# House Price Expectations and Housing Choice

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October 30, 2019

## Preliminary and Incomplete. Comments welcome!

#### Abstract

What is the role of heterogeneous house-price expectations for boom-bust cycles in the housing market? We exploit a unique Dutch panel data set on households' house price expectations and their consumption, savings and housing choices for the period 2003-2016. This period was characterized by a pronounced boom-bust cycle in the housing market. Conditioning the sample on household heads who report non-zero house price expectations, we find that expectations closely track realized house prices. We next develop a structural life-cycle model of the Dutch housing market where we distinguish household types according to their house price expectations. We employ a calibrated model variant to test if observed variations in expectations can account for the housing boom-bust cycle. First results show that our model closely matches the observed fluctuations of the rent-to-price ratio in the data but overshoots the size of the housing boom.

**JEL Classification:** D14,D84,D31,E21,E30,G21, R21

Keywords: Housing, survey expectations, house price cycles, life-cycle model

This paper represents the authors' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank or any other official institution. Alex Ludwig, Jorge Quintana and Nate Vellekoop gratefully acknowledge financial support by the Research Center SAFE, funded by the State of Hessen initiative for research LOEWE. We thank Nicola Fuchs-Schündeln, Leo Kaas, Hitoshi Tsujiyama, Gianluca Violante and Jesús Fernández-Villaverde, as well as participants of the European Economic Association 2018 Annual Meetings and the Bundesbank Banking & Finance Group at the Research Centre seminar, for their thoughtful comments that helped improve this paper. All errors are our own.

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## 1 Introduction

Much of the recent literature on macroeconomics and housing is motivated by the international housing boom-bust episode of the early 2000's and the ensuing economic recession. One of the key questions is to identify the forces behind the increase and subsequent collapse in housing. The existing literature provides two alternative views on this question. While Justiniano et al. (2015) and Favilukis et al. (2017) find that shocks to credit conditions are the determining factor in explaining the housing boom and bust, Kaplan et al. (2017) and Landvoigt (2017) stress the role played by expectations/beliefs about future house prices. Thus, in these papers, as in Bailey et al. (2017), expectations or beliefs about future prices developments are central.

We use a households panel data set that includes house price expectations and choices of Dutch households for the years 2003-2016, a period in which the Netherlands went through a boom-bust cycle in the housing market. In our analysis, we feed the observed expectations into a structural macroeconomic model of the housing market to quantitatively assess the role of expectations for observed house price movements. Unlike the existing literature, we are able to explore explicit measures of expectations and do not have to rely on implicit measures derived from a particular economic model.

As a first step, we document the key characteristics of house price fluctuations and house price expectations in the data. We show that average house price expectations feature the same timing of the boom-bust cycle as the nation wide house price index but with a much smaller amplitude. We next note that a significant number of household heads reports zero house price expectations. Conditioning the sample on household heads who report non-zero expectations we show that their expectations track both the realized own house price growth and the nationwide house price growth very well. However, it is important to note that while the realized house price growth is from period t - 1 to t, expectations are from period t to t+1. Thus, households seem to extrapolate past changes into the future. Further, we find that households that report non-zero expectations are more likely to move than those that report zero expected house price growth when they expect positive house price growth.

Based on this observation we define as *attentive* those households who report non-zero house price growth expectations for at least one survey year and accordingly, as *inattentive* households those who always report zero house price growth expectations or do not respond this question in the survey. With this definition we investigate how heterogeneity in beliefs affects the moving decision. We find that the likelihood of moving of attentive households responds more to aggregate house price fluctuations than that of inattentive households. Thus transaction decisions are closely linked to expectations. These observations lead us to conjecture that observed heterogeneity in expectations may play a key

role for actual house price movements.

To investigate the quantitative role of house price expectations for observed movements in house prices, we proceed by employing a structural macroeconomic model of the Dutch housing market. Our structural model features many elements that are standard in the quantitative macroeconomics housing literature (Berger et al., 2015; Kaplan et al., 2017) such as an owner-occupied home and a rental market, idiosyncratic income shocks, a warm glow bequest motive, and long-term mortgage contracts. We abstract from default as this option is essentially not observed in the Netherlands. The key feature of the model is heterogeneity in expectations which is consistent with the data.

We calibrate the model to the Dutch housing market. Specifically, we adopt our definition of attentive and inattentive households, and calibrate the respective fraction of household types. As their expectation formation we assume that attentive households expect house price growth as measured in the data to continue. That is, they expect house prices to grow from period t to t + 1 the same way that they have grown from period t - 1 to t. In contrast, inattentive households assume zero house price growth. We then adopt the concept of a *temporary equilibrium* approach suggested by Piazzesi and Schneider (2016) to simulate the model forward in time.<sup>1</sup> Accordingly, going from period t - 1 to t we take expectations as exogenous, compute decisions, aggregate and clear the housing market in period t. We then move on to the next period, again taking expectations as exogenous. More precisely, in our baseline experiment, we feed into the model the expectations observed in the data. Accordingly, attentive households assume from any period t to t + 1 that house prices grow at the rate observed from t - 1 to t, whereas inattentive households assume zero house price growth. It is important to note that, conditional on their expectations, households behave fully rationally.

Our first experiment shows that the rent-to-price ratio simulated in the model tracks the observed rent-to-price ratio quite well, with slightly higher fluctuations than in the data. At the same time the model overshoots the observed house price boom. To reconcile these two observations, note that our model also roughly matches the observed upward trend of the rental rate until 2013, but again features stronger fluctuations than what is observed in the data.

In the future, we plan to explore calibrated versions of a rational expectations equilibrium as well and compare these model variants to quantify the role of expectation formation for actual house price movements.

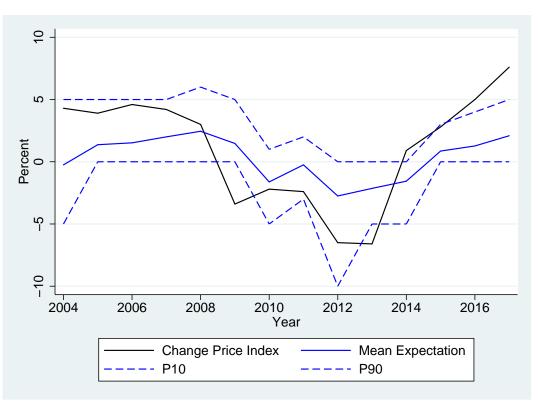
The remainder of this paper is structured as follows. Section 2 describes our data and summarizes the main results of our data analysis. We proceed in Section 3 by developing the quantitative life-cycle model of the Dutch housing market. Section 4 defines the temporary equilibrium approach and describes our choices of functional forms and our calibration. Section 5 documents the results from our first experiments and provides an

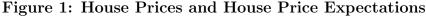
<sup>&</sup>lt;sup>1</sup>Piazzesi and Schneider (2016) base this notion on early work by Grandmont (1977).

outlook on future experiments we aim at conducting in the future. Finally, Section 6 concludes the paper. A separate appendix provides more details on our computational procedures.

## 2 Data: Heterogeneity in Expectations

In this section we explore the panel data on household house-price expectations and house-adjustment decisions. In Figure 1 we plot the time series for house price growth and expectations of house price growth. The Netherlands had very fast house price growth until 2008, negative house price growth between 2009 and 2013, and positive house price growth since 2014. House prices increased by about 7% per year on average over the entire period. The average forecast of house price growth is fairly stable around zero (literally zero in 2004, slightly positive 2005-2009, slightly negative 2010-2014, slightly positive since 2015). This average forecast masks a lot of heterogeneity; about 50% of the respondents expect zero house price growth. The forecasts of the remaining 50% are dispersed and move with realized aggregate house price growth.





In Figure 2 below we plot the distribution of the forecast of aggregate house price growth for the two phases of the boom-bust episode. The data are generated based on households' response to the two following questions: "What kind of price movement do you expect on the housing market **in the next two years**? Will the housing prices increase, decrease or remain about the same?"<sup>2</sup> and "How much percentage points a year will they increase/decrease on average?"<sup>3</sup>

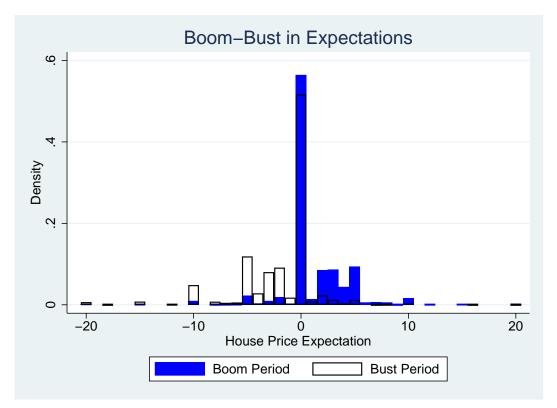


Figure 2: Distribution of Expectations

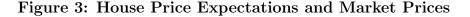
The graph reveals that there is a significant number of households that register an expectation of zero growth in house prices. This is the case both during the boom and bust phases of the housing market cycle. There is also a large number of households that report non-zero house price growth. The distribution of these households is shifted to the right, relative to zero, during the boom phase and to the left during the bust phase.

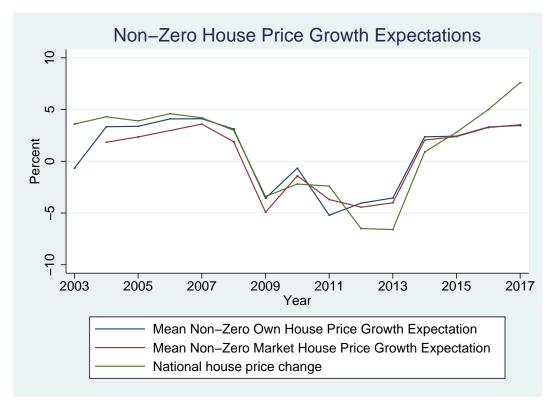
Focusing on households that register non-zero house price growth, in Figure 3 we show the mean expectation for own and market house price growth, as well as the realized change in house prices. In reading the graph, it is important to recall the timing of the data: at ant year t, the data points that are graphed are (i) households' average expected own home house price inflation for the next year<sup>4</sup> reported in year t, (ii) households' average expected aggregate house price inflation for the next year reported in year t, and, if applicable, (iii) realized aggregate house price inflation between year t - 1 and year t; which is in the household's information set when they register their expectation.

 $<sup>^{2}</sup>$ Bold in the survey.

<sup>&</sup>lt;sup>3</sup>We have restricted the data to be within  $\pm 20\%$ .

<sup>&</sup>lt;sup>4</sup>More precisely, their average expected house price growth for the next two years.





We see that households who register non-zero house price growth expectations track the market quite closely and expect realized house price changes to continue in the current period. Hence, we posit the following two types of expectations, which we will refer to as *attentive* and *inattentive* for reasons to be explained below:

$$E^{i}\left(\pi_{t+1}^{H} \mid \pi_{t}^{H}\right) = \begin{cases} 0 & \text{if } i = \text{Inattentive} \\ \pi_{t}^{H} & \text{if } i = \text{Attentive} \end{cases}$$
(1)

Hence, the data can be summarized by two types of households, based on the expectations they report. Inattentive households expect no price-growth in the coming period and attentive households have *extrapolative* expectations.<sup>5</sup> But do these expectations vary systematically with their consumption and savings behavior? Lets focus on household adjustment.

Figure 4 plots the time series for the *proportion* of home-adjusting agents, per type of expectations. We construct a *house-adjustment indicator variable* for a window of five years. The *number of respondents* is the cross-sectional sum of the relevant indicator variable by type: (i) No-growth expectations and (ii) Non-zero expectations. This is done using expectations on market prices, and agents are categorized based on the answer they

<sup>&</sup>lt;sup>5</sup>See Gennaioli et al. (2016) for evidence on the extrapolative nature of expectations; in the sense that "when [agents] observe good or bad [...] realization, they think that similar realization persists into the future and fail to correct for mean reversion." (p. 217).

give in the current year.<sup>6</sup> There is no data on expectations of market prices for 2003 or 2006.



**Figure 4: Housing Adjustment and Expectations** 

The figure reveals a clear above-below-above pattern. When house price growth is positive, non-zero-growth-expectation households are more likely to move than zero-growthexpectations households. When realized house price growth is negative, the opposite is true: non-zero-growth-expectation households are less likely to adjust their housing consumption than zero-growth-expectations households.

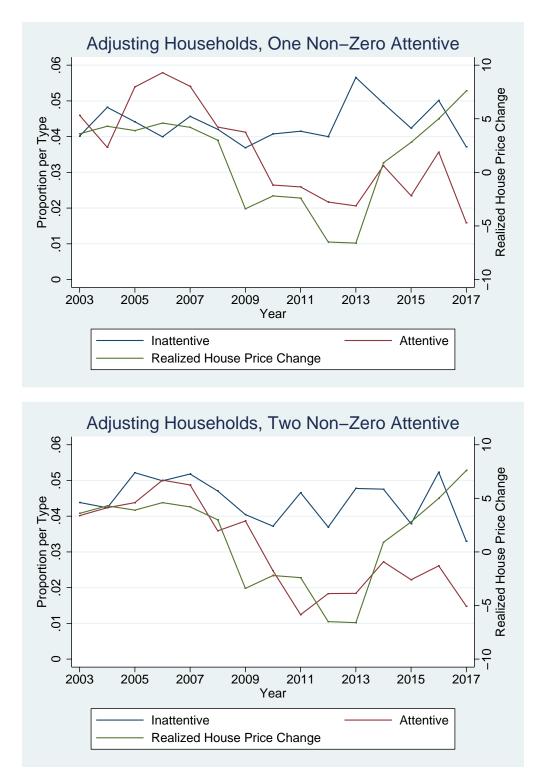
There two natural interpretations of this figure. The first interpretation is that households with higher house price growth expectations are more likely to move. In the boom years (pre-2009 and post-2013), those are on average the non-zero-growthexpectation households. In the bust years (2009-2013), those are on average the zerogrowth-expectations households. The second interpretation is that the non-zero-growthexpectation, or attentive, households respond more with their moving decision to realized house price growth. Therefore, in the boom years they move more, and in the bust year they move less.

<sup>&</sup>lt;sup>6</sup>More specifically, the *indicator variables* are constructed as follows: set 1 if agent purchased its house two years ago, last year, in the current year, next year, or in two years from the current year; set 0 if agent did not purchase its house two years ago, last year, in the current year, next year, or in two years from the current year. The *proportion per type* for type *i* at period *t* is calculated by summing cross-sectionally within type *i* the relevant indicator variable and dividing by the number of type-*i* agents in period-*t*.

To discriminate between these two interpretations, in Figure 5 we focus on the zerogrowth-expectation households. Some households who report a zero expectation may track house prices. Most households who report a zero expectation probably do not track house prices. To distinguish between the two, we look at consecutive answers for the same household and categorize them as follows. We label a household as *attentive* if it reports (i) at least one non-zero expectation, and (ii) at least two *consecutive* non-zero expectations. Every other household is classified as *inattentive*.<sup>7</sup> Note that this implies that a household's type is fixed. For reference, we include the time series for realized market house price changes.

 $<sup>^7\</sup>mathrm{Note}$  that missing expectations are  $de\;facto$  treated as zero-growth expectations with this classification scheme.





Thus, we see that while inattentive households are largely insensitive in their housing choices to changes in aggregate house prices, attentive households respond markedly to movements in house prices by adjusting less their consumption of housing when prices fall. In the quantitative analysis below, we focus on this dimension of heterogeneity.

## 3 A Structural Housing Model

In order to study the potential for observed variations in house-price expectations – along with fluctuations in the distribution of households over states – to account for the boombust cycle in house prices, we turn to a structural model. The model features idiosyncratic income shocks, a warm glow bequest motive, home-ownership and rental markets for housing services, and long-term mortgage contracts. We abstract from default as this option is essentially not observed in the Dutch data set. The model is standard and very close to those of Kaplan et al. (2017) and Berger et al. (2015), but we allow for a small degree of heterogeneity in expectations.

#### 3.1 Households

The household sector is composed of a continuum of households that live for a fixed number of discrete-time periods,  $j = \{1, 2, ..., J\}$ , and are indexed at any period  $t = \{0, 1, ...\}$  by the index *i*. They derive utility from non-durable consumption,  $C_{it}$ , and housing services,  $S\left(H_{it}, \tilde{H}_{it}\right)$ , according to the instant utility function  $U\left(C_{it}, S\left(H_{it}, \tilde{H}_{it}\right)\right)$ . Each household receives during its working life a stochastic income, and a deterministic pension after retirement at age  $J^{ret}$ . We denote by  $Y_{it}$  net earnings from either labor or pension payments. Subject to a no short-selling constraint, households can save in risk-free bonds that pay a net return,  $r_b$ , and in discrete housing units,  $H_{it} \in \mathcal{H}$  where  $\mathcal{H} = \{h^0, ..., h^{N_h}\}$  and  $0 < h^0 < ... < h^{N_h}$ , that sell at unit price  $P_t$ . In purchasing housing units, households have the option to finance part of the purchase through a long-term mortgage contract at a fixed rate,  $r_m$ , that is subject to an intermediation spread such that  $r_m = r_b + \zeta$ , where  $\tilde{\mathcal{H}} = \{\tilde{h}^0, ..., \tilde{h}^{N_{\tilde{h}}}\}$  and  $0 < \tilde{h}^0 < ... < \tilde{h}^{N_{\tilde{h}}}$ , at rental price  $\tilde{P}_t$ .

It is understood that the elements in  $\mathcal{H}$  and  $\mathcal{H}$  represent the size and quality of houses. Further, if a household decides to adjust the size of the house it owns or move into rental housing, it must incur a house-sales transaction cost linked to the size of the house being sold,  $\kappa (H_{t-1}) = \theta (1 - \delta) P_t H_{t-1}$ . Housing units are subject to a per-period depreciation rate,  $\delta$ . Households cannot simultaneously own and rent housing.

A household i born in period t wishes to maximize its expected lifetime utility:

$$E_t^i \left[ \sum_{j=1}^J \beta^j U\left( C_{it+j}, S\left( H_{it+j}, \tilde{H}_{it+j} \right) \right) + \beta^{J+1} B\left( \tilde{W}_{it+J+1} \right) \right]$$

where  $\beta \in (0, 1)$  is the household's subjective discount factor and  $B\left(\tilde{W}_{it+J+1}\right)$  represents utility received from the wealth,  $\tilde{W}_{it+J+1}$ , it bequeaths to younger generations.

Following Landvoigt (2017), note that given  $\zeta > 0$ , households will never choose to take out a mortgage and save in bonds at the same time. Therefore, we need only keep

track of households' net non-housing (liquid) asset position, which we denote by  $A_{it}$ . Mortgage contracts are such that at origination, a *house-adjusting household* is subject to the following borrowing constraint

$$-A_{it} \le \lambda_m P_t H_{it}.$$

After the mortgage contract has been entered into, a *non-adjusting household* must only make a *minimum* payment while the mortgage is outstanding, according to the constraint

$$A_{it} \ge [1 + r_M - \phi(r_M, j)] A_{it-1}.$$

The function  $\phi(r_M, j)$  is defined by the constant-amortization formula

$$\phi(r_M, j) = \frac{r_M (1 + r_M)^{J+1-j}}{(1 + r_M)^{(J+1-j)} - 1}.$$

Renting households are subject to a no-borrowing constraint  $A_{it} \ge 0$ . Hence, there are three types of households: (i) adjusting households, (ii) non-adjusting households, and (iii) renting households.

Idiosyncratic labor income,  $Y_{it}$ , is given by

$$Y_{it} = g(j_{it}) \chi_{it} \varepsilon_{it}$$
  
$$\chi_{it} = \chi^{\rho}_{it-1} \nu_{it}$$

where  $\rho \in [0, 1)$ ,  $g(j_{it})$  is a deterministic age-dependent parameter,  $\varepsilon_{it}$  is a transitory income shock, and  $\nu_{it}$  is a shock to the persistent component of income,  $\chi_{it}$ .

The general budget constraint for households is given by

$$C_{it} + A_{it} + P_{it}H_{it} + \mathbb{1}_{\{H_{it} \neq H_{it-1}\}}\theta(1-\delta) P_{t}H_{it-1} + \mathbb{1}_{\{\tilde{H}_{it}>0\}}\tilde{P}_{t}\tilde{H}_{it} = W_{it}$$

$$W_{it} = Y_{it} + (1+r_{b} + \mathbb{1}_{\{A_{it-1}<0\}}\zeta) A_{it-1} + (1-\delta) P_{t}H_{it-1}$$

$$-A_{it} \leq \begin{cases} \lambda_{m}P_{t}H_{it} & \text{if } H_{it-1} \neq H_{it} > 0\\ -[1+r_{M}-\phi(r_{M},j)] A_{it-1} & \text{if } H_{it-1} = H_{it} > 0 & \text{and } A_{it-1} < 0\\ 0 & \text{else} \end{cases}$$

Where  $W_{it}$  denotes the household's beginning-of-period liquid resources and  $\mathbb{1}_{\{\cdot\}}$  denotes the indicator function.

The Housing Capital Gains Mechanism In line with the stylized fact presented in section 2, we assume two types of households: attentive and inattentive. Households' type is fixed throughout their life. They differ in their forecasts of future house-price growth according to equation (1).

Further, we assume that households abstract from aggregate uncertainty in the sense that they take their central forecast for house price growth to be a certain outcome. Also, income risk is assumed to be orthogonal to house price changes. Hence, suppressing the subscript *i* when unnecessary and letting the household's state vector be given by  $X_t = (Y_t, A_{t-1}, H_{t-1}, j_t)$ , we can sum up the household's problem as any date *t* as follows:

$$V_{t}^{i}(X_{t}) = \max_{\{C_{t},A_{t},d_{t}\}} \left\{ U(C_{t},S(d_{t})) + \beta E^{i} \left[ V_{t+1}^{i}(X_{t+1}) \mid Y_{t},\pi_{t}^{H},A_{t},d_{t} \right] \right\} \\ = \max_{\{C_{t},A_{t},d_{t}\}} \left\{ U(C_{t},S(d_{t})) + \beta E \left[ V_{t+1}^{i} \left( E^{i} \left( X_{t+1} \mid \pi_{t}^{H},A_{t},d_{t} \right) \right) \mid Y_{t} \right] \right\}$$
(2)

where the superscript *i* denotes the household's expectation type,  $i = \{\text{Attentive}, \text{Inattentive}\}, d_t$  is the household's housing choice and  $\pi_t^H$  is the change in house prices from period t-1 to period t.<sup>8</sup>

So, how do house price changes affect the household's state vector,  $X_t$ ? Recall that the budget constraint reads as follows:

$$C_t + A_t + F(d_t) = W_t \tag{3}$$

where  $W_t$  are the household's beginning-of-period liquid resources and  $F(d_t)$  denotes housing-related expenses, given by

$$F(d_t) = \begin{cases} P_t H_t + \theta (1 - \delta) P_t H_{t-1} & \text{if } H_{t-1} \neq H_t > 0 \\ P_t H_t & \text{if } H_{t-1} = H_t > 0 \\ \tilde{P}_t \tilde{H}_t + \theta (1 - \delta) P_t H_{t-1} & \text{else} \end{cases}$$
(4)

In the presence of movements in the house price, the household must account for potential housing capital gains since the law of motion of  $W_t$  is given by

$$W_{t+1} = Y_{t+1} + (1 + r_b + \mathbb{1}_{\{A_t < 0\}}\zeta) A_t + (1 - \delta) P_{t+1}H_t$$
  
=  $Y_{t+1} + (1 + r_b + \mathbb{1}_{\{A_t < 0\}}\zeta) A_t + (1 - \delta) P_t H_t (1 + \pi_{t+1}^H)$  (5)  
=  $Y_{t+1} + (1 + r_b + \mathbb{1}_{\{A_t < 0\}}\zeta) A_t + (1 - \delta) P_t H_t + \underbrace{(1 - \delta) P_t H_t \pi_{t+1}^H}_{\text{Housing Capital Gains}}$ 

Hence, *conditional on a current-period housing and saving choices*, the household will forecast its future beginning-of-period resources as:

$$E^{i}(W_{t+1} | Y_{t}, \pi_{t}^{H}, A_{t}, H_{t}) = E(Y_{t+1} | Y_{t}) + (1 + r_{b} + \mathbb{1}_{\{A_{t} < 0\}}\zeta)A_{t}$$

$$+ (1 - \delta)P_{t}H_{t} + (1 - \delta)P_{t}H_{t}E^{i}(\pi_{t+1}^{H} | \pi_{t}^{H})$$
(6)

where expected housing capital gains – i.e., the last term of equation (6) – varies by expectation type, *i*, according to equation (1). Importantly, whether or not the household receives next-period housing capital gains depends on its *current period housing choice*. Hence, the mechanism through which differing expectations about future house price

<sup>&</sup>lt;sup>8</sup>Throughout we will assume that household's expectations of the rental rate obey the user cost of housing formula, and so are implicitly defined by their expectations of house-price growth.

growth influence consumption and savings decisions works through a wealth/endowment effect due to future capital gains.<sup>9</sup>

This implies that the household's problem in recursive form – following Berger et al. (2015) – is as follows; again, suppressing the subscript *i* when it is not essential.

**Definition 1** Given the environment and vector of state variables defined above,  $X_t$ , the household's dynamic programming problem is given by:

$$V_t^i(X_t) = \max\left\{V_t^{i,adj}(X_t), V_t^{i,noadj}(X_t), V_t^{i,rent}(X_t)\right\}$$

where

$$V_{t}^{i,adj}(X_{t}) = \max_{\{C_{t},A_{t},H_{t}\}} \left\{ U(C_{t},S(H_{t})) + \beta E^{i} \left[ V_{t+1}(X_{t+1}) \mid Y_{t},\pi_{t}^{H} \right] \right\}$$
  
s.t.  $C_{t} + A_{t} + P_{t}H_{t} = Y_{t} + \left( 1 + r_{b} + \mathbb{1}_{\{A_{t-1} < 0\}}\zeta \right) A_{t-1} + (1-\theta)(1-\delta) P_{t}H_{t-1}$   
 $A_{t} \geq -\lambda_{m}P_{t}H_{t}$ 

$$V_{t}^{i,noadj}(X_{t}) = \max_{\{C_{t},A_{t}\}} \left\{ U\left(C_{t}, S\left(H_{t-1}\right)\right) + \beta E^{i}\left[V_{t+1}\left(X_{t+1}\right) \mid Y_{t}, \pi_{t}^{H}\right] \right\}$$
  
s.t.  $C_{t} + A_{t} + \delta P_{t}H_{t-1} = Y_{t} + \left(1 + r_{b} + \mathbb{1}_{\{A_{t-1}<0\}}\zeta\right)A_{t-1}$   
 $A_{t} \geq \left\{ \begin{array}{cc} \left[1 + r_{M} - \phi\left(r_{M}, j\right)\right]A_{t-1} & \text{if } A_{t-1} < 0\\ 0 & \text{if } A_{t-1} \ge 0 \end{array} \right.$ 

$$V_{t}^{i,rent}(X_{t}) = \max_{\{C_{t},A_{t},\tilde{H}_{t}\}} \left\{ U\left(C_{t},S\left(\tilde{H}_{t}\right)\right) + \beta E^{i}\left[V_{t+1}\left(X_{t+1}\right) \mid Y_{t},\pi_{t}^{H}\right] \right\}$$
  
s.t.  $C_{t} + A_{t} + \tilde{P}_{t}\tilde{H}_{t} = Y_{t} + \left(1 + r_{b} + \mathbb{1}_{\{A_{t-1}<0\}}\zeta\right)A_{t-1} + (1-\theta)\left(1-\delta\right)P_{t}H_{t-1}$   
 $A_{t} \geq 0$ 

with the terminal value of the household's value function (at period T + 1) given by:

$$V_{T+1}\left(X_{T+1}\right) = B\left(\tilde{W}_{T+1}\right)$$

where

$$\tilde{W}_{T+1} = (1+r_b) A_T + (1-\delta) P_{T+1} H_T$$

since at T the relevant borrowing constraint is  $A_T \ge 0$  and where income follows the process specified above.

The model is solved using the dicrete-continuous endogenous gird method (DC-EGM) as in Iskhakov et al. (2017). This procedure builds on the EGM of Carroll (2006) and consists in using an exogenous end-of-period (i.e., post-decision) savings grid and the

<sup>&</sup>lt;sup>9</sup>Recall that in solving its dynamic programming problem, the household will compare choice-specific (with regard to housing) value functions in order to select its optimal discrete choice. Thus, the expected capital gains, being a function of the current discrete choice  $(H_t)$ , directly affect the household's current decision on housing.

household's smoothed Euler equation to back out an endogenous grid for beginning-ofperiod financial wealth. The addition of taste shocks to the household's problem helps smooth out kinks in the Euler equation that arise from the presence of discrete choices and, by assuming an i.i.d. extreme value distribution, speeds up computation due to the availability of an analytical expression for the discrete choice probabilities. Lastly, secondary kinds in choice-specific value functions are handled by eliminating segments that fall below the upper envelope of the correspondence. This approach helps eliminate discontinuities in the aggregate demand of housing, which facilitates the computation of equilibrium prices.

## 4 Temporary Equilibria and Price Dynamics

In order to study the implications of household expectations heterogeneity for aggregate price dynamics, we use the structural housing model to look at the sequence of temporary equilibria generated by the observed distribution of household income, wealth, demographics and expectations. A temporary equilibrium for date t, following Piazzesi and Schneider (2016), is defined as "a collection of prices and allocations such that markets clear given beliefs and agents' preferences and endowments" (p. 1587). That is, a temporary equilibrium is a static concept where the cross-sectional distribution of expectations and wealth is kept exogenous, and market-clearing is imposed. In this sense, a *full-information rational expectations equilibrium* is as sequence of temporary equilibria such that agents' price expectations coincide with their physical measure. Further, again following Piazzesi and Schneider (2016), "a sequence of temporary equilibria is a collection of date t temporary equilibria that are connected via the updating of endowments" (p. 1589).

By modeling the dynamic fluctuations of aggregate prices as a sequence of temporary equilibria, we are able to account for the effect of distributional changes (including in expectations) within the household sector on aggregate prices, while remaining agnostic about the source of such changes. Thus, we incorporate the influence of shifting heterogeneous expectations on house-price growth without taking a stand on the specific expectation-formation process that is behind the observed joint distribution of households. Further, by taking the supply of assets directly from the data, we do not need to explicitly model the supply side of the economy. And given that our results arise from a exogenous sequence of joint distributions, they will continue to hold for any model that delivers an identical sequence of equilibrium distributions – regardless of the source of fluctuations and supply-side dynamics. Finally, note that we do not need to treat the distribution of households as a state variable in the household's dynamic programming problem since this is only relevant (in the presence of aggregate risk) when it informs the household's price expectations, which we already directly observe. Next, we provide a formal definition of the temporary equilibrium in the environment we study and assuming a small open economy. Let  $\mathcal{E}$  be the set of possible income realizations,  $\mathcal{A} = \mathbb{R}$  the set of possible non-housing assets held by the household,  $\mathcal{H}$  the set of possible housing assets, as defined above, and  $\mathcal{J}$  the set of possible ages. Let X = (Y, A, H, j) and  $\mathcal{X} = \mathcal{E} \times \mathcal{A} \times \mathcal{H} \times \mathcal{J}$ , where it is understood that timing convention for X is as described in the recursive formulation of the household's problem. Further, let  $\mathcal{P}(\iota)$  and  $\mathcal{B}(\iota)$  denote the power set and the Borel  $\sigma$ -algebra of  $\iota$ , respectively. Finally, let  $\mathcal{M}$  be the set of all probability measures on the measurable space  $(\mathcal{X}, \mathcal{B}(\mathcal{X}))$ .<sup>10</sup>

**Definition 2** Given interest rates  $r_b$  and  $r_m$ , a fixed supply of owner-occupied and rental housing  $-\bar{H}_t$  and  $\bar{H}_t$ , respectively -, and a joint distribution of endowments, assets, housing, ages and expectation types,  $\Phi_t$ , a temporary equilibrium is a set of value and policy functions  $V_t^i : \mathcal{X} \to \mathbb{R}$ ,  $h_t^i : \mathcal{X} \to \mathcal{H}$ , and  $\tilde{h}_t^i : \mathcal{X} \to \tilde{\mathcal{H}}$ , where the superscript i denotes agents' expectations-type, and prices P and  $\tilde{P}$  such that

- 1. The functions  $V_t^i$ ,  $h_t^i$ , and  $\tilde{h}_t^i$  are measurable with respect to  $\mathcal{B}(\mathcal{X})$ , the function  $V_t^i$ satisfies the household's Bellman equation and functions  $h_t^i$  and  $\tilde{h}_t^i$  are the associated policy functions; given P,  $\tilde{P}$ ,  $E^i(P)$  and  $E^i(\tilde{P})$ .
- 2. Markets clear

$$\bar{H}_t = \int h_t^i d\Phi_t$$
$$\bar{\tilde{H}}_t = \int \tilde{h}_t^i d\Phi_t$$

Further, again following Piazzesi and Schneider (2016), "a sequence of temporary equilibria is a collection of date t temporary equilibria that are connected via the updating of endowments" (p 1589). Additional discussion is provided in appendix A.

Computationally, the problem of finding the date t temporary equilibrium is as follows. (i) From the data set construct a matrix where each row represents a household and columns register the year, assets, owned housing value, rental housing value, income state, assets state, housing state, age and expectation type. (ii) Define a function that computes for a given price vector,  $(P_t, \tilde{P}_t)$ , a vector of excess demand,  $(R^H, R^{\tilde{H}})$ , as follows:

$$\begin{aligned} R^{H} &= P_{t} \sum_{n=1}^{N} \left[ h\left(Y_{t}^{n}, A_{t-1}^{n}, H_{t-1}^{n}, j_{t}^{n}, E(\pi_{t+1}^{H} \mid \pi_{t}^{H})^{n} \right) \right] - \sum_{n=1}^{N} (PH)_{t}^{n} \\ R^{\tilde{H}} &= \tilde{P}_{t} \sum_{n=1}^{N} \left[ \underbrace{\tilde{h}}_{\text{Model}} \left( \underbrace{Y_{t}^{n}, A_{t-1}^{n}, H_{t-1}^{n}, j_{t}^{n}, E(\pi_{t+1}^{H} \mid \pi_{t}^{H})^{n}}_{\text{Data}} \right) \right] - \sum_{n=1}^{N} \underbrace{(\tilde{P}\tilde{H})_{t}^{n}}_{\text{Data}} \end{aligned}$$

<sup>10</sup>Where  $\mathcal{B}(\mathcal{X}) = \mathcal{P}(\mathcal{E}) \times \mathcal{B}(\mathcal{A}) \times \mathcal{P}(\mathcal{H}) \times \mathcal{P}(\mathcal{J}).$ 

where  $n \in N$  denotes a household's number in the sample. (iii) Finally, use a root-finding algorithm to derive the price vector,  $(P_t, \tilde{P}_t)$ , such that markets clear, i.e.,  $(R^H, R^{\tilde{H}}) = (0, 0)$ .

In this way, the sequence of temporary equilibria generated by the model allows us to map the *observed* sequence of distributions over expectation types and states,  $\{\Phi_t\}_{t=2003}^{t=2016}$ , to a sequence of price vectors,  $\{(P_t, \tilde{P}_t)\}_{t=2003}^{t=2016}$ , which includes the boom-bust cycle in the housing sector in the Netherlands.

## 4.1 Functional Forms and Calibration

In this section, we specify the functional forms relating to households' preferences and discuss calibration.

**Functional Forms** Households' instantaneous utility function, following Landvoigt (2017) and Berger et al. (2015) is given by:

$$U\left(C_{t}, S\left(H_{t}, \tilde{H}_{t}\right)\right) = \frac{\left[C_{t}^{1-\sigma}S\left(H_{t}, \tilde{H}_{t}\right)^{\sigma}\right]^{1-\gamma} - 1}{1-\gamma}$$

where  $S(\cdot)$  is linear in its first two arguments and given by

$$S\left(H_t, \tilde{H}_t\right) = \omega H_t + \tilde{H}_t + \varpi \quad \text{where} \quad \varpi \ge 0 \quad \text{and} \quad \omega \ge 1$$

where  $\varpi$  captures social housing. The specification for utility due to bequests is taken from Kaplan et al. (2017), who follow De Nardi (2004),

$$B\left(\tilde{W}\right) = \vartheta_1 \frac{\left(\tilde{W} + \vartheta_2\right)^{1-\gamma} - 1}{1-\gamma}$$

**Calibration** The parameter values that are calculated directly from the data are reported in Table 2 and the targeted moments for the parameters chosen within the model are reported in Table 3 below. Further, we take the distribution over assets and house holdings, and the demographic distribution from the data.

The model is specified at an annual frequency, with households starting their working life at age 25, retiring at age 65 and dying at age 80. The replacement rate is set to 0.80,<sup>11</sup> in accordance with the data. The estimation of the income process follows Storesletten et al. (2000, 2004). Finally, the share of housing services is set to 0.15 following Kaplan et al. (2017). The risk-free rate is set to 3% p.a. The mortgage loan mark-up is set to

<sup>&</sup>lt;sup>11</sup>Recall that this does not refer to income received from a government-managed pension system, but captures all of the non-interest income households receive after retirement.

Parameter	Interpretation	Value in Data	
Demographics			
$\overline{j}$	Period length in years	1	
J	Length of life	80	
$J^{ret}$	Retirement age	65	
$J^{born}$	Age of newborns	25	
Income Process			
$\{g(j_t)\}$	Deterministic age profile polynomial order	4	
rr	Replacement rate	0.80	
ho	Autocorrelation of persistent component	0.9669	
$\sigma_{\chi}^2$	Variance of persistent shock	0.0146	
$\sigma_\chi^2 \ \sigma_arepsilon^2$	Variance of transitory shock	0.2908	
Financial Instruments			
$r_b$	Risk-free rate	0.03	
$\zeta$	Mortgage loan markup	0.01	
$\lambda_m$	Maximum LTV ratio on mortgage loans	0.90	

 Table 2: Parameters Fixed to Data-Counterparts

the period average of 1% p.a., and the maximum LTV ratio is set to 0.90 equal to its counterpart in the data.

The parameters to be chosen within the model are (i) the discount factor, (ii) additional utility from home-ownership, (iii) the strength of the bequest motive, (iv) the degree of luxuriousness of bequests, and (v) the rental and owner-occupied housing grids. We assume a small open economy and estimate the temporary equilibrium of the model while targeting the distributional moments obtained from the average over the entire period of analysis, from 2003 to 2016. The parameter values and targeted moments are reported in tables 3 and 4, respectively. The algorithm employed for calibration of the model is detailed in appendix B.

Table 3: Calibrated Parameter Values			
Parameter	Interpretation	Value	
β	Discount factor	1.015	
ω	Additional utility from owning	0.747	
$\vartheta_1$	Strength of bequest motive	1582.326	
$\vartheta_2$	Luxuriousness of bequests	43.063	
${\cal H}$	Owned Housing Bins	in text	
$ ilde{\mathcal{H}}$	Rental Housing Bins	in text	

The housing sector supply side is approximated by discrete housing bins for both rental and owner-occupied housing. Following Kaplan et al. (2017), the owner-occupied housing set is constructed through a log-linear grid where the minimum grid point, the number of grid points and the (fixed) difference between grid points are chosen by targeting the  $10^{th}$ percentile (69.10%), the mean (90.94%) and the  $90^{th}$  percentile (100.00%) percentiles of the distribution of housing net wealth to total net wealth. For rental units of housing, we follow an analogous strategy but target the distribution of rental housing expenditures to net income; where the corresponding percentiles (now using the  $50^{th}$  percentile instead of the mean) are 17.99%, 37.38% and 55.37%, respectively. Additionally, we take from the literature the house depreciation rate,  $\delta = 0.03$ , and the house sales transaction cost,  $\theta = 0.07$ , from studies of the Dutch housing sector (see Van Ommeren and Van Leuvensteijn (2005) and Sánchez and Andrews (2011)). Finally, following Fernandez-Villaverde and Krueger (2011), the parameter governing the value of social housing is set low enough for it to be irrelevant for the quantitative properties of the model.<sup>12</sup>

Table 4: Target Data Moments Targeted Moment Data Model Average financial assets -0.454-0.248Home-ownership rate 0.7480.773Median  $NW_{j=75}$  / Median  $NW_{j=50}$ 1.4441.330Percent of bequest HHs in bottom half of NW dist. 0.080 0.112Housing/Net Worth 10th percentile 0.7000.797Housing/Net Worth mean 0.9160.948 Housing/Net Worth 90th percentile 1.000 1.000 Rent / Income 10th percentile 0.0540.178Rent / Income 50th percentile 0.316 0.133Rent / Income 90th percentile 0.5540.277

The calibration implies a rent-to-price ratio of 0.12. Even though the rent-price ratio is not targeted in our estimation procedure, its value in the model, 0.12, is not that far from the value in the data, 0.07.

## 5 Results

In this section we study the role of household house-price expectations in the housing boom-bust cycle.

# 5.1 Temporary Equilibria Sequence with Extrapolative Expectations

In this section we present the sequence of temporary equilibria generated by the model and compare it to the data. This exercise is reported in Figure 6 below. Specifically, the figure plots the time series for the aggregate house price and rental rate in the Netherlands, along with the sequence of temporary equilibria generated by the model. For every period, we employ the corresponding cross-sectional distribution over states and expectation types,  $\Phi_t$ , and set attentive households' expectations according to equation (1). All series are normalized to their mean. Note that the aggregate series is taken directly from Statistics Netherlands, while the sequence of cross-sectional distributions generating the model prices are constructed from the DNB Household Panel Survey – and recall that none of the plotted series are targeted in calibration.

<sup>&</sup>lt;sup>12</sup>In practice, this is achieved by setting  $\varpi = 0.000001$ , implying that only about 1% of households choose to not consume positive levels of rental or owner-occupied housing.

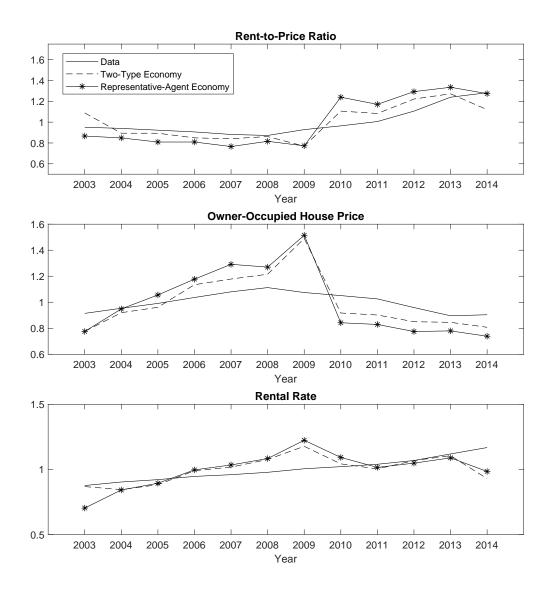
The model does a fairly good job of matching the general pattern in house prices and rental rates, and hence also in the rent-to-price ratio. This serves as a direct validation of the potential for household expectations to play a key role in driving boom-bust cycles in house prices. However, the model overshoots both the boom and bust phases of the episode. This could be because (i) extrapolative expectations lead to too strong an expected capital gains mechanism, that is households expect prices to rise in the future, therefore demand a lot of housing in the current period, thereby driving up current prices or (ii) the absence of aggregate risk in the model leads households to underestimate the risk from fluctuating house prices.



#### Figure 6: Sequence of Temporary Equilibria

## 5.2 Prices and Expectation Heterogeneity

In this section we compare the sequence of temporary equilibria for the two-type economy presented above with that of a representative-agent economy. In this economy, we eliminate heterogeneity in expectations by forcing all households to hold the same expectation of house-price growth. To keep the comparison fair, we set expected house-price growth equal to the average expectation in the two-type economy. Figure 7 below reproduces the graph presented above, but adds the representative-agent economy; all series are normalized to their mean value.



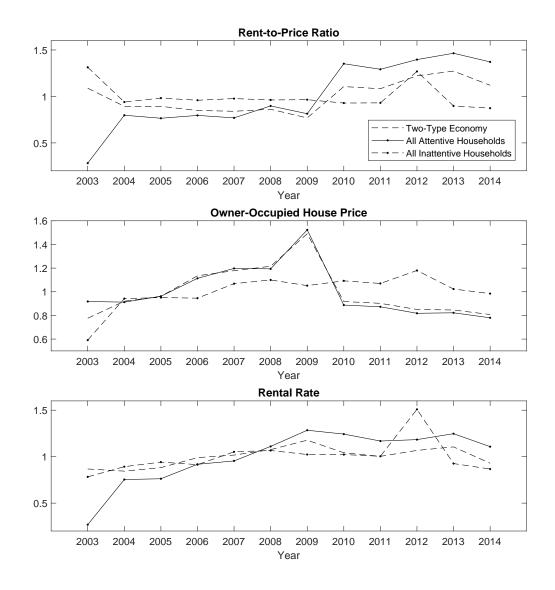
#### Figure 7: Two- vs. One-Type Economy

The plot shows that prices are more volatile in the representative-agent economy and hence, there is a deterioration of the model's fit to the data. Thus, adding a dimension of heterogeneity in expectations allows for a more accurate mapping of the observed distribution of households to aggregate price dynamics. In this sense, heterogeneity in expectations matters for accurately modeling aggregate price dynamics. The mechanism that links heterogeneity in expectations to differences in household policy functions – namely, the capital gains mechanism – seems an important driver of household behavior and a relevant feature of the data.

The greater price volatility generated by the representative-agent version of the model is given by a higher rise in house-prices during the boom phase of the cycle and a deeper fall in the bust phase. This owes to the strength of the capital gains channel coupled with a high rate of home-ownership. Both in the model and the data, attentive households tend to be home-owners. During the boom phase, these households have higher houseprice growth expectations in the two-economy model relative to the representative agent case, so they demand relatively more owner-occupied housing. This requires only a more moderate increase in house-prices to clear the market. During the bust phase, the same mechanism works in the opposite direction so that house-price fluctuations are smoothed out over the cycle in the two-types economy, relative to the representative agent economy. This mechanism is explored in greater detail in the next section.

Figure 8 plots the comparison between the two-type economy and two counter-factual one-type economies: (i) all households are attentive and hold extrapolative expectations and (ii) all households are inattentive.<sup>13</sup> Note that in this exercise, the mean expectation is different in each economy. Once again, the sequence of temporary equilibria generated by the two-type economy provides a better match to the data (not shown) than the counter-factual economies, especially for the rent-to-price ratio. While the qualitative pattern of the *attentive economy* is broadly in line with the data, the amplitude of the cycle for the rent-to-price ration and the rental rate is far greater than in the data. This is due to the fact that it is precisely inattentive households' expectations which are exogenously changed; since inattentive households tend to be renters, it is the rental rate that deviates most from the data.

<sup>&</sup>lt;sup>13</sup>The sequence of temporary equilibria in this case is approximate.



In the case of the *inattentive economy*, the amplitude of the cycle is drastically reduced and the sequence of temporary equilibria largely fail to replicate the boom-bust pattern observed in the data. Thus, the main takeaway of these exercises is a that the data are better matched by the two-type economy, despite not being targeted in calibration. Recall that attentive/inattentive households are identified in the data –without any information from the model– and that the difference in their policy functions comes not from a reduced-form procedure, but from the *expected capital gains mechanism* that was described in section 3. Thus, in order to match the empirical rent-to-price ratio through a sequence of temporary equilibria, one must account for the independent dimension of heterogeneity that is captured in the survey by households' house-price expectations.

## 5.3 Prices and Expectations

In this section we look at the relationship between equilibrium prices and house-price expectations, while holding constant the distribution over states. Specifically, we plot the equilibrium rent-to-price ratio, the price of owner-occupied housing units and the rental rate against the expected house-price growth of attentive households. Note that each households' type is unchanged across equilibria, so the average expectation is also changing. We employ the distribution over the entire period; that is, we pool all observations. In this way, we isolate the effect of house-price expectations on equilibrium prices.

Figure 9 plots the results of this exercise. Focus on the top graph first. The clearest pattern is that the rent-to-price ratio is strictly decreasing in the rate of expected house-price growth of attentive households. This is consistent with the user cost of housing equation, which states that the price of a unit of housing is equal to the expected net present value of the stream of income that would be generated by the property if rented:

$$P_t = E_t \left[ \sum_{\tau=0}^{\infty} \left( \frac{1-\delta}{1+r_b} \right)^{\tau} \tilde{P}_{t+\tau} \right]$$

where  $\delta$  is the house depreciation rate and  $r_t$  is the risk-free interest rate.

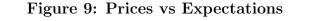
This expression can be written in recursive form as

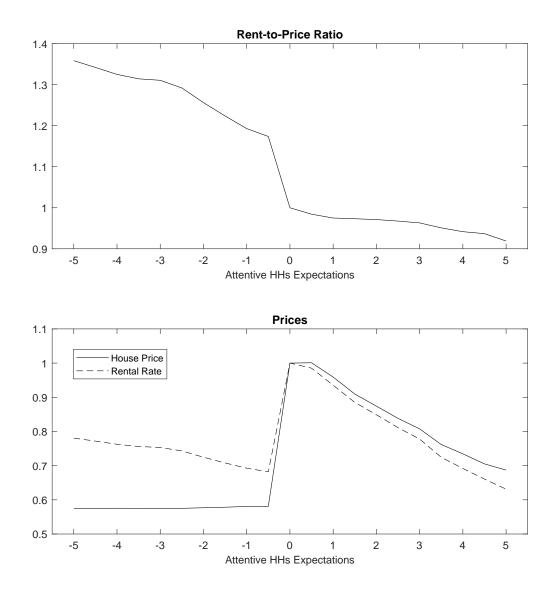
$$P_t = \tilde{P}_t + \left(\frac{1-\delta}{1+r_b}\right) E_t \left(P_{t+1}\right)$$

which implies the following relationship between the rent-to-price ratio and expected future house-price growth:

$$\frac{\tilde{P}_t}{P_t} = 1 - \left(\frac{1-\delta}{1+r_b}\right) E_t \left(1 + \pi_{t+1}^H\right)$$

where  $\pi_{t+1}^{H}$  is the rate of house-price growth between period t and t+1. This establishes the predicted negative relationship between expected house-price growth and the rentto-price ratio that features in the current exercise. However, the graph displays two interesting patterns: (i) a steeper slope in the area where expected house-price growth is negative and (ii) a discontinuity at zero expected house-price growth.





The bottom graph shows the (normalized) level of prices across the different temporary equilibria. Again, we see a discontinuity at the point where all households expect zero house-price growth. When attentive households' expected house-price growth is negative and lower, the price of owner-occupied housing is substantially lower than the zero-growth reference point and decreasing; while the rental rate is increasing for lower expected growth. Lower expected negative growth leads households to demand more rental housing, pushing up the rental while owner-occupied house prices remain low.

When attentive households expect positive house-price growth, on the other hand, owner-occupied house prices are also decreasing in expectations. The rental rate is also decreasing in expectations, and at a faster rate than owner-occupied prices. Hence, the negative relationship between expectations and the rent-to-price ratio apparent in the top graph of Figure 7. But while the negative relationship between the rental rate and expected house-price growth is intuitive, the pattern observed in house prices requires further comment.

To understand the decreasing (in expected house-price growth) level of house prices, consider the temporary equilibrium housing demand condition for a single attentive household, i, in response to an increase in expected house-price growth,  $\uparrow E^i(\pi^H)$ , while keeping the rental rate fixed at  $\tilde{\tilde{P}}$ :

$$\bar{PH}_{TE}^{i} = \downarrow P_{TE} \uparrow h\left(Y_{TE}^{i}, A_{TE}^{i}, H_{TE}^{i}, j_{TE}^{i}, \uparrow E^{i}(\pi^{H}); \bar{\tilde{P}}\right)$$

As the household expects higher house-price growth, it demands more housing,  $\uparrow h(\cdot)$ , due to the capital gains mechanism described in section 3. But since the left hand side of the equation is fixed (i.e., the household's observed level of housing consumption),  $P\bar{H}_{TE}^i$ , the level of house prices must adjust downward to maintain the equality,  $\downarrow P_{TE}$ . This effect will be attenuated, amplified or reversed depending on the size of the adjustment – in equilibrium – of the rental rate. Thus, while the direction of the rent-to-price ratio in response to expected house-price growth is unambiguous, the relationship between the level of house prices and expected house-price growth across temporary equilibrium depends on the sensitivity of the rental rate to expectations.

Alternatively, this result can be rationalized by considering how expected-house price growth influences the household's dynamic programming problem in this setup. Assume, for simplicity, that the marginal household, *i*, consumes positive owner-occupied housing is not credit constrained and that housing is continuous rather than discrete.<sup>14</sup> We can break up the household's problem, equations (2)-(5), into two stages: (i) conditional on a specific housing choice,  $d_t = H_t$ , consumption and savings are chosen, and (ii) housing is optimally chosen given the choice-specific value functions of the problem.<sup>15</sup> So, we can define the choice-specific dynamic programming problem at period *t* as follows:

$$\begin{split} \tilde{V}_{t}^{i} \left( \tilde{W}_{t}, H_{t}; E^{i}(\pi_{t+1}^{H}) \right) &= \max_{\{C_{t}, A_{t}\}} \left\{ U\left( C_{t}, S\left( H_{t} \right) \right) + \beta E^{i} \left[ V_{t+1}\left( W_{t+1} \right) \mid Y_{t} \right] \right\} \\ \text{s.t.} \quad \tilde{W}_{t} &= C_{t} + A_{t} \\ W_{t+1} &= Y_{t+1} + \left( 1 + r_{b} + \mathbb{1}_{\{A_{t} < 0\}} \zeta \right) A_{t} + \left( 1 - \delta \right) P_{t} H_{t} \left( 1 + \pi_{t+1}^{H} \right) \end{split}$$

where  $\tilde{W}_t = W_t - F(H_t)$ ; that is, the level of cash-on-hand once all housing-related expenditures have been carried out. Note that we have made explicit the problem's dependence on the expected house-price growth,  $E^i(\pi_{t+1}^H)$ , which here acts as a parameter similar to  $r_b$  as it defines the expected capital gains on housing. Since the problem is conditional on a level of housing,  $H_t$ , all that is needed to forecast future cash-on-hand,

<sup>&</sup>lt;sup>14</sup>This is a good approximation to our numerical analysis since our solution algorithm involves convexifying the problem by including taste shocks.

<sup>&</sup>lt;sup>15</sup>For details on this solution strategy, see Piazzesi and Schneider (2016).

 $W_{t+1}$ , for any choice of  $\{C_t, A_t\}$  is current income.<sup>16</sup> Lastly, since we assume the household is not credit constrained current cash-on-hand is the only relevant state variable going forward.

Having defined the choice-specific value function,  $\tilde{V}_t^i\left(\tilde{W}_t, H_t; E^i(\pi_{t+1}^H)\right)$ , we can specify the household's optimal housing-choice as follows

$$V_t^i(W_t; E^i(\pi_{t+1}^H)) = \max_{\{H_t \in \mathcal{H}\}} \left\{ \tilde{V}_t^i(W_t - F(H_t), H_t; E^i(\pi_{t+1}^H)) \right\}$$

This establishes the following optimality condition that must hold for the marginal household in equilibrium; recall we have assumed continuous housing.

$$F'(H_t) = P_t = \frac{\tilde{V}_{H,t}^i \left( \tilde{W}_t, H_t; E^i(\pi_{t+1}^H) \right)}{\tilde{V}_{\tilde{W},t}^i \left( \tilde{W}_t, H_t; E^i(\pi_{t+1}^H) \right)}$$

where  $\tilde{V}_{\tilde{W},t}^i$  and  $\tilde{V}_{H,t}^i$  denote the partial derivatives of the choice-specific value functions with respect to financial wealth and house holdings, respectively. So, for the temporary equilibrium price of housing to be decreasing in expected house-price growth, all that is required is that for the marginal household  $\partial \tilde{V}_{\tilde{W},t}^i/\partial E^i(\pi_{t+1}^H) > \partial \tilde{V}_{H,t}^i/\partial E^i(\pi_{t+1}^H)$ . That is, if higher expected house-price growth has a greater impact on the marginal utility of wealth relative to housing, the temporary equilibrium price of housing will be lower. Potentially, this could be due to a strong capital gains channel or simply because of market structure, as an additional unit of wealth can be freely allocated towards any asset or consumption whereas for an additional unit of housing faced transaction costs.

Finally, it is worth stressing that this relationship holds only at the cross-sectional level, since the temporary equilibrium does not impose any relation between expected house-price growth and actual house-price growth. Hence, the influence of house-price expectations on future house-prices is likely to work via its effects on the distribution of households over states.

## 6 Conclusion

We developed a structural model of household consumption and savings featuring a housing market with the availability of rental housing to study the influence of expectations on aggregate house prices. To do so, we allowed for heterogeneity in expectations in the form of two types of households that are distinct in the data: attentive households that hold extrapolative expectations and inattentive households that expect no change in house prices. By computing the sequence of temporary equilibria for the boom-bust episode we are able to map the empirical sequence of distributions of households over assets, housing,

<sup>&</sup>lt;sup>16</sup>To simplify notation, we have left savings and housing out of the conditional expectations operator.

income, age and expectation type onto aggregate house prices while remaining agnostic about the expectation-formation process.

We find that the model is able to closely match the rent-to-price ratio over this period, but overshoots the size of the housing boom. Different household expectations translate into different policy functions through the housing capital gains mechanism that is present in the model. This constitutes an independent dimension of heterogeneity in the data that turns out to improve the fit of the model when compared to its representative-agent counterpart. In the representative-agent economy, expectations are set to be homogeneous and equal to the average expectation of the two-type economy. This leads to a deterioration in the empirical performance of the model, which requires heterogeneity in expectations since inattentive households are relatively unreactive to house-price changes. Forcing every household to hold the same expectation hence leads to counterfactually large variation in prices.

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# Appendix

# A A Note on Equilibrium Concepts

## A.1 Two Related Equilibrium Concepts

There are three equilibrium concepts:

- 1. Temporary equilibrium
- 2. Partial equilibrium
- 3. Modified temporary equilibrium

and two types of agents

- 1. Inattentive agents
- 2. Attentive agents

#### A.1.1 Expectations by Agent Type

Before turning to the equilibrium concepts, let's define expectations of the two types of agents formed in any period t. Inattentive agents have static expectations on house price dynamics whereas attentive agents form expectations on the basis of house price growth. We define expectations on the basis of some current period price  $\tilde{p}_t$ . The equilibrium concepts are distinguished by a definition of  $\tilde{p}_t$  (to be made precise below) and an according cross-sectional distribution  $\tilde{\Phi}_t$ .

Using  $\tilde{p}_t$ , expectations are as follows:

1. Inattentive agents:

$$Ep_{t+1} = \tilde{p}_t$$

2. Attentive agents:

$$E\pi_{t+1} = \rho\pi_t$$
  
$$\pi_t = \ln \tilde{p}_t - \ln \tilde{p}_{t-1}$$
  
$$\Leftrightarrow \quad Ep_{t+1} = \tilde{p}_t + E\pi_{t+1} = \tilde{p}_t + \rho\pi_t$$

#### A.1.2 Equilibrium Concepts

To compute the equilibrium, we assume that agents act under point expectations, i.e., we solve the model under the assumption that  $p_{t+1} = Ep_{t+1}$ . This enables us to compute the equilibrium using methods for models without any aggregate risk.

The three equilibrium concepts are as follows:

1. Temporary equilibrium.

We compute a temporary equilibrium taking as given the data observations on (i) the equilibrium house prices and (ii) the cross-sectional distribution, thus

$$\tilde{p}_t = p_t^d$$
 and  $\tilde{\Phi}_t = \Phi_t^d$ 

where superscript d denotes a data object.

Market clearing in this model gives an equilibrium price  $\hat{p}_t$  which does almost surely not coincide with the data equilibrium price  $p_t^d$ . This implicitly defines a shock  $\epsilon_t$ . Assuming that the post-shock equilibrium price coincides with the data price  $p_t = p_t^d$ , this implies that  $p_t = \hat{p}_t + \epsilon_t = p_t^d + \epsilon_t$ , i.e.,  $\epsilon_t = p_t^d - \hat{p}_t$ , but this shock does play no further role in the model. The reason is that the shock would affect the cross-sectional distribution but since we aggregate with the data distribution  $\Phi_t^d$ , the implicit shock realization does not appear explicitly in the model computations. Thus:

- (a) Given the price expectations of the two types we compute policy functions.
- (b) Given the policy functions we aggregate and compute the equilibrium price in the housing market  $\hat{p}_t$  by aggregating with the data distribution  $\Phi_t^d$ .
- (c) Upon market clearing (=convergence on  $\hat{p}_t$ ), the MIT shock hits the economy such that the post-shock equilibrium price is  $p_t = p_t^d$ , which implies  $p_t = \hat{p}_t + \epsilon_t$ , i.e.,  $\epsilon_t = p_t \hat{p}_t = p_t^d \hat{p}_t$ .
- (d) However, the shock does not play an explicit role in the computations because  $\Phi_t = \Phi_t^d$ .

Thus, in the temporary equilibrium there is no link between time periods t and t+s, for  $s \ge 1$ .

2. Partial equilibrium.

We compute a partial equilibrium, i.e., a market clearing price  $p_t$  in any period t and continue to base expectations on the data price  $p_t^d$ . We aggregate with the crosssectional distribution that is implied by the policy functions and the exogenous laws of motion (shock processes) in the respective period t, thus

$$\tilde{p}_t = p_t^d \quad \text{and} \quad \tilde{\Phi}_t = \hat{\Phi}_t$$

In this variant, a crucial distinction is between a beginning of period and an end of period cross-sectional distribution. The MIT shock  $\epsilon_t = p_t^d - \hat{p}_t$  now plays a crucial role, where again  $\hat{p}_t$  is the pre-shock equilibrium price:

- (a) We enter period t with a cross-sectional distribution  $\Phi_{t-1}$ , which is the end of period cross-sectional distribution from period t-1.
- (b) Given the price expectations of the two types we compute policy functions.
- (c) Given the policy functions we aggregate and compute the equilibrium price in the housing market  $\hat{p}_t$  and associated cross-sectional distribution  $\hat{\Phi}_t$ . Observe that  $\hat{\Phi}_t = G(\Phi_{t-1}, \eta_{t-1}, \eta_t)$ , where  $G(\cdot)$  maps  $\Phi_{t-1}$  into  $\hat{\Phi}_t$  through policy functions (endogenous laws of motion) and income shocks  $\eta_t$  (exogenous laws of motion).
- (d) Upon market clearing (=convergence on  $\hat{p}_t$  implying the equilibrium crosssectional distribution  $\hat{\Phi}_t$ ), the MIT shock hits the economy such that the post-shock equilibrium price is  $p_t = p_t^d$ , which implies  $p_t = \hat{p}_t + \epsilon_t$ , i.e.,  $\epsilon_t = p_t - \hat{p}_t = p_t^d - \hat{p}_t$ .
- (e) This gives rise to a post-shock end of period cross-sectional distribution, which is implied by the distribution  $\hat{\Phi}_t$  and the shock, thus  $\Phi_t = H(\hat{\Phi}_t, \epsilon_t)$ .

Thus, in the partial equilibrium time periods t and t+s, for  $s \ge 1$  are linked. Specifically, the cross-sectional distribution  $\Phi_t$  is  $\Phi_t = H(\hat{\Phi}_t, \epsilon_t) = H(G(\Phi_{t-1}, \eta_{t-1}, \eta_t), \epsilon_t)$ .

3. Modified temporary equilibrium.

In the modified temporary equilibrium we base expectations on the current equilibrium price  $\hat{p}_t$ , i.e.,

$$\tilde{p}_t = \hat{p}_t$$
 and  $\tilde{\Phi}_t = \Phi_t^d$ 

## **B** Calibration Algorithm

In this section, we describe the calibration algorithm.

**Algorithm 1** This algorithm calibrates the model. Note that the ratio of equilibrium prices is not a targeted object.

- 1. Start with an initial guess for the parameter values,  $\phi^0$ .
- 2. Given current guess for parameters,  $\phi^k$ , compute the price vector of the temporary equilibrium,  $\bar{P}^k$ ; where k denotes the iteration step.
  - Note that this step implies finding the price vector that clears the owneroccupied and rental housing markets.
- 3. Compute the relevant moments implied by the equilibrium,  $\{m(\bar{P}^k)\}$ .

- 4. Check the value for the pre-specified metric,  $|| m(\bar{P}^k) m^d ||$ , where  $m^d$  denotes the targeted data moments.
  - i. If the metric is lower than the tolerance level,  $|| m(\bar{P}^k) m^d || < \varepsilon$ , stop.
  - ii. Otherwise,  $|| m(\bar{P}^k) m^d || \ge \varepsilon$ ; update guess as follows.
- 5. Given the current iterate price of housing,  $\bar{P}^k$ , compute  $\phi^{k+1} \quad s.t. \parallel m(\bar{P}^k) - m^d \parallel < \varepsilon.$
- 6. Given the new guess  $\phi^{k+1}$ , go back to Step 2.