International Trade and Innovation Dynamics with Endogenous Markups*

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Abstract

Greater openness to international trade affects both firms' innovation incentives and the markups that they can charge. In this paper, we show that considering the interaction between these two effects yields new insights. Building a two-sector dynamic general equilibrium model with endogenous innovation and endogenous markups, we find that lower trade costs lead to both lower markups and higher innovation effort of incumbent firms. These findings are consistent with firm-level empirical evidence from Spain, which shows that both export and import shocks tend to decrease within-firm markups and increase within-firm productivity. However, our model also suggests that innovation may dampen or amplify the usual pro-competitive effects of trade, depending on the initial level of concentration of the industry: while trade lowers static markups, it also increases domestic firms' incentives to escape competition by innovating if these firms are sufficiently dominant to start with. When this is the case, successful domestic innovators charge higher markups going forward; otherwise, markups are further depressed. These effects are absent in standard models of trade. In a calibrated version of the model, we show that the growth effects of trade shocks are under-estimated if the innovation-competition feedback is ignored.

JEL codes: F43, F60, L13, O31, O32, O33, and O41.

Keywords: International Trade, Markups, Innovation, R&D, Productivity.

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1 Introduction

In the last decade, a growing empirical literature has shown that trade affects firms' innovation and technology adoption choices (see Shu and Steinwender, 2018, for a recent survey). By and large, most studies find that trade liberalizations increase innovation, with important qualifications regarding the type of firms and trade shocks under consideration.[1] This suggests that trade may affect long-run productivity growth, and that its true welfare effects could be larger than the ones emphasized by the classical static trade theories.

However, while innovation and technology adoption obviously affect productivity, they also affect competition and markups. For instance, if trade incentivizes industry leaders to innovate more, the latter may increase their competitive advantage and their markups. In contrast, if trade encourages entry of new innovative firms, this may increase competition and lower markups. Conversely, the competitive structure of an industry alters firms' innovation incentives. However, while there is an extensive literature studying the effects of trade on innovation and markups separately, the feedback effects between them have received less attention. In this paper, we contribute to fill this gap by explicitly studying the interaction between trade-induced shocks to markups and innovation.

As a starting point, we use firm-level data from the Spanish manufacturing sector to document a number of stylized facts. First, we show that industry-level trade shocks (that is, increases in Spanish exports or imports, instrumented with increases in U.S. exports or imports) are associated with within-firm increases in revenue productivity and within-firm decreases in markups. While this finding holds both for export and import shocks, it is stronger for imports. This already underlines the importance of considering innovation and markups jointly: our results suggest that by ignoring markup changes, one would underestimate trade-induced productivity increases at the firm level. Second, we show that firm-level markups are substantially lower in Spain's comparative advantage industries, as identified by the Balassa (1965) *revealed comparative advantage index*.

To further investigate the interaction between trade, markups and innovation, we build a two-country, two-sector general equilibrium model with heterogeneous firms, endogenous innovation and endogenous markups. Both sectors employ specific labor for production, and the only difference between the two countries is their relative endowment of sector-specific labor. This creates a comparative advantage structure, where each country exports more in one industry than in the other, and allows us to distinguish between the effects of trade shocks in export and import sectors. In each country and sector, final goods are assembled as a Cobb-Douglas aggregate of a continuum of products. These products are, in turn, assembled as CES aggregates of domestic and foreign intermediate goods. The behavior of intermediate goods producers is at the heart of our model. At each instant, the domestic and the foreign producer in each product line engage in strategic interaction with oligopolistic competition, modeled along the

[1] For instance, Aghion *et al.* (2017) stress that initially more productive firms increase innovation, while less productive firms reduce it. Shu and Steinwender (2018) show that while most papers find positive innovation effects of higher exports or intermediate input imports, the evidence for shocks to import competition is more mixed.

lines of Atkeson and Burstein (2008). In this framework, markups are endogenous and positively correlated with a firm's market share. Furthermore, both producers can invest into R&D in order to increase their productivity, which is defined on a quality ladder. Successful innovations improve a firm's productivity relative to the competitor's, and by extension they increase the firm's market share and its markup. Note that producers face both fixed and variable trade costs when selling to the other country's market. In the balanced growth path, firms increase their R&D efforts as they approach the relative productivity levels at which they can expect changes in the degree of competition in the markets where they operate. When firms fall back sufficiently far, the foreign competitor starts exporting, at which point markups for domestic firms would decrease. Symmetrically, as firms gain a sufficient lead, they can access the foreign market and set a high markup, so R&D efforts increase. Therefore, R&D and markup decisions are intimately linked through the gains that firms obtain from trading internationally.

To illustrate how this feedback relationship may affect the overall gains from trade, we conduct a series of comparative static exercises with respect to the level of variable trade costs, in a model economy with no intra-industry trade (i.e. with a pure export sector and a pure import sector). We find that lower trade costs stimulate innovation through a market size effect in the export sector. In the import sector, though, there is an increase in competition and therefore a decrease in the the markups of firms that see a foreign competitor enter their market. The entry of high-markup, highly productive foreign firms therefore has an overall ambiguous static effect, which depends on the initial level of concentration of the industry: when the domestic leader is very dominant (high concentration), the static net effects are pro-competitive, lowering the average markup.

Most interestingly, the feedback between R&D and competition gives rise to dynamic effects which may amplify or dampen the effects of trade liberalizations. Highly concentrated industries respond to the increase in foreign competition by increasing their defensive R&D efforts, which leads to changes in the productivity distribution and further increases in the market share of domestic producers. As these are associated with higher markups, the pro-competitive effects are dampened. Conversely, less productive industries in which the domestic leader captures a small share of the market, competition from the foreign market depresses domestic innovative incentives, leading to further decreases in the market share and markups of domestic firms, thus amplifying the pro-competitive effects of the liberalization. In sum, we find that the dynamic effects are dampened or amplified depending on the initial level of concentration of each industry. These effects are absent in standard static models of trade with endogenous markups, as well as dynamic models with exogenous markups. Yet, these effects are relevant as they will affect the growth predictions of trade liberalizations.

To understand the extend to which the interaction between markups and innovation effects concentration and growth, we calibrate the model to Spain using our firm-level data via a Simulated Method of Moments procedure. In the calibrated economy, we find that the innovation-competition interaction amplifies the effects from trade. To show this, we compare the change in the average markup and TFP growth in the baseline economy with a counterfactual economy in which the productivity distribution is kept fixed at its initial values. We find that, without shifts in the distribution, average markups would react by less, and TFP growth would not increase as much. Therefore, ignoring the innovation-competition feedback would lead us to under-estimate the growth effects of trade liberalizations.

Related literature Our paper relates to different strands in the literature on international trade. Theoretical studies on the effect of trade on innovation and growth date back to the pioneering works of Grossman and Helpman (1991) and Rivera-Batiz and Romer (1991), who extended the baseline endogenous growth model to an open-economy setup. More recent contributions include Baldwin and Robert-Nicoud (2008), Atkeson and Burstein (2010), Perla *et al.* (2015), Impullitti and Licandro (2016), and Sampson (2016). This line of research shows that the effect of trade on growth is a priori ambiguous. On the one hand, export opportunities increase market size and encourage firms to innovate more, but on the other hand, more intense competition may depress profits from innovation.[2] Recent quantitative studies such as Bloom *et al.* (2014) or Akcigit *et al.* (2018) find that the positive effects tend to prevail.

These theoretical findings are consistent with evidence from empirical studies, which by and large find that trade leads firms to increase their innovation and technology adoption efforts (see, e.g., Lileeva and Trefler, 2010, Bustos, 2011, Coelli *et al.*, 2016, or Chen and Steinwender, 2019). There are, however, important qualifications: Aghion *et al.* (2017) argue that export opportunities only arise to the most productive firms, which then increase innovation, while less productive firms only feel the squeeze of competition and innovate less. Furthermore, the evidence of the effect of import competition is more mixed: while Bloom *et al.* (2016) find a positive effect of Chinese import competition on innovation for European firms, Autor *et al.* (2016) find a negative effect for firms in the United States.

There is also an extensive literature studying how trade affects markups, dating back at least to Krugman (1979), who argued that trade lowers markups by forcing firms to a more price-elastic part of their demand curve. Melitz and Ottaviano (2008) find a similar effect in a model with heterogeneous firms and a quadratic demand system. More recently, the empirical literature has been greatly influenced by De Loecker and Warzynski (2012), who propose a new methodology to estimate firm-level markups (which we also use in our paper). Using Slovenian data, they show that exporting firms charge higher markups than non-exporting firms, and that firms entering the export market increase markups. In the case of India, De Loecker *et al.* (2016) find that while lower output tariffs lead to lower markups, the dominating effect of trade liberalization has been to increase markups, as firms did not pass the lower prices of imported intermediate inputs on to consumers. Brandt *et al.* (2017) find very similar results for China's manufacturing sector: while lower output tariffs have reduced markups, lower input tariffs have increased them. Lim *et al.* (2018) have found, also for China, that the scale effects of trade increase innovation, while the competition effects tend to decrease it.[3]

[2] Of course, this does not imply that higher competition always reduces innovation incentives. On the contrary, it is well-known that the effects of competition on innovation are non-monotonic (see Aghion *et al.*, 2005).

[3] Other papers have focused not on changes in the level of markups, but on changes in markup dispersion (see, for instance, Epifani and Gancia, 2011 or Edmond *et al.*, 2015). These changes in dispersion affect the efficiency of the resource allocation, and therefore aggregate productivity.

Our paper builds on this extensive literature by considering more explicitly the interaction between markups and innovation. Some papers among the ones mentioned so far also explore these interactions, and it is worth discussing the connections to them in more detail. Bloom *et al.* (2016) show that exposure to China has increased innovation for European firms, and argue that this result could be the direct consequence of higher competition. The theoretical models in Impullitti and Licandro (2016) and Aghion *et al.* (2017) both feature innovation and endogenous markups. Both models have some similarities, emphasizing that trade lowers markups for the least efficient firms (pushing some of them out of the market), while at the same time stimulating the most efficient firms to innovate more (and to increase their markups in the process). However, neither of these papers provides empirical evidence for changes in markups, considers a dynamic endogenous growth model or studies the dynamic feedback effect of innovation on markups. Lim *et al.* (2018) consider this feedback between quality upgrading and markups in a model of trade, but do not explore the effects on economic growth nor the role of strategic interaction between firms. Finally, our model is most similar to Akcigit *et al.* (2018), but extends their framework by considering endogenous markups and a multi-sector economy. The model also shares some similarities with Peters (2018), who however considers a one-sector closed economy, and a different microfoundation for heterogeneous markups.

The remainder of the paper is structured as follows. In Section 2, we describe our data sources and the stylized facts regarding the effects of export and import shocks in the Spanish manufacturing sector. Section 3 describes our model's assumptions, and Section 4 discusses its main results. Section 6 concludes.

2 Stylized facts

2.1 Data

2.1.1 Firm-level data

Our data covers the period from 2000 to 2013, and mainly comes from an unbalanced panel of confidential firm-level data from the Spanish Commercial Registry (*Registro Mercantil Central*), containing annual balance sheet information on around 85% of registered Spanish firms from the non-financial market economy. These data combine information compiled by the Bank of Spain's *Central de Balances*, as well as the SABI dataset, collected by a private entity called Informa. Firms in the dataset include businesses constituted in the form of a Corporation (*Sociedad Anónima*), a Limited Liability Company (*Sociedad Limitada*), or a Cooperative (*Cooperativa*). The data contain information on net operating revenue, material expenditures (including intermediate inputs purchased by the firm), labor expenditures, number of employees, and total fixed assets, as well as sector of activity (4-digit NACE Rev. 2 code) and location (5-digit ZIP code). Almunia *et al.* (2018b) describe the data in greater detail, and show that it is representative of the Spanish economy by comparing the micro-data aggregates with statistics from the National Accounts.

We complement these data with the foreign transaction registry collected by the Bank of Spain, which has firm-level information on total exports and imports by year. We merge this to our main dataset based on firm's fiscal code. Almunia *et al.* (2018a) show that the export information from the micro-data has a good coverage (over 90% throughout the period), and the micro aggregates tracks well the available aggregate data on Spanish exports from Customs.

Throughout, we focus on the manufacturing sector (4-digit industry codes 1011 to 3320 according to the NACE Rev. 2 classification, 214 industries in total), as our trade data is available only for manufacturing. We drop observations with missing or zero sales, employees, materials, or fixed assets, and drop outliers from the material share distribution since markups will be estimated directly using this input. Moreover, we focus on industries that have at least ten firms per year, and use 2-digit-level value-added deflators for materials, sales, and fixed assets, which we take from the Spanish National Accounts.

Our final sample contains 129,222 firms, with a median number of around 68,000 firms per year. Table A.1 in Appendix A shows some summary statistics for our final sample.

2.1.2 Trade data and revealed comparative advantage

Our trade data is taken from the World Bank's World integrated Trade Solutions (WITS) website, which provides export and import data for most countries of the world. We use data on Spanish, U.S. and World imports and exports, for the period 1998-2013. The WITS data use the ISIC Rev. 3 industry classification, which we convert into NACE Rev. 2 industries using correspondence tables from Eurostat and from the Bank of Spain.[4]

For each Spanish industry j, we calculate an index of revealed comparative advantage following the seminal contribution of Balassa (1965). This index is defined as:

$$RCA_{\text{Spain},j} = \frac{\left(\dfrac{X_{\text{Spain},j}}{\sum_{j'} X_{\text{Spain},j'}}\right)}{\left(\dfrac{X_{\text{World},j}}{\sum_{j'} X_{\text{World},j'}}\right)} \tag{1}$$

that is, the ratio between the percentage of Spanish exports due to industry j and the percentage of world exports due to industry j. We calculate these indexes in the beginning of our period of analysis, aggregating export data over the period 1999-2002. Table A.2 in Appendix A lists the 10 industries with the highest and lowest values for this index. Note that Spain's RCA industries appear to be relatively low-tech, with the exception of the car industry. Spain has essentially no exports in electronics and computer industries.

[4] We convert the data from ISIC Rev. 3 to NACE Rev 1.1 using the correspondence table provided by Eurostat. We then use an internal correspondence table from the Bank of Spain to convert the data from NACE Rev. 1.1 to NACE Rev. 2. These conversions unavoidably introduce some noise. For instance, it often happens that several NACE Rev 2. industries get assigned the same trade growth rates. To adjust for this in our regressions, we consider such industry groups as clusters (i.e., we cluster standard errors at the industry group and not at the industry level).

2.1.3 Markup estimation

To understand the empirical relationship between trade and markups, we obtain markups at the firm level using our sample of Spanish micro-data. Markups are famously hard to estimate, because firm-level pricing data is scarce, and marginal costs are a theoretical concept with no clear empirical counterpart.[5] For this reason, we use the methodology developed in De Loecker and Warzynski (2012), who build on insights by Hall (1986), relying on firm-level balance-sheet data instead of direct information on prices. A virtue of this method is that it does not impose a structure on product markets, and it does not require the estimation of specific demand systems.

Firms are assumed to choose both variable and fixed inputs by solving a period-by-period cost minimization problem. The optimality conditions of this problem show that:

$$\varepsilon_{it} = \frac{1}{\Lambda_{it}} \frac{P_{it} V_{it}}{Q_{it}}$$

where Q_{it} is the output of firm i in year t, V_{it} is the variable input (one that is not subject to adjustment costs), and P_{it} is the price of the input. Moreover, $\varepsilon_{it} \equiv \frac{\partial Q_{it}}{\partial V_{it}} \frac{V_{it}}{Q_{it}}$ is the elasticity of output with respect to the variable input, and Λ_{it} is the shadow value of the cost-minimization problem, and therefore the cost of an additional unit of output (i.e. a proxy for the marginal cost). Rearranging, we obtain that the markup can be computed as $\mu_{it} = \frac{\varepsilon_{it}}{\alpha_{it}}$, where $\alpha_{it} = \frac{P_{it} V_{it}}{P^Q_{it} Q_{it}}$ is the input share of total revenues, P^Q_{it} being the (unobserved) output price. While the input share can be computed directly from the data as the ratio of input expenditures to sales, the elasticity ε_{it} requires the estimation of a production function. For this step, the Olley and Pakes (1996) method is used to prevent biased estimates arising from serial correlation in productivity, and possible selection biases. Specifically, following the literature, we pose a production function (in logs) of the form:

$$y_{it} = \omega_{it} + f(m_{it}, \ell_{it}, k_{it}; \boldsymbol{\gamma}) + \epsilon_{it}$$

for each firm i, where $(m_{it}, \ell_{it}, k_{it})$ are material, labor, and capital inputs; $\boldsymbol{\gamma}$ is a vector of technological parameters; ω_{it} is observed TFP; and ϵ_{it} is a term capturing unanticipated productivity shocks and possible measurement error (both of which unobserved by the firm when choosing its inputs). The production function f may but need not feature constant returns to scale, meaning that ε_{it} may be time-varying. Following De Loecker and Warzynski (2012), we proxy observed productivity by current input choices, and assume lagged input values do not respond to current shocks. Therefore, lagged and current input use are correlated because of persistence in TFP. Assuming that ω_{it} follows an AR(1), we estimate $\boldsymbol{\gamma}$ by GMM with a moment condition establishing that innovations to observed productivity should be orthogonal to predetermined input choices.

In terms of our implementation, we use materials as our variable input, as these are arguably

[5] One could think of using average costs as a proxy for marginal costs, but this approximation is likely to be too crude for the case of Spain. Indeed, throughout this period, Spanish manufacturing firms experienced substantial changes in their structure of costs, with varying usage of variable versus fixed costs, for example due to deep institutional reforms in the labor market.

not subject to adjustment costs among Spanish firms.[6] Therefore, the markup at the firm level is:

$$\mu_{it} = \varepsilon_{it}\frac{S_{it}}{M_{it}}, \qquad \text{with } \varepsilon_{it} = \frac{\partial f(m_{it}, \ell_{it}, k_{it}; \widehat{\boldsymbol{\gamma}})}{\partial m_{it}}$$

where S_{it} and M_{it} are sales and material expenditures (in levels), $m_{it} = \log M_{it}$, and $\widehat{\boldsymbol{\gamma}}$ is the GMM estimate described above, estimated at the 4-digit level. Our baseline specification for the production function has constant returns to scale (a Cobb-Douglas), but as robustness we have experimented with a Translog, a common production function in these type of studies, and obtained similar results.

Figure A.1 in Appendix A shows the evolution of the average markup in the manufacturing economy in Spain, where firms' markups are weighted by the firm sales shares relative to the whole manufacturing sector. Markups are fairly stable during this period, ranging between 10 and 14% in the estimation based on a Cobb-Douglas production function. The exception is the increase to 18% during the Great Recession, which is due to a strong decline in the material share of firms during this period. Figure A.2 presents the raw distribution of markups across all firms and years, and it shows that the distribution is right-skewed, with a mode around 5-10%.

2.2 Results

2.2.1 The effect of trade on productivity and markups

We are now ready to discuss our main empirical results, regarding the effects of export and import shocks on firm-level productivity and markups. Our empirical strategy is closely related to Autor *et al.* (2014), who study the response of worker-level outcomes to industry-level import competition shocks. We study a somewhat analogous problem: the response of firm-level outcomes to industry-level trade shocks. Our baseline specification takes the form:

$$\ln\left(\frac{y_{ijt}}{y_{ijt_0}}\right) = \beta_0 + \beta_1 \Delta Trade^t_{jt_0} + \boldsymbol{\beta}_2' \boldsymbol{X}_{j,t_0} + \boldsymbol{\beta}_3' \boldsymbol{X}_{ij,t_0} + \varepsilon_{ijt}. \tag{2}$$

In this equation, the dependent variable is the log difference between an outcome variable for firm i in industry j (markups or revenue productivity) in year t and in a base year t_0. Note that we estimate equation (2) for one year t at the time. In our baseline results shown in the main text, we consider $t = 2008$ (the last pre-crisis year) and $t_0 = 2000$ (the first year with firm-level data), but we also consider various robustness checks around these two values.

The main explanatory variable captures the trade shock experienced by industry j, where $Trade$ is either equal to $Exports$ or to $Imports$. As in Autor *et al.* (2014), the import shock is

[6] The literature has alternatively resorted to labor expenditures, especially when information on material inputs is unavailable. However, labor inputs are likely to suffer from large frictions in the Spanish economy, such as overhead costs or hiring and firing costs, which may preclude firms from being able to costlessly adjustment them. An important advantage of the Spanish firm-level data is that inputs for production are disaggregated into different categories.

defined as:

$$\Delta Imports_{jt_0}^{t} = \frac{Imports_{jt_0}}{Y_{jt_0}} \cdot \frac{Imports_{jt} - Imports_{jt_0}}{Imports_{jt_0}} \tag{3}$$

where Y_{jt_0} stands for the total sales of Spanish firms of industry j in Spain. Thus, the shock measure is the product of the import penetration ratio (imports as a percentage of domestic sales) of industry j in the base year t_0, and the growth rate of industry j imports between year t_0 and year t. Likewise, we define export shocks as:

$$\Delta Exports_{jt_0}^{t} = \frac{Exports_{jt_0}}{Y_{jt_0}} \cdot \frac{Exports_{jt} - Exports_{jt_0}}{Exports_{jt_0}} \tag{4}$$

For both of these measures, we winsorize the initial ratios at 1 and the growth rates at +250%, which corresponds roughly to the 99th percentile of these measures. As the shocks are measured at the industry-level, the standard errors of our regressions are always clustered by industry.

Of course, our measures of export and import shocks are potentially correlated with other demand or supply shocks affecting Spanish industries. Therefore, as in Autor *et al.* (2014), we instrument Spanish export and import growth in equations (3) and (4) with export and import growth in the corresponding industries in the United States (excluding exports to or imports from Spain). These instruments are arguably orthogonal to Spanish supply and demand shocks. However, they may potentially correlate with supply or demand shocks that are common across both countries, but unrelated to trade. To address this concern, we will later introduce control variables for the most relevant alternative shocks. Finally, the regressions also control for a number of industry and firm characteristics contained in the vectors $\boldsymbol{X}_{j,t_0}$ and $\boldsymbol{X}_{ij,t_0}$. The control variables are industry size (the logarithm of total domestic sales), a dummy for whether the industry belongs to the 50% of industries with the highest revealed comparative advantage indeces, firm size (the logarithm of firm sales), and a dummy for whether the firm exports or imports. All of these control variables are defined in the base year t_0.

2.2.2 Trade shocks and revenue productivity

We first consider the effect of export shocks on firm-level (revenue) productivity, proxied by the logarithm of sales per worker.

Table 1 shows results for our baseline specification. The OLS estimates already show a strong positive relationship between export shocks and sales per worker: the industries with the highest export growth have also seen the largest within–firm revenue productivity increases. When we instrument Spanish export growth rates with U.S. growth rates, results get even stronger. One potential concern about these results could be that export growth may be positively correlated with overall industry growth, and that it is the latter than drives our results. However, this is not the case: results are unchanged when we control for the growth rate of overall domestic sales (see columns (3) and (6) of Table 1). Finally, it is also interesting to split our sample into firms that were exporting in 2000 and firms which were not yet exporting in 2000. This is done in columns (7) and (8) of Table 1, and shows that the initially non-exporting firms have seen the largest increases in revenue productivity (even though the difference between the two groups is not statistically significant).

Dependent variable: $\ln\left(\frac{\text{Sales}_{ij,2008}}{\text{Employment}_{ij,2008}}\right) - \ln\left(\frac{\text{Sales}_{ij,2000}}{\text{Employment}_{ij,2000}}\right)$.

	OLS			IV				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
							Exporters	Nonexp.
$\Delta Exports_{j,2000}^{2008}$	0.122**	0.237***	0.194***	0.284***	0.430***	0.366***	0.224**	0.395***
	(0.053)	(0.056)	(0.051)	(0.118)	(0.133)	(0.115)	(0.106)	(0.132)
$\Delta Y_{j,2000}^{2008}$			0.092***			0.086***	0.123***	0.081***
			(0.017)			(0.018)	(0.023)	(0.019)
Controls		✓	✓		✓	✓	✓	✓
R^2	0.00	0.02	0.03	0.00	0.02	0.03	0.03	0.03
# Observations	34707	34707	34707	34707	34707	34707	4520	30187
# Industry clusters	131	131	131	131	131	131	131	130
First-stage F-statistic				16.5	16.7	16.6	11.8	19.0

Table 1: Sales per worker and Export Shocks. *Notes:* All standard errors (in parentheses) are clustered by industry. Controls are the logarithm of firm and industry sales, a dummy for revealed comparative advantage industries and export status (except in columns (7) and (8)), all measured in the year 2000. The first-stage F-statistic is the Kleibergen-Paap F-statistic reported by the Stata routine ivreg2. Significance levels: *=10%, **=5%, ***=1%.

Table 2 repeats the same regressions using import shocks. Results are similar to exports: higher import penetration is also associated with higher increases in within-firm revenue productivity. Again, results are stronger when using instrumental variables. They are also slightly stronger for firms which initially did not import.

Dependent variable: $\ln\left(\frac{\text{Sales}_{ij,2008}}{\text{Employment}_{ij,2008}}\right) - \ln\left(\frac{\text{Sales}_{ij,2000}}{\text{Employment}_{ij,2000}}\right)$.

	OLS			IV				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
							Importers	Nonimp.
$\Delta Imports_{j,2000}^{2008}$	0.134**	0.207***	0.225***	0.476***	0.689***	0.665***	0.535**	0.669***
	(0.059)	(0.066)	(0.062)	(0.155)	(0.217)	(0.193)	(0.215)	(0.203)
$\Delta Y_{j,2000}^{2008}$			0.102***			0.109***	0.151***	0.102***
			(0.017)			(0.018)	(0.024)	(0.019)
Controls		✓	✓		✓	✓	✓	✓
R^2	0.00	0.02	0.03	0.00	0.01	0.02	0.02	0.02
# Observations	34707	34707	34707	34707	34707	34707	4151	30556
# Industry clusters	131	131	131	131	131	131	130	131
First-stage F-statistic				14.1	11.6	11.8	12.6	10.1

Table 2: Sales per worker and Import Shocks. *Notes:* See Notes Table 1.

One problem with the results reported so far could be the fact that export and import growth are correlated (indeed, when regressing $\Delta Exports_{j,2000}^{2008}$ on $\Delta Imports_{j,2000}^{2008}$, we find a positive coefficient with a t-statistic of 1.7). Thus, Table 3 shows our results when including both shocks in the same regression, both in an OLS specification and in a reduced-form IV specification. This shows that the effects of import shocks appear to dominate: they stay positive and strongly significant, while export shocks remain positive, but become insignificant.

Dependent variable: $\ln\left(\frac{\text{Sales}_{ij,2008}}{\text{Employment}_{ij,2008}}\right) - \ln\left(\frac{\text{Sales}_{ij,2000}}{\text{Employment}_{ij,2000}}\right)$.

	OLS		Reduced-form IV	
	(1)	(2)	(3)	(4)
$\Delta Exports_{j,2000}^{2008}$	0.081	0.124**	0.058	0.057
	(0.052)	(0.056)	(0.073)	(0.059)
$\Delta Imports_{j,2000}^{2008}$	0.101**	0.175***	0.169***	0.244***
	(0.049)	(0.043)	(0.054)	(0.046)
$\Delta Y_{j,2000}^{2008}$		0.098***		0.096***
		(0.017)		(0.015)
Controls		✓		✓
R^2	0.00	0.03	0.00	0.03
# Observations	34707	34707	34707	34707
# Industry clusters	131	131	131	131

Table 3: Export and import shocks, joint regressions. *Notes:* See Notes Table 1.

Summing up, our overall results in this section are in line with the literature: trade shocks are associated with within-firm increases of sales per worker. These effects are mostly driven by imports rather than by exports, which is in line with the findings of Bloom *et al.* (2016). However, only considering revenue productivity may of course over- or underestimate true changes in productivity, as this measure combines physical efficiency, product quality, and markups. Thus, we now turn to analyzing the response of markups.

2.2.3 Trade shocks and markups

To assess the effect of trade shocks on markups, we consider the same specification as before, but now use markups instead of sales per worker as the dependent variable. Table 4 shows the effect of export shocks. It indicates that export shocks are associated with falling markups: the industries which experienced the largest export shocks have also experienced the largest reductions in markups. Keeping in mind the results from the previous section, this suggests that the reaction of sales per worker understates the true productivity impact of export shocks: correcting for markups, the change in productivity or product quality would be even higher.

Table 5 shows the same pattern for import shocks. Increases in import penetration are associated with significant within-firm reductions in markups. These results are very much in line with the typical findings in the trade literature showing that import competition reduces markups. They also indicate that for import shocks as well, the true within-firm productivity impact of trade is underestimates when considering only revenue productivity.

Finally, Table 6 again puts both shocks together. As for productivity, import shocks seem have a stronger effect (at least in the reduced-form IV regressions). The negative effect of export

Dependent variable: $\ln\left(\frac{\mu_{ij,2008}}{\mu_{ij,2000}}\right)$.

	OLS			IV				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
							Exporters	Nonexp.
$\Delta Exports^{2008}_{j,2000}$	-0.087***	-0.122***	-0.116***	-0.134**	-0.162***	-0.154***	-0.097**	-0.165***
	(0.031)	(0.034)	(0.035)	(0.057)	(0.057)	(0.057)	(0.050)	(0.064)
$\Delta Y^{2008}_{j,2000}$			-0.013			-0.011	-0.019**	
			(0.013)			(0.013)	(0.010)	
Controls		✓	✓		✓	✓	✓	✓
R^2	0.00	0.01	0.01	0.00	0.01	0.01	0.03	0.01
# Observations	34707	34707	34707	34707	34707	34707	4520	30187
# Industry clusters	131	131	131	131	131	131	131	130
First-stage F-statistic				16.5	16.7	16.6	11.8	19.0

Table 4: Markups and Export Shocks. *Notes:* See Notes Table 1.

Dependent variable: $\ln\left(\frac{\mu_{ij,2008}}{\mu_{ij,2000}}\right)$.

	OLS			IV				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
							Importers	Nonimp.
$\Delta Imports^{2008}_{j,2000}$	-0.034	-0.057*	-0.060*	-0.197**	-0.252***	-0.247***	-0.145**	-0.27**
	(0.032)	(0.032)	(0.031)	(0.083)	(0.098)	(0.093)	(0.068)	(0.108)
$\Delta Y^{2008}_{j,2000}$			-0.017			-0.021	-0.027***	-0.020
			(0.013)			(0.015)	(0.010)	(0.015)
Controls		✓	✓		✓	✓	✓	✓
R^2	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.00
# Observations	34707	34707	34707	34707	34707	34707	4151	30556
# Industry clusters	131	131	131	131	131	131	130	131
First-stage F-statistic				14.1	11.6	11.8	12.6	10.1

Table 5: Markups and Import Shocks. *Notes:* See Notes Table 1.

shocks persists, but it is not significant in the IV specifications.

Dependent variable: $\ln\left(\frac{\mu_{ij,2008}}{\mu_{ij,2000}}\right)$.				
	OLS		Reduced-form IV	
	(1)	(2)	(3)	(4)
$\Delta Exports^{2008}_{j,2000}$	-0.089***	-0.110***	-0.039	-0.036
	(0.033)	(0.037)	(0.035)	(0.036)
$\Delta Imports^{2008}_{j,2000}$	0.003	-0.015	-0.061**	-0.081***
	(0.023)	(0.019)	(0.029)	(0.029)
$\Delta Y^{2008}_{j,2000}$		-0.014		-0.015
		(0.013)		(0.013)
Controls		✓		✓
R^2	0.00	0.01	0.00	0.01
# Observations	34707	34707	34707	34707
# Industry clusters	131	131	131	131

Table 6: Export and import shocks, joint regressions. *Notes:* See Notes Table 1.

Finally, we have also considered our results separately for comparative advantage and disadvantage industries. While results are largely similar for these two subgroups, we do find that in the cross-section, firm-level markups are on average lower in industries in which Spain has revealed comparative advantage. To show this, we run the regression:

$$\mu_{ijt} = \beta_0 + \beta_1 D_j^{RCA} + \beta_2 \ln(y_{it}) + \beta_3 \ln(Y_{jt}) + \delta_t + \varepsilon_{ijt},$$

where μ_{ijt} is the markup of firm i in industry j and year t, and D_j^{RCA} is a dummy variable equal to 1 if industry j belongs to the 50% of Spanish industries with the highest RCA index values, and 0 otherwise. We include the logarithm of firm and industry-level sales (y_{it} and Y_{jt}, respectively), as well as time fixed effects as control variables, and we cluster standard errors by year and industry. Table 7 illustrates the results of this regression: on average, firms in Spain's comparative advantage sectors have around 7.6 percentage points lower markups.

Dependent variable: firm-level markups, μ_{ijt}		
	All firms	
	(1)	(2)
D_j^{RCA}	-0.103***	-0.076**
	(0.040)	(0.036)
Controls		✓
R^2	0.005	0.16
# Observations	684230	684230
# Years	14	14
# Industry clusters	130	130

Table 7: Markups differences by sectoral revealed comparative advantage. *Notes:* All standard errors (in parentheses) are clustered by year and industry.

The results so far indicate that trade shocks have a strong effect on both markups and firm-level productivity. Thus, analyzing one without considering the other at best only gives a highly incomplete picture of the dynamic effects of trade. In the next section, we therefore present a unified framework to study the response of markups and productivity to trade shocks, to make sense of the joint effects observed in the data and to analyze their overall importance for economic growth.

3 Model

3.1 Environment

Preferences Time is continuous, infinite, and indexed by $t \in \mathbb{R}_+$. There exist two large open economies in the world, labeled $k \in \{H, F\}$ for Home (H) and Foreign (F). Each economy is populated by a continuum of identical and infinitely-lived individuals, with common discount rate $\rho > 0$, forming a representative household. The household is endowed with fixed amounts of time $\boldsymbol{L}_s^k$ each instant, which are supplied inelastically in competitive labor markets to the productive sectors of the domestic economy, indexed by s. There are two productive sectors, labeled $s = A, B$. Labor cannot move between countries, nor across sectors within each country.

The representative household has preferences:

$$\boldsymbol{U}_0^k = \int_0^{+\infty} e^{-\rho t} \ln \boldsymbol{C}_t^k \mathrm{d}t \tag{5}$$

where $\boldsymbol{C}_t^k$ is domestic consumption of a single final good. Consumption decisions are subject to a flow budget constraint:

$$\dot{\boldsymbol{A}}_t^k \leq r_t^k \boldsymbol{A}_t^k + \sum_{s=A,B} w_{st}^k \boldsymbol{L}_s^k - \boldsymbol{P}_t^k \boldsymbol{C}_t^k \tag{6}$$

with $\boldsymbol{A}_0^k > 0$ given. Here, w_{st}^k is the (endogenous) wage rate in sector s and country k, paid in units of the final good in country k, and $\boldsymbol{P}_t^k$ is the price of the final good. Labor supplied to sector s is denoted $\boldsymbol{L}_s^k$, and we have $\boldsymbol{L}_A^k + \boldsymbol{L}_B^k = \boldsymbol{L}^k$. The household owns the domestic final good firm. We assume that there are no international capital flows, so the stock of wealth $\boldsymbol{A}_t^k$ is equal to the total value of corporate assets within the country, and hence the (endogenous) rate of return on assets, r_t^k, is country-specific.

Final-good technology The final output is a Cobb-Douglas aggregate of the sectoral outputs, with:

$$\boldsymbol{Y}_t^k = \left(\boldsymbol{Y}_{At}^k\right)^{\beta} \left(\boldsymbol{Y}_{Bt}^k\right)^{1-\beta} \tag{7}$$

where $\boldsymbol{Y}_{st}^k$ denotes the output of sector s in country k at time t. For simplicity, we will assume that the spending shares on each sector are equal, $\beta = \frac{1}{2}$. Moreover, the two countries have the same labor supply, i.e. $\boldsymbol{L}^H = \boldsymbol{L}^F = \boldsymbol{L}$. Countries are, however, asymmetric in their labor endowments across sectors. We assume that H (respectively, F) is labor-abundant in sector A (respectively, B), so that:

$$\boldsymbol{L}_A^H = \boldsymbol{L}_B^F > \boldsymbol{L}_B^H = \boldsymbol{L}_A^F \tag{8}$$

Symmetry in total labor endowments and in aggregate spending shares implies that, in equilibrium, aggregate nominal GDP in both countries is equal along the stationary solution. Asymmetry in the sector-specific endowments across countries gives rise to trade flows on the basis of comparative advantage. In particular, since country H has comparative advantage in sector A, it will become a net exporter in this sector, and a net importer in sector B.

The technology within each sector is similar to the set-up from Atkeson and Burstein (2008). In each country, the final good from each sector is produced in a perfectly competitive market as a bundle of a measure-one continuum of products, indexed by $j \in [0,1]$, with the following Cobb-Douglas technology:

$$\boldsymbol{Y}_{st}^k = \exp\left[\int_0^1 \alpha_{js} \ln(Y_{jst}^k)\mathrm{d}j\right] \tag{9}$$

In this formula, α_{js} is a taste shifter for product j of sector s, with $\int_0^1 \alpha_{js}\mathrm{d}j = 1$, $\forall s \in \{A, B\}$, and Y_{jst}^k is the quantity of product j in sector s of country k. The elasticity of substitution between products within the sector is equal to one.

Each product j is assembled using intermediate goods from domestic producers directly, as well as from the international competitor producing the same intermediate good. Among domestic producers, there is a technological leader and a competitive fringe. The competitive fringe can be thought of as a large mass of price-taking firms operating in perfect competition, making zero profits, and not conducting R&D activities.[7] The output levels from the domestic leader, the domestic competitive fringe, and the foreign producer are aggregated in a CES fashion:

$$Y_{jst}^k = \left[\omega_H^k \left(y_{jsH,t}^k\right)^{\frac{\eta-1}{\eta}} + \omega_C^k \left(y_{jsC,t}^k\right)^{\frac{\eta-1}{\eta}} + \omega_F^k \left(y_{jsF,t}^k\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{10}$$

Here, $\{y_{jsH,t}^k, y_{jsC,t}^k, y_{jsF,t}^k\}$ are the quantities of product j from sector s produced by the domestic leader, the domestic competitive fringe, and the foreign leader, respectively, and consumed in destination market k at time t.[8] The objects $\{\omega_c^k\}$ are preference (or Armington) weights for the intermediate good sold by producer $c \in \{H, C, F\}$, with $\sum_c \omega_c^k = 1$, $\forall k$. For simplicity, we will assume that the two countries are symmetric in their weights, in that $\omega_H^H = \omega_F^F \equiv \omega_{dom}$ (taste for goods of the domestic leader), and $\omega_H^F = \omega_F^H \equiv \omega_{for}$ (taste for goods of the foreign leader), implying that $\omega_C^H = \omega_C^F \equiv \omega_{comp}$ (taste for goods from the compet-

[7] The reason why we add a competitive fringe relative to the Atkeson and Burstein (2008) framework is that, otherwise, the domestic firm would charge an infinite price if the country had no imports, as preferences over intermediates are Cobb-Douglas.

[8] Note that we have omitted imports from the foreign competitive fringe in equation (10). This is without loss of generality, as the competitive fringe will never export in equilibrium, because it will never be willing to pay the fixed cost of exporting.

itive fringe). Finally, η is the elasticity of substitution between the different types of product j, and we assume throughout that $\eta > 1$. Namely, the household is more willing to substitute between the domestic and foreign versions of the same intermediate good than between different intermediates altogether.

Intermediate-good technology The production of intermediate goods by each producer $c = H, C, F$ takes place via a simple linear technology:

$$y^k_{jsc,t} = q^k_{jsc,t} \ell^k_{jsc,t}$$

where $\ell^k_{jsc,t}$ is labor used in the production of good j at time t by producer c in sector s. Here, $q^k_{jsc,t}$ is the quality available to producer c in country k for this product, an object which will be advanced by the leader through a process of innovation, described below.

When the product is shipped abroad for its consumption in the other country, the exporting producer must incur two types of costs over and above the cost of labor. On the one hand, producers must pay a fixed flow cost equal to κ, paid in units of the aggregate domestic output. The existence of this fixed cost will guarantee that not all domestic producers decide to export their product into the foreign market. On the other hand, exporters must pay a flat-rate iceberg cost $\tau > 1$, so that in order for $y_{jsc,t}$ units of good j produced in c to arrive to the other country, $\tau y_{jsc,t}$ units must be produced.

Competition and innovation The leading domestic producer competes in quantities with its foreign counterpart. The two leading firms interact strategically in a static Cournot game, and are separated by a technological gap. This gap expands and contracts over time through the endogenous process of innovation. In addition, the technological frontier in both countries co-moves due to the presence of international knowledge spillovers, which prevent laggard firms from falling too far behind the frontier.

To acquire comparative technological advantage over its foreign competitor, the leading domestic firm invests into R&D in order to advance the quality of the product. In particular, to generate a Poisson rate of innovation equal to z_j in product market j, the firm must pay a cost equal to $r(z_j)$ units of the final good, where r is convex in z_j. A successful innovation improves quality by a step factor $(1 + \lambda) > 1$, so that after a small interval of time $\Delta > 0$, we have:

$$q^k_{jsk,t+\Delta} = (1 + \lambda) q^k_{jsk,t}$$

Therefore, the *technological gap* between a leading firm (in sector s of country k) and its foreign leading competitor (in country $m \neq k$) can be summarized by an integer $n_{js,t}$, satisfying:

$$\frac{q^k_{jsk,t}}{q^m_{jsm,t}} = (1 + \lambda)^{n_{js,t}} \tag{11}$$

We will then say that a country is *leading* in good j of sector s if $n_{js,t} > 0$, *lagging* if $n_{js,t} < 0$, and *neck-to-neck* with the foreign economy if $n_{js,t} = 0$. The technological gap is defined over the support $\mathcal{N} \equiv \{-\overline{n}, \dots, -2, -1, 0, 1, 2, \dots, \overline{n}\}$, and we assume that there is a limit to how far firms may fall back (i.e. $\overline{n} < +\infty$). This implies that there exist knowledge spillovers between the two countries when the competing firms are sufficiently far apart as, in that case, any further innovation from the leader is immediately adopted by the follower of the other country.[9]

Additionally, the domestic firm maintains domestic leadership relative to the competitive fringe. When the domestic leader successfully implements an innovation, this knowledge is instantaneously diffused to the fringe, and thus the leader always keeps the same technological lead over the fringe, so that $q^k_{jsk,t} = (1+\lambda)^{n_C} q^k_{jsC,t}$, for all $t > 0$ and $k = H, F$. Here, $n_C \in \mathbb{N} = \{1, 2, \dots\}$ is therefore treated as a parameter, and is common across countries for simplicity.

Entry and exit Though incumbent firms always maintain their leadership over the competitive fringe, we assume that they may be displaced by new firms (entrants), whenever the latter successfully implement an innovation into the product market. Entry is sector-specific, but undirected across products within the sector. By paying a cost of $r_e(x_s)$ units of the final good, a potential entrant generates a Poisson rate of innovation in sector s equal to x_s.[10] In case of a successful entry, the firm draws a product at random from the $j \in [0, 1]$ continuum, and replaces the previous incumbent producer, who exits the market and becomes a potential entrant. The entrant firm then advances a step relative to the pre-existing quality gap n between the foreign competitor and the previous domestic incumbent, i.e. it increases the lead in case the previous incumbent was leading ($n > 0$), and shrinks the gap if it was lagging ($n < 0$). In case that the gap was already maximum, entry into the leading country will again imply that knowledge is immediately diffused into the disadvantaged country, keeping the gap between the firms unchanged.

Feasibility The final good of the economy is used to pay for final private consumption, R&D investment, and for the fixed costs of trade. The feasibility constraint reads:

$$\boldsymbol{C}^k_t + \boldsymbol{R}^k_t + \boldsymbol{K}^k_t \leq \boldsymbol{Y}^k_t \tag{12}$$

for each country $k \in \{H, F\}$, where $\boldsymbol{R}^k_t$ and $\boldsymbol{K}^k_t$ are, respectively, the total R&D and export costs at time t. Feasibility in the labor market, in turn, requires that labor demand from domestic producers (fringe, selling domestically, and leader, selling in the domestic and foreign markets) equates labor supply in each sector and country, that is:

$$\int_0^1 \Bigg(\sum_{c=k,C} \ell^k_{jsc,t} + \ell^{k'}_{jsk,t} \Bigg) \mathrm{d}j \leq \boldsymbol{L}^k_s \tag{13}$$

for all $s \in \{A, B\}$, $k \in \{H, F\}$, $k' \neq k$, and $t \in \mathbb{R}_+$. Finally, international trade must be

[9] The assumption guarantees that, since each country will partly specialize on the sector in which it has a comparative advantage, in the stationary equilibrium the comparatively disadvantaged sector will never disappear, as the most disadvantaged firms will be pulled along by the foreign country through the international spillovers.

[10] To guarantee the existence of a Balanced Growth Path, we assume that both r and r_e grow linearly with GDP.

balanced on each country, in that:

$$\boldsymbol{M}_t^k = \boldsymbol{X}_t^k \tag{14}$$

where $\boldsymbol{M}_t^k$ are total imports, and $\boldsymbol{X}_t^k$ are total exports, of country k.

3.2 Equilibrium

In this section, we derive the Markov Perfect Equilibrium of the world economy, and later we specialize it to a Balanced Growth Path where all country aggregates grow at a constant rate. The structure of the model allows us to split the equilibrium characterization into a static part (involving output and pricing decisions) and a dynamic part (involving innovation and entry choices).[11]

Final good sector's problem The final consumer in each country k maximizes utility (5) subject to the flow budget constraint (6), and a no-Ponzi condition $\lim_{t\to+\infty} e^{-\int_0^t r_h^k \mathrm{d}h} \boldsymbol{A}_t^k \geq 0$, taking the initial level of wealth as given. This leads to a standard consumption Euler equation:

$$\frac{\dot{\boldsymbol{C}}_t^k}{\boldsymbol{C}_t^k} = r_t^k - \rho \tag{15}$$

Domestic firms are owned by the final consumer, so the value of household wealth is equal to the value of domestic corporate assets, i.e. $\boldsymbol{A}_t^k = \int_0^1 \sum_s V_{jst}^k \mathrm{d}j$, where V_{jst}^k is the value of a domestic firm producing good j in sector s. In equilibrium, the following transversality condition must hold in each country:

$$\lim_{t\to+\infty} e^{-\int_0^t r_h^k \mathrm{d}h} \int_0^1 \sum_{s=A,B} V_{jst}^k \mathrm{d}j = 0 \tag{16}$$

Let us omit time subscripts from now on unless necessary. The final good firm demands input quantities $\left\{\left(y_{jsH}^k, y_{jsC}^k, y_{jsF}^k\right)_{j\in[0,1]}\right\}_{s=A,B}$ from domestic and foreign firms. The cost-minimization problem leads to the demand function:

$$y_{jsc}^k = \alpha_{js}\left(\frac{p_{jsc}^k}{\omega_c^k P_{js}^k}\right)^{-\eta} \frac{\boldsymbol{P}_s^k \boldsymbol{Y}_s^k}{P_{js}^k}, \tag{17}$$

$$\text{where } P_{js}^k = \left[\sum_{c=H,C,F} \omega_c^k \left(\frac{p_{jsc}^k}{\omega_c^k}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \text{ and } \boldsymbol{P}_s^k = \exp\left[\int_0^1 \alpha_{js} \ln\left(\frac{P_{js}^k}{\alpha_{js}}\right) \mathrm{d}j\right]$$

where p_{jsc}^k is the price of the good j in sector s that is produced by producer c and consumed in country k. This equation says that within-product and cross-product demand functions are both iso-elastic, with the former being more price-elastic as $\eta > 1$. The object P_{js}^k is the ideal

[11] We abstract from any equilibria with collusion, and focus instead on a period-by-period optimization of the duopoly.

consumption price index at the product-sector level, defined as a weighted average of the prices of the products produced by the different producers in the two countries. The within-sector allocation of total expenditures across products depends on the product-level price relative to the overall ideal sectoral price index $\boldsymbol{P}_s^k$, as well as the product-sector taste parameter α_{js}. In contrast, the allocation of expenditures within a product group depends on the relative price of each product, p_{jsc}^k, with respect to the overall price index P_{js}^k, as well as the ω_c^k shifter. At the sector and product levels, total expenditures satisfy $\int_0^1 P_{js}^k Y_{js}^k \mathrm{d}j = \boldsymbol{P}_s^k \boldsymbol{Y}_s^k$ and $P_{js}^k Y_{js}^k = \sum_c p_{jsc}^k y_{jsc}^k$, respectively. Thus, the final good firm makes zero profits in both countries, $\boldsymbol{\Pi}_t^k = 0$. Moreover, we have that $P_{js}^k Y_{js}^k = \alpha_{js} \boldsymbol{P}_s^k \boldsymbol{Y}_s^k$, i.e. the total expenditure on each product j is a constant share α_{js} of sectoral expenditures in the economy.[12] Finally, total expenditures at the sectoral level are an equal share of aggregate GDP, i.e. $\boldsymbol{P}_s^k \boldsymbol{Y}_s^k = \frac{1}{2} \boldsymbol{P}^k \boldsymbol{Y}^k$. Across countries, aggregate nominal GDPs are equal, i.e. $\boldsymbol{P}^H \boldsymbol{Y}^H = \boldsymbol{P}^F \boldsymbol{Y}^F$, and so from now on we will write $\boldsymbol{Y}$ (i.e. normalizing $\boldsymbol{P} = 1$) to refer to the world nominal GDP. This result holds because the countries are identical except for their sectoral labor endowments, but the relative endowments are symmetric across sectors (equation (8)).

Intermediate production sector The leading producers of each intermediate product $j \in [0, 1]$ and sector $s \in \{A, B\}$ compete in quantities in both countries, and they take as given (i) the production of the domestic competitive fringe, (ii) the production of the foreign competitor, (iii) the wage rate in the sector, and (iv) the demand function from both domestic and foreign households (equation (17)). Importantly, since each producer has some degree of market power within its product market, they internalize the effect that their production decisions have on equilibrium prices, both at home and abroad.

Throughout, we describe firm behavior in country H, but firms in country F face symmetric problems. The static problem of a firm producing good j in sector s and country H is:

$$\begin{aligned} \pi_{jsH} \equiv \max_{\{y_{jsH}^H, y_{jsH}^F\}} \Bigg\{ & \underbrace{\left(p_H^H(y_{jsH}^H, \widehat{y}_{jsF}^H) - \frac{w_s^H}{q_{jsH}^H} \right) y_{jsH}^H}_{\text{Domestic profits, } \pi_{jsH}^H} \\ & + \underbrace{\left(p_H^F(y_{jsH}^F, \widehat{y}_{jsF}^F) - \tau \frac{w_s^H}{q_{jsH}^H} \right) y_{jsH}^F}_{\text{Foreign profits, } \pi_{jsH}^F} - \underbrace{\kappa \boldsymbol{Y} \mathbb{1}[y_{jsH}^F > 0]}_{\substack{\text{Fixed cost} \\ \text{of exporting}}} \Bigg\} \end{aligned} \tag{18}$$

Here, (y_{jsH}^H, y_{jsH}^F) are the production levels sold domestically and internationally, respectively; a hat $(\widehat{y})$ indicates that the production level of the foreign competitor is taken as given; and $\mathbb{1}[.]$ is an indicator function. Furthermore, by inverting equation (17), we have defined the

[12] As we shall see, assuming Cobb-Douglas preferences at this level of disaggregation is necessary in this model to obtain analytical tractability in the Balance Growth Path characterization of the equilibrium. Otherwise, expenditure shares would not be constant, and we would need to keep track of sectoral outputs (in levels) in the two countries in order to solve the model.

inverse demand function as:

$$p_H^k(y,\hat{y}) = \frac{1}{2}\alpha_{js}\omega_H^k\big(Y_{js}^k(y,\widehat{y})\big)^{-\frac{\eta-1}{\eta}}y^{-\frac{1}{\eta}}\boldsymbol{Y} \tag{19}$$

for destination market $k \in \{H, F\}$, where $Y_{js}^k(y,\widehat{y})$ is the aggregator in equation (10), written here explicitly as a function of the two production levels to make it clear that the domestic firm internalizes the effect that its output has on the total output of the product group. To simplify notation, the production level of the competitive fringe does not feature explicitly in the problem because the domestic leader does not compete in quantities directly with the fringe. Unlike with product-level prices and quantities, aggregate expenditure $\boldsymbol{Y}$ is taken as given because firms are infinitesimal relative to the world economy.

Conveniently, the firm's problems on the domestic and foreign markets are independent from each other, which allows us to analyze them separately. The optimality conditions of problem (18) have the form:

$$\begin{aligned}
&\text{Domestic sales } (y_{jsH}^H): && \frac{w_s^H}{q_{jsH}^H} = p_H^H(y_{jsH}^H,\widehat{y}_{jsF}^H) + \frac{\partial p_H^H(y_{jsH}^H,\widehat{y}_{jsF}^H)}{\partial y_{jsH}^H}y_{jsH}^H\\
&\text{Exports, if any } (y_{jsH}^F): && \tau\frac{w_s^H}{q_{jsH}^H} = p_H^F(y_{jsH}^F,\widehat{y}_{jsF}^F) + \frac{\partial p_H^F(y_{jsH}^F,\widehat{y}_{jsF}^F)}{\partial y_{jsH}^F}y_{jsH}^F
\end{aligned}$$

The optimality conditions equate the marginal cost of production (on the left), to the marginal benefit of rising output by one unit (on the right). The latter has a direct positive effect on revenue (first term), but it also implicitly lowers the output's price, given the competitor's output level, because firms face a downward-sloping demand schedule (second term). Exporters, on the other hand, take into account that their marginal cost is effectively higher due to the trade cost $\tau > 1$.

To derive the best responses, it is convenient to define:

$$\sigma_{jsc}^k \equiv \frac{p_{jsc}^k y_{jsc}^k}{P_{js}^k Y_{js}^k} \tag{21}$$

as the sales share in destination country $k \in \{H, F\}$ of firm $c \in \{H, C, F\}$ producing in market $j \times s$, so that $\sum_c \sigma_{jsc}^k = 1, \forall j, s, k$. Using this definition, we can then write the optimality condition for domestic sales as:

$$\frac{w_s^H}{q_{jsH}^H} = \frac{p_H^H(y_{jsH}^H,\widehat{y}_{jsF}^H)}{\mu_{jsH}^H}, \qquad \text{where } \mu_{jsH}^H \equiv \left(\frac{\eta}{\eta-1}\right)\frac{1}{1-\sigma_{jsH}^H} \tag{22}$$

That is, the firm's production decision in the domestic market leads firms to charge a price markup $\mu_{jsH}^H > 1$ over the marginal cost of production, w_s^H/q_{jsH}^H. The expression for the domestic markup is made of the product of two terms. The first term is the markup that would obtain in a monopolistically competitive environment in which firms had no market power over the input they produce within their market. The second term, which is strictly greater than one, captures the fact that firms exploit their market power by charging higher markups the more market share they have.

As demand is never zero for domestically-produced goods ($\omega_H^H > 0$), producers will always sell in the domestic market. Additionally, they may choose to sell in the foreign market, since foreign households have a preference for goods produced abroad ($\omega_H^F > 0$). If the firm decided to pay κ and export, the optimal level of exports y_{jsH}^F would satisfy:

$$\tau \frac{w_s^H}{q_{jsH}^H} = \frac{p_H^F(y_{jsH}^F, \widehat{y}_{jsF}^F)}{\mu_{jsH}^F}, \qquad \text{where } \mu_{jsH}^F \equiv \left(\frac{\eta}{\eta-1}\right) \frac{1}{1-\sigma_{jsH}^F} \tag{23}$$

is the markup of the firm in the foreign market (always conditional on the firm exporting its product). The firm, however, exports its product in the first place only if the fixed cost of exporting is sufficiently low relative to ex-post profits from exporting. Fortunately, this margin can be characterized via a simple threshold rule. Indeed, one can show that ex-post static profits in market $k \in \{H, F\}$ (i.e. either the domestic or the foreign market) can be generally written as a quadratic function of market shares:

$$\pi_{jsH}^k = \left[\sigma_{jsH}^k + (\eta-1)(\sigma_{jsH}^k)^2\right] \frac{\alpha_{js}}{2\eta} \boldsymbol{Y}$$

The domestic firm exports its product if, and only if, $\pi_{jsH}^F \geq \kappa \boldsymbol{Y}$. Since the left-hand side is quadratic in σ, and static profits are increasing in the market share, this inequality boils down to a threshold rule on the ex-post market share in the foreign country. That is, the firm from country H exports to F if, and only if:

$$\sigma_{jsH}^F \geq \sigma_{js}^*, \qquad \text{where } \sigma_{js}^* \equiv \frac{1}{2(\eta-1)} \left(-1 + \sqrt{1 + 8(\eta-1)\frac{\kappa\eta}{\alpha_{js}}}\right) \tag{24}$$

In this rule, the left-hand side is the market share in the foreign market if the firm decided to export, resulting from the optimality condition in equation (23). Therefore, firms will export their product into the foreign country only if, ex-post, they can capture a sufficiently high share of the market. Conveniently, the threshold σ_{js}^* is only a function of parameters, $(\eta, \kappa, \alpha_{js})$, and is constant over time. A necessary condition for an equilibrium in which countries export their product in some market $j \times s$ is that $\sigma_{js}^* < 1$, a condition that is guaranteed if export costs are not too high, or $\kappa < \frac{1}{2}\alpha_{js}$. We keep this parametric restriction throughout.

Unlike domestic leaders, the competitive fringe on each country is a price-taker, and does not undertake R&D activities. Therefore, the price of their products satisfies $p_{jsC}^H = w_s^H / q_{jsC}^H$, where q_{jsC}^H denotes the product quality available to the fringe, and their profits are $\pi_{jsC}^H = 0$. When the domestic leader implements an innovation, this know-how is immediately transmitted to the rest of firms of the economy, and thus leader and fringe always keep the same technological distance n_C between them. Thus, $q_{jsC}^H = (1+\lambda)^{-n_C} q_{jsH}^H$. Since profits are zero, firms within the competitive fringe will never choose to export their product, as exporting carries a fixed cost $\kappa > 0$ which these firms will never be willing to pay.

To close the static characterization of the equilibrium, we must impose market clearing in the labor market and ensure that trade is balanced in both countries. Domestic labor is employed by technological leaders (for domestic production and, possibly, exports), and by the domestic competitive fringe. The corresponding labor demands are given by:

$$\ell_{jsH}^{H} = \frac{\sigma_{jsH}^{H}}{\mu_{jsH}^{H} w_{s}^{H}} P_{js}^{H} Y_{js}^{H}, \qquad \ell_{jsH}^{F} = \frac{\sigma_{jsH}^{F}}{\mu_{jsH}^{F} \tau w_{s}^{H}} P_{js}^{F} Y_{js}^{F}, \qquad \ell_{jsC}^{H} = \frac{\sigma_{jsC}^{H}}{w_{s}^{H}} P_{js}^{H} Y_{js}^{H}, \tag{25}$$

respectively. The market clearing condition (equation (13)) then reads $\int_0^1 \left(\ell_{jsH}^{H} + \ell_{jsC}^{H}\right) \mathrm{d}j + \int_{j \in \mathcal{X}_s^H} \ell_{jsH}^{F} \mathrm{d}j = \boldsymbol{L}_s^H$, for each sector s, where $\mathcal{X}_s^H \subseteq [0,1]$ is the set of products that are exported in the sector. This yields an expression for the labor share of GDP in country H:

$$\frac{w_s^H \boldsymbol{L}_s^H}{\boldsymbol{Y}} = \frac{1}{2} \int_0^1 \alpha_{js} \left\{ \sigma_{jsC}^{H} + \left(\frac{\eta - 1}{\eta}\right) \left[\sigma_{jsH}^{H}(1 - \sigma_{jsH}^{H}) + \frac{\sigma_{jsH}^{F}(1 - \sigma_{jsH}^{F})}{\tau} \right] \right\} \mathrm{d}j \tag{26}$$

with the understanding that $\sigma_{jsH}^{F} = 0$, $\forall j \notin \mathcal{X}_s^H$. On the other hand, trade must be balanced in both countries, according to equation (14). By definition, total exports are $\boldsymbol{X}^H = \boldsymbol{X}_A^H + \boldsymbol{X}_B^H$, where $\boldsymbol{X}_s^H = \int_{j \in \mathcal{X}_s^H} p_{jsH}^{F} y_{jsH}^{F} \mathrm{d}j$ are exports in sector s, while total imports are $\boldsymbol{M}^H = \boldsymbol{M}_A^H + \boldsymbol{M}_B^H$, where $\boldsymbol{M}_s^H = \int_{j \in \mathcal{X}_s^F} p_{jsF}^{H} y_{jsF}^{H} \mathrm{d}j$ are imports in sector s.[13] The balanced-trade condition can then be written as follows:

$$\sum_{s=A,B} \left(\int_{j \in \mathcal{X}_s^H} \alpha_{js} \sigma_{jsH}^{F} \mathrm{d}j - \int_{j \in \mathcal{X}_s^F} \alpha_{js} \sigma_{jsF}^{H} \mathrm{d}j \right) = 0$$

This concludes the characterization of the static part of the equilibrium. Up until now, we have described how firms make output and pricing decisions across products, sectors, and countries, taking the equilibrium sectoral wage as given. Next, we describe how firms make their R&D choices in order to advance their technology relative to that of their direct foreign competitors, and describe the entry problem and the dynamics of the distribution of firms.

Dynamic R&D and entry problems To derive dynamic behavior, we specialize the equilibrium to a Balanced Growth Path (BGP). In our context, a BGP is defined as an equilibrium in which aggregate output in *both* countries grows at a common and constant rate, or $\dot{\boldsymbol{Y}}^k / \boldsymbol{Y}^k = g$, $\forall k \in \{H, F\}$.

By the resource constraint, and the fact that $\boldsymbol{R}^k$ and $\boldsymbol{K}^k$ grow linearly with the economy (a result that we verify later), consumption must grow at the rate g along a BGP. Using the Euler equation (15), this implies that $r^H = r^F = r$, and:

$$g = r - \rho$$

While the production and exporting choices of countries can be derived on and off the stationary solution, we now describe the R&D choices strictly along the BGP. Again, we focus on firms in country H, but reciprocal problems are being faced by leading firms of country F.

[13] Note that the integral is over $\mathcal{X}_s^F$, since everything that is imported by H is exported by F.

To simplify the analysis, we will assume henceforth that there are no differences in the spending shares of products, i.e. $\alpha_{js} = 1$, $\forall j, s$. This being the case, along the BGP competing firms within a product-sector group will be uniquely identified by the existing technological gap between them. Thus, we may replace the product subscript j by n without loss of generality. As different sectors have different labor endowments, though, wages will be different, and so will static profits, so we must keep indexing all variables by s. However, since the two countries are symmetric in their relative sectoral labor endowments (as per assumption (8)), we can anticipate that the solution of one sector in a country will be identical to the solution of the opposite sector in the other country. Thus, if V_s^k is the value of a firm in sector s and country k, we have $V_A^H = V_B^F$ and $V_B^H = V_A^F$. For this reason, we may index the value function only by sector s, and not country k.

The relevant states of a firm are: (i) the number of technological steps $n \in \mathcal{N}$ that the firm is ahead with respect to the foreign competitor within the product group, where n has been defined in equation (11); and (ii) the size of the world economy, $\boldsymbol{Y}$, determining aggregate demand both at home and abroad. Let $V_s(n, \boldsymbol{Y})$ be the value function of a firm in sector s, and define $\pi_{sc,n}^k \equiv \left(\sigma_{sc,n}^k + (\eta - 1)(\sigma_{sc,n}^k)^2\right)\frac{1}{2\eta}$ as the static profits (per unit of output) given market share $\sigma_{sc,n}^k$ for a firm which is a distance n from its foreign competitor in sector s. The dynamic problem of an incumbent firm consists of choosing an innovation rate z_s^H to maximize the value of the present-discounted future stream of profits. The firm takes as given the sectoral wage, w_s (which determines static profits); the world interest rate, r; the world growth rate, g; the sector-level entry rate in both economies, (x_s^H, x_s^F); and the innovation policy of foreign firms in the same product-sector market, $\{z_{s,n}^F\}_{n=-\overline{n}}^{\overline{n}}$.

The problem can be written as the following Hamilton-Jacobi-Bellman (HJB) equation:

$$
\begin{aligned}
rV_s(n, \boldsymbol{Y}) = \max_{z_s^H \geq 0} \Bigg\{ & \pi_{sH,n}^H \boldsymbol{Y} + \left(\pi_{sH,n}^F - \kappa\right)^+ \boldsymbol{Y} \\
& + x_s^H \Big(\widetilde{V}_s - V_s(n, \boldsymbol{Y})\Big) + z_s^H \Big(V_s(n+1, \boldsymbol{Y}) - V_s(n, \boldsymbol{Y})\Big) \\
& + \left(x_s^F + z_{s,(-n)}^F\right)\Big(V_s\big(n-1, \boldsymbol{Y}\big) - V_s(n, \boldsymbol{Y})\Big) - r(z_s^H)\boldsymbol{Y} \Bigg\} \\
& + \dot{V}_s(n, \boldsymbol{Y})
\end{aligned} \tag{27}
$$

when $\overline{n} > n > -\overline{n}$. The value function has the following parts. On the right-hand side, the first line captures static profits from domestic sales and exports (if any), respectively. The firm exports only if $\sigma_{sH,n}^F \geq \sigma^*$, as captured by the notation $(\cdot)^+ = \max\{\cdot, 0\}$. The second line includes, first, the case in which the incumbent firm is displaced by an entrant in the domestic market, an event which occurs at rate x_s^H. In this case, the firm becomes a potential entrant himself, with value $\widetilde{V}_s$. The second term reflects a successful innovation by the domestic incumbent, which widens (or shrinks) the domestic lead (or lag) by one step. This occurs at rate $z_s^H \geq 0$, chosen by the firm. The third line is the event in which the foreign country implements an innovation, be it via entry or due to the R&D activities of an incumbent producer. In the latter case, the rate of innovation corresponds to the policy of incumbent competitors in F, which are $(-n)$ steps away from H. In either case, the domestic firm loses relative technological advantage. The last term on this line incorporates the costs of innovation for the domestic firm. Finally, the last line of equation (27) captures the change in firm value that is due to economic growth.

When either $n = \overline{n}$ (i.e. H has the maximum lead) or $n = -\overline{n}$ (i.e. F has the maximum lead), innovations in one country are instantly diffused and adopted by the other country, i.e. the leading firm pulls the lagging firm along. Therefore, as the technological gap does not change, the leading firm does not gain value by innovating. This sets a boundary condition for the value function, so that, for any $m \in \mathbb{N}$:

$$V_s(\overline{n} + m, \boldsymbol{Y}) = V_s(\overline{n}, \boldsymbol{Y}) \qquad \text{and} \qquad V_s(-(\overline{n} + m), \boldsymbol{Y}) = V_s(-\overline{n}, \boldsymbol{Y}) \tag{28}$$

Potential entrants in the domestic economy pay a cost $r_e(x_s^H)\boldsymbol{Y}$ to generate a rate of entry into sector s equal to x_s^H. Entry is undirected across products, in the sense that entrants draw a product group $j \in [0, 1]$ at random and displace the domestic incumbent previously producing the good. Since each domestic producer can be summarized by the technological gap n, this is isomorphic to the entrant drawing a step size n from the distribution of steps, $\{\varphi_{s,n}\}_{n=-\overline{n}}^{\overline{n}}$, which is sector-specific and which we derive below.[14] Therefore, the problem of potential entrants in sector s is:

$$r\widetilde{V}_s = \max_{x_s^H > 0} \left\{ x_s^H \left[\sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} V_s(n+1, \boldsymbol{Y}) - \widetilde{V}_s \right] - r_e(x_s^H)\boldsymbol{Y} \right\} \tag{29}$$

We assume that $r_e(x) = \nu x$, with $\nu > 0$. Linearity in x guarantees that potential entrants flood into the domestic economy up until the point in which the ex-ante value of entry is driven to zero. Therefore, in a free-entry equilibrium with positive entry in all sectors and countries, it must be the case that $\widetilde{V}_s = 0$, $\forall s$. Imposing this on equation (29), we find that, in equilibrium:

$$\sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} V_s(n+1, \boldsymbol{Y}) = \nu \boldsymbol{Y}$$

That is, as entry is undirected, the total cost of entry must be equal to the expected value of being an incumbent in sector s.[15]

To solve for the value function, we guess-and-verify that we can write $V_s(n, \boldsymbol{Y}) = v_{s,n}\boldsymbol{Y}$, for some sequence of real numbers $\{v_{s,n}\}_{n=-\overline{n}}^{\overline{n}}$. This allows us to normalize problem (27) everywhere by $\boldsymbol{Y}$. After some straightforward algebra, we find:

[14] By construction, for every domestic firm that is n steps ahead (or behind) of its foreign competitor, there is exactly one foreign firm which is n steps behind (or ahead). Therefore, we have $\varphi_{s,n}^H = \varphi_{s,(-n)}^F$. Hence, we may omit the country superscript to simplify notation.

[15] Another implication of this condition is that the value of household wealth in the country is constant along the BGP, as $\boldsymbol{A}^H = \boldsymbol{A}^F = \int_0^1 \sum_s V_{js} \mathrm{d}j = 2\nu$. The transversality condition (equation (16)) then reduces to $\lim_{t \to +\infty} e^{-rt} 2\nu = 0$, which holds true because $r > 0$.

$$(\rho+x_s^H)v_{s,n} = \max_{z_s^H \geq 0} \left\{ \pi_{sH,n}^H + \left(\pi_{sH,n}^F - \kappa\right)^+ + z_s^H \Big(v_{s,n+1} - v_{s,n} \Big) + \left(x_s^F + z_{s,(-n)}^F\right) \Big(v_{s,n-1} - v_{s,n} \Big) - r(z_s^H) \right\}$$

To arrive at this formula we have used that, under our guess, $\frac{\dot{V}_s(n,\boldsymbol{Y})}{\boldsymbol{Y}} = gv_{s,n}$, and on the left-hand side we have grouped terms by using that $\rho = r - g$. Further, we have already imposed that $\widetilde{V}_s = 0$ by free-entry, so that the entry rate in the domestic economy, x_s^H, effectively acts as a discount rate for firms. The first-order condition for the incumbent's problem is:

$$r'(z_{s,n}^H) = \begin{cases} 0 & \text{if } n = \overline{n} \\ v_{s,n+1} - v_{s,n} & \text{if } -\overline{n} \leq n < \overline{n} \end{cases} \tag{30}$$

where $z_{s,n}^H$ is the innovation policy as a function of the technological gap. The first-order condition states the optimal innovation intensities when the domestic firm has the maximum lead, and for some interior gap $n < \overline{n}$, respectively. In the first case, the gap cannot be increased any further, so firms do not invest into R&D because of the boundary condition (28) on the value function. On the other hand, the free-entry condition now implies:

$$\nu = \sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} v_{s,n+1}$$

stating once again that even though value functions differ across sectors within the country, the expected value of being an incumbent is fixed to the cost of entry, which is common across sectors and countries. The labor share can also be calculated along the BGP, by equation (26):

$$\frac{w_s^H \boldsymbol{L}_s^H}{\boldsymbol{Y}} = \frac{1}{2} \sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} \left\{ \sigma_{sC,n}^H + \left(\frac{\eta - 1}{\eta}\right) \left[\sigma_{sH,n}^H (1 - \sigma_{sH,n}^H) + \frac{\sigma_{sH,n}^F (1 - \sigma_{sH,n}^F)}{\tau} \right] \right\} \tag{31}$$

for both $s = A, B$. Therefore, the wage bill of the sector grows at the rate of the economy, implying a constant labor share along the BGP. Moreover, we find the following result on the growth rate:

Lemma 1 (Growth rate) *In an equilibrium with positive entry in all countries and sectors, the growth rate of output is constant across countries and given by:*

$$g = \lambda \boldsymbol{I}, \qquad \textit{where } \boldsymbol{I} \equiv \frac{1}{2} \sum_{s=A,B} \left(x_s^k + \sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} z_{s,n}^k \right)$$

is the average innovation rate across sectors.

Proof. See Appendix B.1.

Since entrants and incumbents in all countries and sectors are advancing the technological frontier through their R&D spending, the growth rate features all of their innovation efforts. The growth rate is also an increasing function of the step size of innovation, λ, which for simplicity is common across firm types (incumbent or entrant), sectors, and countries.[16] Moreover, notice that the average innovation rate within a country across sectors is the same in both countries, implying a common rate of economic growth. Were this not the case, one country would outgrow the other, and in the limit the two economies could not coexist in equilibrium. Sectors within each country may in principle grow at different rates. Yet, the two sectors can coexist on the BGP because there is a limit to how far firms can fall back (as $\bar{n} < +\infty$), so that the slow-growing sector (if any) in one country is pulled up by the fast-growing sector in the other country through the international knowledge spillovers. To obtain innovation rates in all sectors and countries, we can exploit the symmetry in the relative sectoral endowment of labor (equation (8)), which implies:

$$(x_A^H, x_B^H) = (x_B^F, x_A^F) \qquad \text{and} \qquad (z_{A,n}^H, z_{B,n}^H) = (z_{B,(-n)}^F, z_{A,(-n)}^F) \tag{32}$$

for each $n \in \mathcal{N}$ (where recall that n is always denotes the distance of H from F). As a result, we have $(\boldsymbol{I}_A^H, \boldsymbol{I}_B^H) = (\boldsymbol{I}_B^F, \boldsymbol{I}_A^F)$, where $\boldsymbol{I}_s^k \equiv x_s^k + \sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} z_{s,n}^k$ is the aggregate innovation rate of sector s in country k.

To close the equilibrium, we must impose the resource constraint, and derive the invariant step-size distribution. First, we derive aggregate R&D and trade costs, both of which are paid with units of the final good, along the BGP. The R&D share of GDP is given by:

$$\frac{\boldsymbol{R}}{\boldsymbol{Y}} = \sum_{s=A,B} \left(r_e(x_s^k) + \sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} r(z_{s,n}^k) \right) \tag{33}$$

which, again, is equal in both countries, by symmetry. Similarly, the share of GDP that pays for trade costs is:

$$\frac{\boldsymbol{K}}{\boldsymbol{Y}} = \kappa \sum_{s=A,B} \sum_{n \geq \lceil n_s \rceil} \varphi_{s,n} \tag{34}$$

where $\lceil \cdot \rceil$ is the ceiling function, and n_s satisfies:

$$\sigma_{sH,n_s}^F = \sigma^* \tag{35}$$

or, equivalently, $\sigma_{s'F,(-n_s)}^H = \sigma^*$ for $s' \neq s$. Equation (34) sums fixed exporting costs over those firms that decide to export, i.e. firms that possess a sufficient technological gap that makes them willing to export into the foreign market. Since the right-hand side of equations (33) and (34) is constant along the BGP, these equations then show that aggregate consumption, which can be obtained residually from equation (12), indeed grows at the rate g.

[16] Extending the model to a setting with heterogeneous innovations across firm types and sectors is straightforward. However, symmetry across countries must be preserved to guarantee the existence of a BGP in which both countries grow at the same rate.

Distribution dynamics We have argued above that the market for each product group $j \in [0,1]$ can be characterized by the number of steps n that separate the domestic leader from its direct foreign competitor. Across all product markets j within a given sector s, there is a share $\varphi_{s,n} \geq 0$ of firms on each n bin that are producing in the domestic economy, with $\sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} = 1, \forall s$. Firms gain or lose advantage over their competitors on the basis of successful innovations to the quality of their products.

For any $\overline{n} > n > -\overline{n}$, the flow equation for the share of firms is:

$$\dot{\varphi}_{s,n} = \underbrace{\varphi_{s,n-1}(z^H_{s,n-1} + x^H_s) + \varphi_{s,n+1}(z^F_{s,-(n-1)} + x^F_s)}_{\text{Inflows}} - \underbrace{\varphi_{s,n}(z^H_{s,n} + z^F_{s,(-n)} + x^H_s + x^F_s)}_{\text{Outflows}} \tag{36a}$$

The change in the share of firms in state n is the difference between inflows and outflows. Inflows are given by: (i) incumbent firms flowing from $n-1$ into n due to a successful innovation, (ii) domestic entrants building upon the technology gap of the previous incumbents, and (iii) firms who worsen their relative technology either because of successful innovation or because of entry in the foreign market. Flows out of state n are given by those firms with gap n that (i) advance their technology through successful innovation, (ii) fall back because the foreign firm innovates on the product, (iii) exit the market altogether because of domestic entry, or (iv) fall back because there is entry in the foreign country.

At the boundaries, international knowledge spillovers take effect, preventing laggard firms from falling too far behind. When H is ahead by the maximum possible lead (i.e. $n = \overline{n}$), we have:

$$\dot{\varphi}_{s,n} = \underbrace{\varphi_{s,n-1}(z^H_{s,n-1} + x^H_s)}_{\text{Inflows}} - \underbrace{\varphi_{s,n}(z^F_{s,(-n)} + x^F_s)}_{\text{Outflows}} \tag{36b}$$

In this case, since H's lead cannot increase any further, inflows are limited to those successful innovators at H that improve upon lead $n-1$, whereas outflows are limited to H firms losing the lead due to a foreign innovation (by a foreign incumbent or a foreign entrant). By a similar intuition, when H is behind by the maximum possible lag (i.e. $n = -\overline{n}$), we have:

$$\dot{\varphi}_{s,n} = \underbrace{\varphi_{s,n+1}(z^F_{s,-(n-1)} + x^F_s)}_{\text{Inflows}} - \underbrace{\varphi_{s,n}(z^H_{s,n} + x^H_s)}_{\text{Outflows}} \tag{36c}$$

Equations (36a)-(36b)-(36c) define Markov transitions in the $\mathcal{N}$ space, with reflective barriers at $(-\overline{n})$ and $\overline{n}$. To find the invariant step-size distribution in the sector, we can simply specialize the equations to a BGP equilibrium by imposing that $\dot{\varphi}_{s,n} = 0, \forall(s,n)$.

We are now ready to define a BGP equilibrium:

Definition 1 (BGP Equilibrium) *A Balanced Growth Path Equilibrium with positive entry in all sectors and countries is, for all $k \in \{H,F\}$, $j \in [0,1]$, $s \in \{A,B\}$, and given $\boldsymbol{Y} > 0$: outputs $(y^k_{jsH}, y^k_{jsC}, y^k_{jsF})$; prices $(p^k_{jsH}, p^k_{jsC}, p^k_{jsF})$; aggregates $(\boldsymbol{C}^k, \boldsymbol{K}^k, \boldsymbol{R}^k, \boldsymbol{A}^k)$; a wage rate*

w_s^k; a rental rate r; a growth rate g; a set of value functions $\{v_{s,n} : n \in \mathcal{N}\}$ and innovation policies $\{z_{s,n}^k : n \in \mathcal{N}\}$; an entry rate $x_s^k > 0$; and a step-size distribution $\{\varphi_{s,n} : n \in \mathcal{N}\}$; such that: (i) given prices, the final good sector maximizes profits; (ii) y_{jsc}^k and p_{jsc}^k solve problem (18); (iii) the innovation flows solve equation (30); (iv) the step-size distribution $\{\varphi_{s,n} : n \in \mathcal{N}\}$ solves the flow equations (36a)-(36b)-(36c) with $\dot{\varphi}_{s,n} = 0$; (v) $x_s^k > 0$ solves the entry problem (equation (29)) and implies $\sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} = 1$; (vi) the value functions satisfy the incumbent's problem (equation (27)), and satisfy the free-entry condition $\nu = \sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n} v_{s,n+1}$; (vii) the rental rate satisfies $r = g + \rho$; (viii) the growth rate g is given in Lemma 1; (ix) the wage rate w_s^k satisfies equation (26); (x) wealth is given by $\boldsymbol{A}^k = 2\nu$; (xi) $\boldsymbol{R}^H = \boldsymbol{R}^F$ and $\boldsymbol{K}^H = \boldsymbol{K}^F$ are given by (33) and (34), respectively; and (xii) aggregate consumption $\boldsymbol{C}^k$ satisfies the resource constraint (12).

Note that the static equilibrium conditions are all intrinsic functions of the sectoral wage. Indeed, wages determine marginal costs, and these in turn pin down market shares, markups, and profits. However, equilibrium wages are computed using the aggregate distribution of firms, an equilibrium object that we can only obtain as a result of the dynamic behavior of firms. Therefore, to solve the model numerically, we implement an algorithm that iterates on the value of wages: first, we compute market shares under a guess for wages, we then solve the R&D and entry problems, compute the resulting invariant distribution by imposing free entry in both sectors, and finally update the wage guess by checking whether the labor market clears. For full details of this algorithm, see Appendix C.

4 Model Discussion

4.1 Qualitative Properties

This section presents the qualitative properties of the model, with an emphasis on the interplay between competition and innovation incentives. In the model, R&D allows firms to gain technological advantage over their competitors, increasing their market share and, potentially, granting them access to international markets. However, changes in market shares have implications for markups, both at home and abroad, and these in turn affect the incentives of firms to exert further innovation effort.

To illustrate these mechanisms, we present results from a numerical example. In this example, we assume that $\omega_H^H = \omega_H^F$ so that differences in profits and markups between the domestic and foreign markets for firms are not driven by preferences, but by the forces of comparative advantage, endogenous competition, and innovation, which are the novel channel that we want to explore in this paper.[17] Figure 1 shows the value function $v_{n,s}$ and innovation policies $z_{n,s}^H$

[17] We consider the R&D cost function $r(z) = \chi z^{\psi}$, with $\chi > 0$ and $\psi > 1$. The model parameters are set to the following values. Preference and R&D parameters are $(\rho, \eta, \lambda, \chi, \psi, \nu) = (0.02, 1.5, 0.1, 2, 2, 1)$, trade parameters are $(\kappa, \tau) = (0.1, 1.5)$, implying $\sigma^* = 0.2649$, and the state space uses $(\overline{n}, n_C) = (10, 2)$. Moreover, we assume that H has twice the labor endowment of F in sector A, i.e. $\frac{\boldsymbol{L}_A^H}{\boldsymbol{L}_A^F} = 2$.

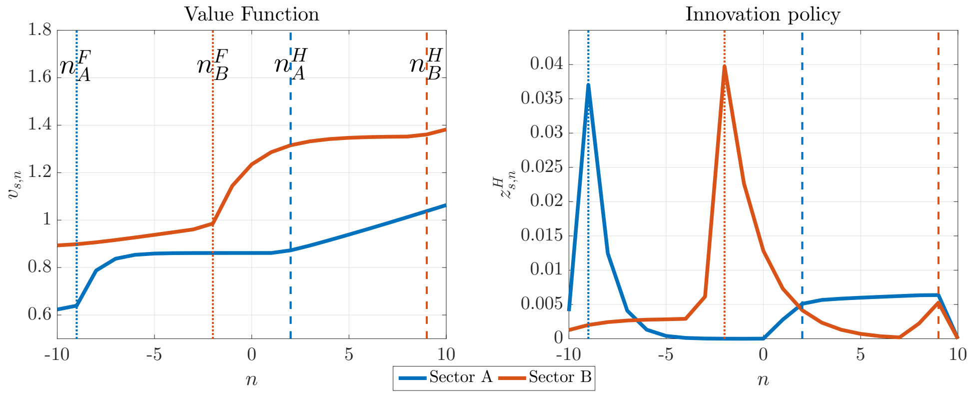

Figure 1: Value functions and innovation policies in both sectors, as functions of the technological step size.

at H for both sectors $s = A, B$ assuming, as in equation (8), that sector A has a higher relative labor endowment than sector B. Figure 2 shows the corresponding static profits and markups, in both the domestic and the foreign markets, and for both sectors. In these figures, the horizontal axis is always the number of steps away that H is relative to F, with $n > 0$ implying that H is ahead.

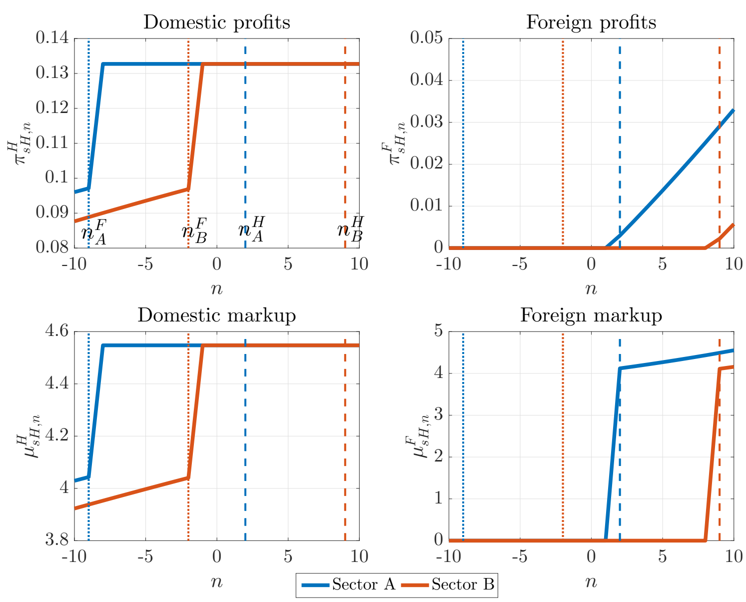

Figure 2: Static profits (top row) and firm-level markups (bottom row), in the domestic (first column) and foreign (second column) markets, for sectors A (in red) and B (in blue).

The value functions are increasing in n in both sectors, and exhibit kinks at those points where firms from either country start to export to the other country (recall the cutoffs defined in equation (35)). That is, H exports to F in sector s for all $n > n^H_s$. In this example, $(n^H_A, n^H_B) = (2, 9)$. Similarly, F exports to H in sector s for all $n < n^F_s$, with $n^F_s = -n^H_{s'}$ for each $s' \neq s$ by symmetry, so that $(n^F_A, n^F_B) = (-9, -2)$. In Figure 3 we summarize the trade patterns for country H along the n space. We see that, when H is sufficiently far ahead from F, the country starts exporting goods from sector A, in which it has a comparative advantage. Only when the technological gap is very large in favor of H does this country decide to export the good for which it has a comparative disadvantage. Therefore, in this example, there is partial trade specialization. When the countries have similar technological levels, they do not trade at all and remain in autarky. Hence, a motive for international trade emerges when the countries become sufficiently dissimilar in productivities through the endogenous process of innovation.

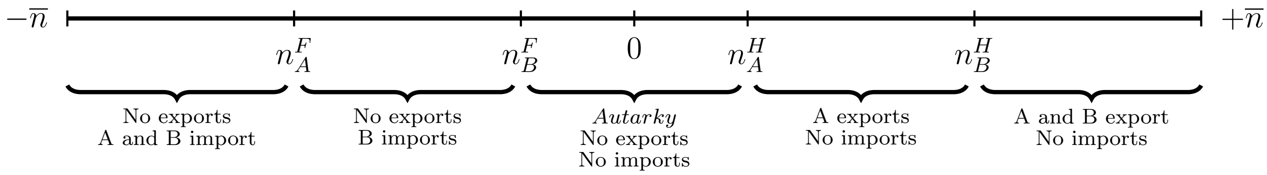

Figure 3: Patterns of trade for country H, as a function of the relative technological distance between domestic and foreign firms, in the numerical example.

Innovation efforts exhibit two local maxima in relative technological distance. On the left end, the innovation effort is higher the closer the foreign firm is from entering the H market (i.e. around n_s^F). Intuitively, as the H firm falls back toward this threshold, its innovative efforts increase to try to escape the prospect of competition with the foreign leader in the domestic market. On the other end, the innovation effort increases close to the threshold at which the H firm penetrates the foreign market (i.e. around n_s^H). Intuitively, as the H firm gains relative technological advantage, the incentives increase for this firm to widen the gap further in order to gain access to the foreign market. Similar patterns in the innovation policy emerge in the model of Akcigit *et al.* (2018), with spikes in the innovation efforts which they label *defensive* and *expansionary* innovation, respectively. Furthermore, they find evidence for this behavior in the data. However, while in their model firms are trying to conserve monopolistic power in their markets, where they are always the sole producers by assumption, in our model the degree of competition is endogenous and taken into account by firms when making innovation decisions.

As a result, firms charge different markups in the domestic and the foreign markets, depending on their relative technological position. Figure 2 shows that, when firms are the sole distributers of a good within their own country ($n > n_s^F$), static profits, market shares, and markups in the domestic market for that sector are flat. This is because, in this case, the firm is only competing with the competitive fringe, with which it maintains a fixed technological distance. However, when the foreign firm penetrates the domestic market in either, or both, sectors ($n \leq n_s^F$), static profits decay the larger the advantage of the foreign firm becomes with respect to the domestic firm. As a result of this competitive force, the domestic leader loses market power in its own market, and is forced to reduce the markup that it charges for its products to the domestic residents of the country.

The reciprocal phenomenon occurs when it is the domestic firm selling in the F market given that it has acquired a sufficiently large lead in the sector ($n > n_s^H$). In this case, the firm charges a markup to the residents of the foreign country which increases in the technological lead of the firm. Because the marginal costs of production are higher for selling in the foreign economy (as $\tau > 1$), the markup that firms set abroad is always lower than that which they set at home. Moreover, foreign markups are higher in the sector in which the country has a comparative advantage (Sector A, in this case), as the country does not need that large a lead to start exporting. Domestic markups are also higher for firms in the comparative advantage sector, because firms do not compete with the foreign firm for a larger segment of the support of technological distances. Indeed, in Sector A, the domestic firm is not competing at all with the foreign firm unless the former is lagging very far behind ($n < n_A^F$). As the domestic leader otherwise enjoys a large market share, it is able to set high markups —and, in any case, higher than those in the comparative disadvantaged sectors.

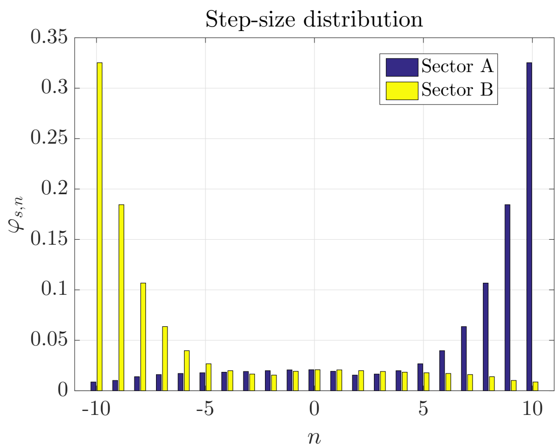

Figure 4: Invariant step-size distribution in country H, in the numerical example.

Figure 4 depicts the step-size distribution $\{\varphi_{s,n}\}$ at H in both sectors $s = A, B$ that results from this numerical example. The figure shows very clearly the patters of specialization that emerge in BGP. In Sector A, H has a comparative advantage, so most firms become very innovative-intensive in order to exploit their productive edge. As a result, they gain market power and enter the foreign market. Overall, firms in H tend to take the technological lead in the production of goods in sector A, and the country thus becomes a heavy exporter of those goods. Symmetrically, foreign firms behave similarly in Sector B, so most firms from H in that sector have poor relative technologies, losing competitive lead against their foreign counterparts. This shows, then, that the model delivers a fat-tailed firm and markup distribution along the BGP.

4.2 The Effects of Trade Shocks

Next, we analyze how reductions in variable trade costs, from some τ_H to $\tau_L < \tau_H$, affect patterns of innovation and trade through changes in the competitive structure of international markets, and how these translate into aggregate effects on productivity, and the average markup. For simplicity, we consider a world with no intra-industry trade (unlike the previous example), in which Sector A (resp. B) only exports (resp. imports). We then compare two such economies in their respective stationary equilibria, with identical primitives except for their value of τ.

4.2.1 Market-Size vs. Competition Effects

Because countries are asymmetric in their sectoral labor endowments, reducing trade costs for all firms in the same way will have asymmetric effects on the different sectors within each country. In the export sector (Sector A), a τ reduction can be seen as an *export shock*: it will increase the size of markets for the industries in that sector that export. In the import sector (Sector B), a τ reduction is an *import shock*: it will increase competition from the foreign country in the industries in that sector that import.

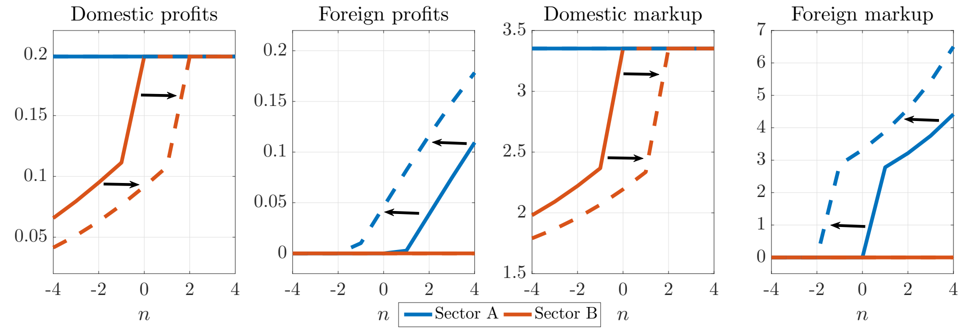

Figure 5: Static profits and markups in both sectors and both markets. Comparison between τ_L (solid lines) and $\tau_H > \tau_L$ (dashed lines).

Figure 5 shows domestic and foreign static profits and markups in both sectors, as a function of relative productivities, under the two τ regimes. As there is no intra-industry trade, Sector B makes no profits in the foreign market, and Sector A faces no competition from abroad in the domestic market. On the extensive margin, the trade thresholds change due to the τ decrease. Because trading is now cheaper, firms in the export sector need a lower productivity advantage relative to the foreign leader to start exporting their product, and profits in the export market are higher because of a pure *market size effect.* As domestic firms capture higher market shares, they are also able to charge higher markups. Symmetrically, less productive exporting firms from the foreign country can now enter the domestic economy, so Sector B now faces stronger competition from abroad. Therefore, domestic profits decrease in Sector B due to a pure *competition effect.* As these firms lose market share to the foreign competitor, they must also lower their markups.

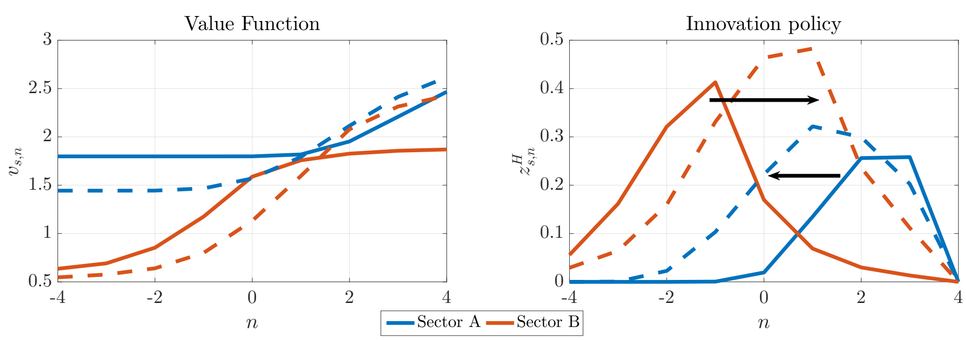

Figure 6: Value functions and innovation policies in both sectors. Comparison between τ_L (solid lines) and $\tau_H > \tau_L$ (dashed lines).

To understand the dynamic implications of these static effects, Figure 6 plots the value functions and innovation policies in both sectors under the two τ regimes. In this example, as there is no intra-industry trade, the innovation policies exhibit only one peak, corresponding to expansionary (for Sector A) and defensive (for Sector B) R&D efforts, respectively. The latter sector now faces stronger competition, and additional entry of relatively less productive foreign firms, so the defensive innovation efforts increase for those firms that are closer to the import threshold. Firms that are further away, however, have lower incentives to exert R&D, as their distance to the threshold has now increased. Therefore, the innovation policy shifts to the right. For Sector A, symmetrically, the curve shifts to the left: less productive firms who were not exporting before now increase their expansionary innovation effort to gain and maintain access into the foreign market, whereas the very top firms need not increase their R&D spending relative to the foreign leader as their position in the foreign economy is has strengthened. Figure 7 then shows that, as a consequence of these changes in innovation policies, there is a hollowing out of the distribution in both sectors.

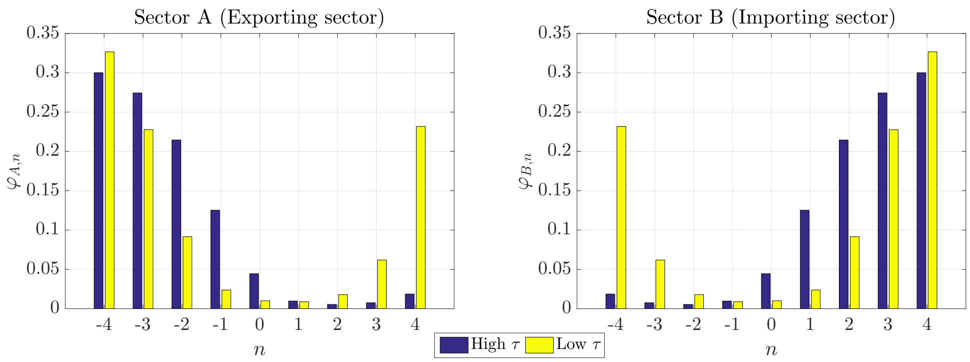

Figure 7: Invariant step-size distribution in both sectors in country H, in the numerical example, for high and low variable trade costs.

4.2.2 Overall effects on average markups

What is the overall effect on the average markup in the domestic economy? In this example, the markup in the export sector remains flat (as this sector does not import), so to answer this question it is sufficient to look at the markup from Sector B. Using equations (22)-(23), we can write the average markup in Sector B as follows:

$$\boldsymbol{\mu}^H \equiv \sum_{n=-\overline{n}}^{\overline{n}} \varphi_n \Big(\underbrace{\sigma_{H,n}^H \mu_{H,n}^H + \sigma_{C,n}^H + \sigma_{F,n}^H \mu_{F,n}^H}_{[A]} \Big), \quad \text{where recall } \mu_{c,n}^H \propto \frac{1}{1-\sigma_{c,n}^H},\ c = H, F$$

where we have omitted the sector subscripts to alleviate notation. To decompose the effects of the τ decrease on $\boldsymbol{\mu}^H$, we can distinguish between static and dynamic effects.

Statically (i.e. ignoring shifts in the productivity distribution), the entry of more productive foreign firms means that these firms now capture a larger share and domestic firms lose out, so $\sigma_{H,n}^H \mu_{H,n}^H$ and $\sigma_{C,n}^H$ are lower, and $\sigma_{F,n}^H \mu_{F,n}^H$ is higher. The total effect on term [A] is therefore ambiguous, and it depends on the initial level of concentration of the industry. In particular, in those industries n where $\sigma_{H,n}^H$ is sufficiently high to start with, term [A] will decrease. Otherwise, [A] will increase. In words, those importing industries with a sufficiently high level of sales concentration by the domestic leader will see their static markups decline (a pro-competitive effect) after a trade liberalization. In less concentrated industries, however, the opposite is true.

Dynamically (i.e. letting the productivity distribution adjust through R&D), these effects are affected by changes in the innovation policy of firms. Here, we must distinguish between the response of productive versus unproductive industries (relative to their foreign counterparts). As we have argued, more productive (i.e. high n) industries increase their innovation efforts relative to the foreign leader. This increases future market shares for domestic leaders ($\sigma^H_{H,n}$), and thus markups. If the industry is sufficiently concentrated to start with, these effects generate an upward pressure on the average markup for the whole sector. However, if the industry leader does not have a sufficiently high share to start with, the increase in her innovation efforts and markups will not translate into higher markups on average. In the first case, therefore, the static pro-competitive gains described above are *dampened* going forward. In the second case, they are *amplified*. In less productive (i.e. low n) industries, where the static effects are anti-competitive (i.e. an upward pressure statically on the average markup), the picture is exactly the opposite, by symmetry: industries respond by lowering their innovation efforts, and the effects are amplified (resp. dampened) if the industry has a high (resp. low) concentration to start with.

Ultimately, whether there is amplification or dampening depends upon the underlying levels of concentration within industries, the productivity distribution across industries, as well as the patterns of trade between the countries, before and after the changes in trade costs. Thus, understanding the overall effects on competition of trade liberalizations calls for a quantitative assessment of the model.

5 Quantitative Analysis [Preliminary]

5.1 Calibration Strategy

We seek to match both aggregate and firm-level moments from the Spanish balance sheet data introduced in Section 2, for the period 2000-2013. There are 14 deep parameters to be identified: the discount rate, ρ; the firms' taste shifters, $(\omega_{dom}, \omega_{comp}, \omega_{for})$; the relative size of Sector A, $\boldsymbol{L}^H_A/\boldsymbol{L}^F_A$; the distance between leader and fringe, and maximum relative distance, $(n_C, \overline{n})$; the within-industry elasticity of substitution, η; the innovation step, λ; the fixed and variable trade costs, (κ, τ); the entry cost, ν; and the scale and curvature parameters in the incumbents' R&D cost function, (χ, ψ).[18]

External identification The parameters $(\rho, \omega_{dom}, \psi)$ are set outside the model. The taste for goods produced by the domestic leader is normalized to unity ($\omega_{dom} = 1$), which comes with no loss of generality as the model is solved in terms of relative outputs (see Appendix C.1). The discount rate is set to $\rho = 0.02$, corresponding to an annual discount factor of approximately 98%. Finally, the curvature of the R&D cost function is set to $\psi = 2$, following Akcigit and Kerr

[18] We choose $r(z) = \chi z^{\psi}$, with $\chi > 0$ and $\psi > 1$.

(2018), who in turn survey the empirical literature estimating the cost curvature of different types of R&D.

Internal identification Because the model is highly non-linear, maximum likelihood methods are not useful to identify each of the remaining parameters separately, so we resort to moment-based identification via simulated method of moments (SMM). To implement this procedure, we use a search algorithm that employs an unweighted minimum-distance criterion function comparing empirical moments to their model-generated counterparts.

The set of moments can be grouped into aggregate and firm-level moments. At the aggregate level, we target an annual growth rate of TFP in the Spanish manufacturing sector equal to 1.7%, which we obtain from KLEMS. This helps us identify the innovation step λ, as the growth rate is shifted by this parameter. We also target the aggregate share of sales that goes into R&D investment in the manufacturing sector, equal to 1.4%, which we obtain from the Spanish National Statistical Institute (INE). This helps us pin down a value for χ, as this parameter scales the cost of R&D in the same way for all incumbents, regardless of their relative productivity. Finally, we target the average export-to-import ratio in RCA industries, where an RCA industry is defined as one with an RCA index (after equation (1)) above the median. We obtain this number (equal to 1.039) from the World Bank's WITS database, and it helps us identify the relative size of Sector A, as this sector has the comparative advantage in the model.

Param.	**Value**	**Description**	**Source / Target**
		Calibrated externally	
ρ	0.02	Discount rate	Standard
ω_{dom}	1	Domestic leader shifter	Normalization
ψ	2	R&D cost elasticity	Akcigit & Kerr (2018)
		Estimated internally	
λ	0.3622	Innovation step	TFP growth
χ	8.2362	R&D cost scale	R&D share
$\boldsymbol{L}_A^H/\boldsymbol{L}_A^F$	1.6494	Relative size of Sector A	Exports/Imports ratio
η	14.5	Within-sector EoS	Average markup
τ	1.16	Variable trade cost	Export share
κ	0.068	Fixed trade cost	% Exporting firms
$\overline{n}$	6	Maximum relative distance	Relative size of leader
n_C	1.4515	Distance leader from fringe	Relative size of leader
ν	0.9130	Entry cost	Turnover of the leader
ω_{for}	0.2388	Foreign leader shifter	Markup dispersion
ω_{comp}	1.0735	Competitive fringe shifter	Domestic HHI
Period	1 year		

Table 8: Full set of calibrated parameters in the baseline estimation.

The remaining moments are obtained from our sample of Spanish firms. To identify the within-industry elasticity of substitution η, we target the sales-weighted average markup in the Spanish manufacturing sector during this period, equal to 12.8%.[19] To identify the trade cost parameters, τ and κ, we target the export share of sales in manufacturing, and the share of exporting firms, which in the data are equal to 43.43% and 21.29%, respectively. To identify the maximum distance between domestic leader and fringe (n_C), and between domestic and foreign leaders ($\overline{n}$), we target the relative size of the leader in the data, which is defined as the average sales share of the top firm throughout the period (equal to 0.175). The entry cost ν is calibrated to match the turnover of the leader in the data, defined as the average yearly probability with which the identity of the top firm changes within each 4-digit industry, equal to 25.65% in the data. Finally, the relative taste shifters ($\omega_{comp}, \omega_{for}$) are jointly identified by different measures of dispersion: the average standard deviation of sales-weighted markups, equal to 31.7% in the data; and the average Herfindahl index among all manufacturing firms in our sample, equal to 0.1157 (on a scale from zero to one).

Moment	**Model**	**Data**	**Data Source**
A. From aggregate data			
TFP growth	1.8%	1.7%	EU KLEMS
R&D share	8.56%	1.4%	INE
Exports/Imports in RCA	100	1.039	WITS
B. From firm-level data			
Average markup	1.252	1.128	Firm-level data
Export share	10.18%	43.42%	Firm-level data
% Exporting firms	22.65%	21.29%	Firm-level data
Relative size of the leader	0.338	0.175	Firm-level data
Turnover of industry leader	5.9%	25.65%	Firm-level data
Markup dispersion	33.2%	31.7%	Firm-level data
Domestic HHI	0.1315	0.1157	Firm-level data

Table 9: Targeted moments: model versus data. Notes: All empirical moments based on the Spanish manufacturing sector. R&D and export shares are relative to total sales in manufacturing. An RCA industry is defined to belong to the top 50% of industries with the highest RCA index values. The average markup is weighted by firm-level sales shares relative to the whole manufacturing sector.

Table 8 shows the full set of parameter values, and Table 9 the results of the moment-matching exercise. For now, the model is able to capture only certain features of the data. The model predicts well the rate of TFP growth, average markups, the share of exporting firms, the relative size of the leader, and both measures of dispersion, but fails to deliver the correct exports-to-imports ratio, R&D and export shares, and the turnover rate of the industry leader. Indeed, in the current calibration, there is hardly any intra-industry trade, not enough exporting

[19] While the cross-industry elasticity is one by assumption, here we obtain a within-industry elasticity of $\eta = 14.5$. By comparison, the within-industry elasticity obtained by Atkeson and Burstein (2008) is 10, and the elasticity across industries is 1.01.

and insufficient leader turnover. This is very much work in progress, and will be improved in future versions of this paper.

5.2 Quantitative Evaluation of the Dynamic Effects from Trade

Using the estimated economy, and following our discussion in Section 4.2, we can now quantitatively assess the dynamic effects of trade cost reductions on markups through innovation. In this exercise, we fix all parameters to their estimated values, and introduce a decrease in the variable trade cost, τ. We then compare across BGP solutions under two scenarios: (i) the baseline economy, in which changes in innovation incentives give rise to shifts in the relative productivity distribution; (ii) a counterfactual economy, in which these shifts are ignored, and the distribution is kept fixed at its initial levels through the different τ regimes.

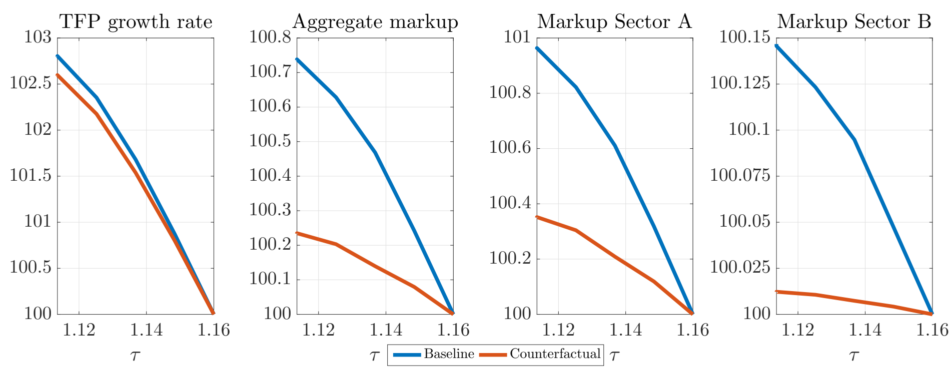

Figure 8: Changes in growth and markups in BGP across different levels of τ, in the baseline economy (blue lines) and a counterfactual economy in which the productivity distribution does not adjust to changes in innovation policies (red lines).

Figure 8 shows the results of this exercise. In this case, a decrease in trade costs leads to increases in markups in both sectors. However, these increases would have been smaller had we ignored how innovation feeds into markups through changes in the distribution. Consequently, the growth rate increases more in a world in which there is an interaction between innovation and competition. The main result is that, under this (imperfect) calibration, the effects of trade liberalization episodes on markups and growth would be under-estimated if this feedback were to be ignored. In the future, we plan to improve the estimation in order to provide a more reliable number of this measurement gap in the dynamic effects of trade on markups.

6 Conclusion

This paper studies the effect of greater openness to trade on innovation and markups, and the interaction between the two. We start by providing empirical evidence on the relationship between trade shocks, productivity growth and markups using Spanish firm-level data. We find that both positive import and export shocks foster productivity growth and decrease markups in the domestic market. Interestingly, these effects are stronger for import shocks. We then build an endogenous growth model of trade with two countries and two sectors with strategic interaction between domestic and foreign firms. Our model features both so-called market-size and competition effects of trade liberalizations. By innovating, firms can increase their market share and charge higher markups, and endogenous changes in market power in turn shape the innovation incentives of competitors who are trying to escape competition. Therefore, there is a bidirectional relationship between R&D and markups.

Our model is able to qualitatively replicate the observed relationship between trade openness, productivity growth, and markups. In addition, we show that the response of innovation to decreased trade costs may dampen or amplify the effects of trade on markups through a shift in the firm (relative) size distribution. In a quantitative exercise, we find that the effects are amplified: ignoring shifts in the productivity distribution would lead us to under-estimate the effects that reductions in trade costs have on markups and productivity growth.

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Appendix

A Figures and Tables

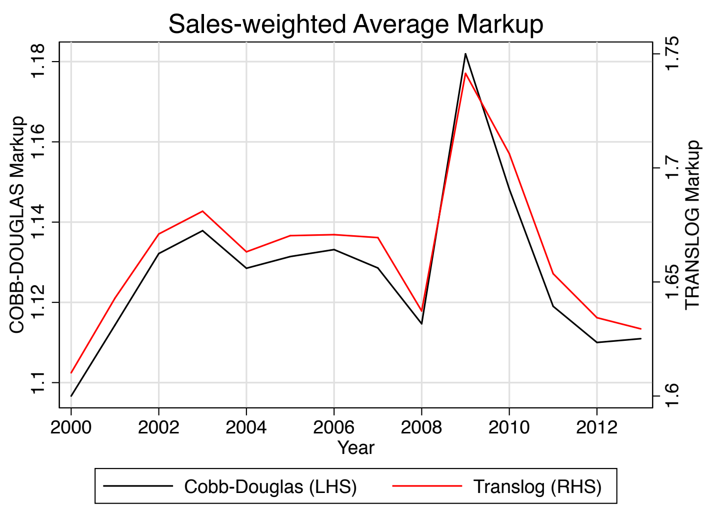

Figure A.1: Sales-weighted average markup in the Spanish manufacturing sector (2000-2013). *Notes:* The average markup is defined as: $M_t = \sum_i \omega_{it}\mu_{it}$, where μ_{it} is firm i's markup, and $\omega_{it} \equiv \frac{Sales_{it}}{\sum_{i=1}^{N_t} Sales_{it}}$, where N_t is the total number of manufacturing firms at time t. The black line (left axis) corresponds to a Cobb-Douglas production function. The red line (right axis) corresponds to a Translog production function (with non-constant returns to scale).

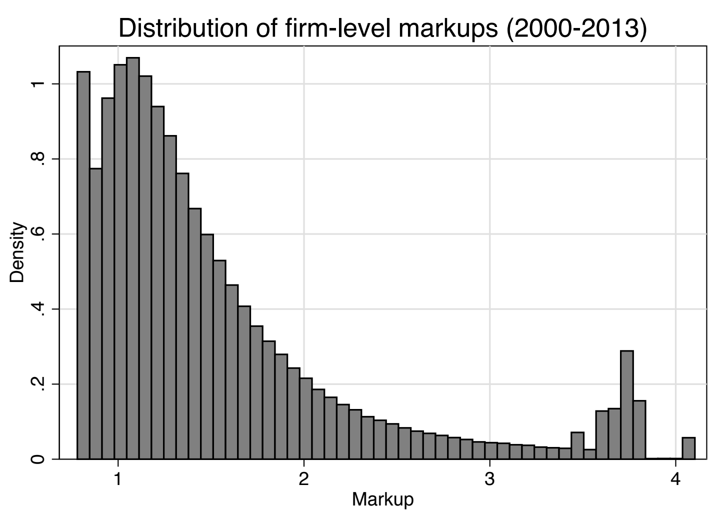

Figure A.2: Markup distribution (2000-2013). *Notes:* Raw distribution of firm-level markups, across time and firms, in the Spanish manufacturing sector (2000-2013).

	Panel A: All firms (2004-2013)				
	Mean	Std. Dev.	25th prctile	Median	75th prctile
Operating revenue	4444.11	55637.67	208.86	542.41	1643.43
Employees	23.19	126.43	4	8	17
Wages	22.29	10.93	14.69	20.51	28.12
Fixed Assets	2189.03	69107.2	38.22	148.75	556.52
Total Assets	4654.1	123063.7	184.23	500.68	1586.8
Material inputs	2681.65	42502.65	75.88	233.2	821
Material share of sales	0.466	0.1924	0.332	0.472	0.607
Labor share of sales	0.342	0.195	0.199	0.31	0.446
Markups	1.77	2.54	1.02	1.29	1.73
# Obs. 721,931					

	Panel B: Exporting firms (2004-2013)				
	Mean	Std. Dev.	25th prctile	Median	75th prctile
Operating revenue	19122.08	125961.9	1584.72	1584.72	10837.87
Employees	75.85	280.74	14	28	61
Wages	31.17	11.47	22.96	29.94	37.80
Fixed Assets	9682.03	157806.8	325.15	1183.78	4063.03
Total Assets	20195.51	280862.8	1489.11	3791.79	11191.1
Material inputs	11841.53	96565.81	727.98	2024	6024.42
Material share of sales	0.534	0.171	0.423	0.543	0.656
Labor share of sales	0.245	0.153	0.137	0.214	0.317
Markups	1.27	1.22	0.91	1.08	1.35
# Obs. 137,951					

	Panel C: Importing firms (2004-2013)				
	Mean	Std. Dev.	25th prctile	Median	75th prctile
Operating revenue	19718.92	126883.5	1837.695	4259.375	11621.24
Employees	78.15	283.1	15	30	64
Wages	30.93	11.44	22.67	29.51	37.48
Fixed Assets	10025.72	158979.6	413.12	1353.44	4456.86
Total Assets	20820.9	282925.6	1738.19	4219.66	11837.78
Material inputs	12220.79	97274.69	883.09	2288.22	6532.44
Material share of sales	0.544	0.17	0.438	0.557	0.666
Labor share of sales	0.231	0.142	0.132	0.201	0.295
Markups	1.25	1.33	0.89	1.06	1.31
# Obs. 135,940					

Table A.1: Summary statistics for all firms, exporting firms, and importing firms, over the 2004-2013 period. *Notes:* All monetary values in thousands of euros. *Wages* are average annual salary per worker. *Materials* are net purchases of raw materials used in production. *Markups* are the baseline estimates using the De Loecker and Warzynski (2012) method (see Section 2.1.3) with a Cobb-Douglas production function.

Top 10
2331/2332 Ceramic tiles, flags, bricks and construction products, in baked clay
2370 Cutting, shaping and finishing of stone
1102/1103/1104 Wine, Cider and other non-distilled fermented beverages
1420 Manufacture of articles of fur
1031/1032/1039 Processing and preserving of fruit and vegetables
3012 Building of pleasure and sporting boats
2211 Rubber tyres and tubes; retreading and rebuilding of rubber tyres
2910 Motor vehicles
1520 Footwear
1041/1042 Vegetable and animal oils and fats
Bottom 10
2611/2612 Electronic components and boards
3099 Other transport equipment n.e.c.
2670 Optical instruments and photographic equipment
1081 Sugar
2811 Engines and turbines, except aircraft, vehicle and cycle engines
1610 Sawmilling and planing of wood
3211/3212 Striking of coins, jewellery and related articles
2620 Computers and peripheral equipment
2823 Office machinery and equipment (except computers and peripheral equipment)
1200 Tobacco Products

Table A.2: Spain's export industries according to Balassa's RCA index (equation (1)).

B Derivations and Proofs

B.1 Proof of Lemma 1: Growth rate

Consider country H throughout. Using definition (7) and $\beta = \frac{1}{2}$, we have:

$$g^H \equiv \frac{\dot{\boldsymbol{Y}}_t^H}{\boldsymbol{Y}_t^H} = \frac{1}{2}\left(\frac{\dot{\boldsymbol{Y}}_{At}^H}{\boldsymbol{Y}_{At}^H} + \frac{\dot{\boldsymbol{Y}}_{Bt}^H}{\boldsymbol{Y}_{Bt}^H}\right) \tag{B.1}$$

By definitions (9)-(10) with $\alpha_{js} = 1$, note that:

$$\begin{aligned}
\ln \boldsymbol{Y}_{st}^H &= \int_0^1 \ln\left(Y_{jst}^H\right)\mathrm{d}j = \left(\frac{\eta}{\eta-1}\right)\int_0^1 \ln\left(\sum_{c=H,C,F} \omega_c^H \left(y_{jsc,t}^H\right)^{\frac{\eta-1}{\eta}}\right)\mathrm{d}j \\
&= \left(\frac{\eta}{\eta-1}\right)\int_0^1 \left[\ln\left(\left(y_{jsH,t}^H\right)^{\frac{\eta-1}{\eta}}\right) + \ln\left(\sum_{c=H,C,F}\omega_c^H\left(\frac{y_{jsc,t}^H}{y_{jsH,t}^H}\right)^{\frac{\eta-1}{\eta}}\right)\right]\mathrm{d}j \\
&= \int_0^1 \ln(y_{jsH,t}^H)\mathrm{d}j + \underbrace{\left(\frac{\eta}{\eta-1}\right)\int_0^1 \ln\left(\sum_{c=H,C,F}\omega_c^H\left(\frac{y_{jsc,t}^H}{y_{jsH,t}^H}\right)^{\frac{\eta-1}{\eta}}\right)\mathrm{d}j}_{\equiv \Psi_{st}^H} \\
&= \int_0^1 \ln(y_{jsH,t}^H)\mathrm{d}j + \Psi_{st}^H
\end{aligned} \tag{B.2}$$

Using the optimality conditions from the firms' pricing problems, and the inverse demand function, we know that:

$$y_{jsc,t}^H = \left(\Phi_{jsc,t}^H\right)^\eta \left(Y_{jst}^H\right)^{1-\eta} \tag{B.3}$$

$$\text{where } \Phi_{jsc,t}^H = \begin{cases} \frac{1}{2}\left(\frac{\eta-1}{\eta}\right)\frac{\boldsymbol{Y}_t}{w_{st}^H}\omega_H^H(1-\sigma_{jsHt}^H)q_{jsH,t}^H & \text{for } c = H \text{ (domestic leader)} \\ \frac{1}{2}\frac{\boldsymbol{Y}_t}{w_{st}^H}\omega_C^H q_{jsC,t}^H & \text{for } c = C \text{ (domestic fringe)} \\ \frac{1}{2}\left(\frac{\eta-1}{\eta}\right)\frac{\boldsymbol{Y}_t}{w_{st}^F}\omega_F^H(1-\sigma_{jsFt}^H)\frac{q_{jsF,t}^F}{\tau} & \text{for } c = F \text{ (foreign leader)} \end{cases}$$

Consequently, taking output ratios:

$$\frac{y_{jsC,t}^H}{y_{jsH,t}^H} = \left(\frac{\omega_C^H}{\omega_H^H}\right)^\eta \left(\frac{\eta}{\eta-1}\right)^\eta \left(\frac{1}{1-\sigma_{jsH,t}^H}\right)^\eta (1+\lambda)^{-\eta n_C} \tag{B.4a}$$

$$\frac{y_{jsF,t}^H}{y_{jsH,t}^H} = \left(\frac{\omega_F^H}{\omega_H^H}\right)^\eta \left(\frac{w_{st}^H}{w_{st}^F}\right)^\eta \left(\frac{1-\sigma_{jsF,t}^H}{1-\sigma_{jsH,t}^H}\right)^\eta \left(\frac{q_{jsF,t}^F}{\tau q_{jsH,t}^H}\right)^\eta \tag{B.4b}$$

Therefore, along the BGP, all the terms inside Ψ_{st}^H are either constant or depend directly on the step size distribution, $\{\varphi_{s,n}\}$, so we can write Ψ_{st}^H along the BGP as:

$$\Psi_s^H = \left(\frac{\eta}{\eta-1}\right)\sum_{n=-\underline{n}}^{\overline{n}} \ln\left(\sum_{c=H,C,F}\omega_c^H\left(\frac{y_{sc,n}^H}{y_{sH,n}^H}\right)^{\frac{\eta-1}{\eta}}\right)$$

As the φ distribution is time-invariant, then Ψ_s^H is constant in time. Moreover, using the production function, the first term on the right-hand side of equation (B.2) is:

$$\int_0^1 \ln(y_{jsH,t}^H)\mathrm{d}j = \int_0^1 \ln(q_{jsH,t}^H)\mathrm{d}j + \int_0^1 \ln(\ell_{jsH,t}^H)\mathrm{d}j$$

By equation (25), the second term of the last equation is also constant along the BGP, as $\ell_{jsH,t}^H$ depends on variables that are either constant or a function of the time-invariant distribution. Putting things together, we have found that sectoral growth in country H along the BGP is:

$$\frac{\dot{\boldsymbol{Y}}_{st}^H}{\boldsymbol{Y}_{st}^H} = \int_0^1 \frac{\dot{q}_{jsH,t}^H}{q_{jsH,t}^H}\mathrm{d}j$$

To find $\frac{\dot{q}_{jsH,t}^H}{q_{jsH,t}^H}$, consider a small time step of size $\Delta > 0$. Then, we have:

$$\begin{aligned} q_{jsH,t+\Delta}^H = & \underbrace{\Big[(z_{jst}^H + x_{st}^H)\Delta + o(\Delta)\Big](1+\lambda)q_{jsH,t}^H}_{\text{One innovation}} \\ & + \underbrace{\Big[1-(z_{jst}^H + x_{st}^H)\Delta - o(\Delta)\Big]q_{jsH,t}^H}_{\text{No innovations}} + \underbrace{o(\Delta)}_{\substack{\text{Two or more}\\ \text{innovations}}} \end{aligned}$$

Subtracting $q_{jsH,t}^H$ from both sides, dividing through by $o(\Delta)$, and letting $\Delta \to 0$, yields:

$$\frac{\dot{q}_{jsH,t}^H}{q_{jsH,t}^H} = \lambda(z_{jst}^H + x_{st}^H)$$

where we have used that $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta} = 0$. By equations (B.1), we then get that country-level economic growth is:

$$g^H = \frac{\lambda}{2}\boldsymbol{I}^H, \quad \text{where } \boldsymbol{I}^H = \sum_{s=A,B}\boldsymbol{I}_s^H, \quad \text{and} \quad \boldsymbol{I}_s^H \equiv x_s^H + \sum_{n=-\overline{n}}^{\overline{n}}\varphi_{s,n}z_{s,n}^H$$

Here, $\boldsymbol{I}_s^H$ is the innovation rate of sector s in country H, and $\boldsymbol{I}^H$ is the aggregate innovation rate in the country. However, by symmetry (equation (32)), we get that $\boldsymbol{I}^H = \boldsymbol{I}^F = \boldsymbol{I}$. Therefore, the growth rate is the same in both countries. ■

B.2 Exogenous markups

In this section, we assume that firms do not behave strategically, so that their markups do not reflect that they possess some market power. The static problem is now::

$$\pi_{jsH} \equiv \max_{\{y^H_{jsH}, y^F_{jsH}\}} \left\{ \left(p^H_H(y^H_{jsH}) - \frac{w^H_s}{q^H_{jsH}} \right) y^H_{jsH} + \left(p^F_H(y^F_{jsH}) - \tau \frac{w^H_s}{q^H_{jsH}} \right) y^F_{jsH} - \kappa \boldsymbol{Y} \mathbb{1}[y^F_{jsH} > 0] \right\}$$

where $p^k_H(y) = \frac{1}{2}\alpha_{js}\omega^k_H \left(Y^k_{js}\right)^{-\frac{\eta-1}{\eta}} y^{-\frac{1}{\eta}} \boldsymbol{Y}$, for $k = H, F$. Now, Y^k_{js} is taken as given by the firm, who does not internalize the effect of its own output on the total output of the product group. The optimality conditions lead to prices for domestic sales and exports (if any) of:

$$p^H_{jsH} = \left(\frac{\eta}{\eta - 1}\right) \frac{w^H_s}{q^H_{jsH}} \quad \text{and} \quad p^F_{jsH} = \left(\frac{\eta}{\eta - 1}\right) \frac{\tau w^H_s}{q^H_{jsH}}$$

That is, the price is a constant markup over the marginal cost, as in the standard monopolistically competitive environment. The static profit made in marker k can now be written:

$$\pi^k_{jsH} = \sigma^k_{jsH} \frac{\alpha_{js}}{2\eta} \boldsymbol{Y}$$

Therefore, the firm decides to export if, and only if:

$$\sigma^F_{jsH} \geq \sigma_{js}, \qquad \text{where } \sigma^*_{js} \equiv \frac{2\eta\kappa}{\alpha_{js}}$$

One can easily check that the export threshold with exogenous markups is always higher than with endogenous markups (equation (24)).

C Numerical Appendix

This appendix explains the necessary steps to numerically solve the model. First, we describe how to find the static equilibrium allocation for given wages. Then, we explain how to solve for the dynamic R&D problems and find wages, given the static equilibrium outcomes.

C.1 Static Solution

To solve for the static block of the equilibrium, first we express the optimality conditions in terms of relative outputs. Since we solve for the BGP equilibrium (under the assumption that $\alpha_{js} = 1$), let us replace j subscripts by n subscripts. Equation (B.3) gives us the optimal levels of output for each producer c selling in sector s and country k. Consider, for instance, country H. Then, take output ratios of the domestic leader relative to imports and to sales from the competitive fringe. First, if the foreign firm exports ($y^H_{sF,n} > 0$), then:

$$\left(\frac{y^H_{sF,n}}{y^H_{sH,n}}\right)^{-\frac{1}{\eta}} = (1+\lambda)^n \cdot \tau \cdot \left(\frac{\omega^H_{sH}}{\omega^H_{sF}}\right) \cdot \left(\frac{w^F_s}{w^H_s}\right) \cdot \left(\frac{1 - \sigma^H_{sH,n}}{1 - \sigma^H_{sF,n}}\right) \tag{C.1}$$

where we have used that $q_{sH}/q_{sF} = (1+\lambda)^n$, when the countries are $n \in \mathcal{N}$ steps away from each other. This equation states that differences in domestically-consumed output between domestic and foreign producers are due to (i) technological differences, (ii) trade costs, (iii) tastes, (iv) differences in the relative wage, and (v) differences in market shares in the domestic economy between the different producers.

Similarly, between domestic leaders and the competitive fringe, we have:

$$\left(\frac{y^H_{sC,n}}{y^H_{sH,n}}\right)^{-\frac{1}{\eta}} = (1+\lambda)^{n_C} \cdot \left(\frac{\omega^H_{sH}}{\omega^H_{sC}}\right) \cdot \left(\frac{\eta-1}{\eta}\right) \cdot (1-\sigma^H_{sH,n}) \tag{C.2}$$

where we have used that $q_{sH}/q_{sC} = (1+\lambda)^{n_C}$, as the leader is $n_C \in \mathbb{N}$ steps ahead of the fringe. Thus, differences in the leader's output relative to the competitive fringe arise from (i) technological differences, which are here given exogenously, (ii) tastes, and (iii) differences in market power between the leader and the fringe.

Note that market shares are themselves functions of relative outputs, as:

$$\sigma^k_{sc,n} = \omega^k_{sc} \left(\frac{y^k_{sc,n}}{Y^k_{sc}}\right)^{\frac{\eta-1}{\eta}}$$

Therefore, for a given relative wage $\frac{w^H_s}{w^F_s}$, equations (C.1)-(C.2) are a system of two equations and two unknowns, $\left(\frac{y^H_{sF,n}}{y^H_{sH,n}}, \frac{y^H_{sC,n}}{y^H_{sH,n}}\right)$, but not functions of output or quality *in levels*. To solve for the static equilibrium under a guess for $\frac{w^H_s}{w^F_s}$, we then follow a simple algorithm:

Step 1. For each $n \in \mathcal{N}$, solve for $\left(\frac{y^H_{sF,n}}{y^H_{sH,n}}, \frac{y^H_{sC,n}}{y^H_{sH,n}}\right)$ under the assumption that the foreign firm exports its product (i.e. $y^H_{sF,n} > 0$) using the system of equations (C.1)-(C.2). Compute the resulting market shares, $(\sigma^H_{sH,n}, \sigma^H_{sC,n}, \sigma^H_{sF,n})$, using definition (21).

Step 2. Compute the market shares in the other sector, using the fact that $\frac{w^F_{s'}}{w^H_{s'}} = \frac{w^H_s}{w^F_s}$ for $s' \neq s$, by symmetry.

Step 3. For each s and n, check whether $\sigma^H_{sF,n} \geq \sigma^*_s$, where σ^*_s is defined in equation (24). If not, set $y^H_{sF,n} = \sigma^H_{sF,n} = 0$, and recompute $(\sigma^H_{sH,n}, \sigma^H_{sC,n})$ using (C.2).

Step 4. Compute static profits and markups in the domestic and foreign market, for all sectors and step sizes.

As a result of this procedure we can also get equilibrium outcomes at F, since we can exploit symmetry once again and say that, for $s' \neq s$, we have $\sigma^k_{sc,n} = \sigma^c_{s'k,(-n)}$, $\forall (c,k,n)$.

C.2 Dynamic Solution

The static block of the equilibrium gives us static profits and market shares $\{(\pi^k_{sH,n}, \sigma^k_{sH,n})\}$ in sector s, for all step sizes n and destination markets $k \in \{H, F\}$, under a guess for $\frac{w^H_s}{w^F_s}$. Next, we solve for the value function, the R&D problem, the entry problem, and the invariant step-size distribution, and update our guess of wages using the labor market clearing condition.

The steps of our algorithm are the following:

Step 1. Choose guesses $\widetilde{w}_A^{(0)} \equiv \frac{w_A^{H(0)}}{w_A^{F(0)}}$, and set $\widetilde{w}_B^{(0)} = \frac{1}{\widetilde{w}_A^{(0)}}$, by symmetry.

Step 2. For any given $i \in \mathbb{N}$ and guess $\widetilde{w}_s^{(i)}$, solve for the static equilibrium (see Section C.1) and obtain the collection $\{(\pi_{sH,n}^{(i)}, \sigma_{sH,n}^{(i)})\}$, $\forall(n,s)$.

Step 3. Solve the entry problem using a bisection method and a VFI algorithm:

Step 3.1 Pick vectors $\underline{\vec{x}}_s^{(i)} \equiv \left[\underline{x}_s^{H(i)}, \underline{x}_s^{F(i)}\right]$ and $\overline{\vec{x}}_s^{(i)} \gg \underline{\vec{x}}_s^{(i)}$ and set the entry rates to:

$$\vec{x}_s^{(i)} = \frac{1}{2}\left(\underline{\vec{x}}_s^{(i)} + \overline{\vec{x}}_s^{(i)}\right)$$

Step 3.2 For all s, use VFI to find the fixed point $v_{s,n}^{(i)}$ of:

$$\begin{aligned}(\rho + x_s^{H(i)})v_{s,n}^{(i)} =& \pi_{sH,n}^{H(i)} + \left(\pi_{sH,n}^{F(i)} - \kappa\right)^+ + z_{s,n}^{H(i)}\left(v_{s,n+1}^{(i)} - v_{s,n}^{(i)}\right)\\ &+ (x_s^{F(i)} + z_{s,(-n)}^{F(i)})\left(v_{s,n-1}^{(i)} - v_{s,n}^{(i)}\right) - r\left(z_{s,n}^{H(i)}\right)\end{aligned}$$

where $z_{s,n}^{H(i)} = (r')^{-1}\left(v_{s,n+1}^{(i)} - v_{s,n}^{(i)}\right)$, and $z_{s,(-n)}^{F(i)} = z_{s',n}^{H(i)}$ for $s' \neq s$.

Step 3.3 Compute the invariant step-size distribution, $\{\varphi_{s,n}^{(i)}\}_{n=-\overline{n}}^{\overline{n}}$, for all s, using equations (36a)-(36b)-(36c) with $\dot{\varphi}_{s,n} = 0$.

Step 3.4 Check the free-entry condition. That is, for each s, compute the object:

$$\Delta_s^{(i)} \equiv \nu - \sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n}^{(i)} v_{s,n+1}^{(i)}$$

Stop if $\left|\Delta_s^{(i)}\right| < \varepsilon_x$, $\forall s$, with some small $\varepsilon_x > 0$. Otherwise, set:

$$\vec{x}_s^{(i+1)} = \frac{1}{2}\left(\underline{\vec{x}}_s^{(i+1)} + \overline{\vec{x}}_s^{(i+1)}\right)$$

where:

(a) If $\Delta_s^{(i)} > \varepsilon_x$, then $\underline{\vec{x}}_s^{(i+1)} = \underline{\vec{x}}_s^{(i)}$ and $\overline{\vec{x}}_s^{(i+1)} = \vec{x}_s^{(i)}$.

(b) If $\Delta_s^{(i)} < -\varepsilon_x$, then $\underline{\vec{x}}_s^{(i+1)} = \vec{x}_s^{(i)}$ and $\overline{\vec{x}}_s^{(i+1)} = \overline{\vec{x}}_s^{(i)}$.

and go back to Step 3.2. Iterate on 3.2–3.4 until convergence.

Step 4. Compute the relative wage $\widetilde{w}_s^{(i+1)}$ using equation (31), that is:

$$\widetilde{w}_s^{(i+1)} = \left(\frac{\boldsymbol{L}_s^F}{\boldsymbol{L}_s^H}\right)\frac{\sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,n}^{(i)}\left\{\sigma_{sC,n}^{H(i)} + \left(\frac{\eta-1}{\eta}\right)\left[\sigma_{sH,n}^{H(i)}\left(1-\sigma_{sH,n}^{H(i)}\right) + \frac{1}{\tau}\sigma_{sH,n}^{F(i)}\left(1-\sigma_{sH,n}^{F(i)}\right)\right]\right\}}{\sum_{n=-\overline{n}}^{\overline{n}} \varphi_{s,(-n)}^{(i)}\left\{\sigma_{sC,(-n)}^{F(i)} + \left(\frac{\eta-1}{\eta}\right)\left[\sigma_{sF,(-n)}^{F(i)}\left(1-\sigma_{sF,(-n)}^{F(i)}\right) + \frac{1}{\tau}\sigma_{sF,(-n)}^{H(i)}\left(1-\sigma_{sF,(-n)}^{H(i)}\right)\right]\right\}}$$

Stop if $\left|\widetilde{w}_s^{(i+1)} - \widetilde{w}_s^{(i)}\right| < \varepsilon_w$, for some small tolerance $\varepsilon_w > 0$. Otherwise, go back to Step 2. with $[i] \leftarrow [i+1]$, using a convex combination between the old and new relative-wage guesses.