$

subject to the flow budget constraint

$$C_t + K_{t+1} + B_{t+1} + T_t = \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right) K_t + W_t H_t \left(1-\tau^H\right) + K_t + R_t B_t.$$

$\widehat{E}_t$ denotes the subjective expectations of agents for the future, which agents hold in the absence of RE. RE analysis is standard. $C_t, L_t, H_t, K_t, U_t,\ B_t, T_t$ are consumption, leisure, hours, capital, capacity utilization rate, bond holdings and lump-sum taxes. τ^K and τ^H are capital and labor income tax rate, respectively. W_t, R_t^K and R_t are the wage rate, rental rate for capital services and interest rate. β is the discount rate between 0 and 1. $\sigma > 1$ and $v', v'' > 0$. Capacity utilization in the data displays pronounced procyclical variability (see King and Rebelo (1999)) and is used to improve the fit of the model. Capital depreciation is assumed to increase with capacity utilization U_t according to the function $\delta(U_t) = \theta^{-1} U_t^{\theta}$ where $\theta > 0$.

There are a continuum of identical competitive firms of mass one. Each produces the economy's only good Y_t using capital K_t and labor H_t as inputs according to the production function $Y_t = (U_t K_t)^{\alpha} (X_t H_t)^{1-\alpha}$, where $0 < \alpha < 1$. Each firm maximizes profits, $\Pi_t = Y_t - R_t^K U_t K_t - W_t H_t$, choosing labor and capital inputs and taking factor prices as given. Stochastic variations in the technology factor are the source of aggregate fluctuations and we assume that the technology factor X_t is a random walk with drift, i.e.

$$\log(X_t / X_{t-1}) = \gamma_t = \log(\overline{\gamma}) + \widehat{\gamma}_t, \tag{2}$$

where $\widehat{\gamma}_t$ is an independently and identically distributed (i.i.d) random variable with zero mean and standard deviation σ_{γ} and $\overline{\gamma} > 0$. We assume no shocks to trend growth rates $\log(\overline{\gamma})$ as a benchmark and in most of our analysis. This is motivated by the evidence for the eurozone provided in e.g., Jarocinski and Lenza (2018) which supports the view of no decline in the trend growth rate following the recession. Section 9.3 studies an extension with shocks to both the level and trend growth rate of productivity.

The government budget constraint is

$$B_{t+1} = B_t R_t - \left(R_t^K U_t - \delta(U_t)\right) K_t \tau^K - W_t H_t \tau^H + G_t - T_t, \tag{3}$$

Denote by Y_t^P and OG_t^S policymakers' *real-time* estimate of potential output and output gaps under subjective beliefs. Government expenditure (G_t) is assumed to follow a fiscal rule

$$G_t/Y_t = \lambda_0 + \lambda_1 OG_t^S + \lambda_2 B_t/Y_t, \tag{4}$$

$$\text{where } OG_t^S = Y_t/Y_t^P - 1. \tag{5}$$

Thus, government expenditure to GDP ratio (G_t/Y_t) responds to debt-to-GDP ratio (B_t/Y_t) and output gaps, following Leeper, Plante and Traum (2010). A similar fiscal rule for government expenditure is used in e.g. Corsetti, Meier and Mueller (2012). Both λ_1 and λ_2 are assumed to be negative. $\lambda_1 < 0$ means G_t/Y_t increases (or decreases) in response to a decline (or increase) in output gaps. $\lambda_2 < 0$ implies that the policymaker cuts G_t/Y_t in response to a rise in B_t/Y_t.

Balanced growth requires consumption, investment, output, the capital stock, and real wages to grow at the rate of the stochastic trend so that $k_t = K_t/X_{t-1}, y_t = Y_t/X_t, c_t = C_t/X_t$ etc are stationary. Hours and the rental rate of capital are stationary. Hatted variables are percentage deviations from the steady state. Details of the first order conditions of the household and firm, steady state and the log-linear approximation are presented in the Appendix D. Utilizing the consumption Euler equation and the intertemporal budget constraint of households, we obtain the linearized consumption decision rule

$$\begin{aligned}\widehat{c}_t + \sigma^{-1}\psi(1-\sigma)\widehat{H}_t = {} & (1-\chi)(1-\widetilde{\beta})\varepsilon_c^{-1}\left[\widetilde{\beta}^{-1}\widehat{a}_t + \widetilde{R}\widehat{R}_t^K - \widetilde{\beta}^{-1}\widehat{\gamma}_t + (\varepsilon_w + \varepsilon_c \chi (1-\chi)^{-1})\widehat{w}_t\right] \\ & + \widetilde{\beta}\widehat{E}_t \sum_{T=t}^{\infty} \widetilde{\beta}^{T-t}\Gamma_1 \widehat{R}_{T+1}^K + \widetilde{\beta}\widehat{E}_t \sum_{T=t}^{\infty} \widetilde{\beta}^{T-t}\Gamma_2 \widehat{w}_{T+1},\end{aligned} \tag{6}$$

$\widetilde{\beta}$ is growth-adjusted discount factor and $\widehat{a}_t$ household's wealth (i.e., the sum of bond and capital holdings). The composite parameters ε_w, $\varepsilon_c, \psi, \Gamma_1, \Gamma_2, \chi$ are explained in the Appendix. Equation (6) says that the linear combination of consumption and labor supply depends on the forecast of the discounted sum of future rental rates, wage rates, and productivity as well as current wealth, productivity and market prices. We do not assume that households have knowledge of the government budget constraint. Note the forecast of taxes are functions of forecasts of wage rates and rental rates and do not explicitly appear in the decision rule. We consider a symmetric equilibrium in what follows. To determine equilibrium prices and quantities, the learning model needs to be augmented by belief specification and updating presented in Section 6.

6 Beliefs and learning

This section discusses beliefs and learning by the policymaker and households as well as potential mis-measurement of the output gap and cyclically-adjusted budget balances (CABs) by the policymaker in the model.

6.1 Learning by the policymaker

This section explains the learning behavior by the policymaker in our business cycle model and how it helps to replicate the evidence we discussed in Section 3. The policymaker is assumed to face real-time uncertainty about the potential growth rates of the economy. Denote by μ_t^y the policymaker's beliefs about the growth rate of real potential output. We follow Stock and Watson (1998) and Holston, Laubach and Williams (2017) by specifying the policymaker's perceived log potential output as a random walk with a stochastic drift that itself follows a random walk. Given this belief, μ_t^y is updated optimally according to

$$\mu_t^y = \mu_{t-1}^y + \widetilde{g}\left(\Delta \log Y_{t-1} - \mu_{t-1}^y\right), \tag{7}$$

where $\Delta \log Y_{t-1}$ is the GDP growth rate in period $t-1$. $\widetilde{g} > 0$ is the gain parameter and determines the weight that the policymaker places to new information when forming the estimates of potential output growth. μ_t^y amounts to a weighted average of past GDP growth with exponentially decaying weights.[18] Time-varying estimates of potential output growth rates and their revisions shown in Section 3 and 4 is captured by introducing misperception and learning about potential output growth rates as in (7) (and cannot be captured by introducing learning about the level of potential output only). Considering learning about potential output growth also facilitates cmparison across member states. The implications of learning for the level of potential output, output gap and CAB are analyzed later in this Section.

We make two remarks about the choice of $\widetilde{g}$. First, real-time estimates of potential output made by the EC are used to discipline the choice of $\widetilde{g}$ which in our model becomes the actual gain parameter used by the policymaker. The large downward revisions to estimates of potential output growth following the Great Recession made by the EC requires a relatively big value of $\widetilde{g}$. Second, Section 9 explores an alternative choice of $\widetilde{g}$ by the policymaker which minimizes real-time mis-measurement of the output gap (and hence structural balance); we call this the optimal gain parameter there. This optimal gain is smaller than the actual gain used by the policymaker and we use this optimal gain to provide a counterfactual analysis.

This belief updating rule (7) approximates reasonably well the real-time estimates of potential output growth rates made by the EC; the EC's methodology is briefly explained in Section 2.1. Coibion Gorodnichenko and Ulate (2018) find that the estimates of potential output growth rates

[18] Learning about growth rates of asset prices (stock and house prices) turns out to be important to match survey expectations and asset boom-bust cycles in Adam, Marcet and Nicolini (2016), Adam, Marcet and Beutel (2017) and Adam, Kuang and Marcet (2012).

for the US and a large number of industrialized countries made by a wide range of public institutions and private forecasters are well approximated by a weighted moving-average of recent GDP changes.

Learning via (7) by the policymaker implies cyclical fluctuations in output may lead to changes to potential output estimates. This produces pro-cyclical potential output estimates and helps to replicate the evidence presented in Section 3. First, following a large economic recession, estimates of potential output growth rates based on (7) will be revised downward. The learning model generates secular-stagnation-like slowdown according to the policymaker's estimates. Accordingly, estimates of the output gap in the past will be revised upward and estimates of CAB downward due to upward revisions to the output gap. Moreover, a country which experiences a larger economic recession (or lower realizations of output growth) tends to associate with a bigger downward revision to estimates of potential GDP growth rates. These are consistent with Evidence 1 in Section 3.1. Second, a positive (or negative) surprise to output growth tends to associate with an upward (or downward) revision to the trend growth of output, consistent with Evidence 3 in Section 3.3. Third, during the recovery phase, improving economic performance for a country tends to associate with upward revisions to estimates of potential output growth, consistent with Evidence 2 in Section 3.2. Fourth, Coibion, Gorodnichenko and Ulate (2018) find that the estimates of potential output made by a range of public institutions decline (or rise) persistently in response to an i.i.d negative (or positive) TFP shock. Learning by the policymaker reproduces this evidence. In the learning model, a negative productivity shock leads to a fall in output and the policymaker uses the new observation of output to revise downward their potential output estimate. Moreover, the interaction between belief revision and equilibrium outcomes helps to produce this persistent response.

Now turning to the estimate of the output gap and structural balance made by the policymaker. The policymaker's imperfect knowledge of the economy gives rise to potential mis-measurement of the output gap and structural balance unlike full-information RE models. We make a distinction between true output gaps (OG_t^T) and subjective output gaps (OG_t^S). If the policymaker had full information, she would know the evolution of true potential output $\log Y_t^{P,T}$ is determined by the productivity process $\log Y_t^{P,T} = \log Y_{t-1}^{P,T} + \log(\overline{\gamma}) + \widehat{\gamma}_t$. OG_t^T is calculated using the true trend output and given by $OG_t^T = \log Y_t - \log Y_t^{P,T}$. Subjective trend output ($\log Y_t^{P,S}$) and OG_t^S are $\log Y_t^{P,S} = \log Y_{t-1}^{P,S} + \mu_t^y + \widehat{\gamma}_t$ and $OG_t^S = \log Y_t - \log Y_t^{P,S}$. The mis-measurement of the output gap is

$$OG_t^S - OG_t^T = \log Y_t^{P,T} - \log Y_t^{P,S}. \tag{8}$$

Denote by $Est\_error_{CAB}(t)$ the estimation error of CAB in period t. It can be calculated based on equation (1) and is proportional to the mis-measurement of the output gap

$$Est\_error_{CAB}(t) = -\epsilon\left(OG_t^S - OG_t^T\right). \tag{9}$$

6.2 Learning by households

This section introduces learning by households. Like the policymaker, households are likely to have imperfect knowledge of the trend and cycle and face uncertainty about the long-run growth rate of endogenous variables. For instance, the forecasts of US long-run output (or productivity) growth made by agents are pro-cyclical as is shown in Kuang and Mitra (2016) using Survey of Professional Forecasters data and in Coibion, Gorodnichenko and Ulate (2018).

Introducing housholds' learning serves at least the following purposes. First, typical real business cycle models usually have small government spending multipliers, see e.g., Woodford (2011). We show later that learning and households' pessimism following the recession amplify the impact of fiscal austerity and the recession in our model. The model generates a realistic magnitude of the goverment spending multiplier in line with e.g. House, Proebsting and Tesar (2019). Second, households' learning helps to generate a larger government spending multiplier during recessions, consistent with evidence provided by Auerbach and Gorodnichenko (2012). This is because the impact of reducing government expenditures tends to be larger when agents are more pessimistic during recessions. Third, while private investment comoves negatively with government expenditures in RE RBC models, households' pessimism helps to produce positive comovement between government expenditures and private investment as is shown later. Fourth, as in shown later, learning by households helps to deliver consistency between the model and survey expectations data, such as very strong autocorrelation in forecast errors for macroeconomic variables.

Households need to forecast wage rates and rental rates up to the indefinite future to make consumption and other decisions; see (6). They are assumed to have a simple econometric model, relating wages and the capital rental rate to the aggregate stock of capital and bond holdings

$$\begin{aligned}
\Delta \log R_t^K &= \omega_0^r + \omega_1^r \Delta \log K_t + \omega_2^r \Delta \log B_t + e_t^r, && (10)\\
\Delta \log W_t &= \omega_0^w + \omega_1^w \Delta \log K_t + \omega_2^w \Delta \log B_t + e_t^w, && (11)\\
\Delta \log K_{t+1} &= \omega_0^k + \omega_1^k \Delta \log K_t + \omega_2^k \Delta \log B_t + e_t^k, && (12)\\
\Delta \log B_{t+1} &= \omega_0^b + \omega_1^b \Delta \log K_t + \omega_2^b \Delta \log B_t + e_t^b, && (13)
\end{aligned}$$

where e_t^r, e_t^w, e_t^k, and e_t^b are regression errors.[19] The beliefs have the same functional form as the linearized minimum-state-variable RE solution to the model (reformulated in levels). This belief specification follows Kuang and Mitra (2016) which is in contrast to the belief specification in existing business cycle models with AL that agents learn about detrended variables. The latter implies that agents have exact knowledge of the stochastic and deterministic component of the trend growth rate of endogenous variables, as elaborated in Section 7 of Kuang and Mitra (2016).

Let $\omega^{i'} = (\omega_0^i, \omega_1^i, \omega_2^i)$ for $i = r, w, k, b$. $z_t^r = \Delta \log R_t^K, z_t^w = \Delta \log W_t, z_t^k = \Delta \log K_{t+1}, z_t^b = \Delta \log B_{t+1}$ and $q_{t-1}^{'} = (1, \Delta \log K_t, \Delta \log B_t)$. Beliefs at period t, ω_t^i are updated recursively by the

[19] We note that under RE the residuals in the perceived law of motion are correlated but that agents ignore possible correlations when estimating the forecast functions. This approach under AL is consistent with the standard approach in the literature; see eg. Eusepi and Preston (2011, 2018) and Sargent et al. (2009).

constant-gain Generalized Stochastic Gradient (GSG) learning algorithm as in Evans, Honkapohja and Williams (2010, henceforth EHW)

$$\widehat{\omega}_t^i = \widehat{\omega}_{t-1}^i + g^i \Gamma q_{t-1} \left(z_t^i - \omega_{t-1}^{i\prime} q_{t-1} \right), \tag{14}$$

where $\widehat{\omega}_t^i$ denotes the current-period's coefficient estimate.[20] For generality, we allow g^i used by households to be different from the gain $\widetilde{g}$ used by the policymaker. Γ controls the direction of belief updating and $g^i \in (0,1)$, the constant gain, determining the rate at which older observations are discounted. Bayesian and robustness justification for the GSG algorithm are provided in EHW. In particular, they show that (1) GSG learning algorithm asymptotically approximates the Bayesian optimal estimator when agents allow for drifting coefficients models, and (2) it is also the "maximally robust" estimator when agents allow for model uncertainty. As is standard in the literature, beliefs at t are updated using data up to period $t-1$.

7 Quantitative results: eurozone

This section examines the role for subjective beliefs of households and the policymaker in the recession. In particular, we analyze the implications of mis-measuring potential output and structural balance for fiscal policy decisions and the recession.

7.1 Calibration

Parameters are calibrated for the euro area. Capital share α is set to 0.38 and σ in the utility function 2, as in Trabandt and Uhlig (2011). The discount factor $\tilde{\beta}$ is set to 0.995, which corresponds to a 2% real interest rate at the steady state. The unconditional mean of the growth rate of productivity is 2% per year (i.e., $\overline{\gamma} = 1.005$). The depreciation rate δ is set to 2.5%. The inverse of Frisch elasticity of labor supply is set to 0.1.[21] The steady state of government expenditure-to-output ratio $\overline{g}/\overline{y}$ is 0.18. The debt-to-GDP ratio is set to 75%. Capital income tax rate τ^K is set to 0.4 and labor income tax rate $\tau^H = 0.42$. The budgetary semi-elasticity ϵ for computing CAB is set to the EU average value 0.53 taken from Mourre et al (2013). In the fiscal rule, debt sensitivity of government spending λ_2 is set to -0.02 taken from Corsetti, Meier, and Mueller (2012) and output gap sensitivity of spending λ_1 is set to -0.002.[22]

[20] An alternative learning rule is the constant-gain recursive least squares (CG-RLS) algorithm. The impulse response functions of our model with CG-RLS learning are similar to the results with GSG learning. However, CG-RLS learning often imposes a projection facility on beliefs and/or generates singularity problem in inverting the moment matrix; this is perhaps not ideal and we prefer presenting our results with GSG learning where the projection facility is not invoked.

[21] This is the benchmark value in KM. If a smaller Frisch elasticity of labor supply is used in the current model, e.g. 5 or 7, our results are quantitatively similar. If we use a Frisch elasticity as low as say 0.5 or 1, our main results are still preserved except that the learning model does not produce the comovement between forecasted changes in consumption and working hours discussed in Rotemberg and Woodford (1996), see Section C2 of the Online Appendix of KM for a discussion.

[22] Similar tax rates have been used in Trabandt and Uhlig (2011). Note the 75% debt ratio corresponds to the average of eurozone countries during 1993 - 2012. Detrended lump-sum taxes are assumed to be constant and chosen

Using data from the European Central Bank's Survey of Professional Forecasters, Andrade and Le Bihan (2013) provide evidence on strong autocorrelation in the (average) 1−year ahead forecast errors of euro area real GDP growth rates, inflation rate, and unemployment rate across forecasters; see their Table 1. In particular, the autocorrelation coefficient of the forecast errors of real GDP growth rate is 0.849. This evidence is used to impose discipline on the choice of the gain parameter for households. The gain parameter g is chosen as $g^w = g^K = g^b = 0.004$ and $g^R = 0.015$, which leads to the corresponding autocorrelation coefficient of 0.92 in the learning model.[23] Our choice of gain is consistent with values found in the literature, which range from $0.002 - 0.05$; see eg. Eusepi and Preston (2011). The moment matrix in (14) is set to the identity matrix, corresponding to the classical Stochastic Gradient (SG) learning.[24]

The following assumption is made on the policymaker's initial beliefs. In reality policymakers usually have large uncertainty in measuring output gaps in the current period and recent past. However, when the past is sufficiently distant from current period, they are likely to have much less uncertainty in decomposing historical aggregate output. Therefore, for simplicity, we assume that the policymaker's belief about the trend output N periods ago is fixed at the true value where N is relatively large. N is set to 16 below which means that the policymaker does not update the trend output level four years ago. The choice of $N = 16$ is motivated by Figure 3 which shows that after the Great Recession and for almost all countries, the revision to potential growth rates in 2003 (i.e., 16 periods ago) from 2008 Spring to 2011 Spring is very small. We also assume that before the shock, the economy was at the long-run growth path (and hence at the steady state) and the policymaker's initial belief about the growth rate of trend output is at the RE value $log(\overline{\gamma})$.[25]

We take a broad interpretation of productivity shocks and model the Great Recession in terms of a sequence of negative productivity shocks in our RBC economy with distortionary taxes.[26] While we do not deny that financial frictions may be a cause of this recession, we think we may nevertheless plausibly represent the aggregate consequences of these frictions by a productivity shock. This belief is backed by Chari, Kehoe and McGrattan (2007) who show that suitable input financing frictions are observationally equivalent to negative productivity shocks. In particular, they set up a detailed economy with input-financing frictions across firms leading to inefficient utilization of factor inputs. They show that an economy in which technology is constant but input financing frictions vary over time is observationally equivalent to a neoclassical model with productivity shocks in the sense that the economies display the same aggregate properties. We are

to yield this value of the debt ratio. The results are not sensitive to the choice of λ_1 and λ_2.

[23] This way of disciplining gain parameters has similarly been used in Eusepi and Preston (2011), Kuang and Mitra (2016) and Adam, Marcet and Beutel (2017). Heterogeneous gain parameters have been used in Branch and Evans (2011) and Kuang and Mitra (2016).

[24] Note even if the SG learning algorithm is adopted here, the quantitative results of our learning model are scale invariant and remain exactly the same in response to a change in the units of regressors; see Footnote 13 of KM. The quantitative results of the AL model are robust to alternative moment matrices and to labor elasticities but we do not report them here for economy of space. The Online Appendix of KM contains similar robustness exercises.

[25] A difference of our learning model from most existing adaptive learning models is that we also seek to replicate the data when policy makers look back, i.e., revisions to historical estimates.

[26] A similar technique has been used in Mitra, Evans and Honkapohja (2016).

not interested in the causes of the recession *per se.* One may thus view the reduced productivity as a convenient short-cut for modelling distortions associated with the financial crisis by appealing to the equivalence result of Chari, Kehoe and McGrattan (2007).[27]

The real output growth rate for eurozone in year 2008 and 2009 are 0.5% and −4.5%, respectively. The Great Recession is modeled as a sequence of negative productivity shocks for eight periods, the size of which is chosen to match these output growth rates in the AL model; see top left plot of Figure 11.[28] From 2008 to 2010, the estimate of the growth rate of potential output in the eurozone was revised downward from 2% to 0.6%. The gain parameter for the policymaker is chosen to match the size of this revision, which yields $\widetilde{g} = 0.034$.

7.2 The Great Recession scenario

We discuss the analysis of impulse response to a productivity shock in Appendix C where the intuition is provided (this is similar to that in KM). We note that our learning model is able to reproduce the evidence from CGU (2018) that the estimates of potential output made by a range of public institutions decline (or rise) persistently in response to a i.i.d negative (or positive) TFP shock. In the learning model, a negative productivity shock leads to a fall in output and the policymaker uses the new observation of output to revise downward their potential output estimate. In addition, the interaction between belief revision and equilibrium outcomes helps to produce this persistent response; see the top left plot of Figure A.5. By contrast, the RE version of the model is inconsistent with this evidence because it does not produce a *persistent* response of potential output estimate to the i.i.d shock.

In Figure 12, percentage deviations from the unshocked balanced growth path are plotted for output (Y), consumption (C) and investment (I). For hours (H), deviations from the steady state are plotted. For G/Y ratio and debt-to-GDP ratio, percentage changes are plotted. This figure shows that in the AL model, the shocks lead to a large decline in Y, C, I, H and a rise in debt-to-GDP ratio. G/Y ratio declines initially in response to rising debt-to-GDP ratios and rise gradually later. Relative to the RE model, the learning model generates large amplification arising mainly from the downward revision to households' wage (and income) forecasts and mutual reinforcement between households' pessimism and market outcomes. For example, the maximum response of output (i.e. at the trough) in the learning model is about twice as large as that in the RE model.

[27] In particular, Chari, Kehoe and McGrattan (2007) consider a special case where on average financing frictions across firms are constant but relative distortions (interest rate spreads) fluctuate so that capital and labor taxes are constant while only the technology fluctuates. Periods in which relative distortions increase would show up as periods of technological regress (see their discussion on page 790-791).

[28] The sequence of eight productivity shocks are −0.24% for quarter 1 to 4 and −1.02% from quarter 5 to 8.

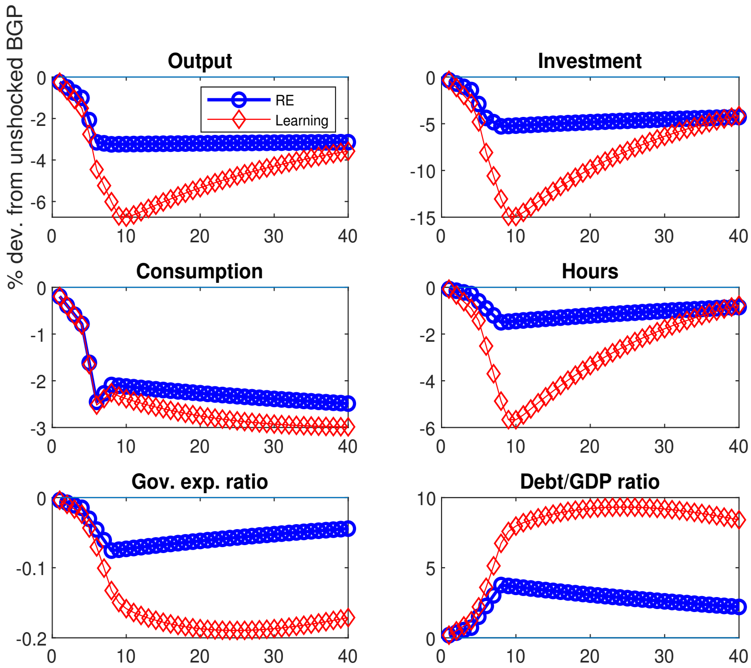

Figure 12: Response: the Great Recession scenario

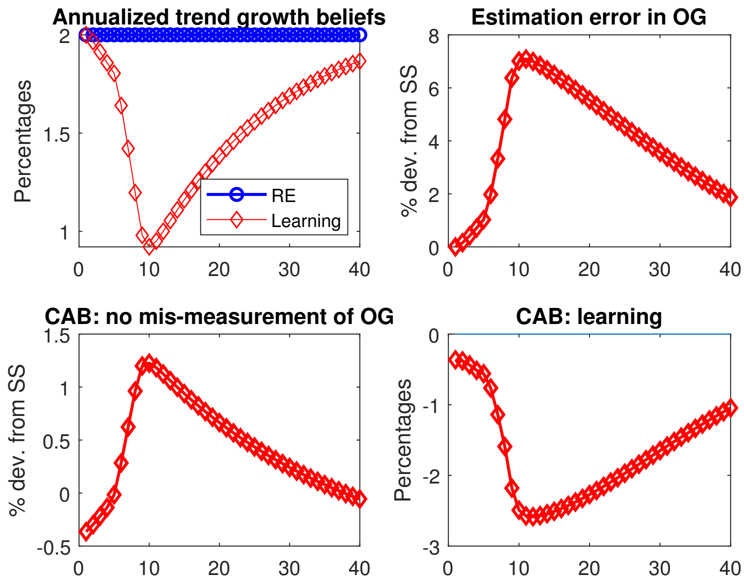

Figure 13: Policy maker's beliefs: the Great Recession scenario. Note the lower two panels shows the change in the CAB *vis-a-vis* the steady state.

The recession leads to a large downward revision to estimates of potential output growth made by the policymaker (top left plot of Figure 13) as in the eurozone data while the (deterministic component of) true trend growth remains at 2% per year in the model. In addition, the recession produces a 2.5% decline in the subjective estimate of CAB at the trough (the bottom right plot) which is computed using the EC's formula (1). This is because the recession reduces tax revenues and generates government budget deficits as the decline in revenues dominates the decline in government expenditures. After cyclical adjustment with subjective estimates of output gaps, CAB continues to be negative and declining. Note the size of the decline in CAB approximately matches the data as the structural balance ratio for euro area in 2010 minus the average structural balance ratio over 2007 is 2.7%.[29] However, pessimism about potential output leads to a large underestimation of the size of output gap and structural balances which at the trough is 7.1% and 3.8%, respectively (3.8% is computed using equation (9)).[30]

Due to structural balance targeting in the EU fiscal policy framework, the underestimation of CABs will in turn lead to excessive requirement to reduce structural deficits and output losses, which is analyzed in Section 7.3. The bottom left plot of Figure 12 displays estimates of CAB when there is no mis-measurement of output gaps, i.e., when the true (instead of subjective) output gaps are used for computing CABs. This measure of CABs declines a bit initially and then rises for some periods. They are much higher than the subjective estimates of CAB (on the bottom right plot).

7.3 Effects of "undue" fiscal consolidation

The Great Recession led to a large fall in the subjective estimates of structural budget balances made by policymakers and debt-to-GDP ratio increased rapidly for almost all countries in the sample. For example, the 2010 Report on Public Finances in EMU (p.15) states that none of the EU countries in our sample are expected to achieve their medium-term objective of structural budget balances in 2010 and 2011. Targeting structural balance in the context of the Stability and Growth Pact requires fiscal actions and EU policymakers indeed responded by adopting consolidation programmes.

The learning model in Section 7.2 reproduces a large decline in subjective estimates of CABs by policymakers (see the lower right panel of Figure 13) and a large increase in debt-to-GDP ratio (see the lower right panel of Figure 12) following the recession. The AL model suggests that the recession leads to potential output pessimism which translates as underestimation of structural balances and undue requirement in reducing the structural deficits due to targeting structural balance. We call the fiscal consolidation programmes undue because the AL model suggests the recession would only reduce CABs a bit initially and then improve CABs if there was no mis-measurement of output gaps (see the bottom left plot of Figure 13). We now examine the macroeconomic consequences of fiscal consolidation programmes which respond to the deterioration of subjective estimates of CABs and debt ratios in an effort to capture the actual scenarios in EU countries. The model suggests the mutual reinforcement between growth expectations and undue fiscal consolidation leads to further substantial output declines.

[29] See Table 39, Statistical Annex of the European Commission's Economic Forecast, Spring 2011.

[30] Under RE, structural balances remain constant following the productivity shocks, as opposed to large changes in the data on the subjective estimates of CAB.

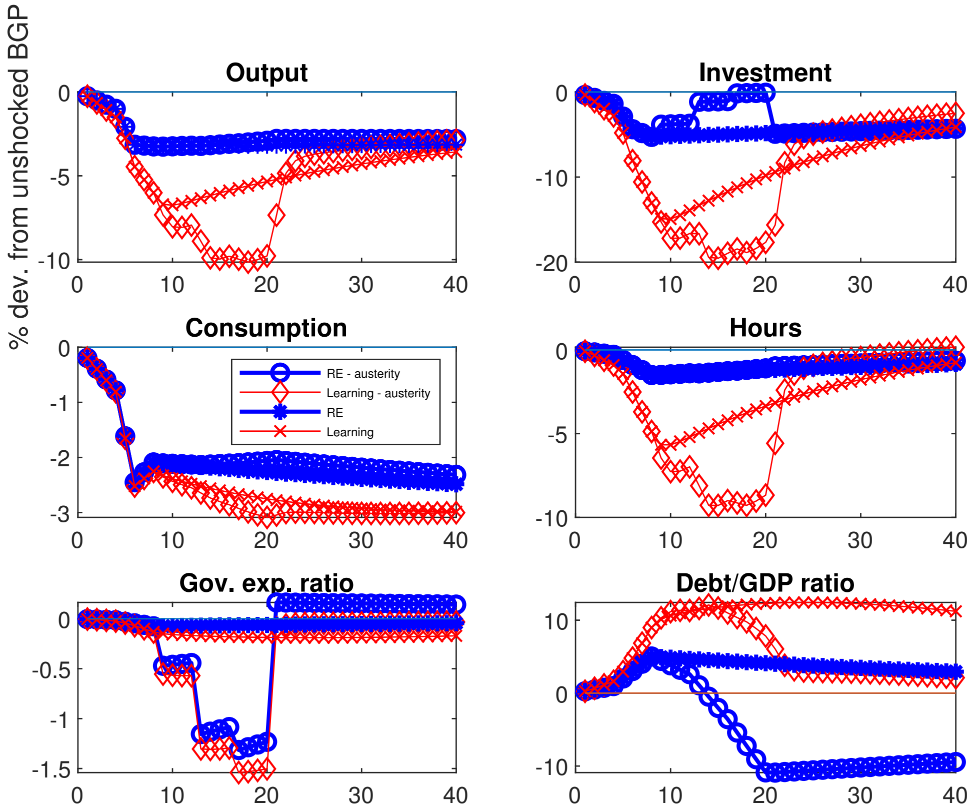

Figure 14: Impact of undue fiscal consolidation

Undue fiscal consolidation is modeled as unexpected declines in the steady state of government expenditures for three years from 2010 onwards (i.e., period 9 onwards). This may be justified as e.g., House, Proebsting and Tesar (2019) find that other austerity policies – such as cutting transfer payments or increasing taxes – do not explain the cross-variation in output. Morever, they find there is little evidence that austerity in government purchases is a consequence of the run-up of government debt during the Great Recession. The size of these declines in the model is chosen to produce the size of cut in government expenditures to GDP ratio in 2010, 2011 and 2012 relative to year 2009, i.e., -0.47%, -1.17%, -1.37%; see Table A.2. Figure 14 shows that under RE, the undue fiscal consolidation does not produce much difference in output and working hours (comparing the line labeled "RE - austerity" and the line labeled "RE"). Note increases in private investment are associated with reductions in government expenditures. Moreover, fiscal consolidation leads to immediate declines in debt-to-GDP ratio.

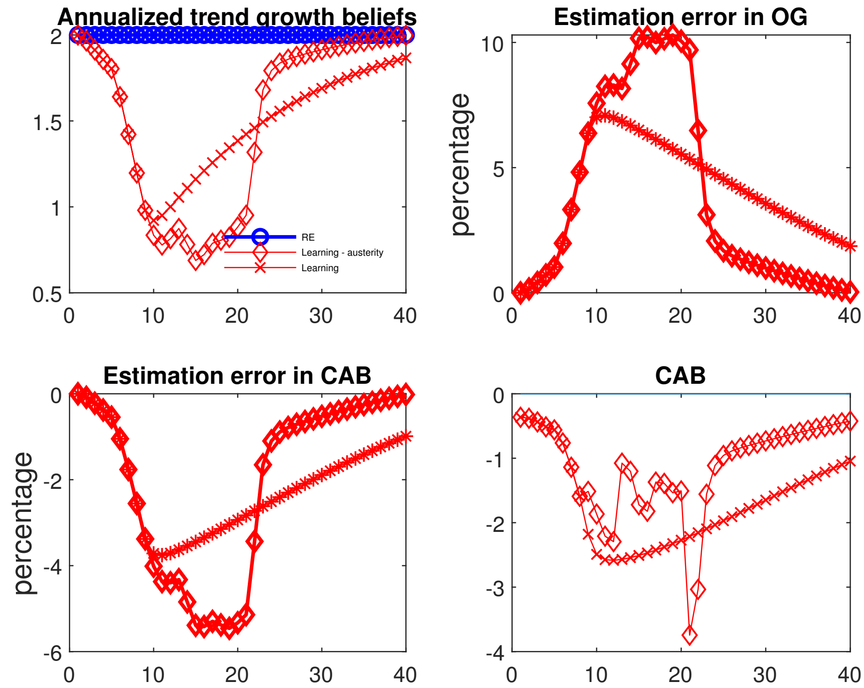

Figure 15: Impact of undue fiscal consolidation (continued)

In contrast, the same austerity programme in the AL model has a much larger impact and leads to further declines in output, investment, hours and consumption (comparing the line labeled "Learning - austerity" with the line labeled "Learning").[31] Following Auerbach and Gorodnichenko (2012), we calculate the cumulative government spending multiplier over 20 quarters from the start of the consolidation period, i.e., $\sum_{t=9}^{T} Y_t / \sum_{t=9}^{T} G_t$ in the learning model with $T = 28$. This gives us a multiplier of 2.0.[32] This is consistent with evidence provided in House, Proebsting, Tesar (2019) which finds a multiplier of roughly 2 for European Economies during 2010 - 2014.

Note private investment declines are associated with declining government expenditures unlike the RE model. During the consolidation period, the average output loss per year is 3.1%. The large amplification of the effect of fiscal austerity under learning is because the consolidation is implemented when households have pessimistic beliefs about future wage rates (and income) and the subsequent interaction between households' pessimistic beliefs and equilibrium outcomes. Based on the pessimism, households reduce I, H, and C following the consolidation, which leads to lower aggregate output. The latter reinforces households' pessimistic beliefs about future wages (and income).

Lower aggregate output due to consolidation yields significantly lower estimates of potential output growth rates made by the policymaker, see the top left plot of Figure 15. During the consolidation period, lower estimates of potential output produce a substantially larger estimation

[31] It seems after the consolidation period, output recovers quicker than without consolidation.

[32] The government spending multiplier is in the interval of $[1.7, 2.1]$ when T varies in the interval of $[10, 40]$.

error in output gaps and CABs (top right and bottom left plots of Figure 15), magnifying the measurement problem.

In the AL model, declines in G/Y ratio lead to lower aggregate output and tax revenues relative to RE. The decline in tax revenues dominates the reduction in government expenditures, which leads to a (small) rise in debt/GDP ratio for some periods (in contrast with declines in debt ratios in the RE model). This is consistent with the observation in the data. In the eurozone crisis, most EU countries, such as Spain and Ireland, have cut government spending, in order to reduce their budget deficits. However, these spending cuts were associated with a decline in economic growth, lower tax revenues and rising debt to GDP ratio.[33]

7.4 Additional properties of the EC's estimates

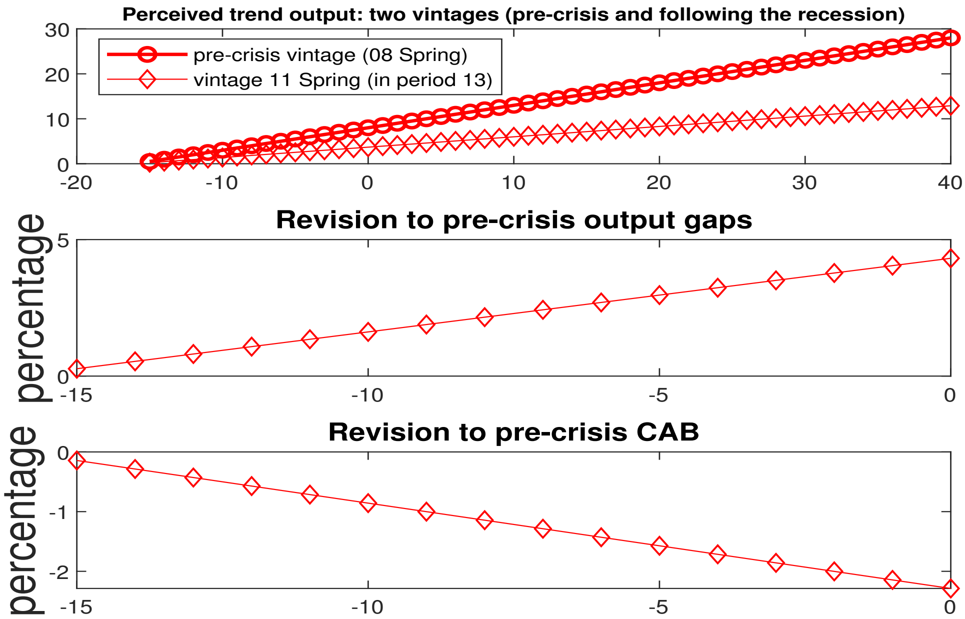

Figure 16: Changes to policy makers' view following the Great Recession.

The learning model reproduces two additional properties of the EC's estimates. First, Figure 16 displays changes to the policymaker's view following the recession in the model. The top plot displays the policymaker's perceived evolution of (the deterministic component of) potential output of two vintages: one before the shocks and the other in period 13 (three years since the start of the recession). In this figure, period $-n$ on the horizontal axis corresponds to $(n+1)$ quarter before the crisis period for $n = 0, 1, 2, ...$ and the trend output in period -15 is normalized to zero. Before the shocks, trend output (in the past and future) is perceived to grow at the rate of $log(\overline{\gamma})$. Following the

[33]In reality, the over-pessimistic estimate of potential output and structural balances are likely to play a role in amplifying the increase in government bond risk premium and output decline for countries concerned following the recession. This mechanism is not captured by the current model but can be incorporated in an extension which is left for future research.

recession, the policymaker perceives a flatter path for future and past trend output, consistent with Figure 3. The middle and bottom plot of Figure 16 shows that the lower perceived potential output leads to upward revision to estimates of pre-crisis output gaps and downward revisions to estimates of pre-crisis structural budget balances, consistent with the evidence in Figure 5 and 6. Note the general pattern in Figure 5 and 6: the size of the revision to output gaps and structural balance for a year is larger, closer it is to the crisis year (2008). For example, the revision to the output gap in 2007 is larger than in 2006 for all countries (similarly, for structural balances).[34] Interestingly, our learning model is consistent with this feature of the revisions. For instance, the revision to the output gap is 2.6% in period -5 while it is 1.5% in period -10. Note standard full-information RE models cannot explain why the arrival of new data observations (e.g., sustained declines in output) will lead to a revision of the *historical* estimate of the trend and cycle of *endogenous* variables (e.g., aggregate output and budget balances). This is because agents can deduce the evolution of endogenous variables and have exact knowledge of the trend and cycle of endogenous variables.

Second, the learning model can reproduce a strong positive correlation between surprises to output growth rates and revisions to forecasts of potential output growth rates documented in 3.3 (Evidence 3). Surprises in output growth for year t are measured by output growth rate of year t minus the forecast of year t output growth rate made in the first or third quarter of year t. Revisions to forecasts of potential output growth rates are measured by forecasts of year $t+1$ potential output growth rates in the first or third quarter of year t. The stationary distribution of agents' and the policymaker's beliefs is obtained by simulating the learning model for T periods and N repetitions. We set $T = 2000$ and $N = 500$. After obtaining the stationary distribution, we simulate the learning model for 48 periods, which corresponds to the length of our sample (12 years). Using semi-annual frequency as in the survey evidence in Section 3.3, we calculate the median correlation coefficient over repetitions. The learning model produces a correlation coefficient of 0.87, which is a bit higher than the corresponding number in the data of countries like France and Greece. Consider a positive (or negative) surprise to output growth rate, i.e., outturn higher (or lower) than prediction. This tends to lead to more optimistic (or pessimistic) forecast of the growth rate of potential output in the following year, i.e., an upward (downward) revision to the forecast of potential output growth rate. The correlation between surprises to output growth rates and revisions to forecasts of potential growth rates is usually zero in full-information RE models, as opposed to a strong positive correlation in the data. This is because the forecasts of potential output growth rates are usually constant, given the (trend) productivity process (2).

8 Quantitative results: individual countries

This section provide simulations for individual countries with a view of capturing a large number of evidence we document.

[34] Greece is the only exception for output gaps (though not for structural balances).

8.1 Country-specific calibration

Table 3 reports calibration of parameters for individual countries. The trend output growth rates $log(\overline{\gamma})$ are calculated as the average growth rate of real GDP over 2003 – 2007. The remaining parameters are taken from Trabandt and Uhlig (2011) with a few exceptions. The steady state debt-to-GDP ratio for Portugal is set to 95% to capture high debteness of the Portugese economy. The discount factor $\tilde{\beta}$ is set to 0.995 for individual country except it is 0.992 for Greece and 0.99 for Ireland, Portugal and Netherlands.[35]

Table 3: Calibration for individual countries (all in %)

	τ^H	τ^K	$log(\overline{\gamma})$	$\overline{b}/\overline{y}$	$\overline{g}/\overline{y}$	α	δ	ϵ
Belgium	49	42	2.5	107	24	39	8.4	55
Germany	41	23	1.6	62	21	37	6.7	56
Greece	41	16	4.1	100	20	40	6.1	47
Spain	36	30	3.6	54	21	42	8.5	48
France	46	35	2.0	60	27	41	6.9	55
Ireland	27	21	5.3	43	19	36	8.6	51
Italy	47	34	1.2	110	21	39	7.0	55
Netherlands	44	29	2.3	58	27	38	7.7	57
Austria	50	24	2.5	65	20	39	7.1	49
Portugal	31	23	1.1	95	23	39	9.8	46
Finland	49	31	3.6	46	24	34	7.0	53
UK	28	46	2.8	44	21	36	6.4	48

Notes: τ^H and τ^K are labor and capital tax rates. $\log(\overline{\gamma})$: pre-crisis trend growth rates of output (measured by average annual output growth rates over 2003 – 2007). $\overline{b}/\overline{y}$ and $\overline{g}/\overline{y}$ are steady state debt to GDP ratio and government expenditures to GDP ratio, respectively. α : capital share. δ : depreciation rate. ϵ : budgetary semi-elasticity taken from Mourre et al (2013).

A sequence of productivity shocks is chosen to match the real GDP growth rates during 2008 and 2009. Table A1 provides the targeted real GDP growth rates in the data for individual countries and their counterpart in the model. Fiscal consolidation is modeled as unexpected declines in the steady state of government expenditures for three years. The size of the declines are chosed to match the data, i.e., the change in government spending to GDP ratios in year 2010, 2011 and 2012 relative to year 2009. Table A2 reports the changes in government spending to GDP ratios for each country in the data and their model counterpart. Both targeted output growth and changes in government spending ratios are matched very well in the model.

[35] A lower discount factor is chosen for these countries due to too strong reaction of the economy to the shocks.

8.2 Quantitative properties

We discuss how the learning model can replicate a large number of properties of country-specific estimates made by policymakers. First, the learning model helps to produce the heterogeneity in the downward revisions to potential output growth rates and the strong positive correlation between the severity of recession and the size of revision to potential growth across EU countries documented in Section 3.1. The severity of economic recession is calculated as the difference between the average growth rate of real GDP during 2008 – 2010 (period 1 – 12) and the steady state real GDP growth rate (i.e., the average real GDP growth rate during 2003 – 2007). We also compute the size of revisions to the average growth rate of potential output during 2008 – 2010 from 2008 Spring to 2011 Spring. In the model, the correlation coefficient between the two variables is strongly positive and close to 1, which is somewhat higher than that in the data (see 0.86 in Evidence 1).[36] Figure 17 provides an illustration using the simulation for two countries: Spain and France as an example. Spain experienced a more severe recession. The average growth rate of Spanish real GDP during 2008 – 2010 is 4.31% lower than that during 2003 – 2007. The average growth rate of Spanish real potential GDP during 2008 – 2010 is revised downward by 1.5% from 2008 Spring to 2011 Spring, as the policymaker uses new observation of output to revise beliefs. By contrast, France experienced a less severe recession and smaller downward revisions to real potential GDP growth rates.

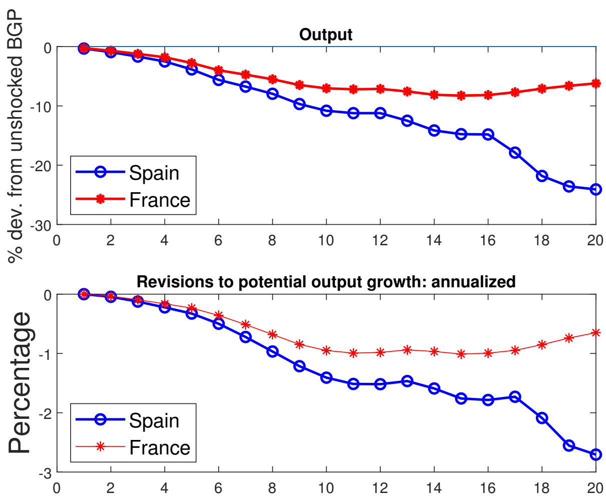

Figure 17: Relation between the severity of recession during 2008 – 2010 and the size of downward revisions to estimates of potential output growth rates

Second, we compute the correlation between real-time estimate of CAB in 2009 and the change in real-time estimates of CAB during 2010 – 2012 for each country in the model. This correlation coefficient is −0.52 which (in absolute value) is a bit smaller than the data (−0.77 and Evidence 4).

[36] This is because equation (7) is an approximation of the way how policymakers form estimates of potential output growth rates in reality.

In the model, a country which experiences a bigger recession tends to associate with a bigger increase in estimates of structural deficits ratio. Recall changes to government spending ratios during the consolidation period are matched with the data. The country which experienced a bigger recession (and a bigger increase in structural deficits) tended to associate with larger government spending cuts and larger adjustment in CAB subsequently.

Third, the learning model generates a strong negative cross-country correlation between fiscal consolidation and changes in real-time estimate of potential output growth rates. We calculate the correlation between changes in real-time estimate of CAB and potential output growth rates from 2009 to 2012 across the member states. This correlation coefficient is -0.82 in the model, while it is -0.86 in the data as shown in Section 4.2 (Evidence 5). In the model, a bigger fiscal consolidation tends to associate with larger declines in output growth rates. The latter in turn translates into larger decreases in real-time estimates of potential output growth rates as the policymaker continually learns about potential output growth rates using new data observations.

Fourth, we obtain a correlation coefficient of -0.87 between fiscal consolidation and growth forecast errors in the model, while the corresponding number in the data is -0.89 (Evidence 6). Fiscal consolidation is measured by changes in real-time estimates of CAB during 2010 − 2012. Growth forecast errors are cumulative annualized real GDP growth rates during 2010 − 2012 minus the corresponding forecast made in 2009 Q4 (period 8). Households and the policymaker are assumed to use an econometric forecasting model $\Delta \log Y_t = \omega_0^y + \omega_1^y \Delta \log K_t + \omega_2^y \Delta \log B_t + e_t^y$ (which nests the RE solution for output) to forecast output growth; similar as in (10) – (13). They learn about the parameters of the forecasting model over time using a constant gain learning rule like (14) with a gain parameter of 0.004.

In the AL model, the policymaker and households have imperfect knowledge and do not fully understand the general equilibrium consequences of fiscal consolidations. They extrapolate historical patterns (i.e. the estimated perceived law of motion) in output to forecast the growth rate of output. A country which implements a bigger fiscal consolidation tends to associate with a bigger growth forecast errors as the forecasts fail to be adequately dependent on the size of the consolidation. The learning model in this way helps to explain the correlation between the size of fiscal consolidations and growth forecast errors.

Fifth, the learning model reproduces a strong positive cross-country correlation between the pace of recovery and revisions to potential GDP growth rates during the recovery phase. This correlation coefficient is 0.86 in the model, while it is 0.91 in the data. In the model, the pace of recovery is calculated as the average real GDP growth rates during 2013 − 16 minus that during 2007 − 2012. We also compute the revision to 2012 real potential GDP growth rates from 2012 Q4 to 2016 Q3. A country which experiences a quicker recovery tends to associate with a larger upward revision to 2012 potential GDP growth rates as the policymaker uses output growth data during 2013 − 2016 to revise the estimate of 2012 potential output growth rates according to equation (7). Moreover, in the AL model, upward (or downward) revisions to potential output tends to associate with downward (or upward) revisions to output gaps and upward (or downward) revisions to CABs, consistent with the evidence in Figure A2 and 8.

9 Minimizing real-time mis-measurement of the output gap (and CAB) and counterfactual analysis

Estimates of potential output are put to work in macroeconomic policy decisions which in turn influence aggregate economic outcomes. We explore how policymakers can minimize real-time mis-measurement of output gaps (and CABs) which is of interest to EU fiscal policymakers. Standard full information RE models do not have room to analyze this issue given the assumption of policymakers' exact knowledge of the economy.

9.1 Minimizing mis-measurement

Real-time mis-measurement in output gaps and CABs is inevitable due to the uncertainty in measuring cyclical conditions. How can the policymaker minimize the real-time estimation errors? This section explores whether there exists a value of the gain parameter that minimizes the estimation errors. The gain parameter determines the weight placed on recent data observations relative to more distant observations (or degree of extrapolation). We focus on minimizing the estimation errors in output gaps (which then implies minimization of the estimation errors in CABs). We assume that the policymaker wishes to minimize the distance between the real-time estimates of the output gap and the true output gap.

Three issues deserve some discussion before we spell out the exact objective function of the policymaker. First, the true output gap is, of course, not observed. However, to implement this exercise in practice, policymakers may proxy this by the *ex post* estimate of output gap from the distant past. For example, the policymaker in 2018 has much less uncertainty about the output gap in 1990s; thus the *ex post* estimate of output gap in the distant past may be close to the true output gap. Second, as the policymaker learns using a constant gain parameter, the beliefs of trend growth do not converge pointwise to the RE value. In the exercise below, initial beliefs are drawn from a distribution.[37] Given initial beliefs, the policymaker considers alternative choices of the value of the gain parameter which in turn will yield different estimates of output gaps, fiscal policy decisions and aggregate outcomes. Third, it would be interesting to explore in future if policymakers can further minimize mis-measurement of the output gap by using the information in other variables, like inflation, wages, consumption or hours to draw inferences about potential output. However, it is likely that real-time estimates of potential output produced with this additional information may be reasonablly approximated by a simple weighted average of past real GDP growth. Thus, we focus on the issue of choosing an optimal weighting scheme or gain parameter.

We assume that this (alternative) choice of the policymaker's gain parameter does not influence households' gain parameter in their belief formation (i.e. household gain parameter is fixed in

[37] The distribution is the stationary distribution of beliefs about trend output growth rates using the actual gain parameter of the policy maker.

this exercise).[38] We think this is a plausible assumption. Consider the following scenario: around 2002, the EC switched from using HP filtering to the production function approach as the main method for estimating potential output (conceptually this is similar to alternative choices of gain parameter). Realistically, households would not understand the implications of this change by the EC and are unlikely to discount past data differently as a result. Nevertheless, there is an indirect impact from this alternative choice of the policymaker's gain parameter on households' belief formation in the AL model: this alternative gain parameter leads to different macroeconomic outcomes which in turn is used by households to form their beliefs.

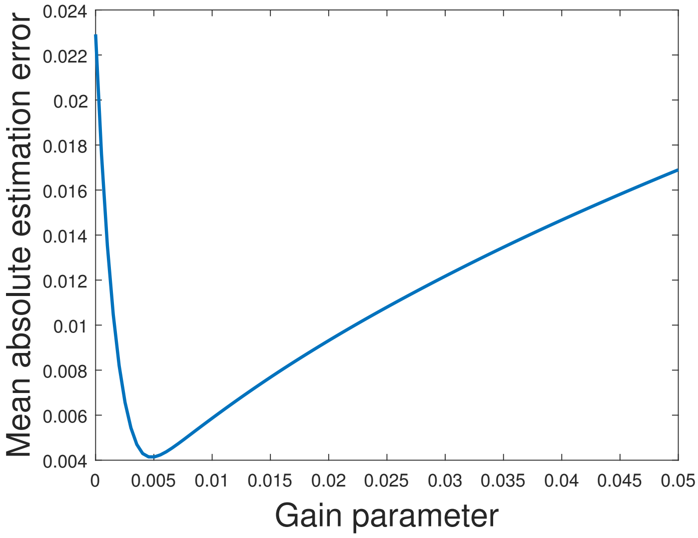

Figure 18: The mean absolute estimation error as a function of the gain parameter chosen by the policy maker

Specifically, we calculate the gain parameter which minimizes the mean absolute estimation error over a number of periods (or an evaluation span T_1) and across different initial beliefs as follows

$$Est\_error = T_1^{-1} T_2^{-1} \sum_{t=1}^{T_1} \sum_{i=1}^{T_2} \left| OG_t^S(i) - OG_t^T(i) \right| \tag{15}$$

where $OG_t^S(i)$ stands for the output gap under subjective beliefs in replication i and period t; similarly $OG_t^T(i)$ the true output gap. T_2 is the number of replications. The size of the errors in estimating output gap and the derived CAB depends on the gain parameter used by the policy-maker.

Figure 18 displays the estimation errors measured by (15) as a function of the policymaker's gain parameter in the interval $[0, 0.05]$ with $T_1 = 40$ periods (10 years) and $T_2 = 500$. The mean

[38] Similar assumptions are widespread in the AL literature eg. Adam, Beutel, Marcet and Merkel (2015) and Mitra, Evans and Honkapohja (2016).

absolute estimation error is decreasing in the gain parameter initially and then increasing. The optimal gain parameter yielding the minimum estimator error is 0.45% which is relatively small (appearing robust to the choice of T_1); the associated mean estimation error in output gaps is 0.42%.

There are two things to note here. First, the gain parameter which minimizes the estimation errors (0.0045) is much less than the calibrated gain parameter (0.034). This suggests actual policymakers may have over-reacted to the sustained output declines during the Great Recession by placing too large a weight on this recession data; note that this observation is consistent with Krugman's (2013) quote discussed in the Introduction. In ordinary least squares regressions, all data points receive equal weights and we can normalize the weight received by each data point to 1. A calibrated gain parameter of 0.034 for the policymaker implies that the weight received by the output growth data of (say) 20 years ago is $(1 - 0.034)^{80} \simeq 0.063$. In contrast, the optimal gain parameter of 0.0045 implies the weight received by the output growth data 20 years ago is much larger and equals $(1 - 0.0045)^{80} \simeq 0.70$.

Second, a moderate speed of learning (or not discounting past data too much) by the policymaker minimizes the estimation errors when forming potential output estimates. On the one hand, a large gain parameter (like 0.034) leads to a large estimation errors because potential output estimates will be quite sensitive to random innovations which leads to more frequent and larger deviations from the true value. On the other hand, a small gain (close to zero) is not optimal because potential output estimates could remain far from the true value for long periods of time since beliefs are revised slowly; this produces a substantial estimation error.[39]

9.2 Counterfactual analysis

This section provides a counterfactual analysis when the policymaker adopts the optimal gain parameter of 0.0045 from the previous section (rather than the calibrated gain parameter 0.034). With the optimal and smaller gain parameter, the learning model is simulated using the same sequence of negative productivity shocks as in Section 7. A smaller gain parameter implies placing a smaller weight to data during the Great Recession period, which yields less pessimistic estimates of trend output and CAB following the Great Recession. The latter is above the -0.5% threshold, as can be seen in the left plot of Figure 19. Since the CAB is above the -0.5% threshold, fiscal austerity due to the EU debt brake (considered in Section 7.3) is not necessary. Figure 19 shows the counterfactual path for CAB and aggregate output (along with the simulated learning paths from Section 7.3). The real output, without implementing the fiscal consolidation plans, would be much higher for the consolidation period (i.e., period $9 - 20$) relative to the case where the gain parameter 0.034 is adopted.

[39] Our model does not feature shocks to trend productivity growth rates for two reasons. First, large shocks to the trend growth rate are rarely observed in industrialized economies. Second, while data on estimates of the trend growth rate of output and output per hour may display high positive autocorrelation, this autocorrelation can be *endogenously* obtained in learning models with *zero* correlation in trend productivity growth; see the discussion at the end of Section 6 in Kuang and Mitra (2016).

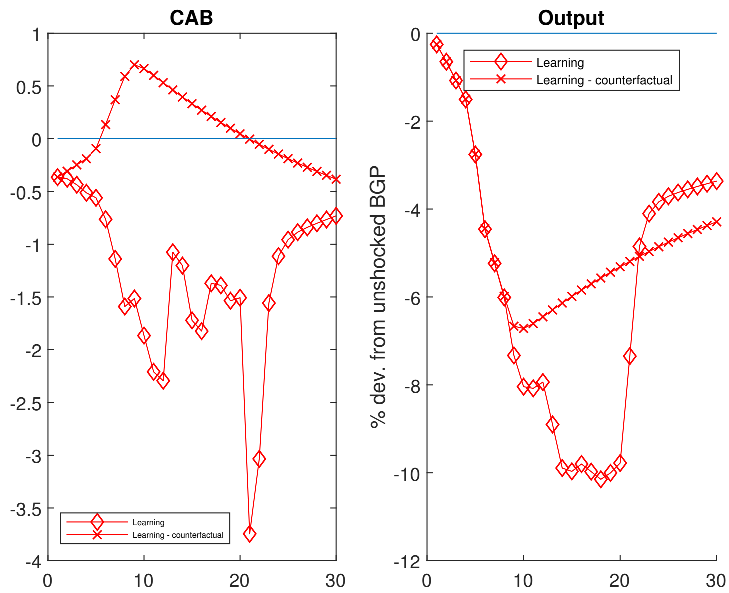

Figure 19: Couterfactual path for CAB and output (line with "x"). The lines with diamonds illustrate the learning path from section 7.3.

9.3 Shocks to trend growth rates

While the trend productivity process (2) does not contain shocks to the trend growth rate motivated by the evidence in e.g., Jarocinski and Lenza (2018), this section re-calculates the optimal gain parameter which minimizes the mis-measurement of the output gap (and CAB) by allowing for shocks to the growth rate of trend productivity. Specifically, the process (2) is modified as

$$\begin{aligned} \log(X_t/X_{t-1}) &= \gamma_t = \overline{\gamma}_t + \widehat{\widehat{\gamma}}_t, \\ \overline{\gamma}_t &= (1-\rho_\gamma)\overline{\gamma} + \rho_\gamma \overline{\gamma}_{t-1} + \widehat{\overline{\gamma}}_t, \end{aligned}$$

where $\widehat{\widehat{\gamma}}_t$ and $\widehat{\overline{\gamma}}_t$ are shocks to the level and growth rate of trend productivity, respectively. The standard deviation of $\widehat{\widehat{\gamma}}_t$ is calibrated to match the standard deviation of HP-filtered output volatility in the euro area, which yields a value of 0.3%.[40] Table 4 reports the optimal gain parameter when we vary the value of ρ_γ and the standard deviation for the trend growth shocks ($\sigma_{\widehat{\overline{\gamma}}}$). The optimal gain parameter appears somewhat higher relative to the benchmark case. But they remain much smaller than the calibrated actual gain parameter 0.034.

The counterfactual analysis is affected little by considering shocks to trend growth rates and a bit higher optimal gain parameter. Suppose the policymaker uses an optimal gain parameter of 0.0071 (say row 3 of Table 4) instead of 0.0045. In the aftermath of the Great Recession, the

[40] Using the real GDP data for Euro area (19 countries) from 1995Q1 to 2018Q3, the standard deviation is calculated as 1.13%.

subjective estimate of CAB will be slightly lower than that under $\widetilde{g} = 0.0045$. The estimate of CAB declines a bit initially and increase subsequently. The fiscal austerity in reality remains undue and there would be a less severe recession too. The counterfactual path for output remains the line with "x" in the right panel of Figure 19.

Table 4: Optimal gain parameters

Parameterization	Optimal gain parameter
$\rho_\gamma = 0.6,\ \sigma_{\widehat{\widetilde{\gamma}}} = 0.001$	0.0056
$\rho_\gamma = 0.6,\ \sigma_{\widehat{\widetilde{\gamma}}} = 0.002$	0.0061
$\rho_\gamma = 0.8,\ \sigma_{\widehat{\widetilde{\gamma}}} = 0.001$	0.0071
$\rho_\gamma = 0.8,\ \sigma_{\widehat{\widetilde{\gamma}}} = 0.002$	0.0071

10 Conclusion

The paper documents two sets of new evidence for twelf EU member states using the EU fiscal policymakers' real-time estimates which are major inputs for fiscal policy making in reality. The first set contains the pattern of revisions to real-time estimates of potential output, output gaps and structural balances. The estimates of potential output and structural balance made by the EC appear to be pro-cyclical and too sensitive to latest economic data. They were revised massively downward in the wake of the Great Recession and subsequently revised upward during the recovery phase, particularly for crisis countries. There have been large and corresponding revisions to estimates of output gaps and structural balance. Moreover, surprises to estimates of real output growth rates are strongly positively correlated with revisions to forecasts of potential output growth rates.

The second set of new evidence concerns fiscal consolidation in the aftermath of the Great Recession and its relation to economic performance in the member states. First, a country with a higher level of structural deficits (according to the European Commission's estimates) in the aftermath of the Global Financial Crisis tends to associate with fiscal consolidation with a bigger size and faster pace. Second, a country which implemented a bigger fiscal consolidation tends to associate with a bigger reduction in real-time estimates of potential output growth and a bigger growth forecast error.[41]

Targeting structural balance has become a central element in the EU fiscal policy surveillance since 2005. A major challenge to implementing this policy is the unreliability of real-time estimates of output gaps and structural balance. The concept of structural balance and this major policy are rarely analyzed in the context of macroeconomic models. The paper develops a business cycle model to analyze the implications of mis-measuring potential output and structural balance for fiscal policy decisions and the EU recession. Moreover, the model provides recommendations for minimizing real time mis-measurement of output gap and structural balance.

[41] The evidence on the relation between fiscal consolidation and growth forecast errors across EU countries is provided by following Blanchard and Leigh (2013) but using the EC data.

In the model, the policymaker faces real-time uncertainty and learns about potential output. It features a negative feedback loop created by the dynamic interaction between pessimism about potential output and structural balance, fiscal austerity and macroeconomy. The policymaker overreacted to sustained declines in aggregate output following the Great Recession with too aggressive downward revisions to potential output. The model displays secular-stagnation-like slowdown according to the policymaker's estimates. The pessimism about potential output yields large underestimation of structural fiscal balances which then triggers undue austerity due to targeting structural balance in the EC fiscal policy framework. The mutual reinforcement between potential output pessimism and undue austerity contributed significantly to further recession in the learning model.

The model generates a realistic magnitude of government spending multiplier and a larger multiplier during recessions. Uniquely, the model reproduces the two sets of evidence we documented. Moreover, it replicates another evidence that potential output estimates respond persistently to transitory TFP shocks which standard full-information RE models cannot reproduce.

We explored how policymakers can reduce real-time mis-measurement of output gap and structural balance which is of interest for EU fiscal policy making. The model suggests that when forming beliefs about potential output, policymakers should place a moderate weight on recent data, which is much smaller than that used by policymakers in reality (calibrated to match the massive downward revisions to the potential output estimates in response to the Great Recession). If policymakers placed a much smaller weight to the data during the Great Recession period, there would be higher estimate of structural budget balances, less fiscal austerity and a less severe economic recession.

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Appendix (For Online Publication)

A Additional evidence

A.1 Relation between the severity of recession and revisions to pre-crisis real potential GDP growth rates

Figure A.1 shows that the size of the revision to the average growth rate of real potential output of a country during the *pre-crisis* period (e.g., $2005-07$) is strongly positively correlated with the severity of the recession during $2008-10$ measured by the average growth rate of real output over $2008-10$ minus that over a previous period (i.e., $2003-07$); the correlation coefficient is 0.77 and statistically significant at 1% level. A country which has a larger recession during $2008-10$ tends to have a larger downward revision to average growth rate of real potential output over the pre-crisis years $2005-07$.

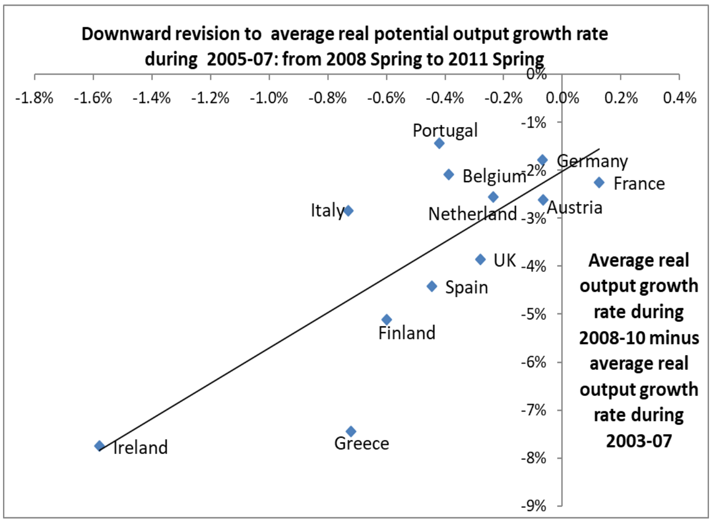

Figure A.1: Relation between severity of the recession and revisions to the growth rate of pre-crisis real potential output across countries

A.2 Revision to potential output growth rates and output gaps during the recovery phase

Figure A.2 shows revisions to the estimate of 2012 potential growth rates and output gaps: from 2013 Autumn to 2016 Autumn. This figure is referred to in Section 3.2 and discussed there.

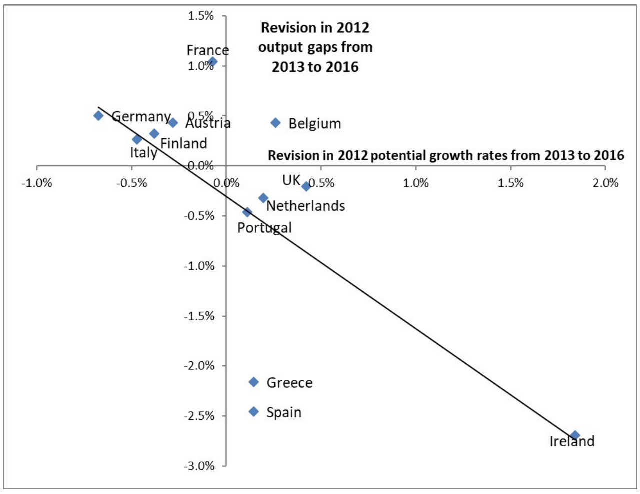

Figure A.2: Revision to the estimate of 2012 potential growth rates and output gaps: from 2013 Autumn to 2016 Autumn

A.3 Structural deficits in 2009 and subsequently planned fiscal consolidation

Using data in Table 2, Figure A.3 plots the change in real-time estimates of real potential GDP growth rates from 2009 to 2012 against the change in real-time estimate of structural balance from 2009 to 2012 (during the consolidation period of 2010 - 2012). The cross-country correlation between planned fiscal consolidation during 2010 - 2012 and changes to real-time estimate of real potential GDP growth rates during the same period is -0.85.

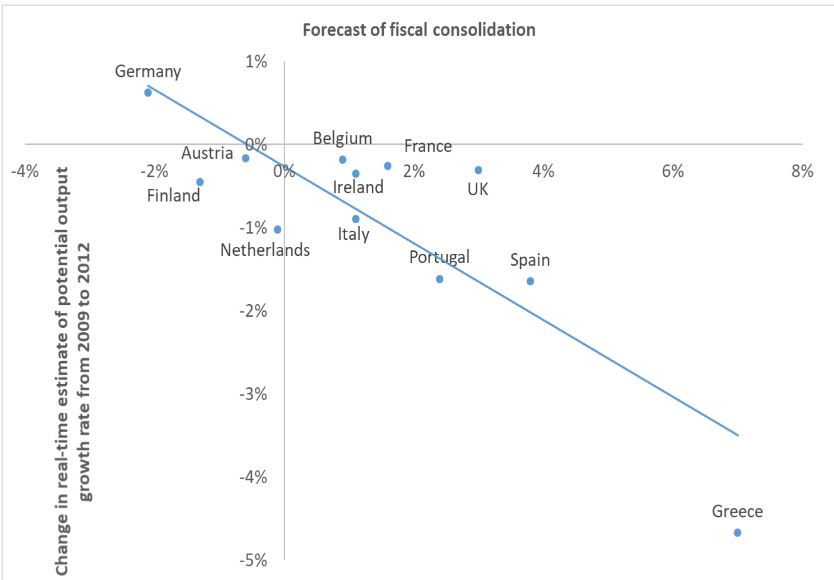

Figure A.3: Forecasts of fiscal consolidation and revisions to real-time estimates of potential GDP growth rates

A.4 Planned fiscal consolidation and growth forecast errors

Figure A.4 plots cumulative forecast error for real GDP growth in 2010, 2011 and 2012 relative to the forecasts of (or planned) fiscal consolidation for 2010, 2011 and 2012 made in Spring of year 2010.

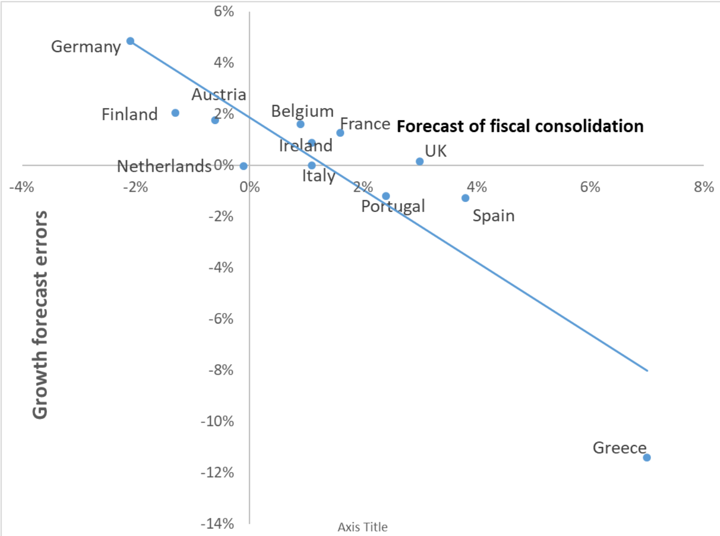

Figure A.4: Growth forecast errors versus forecasts of fiscal consolidation

B Target output growth rates and changes in government spending to GDP ratios: data vs model

The size of productivity shocks is chosen to match the recession in 2008 – 09. Table A.1 reports the 2008 and 2009 output growth rate for the eurozone and individual countries in the data and model. The size of fiscal consolidation is chosen to match the changes in government spending to GDP ratios during 2010 – 2012 relative to 2009. Table A.2 reports government spending to GDP ratios in 2010, 2011 and 2012 (minus that of 2009) in the data and model.

Table A.1: Target output growth: data vs model (%)

	Data		Model	
	2008	2009	2008	2009
Euro-19	0.5	−4.5	0.49	−4.5
Belgium	0.75	−2.28	−0.75	−2.2
Germany	1.08	−5.61	1.08	−5.67
Greece	−0.34	−4.3	−0.33	−4.3
Spain	1.12	−3.57	1.10	−3.55
France	0.20	−2.94	0.20	−2.95
Ireland	−2.16	−5.64	−2.16	−5.63
Italy	−1.05	−5.48	−1.07	−5.48
Netherlands	1.70	−3.77	1.69	−3.76
Austria	1.55	−3.80	1.57	−3.75
Portugal	0.20	−2.98	0.19	−2.98
Finland	0.72	−8.27	0.73	−8.21
UK	−0.47	−4.19	−0.47	−4.19

Table A.2: changes in government expenditure to GDP ratios (relative to year 2009 and in percentage points)

	Data			Model		
	2010	2011	2012	2010	2011	2012
Euro-19	−0.47	−1.17	−1.37	−0.47	−1.17	−1.37
Belgium	−0.47	−0.15	0.44	−0.47	−0.15	0.44
Germany	−0.54	−0.95	−0.92	−0.54	−0.96	−0.92
Greece	−3.10	−4.72	−4.77	−3.16	−4.71	−4.77
Spain	−0.37	−1.36	−3.34	−0.38	−1.36	−3.31
France	−0.19	−0.64	−0.43	−0.20	−0.64	−0.44
Ireland	−1.75	−3.13	−4.27	−1.75	−3.16	−4.29
Italy	−0.71	−1.63	−1.87	−0.71	−1.62	−1.88
Netherlands	0.15	−0.41	−0.47	0.15	−0.42	−0.47
Austria	−0.39	−1.19	−1.32	−0.37	−1.19	−1.32
Portugal	0.48	−2.17	−4.52	0.49	−2.18	−4.54
Finland	−0.63	−0.80	0.15	−0.63	−0.80	0.16
UK	−0.40	−1.37	−1.78	−0.40	−1.38	−1.77

C Impulse response analysis

Figure A.5 depicts the impulse response functions under RE and learning for 40 quarters in response to an unexpected negative productivity shock of 0.45% in period 1. Percentage deviations from the unshocked balanced growth path are plotted for output, consumption and investment. Therefore, they will not return to zero in this figure. For hours, deviations from the steady state are plotted. For G/Y ratio and debt-to-GDP ratio, percentage changes are plotted.

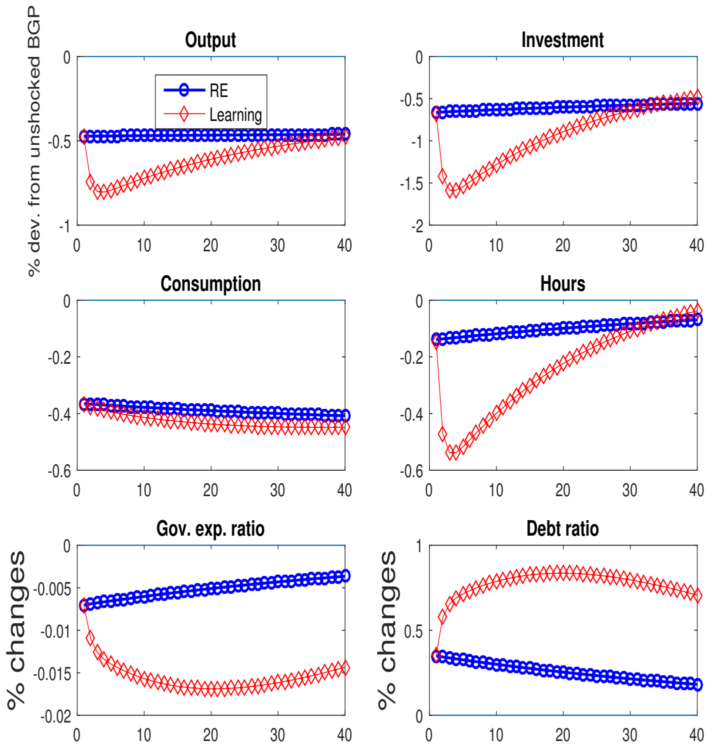

Figure A.5: Response to a 0.45% unexpected negative i.i.d productivity shock.

We first describe the dynamic effects under RE. There is an abundance of capital after the shock which decreases the marginal product of capital and hence the real interest rate. This implies the marginal utility of consumption must increase over time. Capacity utilization declines on impact due to the decline in rental rates. Output decreases initially because working hours, capacity utilization and productivity decrease initially. The low interest rates induce falling consumption and leisure over time. Output, on the other hand, increases slightly over time: this is because capacity utilization and working hours increase over time and this is sufficiently pronounced to offset the effect on output from the decline in capital stock. Debt-to-GDP ratio increases initially and then decreases gradually. Government expenditures to GDP ratio responds negatively to a rising debt-to-GDP ratio and positively to a negative output gap. However, the former effect dominates so that government expenditures to GDP ratio falls initially. Subsequently, in response to the fall in the debt ratio, government expenditures to GDP ratio increases gradually.

We now turn to the dynamics under AL. After the impact period, the low realization of rental rates and wage rates leads to downward revisions to the forecasts of the discounted sum of wages and rental rates. The dominance of the pessimism about future wage rates produces a further decline in consumption. The response of these forecasts following a productivity shock in our model is similar to those in the model of KM; see Figure 3 of KM which considers a *positive* shock (in contrast to the negative shock here). We suppress the plots of the forecast of discounted sum of wage rates and rental rates here. Working hours decrease further as firms' labor demand decreases and the constant-consumption labor supply shifts inward. The decline in the return to capital due to declining productivity and working hours induces an decline in investment. Output also decreases due to decreases in productivity, working hours, investment and capacity utilization. The decrease in output reduces tax revenues, which offsets the decline in government expenditures and leads to a persistent rise in debt-to-GDP ratio. At some point, investment and hours revert and move towards the steady state. During the initial periods of the reversion of output, the effect of lower tax revenues dominates so that the debt ratio continue to increase. Overall, the amplification in the response of output, hours and investment under learning arises mainly from households' belief revisions and the mutual reinforecement between households' wages (and income) forecasts and equilibrium outcomes.

CGU (2018) find that the estimates of potential output made by a range of public institutions decline (or rise) persistently in response to a negative (or positive) TFP shock. The RE version of the model is inconsistent with this finding because it does not produce a *persistent* response of estimate of potential output made by the policymaker to the i.i.d shock (due to her exact knowledge of the evoluation of potential output). In contrast, our learning model is consistent with this finding. The negative productivity shock leads to a fall in aggregate output and the policymaker uses the new observation of output to revise downward her estimate of potential output; see equation (7) and the top left plot of Figure A.6.

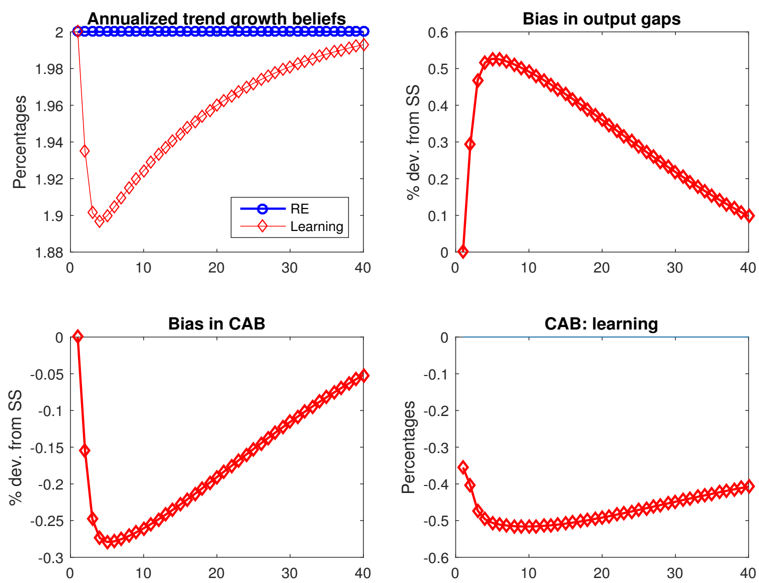

Figure A.6: Policy maker's beliefs: following a 0.45% unexpected negative i.i.d productivity shock.

The downward revision to subjective estimates of trend output made by the policymaker is associated with an underestimation of the size of output gaps, so the estimation error in the output gap is positive (recall this bias is given by subjective minus true output gaps, see equation (8)). The underestimation of the output gap in turn generates an underestimation of CAB, as can be seen from the lower left plot. The size of the estimation error in CAB is calculated by equation (9). Subjective estimates of CAB made by the policymaker become negative following the shock and decline further for several periods and then increase towards the steady state value, see the bottom right plot.

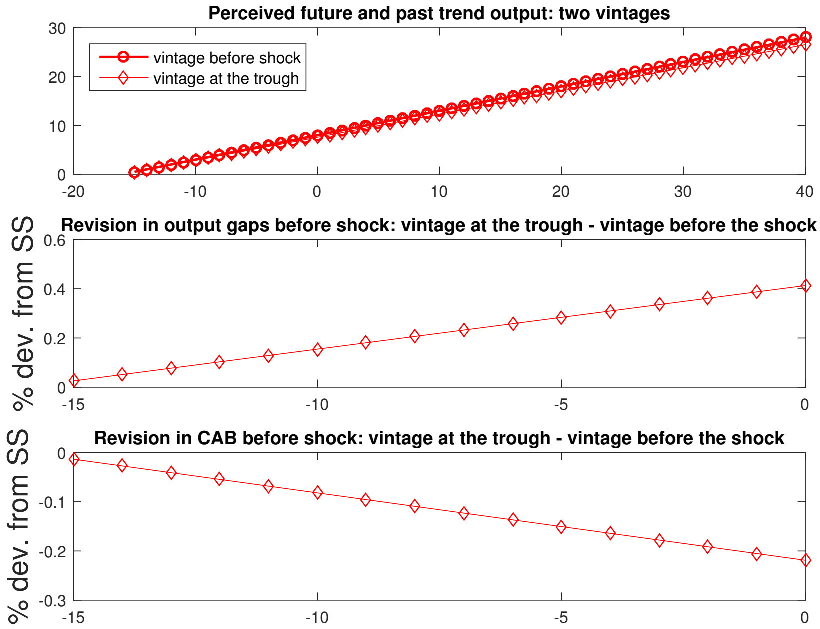

Figure A.7: Changes to policy maker's view following a 0.45% unexpected negative i.i.d productivity shock.

Note: Period $-n$ in Figure A.7 corresponds to $(n+1)$ quarters before the shock for $n = 0, 1, 2...$

We now study how the negative shock changes policymakers' beliefs about the evolution of trend output and historical estimates of output gaps and structural fiscal balances. We consider two vintages: one before the shock and the other when aggregate output reaches its trough (which is period 4 in this simulation). The top plot of Figure A.7 displays the evolution of the deterministic component of trend output before and after the shock (with initial period trend output normalized to zero). The quantitative difference is not that apparent in the top plot since the shock is only one-off. Before the shock, trend output (in the past and future) are perceived to grow at the rate of $log(\overline{\gamma})$. When output reaches its trough, policymakers perceive a lower growth rate of trend output for future and past (since policymakers use lower than expected growth rates to revise estimates of growth rates of trend output by equation (7)). Trend output up to 4 years backward (recall $N = 16$) and in the future are revised downward. The downward revision to the historical estimate of trend output is associated with an upward revision to historical estimate of output gaps (the middle plot) and a downward revision to historical estimate of CABs (the lower plot).

Note under RE, the negative productivity shock will not lead to revisions to historical estimate of potential output, output gaps and CABs due to the policymaker's exact knowledge of the economy.

D The Model

The variables with a bar are the non-stochastic steady state values while the variables with a hat denote log-linearized variables around the non-stochastic steady state i.e. $x_t = \log \frac{X_t}{\bar{X}}$. Capital letters denote levels while small case letters denote their stationary counterpart.

D.1 Model Setup

Household's problem

The representative household maximizes

$$\widehat{E}_t \sum_{t=0}^{\infty} \beta^t u(C_t, L_t);$$

$$u(C_t, L_t) = \frac{C_t^{1-\sigma} v(1-L_t)}{1-\sigma}$$

subject to the flow budget constraint

$$\begin{aligned} C_t + K_{t+1} + B_{t+1} &= \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right) K_t \\ &+ W_t H_t \left(1-\tau^H\right) + K_t + R_t B_t - T_t \qquad (16) \\ L_t &= 1 - H_t \qquad (17) \end{aligned}$$

$\delta(U_t) = \frac{1}{\theta} U_t^{\theta}$. Define $A_t = B_t + K_t$.The arbitrage condition implies

$$R_t = 1 + \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right). \qquad (18)$$

The budget constraint can be simplified as

$$C_t + A_{t+1} = R_t A_t + W_t H_t \left(1-\tau^H\right) - T_t \qquad (19)$$

Evolution of capital $K_{t+1} = (1-\delta(U_t))K_t + I_t$.

Firm's problem

The firm's problem is

$$\max_{U_t K_t, H_t} Y_t - W_t H_t - R_t^K (U_t K_t) \qquad (20)$$

subject to the production technology

$$Y_t = (U_t K_t)^{\alpha} (X_t H_t)^{1-\alpha} \tag{21}$$

Resource constraint

$$Y_t = C_t + I_t + G_t \tag{22}$$

Fiscal rule

Government budget constraint

$$B_{t+1} = B_t R_t - \left(R_t^K U_t - \delta(U_t)\right) K_t \tau^K - W_t H_t \tau^H + G_t - T_t \tag{23}$$

Government expenditures follow

$$\begin{aligned} G_t/Y_t &= \lambda_0 + \lambda_1 OG_t + \lambda_2 B_t/Y_t + u_t^G, \\ \text{where } OG_t &= Y_t/Y_t^P - 1 \end{aligned} \tag{24}$$

Y_t^P policymakers' estimate of trend output.

D.2 First Order Condition

Households' first-order conditions

The Lagrangian for the household problem is

$$\widehat{E}_t \sum_{t=0}^{\infty} \beta^t \left(u(C_t, L_t) + \Lambda_t \left(\begin{array}{c} \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right) K_t + W_t H_t \left(1-\tau^H\right) + \\ K_t + R_t B_t - C_t - K_{t+1} - B_{t+1} - T_t \end{array} \right) \right) \tag{25}$$

Household optimization yields the following first-order conditions:

$$C_t \; : \; u_C(C_t, L_t) = \Lambda_t \tag{26}$$

$$K_{t+1} \; : \; \beta \widehat{E}_t \Lambda_{t+1} R_{t+1} - \Lambda_t = 0 \tag{27}$$

$$L_t \; : \; u_L(C_t, L_t) = \Lambda_t W_t \left(1-\tau^H\right) \tag{28}$$

$$U_t \; : \; R_t^K = \delta'(U_t) \tag{29}$$

$$B_{t+1} \; : \; \beta \widehat{E}_t \Lambda_{t+1} R_{t+1} = \Lambda_t \tag{30}$$

where Λ_t is the Lagrangian multiplier associated with period t budget constraint.

Firm's first-order conditions

The firm's first order condition with respect to hours is $W_t = (1-\alpha)(U_t K_t)^{\alpha} \; (X_t)^{1-\alpha} H_t^{-\alpha}$. The capital input decision gives $R_t^K = \alpha(U_t \frac{K_t}{X_t})^{\alpha-1}(H_t)^{1-\alpha}$.

D.3 Detrended Equations

For consumption,

$$\lambda_t \equiv X_t^{\sigma}\Lambda_t = X_t^{\sigma}u_c\left(C_t, L_t\right) = X_t^{\sigma}C_t^{-\sigma}\upsilon\left(H_t\right) = c_t^{-\sigma}\upsilon\left(H_t\right)$$

For capital,

$$\begin{aligned} 1 &= \beta\widehat{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(1+\left(R_{t+1}^{K}U_{t+1}-\delta(U_{t+1})\right)\left(1-\tau^{K}\right)\right)\right] \\ &= \beta\widehat{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{X_{t+1}^{\sigma}}{X_{t+1}^{\sigma}}\frac{X_t^{\sigma}}{X_t^{\sigma}}\left(1+\left(R_{t+1}^{K}U_{t+1}-\delta(U_{t+1})\right)\left(1-\tau^{K}\right)\right)\right] \\ &= \beta\widehat{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\gamma_{t+1}^{\sigma}}\left(1+\left(R_{t+1}^{K}U_{t+1}-\delta(U_{t+1})\right)\left(1-\tau^{K}\right)\right)\right] \qquad (31) \end{aligned}$$

For leisure,

$$\begin{aligned} \lambda_t w_t\left(1-\tau^{H}\right) &= X_t^{\sigma-1}\Lambda_t W_t\left(1-\tau^{H}\right) = X_t^{\sigma-1}u_L\left(C_t, L_t\right) \\ &= -X_t^{\sigma-1}\frac{C_t^{1-\sigma}}{1-\sigma}\upsilon'\left(H_t\right) = -\frac{c_t^{1-\sigma}}{1-\sigma}\upsilon'\left(H_t\right) \end{aligned}$$

Evolution of capital

$$\begin{aligned} \frac{K_{t+1}}{X_t} &= (1-\delta(U_t))\frac{K_t}{X_{t-1}}\frac{X_{t-1}}{X_t}+\frac{I_t}{X_t} \\ k_{t+1} &= (1-\delta(U_t))\frac{k_t}{\gamma_t}+i_t \qquad (32) \end{aligned}$$

The household budget constraint becomes

$$\begin{aligned} \frac{C_t}{X_t}+\frac{K_{t+1}}{X_t}+\frac{B_{t+1}}{X_t} &= \left[1+\left(R_t^{K}U_t-\delta(U_t)\right)\left(1-\tau^{K}\right)\right]\frac{K_t}{X_{t-1}}\frac{X_{t-1}}{X_t} \\ &\quad +\frac{W_t}{X_t}H_t\left(1-\tau^{H}\right)+R_t\frac{B_t}{X_{t-1}}\frac{X_{t-1}}{X_t}-\tau \\ c_t+k_{t+1}+b_{t+1} &= \left[1+\left(R_t^{K}U_t-\delta(U_t)\right)\left(1-\tau^{K}\right)\right]\frac{k_t}{\gamma_t} \\ &\quad +w_tH_t\left(1-\tau^{H}\right)+R_t\frac{b_t}{\gamma_t}-\tau \end{aligned}$$

where we assume T_t/X_t is a constant τ.

The simplified household budget constraint (19) becomes

$$\frac{C_t}{X_t} + \frac{A_{t+1}}{X_t} = R_t \frac{A_t}{X_{t-1}} \frac{X_{t-1}}{X_t} + \frac{W_t H_t}{X_t} \left(1 - \tau^H\right) - \tau \tag{33}$$

$$c_t + a_{t+1} = R_t a_t \gamma_t^{-1} + w_t H_t \left(1 - \tau^H\right) - \tau \tag{34}$$

The production function

$$\begin{aligned} Y_t &= (U_t K_t)^\alpha (X_t H_t)^{1-\alpha} \\ \frac{Y_t}{X_t} &= (U_t \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t})^\alpha (H_t)^{1-\alpha} \\ y_t &= (U_t k_t \gamma_t^{-1})^\alpha (H_t)^{1-\alpha} \end{aligned}$$

For capital demand,

$$\begin{aligned} R_t^K &= \alpha (U_t \frac{K_t}{X_t})^{\alpha-1} (H_t)^{1-\alpha} \\ &= \alpha (U_t \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t})^{\alpha-1} (H_t)^{1-\alpha} \\ &= \alpha (\frac{U_t k_t}{\gamma_t})^{\alpha-1} (H_t)^{1-\alpha} \end{aligned} \tag{35}$$

Using the definition of output yields

$$R_t^K = \alpha \gamma_t \frac{y_t}{U_t k_t} \tag{36}$$

For labor demand,

$$\begin{aligned} W_t &= (1-\alpha)\,(U_t K_t)^\alpha (X_t)^{1-\alpha} H_t^{-\alpha} \\ \frac{W_t}{X_t} &= (1-\alpha)\,(U_t \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t})^\alpha H_t^{-\alpha} \\ w_t &= (1-\alpha) \gamma_t^{-\alpha} (U_t k_t)^\alpha H_t^{-\alpha} \end{aligned}$$

Using the definition of output gives

$$w_t = (1-\alpha) \frac{y_t}{H_t} \tag{37}$$

Government budget constraint

$$\begin{aligned} B_{t+1} &= B_t R_t - \left(R_t^K U_t - \delta(U_t)\right) K_t \tau^K - W_t H_t \tau^H + G_t - T_t \\ \frac{B_{t+1}}{X_t} &= \frac{B_t}{X_{t-1}} \frac{X_{t-1}}{X_t} R_t - \left(R_t^K U_t - \delta(U_t)\right) \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t} \tau^K - \frac{W_t}{X_t} H_t \tau^H + \frac{G_t}{X_t} - \frac{T_t}{X_t} \\ b_{t+1} &= \frac{b_t}{\gamma_t} R_t - \left(R_t^K U_t - \delta(U_t)\right) \frac{k_t}{\gamma_t} \tau^K - w_t H_t \tau^H + g_t - \tau \end{aligned}$$

Resource constraint (22)

$$\begin{aligned}\frac{Y_t}{X_t} &= \frac{C_t}{X_t} + \frac{I_t}{X_t} + \frac{G_t}{X_t} \\ y_t &= c_t + i_t + g_t \end{aligned} \tag{38}$$

Government expenditure

$$\frac{G_t/X_t}{Y_t/X_t} = \lambda_0 + \lambda_1 OG_t + \lambda_2 \frac{\frac{B_t}{X_{t-1}}\frac{X_{t-1}}{X_t}}{\frac{Y_t}{X_t}} + u_t^G \tag{39}$$

$$\frac{g_t}{y_t} = \lambda_0 + \lambda_1 \frac{y_t - y_t^P}{y_t^P} + \lambda_2 \frac{b_t}{y_t}\gamma_t^{-1} + u_t^G \tag{40}$$

$$OG_t = \frac{y_t - y_t^P}{y_t^P} \tag{41}$$

The definition of A_t

$$\begin{aligned} A_t &= B_t + K_t \\ \frac{A_t}{X_{t-1}} &= \frac{B_t}{X_{t-1}} + \frac{K_t}{X_{t-1}} \\ a_t &= b_t + k_t \end{aligned} \tag{42}$$

D.4 Steady State

The definition of interest rates (18) yields

$$\overline{R} = \left[1 + \left(\overline{R}^K \overline{U} - \overline{\delta}\right)\left(1 - \tau^K\right)\right] \tag{43}$$

The Euler equation (31) implies

$$1 = \beta \frac{1}{\overline{\gamma}^{\sigma}} \overline{R}$$

and

$$\overline{R} = \frac{\overline{\gamma}^{\sigma}}{\beta}$$

Define $\widetilde{\beta} \equiv \overline{R}^{-1}\overline{\gamma}$. So

$$\overline{R} = \frac{\overline{\gamma}}{\widetilde{\beta}}$$

and

$$\beta = \widetilde{\beta}\overline{\gamma}^{\sigma-1}$$

(43) gives

$$\overline{R}^K \overline{U} = \frac{\overline{R} - 1}{1 - \tau^K} + \overline{\delta} \tag{44}$$

The capacity utilization equation (29) implies $\overline{R}^K = \delta'(\overline{U}) = \overline{U}^{\theta-1} = \frac{\theta\overline{\delta}}{\overline{U}}$ or equivalently

$$\theta = \frac{\overline{R}^K\overline{U}}{\overline{\delta}} = \frac{\overline{R}-1}{\overline{\delta}\left(1-\tau^K\right)} + 1 \tag{45}$$

The capital demand equation (36) implies

$$\begin{aligned} R_t^K &= \alpha\gamma_t\frac{y_t}{U_t k_t} \\ \frac{\overline{R}^K\overline{U}\overline{k}}{\overline{y}\overline{\gamma}} &= \alpha \end{aligned}$$

Note

$$\frac{\overline{k}}{\overline{y}} = \frac{\alpha\overline{\gamma}}{\overline{R}^K\overline{U}} = \frac{\alpha\overline{\gamma}}{\frac{\overline{R}-1}{1-\tau^K} + \overline{\delta}} \tag{46}$$

The labor demand equation (37) implies

$$\frac{\overline{w}\overline{H}}{\overline{y}} = 1-\alpha$$

The household budget constraint (34) implies

$$\overline{c} + \overline{a} = \overline{R}\overline{a}\overline{\gamma}^{-1} + \overline{w}\overline{H}\left(1-\tau^H\right) - \tau \tag{47}$$

$$\frac{\overline{c}}{\overline{a}} + 1 = \overline{R}\overline{\gamma}^{-1} + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right) - \frac{\tau}{\overline{a}} \tag{48}$$

The capital accumulation equation (32) yields

$$k_{t+1} = (1-\delta(U_t))\frac{k_t}{\gamma_t} + i_t \tag{49}$$

$$\overline{k} = (1-\overline{\delta})\frac{\overline{k}}{\overline{\gamma}} + \overline{i}$$

$$\frac{\overline{i}}{\overline{k}} = 1 - \frac{(1-\overline{\delta})}{\overline{\gamma}} \tag{50}$$

Investment to output ratio is determined by combining (46) and (50)

$$\begin{aligned} \frac{\overline{i}}{\overline{y}} &= \frac{\overline{i}/\overline{k}}{\overline{y}/\overline{k}} \\ &= \frac{1-\frac{(1-\overline{\delta})}{\overline{\gamma}}}{\frac{\overline{R}^K\overline{U}}{\alpha\overline{\gamma}}} \end{aligned} \tag{51}$$

The resource constraint (38) yields

$$1 = \frac{\overline{c}}{\overline{y}} + \frac{\overline{i}}{\overline{y}} + \frac{\overline{g}}{\overline{y}} \tag{52}$$

So

$$\frac{\overline{c}}{\overline{y}} = 1 - \frac{\overline{i}}{\overline{y}} - \frac{\overline{g}}{\overline{y}} \tag{53}$$

The government budget constraint yields

$$\begin{aligned}
b_{t+1} &= \frac{b_t}{\gamma_t}R_t - \left(R_t^K U_t - \delta\left(U_t\right)\right)\frac{k_t}{\gamma_t}\tau^K - w_t H_t \tau^H + g_t - \tau \\
\overline{b} &= \frac{\overline{b}}{\overline{\gamma}}\overline{R} - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\frac{\overline{k}}{\overline{\gamma}}\tau^K - \overline{w}\overline{H}\tau^H + \overline{g} - \tau \\
\frac{\overline{b}}{\overline{y}}\left(1 - \frac{\overline{R}}{\overline{\gamma}}\right) &= -\frac{\overline{R}^K\overline{U} - \overline{\delta}}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\tau^K - \frac{\overline{w}\overline{H}}{\overline{y}}\tau^H + \frac{\overline{g}}{\overline{y}} - \tau \qquad (54) \\
&= -\alpha\tau^K - (1-\alpha)\tau^H + \frac{\overline{g}}{\overline{y}} + \frac{\overline{\delta}}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\tau^K - \tau \qquad (55)
\end{aligned}$$

The fiscal rule (40) yields

$$\frac{\overline{g}}{\overline{y}} = \lambda_0 + \lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1} \tag{56}$$

The definition of A_t (42) yields

$$\frac{\overline{a}}{\overline{y}} = \frac{\overline{b}}{\overline{y}} + \frac{\overline{k}}{\overline{y}} \tag{57}$$

The marginal utility of consumption gives

$$\overline{\lambda} = \overline{c}^{-\sigma}\upsilon(\overline{H})$$

and the first order condition for labor supply yields

$$\overline{\lambda}\overline{w}(1-\tau^H) = -\frac{\overline{c}^{1-\sigma}}{1-\sigma}\upsilon'(\overline{H})$$

Combining the above two equations delivers

$$\begin{aligned}
\overline{c}^{-\sigma}\upsilon(\overline{H})\overline{w}(1-\tau^H) &= -\frac{\overline{c}^{1-\sigma}}{1-\sigma}\upsilon'(\overline{H}) \\
\frac{\overline{w}\overline{H}(1-\tau^H)}{\overline{c}} &= -\frac{\upsilon'(\overline{H})\overline{H}}{\upsilon(\overline{H})}(1-\sigma)^{-1}
\end{aligned}$$

Define

$$\psi \equiv -\frac{\overline{H}v'(\overline{H})}{v(\overline{H})}(1-\sigma)^{-1} = \frac{\overline{w}\overline{H}(1-\tau^H)}{\overline{c}}$$
$$= \frac{\overline{w}\overline{H}}{\overline{y}}\frac{\overline{y}}{\overline{c}}(1-\tau^H) = (1-\alpha)\frac{\overline{y}}{\overline{c}}(1-\tau^H)$$

D.5 Log-linearization

Households

1. Marginal utility of consumption

$$\widehat{\lambda}_t = -\sigma\widehat{c}_t - \psi(1-\sigma)\widehat{H}_t \tag{58}$$

where in steady state

$$\psi \equiv -\frac{\overline{H}v'(\overline{H})}{v(\overline{H})}(1-\sigma)^{-1} = \frac{\overline{w}\overline{H}}{\overline{c}}\left(1-\tau^H\right)$$

2. Euler equation. Starting from

$$1 = \beta\widehat{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\gamma_{t+1}^{\sigma}}\left(1+\left(R_{t+1}^K U_{t+1} - \delta(U_{t+1})\right)\left(1-\tau^K\right)\right)\right]$$
$$0 = \frac{\beta}{\overline{\gamma}^{\sigma}}\widehat{E}_t\left[\overline{R}\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \sigma\widehat{\gamma}_{t+1}\right) + \overline{R}^K\overline{U}\left(1-\tau^K\right)(\widehat{R}_{t+1}^K + \widehat{U}_{t+1}) - \overline{\delta}\theta\left(1-\tau^K\right)\widehat{U}_{t+1}\right]$$
$$0 = \widehat{E}_t\left[\overline{R}\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \sigma\widehat{\gamma}_{t+1}\right) + \overline{R}^K\overline{U}\left(1-\tau^K\right)\widehat{R}_{t+1}^K\right]$$
$$0 = \widehat{E}_t\left[\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \sigma\widehat{\gamma}_{t+1}\right) + \frac{\beta}{\overline{\gamma}^{\sigma}}\overline{R}^K\overline{U}\left(1-\tau^K\right)\widehat{R}_{t+1}^K\right]$$

Note $\beta\frac{\overline{R}}{\overline{\gamma}^{\sigma}} = 1$, $\delta'(\overline{U})\overline{U} = \delta\theta$.

3. Labor-leisure choice

$$\lambda_t w_t\left(1-\tau^H\right) = -\frac{c_t^{1-\sigma}}{1-\sigma}\upsilon'(H_t)$$
$$\overline{\lambda}\overline{w}\left(\widehat{\lambda}_t + \widehat{w}_t\right) = -\frac{\overline{c}^{1-\sigma}}{(1-\sigma)\left(1-\tau^H\right)}\upsilon'\left(\overline{H}\right)\left((1-\sigma)\widehat{c}_t + \frac{\upsilon''\left(\overline{H}\right)\overline{H}}{\upsilon'\left(\overline{H}\right)}\widehat{H}_t\right)$$
$$(1-\sigma)\widehat{c}_t + \epsilon_v\widehat{H}_t = \widehat{\lambda}_t + \widehat{w}_t$$
$$\epsilon_v = \frac{\upsilon''\left(\overline{H}\right)\overline{H}}{\upsilon'\left(\overline{H}\right)} > 0$$

Combining the second equation above with the expression for marginal utility, gives

$$\sigma^{-1}\widehat{\lambda}_t + \widehat{w}_t = \epsilon_H\widehat{H}_t \tag{59}$$

where

$$\epsilon_H = \epsilon_v - \frac{(\sigma-1)^2}{\sigma}\psi > 0$$

is the inverse Frisch elasticity of labor supply.

4. Capital utilization

$$\begin{aligned} R_t^K &= \delta'(U_t) \\ \widehat{R}_t^K \overline{R}^K &= \delta' \frac{\delta'' \overline{U}}{\delta'} \widehat{U}_t \\ \widehat{U}_t &= \frac{1}{\theta-1} \widehat{R}_t^K \end{aligned}$$

Note $\frac{\delta''\overline{U}}{\delta'} = \theta - 1$.

Using the expression for marginal utility of consumption, the Euler equation yields

$$\begin{aligned} 0 &= \widehat{E}_t \left[\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \sigma\widehat{\gamma}_{t+1} \right) + \beta \frac{\overline{R}^k \overline{U} \left(1-\tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K \right] \\ \widehat{\lambda}_t &= \widehat{E}_t \left[\left(\widehat{\lambda}_{t+1} - \sigma\widehat{\gamma}_{t+1} \right) + \beta \frac{\overline{R}^k \overline{U} \left(1-\tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K \right] \\ -\sigma\widehat{c}_t - \psi(1-\sigma)\widehat{H}_t &= \widehat{E}_t \left[-\sigma\widehat{c}_{t+1} - \psi(1-\sigma)\widehat{H}_{t+1} \right] - \sigma\widehat{E}_t\widehat{\gamma}_{t+1} + \widehat{E}_t\beta \frac{\overline{R}^k \overline{U} \left(1-\tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K \\ \widehat{Q}_t &= -\widehat{E}_t\beta \frac{\overline{R}^k \overline{U} \left(1-\tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K + \widehat{E}_t\widehat{Q}_{t+1} + \sigma\widehat{E}_t\widehat{\gamma}_{t+1} \end{aligned}$$

where

$$\widehat{Q}_t = \sigma\widehat{c}_t + \psi(1-\sigma)\widehat{H}_t.$$

Combining the marginal utility of consumption and labor-leisure choice yields

$$\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi \right) \widehat{H}_t = \widehat{w}_t - \widehat{c}_t \tag{60}$$

Linearizing the interest rate

$$\begin{aligned} R_t &= 1 + \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right) \\ \overline{R}\widehat{R}_t &= \left(\overline{R}^K \overline{U} \left(\widehat{R}_t^K + \widehat{U}_t \right) - \overline{\delta}\theta\widehat{U}_t \right) \left(1-\tau^K\right) \\ &= \overline{R}^K \overline{U} \left(1-\tau^K\right) \widehat{R}_t^K \\ \widehat{R}_t &= \frac{\overline{R}^K \overline{U}}{\overline{R}} \left(1-\tau^K\right) \widehat{R}_t^K \end{aligned}$$

Firms

Labor demand equation becomes

$$\widehat{w}_t = \widehat{y}_t - \widehat{H}_t \tag{61}$$

Using the definition of output yields

$$R_t^K = \alpha\gamma_t \frac{y_t}{U_t k_t}$$

which in log-linear form is

$$\widehat{R}_t^K = \widehat{\gamma}_t + \widehat{y}_t - \widehat{U}_t - \widehat{k}_t$$

The evolution of capital is

$$\widehat{k}_{t+1} = \frac{\overline{i}}{\overline{k}}\widehat{i}_t + \frac{(1-\delta)}{\overline{\gamma}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) - \frac{\delta\theta}{\overline{\gamma}}\widehat{U}_t$$

The production function

$$\begin{aligned} y_t &= (U_t k_t \gamma_t^{-1})^{\alpha}(H_t)^{1-\alpha} \\ \widehat{y}_t &= \alpha\widehat{U}_t + \alpha\widehat{k}_t - \alpha\widehat{\gamma}_t + (1-\alpha)\widehat{H}_t \end{aligned}$$

Good market clearing condition

$$\begin{aligned} y_t &= c_t + i_t + g_t \\ \widehat{y}_t &= \frac{\overline{c}}{\overline{y}}\widehat{c}_t + \frac{\overline{i}}{\overline{y}}\widehat{i}_t + \frac{\overline{g}}{\overline{y}}\widehat{g}_t \end{aligned}$$

The definition of a_t

$$\begin{aligned} a_t &= b_t + k_t \\ \frac{\overline{a}}{\overline{y}}\widehat{a}_t &= \frac{\overline{b}}{\overline{y}}\widehat{b}_t + \frac{\overline{k}}{\overline{y}}\widehat{k}_t \end{aligned}$$

Government budget constraint

$$
\begin{aligned}
b_{t+1} &= \frac{b_t}{\gamma_t}R_t - \left(R_t^K U_t - \delta\left(U_t\right)\right)\frac{k_t}{\gamma_t}\tau^K - w_t H_t \tau^H + g_t - \tau \\
\overline{b}\widehat{b}_{t+1} &= \overline{b}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t - \widehat{\gamma}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{\overline{k}}{\overline{\gamma}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) \\
&\quad - \left(\overline{R}^K\overline{U}\left(\widehat{R}_t^K + \widehat{U}_t\right) - \overline{\delta}\theta\widehat{U}_t\right)\tau^K\frac{\overline{k}}{\overline{\gamma}} - \overline{w}\overline{H}\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \overline{g}\widehat{g}_t \\
\overline{b}\widehat{b}_{t+1} &= \overline{b}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t - \widehat{\gamma}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{\overline{k}}{\overline{\gamma}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) \\
&\quad -\tau^K\frac{\overline{k}}{\overline{\gamma}}\overline{R}^K\overline{U}\widehat{R}_t^K - \overline{w}\overline{H}\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \overline{g}\widehat{g}_t \\
\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1} &= \frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t - \widehat{\gamma}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) \\
&\quad -\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K - \frac{\overline{w}\overline{H}}{\overline{y}}\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \frac{\overline{g}}{\overline{y}}\widehat{g}_t \\
\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1} &= \frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t \\
&\quad -\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K - (1-\alpha)\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \frac{\overline{g}}{\overline{y}}\widehat{g}_t \\
&\quad + \left(\left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}} - \frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t
\end{aligned}
$$

Under RE, government expenditure (fiscal rule)

$$
\begin{aligned}
\frac{g_t}{y_t} &= \lambda_0 + \lambda_1\frac{y_t - y_t^P}{y_t^P} + \lambda_2\frac{b_t}{y_t}\gamma_t^{-1} \\
\frac{\overline{g}}{\overline{y}}\left(\widehat{g}_t - \widehat{y}_t\right) &= \lambda_1\widehat{y}_t + \lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}\left(\widehat{b}_t - \widehat{y}_t - \widehat{\gamma}_t\right) \\
\frac{\overline{g}}{\overline{y}}\widehat{g}_t &= \left(\frac{\overline{g}}{\overline{y}} + \lambda_1 - \lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}\right)\widehat{y}_t + \lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}\left(\widehat{b}_t - \widehat{\gamma}_t\right)
\end{aligned}
$$

D.6 Consumption decision rule

Linearizing the budget constraint

$$
c_t + a_{t+1} = R_t a_t \gamma_t^{-1} + w_t H_t\left(1 - \tau^H\right) - \tau
$$

yields

$$
\begin{aligned}
\overline{c}\widehat{c}_t + \overline{a}\widehat{a}_{t+1} &= \overline{R}\overline{a}\overline{\gamma}^{-1}\left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t\right) + \overline{w}\overline{H}\left(1 - \tau^H\right)\left(\widehat{w}_t + \widehat{H}_t\right) \\
\frac{\overline{c}}{\overline{a}}\widehat{c}_t + \widehat{a}_{t+1} &= \overline{R}\overline{\gamma}^{-1}\left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t\right) + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1 - \tau^H\right)\left(\widehat{w}_t + \widehat{H}_t\right) \qquad (62)
\end{aligned}
$$

where $\frac{\overline{w}\overline{H}}{\overline{a}} = \frac{\overline{w}\overline{H}}{\overline{y}} \div \frac{\overline{a}}{\overline{y}}$.

Combining (60) and (62) yields

$$\begin{aligned}
\frac{\overline{c}}{\overline{a}}\widehat{c}_t + \widehat{a}_{t+1} &= \overline{R}\overline{\gamma}^{-1}\left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t\right) + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\widehat{w}_t + \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}(\widehat{w}_t - \widehat{c}_t)\right) \\
\frac{\overline{c}}{\overline{a}}\widehat{c}_t + \widehat{a}_{t+1} &= \overline{R}\overline{\gamma}^{-1}\left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t\right) \\
&\quad + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\left(1 + \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)\widehat{w}_t - \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\widehat{c}_t\right)
\end{aligned}$$

$$\begin{aligned}
\widehat{a}_t &= \widehat{\gamma}_t - \widehat{R}_t + \overline{R}^{-1}\overline{\gamma}\left(\frac{\overline{c}}{\overline{a}}\widehat{c}_t + \widehat{a}_{t+1}\right) \\
&\quad - \overline{R}^{-1}\overline{\gamma}\frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\left(1 + \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)\widehat{w}_t - \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\widehat{c}_t\right) \\
&= \widehat{\gamma}_t - \widehat{R}_t + \overline{R}^{-1}\overline{\gamma}\widehat{a}_{t+1} + \overline{R}^{-1}\overline{\gamma}\left(\frac{\overline{c}}{\overline{a}} + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)\widehat{c}_t \\
&\quad - \overline{R}^{-1}\overline{\gamma}\frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(1 + \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)\widehat{w}_t \\
&= \widehat{\gamma}_t - \widehat{R}_t + \widetilde{\beta}\widehat{a}_{t+1} + \widetilde{\beta}\epsilon_c\widehat{c}_t - \widetilde{\beta}\epsilon_w\widehat{w}_t
\end{aligned}$$

where $\widetilde{\beta} \equiv \overline{R}^{-1}\overline{\gamma}$, $\epsilon_c \equiv \frac{\overline{c}}{\overline{a}} + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1} = \frac{\overline{c}/\overline{y}}{\overline{a}/\overline{y}} + \frac{(1-\alpha)}{\overline{a}/\overline{y}}\left(1-\tau^H\right)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}$, and $\epsilon_w \equiv \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(1 + \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right) = \frac{1-\alpha}{\overline{a}/\overline{y}}\left(1-\tau^H\right)\left(1 + \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)$.

Iterating forward and taking expectations

$$\begin{aligned}
\widehat{a}_t &= \widehat{\gamma}_t - \widehat{R}_t + \widetilde{\beta}\widehat{a}_{t+1} + \widetilde{\beta}\epsilon_c\widehat{c}_t - \widetilde{\beta}\epsilon_w\widehat{w}_t \\
&= \widehat{\gamma}_t - \widehat{R}_t + \widetilde{\beta}\left(\widehat{\gamma}_{t+1} - \widehat{R}_{t+1} + \widetilde{\beta}\widehat{a}_{t+2} + \widetilde{\beta}\epsilon_c\widehat{c}_{t+1} - \widetilde{\beta}\epsilon_w\widehat{w}_{t+1}\right) \\
&\quad + \widetilde{\beta}\epsilon_c\widehat{c}_t - \widetilde{\beta}\epsilon_w\widehat{w}_t \\
&= \ldots \\
&= \sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widehat{\gamma}_T - \widehat{R}_T\right) + \sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widetilde{\beta}\epsilon_c\widehat{c}_T - \sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widetilde{\beta}\epsilon_w\widehat{w}_T \\
\widehat{a}_t &= \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widehat{\gamma}_T - \widehat{R}_T\right) + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widetilde{\beta}\left(\epsilon_c\widehat{c}_T - \epsilon_w\widehat{w}_T\right) \\
\epsilon_c\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widehat{c}_T &= \widetilde{\beta}^{-1}\widehat{a}_t + \widetilde{\beta}^{-1}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widehat{R}_T - \widehat{\gamma}_T\right) + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\epsilon_w\widehat{w}_T \\
&= \widetilde{\beta}^{-1}\widehat{a}_t + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widetilde{\beta}^{-1}\widehat{R}_T - \widetilde{\beta}^{-1}\widehat{\gamma}_T + \epsilon_w\widehat{w}_T\right)
\end{aligned}$$

Recall the Euler equation

$$\widehat{Q}_t = -\widehat{E}_t \beta \frac{\overline{R}^k \overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}^K_{t+1} + \widehat{E}_t \widehat{Q}_{t+1} + \sigma \widehat{E}_t \widehat{\gamma}_{t+1}$$

Define $\widetilde{R}$ such that $\beta \frac{\overline{R}^k \overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^\sigma} = \widetilde{\beta}\widetilde{R}$. Solving backward from time T gives

$$\begin{aligned} \widehat{E}_t \widehat{Q}_T &= \widehat{Q}_t + \widehat{E}_t \left(\sum_{i=t}^{T-1} \left(\widetilde{\beta}\widetilde{R}\widehat{R}^K_{i+1} - \sigma \widehat{\gamma}_{i+1} \right) \right) \\ \widehat{E}_t \left(\sigma \widehat{c}_T + \psi(1-\sigma)\widehat{H}_T \right) &= \sigma \widehat{c}_t + (1-\sigma)\psi \widehat{H}_t + \widehat{E}_t \left(\sum_{i=t}^{T-1} \left(\widetilde{\beta}\widetilde{R}\widehat{R}^K_{i+1} - \sigma \widehat{\gamma}_{i+1} \right) \right) \end{aligned}$$

Substituting for the constant-consumption labor supply yields

$$\begin{aligned} & \widehat{E}_t \left(\sigma \widehat{c}_T + \psi(1-\sigma) \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi \right)^{-1} \left(\widehat{w}_T - \widehat{c}_T \right) \right) \\ = \quad & \sigma \widehat{c}_t + (1-\sigma)\psi \widehat{H}_t + \widehat{E}_t \left(\sum_{i=t}^{T-1} \left(\widetilde{\beta}\widetilde{R}\widehat{R}^K_{i+1} - \sigma \widehat{\gamma}_{i+1} \right) \right) \end{aligned}$$

which can be simplified to

$$\begin{aligned} & \widehat{E}_t \left(\left(1-\chi\right) \sigma \widehat{c}_T + \chi \sigma \widehat{w}_T \right) \\ = \quad & \sigma \widehat{c}_t + (1-\sigma)\psi \widehat{H}_t + \widehat{E}_t \left(\sum_{i=t}^{T-1} \left(\widetilde{\beta}\widetilde{R}\widehat{R}^K_{i+1} - \sigma \widehat{\gamma}_{i+1} \right) \right) \end{aligned}$$

where

$$\chi \equiv \sigma^{-1}\psi(1-\sigma) \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi \right)^{-1}$$

Rearranging in terms of expected consumption

$$\widehat{E}_t \widehat{c}_T = \frac{1}{1-\chi} \left[\widehat{c}_t + \sigma^{-1}\psi(1-\sigma)\widehat{H}_t + \widehat{E}_t \left(\sum_{i=t}^{T-1} \left(\sigma^{-1}\widetilde{\beta}\widetilde{R}\widehat{R}^K_{i+1} - \widehat{\gamma}_{i+1} \right) \right) - \chi \widehat{w}_T \right]$$

We now substitute the above equation into the intertemporal budget constraint

$$\epsilon_c \widehat{E}_t \sum_{T=t}^{\infty} \widetilde{\beta}^{T-t} \widehat{c}_T = \widetilde{\beta}^{-1} \widehat{a}_t + \widehat{E}_t \sum_{T=t}^{\infty} \widetilde{\beta}^{T-t} \left(\widetilde{\beta}^{-1} \widehat{R}_T - \widetilde{\beta}^{-1} \widehat{\gamma}_T + \epsilon_w \widehat{w}_T \right) \tag{63}$$

Note $\widetilde{\beta}\widetilde{R}\widehat{R}^K_T = \beta \frac{\overline{R}^k \overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}^K_T = \widehat{R}_T$. So (63) becomes

$$\epsilon_c \widehat{E}_t \sum_{T=t}^{\infty} \widetilde{\beta}^{T-t} \widehat{c}_T = \widetilde{\beta}^{-1} \widehat{a}_t + \widehat{E}_t \sum_{T=t}^{\infty} \widetilde{\beta}^{T-t} \left(\widetilde{R}\widehat{R}^K_T - \widetilde{\beta}^{-1} \widehat{\gamma}_T + \epsilon_w \widehat{w}_T \right)$$

Based on the derivation of Eusepi and Preston (2011), we get

$$\begin{aligned}\widehat{c}_t + \sigma^{-1}\psi(1-\sigma)\widehat{H}_t &= \frac{(1-\chi)(1-\widetilde{\beta})}{\varepsilon_c}\left[\widetilde{\beta}^{-1}\widehat{a}_t + \widetilde{R}\widehat{R}_t^K - \widetilde{\beta}^{-1}\widehat{\gamma}_t + \left(\varepsilon_w + \varepsilon_c\frac{\chi}{1-\chi}\right)\widehat{w}_t\right] \\ &+ \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left[\widetilde{\beta} - \frac{(1-\chi)\left(1-\widetilde{\beta}\right)}{\varepsilon_c}\right]\widehat{\gamma}_{T+1} \\ &+ \widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_1\widehat{R}_{T+1}^K + \widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_2\widehat{w}_{T+1}, \qquad (64)\end{aligned}$$

where $\Gamma_1 = \left[\frac{(1-\chi)(1-\widetilde{\beta})}{\varepsilon_c} - \widetilde{\beta}\sigma^{-1}\right]\widetilde{R}$ and $\Gamma_2 = \frac{(1-\chi)(1-\widetilde{\beta})}{\varepsilon_c}\left(\varepsilon_w + \varepsilon_c\frac{\chi}{1-\chi}\right)$.

D.7 RE system of equations

$$\begin{gathered}\widehat{w}_t - \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)\widehat{H}_t - \widehat{c}_t = 0 \\ -\widehat{w}_t + \widehat{y}_t - \widehat{H}_t = 0 \\ -\widehat{R}_t^K + \widehat{y}_t - \widehat{U}_t - \left(\widehat{k}_t - \widehat{\gamma}_t\right) = 0 \\ -\widehat{y}_t + \frac{\bar{i}}{\bar{y}}\widehat{i}_t + \left(1 - \frac{\bar{i}}{\bar{y}} - \frac{\bar{g}}{\bar{y}}\right)\widehat{c}_t + \frac{\bar{g}}{\bar{y}}\widehat{g}_t = 0 \\ -\widehat{y}_t + (1-\alpha)\widehat{H}_t + \alpha\widehat{U}_t + \alpha\left(\widehat{k}_t - \widehat{\gamma}_t\right) = 0 \\ -\frac{\delta\theta}{\bar{\gamma}}\widehat{U}_t - \widehat{k}_{t+1} + \frac{\bar{i}}{\bar{k}}\widehat{i}_t + \frac{(1-\delta)}{\bar{\gamma}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) = 0\end{gathered}$$

$$-\frac{\widehat{R}_t^K}{\theta-1}+\widehat{U}_t=0$$

$$(\sigma\widehat{c}_t+\psi(1-\sigma)\widehat{H}_t)-\hat{E}_t[((\sigma\widehat{c}_{t+1}+\psi(1-\sigma)\widehat{H}_{t+1})+\sigma\widehat{\gamma}_{t+1})]+\beta\frac{\overline{R}^k\overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^\sigma}\hat{E}_t\widehat{R}_{t+1}^K=0$$

$$\widehat{R}_t-\frac{\overline{R}^K\overline{U}\left(1-\tau^K\right)}{\overline{R}}\widehat{R}_t^K=0$$

$$\frac{\overline{a}}{\overline{y}}\widehat{a}_t-\frac{\overline{b}}{\overline{y}}\widehat{b}_t-\frac{\overline{k}}{\overline{y}}\widehat{k}_t=0$$

$$\begin{aligned}\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t+\widehat{R}_t\right)-\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t-\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K\\-\left(1-\alpha\right)\tau^H\left(\widehat{w}_t+\widehat{H}_t\right)+\frac{\overline{g}}{\overline{y}}\widehat{g}_t\\+\left(\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}-\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t-\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1}=0\end{aligned} \tag{65}$$

$$\left(\frac{\overline{g}}{\overline{y}}+\lambda_1-\lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}\right)\widehat{y}_t+\lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}\left(\widehat{b}_t-\widehat{\gamma}_t\right)+u_t^G-\frac{\overline{g}}{\overline{y}}\widehat{g}_t=0$$

We now eliminate $\widehat{R}_t$ and $\widehat{a}_t$ from the system of equations for the learning model. Equation (65) becomes

$$\begin{aligned}\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t+\frac{\overline{R}^K\overline{U}\left(1-\tau^K\right)}{\overline{R}}\widehat{R}_t^K\right)-\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t\\-\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K-\left(1-\alpha\right)\tau^H\left(\widehat{w}_t+\widehat{H}_t\right)\\+\frac{\overline{g}}{\overline{y}}\widehat{g}_t+\left(\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}-\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t-\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1}=0\end{aligned}$$

and

$$\begin{aligned}\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\widehat{b}_t+\left(\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\frac{\overline{R}^K\overline{U}\left(1-\tau^K\right)}{\overline{R}}-\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\right)\widehat{R}_t^K-\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t\\-\left(1-\alpha\right)\tau^H\left(\widehat{w}_t+\widehat{H}_t\right)\\+\frac{\overline{g}}{\overline{y}}\widehat{g}_t+\left(\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}-\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t-\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1}=0\end{aligned}$$

Equation (64) becomes

$$
\begin{aligned}
& \widehat{c}_t + \sigma^{-1}\psi(1-\sigma)\widehat{H}_t \\
= \quad & \frac{(1-\chi)(1-\widetilde{\beta})}{\varepsilon_c}\left[\widetilde{\beta}^{-1}\left(\frac{\frac{\overline{b}}{\overline{y}}}{\frac{\overline{a}}{\overline{y}}}\widehat{b}_t + \frac{\frac{\overline{k}}{\overline{y}}}{\frac{\overline{a}}{\overline{y}}}\widehat{k}_t\right) + \widetilde{R}\widehat{R}^K_t - \widetilde{\beta}^{-1}\widehat{\gamma}_t + \left(\varepsilon_w + \varepsilon_c\frac{\chi}{1-\chi}\right)\widehat{w}_t\right] \\
& + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left[\widetilde{\beta} - \frac{(1-\chi)\left(1-\widetilde{\beta}\right)}{\varepsilon_c}\right]\widehat{\gamma}_{T+1} \\
& + \widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_1\widehat{R}^K_{T+1} + \widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_2\widehat{w}_{T+1}
\end{aligned}
$$