

Trade Flows and Fiscal Multipliers*

Matteo Cacciatore[†]
HEC Montréal, Bank of Canada, and NBER

Nora Traum[‡]
HEC Montréal

April 23, 2019

Abstract

We present novel insights on the role of international trade following unanticipated government spending and income tax changes in a flexible exchange rate environment. In a simple two-country, two-good model, we show analytically that fiscal multipliers can be larger in economies more open to trade, even when fiscal expansions imply a trade deficit. Cross-country comovement can be positive or negative. Three factors determine how trade linkages affect fiscal multipliers: the relative import share of public and private goods, how the government finances its budget, and the currency invoicing of exports. A Bayesian prior-predictive analysis shows a quantitative international business-cycle model that includes a rich fiscal specification and microfounded trade structure bears the same agnostic predictions. We estimate the model on Canadian and U.S. data and find that trade linkages increase Canadian government spending multipliers but lower income tax multipliers. Cross-country comovement is positive across U.S. fiscal instruments.

JEL Codes: E62, F41, C11

Keywords: Trade integration, government spending, income tax, Bayesian estimation

*For helpful comments, we thank our discussant Linda Tesar, as well as Giancarlo Corsetti, Michael Devereux, Fabio Ghironi, Eric Leeper, Campbell Leith, Evi Pappa, Paolo Pesenti, Morten Ravn, and seminar and conference participants at the Chicago Federal Reserve, Kansas City Federal Reserve, King's College University, McGill University, New York Federal Reserve, Richmond Federal Reserve, St. Louis Federal Reserve, University College London, University of Glasgow, University of Texas at Austin, the 2018 NBER Summer Institute, the 2017 CFE Conference, the the 2018 CEA Conference, 2017 EEA-ESEM Conference, and the 2018 LACEA-LAMES Conference.

[†]HEC Montréal, Institute of Applied Economics, 3000, chemin de la Côte-Sainte-Catherine, Montréal (Québec). E-mail: matteo.cacciatore@hec.ca. URL: <http://www.hec.ca/en/profs/matteo.cacciatore.html>.

[‡]HEC Montréal, Institute of Applied Economics, 3000, chemin de la Côte-Sainte-Catherine, Montréal (Québec). E-mail: nora.traum@hec.ca. URL: <https://sites.google.com/view/noratraum>.

1 Introduction

Since the onset of the Great Recession, debates over a need for globally-coordinated fiscal stimulus feature prominently in policy debates (e.g., the 2009 G-20 London Summit and G-20 Brisbane meeting in November 2014). Discussions stem from varying perceptions on the sign of fiscal spillovers and the domestic effectiveness of a fiscal stimulus in open economies. In the policy narrative, incentives to coordinate fiscal policy interventions depend on trade balance dynamics (e.g., [Frankel, 2016](#)), implying the issue ultimately centers around a key question: How do trade linkages affect the transmission of fiscal policy? Conventional wisdom dictates real exchange rate appreciation and/or rising aggregate income lowers net exports, mitigating the domestic effectiveness of fiscal policy by shifting stimulus abroad to trading partners.¹ In this conventional view, net export crowding out is the dominant, negative force through which international trade affects fiscal multipliers and incentives to coordinate fiscal interventions across countries.²

This paper shows this view offers only a partial characterization of the effects of trade linkages on the transmission of fiscal policy. We show that following unanticipated government spending and income tax changes, multipliers can be larger in economies more open to trade, even when fiscal expansions imply a trade deficit, and cross-country spillovers can be positive or negative. Domestic multipliers can be larger since private consumption and investment can increase (i.e., be crowded-in) relative to less open economies. Moreover, the total trade-to-GDP ratio does not intrinsically determine these results. Holding the trade share and trade elasticity constant, we show that countries can have higher or lower fiscal multipliers with stronger trade linkages depending on 1) the private-sector import intensity relative to the public-sector, 2) how the government finances fiscal expansions, and 3) the invoicing of import and export prices.

We first demonstrate these results analytically in a simple two-good, two-country model. Then, using a Bayesian prior-predictive analysis, we show a quantitative international business-cycle model featuring a rich fiscal specification and microfounded trade structure bears the same ambiguous predictions. We estimate two versions of the model on Canadian and U.S. data—one abstracting from and one including endogenous tradability and firm heterogeneity. We find support in both estimated models for larger domestic multipliers in economies more open to trade, yet the effects of public spending increases and tax cuts are asymmetric. In particular, Canadian government spending

¹See [Frankel and Mussa \(1981\)](#) and [Chinn \(2013\)](#) for a more formal discussion.

²In this paper, we define a fiscal multiplier as measuring the change in GDP relative to the change in a fiscal instrument, e.g., public spending or tax revenue.

multipliers are higher than in a counterfactually closed economy, while income tax multipliers are lower. Both fiscal expansions induce positive cross-country comovement, albeit public expenditures induce stronger yet shorter-lived spillovers.

We illustrate the core intuition about the role of trade linkages in a simple open-economy variant of Woodford (2011) with endogenous physical capital. Analytical solutions show the effects of trade linkages on fiscal multipliers depend on the relative price of domestic to imported goods, which we define as the domestic terms of consumption. This measure coincides with the terms of trade with complete exchange rate pass-through, whereas it features a markup-adjustment otherwise.³ When fiscal policy induces an appreciation of the domestic terms of consumption—e.g., following an increase in government spending that raises world demand for domestic goods—trade linkages increase domestic multipliers provided that the positive wealth effect stemming from the favorable relative price movement more than offsets its negative substitution effect.⁴ Alternatively, when fiscal policy induces a deterioration of the domestic terms of consumption—e.g., following a decrease in the income tax rate that raises the relative supply of the domestic good—the positive substitution effect must more than offset the negative wealth effect. Conditional on a given trade elasticity, the composition of government spending, the financing of the government budget, and the currency invoicing of trade shape terms-of-consumption dynamics and, in turn, whether multipliers are larger in the open economy.

To build intuition, consider an increase in government spending under flexible prices. Provided that higher public expenditure raises the world demand for domestic goods, domestic prices increase relative to the rest of the world. The resulting domestic terms-of-consumption appreciation (which coincides with an appreciation of the terms of trade with flexible prices) can lead to higher fiscal multipliers in more open economies depending on the relative import shares of the private and public sectors. Other things equal, a higher private import share implies that domestic households benefit more from the positive wealth effect of a terms-of-consumption appreciation, boosting private demand. In contrast, a higher public import share implies the increase in public demand falls more on imported goods, all else constant, raising the trade deficit. When the relative share

³With incomplete exchange rate pass-through, the domestic terms of consumption equals the terms of trade multiplied by the ratio of domestic to export mark-ups. Intuitively, when export prices are sticky in a foreign currency, lack of markup synchronization affects domestic wealth even if the terms of trade were constant, since a unit of export revenue does not yield one unit of domestic consumption once expressed in the same consumption units.

⁴The positive wealth effect occurs since domestic households give up fewer imports to consume one unit of the domestic good when the domestic terms of consumption increase, while the negative substitution effect occurs as households switch to cheaper imports.

of private-sector imports is sufficiently high (and imports' demand is not too elastic), the trade deficit following the increase in public spending is more than offset by the crowding-in of private consumption and investment relative to a closed economy. As a result, larger domestic multipliers can coexist with a trade deficit and positive cross-country comovement.

Distortionary financing can increase the likelihood that trade linkages enhance fiscal multipliers. When public expenditures are financed with higher income taxes, the appreciation of the domestic terms of consumption implied by higher taxes can partly offset the decline in the domestic labor and capital supply, raising multipliers relative to a closed economy. When an income tax cut is financed with lower public spending, multipliers can be higher since part of the reduction in public demand falls on trading partners. Nevertheless, consistent with the literature, distortionary financing worsens the domestic effectiveness of a fiscal stimulus relative to lump-sum financing, for a given trade openness.

Finally, the currency denomination of export prices also affects the dynamics of domestic terms of consumption, and in turn, fiscal multipliers. For instance, in the limiting case of fully rigid prices, the domestic terms of consumption is tied to the response of the real exchange rate under local currency pricing (LCP, i.e., exports prices are sticky in the destination-market currency). In contrast, in the empirically relevant scenario of dollar currency pricing (DCP, i.e., exports prices sticky everywhere in U.S. dollars), what matters is the relative price of domestic to imported goods, despite the terms of trade being constant. In turn, when a fiscal expansion increases the real price of Home goods relative to Foreign goods—e.g., with an increase in government spending—multipliers are more likely to be larger under DCP relative to LCP, for a given response of the real exchange rate.

Having demonstrated the importance of the fiscal environment and export prices' currency invoicing analytically, we then assess and quantify how trade linkages shape fiscal multipliers in a state-of-the-art international business-cycle model that includes additional, competing forces for the fiscal transmission—wage-setting frictions, intertemporal trade in assets, and complementarity between private and public consumption.⁵ In addition, we consider a distinct version that also features endogenous goods' tradability and firm heterogeneity, capturing key micro features of international trade, as highlighted by [Melitz \(2003\)](#), [Ghironi and Melitz \(2004\)](#), and the subsequent literature. When taking the model to the data, we focus on a benchmark small open economy,

⁵The literature has shown these features are key determinants of fiscal multipliers in closed economy models. See for instance [Leeper et al. \(2017\)](#), [Uhlig \(2010\)](#), and [Woodford \(2011\)](#).

Canada. Since 80% of Canadian trade occurs with the U.S., the latter provides a realistic characterization of the rest of the world from the perspective of Canada. Accordingly, we consider a two-country model in which one country (Canada) is of measure zero relative to the rest of the world (U.S.). We assume U.S. dollar export invoicing in both countries ([Goldberg and Tille, 2008](#) and [Gopinath, 2015](#)).

We employ a Bayesian prior-predictive analysis to first uncover the full range of fiscal outcomes implied by the model structure before confronting the data (see [Geweke, 2010](#)). This exercise shows that the model does not restrict fiscal outcomes along any dimension a priori. In particular, the model is agnostic about the role of trade linkages for fiscal multipliers, as well as the sign and size of the responses of the terms of consumption, real exchange rate, and international macroeconomic spillovers following discretionary fiscal interventions. To discern which predictions are favored by the data, we estimate the model with Bayesian inference, notably including data on fiscal measures and bilateral trade flows.

We first estimate a version of the model abstracting from endogenous tradability and firm heterogeneity. We then estimate the model incorporating endogenous entry of heterogeneous producers in the domestic and export markets. We augment our measurement equation with two additional annual series: the number of varieties exported from the U.S. to Canada and imported to the U.S. from Canada and employ mixed-frequency estimation. Across models, posterior estimates imply government spending multipliers are larger with stronger trade linkages. Central for this result is the crowding-in of investment and consumption relative to a closed economy. Endogenous fluctuations along the extensive margin of trade quantitatively affect fiscal multipliers, as they dampen movements in the domestic terms of consumption in the short-run. Both U.S. public spending increases and tax cuts induce positive cross-country comovement for Canada. We show that direct trade linkages are central for the positive spillover from a government spending increase. For a U.S. income tax cut, the appreciation of the Canadian terms of consumption is key for the positive co-movement, as it dominates in equilibrium motives to relocate investment to the U.S., which could induce negative co-movement.

Related Literature This paper is related to several parts of the literature. First, it is related to two strands of the literature that carry out likelihood-based analyses: on fiscal policy in closed economies (e.g., [Drautzburg and Uhlig, 2015](#) and [Leeper et al., 2017](#)) and on the international transmission of business-cycles (e.g., [Adolfson et al., 2005](#), [Justiniano and Preston, 2010b](#), and [Lubik](#)

and Schorfheide, 2005). While the previous literature has emphasized the difficulty in reproducing cross-country correlations (e.g., Justiniano and Preston, 2010a), our model generates positive cross-country correlations for several international variables, stemming from U.S. shocks accounting for a nontrivial portion of the variability of Canadian series. Moreover, to our knowledge, we are the first to estimate a model incorporating a micro-founded trade structure with endogenous entry of heterogeneous producers in domestic and export markets. Furthermore, we exploit data on the number of varieties bilaterally traded together with measures of trade flows and fiscal aggregates.

Seminal theoretical contributions focused on whether fiscal policies are beggar or prosper-ty-neighbor (e.g. Betts and Devereux, 2001, Corsetti and Pesenti, 2001, Mendoza and Tesar, 1998, and Obstfeld and Rogoff, 1995). In addition, a few early quantitative studies examine government spending and tax changes in flexible exchange-rate models (e.g. Baxter, 1995 and Erceg et al., 2005). In contrast to these studies, we focus on the role of trade linkages (and their determinants) for the domestic and international transmission of fiscal policy.

Our analysis does not consider the role of fixed exchange rates nor cross-country strategic interactions in the design of fiscal policy. Without addressing the specific role of trade linkages, several recent works address how monetary and exchange rate policies affect the fiscal transmission (e.g., Beetsma and Jensen, 2005, Born et al., 2013, Corsetti et al., 2013, Erceg and Linde, 2012, Gali and Monacelli, 2008, and Muller, 2008). Farhi and Werning (2016) also compare analytically fiscal multipliers in open and closed economies but focus on a currency union in a liquidity trap. Mendoza et al. (2014) study the macroeconomic effects of tax adjustments in response to large public debt shocks in highly integrated economies. They also consider Nash tax competition, studying features of the cooperative and non-cooperative equilibria and their relationships to the closed economy.

A few recent empirical studies examine cross-country spillovers from expansionary fiscal policies (Auerbach and Gorodnichenko, 2013 and Faccini et al., 2016). Our estimates are consistent with this literature, which generally finds positive international spillovers. The role of trade openness for the domestic transmission of government spending has been explored in the context of structural panel vector autoregressions. For instance, Ilzetzi et al. (2013) find that public spending multipliers are smaller on average in economies with trade-to-GDP ratios exceeding 60%.⁶ Our theoretical analysis shows that behind these average effects, there can be cross-sectional heterogeneity depending on the composition of private and public imports, the financing of the government’s budget, the currency

⁶While Canada and the U.S. are included in their analysis, both countries fall in the “closed” classification (less than 60% trade-to-GDP ratios).

invoicing of trade, and the trade elasticity. In light of our results, an important consideration for future empirical work is to include time-varying controls along these dimensions. Moreover, our results suggest that the effects of trade openness can vary across fiscal instruments.

The empirical literature also has examined the real exchange rate response to government spending shocks. The VAR evidence tends to find an increase in government spending leads to a real depreciation of the exchange rate, although for Canada there is evidence supporting a real appreciation.⁷ Our analysis shows that, a priori, following a government spending increase, gains from trade linkages (i.e., more trade openness increases domestic effectiveness) can coincide with a real exchange rate appreciation or depreciation. Our posterior estimates imply the real exchange rate appreciates in Canada following an increase in government spending. Moreover, the response of the domestic terms of consumption—and not the real exchange rate per se—is the key international price dictating the size of fiscal multipliers.⁸

Finally, this paper also is related to the literature on the “twin deficits,” which studies the causal role of primary fiscal deficits on current account deficits. Model-based evaluations often assign a small role for fiscal policy, see [Chen et al. \(2009\)](#), [Erceg et al. \(2005\)](#), and [Ferrero \(2010\)](#). Empirical evidence is mixed but typically attributes a more significant role for fiscal shocks, e.g. [Chinn \(2017\)](#), [Kim and Roubini \(2008\)](#), [Monacelli and Perotti \(2010\)](#), and [Normandin \(1999\)](#). Our focus is not on providing a structural interpretation of current account dynamics. We show that fiscal induced trade deficits can coincide with higher fiscal stimulus.

Outline The rest of the paper is organized as follows. Section 2 presents the simple analytical model and a full characterization of the equilibrium dynamics following a public spending increase or a tax cut. Section 3 describes the quantitative business-cycle model, while section 4 presents the prior-predictive analysis, estimation details, and posterior analysis. Section 5 describes the results from an extension with an endogenously determined trade structure, while section 6 concludes.

⁷For studies that find a real depreciation, see [Bouakez et al. \(2014\)](#), [Bouakez and Eyquem, 2015](#), [Corsetti et al. \(2012\)](#), [Enders et al. \(2011\)](#), [Monacelli and Perotti \(2010\)](#), [Muller \(2008\)](#), and [Ravn et al. \(2012\)](#). Canada is the only country for which [Monacelli and Perotti \(2010\)](#) do not find depreciation of the exchange rate, while [Kim \(2010\)](#) finds the real exchange rate appreciates significantly following an increase in Canadian government spending.

⁸While there is no direct evidence on the response of the terms of consumption, [Monacelli and Perotti \(2008\)](#) and [Muller \(2008\)](#) find the terms of trade appreciates with an increase in public spending.

2 Building Intuition: A Simple Model

To understand the core implications of trade linkages for fiscal multipliers, we study a benchmark two-country model. The closed-economy version adds physical capital into Woodford's (2011) model, whose dynamics have been extensively studied in the literature.

We start with the stark assumptions of flexible prices, financial autarky, and full home bias in government spending. In this context, the terms of trade summarize the qualitative effects of trade linkages on fiscal multipliers. We then discuss how the composition of government spending, distortionary financing, net-exports dynamics, and price stickiness affect the core intuition in this simple environment. The key message of this section is that trade linkages can increase fiscal multipliers either by crowding-in private demand relative to a closed economy (following an increase in government spending) or by inducing expenditure switching towards domestic goods (following a tax cut).

Flexible Prices and Financial Autarky

Consider two symmetric countries, Home and Foreign. A representative agent at Home maximizes $E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\omega}}{1+\omega} \right] \right\}$, where C_t is consumption and L_t denotes hours worked. International trade is frictionless and trade is balanced in each period (i.e., financial autarky).

We use the subscript D to denote quantities and prices of a country's own tradable good consumed domestically, and the subscript X to denote quantities and prices of exports. We focus on a standard calibration considered in the literature with logarithmic utility in consumption and a unitary Frisch elasticity ($\gamma = \omega = 1$).

Each country produces a distinct, homogeneous good using a constant-returns to scale technology $Y_t = K_t^\alpha L_t^{1-\alpha}$, where K_t denotes physical capital. Households supply competitively both input factors. Let $r_{K,t}$ denote the rental rate of capital and w_t the real wage. Optimal demand for factors of production requires $r_{K,t} = \alpha \rho_{D,t} Y_t / K_t$ and $w_t = (1 - \alpha) \rho_{D,t} Y_t / L_t$, where $\rho_{D,t}$ is the real price (in units of Home consumption) of Home output.

Households consume the final good C_t and accumulate physical capital. We make two assumptions that simplify the derivations below: (i) full capital depreciation and (ii) no time-to-build delays. This implies the model is static: in each period capital is equal to investment, i.e. $I_t = K_t$.

Consumption C_t aggregates Home and Foreign tradable consumption sub-baskets in Armington

form with an exogenous elasticity of substitution $\phi > 0$:

$$C_t = \left[(1 - \alpha_X)^{\frac{1}{\phi}} (C_{D,t})^{\frac{\phi-1}{\phi}} + \alpha_X^{\frac{1}{\phi}} (C_{X,t}^*)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad 0 \leq \alpha_X \leq 1, \quad (1)$$

where $1 - \alpha_X$ is the weight attached to the country's own good. Preferences are biased in favor of domestic goods whenever $\alpha_X < 1/2$. The price index that corresponds to the basket C_t is given by $P_t = \left[(1 - \alpha_X) (P_{D,t})^{1-\phi} + \alpha_X (P_{X,t}^*)^{1-\phi} \right]^{1/(1-\phi)}$. Foreign households derive utility from an analogous consumption bundle of domestic and imported consumption goods, $C_{D,t}^*$ and $C_{X,t}$. Let $\rho_{D,t} \equiv P_{D,t}/P_t$ and $\rho_{X,t}^* \equiv P_{X,t}^*/P_t$ respectively denote the real prices of the domestic and imported goods expressed in Home consumption units. Home private demand for domestic and imported goods is, respectively, $C_{D,t} = (1 - \alpha_X) \rho_{D,t}^{-\phi} C_t$ and $C_{X,t} = \alpha_X \rho_{X,t}^{-\phi} C_t^*$. The law of one price holds, implying: $\rho_{X,t} = \rho_{D,t}/Q_t$, where Q_t is the real exchange rate (in units of Home consumption per unit of Foreign consumption).

Investment in physical capital, $I_{K,t}$, requires purchasing a bundle that has the same composition of final consumption C_t .⁹ The demand for domestic and imported capital goods are given by $I_{D,t} = (1 - \alpha_X) \rho_{D,t}^{-\phi} I_{K,t}$ and $I_{X,t} = \alpha_X \rho_{X,t}^{-\phi} I_{K,t}^*$.

The household's budget constraint is $C_t + I_{K,t} = (1 - \tau_t) (w_t L_t + r_{K,t} K_t) + T_t$, where τ_t is an exogenous tax on income and T_t is a lump-sum transfer from the government. The optimal labor supply implies $L_t^\omega / C_t = (1 - \tau_t) w_t$, while optimality in investment requires $r_{K,t} = 1/(1 - \tau_t)$.

The government uses income taxes and transfers to finance an exogenous level of public expenditures G_t . The domestic government also has preference to consume both domestic and imported goods:

$$G_t = \left[(1 - \alpha_X^g)^{\frac{1}{\phi}} (G_{D,t})^{\frac{\phi-1}{\phi}} + (\alpha_X^g)^{\frac{1}{\phi}} (G_{X,t}^*)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad 0 \leq \alpha_X^g \leq 1, \quad (2)$$

where we allow the share of Foreign goods in government consumption, α_X^g , to vary from private demand. The corresponding price index is $P_{G,t} = \left[(1 - \alpha_X^g) (P_{D,t})^{1-\phi} + \alpha_X^g (P_{X,t}^*)^{1-\phi} \right]^{1/(1-\phi)}$. Government demand for domestic and imported goods is

$$G_{D,t} = (1 - \alpha_X^g) \left(\frac{\rho_{D,t}}{\rho_{G,t}} \right)^{-\phi} G_t, \quad G_{X,t} = \alpha_X^g \left(\frac{\rho_{X,t}}{\rho_{G,t}^*} \right)^{-\phi} G_t^*,$$

where $\rho_{G,t} \equiv P_{G,t}/P_t$ ($\rho_{G,t}^*$) denotes the Home (Foreign) price of government consumption relative

⁹In the quantitative model of the next section, we allow for the relative import shares of investment and consumption to differ, i.e., $\alpha_X \neq \alpha_X^I$, consistent with the data.

to Home (Foreign) consumption. The Home government budget constraint is given by $\rho_{G,t}G_t + T_t = \tau_t(w_tL_t + r_{K,t}K_t)$.

Let TB_t denote the trade balance: $TB_t \equiv Q_t\rho_{X,t}(C_{X,t} + I_{X,t} + G_{X,t}) - \rho_{X,t}^*(C_{X,t}^* + I_{X,t}^* + G_{X,t}^*)$. Financial autarky implies balanced trade in every period: $TB_t = 0$. The resource constraint is $L_t = C_{D,t} + C_{X,t} + I_{D,t} + I_{X,t} + G_{D,t} + G_{X,t}$.

For future reference, we define the terms of trade as the Home price of exports relative to the price of imports (both expressed in Home currency): $TOT_t \equiv Q_t\rho_{X,t}/\rho_{X,t}^*$. With complete exchange-rate pass through, the terms of trade also is equal to the relative price of domestic and imported goods, i.e., $TOT_t = \rho_{D,t}/(\rho_{D,t}^*Q_t)$. Moreover, in the symmetric steady state (where $Q = 1$), the total trade-to-GDP ratio is given by

$$\text{trade/GDP} \equiv \frac{2 \left[\rho_X (C_X + I_{KX}) + \frac{\rho_X}{\rho_G} G_X \right]}{Y} = 2 \left[(1 - s_G) \alpha_X + \alpha_X^g s_G \right],$$

where $s_G \equiv G/Y$ is the steady-state ratio of government spending to GDP. Consequently, trade openness increases monotonically in either α_X or α_X^g .

We log-linearize the equilibrium conditions and use the method of undetermined coefficients to solve the model.¹⁰ Hats denote variables in percentage deviations from steady state. We consider solutions following exogenous changes to either domestic government spending or taxes, $\hat{G}_t \neq 0$ or $\hat{\tau}_t \neq 0$, while holding foreign government spending and taxes constant, $\hat{G}_t^*, \hat{\tau}_t^* = 0$ for all t .

We focus on the effects of fiscal expansions on real GDP, defined in units of Home output $Y_t \equiv K_t^\alpha L_t^{1-\alpha} = \tilde{Y}_t/\rho_{D,t}$, where $\tilde{Y}_t \equiv (C_t + I_{K,t} + \rho_{G,t}G_t)$ is real GDP in units of Home consumption. While the latter measure is identical to output in a closed economy (since $\rho_{D,t} = 1$), in the open economy the two can differ. As discussed in section 3, our quantitative results are even stronger when considering CPI-deflated responses, arguably the metric closer to welfare.¹¹

¹⁰Appendix A lists the complete set of log-linearized equilibrium conditions.

¹¹To simplify the analytical exposition, we focus directly in this section on the GDP response. Nevertheless, the effects of trade openness are similar when considering GDP multipliers. The only caveat concerns tax multipliers. While output always improves with a tax cut, tax revenue can either rise or fall, depending on the position of the income tax rate on the Laffer curve.

Closed Economy

Assuming that both private- and public-consumption import shares are zero, i.e., $\alpha_X = \alpha_X^g = 0$, yields the following GDP response:

$$\hat{Y}_t^{closed} = \frac{s_G}{1 + s_C - s_I} \hat{G}_t + \frac{(s_I - \alpha)(1 - s_G + \alpha(1 + s_C - s_I))}{s_I(1 + s_C - s_I)(1 - \alpha)} \hat{\tau}_t,$$

where $s_C \equiv 1 - \alpha(1 - \tau) - s_G$ is the steady-state ratio of consumption to GDP and $s_I \equiv \alpha(1 - \tau)$ is the steady-state ratio of investment to GDP.

Since $0 < s_I < \alpha < 1$, GDP increases following either an increase in government spending or a reduction in income taxes. Moreover, for an increase in public spending, GDP increases less than one-for-one, reflecting the crowding-out of private consumption. Notice that in this simple, static model, investment increases following an increase in public spending. This reflects the fact that the rental rate of capital is constant absent changes in the tax rate, increasing the firm's demand for both production inputs.¹² Absent capital ($\alpha = 0$), labor and GDP still rise with an increase in public spending, while consumption still declines (see [Woodford, 2011](#)). Following an income tax cut, GDP is higher, reflecting a higher after tax-return to labor and a lower cost of capital for firms.

Full Home-Bias in Government Spending

To study the effects of trade linkages, we first consider the case of full home-bias in government spending, $\alpha_X^g = 0$, a simplifying assumption often adopted in the literature.¹³ We also assume a unitary trade elasticity ϕ , another benchmark value in the literature. Under these knife-edge assumptions, terms-of-trade fluctuations ensure full international risk sharing, i.e., the real exchange rate equals the ratio of the marginal utilities of consumption across the two countries ([Cole and Obstfeld, 1991](#)).

Proposition 1 *Assume that $\phi = 1$ and $\alpha_X^g = 0$. Following a fiscal expansion:*

¹²Notice that $\hat{I}_t = \hat{L}_t = -\hat{C}_t$, as implied by the firm's first-order conditions, the labor supply equation, and the fact that $\hat{\rho}_{D,t} = 0$ in the closed economy.

¹³See for instance [Corsetti and Pesenti \(2001\)](#), [Erceg et al. \(2005\)](#), [Corsetti et al. \(2012\)](#), [Born et al. \(2013\)](#), [Cook and Devereux \(2013\)](#), and [Farhi and Werning \(2016\)](#).

1. The response of the terms of trade is given by:

$$\widehat{TOT}_t = \frac{s_G(1 - s_G - 2s_I)(1 - \alpha)}{(1 - s_G)(1 + s_C - s_I)(1 - \alpha) + 4\alpha\alpha_X(1 - s_G - s_I)} \hat{G}_t - \frac{(s_I - \alpha)(1 - s_G + \alpha(1 - s_G - 2s_I))}{s_I[(1 - s_G)(1 + s_C - s_I)(1 - \alpha) + 4\alpha\alpha_X(1 - s_G - s_I)]} \hat{\tau}_t.$$

It follows that:

- (a) For an increase in government spending, $\widehat{TOT}_t > 0$ provided that $s_I < s_C$.
- (b) For a decrease in income taxes, $\widehat{TOT}_t < 0$.

2. Provided that $\widehat{TOT}_t > 0$, it follows that

- (a) The responses of domestic output, consumption, and investment are increasing in openness (measured by α_X since $\alpha_X^g = 0$); the response of domestic hours is decreasing in openness.
- (b) The responses of foreign output, consumption, and investment are decreasing in openness (measured by α_X since $\alpha_X^g = 0$); the response of foreign hours is increasing in openness.
- (c) The real exchange rate appreciates only when $\alpha_X < 0.5$.

Proposition 1 (proved in Appendix A) states the response of the terms of trade, TOT_t , determines whether or not stronger trade linkages imply a larger fiscal stimulus. In particular, when the terms of trade improve, GDP in the open economy, $Y_t \equiv K_t^\alpha L_t^{1-\alpha}$, is higher than GDP in a counterfactually closed economy, $Y_t^{closed} \equiv (K_t^{closed})^\alpha (L_t^{closed})^{1-\alpha}$. As shown in Proposition 1, the improvement of TOT_t leads to higher investment (and thus capital) relative to a closed economy, boosting output. Under the knife-edge assumptions of full home-bias in government spending, $\alpha_X^g = 0$ and a unitary trade elasticity ϕ , hours per worker, L_t , decline. However, imposing $\alpha_X^g > 0$ is sufficient to overturn this result.

What is the intuition for the results in Proposition 1? Higher government expenditure raises the relative price of the Home good $\rho_{D,t}$, which, ceteris paribus, crowds out private consumption (lowering demand for domestic goods). At the same time, as in the closed economy, investment rises, increasing private demand. Since a share of both investment and consumption goods are imported, import demand also changes. When $s_C > s_I$, imports fall relative to exports, and the terms of

trade appreciate to maintain balanced trade. This favorable relative price movement induces a positive wealth effect for Home as more foreign goods are traded per domestic good. In turn, this allows domestic households to purchase more Home goods, increasing private consumption and investment relative to the closed economy. The same positive wealth effect reduces hours supply relative to the closed economy, leading to a lower (albeit positive) response of L_t relative to L_t^{closed} . In equilibrium, the crowding-in of consumption and investment dominates, and output increases. By contrast, Foreign GDP and its components decline.

While the terms of trade response is a sufficient condition for the effects of trade linkages on GDP and its components, the real exchange rate response is not. For instance, when the terms of trade improve, the real exchange rate can still appreciate or depreciate, depending on the degree of Home bias in private demand.

Under our knife-edge parameterization of Proposition 1, an income tax cut unambiguously depreciates the terms of trade. Following the tax cut, domestic households are willing to work more, leading wages and the domestic price to decrease. Moreover, the rental rate of capital falls—since $\hat{r}_k = \tau/(1 - \tau)\hat{\tau}_t$ —increasing demand for capital. Both effects lead to a deterioration in the terms of trade, which results in a negative (positive) wealth effect domestically (abroad). In turn, this lowers the domestic effectiveness of the stimulus relative to the closed economy. At the same time, Foreign GDP and its components increase.

Generalizing the Composition of Government Spending and the Trade Elasticity

In practice, governments import goods. For instance, import shares for Australia, the U.K., and Canada are approximately 11% (see [Corsetti and Muller, 2006](#)). We now show that the effectiveness of public spending depends on the *relative* size of public-private import shares and not on trade openness per se. Put differently, the relative composition of public and private imports and exports, $\nu \equiv \alpha_X^g/\alpha_X$ (and not α_X^g or α_X per se) is key for the effects of government spending.

We continue to assume a unitary trade elasticity.¹⁴ To obtain analytical results, we also assume $s_C > s_I$ (that is, the GDP consumption share is larger than the investment share), the restriction in Proposition 1 that ensures GDP increases with stronger trade linkages following an increase in public spending, together with $\tau = 0$ and $\nu \leq 1$. We then show numerically the results generalize when these restrictions are relaxed.

¹⁴Notice that when $\alpha_X^g > 0$, a unitary trade elasticity no longer ensures perfect international risk sharing.

Proposition 2 *Assume $\phi = 1$, $\tau = 0$, $\nu \leq 1$, and $s_C > s_I$. Then for an increase in Home government spending, there exists a ν^* , given by*

$$\nu^* \equiv \frac{1 - s_G - 2\alpha}{2 - s_G - 2\alpha},$$

such that for any $\nu < \nu^$:*

$$(i) \widehat{TOT}_t > 0; \quad (ii) \hat{Y}_t > \hat{Y}_t^{closed}, \quad \text{and} \quad (iii) \hat{Y}_t^* < 0,$$

provided that $s_G < 1 - 2s_I$ and $0 < s_I < 1/2$.

Proposition 2 shows that countries with a non-zero public import share can benefit from trade linkages provided that the private-sector consumes a sufficiently high share of total imports. As in Proposition 1, when the terms of trade improve, a sufficiently high private-sector import share (α_X) reduces consumption crowding out and increases investment, leading to higher GDP relative to the closed economy. In contrast, for a given private import share, Home GDP is decreasing in the public import share (α_X^g). As the government consumes more Foreign goods, Foreign prices increase, dampening the Home household's demand for imports. For a sufficiently high value of ν , the reduced private-import demand implies a reduction in Home total imports, resulting in a terms-of-trade depreciation to maintain balanced trade. The negative wealth effect from this depreciation generates more crowding out of domestic private demand relative to the closed economy.

Figure 1a demonstrates the robustness of proposition 2 for various combinations of ϕ and ν . The figure depicts the Home and Foreign GDP responses as well as the terms of trade response following a 1% increase in government spending for various ν and ϕ values. The figure holds the overall level of openness (i.e. the trade share) constant at 0.5 and considers a grid for $\alpha_X^g \in [0, 0.35]$; α_X adjusts to maintain a constant trade share.¹⁵ The plane in each panel plots the response in the closed economy. The relative size of public-private import shares continues to determine outcomes once relaxing the parametric restrictions on ϕ . Consistent with proposition 2, Home GDP responses decline in ν . Home GDP and the terms-of-trade responses also are decreasing in ϕ , since a higher trade elasticity dampens the terms of trade appreciation.

¹⁵In addition, we set $\alpha = 0.33$, $s_G = 0.2$, and $\tau = 0.25$.

Distortionary Fiscal Financing

So far our analysis has assumed lump-sum transfers finance the government budget. However, in practice, governments do not finance fiscal expansions with lump-sum transfers. In a closed economy, distortionary financing can lower the domestic effectiveness of expansionary fiscal policies (e.g. [Leeper et al., 2010](#) and [Woodford, 2011](#)). Here we demonstrate that distortionary financing can increase the likelihood that trade linkages enhance fiscal multipliers.

To assess the role of distortionary financing in an open economy, we resort to numerical simulations.¹⁶ Figure 1b repeats the numerical analysis of figure 1a following a 1% government spending increase assuming income taxes finance the government budget (i.e., lump-sum transfers remain constant). The plane in each panel plots the response in the closed economy. Absent trade linkages, Home GDP falls when government spending increases, due to the negative effect of higher taxes on capital and labor supply. Trade linkages reduce this effect for all combinations of the trade elasticity ϕ and the relative share of public and private imports ν —the GDP response is above the closed economy in all cases. *Ceteris paribus*, the increase in the income tax rate raises the terms of trade, and this additional wealth effect partly offsets the negative response of capital and labor supply. As explained before, the terms-of-trade appreciation is stronger for lower ν and ϕ combinations.

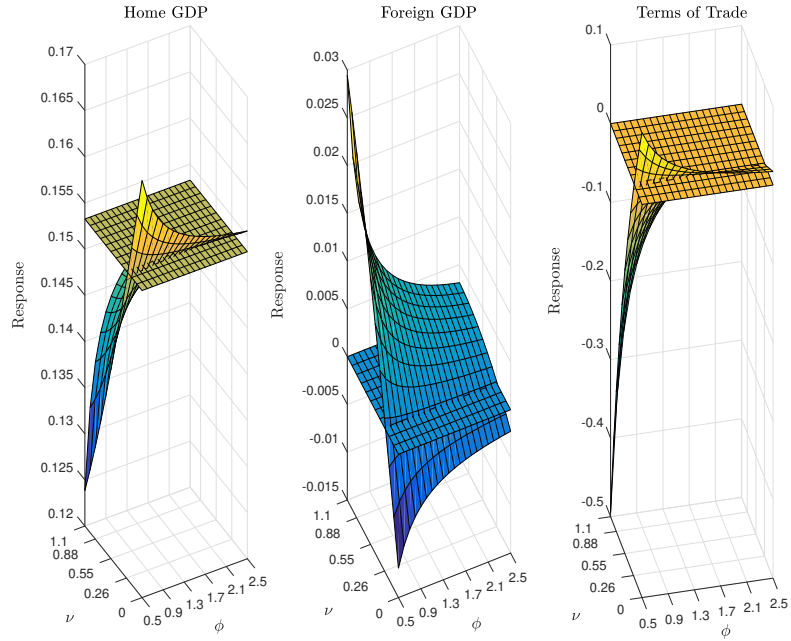
When an income tax cut is financed by lower government expenditures, again the relative share of public and private imports, ν , determines the size of GDP responses. Less home bias in public goods (i.e., a higher ν) raises domestic responses since the reduction in public demand affects more heavily the Foreign economy. Provided that the decrease in government spending falls sufficiently on Foreign imports, the terms of trade can improve in equilibrium, boosting Home GDP relative to the closed economy (see appendix A).

International Trade in Financial Assets

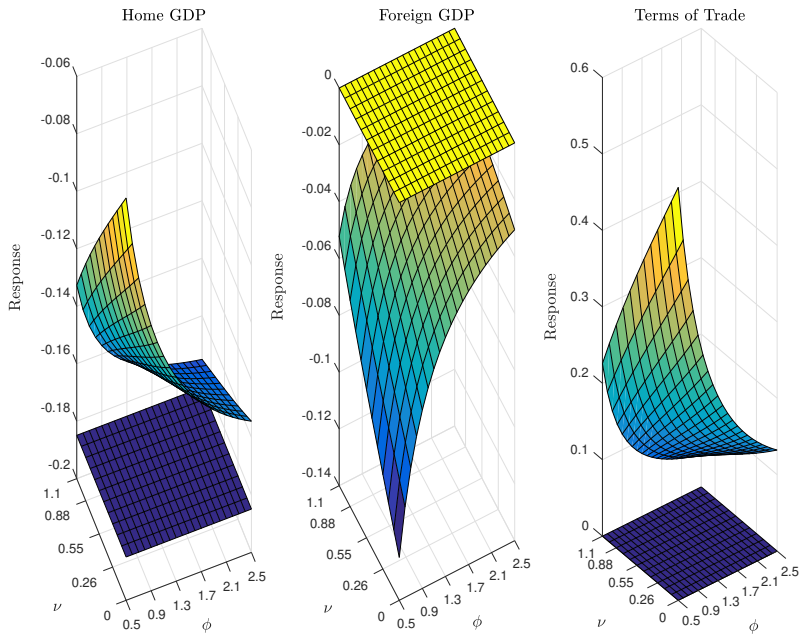
Thus far, balanced trade ruled out net-export dynamics in response to fiscal expansions. We now relax the assumption of financial autarky and (for simplicity) assume complete international financial markets. Full risk-sharing implies that cross-country consumption levels are tied to the real exchange rate, $Q_t = C_t/C_t^*$, replacing the balanced-trade condition from the benchmark model.

To obtain analytical results, we impose $\alpha_X^g = 0$. In addition, we set $\tau = 0$ following an increase

¹⁶Analytical solutions are only possible under the knife-edge assumptions of Proposition 1.



(a) Government Spending Increase Financed by Lump-Sum Transfers



(b) Government Spending Increase Financed by Income Taxes

Figure 1. Impact GDP (Home and Foreign) and the terms of trade responses following a 1% increase in government spending. The plane in each panel denotes the response in the closed economy. In all cases, $\alpha = 0.33$, $s_G = 0.2$, $\tau = 0.25$, and the trade share = 0.5.

in public spending and $s_G = 0$ following an income tax cut. In appendix B, we show numerically the results generalize when relaxing these restrictions.

Proposition 3 *Following a fiscal expansion, trade linkages can increase domestic GDP even when net exports decline.*

1. Assuming $\tau = \alpha_X^g = 0$, following a 1% increase in Home government spending,

(a) $\hat{Y}_t > \hat{Y}_t^{closed}$ and $\hat{Y}_t^* < 0$ provided that $s_G < 1 - 2s_I$, $0 < s_I < 1/2$ and

$$\alpha_X < \frac{1 - s_G - \alpha}{(1 - s_G)(1 - \alpha) + 2\alpha^2},$$

$$\phi < \frac{(1 - s_G)[1 - \alpha_X(1 - \alpha)] - \alpha(1 + 2\alpha\alpha_X)}{(1 - s_G)(1 - \alpha)(1 - \alpha_X)}.$$

(b) $\hat{T}B_t \leq 0$ when $\hat{Y}_t > \hat{Y}_t^{closed}$.

(c) $\widehat{TOT}_t > 0$ as long as $s_I < 1/2$.

2. Assuming $s_G = 0$, following a 1% decrease in Home income taxes,

(a) $\hat{Y}_t > \hat{Y}_t^{closed}$ and $\hat{Y}_t^* < 0$ provided that $\alpha < 1/2$ and

$$\phi > \frac{(1 - s_I)(1 + 4s_I\alpha\alpha_X) - (1 - 2s_I)[\alpha_X(1 - \alpha^2) - \alpha(1 - s_I)]}{(1 - \alpha_X)(1 - \alpha)(1 + \alpha - 2s_I)}.$$

(b) $\hat{T}B_t \leq 0$ when $\hat{Y}_t > \hat{Y}_t^{closed}$.

(c) $\widehat{TOT}_t < 0$.

Proposition 3 shows that even with net export crowding out, trade linkages can increase domestic GDP responses. Following an increase in government spending, GDP is higher than in the closed economy when private home bias and the trade elasticity are below threshold values. In contrast, for a decrease in income taxes, when the trade elasticity is higher than a threshold value, GDP is higher than in the closed economy. Different threshold values determine whether the trade balance is positive or negative (see Appendix B for the analytical expressions).

In addition, Proposition 3 demonstrates trade-balance dynamics imply that an appreciation of the terms of trade may not coincide with gains from trade. In particular, when fiscal policy induces

an appreciation of the domestic terms of trade—e.g., following an increase in public spending—trade linkages increase domestic GDP, provided that the positive wealth effect stemming from the favorable relative price movement more than offsets its negative substitution effect. Alternatively, when fiscal policy induces a deterioration of the domestic terms of trade—e.g., following a decrease in the income tax rate—the positive substitution effect must more than offset the negative wealth effect.

Appendix B shows the qualitative responses of Home and Foreign GDP remain consistent with Proposition 3 once relaxing the parametric restrictions on τ , s_G , and α_X^g . Moreover, consistent with Proposition 2, GDP responses depend on the relative share of public and private imports, ν .

The Role of Price Stickiness

Finally, we discuss the role of price-setting frictions, which can affect international relative price dynamics. In the presence of nominal rigidities, there are three scenarios depending on the invoicing of export prices: (i) producer currency pricing (PCP), in which both domestic and export prices are sticky in the domestic currency; (ii) local currency pricing (LCP), in which export prices are sticky in the foreign currency; and (iii) dollar currency pricing (DCP), in which export prices are sticky everywhere in the Foreign currency (we treat Foreign as the U.S. economy).

In the limiting case of complete price stickiness, it is possible to derive analytical solutions. We present the derivations for this knife-edge scenario under financial autarky in Appendix B. Here we highlight two main insights. First, incomplete exchange-rate pass-through (e.g., LCP and DCP) creates a wedge between terms-of-trade fluctuations and cross-country wealth effects. For instance, with fixed prices, the terms of trade are constant under DCP ($TOT_t = \bar{P}_X/\bar{P}_D^*$). Nevertheless, trade linkages affect fiscal multipliers. Independent of export invoicing, a markup-adjusted terms of trade,

$$TOC_t \equiv TOT_t \frac{\mu_{D,t}/\mu_{D,t}^*}{\mu_{X,t}/\mu_{X,t}^*} = \frac{\rho_{D,t}}{Q_t \rho_{D,t}^*},$$

summarizes the domestic effects of trade linkages following a fiscal expansion (see Appendix B for the proof). We refer to this as the domestic terms of consumption. Intuitively, when export prices are sticky in a foreign currency, lack of markup synchronization affects domestic wealth even if the terms of trade are constant, since a unit of export revenue does not yield one unit of domestic consumption once expressed in the same consumption units. Mirroring the intuition of the flexible price model (where $TOC_t = TOT_t$), an increase in TOC_t implies that Home agents can consume

more Foreign goods per unit of the Home good. Other things equal, this positive wealth effect reduces private demand crowding out relative to a closed economy.

Second, the currency invoicing of imports and exports affects the behavior of the domestic terms of consumption. For instance, with complete price stickiness, $TOC_t = 1/Q_t$ under LCP, whereas $TOC_t = \rho_{D,t}/\rho_{X,t}^*$ under DCP. Thus, while under LCP the effects of a fiscal expansion are simply tied to the response of the real exchange rate, under DCP what matters is the relative price of domestic to imported goods (since in this case the aggregate price indices P_t and P_t^* are not constant). In turn, when a fiscal expansion increases the real price of Home goods relative to Foreign goods—e.g., with an increase in government spending—the gains from trade are more likely to materialize under DCP for a given response of the real exchange rate. We explore the quantitative significance of price-setting frictions and the invoicing of export prices when studying the quantitative model in section 4.

3 A Quantitative Model

We turn to a quantitative exploration using a state-of-the-art, international business-cycle model that introduces additional competing forces for the fiscal transmission absent from the simple model. First, we specify a rich fiscal environment, including government debt, public spending, and consumption and income taxes. Second, we include features proven to be important for the transmission of fiscal shocks in a closed economy—non-separable utility between public and private consumption and wage-setting frictions. Third, we introduce intertemporal investment dynamics and incomplete international asset markets, thus allowing fiscal shocks to affect the current account position of trading partners without implying complete international risk sharing. Finally, consistent with recent empirical evidence (e.g., [Goldberg and Tille, 2008](#) and [Gopinath, 2015](#)), we assume that both exports and imports are sticky in U.S. dollars (the dominant currency). In section 5, we introduce an endogenously determined trade structure along the lines of [Melitz \(2003\)](#), addressing the role of producer heterogeneity and extensive margin dynamics for the international transmission of fiscal policy.

In our quantitative exercises, we focus on Canada and the U.S. This country pair is particularly suited for the analysis, since 80% of Canadian trade occurs with the U.S., implying that the latter provides a realistic characterization of the rest of the world from the perspective of Canada. Since Canada is small relative to the U.S., we follow the standard approach in the literature and consider a two-country model in which one country (the small open economy, also referred to as Home) is

of measure zero relative to the rest of the world (Foreign henceforth). As a consequence, the policy decisions and macroeconomic dynamics of the small open economy have no impact on Foreign. The small open economy's terms of trade fluctuate endogenously in response to aggregate shocks due to the presence of firm monopoly power in both countries. We present in detail below the problems facing households and firms in the small open economy. Variables without a time subscript denote non-stochastic values along the balanced growth path.

Households

The representative household, indexed by $j \in [0, 1]$, maximizes the expected intertemporal utility function

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \bar{\beta}_t \left[\log \left(\tilde{C}_{jt} - h_C \tilde{C}_{t-1} \right) - \bar{h}_t \frac{L_{jt}^{1+\omega}}{1+\omega} \right] \right\}, \quad (3)$$

where $\beta \in (0, 1)$ is the discount factor, \tilde{C}_{jt} is a consumption basket that consists of private and public consumption as described below, and L_{jt} is the number of hours worked. To introduce wage stickiness, we assume that each household is a monopolistic supplier of its differentiated labor input L_{jt} . The household values consumption relative to a habit stock defined in terms of lagged aggregate consumption $h_C \tilde{C}_{t-1}$, where $h_C \in [0, 1)$. $\bar{\beta}_t$ denotes an exogenous shock to the discount factor, which evolves according to $\log \bar{\beta}_t = \rho_{\bar{\beta}} \log \bar{\beta}_{t-1} + \varepsilon_{\bar{\beta}t}$ with $\varepsilon_{\bar{\beta}t} \stackrel{iid}{\sim} N(0, \sigma_{\bar{\beta}}^2)$; \bar{h}_t denotes an exogenous shock to the marginal disutility of hours worked, which evolves according to $\log \bar{h}_t = \rho_{\bar{h}} \log \bar{h}_{t-1} + \varepsilon_{\bar{h}t}$ with $\varepsilon_{\bar{h}t} \stackrel{iid}{\sim} N(0, \sigma_{\bar{h}}^2)$. Consumption utility is logarithmic to ensure balanced growth in the presence of non-stationary technological progress.

To allow for an agnostic response of private consumption following a government spending shock, we follow [Fève et al. \(2013\)](#) and [Leeper et al. \(2017\)](#) and allow non-separable preferences between private and public consumption.¹⁷ Total consumption, \tilde{C}_{jt} is the sum of private C_{jt} and public G_t consumption goods, $\tilde{C}_{jt} = C_{jt} + \omega_G G_t$. Parameter ω_G governs the degree of substitutability of the consumption goods: when $\omega_G < 0$, private and public consumption are complements; when $\omega_G > 0$, the goods are substitutes.

Market consumption C_t aggregates Home and Foreign consumption sub-baskets as described by equation (1) in the previous section. We follow [Mukhin and Itskhoki \(2016\)](#) and [Pavlova and Rigobon \(2007\)](#) and allow for exogenous fluctuations in the elasticity of substitution across Home

¹⁷The literature has found evidence for both complementarity and substitutability of public and private goods. For instance, for the U.S., [Aschauer \(1985\)](#) and [Ercolani and e Azevedo \(2014\)](#) find substitutability between private and government consumption while [Bouakez and Rebei \(2007\)](#) and [Fève et al. \(2013\)](#) find the two goods are complements.

and Foreign baskets, $\bar{\phi}_t$, capturing in reduced form fluctuations in the relative price of imported goods. We assume that $\log \bar{\phi}_t = \rho_{\bar{\phi}} \log \bar{\phi}_{t-1} + \varepsilon_{\bar{\phi}t}$, where $\varepsilon_{\bar{\phi}t} \stackrel{iid}{\sim} N(0, \sigma_{\bar{\phi}}^2)$.

The sub-baskets $C_{D,t}$ and $C_{X,t}^*$ now aggregate differentiated varieties in Dixit-Stiglitz form. The basket $C_{D,t}$ aggregates domestic consumption varieties $C_{D,t}(i)$: $C_{D,t} = \left[\int_0^1 C_{D,t}(i)^{(\bar{\theta}_t-1)/\bar{\theta}_t} di \right]^{\bar{\theta}_t/(\bar{\theta}_t-1)}$, where $\bar{\theta}_t > 1$ is the exogenous elasticity of substitution across domestic goods, which following the literature, we refer to as a price markup shock. We assume that $\log \bar{\theta}_t = \rho_{\bar{\theta}} \log \bar{\theta}_{t-1} + \varepsilon_{\bar{\theta}t}$, where $\varepsilon_{\bar{\theta}t} \stackrel{iid}{\sim} N(0, \sigma_{\bar{\theta}}^2)$. A similar basket describes consumption of Foreign goods: $C_{X,t}^* = \left[\int_0^1 C_{X,t}^*(i)^{(\bar{\theta}_t^*-1)/\bar{\theta}_t^*} di \right]^{\bar{\theta}_t^*/(\bar{\theta}_t^*-1)}$. The corresponding price indexes are $P_{D,t} = \left[\int_0^1 P_{D,t}(i)^{1-\bar{\theta}_t} di \right]^{1/(1-\bar{\theta}_t)}$ and $P_{X,t}^{T*} = \left[\int_0^1 P_{X,t}^*(i)^{1-\bar{\theta}_t^*} di \right]^{1/(1-\bar{\theta}_t^*)}$, both expressed in Home currency.

International asset markets are incomplete, as non-contingent nominal bonds denominated in Foreign currency are the only internationally traded asset. The representative household also can invest in non-contingent nominal bonds denominated in Home currency, which are traded domestically. Let A_t^j and A_t^{*j} denote, respectively, nominal holdings of Home and Foreign bonds for the representative Home household j . Households also have access to one-period, riskless nominal government bonds B_t^j .¹⁸ Moreover, the household accumulates physical capital and rents it to intermediate input producers in a competitive capital market. Investment aggregates domestic and imported investment goods $I_{KD,t}$ and $I_{KX,t}$, in Armington form:

$$I_{K,t} = \left[(1 - \alpha_X^I)^{\frac{1}{\bar{\phi}_t}} (I_{KD,t})^{\frac{\bar{\phi}_t-1}{\bar{\phi}_t}} + (\alpha_X^I)^{\frac{1}{\bar{\phi}_t}} (I_{KX,t}^*)^{\frac{\bar{\phi}_t-1}{\bar{\phi}_t}} \right]^{\frac{\bar{\phi}_t}{\bar{\phi}_t-1}}, \quad 0 \leq \alpha_X^I \leq 1,$$

where $1 - \alpha_X^I$ is the weight attached to the country's own investment good. The investment sub-baskets $I_{KD,t}$ and $I_{KX,t}^*$ have the same composition as the private consumption sub-baskets $C_{D,t}$ and $C_{X,t}^*$. Thus, the price index for $I_{K,t}$ is given by $P_t^{IK} = \left[(1 - \alpha_X^I) (P_{D,t})^{1-\bar{\phi}_t} + \alpha_X^I (P_{X,t}^*)^{1-\bar{\phi}_t} \right]^{1/(1-\bar{\phi}_t)}$.

We introduce convex adjustment costs in physical investment and variable capital utilization. The utilization rate of capital is set by the household. Thus, effective capital rented to firms, K_t^j , is the product of physical capital, \tilde{K}_t^j , and the utilization rate, $u_{K,t}^j$: $K_t^j = u_{K,t}^j \tilde{K}_t^j$. Utilization incurs a cost of $\Psi(u_{K,t}^j)$ per unit of physical capital. In steady state, $u_K = 1$ and $\Psi(1) = 0$. We define the parameter $\psi \in [0, 1)$ such that $\Psi''(1)/\Psi'(1) \equiv \psi/(1-\psi)$. Physical capital, \tilde{K}_t , obeys a

¹⁸We assume all government debt is issued in domestic currency and held by domestic households. This reflects the low share of Canadian debt held by foreigners (approximately 20%) and the low share of Canadian debt issued in foreign currency (approximately 10%). See Broner et al. (2018) and Priftis and Zimic (2018) for a discussion of how foreign debt holdings affect fiscal multipliers.

standard law of motion:

$$\tilde{K}_{t+1}^j = (1 - \delta_K) \tilde{K}_t^j + \bar{P}_{K,t} \left[1 - \frac{\nu_K}{2} \left(\frac{I_{K,t}^j}{I_{K,t-1}^j} - \bar{z} \right)^2 \right] I_{K,t}^j, \quad (4)$$

where $\nu_K > 0$ is a scale parameter, and $\bar{P}_{K,t}$ is an investment specific shock with $\log \bar{P}_{K,t} = \rho_{\bar{P}_K} \log \bar{P}_{K,t-1} + \varepsilon_{\bar{P}_K,t}$ and $\varepsilon_{\bar{P}_K,t} \stackrel{iid}{\sim} N(0, \sigma_{\bar{P}_K}^2)$. This shock is a source of exogenous variation in the efficiency with which the final good can be transformed into physical capital, and thus into tomorrow's capital input.

Household's income (the sum of rental capital and labor income) is taxed at the rate τ_t^I . Moreover, the household pays consumption taxes τ_t^C . The household's period budget constraint is:

$$\begin{aligned} B_t^j + A_t^j + \varepsilon_t A_t^{*j} + P_t C_t^j (1 + \tau_t^C) + P_t I_{K,t}^j + \Psi(u_{K,t}^j) \tilde{K}_t^j \\ = \\ (1 + i_{t-1}) B_{t-1}^j + (1 + i_{t-1}) A_{t-1}^j + (1 + i_{t-1}^*) \Gamma_{t-1} A_{t-1}^{*j} \varepsilon_t + (1 - \tau_t^I) \left(w_{jt}^n L_{jt} + P_t r_{K,t} K_t^j \right) + P_t \left(T_{d,t}^j + T_{G,t}^j \right), \end{aligned} \quad (5)$$

where i_t and i_t^* are, respectively, the nominal interest rates on Home and Foreign private bond holdings between $t-1$ and t , known with certainty as of $t-1$. Following [Adolfson et al. \(2005\)](#), the term $\Gamma_t \equiv \exp \left\{ -\gamma \frac{Q_t A_t^*}{Y_t} \right\} \bar{\Lambda}_{at}$, where $\gamma > 0$, denotes the Home country's endogenous risk premium. The risk premium increases with the economy's aggregate level of debt ($-A_t^*$) as a percentage of GDP (Y_t). The term $\bar{\Lambda}_{at}$ captures exogenous fluctuations in the risk-premium. We assume that $\bar{\Lambda}_{at}$ follows a zero-mean autoregressive process: $\bar{\Lambda}_{at} = \rho_{\bar{\Lambda}_a} \bar{\Lambda}_{at-1} + \varepsilon_{\bar{\Lambda}_a,t}$, where $\varepsilon_{\bar{\Lambda}_a,t} \stackrel{iid}{\sim} N(0, \sigma_{\bar{\Lambda}_a}^2)$. Up to a first-order approximation, $\bar{\Lambda}_{at}$ is isomorphic to the financial shock studied by [Mukhin and Itshkhoki \(2016\)](#). Finally, $T_{A,t}^j$ denotes a lump-sum rebate of the cost of adjusting bond holdings, $T_{d,t}^j$ is a lump-sum rebate of profits from final producers, and $T_{G,t}^j$ is a lump-sum transfer from the government.

The household sets the nominal wage w_{jt}^n subject to a quadratic adjustment cost:

$$\frac{\nu_w}{2} \left[\frac{1}{\bar{z}} \frac{w_{jt}^n}{w_{jt-1}^n} (1 + \pi_{C,t-1})^{-\iota_w} - 1 \right]^2 w_{jt}^n L_{jt},$$

where \bar{z} is the growth rate of productivity along the balanced growth path. Households index wage changes to past CPI inflation, $\pi_{C,t} \equiv P_t/P_{t-1} - 1$.

The household maximizes its expected intertemporal utility subject to (4) and (5). We appeal to

symmetry in household preferences and omit the index j henceforth. The real wage, $w_t \equiv w_t^n/P_t$, is a time-varying markup, $\mu_{w,t}$, over the marginal rate of substitution between hours and consumption: $w_t = \mu_{w,t} \bar{\beta}_t \bar{h}_t L_t^\omega (1 - \tau_t^I) / \bar{C}_t (1 + \tau_t^C)$. See Appendix C for the definition of $\mu_{w,t}$.

Let $a_t \equiv A_t/P_t$ denote real holdings of Home bonds (in units of Home consumption) and let $a_t^* \equiv A_t^*/P_t^*$ denote real holdings of Foreign bonds (in units of Foreign consumption). The Euler equations for bond holdings are:

$$1 = (1 + i_t) E_t \left(\frac{\beta_{t,t+1}}{1 + \pi_{C,t+1}} \right), \quad 1 = (1 + i_t^*) \Gamma_t E_t \left(\frac{\beta_{t,t+1}}{1 + \pi_{C,t+1}^*} \frac{Q_{t+1}}{Q_t} \right).$$

We report the remaining, standard first-order conditions in Appendix C.

Production

In each country, there are two vertically integrated production stages. At the upstream level, perfectly competitive firms use capital and labor to produce a non-tradable intermediate input. At the downstream level, monopolistically competitive firms use the intermediate input to produce tradable final consumption goods.

Homogeneous Intermediate Input Production

There is a unit mass of perfectly competitive intermediate producers. The representative intermediate firm produces output $Y_t = K_t^\alpha (\bar{Z}_t L_t)^{1-\alpha}$, where \bar{Z}_t is exogenous productivity, K_t is physical capital, and L_t is a bundle of the labor inputs supplied by individual households. \bar{Z}_t and \bar{Z}_t^* are non-stationary and cointegrated stochastic processes.¹⁹ The growth rate of Foreign productivity $\bar{z}_t^* \equiv \bar{Z}_t^*/\bar{Z}_{t-1}^*$ follows $\log \bar{z}_t^* = \rho_z^* \log \bar{z}_{t-1}^* + (1 - \rho_z^*) \log \bar{z}^* + \varepsilon_{zt}^*$, where $\varepsilon_{zt}^* \stackrel{iid}{\sim} N(0, \sigma_{z^*}^2)$. Home productivity \bar{Z}_t features the same stochastic trend up to a stationary stochastic disturbance $\bar{\zeta}_t$: $\bar{Z}_t^*/\bar{Z}_t = \bar{\zeta}_t$, where $\log \bar{\zeta}_t = \rho_{\bar{\zeta}} \log \bar{\zeta}_{t-1} + \varepsilon_{\bar{\zeta}t}$ and $\varepsilon_{\bar{\zeta}t} \stackrel{iid}{\sim} N(0, \sigma_{\bar{\zeta}}^2)$. As a result, the growth rate of Home productivity $\bar{z}_t \equiv \bar{Z}_t/\bar{Z}_{t-1}$ evolves according to $\log \bar{z}_t = \log \bar{z}_t^* + \log \bar{\zeta}_{t-1} - \log \bar{\zeta}_t$.

The composite labor input aggregates in Dixit-Stiglitz form the differentiated labor inputs provided by domestic households: $L_t \equiv \left[\int_0^1 (L_{jt})^{(\eta-1)/\eta} dh \right]^{\eta/(\eta-1)}$ where $\eta > 0$ is the elasticity of substitution, and L_{jt} denotes the labor hired from household j .

Let φ_t be the real price (in units of final consumption) of the intermediate input. The Home

¹⁹Rabanal and Rubio-Ramirez (2015) and Rabanal et al. (2011) show this specification for TFP improves a business-cycle model's ability to match properties of real exchange rate data.

firm chooses L_t and K_t to maximize the value of per-period profit: $\varphi_t Y_t^I - (w_t^n/P_t) L_t - r_{K,t} K_t$, where $w_t^n \equiv \left[\int_0^1 (w_{j,t}^n)^{1-\eta} dh \right]^{1/(1-\eta)}$ is the nominal wage bill, and $r_{K,t}$ is the real rental rate of capital. The first-order condition for L_t equates the value of the marginal product of labor to the real wage: $(1 - \alpha) \varphi_t Y_t^I / L_t = w_t^n / P_t$. The first-order condition for capital yields $\alpha \varphi_t Y_t^I / K_t = r_{K,t}$.

Final Producers

A continuum of symmetric firms produce tradable consumption varieties indexed by $i \in (0, 1)$. Domestic demand for producer i is $Y_{D,t}^i = \left(P_{D,t}^i / P_{D,t} \right)^{-\bar{\theta}_t} Y_{D,t}$, while export demand is $Y_{X,t}^i = \left(P_{X,t}^i / P_{X,t} \right)^{-\bar{\theta}_t} Y_{X,t}$, where $P_{X,t}^i$ denotes the price in Foreign currency of the exported variety. The terms $Y_{D,t}$ and $Y_{X,t}$ denote, respectively, domestic and export aggregate demand for the Home basket.

Final producers pay a quadratic adjustment cost when changing domestic and export prices, which in Canada are both denominated in U.S. dollars.²⁰ Final producers index export price changes to past CPI inflation in U.S. dollars. With zero trend inflation, the cost of adjusting the domestic price is $(v_p/2) \left[\frac{P_{D,t}^i}{P_{D,t-1}^i} (1 + \pi_{C,t-1})^{-\iota_p} - 1 \right]^2 P_{D,t}^i Y_{D,t}^i$, where $v_p \geq 0$ determines the size of the adjustment costs. The cost (in units of Home currency) of adjusting the export price is $(v_p/2) \left[\frac{P_{X,t}^i}{P_{X,t-1}^i} \left(1 + \pi_{C,t-1}^* \right)^{-\iota_p} - 1 \right]^2 \varepsilon_t P_{X,t}^i Y_{X,t}^i$, where ε_t is the nominal exchange rate (in units of Home currency per unit of Foreign currency).

In the symmetric equilibrium, the price of Home output for domestic sales is a time-varying domestic markup $\mu_{D,t}$ over the marginal cost φ_t : $\rho_{D,t} \equiv P_{D,t} / P_t = \mu_{D,t} \varphi_t$. The export price is a time-varying export markup $\mu_{X,t}$ over the marginal cost: $\rho_{X,t} \equiv P_{X,t} / P_t^* = \mu_{X,t} (1 + \tau_t) \varphi_t / Q_t$, where Q_t denotes the real exchange rate, and $\tau_t \geq 0$ is an iceberg trade cost. The definitions of the markups are standard, which we report in Appendix C.

Monetary and Fiscal Policy

The monetary authority follows a Taylor-type rule, in which the nominal interest rate responds to its lagged value, deviations of CPI inflation, and GDP from their long-run targets. We denote a

²⁰Up to a first-order approximation and with zero trend inflation, Rotemberg and Calvo price adjustment yield identical dynamics. We opt for a quadratic adjustment cost because it simplifies the model aggregation in the presence of firm heterogeneity in section 5.

variable in percentage deviations from steady state by a hat. The interest rate obeys

$$\frac{\hat{i}_t}{1+i} = \varrho_i \frac{\hat{i}_{t-1}}{1+i} + (1 - \varrho_i) \left[\varrho_\pi \hat{\pi}_{C,t} + \varrho_Y \hat{Y}_t^{nipa} \right] + \varepsilon_{i,t}, \quad (6)$$

where $1+i$ is the steady state of the gross nominal interest rate and $\varepsilon_i \stackrel{iid}{\sim} N(0, \sigma_i^2)$. To be consistent with the data used for estimation, we follow NIPA accounting and define real GDP by evaluating expenditures at fixed (steady-state) relative prices (e.g. [Laxton and Pesenti, 2003](#)):

$$Y_t^{nipa} \equiv C_t + \rho_{I_K} I_{K,t} + \rho_G G_t + Q \rho_X Y_{X,t} - \rho_X^* Y_{X,t}^*$$

The government collects tax revenues from capital, labor, and consumption taxes, and sells the nominal bond portfolio, B_t , to finance its interest payments and expenditures, G_t and $T_{G,t}$. As in the one-period model of section 2, government consumption G_t aggregates Home and Foreign government consumption sub-baskets. The government consumption sub-baskets $G_{D,t}$ and $G_{X,t}^*$ have the same composition as the private consumption sub-baskets. The price index for the government consumption basket is therefore: $P_t^G = \left[(1 - \alpha_X^g) (P_{D,t})^{1-\bar{\phi}_t} + \alpha_X^g (P_{X,t})^{1-\bar{\phi}_t} \right]^{1/(1-\bar{\phi}_t)}$.

Fiscal choices satisfy the government's per-period budget constraint:

$$B_t + \tau_t^I (r_{K,t} P_t K_t + w_t^n L_t) + P_t \tau_t^C C_t = (1 + i_{t-1}) B_{t-1} + P_{G,t} G_t + P_t T_{G,t}.$$

Fiscal rules dictate the evolution of policy instruments, $X = \{G, \tau^C, \tau^L\}$, and include an autoregressive term to allow for serial correlation and a response to the market value of the debt-to-GDP ratio $S_t \equiv B_t / (P_t Y_t)$ —to ensure that policies stabilize debt. Specifically, $\hat{X}_t = \varrho_X \hat{X}_{t-1} - (1 - \varrho_X) \gamma_X \hat{S}_{t-1} + \varepsilon_{X,t}$, where $\varepsilon_X \stackrel{iid}{\sim} N(0, \sigma_X^2)$. Since our observables do not include transfers and transfers are Ricardian in the model, $T_{G,t}$ can be thought of as capturing all movements in government debt that are not explained by the model or the government spending and tax processes. We model $T_{G,t}$ as an AR(1) process.

Net Foreign Assets and Model Solution

We present the details of the symmetric equilibrium in Appendix C, and we limit ourselves to presenting the law of motion for net foreign assets below. Private Home bonds are in zero net supply, which implies the equilibrium condition $a_t = a_{t-1} = 0$ in all periods. Home net foreign

assets are determined by:

$$Q_t a_{*,t} = Q_t \frac{(1 + i_{t-1}^*) \Gamma_{t-1}}{1 + \pi_{C,t}^*} a_{*,t-1} + TB_t,$$

where $TB_t \equiv Q_t \rho_{X,t} Y_{X,t} - \rho_{X,t}^* Y_{X,t}^*$ is the trade balance. The change in net foreign assets is determined by the current account:

$$Q_t \left(a_{*,t} - \frac{a_{*,t-1}}{1 + \pi_{C,t}^*} \right) = CA_t \equiv Q_t r_{t-1} a_{*,t-1} + TB_t,$$

where $\pi_{C,t}^* \equiv P_t^*/P_{t-1}^* - 1$ and $r_{t-1} \equiv [(1 + i_{t-1}^*) \Gamma_{t-1} - 1] / (1 + \pi_{C,t}^*)$ denotes the real interest rate.

Eight Foreign variables directly affect macroeconomic dynamics in the small open economy: C_t^* , i_t^* , $\pi_{C,t}^*$, $I_{K,t}^*$, G_t^* , $\rho_{X,t}^*$, $Y_{X,t}^*$. Foreign consumption, C_t^* , government consumption G_t^* , the nominal interest rate, i_t^* , and inflation, π_t^{C*} , are determined by treating the rest of the world (Foreign) as a closed economy that features the same production structure, technology, and frictions that characterize the small open economy. To determine price and quantities related to Foreign exports and imports, we assume that Foreign producers solve a profit maximization problem that is equivalent to that faced by Home producers.

We rewrite the model in terms of detrended variables and compute the log-linear approximation around the non-stochastic steady state. We link the data to the model with an observation equation to form the state-space model used for estimation.

4 Assessing the Role of Trade Linkages

Model Predictions

We employ a prior-predictive analysis in the spirit of [Geweke \(2010, Chapter 3\)](#) to identify the entire range of fiscal outcomes admitted by the model before confronting it with data. This procedure allows us to show that the model does not restrict fiscal outcomes along any dimension a priori.

We propose independent prior density functions for structural parameters, which define the range and probability of values for each parameter. Table 1 lists our priors, which are similar to those employed for Bayesian estimation of structural models (e.g. [Lubik and Schorfheide, 2005](#), [Forni et al., 2009](#), [Justiniano and Preston, 2010a](#), [Leeper et al., 2010](#), and [Drautzburg and Uhlig,](#)

2015). We set the price stickiness parameter, ν , to reproduce a given frequency of price adjustment in a log-linearized Phillips curve using a Calvo parameter ξ .²¹ For comparability with the literature, we directly estimate this Calvo parameter. Likewise, we set the wage stickiness parameter, ν_w , to reproduce the slope of the log-linearized wage Phillips curve derived using a Calvo parameter ξ_w and estimate ξ_w .²² We adopt a uniform distribution for the elasticity of substitution between Home and Foreign goods (ϕ) over the interval 0.05 to 6. A uniform prior ensures the data fully inform the parameter's estimate. Likewise, we adopt a uniform prior for the elasticity of substitution between public and private goods (ω_G). For the parameter governing the endogenous risk premium (γ), we adopt an inverse gamma distribution for 10γ with a mean of 0.75 and standard deviation of 1.5. The low mean reflects the low values of this parameter usually estimated and adopted in the literature (e.g., Lane and Milesi-Ferretti, 2002), while the inverse gamma distribution allows for a long right tail encompassing higher estimates in the literature (e.g., Adolfson et al., 2007).

Our prior means for the AR(1) coefficients for government spending, income and consumption tax processes are centered at 0.8, reflecting the high persistence in fiscal measures. For all other shocks, the priors for the AR(1) coefficients are fairly diffuse with a mean of 0.5 and standard deviation of 0.2. We normalize the shocks to investment, hours, lump-sum transfers, and the elasticity of substitution between home and foreign goods to enter with a unitary coefficient in the log-linearized equations that determine investment, wages, and government debt, as well as the equation that defines exports.

In addition, we impose dogmatic priors for a few parameters. We set the discount factor β equal to 0.99, the share of capital in the Cobb-Douglas production function of the upstream intermediate sector α equal to 0.33, and the capital depreciation rate δ_K equal to 0.025. We set the elasticity of substitution of tradable varieties θ equal to 6 to generate a 20 percent markup in steady state. We set the elasticity of substitution of domestic labor varieties η to 11 to generate a 10 percent wage markup in steady state. We set the steady-state trade-to-GDP ratio to 35 percent, consistent with bilateral trade data between the U.S. and Canada. In addition, we set the import intensity of public consumption to be 1/3 that of private consumption, and the import intensity of investment to be 4/3 that of private consumption, consistent with historical measures of Canadian import shares from Dion et al. (2005) and the World Bank. Finally, we set iceberg trade costs, τ and τ^* , equal to zero, reflecting the absence of tariff barriers between Canada and the U.S.

²¹The parameter ξ is related to ν via the mapping $\nu = [(\bar{\theta} - 1) / \bar{\theta}] \xi / (1 - \xi)(1 - \xi\beta)$.

²²The parameter ξ_w is related to ν_w via the mapping $\nu_w = \xi_w(\eta_w - 1)(1 + \eta_w\omega) / [(1 - \xi_w)(1 - \xi_w\beta)]$.

Steady-state fiscal variables are calibrated to the mean values from Canadian and U.S. data for the general government over the estimation sample. See appendix E for more details on the data. Government consumption as a share of GDP is set to 0.22 in Canada and 0.18 in the U.S. The annualized market-value of government debt to GDP ratio is set to 0.73 in Canada and 0.61 in the U.S. Consumption and income tax rates are set to match average consumption and income tax rates, 0.14 and 0.25 respectively for Canada and 0.12 and 0.27 respectively for the U.S.

Table 1: Prior and Posterior Distributions for Structural Parameters.

Parameter	Prior			Posterior			
	Dist.*	Mean	Std.	Mean	Canada 90% Int	U.S. Mean	U.S. 90% Int
Preferences							
h_C , habit formation	B	0.7	0.1	0.72	[0.65, 0.79]	0.84	[0.76, 0.91]
ω , inverse Frisch	G	2	0.5	1.96	[1.28, 2.79]	1.72	[1.08, 2.52]
ω_G , substitutability of private/public cons.	U	0	1.01	0.10	[-0.14, 0.34]	-0.53	[-0.83, -0.23]
ϕ , substitutability of home/foreign	U	3.03	1.59	1.06	[0.91, 1.23]		
Frictions and Production							
$100 \log \bar{z}$, growth rate	N	0.45	0.03	0.45	[0.40, 0.50]		
10γ , endog. risk premium	IG	0.75	1.5	0.39	[0.23, 0.64]		
ν_K , investment adj. cost	N	4	1.5	4.82	[3.06, 6.78]	6.58	[5.58, 7.75]
ζ , capital utilization	B	0.5	0.2	0.12	[0.02, 0.28]	0.87	[0.75, 0.96]
ξ , Calvo price stickiness	B	0.66	0.1	0.65	[0.52, 0.75]	0.91	[0.89, 0.93]
ξ_w , Calvo wage stickiness	B	0.66	0.1	0.46	[0.35, 0.57]	0.65	[0.56, 0.72]
ι_p , price partial indexation	B	0.5	0.15	0.69	[0.47, 0.87]	0.34	[0.17, 0.52]
ι_w , wage partial indexation	B	0.5	0.15	0.47	[0.24, 0.70]	0.51	[0.28, 0.75]
Monetary Policy							
ϱ_i , resp. to lagged interest rate	B	0.75	0.1	0.82	[0.77, 0.87]	0.77	[0.69, 0.83]
ϱ_π , interest resp. to inflation	N	1.7	0.3	2.12	[1.73, 2.51]	1.87	[1.52, 2.25]
ϱ_Y , interest resp. to Y	G	0.15	0.1	0.04	[0.01, 0.09]	0.05	[0.02, 0.09]
Fiscal Policy							
γ_G , debt response for G	N	0.3	0.1	0.38	[0.25, 0.51]	0.34	[0.20, 0.46]
γ_{τ^I} , debt response for τ^I	N	0.3	0.1	0.15	[0.08, 0.23]	0.36	[0.20, 0.52]
γ_{τ^C} , debt response for τ^C	N	0.3	0.1	0.22	[0.07, 0.39]	0.32	[0.15, 0.48]
ϱ_G , lagged response for G	B	0.8	0.1	0.95	[0.93, 0.97]	0.83	[0.74, 0.92]
ϱ_{τ^I} , lagged response for τ^I	B	0.8	0.1	0.70	[0.55, 0.83]	0.86	[0.80, 0.91]
ϱ_{τ^C} , lagged response for τ^C	B	0.8	0.1	0.96	[0.92, 0.99]	0.94	[0.89, 0.98]
ϱ_{T_G} , lagged response for T_G	B	0.5	0.2	0.29	[0.13, 0.46]	0.18	[0.06, 0.33]
Shock Processes							
$\rho_{\bar{\lambda}_a}$, risk premium	B	0.5	0.2	0.95	[0.90, 0.98]		
$\rho_{\bar{\beta}}$, preference	B	0.5	0.2	0.77	[0.65, 0.87]	0.73	[0.57, 0.84]
ρ_ϕ , subst home/foreign	B	0.5	0.2	0.98	[0.96, 0.99]	0.92	[0.87, 0.96]
$\rho_{\bar{c}}$, tfp growth	B	0.5	0.2	0.98	[0.96, 0.99]	0.35	[0.22, 0.47]
$\rho_{\bar{P}_K}$, investment	B	0.5	0.2	0.34	[0.16, 0.52]	0.73	[0.62, 0.83]
$\rho_{\bar{\theta}}$, price mark-up	B	0.3	0.1	0.59	[0.39, 0.78]	0.31	[0.16, 0.49]
$\rho_{\bar{h}}$, hours supply	B	0.3	0.1	0.24	[0.13, 0.36]	0.22	[0.12, 0.33]
$100\sigma_{\bar{\lambda}_a}$, risk premium	IG	1	1	0.29	[0.23, 0.36]		
$100\sigma_{\bar{\beta}}$, preference	IG	1	1	2.08	[1.58, 2.73]	2.90	[1.91, 4.43]
$100\sigma_\phi$, subst home/foreign	IG	1	1	3.42	[2.89, 4.04]	2.83	[2.45, 3.27]
$100\sigma_{\bar{c}}$, tfp growth	IG	0.5	1	1.26	[1.08, 1.46]	0.71	[0.61, 0.82]
$100\sigma_{\bar{P}_K}$, investment	IG	1	1	0.96	[0.74, 1.20]	0.37	[0.29, 0.47]
$100\sigma_{\bar{\theta}}$, price mark-up	IG	1	1	3.36	[1.51, 6.31]	9.17	[5.58, 13.70]
$100\sigma_{\bar{h}}$, hours supply	IG	1	1	0.32	[0.28, 0.39]	0.42	[0.35, 0.50]
$100\sigma_i$, monetary policy	IG	1	1	0.24	[0.20, 0.28]	0.19	[0.16, 0.22]
$100\sigma_G$, gov spending	IG	1	1	1.17	[1.00, 1.37]	0.74	[0.64, 0.87]
$100\sigma_{\tau^I}$, income tax	IG	1	1	2.58	[2.22, 3.01]	2.00	[1.72, 2.32]
$100\sigma_{\tau^C}$, income tax	IG	1	1	1.04	[0.90, 1.22]	0.61	[0.52, 0.71]
$100\sigma_{T_G}$, transfer	IG	1	1	2.29	[1.97, 2.66]	1.71	[1.47, 1.99]

*Distributions: N: Normal; G: Gamma; B: Beta; U: Uniform;

IG: Inverse Gamma with distribution as in Zellner (1971): $p(y|\gamma, \alpha) = 2/(\Gamma(\alpha)\gamma^\alpha y^{2\alpha+1}) \exp^{-1/\gamma y^2}$.

We take 10,000 draws from our priors and calculate model-implied impulse response functions

and present-value multipliers, defined as:

$$\text{Present Value Multiplier}(k) = \frac{E_t \sum_{j=0}^k \left[\prod_{i=0}^k (1+r)^{-1} \right] \Delta P_{t+j} X_{t+j}}{E_t \sum_{j=0}^k \left[\prod_{i=0}^k (1+r)^{-1} \right] \Delta P_{t+j} F_{t+j}} \quad (7)$$

where k is the time horizon, r is the steady-state real interest rate, X denotes the variable of interest (i.e., GDP, consumption or investment), and F denotes government spending or income tax revenue, where all values are expressed in NIPA units. Additionally, we report GDP multipliers expressed in CPI units, $\tilde{Y}_t = C_t + \rho_{I_{K,t}} I_{K,t} + \rho_{G,t} G_t + TB_t$.²³ These present-value multipliers measure the present value change over the k -horizon in the variable of interest when government spending or income tax revenue increases in present value by one unit over the same horizon.²⁴

Figure 2 displays 90-percent probability bands for fiscal multipliers and various international variables following Canadian and U.S. government spending increases and tax decreases. For Canadian fiscal expansions, Canadian multipliers can be larger or smaller than a counterfactually closed economy (formally defined as $\alpha_X = \alpha_X^g = \alpha_X^{Ik} = 0$), while spillovers from U.S. fiscal shocks to Canada also can be positive or negative.

The model also is agnostic about the sign and size of key international variables including the real exchange rate, the trade balance, and the domestic terms of consumption following any fiscal shock. In particular, a priori, the real exchange rate can depreciate following an increase in government spending. Moreover, gains from trade can coincide with a depreciation of the real exchange rate. For instance, two years after an increase in government spending, GDP multipliers are higher relative to the closed economy in about 50% of the cases in which the real exchange rate depreciates. The agnostic predictions about the real exchange-rate response depend on the presence of a debt-elastic risk premium (Bouakez and Eyquem, 2015), the diffuse prior for the trade elasticity encompassing both micro and macro estimates (Enders et al., 2011), and the diffuse prior for the degree of complementarity between public and private goods.²⁵ The latter shapes the response of private consumption following a public expenditure increase and affects the real exchange rate

²³In Appendix D, we show multipliers constructed using real variables expressed in production units are virtually identical to those reported here.

²⁴For an income tax cut, a negative change in tax revenue implies a positive change in the variable of interest. When analyzing this scenario, we multiply the fiscal multipliers by -1, so that a positive GDP multiplier implies GDP increases, while a negative multiplier implies GDP declines.

²⁵Bouakez and Eyquem (2015) show that a plausible elasticity of the interest rate premium to debt alone can generate a real exchange rate depreciation following an increase in government spending. Enders et al. (2011) show that the trade elasticity also can affect the sign of the real exchange rate following a government spending shock. Ravn et al. (2012) show deep habits also can alter the exchange rate response, while Corsetti et al. (2012) generate a depreciation through spending reversals.

through the uncovered interest parity condition.

Also consistent with the analytical model, the responses of the terms of trade and domestic terms of consumption can differ a priori (even qualitatively). Given our assumption of dollar currency pricing, the response of the terms of trade is always smaller in magnitude, and the range of GDP multipliers correlates more closely with the response of the domestic terms of consumption. To discern which channels are most favored empirically, we now turn to the estimation of the model.

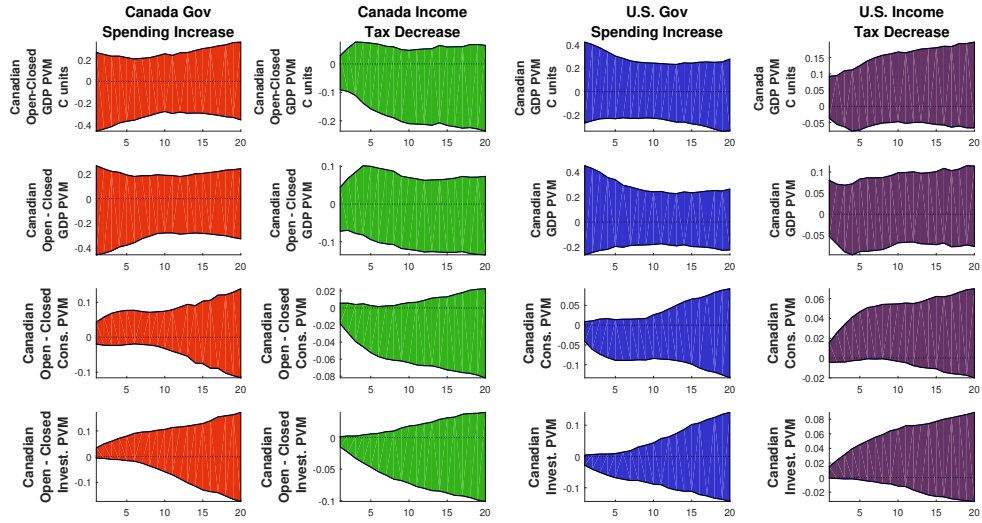
Estimation

We estimate the model using quarterly Canadian and U.S. data over the period 1992-2007. For each country, we include the log first-difference of aggregate consumption, investment, and the consumer price index, as well as the log of hours worked and the short-term interest rate. We include the log demeaned series for the ratios of real government consumption to GDP, income tax revenue to GDP, consumption tax revenue to GDP, and the real market-value of government debt to GDP. In addition, we include the log first-difference of the bilateral real exchange rate between Canada and the U.S., U.S. bilateral exports in goods to Canada, and U.S. bilateral imports in goods to Canada. Details of the data construction and linkage to observables are in Appendix E.

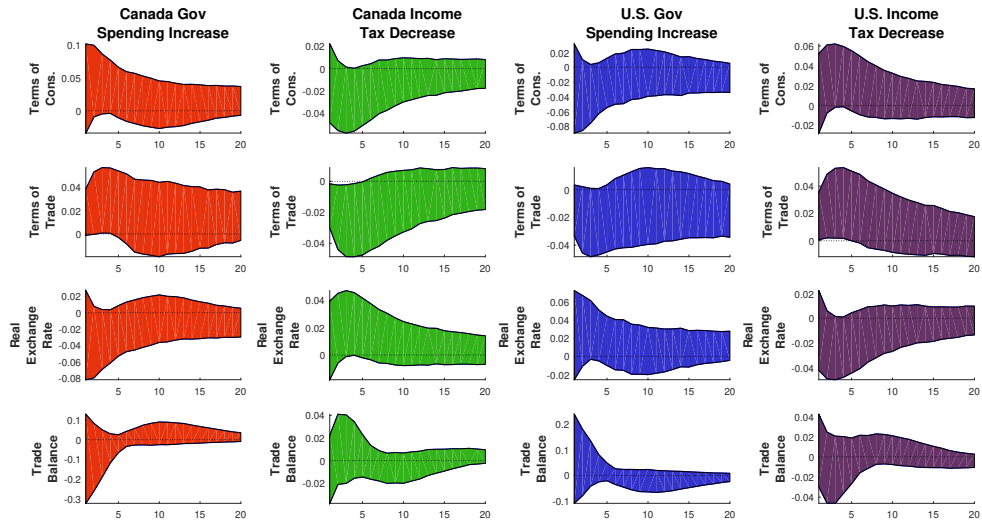
We use Bayesian methods, whereby the data are used to update our priors through the likelihood function, calculated using the Kalman filter. These updates give us draws from the posterior distribution. We take 1.5 million draws from the posterior distribution using the random walk Metropolis-Hastings algorithm. For inference, we discard the first 500,000 draws and keep one out of 100 draws to obtain a sample of 10,000, equivalent to our prior-predictive sample.²⁶

Table 1 displays the posterior mean and 90 percent credible sets for parameters. Posterior credible sets are tighter than the priors, and in line with estimates from the literature. Public and private goods in the U.S. are complements, as in [Bouakez and Rebei \(2007\)](#) and [Fève et al. \(2013\)](#). For Canada, the 90-percent posterior interval includes zero, suggesting public and private goods are neither substitutes nor complements. Notably, the elasticity of substitution between Home and Foreign goods is tightly estimated to be around one. [Lubik and Schorfheide \(2005\)](#) show estimates of this parameter are sensitive to its prior when bilateral exports and imports are not included as observables. In contrast, we find the data to be informative about this parameter once including trade series as observables.

²⁶Our step size implies an acceptance rate of 28 percent. Diagnostics to determine chain convergence include cumulative sum of the draws (CUMSUM) statistics and Geweke's Separated Partial Means (GSPM) test.



(a) Present-Value Multipliers



(b) Select Impulse Responses

Figure 2. 90-percentile intervals from prior-predictive analysis.

Before studying the transmission of fiscal policy, we assess the ability of the model to account for salient features of the data. We focus here on international price movements and cross-country relationships and discuss the model’s fit with respect to domestic variables in Appendix F. Figure 3 plots correlograms from the data (solid lines) and the 90-percent posterior intervals (shaded areas) implied from the estimated model.²⁷ The model accounts for several features of the data that typically are challenging to match. The first four panels of figure 3 plot the cross-correlations for Canadian and U.S. GDP, consumption, investment, and inflation. The model bands encompass the data in all four cases. [Justiniano and Preston \(2010a\)](#) stress the difficulty of international business-cycle models in accounting for these features of the data. Relative to [Justiniano and Preston \(2010a\)](#), two key model features account for our results: investment in physical capital (which has the highest import share in the data) and the common stochastic trend in Canadian and U.S. TFP (which accounts for a nontrivial component of the Canadian series’ variance shares—see Appendix F for variance decompositions).

The next three panels of figure 3 plot the correlogram of the nominal and real exchange rates, the real exchange rate with itself, and the real exchange rate with the Canadian and U.S. consumption growth differential. We focus on these three relationships as they are central in the literature on real exchange rate disconnect (see [Mukhin and Itskhoki, 2016](#) and the references therein). For the most part, the model accounts for these relationships at all horizons, despite the fact that the growth rate of the nominal exchange rate is not an observable for estimation. Moreover, the model accounts for the volatility of these series; see Appendix F for a comparison of model and data volatilities. The model fit along these dimensions mostly reflects the inclusion of a financial shock (the risk premium shock), as highlighted by [Mukhin and Itskhoki \(2016\)](#).²⁸ Finally the last two panels of figure 3 show the model also accounts for the cyclicalities of trade flows. Appendix F shows the model also matches the volatilities of the growth rates of imports and exports.

Posterior Analysis

Figure 4a displays posterior 90-percent intervals for present-value multipliers for GDP, consumption, and investment. We also report GDP multipliers constructed using real variables in units of consumption—a measure that is arguably closer to welfare. The figure plots the level of the

²⁷We sample 5,000 draws from the posterior. For each draw, we simulate 100 samples from the model with the same length as our dataset, after first discarding 100 initial observations. We compute statistics for each of these samples.

²⁸This shock also accounts for a non-trivial share of the variance of both real and nominal Canadian series.

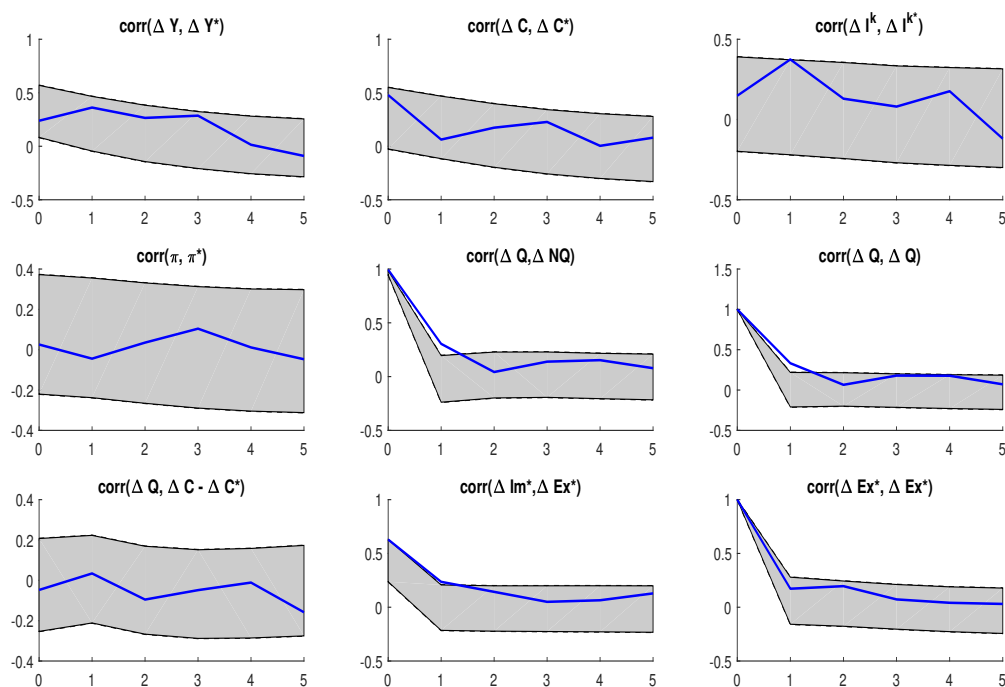


Figure 3. Selected correlograms from the data (solid lines) and 90 percent posterior intervals (shaded areas) implied from the estimated model.

estimated GDP multipliers, as well as the difference in the estimated multipliers relative to a counterfactually closed economy. Figure 4b displays the posterior 90-percent intervals for the impulse responses for the domestic terms of consumption, terms of trade, real exchange rate, and the trade balance following various fiscal shocks. As demonstrated from the first two rows, we find trade linkages increase Canadian government spending multipliers but reduce income tax multipliers. Cross-country comovement is positive across U.S. fiscal instruments.

Column 1 presents responses to a 1% increase in government spending in Canada. Despite the significantly persistent deterioration of the trade balance, the GDP multiplier is unambiguously larger relative to the counterfactually closed economy, and the gains materialize well before the reversal of the trade deficit. The dynamic gain is notable: 6-15 cents higher after 20 quarters when measured in NIPA units or 14-26 cents higher measured in consumption units. In line with the intuition of the analytical model in section 2, the gains from trade reflect the persistent appreciation of the domestic terms of consumption and terms of trade.²⁹ This appreciation crowds in private

²⁹Basu and Kollmann (2013) show that increases in government investment can lead to a deterioration in the terms of trade, as its transmission is similar to an increase in exogenous total factor productivity. Over our sample, government investment is only 14% of total government purchases in Canada and 21% in the U.S.

consumption and investment relative to the closed economy. Finally, there is an appreciation, albeit modest, of the real exchange rate.³⁰

Column 2 plots the responses following a reduction in the Canadian income tax rate. In contrast to the effects of a public spending increase, trade linkages reduce the effectiveness of a tax stimulus at all horizons. Cumulative GDP multipliers are lower than in the counterfactually closed economy, consistent with the deterioration of the domestic terms of consumption, which crowds out private consumption and investment.

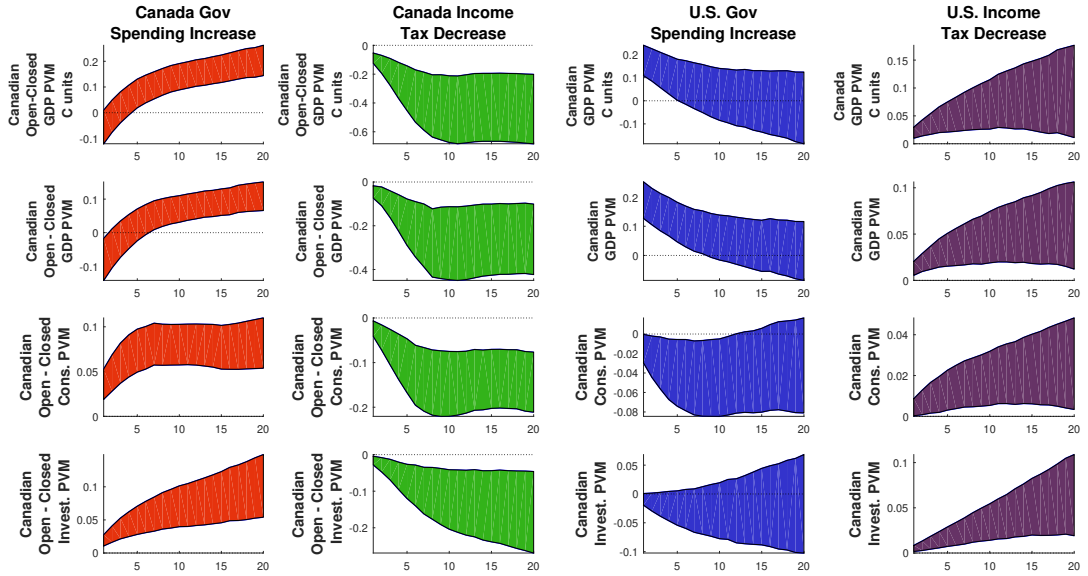
Turning to the spillover effects of U.S. policy, a 1% increase in U.S. public spending (column 3) and a 1% cut in U.S. income taxes (column 4) induce a positive spillover on Canadian GDP. This gain is quantitatively stronger, albeit shorter lived, for an increase in U.S. public spending.

Key Transmission Channels

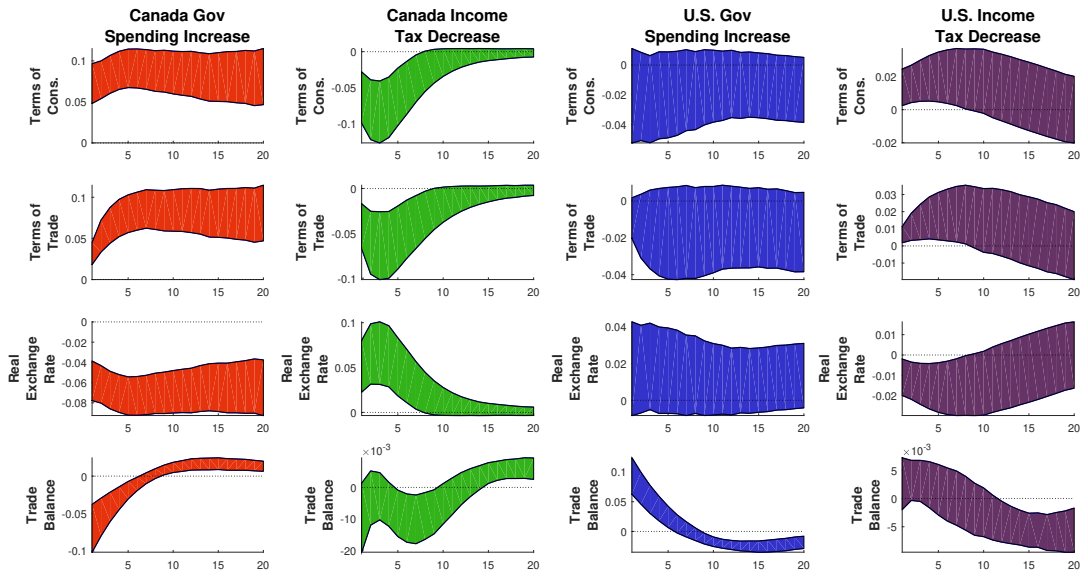
To understand the role of international trade for the transmission of fiscal expansions, we consider alternative counterfactual scenarios that disentangle how competing international linkages affect present-value GDP multipliers. In all experiments, we condition the model on the posterior mean estimates, unless otherwise specified, and report multipliers measured in NIPA units. Following the insights of the analytical model, we first consider a counterfactual economy where the domestic terms of consumption are constant. We induce this outcome by assuming U.S. firms receive a per-period, lump-sum export subsidy such that price adjustments leave the domestic terms of consumption unchanged. We then assess the importance of direct trade linkages related to government expenditures by considering: 1) full home bias in public goods ($\alpha_X^g = 0$) and 2) no complementarity between public and private consumption ($\omega_G = 0$)—since public/private complementarities raise private import demand following public spending increases. Finally, we address the role of trade in financial assets and price setting frictions by considering financial autarky, local currency pricing (LCP), and fully flexible prices.

For all these counterfactuals, the top row of figure 5 shows responses following a 1% increase in Canadian government spending while the bottom row displays responses to a 1% decline in Canadian income taxes. In all the panels, the solid squared line refers to the difference between open- and closed-economy GDP multipliers at the posterior mean. The solid-plus line in the first

³⁰For Canada there is VAR evidence supporting a real appreciation following an increase in government spending (see, for instance, [Kim, 2010](#)).



(a) Present-Value Multipliers



(b) Select Impulse Responses

Figure 4. 90-percentile intervals from posterior estimates.

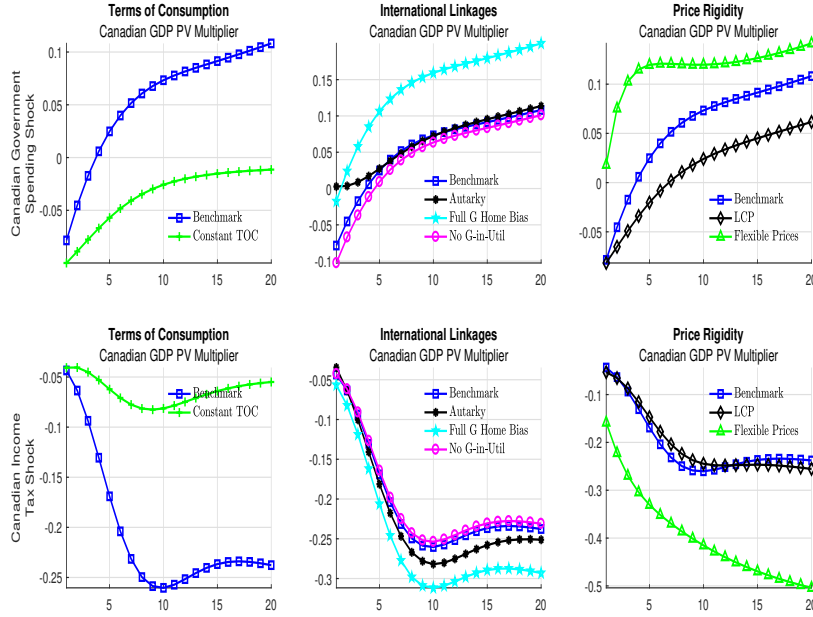


Figure 5. Canadian GDP PV Multiplier response following a change in Canadian (top row) and U.S. (bottom row) government spending. Benchmark response at posterior mean denoted by blue solid lines in all panels, while counterfactuals vary parameter/model structure as described in the legends. X-axis measures quarters.

panel plots the difference in the GDP multipliers in the counterfactual economy with constant terms of consumption. Absent the appreciation of domestic prices relative to import prices, the domestic effectiveness of an increase in public spending is smaller than in a closed economy at all horizons. In contrast, the losses from trade linkages following an income tax cut disappear without the depreciation of the terms of consumption. Consistent with the insight from the analytical model, these results highlight the central role of terms-of-consumption movements for the domestic transmission of fiscal shocks through trade.

The solid-circled and the solid-starred lines in the second panel of figure 5 display the importance of direct trade linkages related to government expenditures. As in the analytical model, lowering the public-to-private import ratio ν to zero increases government-spending multipliers relative to the closed economy—with full home bias in public goods, trade linkages induce an additional increase in GDP multipliers equal to 15 cents after two years (solid-starred line). Also consistent with the results of section 2, the public-to-private import share, ν , is central for income tax multipliers. With full home bias in public goods (solid-starred line), trade linkages result in a much smaller income-tax GDP multiplier relative to the closed economy (30 cents lower after two years). Since the posterior estimates assign a large role to government spending for debt-financing in Canada (see

Table 1), more public home bias implies a larger reduction in domestic aggregate demand to finance the tax cut. Moreover, home bias in public goods implies the deterioration of the domestic terms of consumption following the tax cut leads to a larger reduction in domestic demand.³¹ Finally, the solid-circled lines show no substantial role for government spending complementarities with private consumption, as the posterior mean for ω_G is close to zero for Canada.

The solid-asterisk line in the second panel of figure 5 shows that with a closed current account (i.e., the absence of net export dynamics) the beneficial effects of trade for Canadian government spending multipliers would be larger on impact. This reflects the fact that there is no net-export crowding out under financial autarky. For an income tax cut, the detrimental effects of trade on GDP multipliers also would be larger under financial autarky. In this case, lack of international borrowing has a stronger negative effect on GDP multipliers over time, since borrowing mostly finances domestic investment. Nevertheless, for both income tax and public expenditure changes, the quantitative effects of financial autarky are modest.

Finally, the third panel of figure 5 addresses the role of price-setting frictions. The solid-triangle lines plot the difference in the open and closed economy GDP multipliers for a counterfactual model with fully flexible prices. Larger price adjustments translate into more sizable terms of consumption movements—government spending multipliers in the open economy increase with the appreciation of the terms of consumption while tax multipliers decrease with the stronger depreciation. The solid-diamond lines plot the difference in GDP multipliers under LCP. Once again, following the insights from the analytical model, under LCP the terms of consumption respond less to fiscal shocks. In turn, the beneficial effects of trade for government spending multipliers and the detrimental effects on income tax multipliers are both reduced.

Figure 6 repeats the same counterfactuals as figure 5, but displays the Canadian GDP multipliers following a 1% increase in U.S. government spending (top panel) and a 1% decline in U.S. income taxes (bottom panel). The figure explains why an increase in U.S. government spending generates a stronger yet shorter-lived increase in Canadian GDP relative to a U.S. tax cut. The first panel of the top row shows that without the depreciation in the Canadian terms of consumption following a U.S. public spending increase (solid-plus line), the increase in Canadian GDP would last longer.

The solid-circled and the solid-starred lines in the second panel show that direct trade linkages are key to account for the increase in Canadian GDP following a rise in U.S. public spending.

³¹While total public spending is constant on impact of a tax shock, the value of government spending in consumption units can vary.

Without the demand complementarities between U.S. private and public goods, as well as the direct U.S. public demand for Canadian goods, the spillover to Canadian GDP would be markedly reduced. The solid-triangle line of the third panel also shows the importance of nominal rigidities for the positive spillover following an increase in U.S. government expenditures; with flexible prices, the Canadian terms of consumption would deteriorate more, lowering the Canadian GDP multiplier.

Turning to a U.S. income tax cut, the first panel of the bottom row of figure 6 shows that the appreciation in the Canadian terms of consumption is essential for generating the positive, long-lasting co-movement between U.S. and Canadian GDP. The solid-plus line shows that without the appreciation, the spillover is virtually zero at all horizons. To understand this result, note that a U.S. income tax cut generates two opposing forces for Canada. On the one hand, the increased after-tax return in the U.S. encourages capital to shift from Canada to the U.S., where it is more profitable. *Ceteris paribus*, this lowers Canadian GDP and increases U.S. GDP, generating a negative spillover (see [Mendoza et al. \(2014\)](#) for a discussion of this mechanism in a model of the Euro Area). At the same time, the decrease in U.S. income taxes leads to an appreciation of the Canadian domestic terms of consumption. The positive wealth effect from the appreciation encourages more consumption and investment in Canada, raising Canadian GDP. Given our estimate of the trade elasticity, the terms-of-consumption effect dominates in equilibrium and is essential for generating positive co-movement, as seen from the counterfactual with constant terms-of-consumption (solid-plus line).

5 Endogenous Tradability and Firm Heterogeneity

Model Extension

Despite accounting well for the dynamics of aggregate trade flows, the model in the previous section does not distinguish between the extensive and intensive margins of trade—the number of exporters and the average quantity exported by a producer. However, the international propagation of fiscal shocks depends on the response of both margins, and in turn, extensive-margin dynamics affect endogenously the composition of goods that are traded, i.e., the relative size of the tradable sector and the price of traded goods. To address this issue we incorporate in the model endogenous entry of heterogeneous producers into the domestic and export markets. Our approach follows [Melitz \(2003\)](#), [Ghironi and Melitz \(2004\)](#) and [Cacciatore and Ghironi \(2014\)](#).

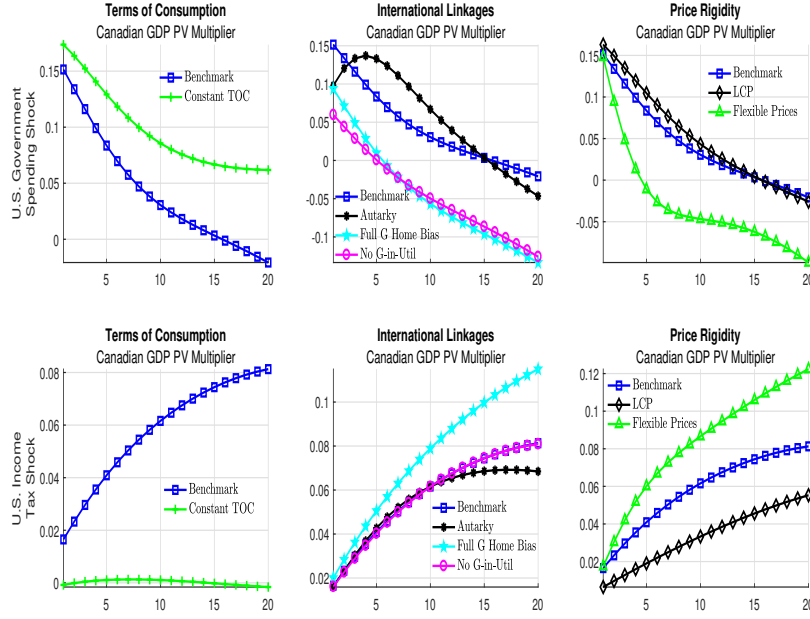


Figure 6. Canadian GDP PV Multiplier response following a change in U.S. government spending (top row) and U.S. income taxes (bottom row). Benchmark response at posterior mean denoted by blue solid-squared lines in all panels, while counterfactuals vary parameter/model structure as described in the legends. X-axis measures quarters.

The only change involves the production of tradable goods. We now assume there is a continuum of symmetric tradable sectors of measure zero relative to the aggregate size of the economy. In each sector i , a representative, monopolistically competitive firm produces a bundle $Y_t^i = \left(\int_{\omega \in \Omega} Y_t^i(\omega)^{(\theta_\omega - 1)/\theta_\omega} d\omega \right)^{\theta_\omega / (\theta_\omega - 1)}$ of differentiated product varieties indexed by ω .³² The number of products (or features) created and commercialized by the producer is endogenous. At each point in time, only a subset of varieties $\Omega_t \subset \Omega$ is actually available to consumers.

To create a new product, the producer pays a sunk investment cost, $f_{E,t} = \bar{Z}_t f_E$, in units of the intermediate input. Each product is produced by a plant, and plants produce with different technologies indexed by relative productivity z .³³ To save notation, we identify a variety with the corresponding plant productivity z , omitting ω . Upon product creation, the productivity level of the new plant z is drawn from a common distribution $G(z)$ with support on $[z_{\min}, \infty)$. This relative productivity level remains fixed thereafter. Each plant uses intermediate inputs to produce its differentiated product variety, with real marginal cost $\varphi_t(z) = \varphi_t/z$.

³² Y_t^i also can be interpreted as a bundle of product features that characterize the final product i .

³³Alternatively, we could decentralize product creation by assuming that monopolistically competitive firms produce product varieties (or features) that are sold to final producers, in this case interpreted as retailers. The two models are isomorphic. Details are available upon request.

At time t , the Home producer i commercializes $N_{D,t}^i$ varieties and creates $N_{E,t}^i$ new products that will be available for sale at time $t + 1$. New and incumbent plants can be hit by a “death” shock with probability $\delta \in (0, 1)$ at the end of each period. The law of motion for the stock of producing plants is $N_{D,t+1}^i = (1 - \delta) \left(N_{D,t}^i + N_{E,t}^i \right)$.

When serving the Foreign market, each producer faces per-unit iceberg trade costs, $\tau_t > 1$, and fixed export costs (in units of intermediate inputs) $f_{X,t} = \bar{Z}_t \bar{f}_{X,t}$, where $\log \bar{f}_{X,t} = \rho_{\bar{f}_X} \log \bar{f}_{X,t-1} + \varepsilon_{\bar{f}_X t}$, where $\varepsilon_{\bar{f}_X t} \stackrel{iid}{\sim} N\left(0, \sigma_{\bar{f}_X}^2\right)$. Fixed export costs are paid for each exported variety. The total fixed cost is then $N_{X,t}^i f_{X,t}$, where $N_{X,t}^i$ denotes the number of product varieties exported to Foreign. Fixed export costs imply that only varieties produced by plants with sufficiently high productivity (above a cutoff level $z_{X,t}$, determined below) are exported. The share of exporting plants is given by $N_{X,t}^i \equiv \left[1 - G(z_{X,t}^i) \right] N_{D,t}^i$.

Assume that $G(\cdot)$ is Pareto with shape parameter $k_p > \theta_\omega - 1$ and define two average productivity levels (weighted by relative output shares): an average \tilde{z}_D for all producing plants and an average $\tilde{z}_{X,t}$ for all plants that export:

$$\tilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\theta_\omega - 1} dG(z) \right]^{\frac{1}{\theta_\omega - 1}} = \left(\frac{k_p}{k_p - \theta_\omega + 1} \right)^{\frac{1}{\theta_\omega - 1}} z_{\min},$$

$$\tilde{z}_{X,t}(i) \equiv \left[\frac{1}{1 - G(z_{X,t}^i)} \right] \left[\int_{z_{X,t}^i}^{\infty} z^{\theta_\omega - 1} dG(z) \right]^{\frac{1}{\theta_\omega - 1}} = \left(\frac{k_p}{k_p - \theta_\omega + 1} \right)^{\frac{1}{\theta_\omega - 1}} z_{X,t}^i,$$

We continue to assume that producers must pay quadratic price adjustment costs when changing domestic and export prices. The producer’s optimal choices for the prices of domestic and export bundles are identical to the model of the previous section.³⁴ Here we briefly discuss the determination of $N_{E,t}^i$, and $z_{X,t}^i$. In the symmetric equilibrium, the optimal export cut-off $z_{X,t}$ satisfies:

$$\frac{k_p - (\theta_\omega - 1)}{(\theta_\omega - 1)k_p} \varphi_{X,t} \frac{Y_{X,t}}{N_{X,t}} \tau_t = f_{X,t} \varphi_t,$$

where $\varphi_{X,t} \equiv N_{X,t}^{1/(1-\theta_\omega)} \varphi_t / \tilde{z}_{X,t}$ is the real marginal cost of producing the export output $Y_{X,t}$. Intuitively, at the optimum, the marginal revenue from adding a variety with productivity $z_{X,t}$ to the export bundle has to be equal to the fixed cost. Thus, varieties produced by plants with productivity below $z_{X,t}$ are distributed only in the domestic market.

³⁴With flexible prices, the tradable sector becomes isomorphic to [Ghironi and Melitz \(2004\)](#). See [Cacciatore and Ghironi \(2014\)](#) and for details.

The optimality condition for $N_{D,t+1}$ determines product creation:

$$\varphi_t f_{E,t} = E_t \left\{ (1 - \delta) \beta_{t,t+1} \left[\begin{aligned} & \varphi_{t+1} \left(f_{E,t+1} - \frac{N_{X,t+1}}{N_{D,t+1}} f_{X,t+1} \right) \\ & + \frac{1}{\theta_\omega - 1} \left(\varphi_{D,t+1} \frac{Y_{D,t+1}}{N_{D,t+1}} + \varphi_{X,t+1} \frac{Y_{X,t+1}}{N_{X,t+1}} \frac{N_{X,t+1}}{N_{D,t+1}} \tau_{t+1} \right) \right] \right\}, \end{aligned}$$

where $\varphi_{D,t} \equiv N_{D,t}^{1/(1-\theta_\omega)} \varphi_t / \tilde{z}_D$ is the real cost of producing the domestic bundle $Y_{D,t}$. The cost of producing an additional variety, $\varphi_t f_{E,t}$, must equal its expected benefit—the expected savings on future sunk investment costs augmented by the marginal revenue from commercializing the variety, net of fixed export costs, if the good is exported. Overall, the model features three new variables ($N_{E,t}$, $N_{X,t}$, and $\tilde{z}_{X,t}$). In addition, the presence of sunk and fixed export costs modifies the goods market clearing condition: $K_t^\alpha (\bar{Z}_t L_t)^{1-\alpha} = Y_{D,t} + Y_{X,t} + N_{E,t} f_{E,t} + N_{X,t} f_{X,t}$.

Estimation and Posterior Analysis

We estimate the model using the same Canadian and U.S. time series as the previous section. We augment this data set with two additional measures: the number of varieties exported from the U.S. to Canada and imported from Canada to the U.S. These measures are available at annual frequency from the U.S. Census, and we conduct inference with mixed frequency data, combining these annual measures with the other quarterly observables. Details of the data construction are in Appendix E.

We calibrate four parameters introduced into this version of the model. Following [Ghironi and Melitz \(2005\)](#), we set the elasticity of tradable varieties, θ_ω , equal to $\bar{\theta}$. The shape parameter κ of the Pareto distribution of plant productivity draws equals 3.4. We normalize z_{mean} and f_e to 1. We choose the fixed export costs f_X such that 21% of plants export abroad in steady state. Table 3 in Appendix G presents the posterior estimates. Several parameter estimates fall within the range estimated in the previous model that abstracted from the extensive margin of trade. Notably, the estimate of the elasticity of substitution between home and foreign goods (ϕ) remains tightly estimated around one.

Figure 7a displays posterior 90-percent intervals for present-value multipliers for GDP (in both consumption and NIPA units), consumption, and investment. The figure plots the difference in the estimated multipliers relative to a counterfactually closed economy. Figure 7b displays the posterior 90-percent intervals for the impulse responses for the domestic terms of consumption, terms of trade, real exchange rate, and the trade balance following various fiscal shocks. The impulse responses show the main message of the previous section is preserved. An expansionary

government spending shock produces favorable terms of consumption dynamics for the domestic economy, while a domestic income tax cut implies a deterioration in the terms of consumption. U.S. public spending increases give larger positive impact spillovers than tax reductions, but they are more short-lived.

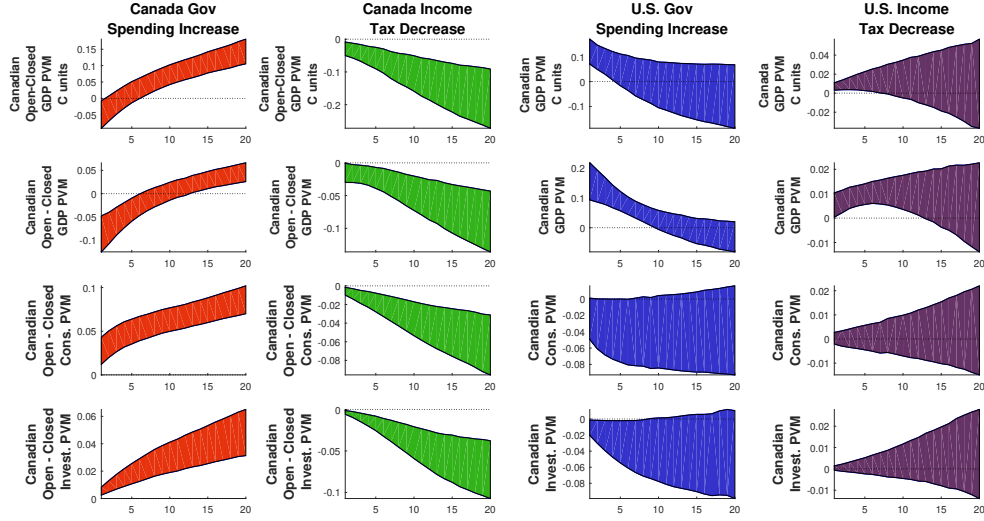
From a quantitative standpoint, both terms of consumption and terms of trade dynamics are dampened relative to the model of section 3. This, in turn, affects differences in open and counterfactually-closed economy multipliers. To understand this result, notice first that in the presence of endogenous tradability and firm heterogeneity, the terms of trade is given by

$$TOT_t \equiv \frac{\epsilon_t P_{X,t}}{P_{X,t}^*} = \left(\frac{N_{X,t}}{N_{X,t}^*} \right)^{\frac{1}{1-\theta\omega}} \frac{\mu_{X,t} \varphi_t \tilde{z}_{X,t}^*}{\mu_{X,t}^* \varphi_t^* \tilde{z}_{X,t}} Q_t,$$

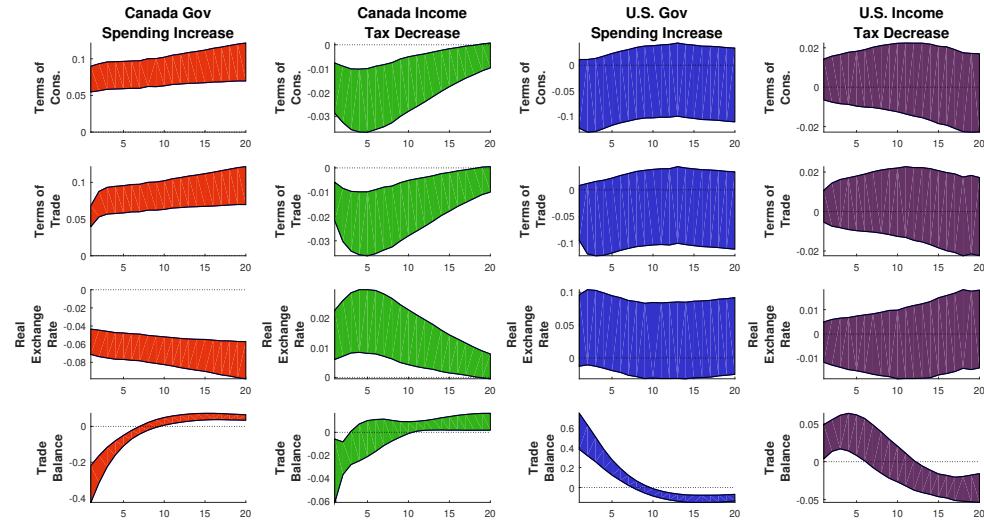
which shows that $\partial TOT_t / \partial N_{X,t} < 0$ and $\partial TOT_t / \partial N_{X,t}^* > 0$ i.e., the terms of trade (and thus the terms of consumption) depreciate following an increase in the number of Home exporters, since a higher $N_{X,t}$ lowers the marginal cost of producing the export bundle $Y_{X,t}$. Similarly, $\partial TOT_t / \partial \tilde{z}_{X,t} < 0$ and $\partial TOT_t / \partial \tilde{z}_{X,t}^* > 0$, e.g., a higher average productivity of Home exporters reduces Home export prices.

The above relationships show why fluctuations in the number of traded goods affect the transmission of fiscal shocks. Consider an increase in government spending. As in the benchmark model, higher government spending increases demand for domestic goods, resulting in a higher real marginal cost. In turn, the rise in the real marginal cost makes sunk and fixed costs more costly, reducing the number of traded goods relative to the Foreign economy. The negative response of the extensive margin dampens the appreciation of the terms of consumption in the short-run (by dampening the response of the terms of trade). For this reason, following a government spending increase, the open-economy GDP multiplier is slightly lower relative to the model of section 3.

A similar logic applies for a tax cut. In this case, the tax cut reduces the real marginal cost of production, stimulating producer entry in both domestic and export markets. The increase in $N_{X,t}$ (partly) offsets the depreciation of the terms of consumption brought about by the decline in the real marginal cost.



(a) Present-Value Multipliers



(b) Select Impulse Responses

Figure 7. 90-percentile intervals from posterior distribution for the model with endogenous tradability and firm heterogeneity.

6 Conclusion

This paper shows fiscal multipliers can be larger in economies more open to trade, even when fiscal expansions imply a trade deficit. Holding the trade share and trade elasticity constant, countries can have higher or lower fiscal multipliers relative to a counterfactually closed economy depending on 1) the private sector import intensity relative to the public sector, 2) how the government finances fiscal expansions, and 3) the invoicing of import and export prices. We demonstrate these ambiguous effects analytically in a simple two-good, two-country model.

We then combine Bayesian prior and posterior analyses on Canadian and U.S. data to demonstrate the results prevail in estimated quantitative international business-cycle models, including a version that features a microfounded trade structure with endogenous tradability and firm heterogeneity. Across models, posterior estimates imply Canadian government spending multipliers are higher than in a counterfactually closed economy. Income tax cuts generate lower multipliers but are more effective in inducing positive cross-country comovement.

Our results have direct implications for the effectiveness of fiscal policy in the global economy, including incentives for international fiscal policy coordination and fiscal consolidations. Moreover, our analysis suggests important considerations for future empirical work on the transmission of fiscal policy.

References

- ADOLFSON, M., S. LASEEN, J. LINDE, AND M. VILLANI (2005): “Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through,” Sveriges Riksbank Working Paper Series No. 179, March.
- (2007): “Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through,” *Journal of International Economics*, 72, 481–511.
- ASCHAUER, D. A. (1985): “Fiscal Policy and Aggregate Demand,” *American Economic Review*, 75, 117–127.
- AUERBACH, A. J. AND Y. GORODNICHENKO (2013): “Output Spillovers from Fiscal Policy,” *American Economic Review*, 103, 141–146.
- BASU, P. AND R. KOLLMANN (2013): “Productive Government Purchases and the Real Exchange Rate,” *The Manchester School*, 81, 461–469.
- BAXTER, M. (1995): “International Trade and Business Cycles,” in *Handbook of International Economics*, ed. by G. M. Grossman and K. Rogoff, Amsterdam: North Holland, vol. 3, 1801–1864.
- BEETSMA, R. M. AND H. JENSEN (2005): “Monetary and Fiscal Policy Interactions in a Micro-Founded Model of a Monetary Union,” *Journal of International Economics*, 67, 320–352.
- BETTS, C. AND M. B. DEVEREUX (2001): “The International Monetary Transmission of Monetary and Fiscal Policies in a Two Country Model,” in *Essays in Honor of Robert A. Mundell*, ed. by M. Obstfeld and G. Calvo, Cambridge, Massachusetts: MIT Press.
- BORN, B., F. JUESSEN, AND G. J. MULLER (2013): “Exchange Rate Regimes and Fiscal Multipliers,” *Journal of Economic Dynamics and Control*, 37, 446–465.
- BOUAKEZ, H., F. CHIHI, AND M. NORMANDIN (2014): “Fiscal policy and external adjustment: New evidence,” *Journal of International Money and Finance*, 40, 1–20.
- BOUAKEZ, H. AND A. EYQUEM (2015): “Government spending, monetary policy, and the real exchange rate,” *Journal of International Money and Finance*, 56, 178–201.
- BOUAKEZ, H. AND N. REBEI (2007): “Why Does Private Consumption Rise After a Government Spending Shock?” *Canadian Journal of Economics*, 40, 954–979.
- BRONER, F., D. CLANCY, A. MARTIN, AND A. ERCE (2018): “Fiscal Multipliers and Foreign Holdings of Public Debt,” European Stability Mechanism Working Paper No. 30-2018.
- CACCIATORE, M. AND F. GHIRONI (2014): “Trade, Unemployment, and Monetary Policy,” *mimeo*.
- CHEN, K., A. IMROHOROGLU, AND S. IMROHOROGLU (2009): “A Quantitative Assessment of the Decline in the U.S. Current Account,” *Journal of Monetary Economics*, 56, 1135–1147.
- CHINN, M. (2013): “Fiscal Multipliers,” in *The New Palgrave Dictionary of Economics*, ed. by S. N. Durlauf and L. E. Blume, Basingstoke: Palgrave Macmillan.

- CHINN, M. D. (2017): “The Once and Future Global Imbalances? Interpreting the Post-Crisis Record,” Prepared for the Kansas City Fed Jackson Hole Symposium “Fostering a Dynamic Global Economy”.
- COLE, H. L. AND M. OBSTFELD (1991): “Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?” *Journal of Monetary Economics*, 28, 3–24.
- COOK, D. AND M. B. DEVEREUX (2013): “Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap,” *American Economic Journal: Macroeconomics*, 5, 190–228.
- CORSETTI, G., K. KUESTER, AND G. J. MULLER (2013): “Floats, Pegs, and the Transmission of Fiscal Policy,” in *Fiscal Policy and Macroeconomic Performance*, ed. by L. Cespedes and J. Gali, Central Bank of Chile.
- CORSETTI, G., A. MEIER, AND G. J. MULLER (2012): “Fiscal Stimulus with Spending Reversals,” *Review of Economics and Statistics*, 94, 878–895.
- CORSETTI, G. AND G. J. MULLER (2006): “Twin deficits: squaring theory, evidence and common sense,” *Economic Policy*, 21, 597–638.
- CORSETTI, G. AND P. PESENTI (2001): “Welfare and Macroeconomic Interdependence,” *Quarterly Journal of Economics*, 116, 421–445.
- DION, R., M. LAURENCE, AND Y. ZHENG (2005): “Exports, Imports, and the Appreciation of the Canadian Dollar,” *Bank of Canada Review*, Autumn, 5–18.
- DRAUTZBURG, T. AND H. UHLIG (2015): “Fiscal Stimulus and Distortionary Taxes,” *Review of Economic Dynamics*, 18, 894–920.
- ENDERS, Z., G. J. MULLER, AND A. SCHOLL (2011): “How Do Fiscal and Technology Shocks Affect Real Exchange Rates? New Evidence for the United States,” *Journal of International Economics*, 83, 53–69.
- ERCEG, C. J., L. GUERRIERI, AND C. GUST (2005): “Expansionary Fiscal Shocks and the U.S. Trade Deficit,” *International Finance*, 8, 363–397.
- ERCEG, C. J. AND J. LINDE (2012): “Fiscal Consolidation in an Open Economy,” *American Economic Review*, 102, 186–191.
- ERCOLANI, V. AND J. V. E AZEVEDO (2014): “The Effects of Public Spending Externalities,” *Journal of Economic Dynamics and Control*, 46, 173–199.
- FACCINI, R., H. MUMTAZ, AND P. SURICO (2016): “International Fiscal Spillovers,” *Journal of International Economics*, 99, 31–45.
- FARHI, E. AND I. WERNING (2016): “Fiscal Multipliers: Liquidity Traps and Currency Unions,” in *Handbook of Macroeconomics*, ed. by J. B. Taylor and H. Uhlig, Amsterdam: Elsevier Press, vol. 2, 2417–2492.
- FERRERO, A. (2010): “A Structural Decomposition of the U.S. Trade Balance: Productivity, Demographics and Fiscal Policy,” *Journal of Monetary Economics*, 57, 478–490.
- FÈVE, P., J. MATHERON, AND J.-G. SAHUC (2013): “A Pitfall with Estimated DSGE-Based Government Spending Multipliers,” *American Economic Journal: Macroeconomics*, 5, 141–178.

- FORNI, L., L. MONTEFORTE, AND L. SESSA (2009): “The General Equilibrium Effects of Fiscal Policy: Estimates for the Euro Area,” *Journal of Public Economics*, 93, 559–585.
- FRANKEL, J. A. (2016): “International Coordination,” National Bureau of Economic Research Working Paper 21878.
- FRENKEL, J. A. AND M. L. MUSSA (1981): “Monetary and Fiscal Policies in an Open Economy,” *American Economic Review*, 71, 253–258.
- GALI, J. AND T. MONACELLI (2008): “Optimal Monetary and Fiscal Policy in a Currency Union,” *Journal of International Economics*, 76, 116–132.
- GEWEKE, J. (2010): *Complete and Incomplete Econometric Models*, Princeton, NJ: Princeton University Press.
- GHIRONI, F. AND M. J. MELITZ (2004): “International Trade and Macroeconomic Dynamics with Heterogeneous Firms,” NBER Working Papers 10540, National Bureau of Economic Research, Inc.
- (2005): “International Trade and Macroeconomic Dynamics with Heterogeneous Firms,” *The Quarterly Journal of Economics*, 120, 865–915.
- GOLDBERG, L. S. AND C. TILLE (2008): “Vehicle currency use in international trade,” *Journal of International Economics*, 76, 177–192.
- GOPINATH, G. (2015): “The International Price System,” Jackson Hole Symposium Proceedings.
- ILZETZKI, E., E. G. MENDOZA, AND C. A. VEGH (2013): “How big (small?) are fiscal multipliers?” *Journal of Monetary Economics*, 60, 239–254.
- JUSTINIANO, A. AND B. PRESTON (2010a): “Can Structural Small Open-Economy Models Account for the Influence of Foreign Disturbances?” *Journal of International Economics*, 81, 61–74.
- (2010b): “Monetary Policy and Uncertainty in an Empirical Small Open-Economy Model,” *Journal of Applied Econometrics*, 25, 93–128.
- KIM, S. (2010): “Country Characteristics and the Effects of Government Consumption Shocks on the Current Account and Real Exchange Rate,” Seoul national university manuscript.
- KIM, S. AND N. ROUBINI (2008): “Twin Deficit or Twin Divergence? Fiscal Policy, Current Account, and Real Exchange Rate in the U.S,” *Journal of International Economics*, 74, 362–383.
- LANE, P. AND G. MILESI-FERRETTI (2002): “Long-Term Capital Movements,” in *NBER Macroeconomics Annual 2001*, National Bureau of Economic Research, Inc, vol. 16, 73–136.
- LAXTON, D. AND P. PESENTI (2003): “Monetary Rules for Small, Open, Emerging Economies,” *Journal of Monetary Economics*, 50, 1109–1146.
- LEEPER, E. M., M. PLANTE, AND N. TRAUM (2010): “Dynamics of Fiscal Financing in the United States,” *Journal of Econometrics*, 156, 304–321.
- LEEPER, E. M., N. TRAUM, AND T. B. WALKER (2017): “Clearing Up the Fiscal Multiplier Morass,” *American Economic Review*, 107, 2409–2454.

- LUBIK, T. AND F. SCHORFHEIDE (2005): “A Bayesian Look at the New Open Economy Macroeconomics,” in *NBER Macroeconomics Annual*, ed. by M. Gertler and K. Rogoff, The University of Chicago Press, vol. 20, 313–366.
- MELITZ, M. J. (2003): “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity,” *Econometrica*, 71, 1695–1725.
- MENDOZA, E. G. AND L. L. TESAR (1998): “The International Ramifications of Tax Reforms: Supply-Side Economics in a Global Economy,” *American Economic Review*, 88, 226–245.
- MENDOZA, E. G., L. L. TESAR, AND J. ZHANG (2014): “Saving Europe?: The Unpleasant Arithmetic of Fiscal Austerity in Integrated Economies,” National bureau of economic research working paper no. 20200.
- MONACELLI, T. AND R. PEROTTI (2008): “Openness and the Sectoral Effects of Fiscal Policy,” *Journal of the European Economic Association*, 6, 395–403.
- (2010): “Fiscal Policy, the Real Exchange Rate, and Traded Goods,” *Economic Journal*, 120, 437–461.
- MUKHIN, D. AND O. ITSKHOKI (2016): “Exchange Rate Disconnect,” Tech. rep.
- MULLER, G. J. (2008): “Understanding the Dynamic Effects of Government Spending on Foreign Trade,” *Journal of International Money and Finance*, 27, 345–371.
- NORMANDIN, M. (1999): “Budget Deficit Persistence and the Twin Deficits Hypothesis,” *Journal of International Economics*, 49, 171–193.
- OBSTFELD, M. AND K. ROGOFF (1995): “Exchange Rate Dynamics Redux,” *Journal of Political Economy*, 103, 624–660.
- PAVLOVA, A. AND R. RIGOBON (2007): “Asset Prices and Exchange Rates,” *Review of Financial Studies*, 20, 1139–1180.
- PRIFTIS, R. AND S. ZIMIC (2018): “Sources of Borrowing and Fiscal Multipliers,” Bank of Canada manuscript.
- RABANAL, P. AND J. F. RUBIO-RAMIREZ (2015): “Can International Macroeconomic Models Explain Low-Frequency Movements of Real Exchange Rates?” *Journal of International Economics*, 96, 199–211.
- RABANAL, P., J. F. RUBIO-RAMIREZ, AND V. TUESTA (2011): “Cointegrated TFP Processes and International Business Cycles,” *Journal of Monetary Economics*, 58, 156–171.
- RAVN, M., S. SCHMITT-GROHE, AND M. URIBE (2012): “Consumption, Government Spending, and the Real Exchange Rate,” *Journal of Monetary Economics*, 59, 215–234.
- UHLIG, H. (2010): “Some Fiscal Calculus,” *American Economic Review*, 100, 30–34.
- WOODFORD, M. (2011): “Simple Analytics of the Government Expenditure Multiplier,” *American Economic Journal: Macroeconomics*, 3, 1–35.
- ZELLNER, A. (1971): *An Introduction to Bayesian Inference in Econometrics*, New York: John Wiley and Sons, Inc.