Learning from House Prices: Amplification and Business Fluctuations*

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Abstract

We provide a new theory of demand-driven business cycles based on learning from prices in an otherwise frictionless neoclassical model. In our model, house price increases caused by aggregate disturbances may be misinterpreted as a signal of higher local consumption prospects, leading households to expand their demand for current consumption and housing. Higher demand reinforces the initial house price increase in an amplification loop that drives comovement in output, labor, residential investment, and house prices even in response to aggregate supply shocks. The qualitative implications of the model are consistent with observed business cycles, and it can explain apparently autonomous changes in sentiment without resorting to non-fundamental shocks.

Keywords: demand shocks, house prices, incomplete information, animal spirits.

JEL Classification: D82, D83, E3.

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"*Housing* is *the Business Cycle.*"

Edward E. Leamer,
Jackson Hole Symposium, 2007.

1 Introduction

House prices provide valuable information about ongoing changes in economic activity, both at the aggregate and regional levels.[1] Over the last half century, real house prices and output have moved together at least half of the time in the US (Figure 1). However, people likely have very different real-time information about these two variables. Precise information about *local* house prices is readily available and relevant to individuals, while the earliest measures of GDP are imprecise, released with delay, and may be less relevant to individual choices. Therefore, an increase in house prices may be seen by people as good news about their economic prospects, generating fluctuations the economy that would not occur in a world with perfect information.

This paper proposes a new model of housing's *informational* role in generating and amplifying demand-driven business fluctuations. The essence of the model is a price-optimism feedback channel: higher house prices beget economic optimism, which begets even higher house prices, and so on. Since house prices reflect all local and aggregate developments, any aggregate shock that is not common knowledge can activate this loop, potentially driving comovement even in response to supply shocks. In this way, the learning feedback channel that we propose offers a new source of amplification for fundamental shocks and breaks the strict dichotomy between disturbances to supply and demand.

We embed our learning mechanism within a neoclassical model with housing. Households are located on islands and consume an aggregate consumption good and local housing. Traded consumption is produced using labor from all islands, while local housing is produced using land, local labor, and a traded productive factor (commodity good) whose supply is fixed. Local house prices can move either because of an increase in the future product of local labor, or because of a current aggregate disturbance to housing production.

[1] Regarding aggregates, Leamer (2007) and Leamer (2015) make the point forcefully, and Campbell and Cocco (2007) and Miller et al. (2011) provide similar evidence at the regional level.

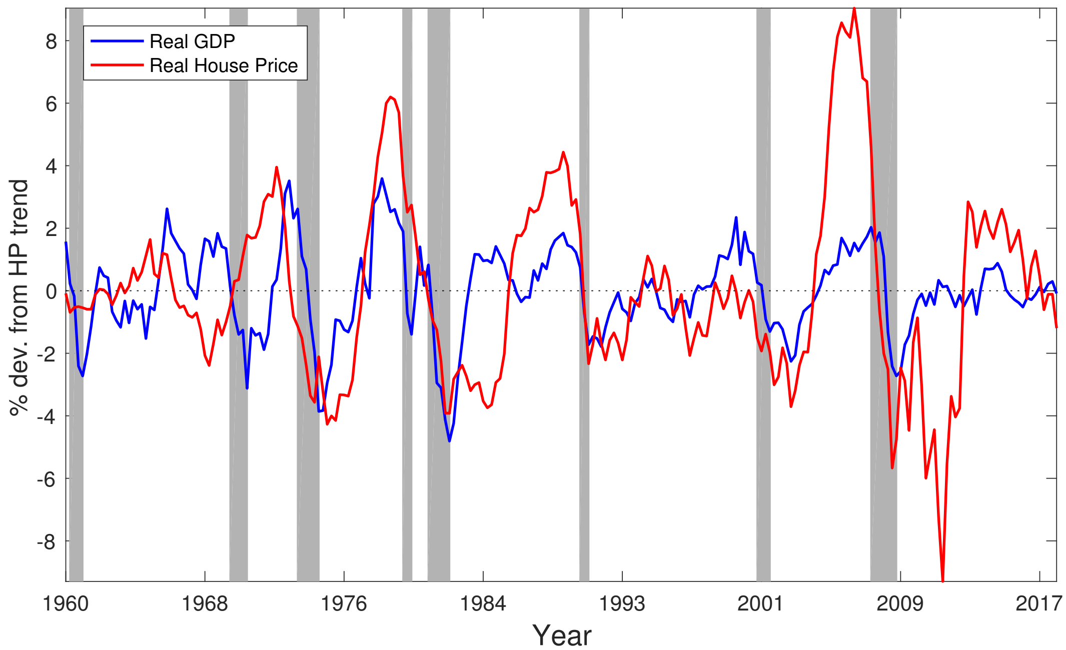


Figure 1: Real gross domestic product and the Shiller national house price index.

Most fluctuations in local house prices are driven by local labor productivity, so people observing high house prices become optimistic about their own labor income prospects. However, a fall in the productivity (or availability) of the common productive factor can also drive an increase in house prices across islands. In this case, the increase in house prices is interpreted by households as a positive signal about wages, increasing demand for both consumption and housing on all islands. Higher aggregate demand further increases house prices, and consequently the price of the common productive factor, reinforcing the initial price increase. In equilibrium, what started as a (possibly small) change in housing supply leads to a generalized increase in house prices, a boom in aggregate demand for both consumption and housing, and a spike in the price of the traded input factor.

Our model of learning from prices has several features that make it an appealing model of the business cycle. First, we embed our mechanism in a flexible price model with competitive markets. This means that endogenous fluctuations in housing demand, and their real effects, are not driven by competitive or nominal frictions, or by suboptimal monetary policy. Indeed, our real economy can be interpreted as a monetary economy in which the nominal price level is fixed. Hence, our model aligns well with recent experience in developed economies, where substantial real macroeconomic fluctuations have coincided with small and largely acyclical fluctuations in inflation (Angeletos et al., 2016).

Second, the signal structure faced by households is fully microfounded. All shocks are fundamental and we explicitly derive the signals used by agents as the outcome of competitive markets. Hence, the model provides an explicit description of how people's beliefs become coordinated and does not require an exogenous assumption, as do the literatures on sunspots (e.g. Cass and Shell, 1983) and sentiments (e.g. Angeletos and La'O, 2013; Benhabib et al., 2015) regarding how people coordinate their beliefs.

Third, while we introduce our mechanism in the context of house prices and housing productivity, its logic extends to any local price and to other sorts of macroeconomic fundamentals. Hence, the mechanism we propose can be a general source of macroeconomic comovement, not just in response to a single shock. We show this generality by introducing a shock to consumption productivity, but also refer the reader to earlier drafts of this paper that demonstrate how the mechanism works for local consumption prices, and for shocks to the nominal money supply.[2]

The microfoundation of our signal structure as a *price* is crucial to our mechanism for two reasons. First, the fact that information comes from market prices, rather than from exogenously specified signals, means that higher house prices can increase demand for both consumption and housing in our model; the price learning channel causes housing demand to be upward sloping. Upward sloping demand, in turn, causes prices and quantities comove in our model.

Second, the feedback of the global factor price into local housing prices allows the model to deliver strong amplification. For some calibrations, the feedback channel can be so strong that the economy has equilibria with an unusual feature: house prices and aggregate quantities exhibit sizable correlated fluctuations in the limit of arbitrarily small aggregate shocks. To an econometrician, the fluctuations emerging at the limit of no aggregate shocks would appear to be driven by something akin to "animal spirits" (Shiller, 2007), "noise" (Gazzani, 2019), or "sentiment" (Benhabib et al., 2015).[3]

[2]In earlier drafts of this paper, (Chahrour and Gaballo, 2017) in which the leading price is local consumption, we also show that total factor productivity shocks can be weakly correlated with business cycle variables all horizons—as they are in the data of Angeletos et al. (2014) and Angeletos et al. (2016)—and still be the single driver of the cycle.

[3]In the limit of zero variance of supply shocks, our equilibria have the same stochastic properties as the

After characterizing equilibrium, we examine the qualitative features of the economy. We show that the model implies positive comovement between output, employment, hours in both the consumption and housing sector, house prices and land prices *for any calibration and any equilibrium* so long as aggregate shocks are small enough. Hence the model provides a foundation for macroeconomic comovement across a wide range of variables.

We then enrich the model to allow a portion of housing productivity to be common knowledge. This allows the model to exhibit standard real business cycle comovement in response to the common knowledge portion of the shock, but continue to exhibit more "demand-like" fluctuations in response to the portion of the shock that households learn about via prices.

In a simple calibration, we show that the extended model delivers qualitatively realistic (i.e. positive but imperfect) correlations between all real variables in the economy. Moreover, even when the model has a unique equilibrium, it delivers substantial amplification of price and quantity fluctuations in the housing market and non-trivial fluctuations in consumption, even though the full information version of the model exhibits constant consumption.

We augment our discussion of real comovements with some non-structural evidence favoring the house price as source of peoples' economic learning. In this exercise, we use Michigan Survey of Consumer Expectations data to show that peoples' past house price experiences are a far better predictor of their expectations of their own income than are peoples' reports about aggregate economic news that they have heard. Moreover, house price experiences modestly lead income expectations, a timing that is consistent with learning flowing from house prices to income expectations. While this evidence is certainly not dispositive, it suggests the model we propose could guide fruitful and more structural interpretations of the expectations survey data.

We conclude the analysis of the paper with several extensions that indicate the robustness of the basic mechanism. Among these, we show that aggregate consumption productivity shocks are isomorphic to exogenous demand shocks in the housing market. When they become sufficiently small, consumption productivity shocks deliver the very same comovements as our baseline model.

sentiment equilibria characterized by Benhabib et al. (2015). Yet, they are not sentiment equilibria in the sense intended by those authors. We discuss this issue in more detail in Section 5.

Literature review

The expectation channel in our economy contrasts with the channel through which news or noise shocks drive demand-driven fluctuations in New-Keynesian models, as in Lorenzoni (2009) and subsequent literature. Without nominal rigidities, these models would imply that higher anticipated *aggregate* productivity drives a change in the real interest rate rather than consumption (see Angeletos, 2018, for a nice discussion.) This does not happen in our real model because local house prices incorporate imperfect news about *local* productivity, which cannot be offset by moves in the *aggregate* real interest rate. Still, aggregate demand cannot move in our economy without our learning mechanism, which correlates forecasts of consumption across islands.

This paper is the first to demonstrate that learning from prices might play a central role in explaining business cycle comovements in the context of a rational expectations model. Nevertheless, endogenous signal structures have also appeared in macroeconomic contexts, starting with Lucas (1972). More recent examples include Amador and Weill (2010), Venkateswaran (2013), and Benhima (2018); Benhima and Blengini (2017). Most recently, Gaballo (2018) presents a learning-from-prices mechanism that can explain aggregate price rigidity in an otherwise frictionless monetary model, while Angeletos and Lian (2019) presents a flexible price model where noisy observations of the intertemporal price lead discount rate shocks to drive real variables. Departing from rational expectations, Eusepi and Preston (2011) show that adaptive learning can generate realistic business cycle co-movements.

Our focus on the informative role of prices also echoes a long tradition in finance, starting with Grossman and Stiglitz (1976, 1980).Several authors have shown that this mechanism can deliver price amplification and/or multiple equilibria, including Burguet and Vives (2000) and Barlevy and Veronesi (2000), Albagli et al. (2014), Manzano and Vives (2011), and Vives (2014).[4] Unlike these papers, which usually include noise traders or exogenous shocks to information, every shock in our model is fundamental and we are the first to show the potential for extreme amplification in limit cases.

[4]The literature on price revelation in auction markets following Milgrom (1981) also features a dual informational/allocative role for prices. For recent examples, see Rostek and Weretka (2012); Lauermann et al. (2012); Atakan and Ekmekci (2014).

With respect to the housing market, our theory is consistent with a range of empirical evidence on housing and the business cycle. Our model of the housing market is very similar to Davis and Heathcote (2005) and, like in that paper, productivity shocks are the main drivers of the housing market. In contrast to that paper, however, prices and quantities in the housing market can positively co-move as observed in the data.[5] Our model also qualitatively accounts for the high volatility of the price of land (Davis and Heathcote, 2007) and for its negative co-movement with labor market variables (Liu et al., 2016).

The demand-like effects of productivity shocks in our economy also challenge conventional restrictions use to identify supply and demand shocks. For example, Iacoviello and Neri (2010) estimate that about 70% of housing price changes are due to housing preference shocks, primarily because of positive price-quantity comovement in the housing market.

Finally, our paper also contributes to a longstanding debate about the nature and size of house price wealth effects. Frictionless models typically imply that house prices should have no causal impact on consumption (e.g. Buiter, 2010) but empirical studies often suggest otherwise. For example, Muellbauer and Murphy (1990) argue the spike in UK consumption in the late 1980's was driven directly by rising house prices, while King (1990), Pagano (1990), and Attanasio and Weber (1994) argue consumption and house prices likely reflected peoples' changing perceptions of their permanent income. These competing views can coexist in our model: rising housing prices drive increased consumption not because consumers expect to sell their houses at a higher price, but because consumers interpret them as a sign of higher labor income.

Other theoretical mechanisms for a direct consumption effect of housing have been proposed more recently in the literature, including borrowing constraints (Iacoviello, 2005), wealth heterogeneity (Kaplan et al., 2017), or incomplete markets (Berger et al., 2017). The learning channel we formalize here offers a complimentary explanation to these mechanisms. One difference is that our channel does not depend on actual new house sales or credit contracts, which might imply a longer delay between house prices and their effects on consumption.

[5]Recently, Nguyen (2018) and Fehrle (2019) have also proposed solutions to related comovement challenges, at the cost of introducing further segmentation in the capital market.

Evidence from from disaggregated levels is also largely consistent with our theory. For example, Miller et al. (2011) finds a positive effect of local house price changes to local per capita growth in US metropolitan-level data, with effects that last about two years; Campbell and Cocco (2007) finds that a 1% increase in the value an individual's house is associated with a 1.22% increase in their real non-durable consumption in the UK, with the unpredictable component of housing prices having an impact both at national and regional level. The recent studies by Mian et al. (2013) and Mian and Sufi (2014) also present regional evidence that falling house prices during the Great Recession are associated with consumption reductions at the ZIP code level.

2 A Housing Model with Learning from Prices

In this section, we present a simple real business cycle model with housing. We aim as much as possible to provide analytical results regarding the mechanism and make simplifying assumptions to this end. Most of these assumptions can be relaxed; we discuss when and how as we proceed.

2.1 Preferences and technology

The economy consists of a continuum of islands, indexed by $i \in (0, 1)$. Each island is inhabited by a continuum of price-taking households who consume local housing and a traded numeraire consumption good. Households provide local labor which is used in the production of both goods. On each island, a mass of competitive housing investment firms combine local labor and land with a traded commodity good to construct new houses, while an aggregate consumption sector combines all islands' labor to produce the traded consumption good.

Households

The representative household on island i chooses consumption, labor supply, and savings in a risk-free nominal bond to maximize the utility function:

$$U_{i0} \equiv \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_{it}^{\phi} \mathcal{H}_{it}^{1-\phi}) - vN_{it} \right\} \tag{1}$$

In the above utility function, C_{it} denotes household i's consumption of the tradable consumption good, $\mathcal{H}_{it}$ measures the total quantity of housing consumed, and N_{it} is the household's supply of labor. The household discount factor is $\beta \in (0,1)$, the share of housing in the consumption basket is $\phi \in (0,1)$, and v parameterizes the household's linear disutility of labor.[6]

We assume that housing consumption is composed of a sequence of housing vintages, $\Delta_{i\tau|k}$, constructed at time k and combined according to the Cobb-Douglas aggregator

$$\mathcal{H}_{it} \equiv \prod_{k=-\infty}^{t} \Delta_{it|k}^{(1-\psi)\psi^{t-k}}, \tag{2}$$

where $\psi \in (0,1)$. This formulation for housing utility adds a realistic dimension to the model, since housing vintages can have very different characteristics and are not perfectly substitutable. More importantly for our purposes, however, this formulation in conjunction with log-utility implies that every housing vintage has an additive-separable impact on intertemporal utility, allowing us to analyze the dynamic model in closed form. The housing aggregate can be written recursively as $\mathcal{H}_{it} = \Delta_{it|t}^{1-\psi}(1-d)\mathcal{H}_{it-1}^{\psi}$, a result we use going forward.

Each vintage of housing depreciates at a constant rate $d \in (0,1)$, so that

$$\Delta_{i\tau+1|k} = (1-d)\Delta_{i\tau|k}$$

for $\tau \geq k$ (while, of course, $\Delta_{i\tau|k} = 0$ for $\tau < k$). The aggregate housing stock, defined as $H_{it} = \sum_{k=-\infty}^{t} \Delta_{it|k}$, then evolves according to a standard evolution equation,

$$H_{it} = \Delta_{it|t} + (1-d)\, H_{it-1}. \tag{3}$$

The choices of the household are subject to the following budget constraint,

$$\mathcal{B}_{it} \equiv \frac{B_{it}}{R_t} + C_{it} + P_{it}\Delta_{it|t} - W_{it}N_{it} - B_{it-1} - \Pi_t^c - \Pi_{it}^h \leq 0 \tag{4}$$

for $t \in \{0, 1, 2...\}$ with $B_{i-1} = 0$. Household resources come from providing local labor at wage W_{it}, from past bond holdings, from profits Π_{it}^h of locally-owned housing firms, and from profits Π_t^c of the aggregate consumption firm, which is evenly held across islands. The household uses its funds to purchase numeraire consumption, to acquire new housing at price P_{it}, and to save in a zero-net-supply aggregate bond with a real risk-free return R_t.

[6]In the Appendix, we allow for convex disutility of labor.

We denote the price of the local housing vintages as $P_{it|k}$ and define the price of the total housing stock as $P_{it}^H = \sum_{k=-\infty}^{t} P_{it|k}\Delta_{it|k}/H_{it}$. Notice, however, that only the price of the current vintage, $P_{it} \equiv P_{it|t}$, shows up in the budget constraint in (4). This happens because we have already anticipated an implication of housing market clearing: since the local household is the only potential buyer and seller of past vintages, trade in them can never generate net resources for the island. Hence, in our model, housing wealth is not wealth in the sense of Buiter (2010). The literature has proposed several ways to break this irrelevance, for example by introducing borrowing constraints (Iacoviello, 2005), wealth heterogeneity (Kaplan et al., 2017), or incomplete markets (Berger et al., 2017). Here, we describe a new and distinct mechanism through which housing price affect consumption that is potentially complementary to these other mechanisms.

Housing Firms

Housing firms produce new houses using a Cobb-Douglas technology,

$$\Delta_{it} = L_{it}^{1-\alpha} X_{it}^{\alpha}, \tag{5}$$

that combines land (L_{it}) with new residential structures (X_{it}) to generate new residential units $\Delta_{it} \equiv \Delta_{it|t}$ with structures share $\alpha \in (0, 1)$. Residential structures, in turn, are produced via a Cobb-Douglas production function

$$X_{it} = (N_{it}^h)^{\gamma}(e^{-\tilde{\zeta}_t} Z_{it})^{1-\gamma} \tag{6}$$

that combines local labor (N_{it}^h) with a traded commodity good (Z_{it}) according to the share parameter $\gamma \in (0, 1)$.

The housing producer maximizes profits,

$$\Pi_{it}^h \equiv P_{it}\Delta_{it} - W_{it}N_{it}^h - Q_t\left(Z_{it} - Z\right) - V_{it}L_{it}$$

subject to (5) and (6). In the above, V_{it} is the local price of land, W_{it} is the price of local labor, and Q_t is the price of the commodity good sold across islands. We assume that housing firms are endowed each period with Z units of the commodity good, which trades freely across islands at a common price and depreciates fully at the end of the period.[7] Land supply is

[7] We interpret Z as a commodity good just for simplicity, to fix ideas. For example, it could also be

exogenous, as each period a fixed amount of residential land — normalized to one — becomes available to housing producers on the island.[8] Without any loss of generality, we assume that new land is allocated to local firms.

The only aggregate shock affecting our baseline economy is a shock to productivity of the commodity good, $\tilde{\zeta}_t$. This shock evolves according to a random walk, $\tilde{\zeta}_t = \tilde{\zeta}_{t-1} + \zeta_t$, with i.i.d. innovation ζ_t distributed according to $N\left(0, \sigma_\zeta^2\right)$. We focus our presentation on this shock because it has no effect on consumption under full information. Still, other shocks could play a similar role: for example, in the Appendix we show that ζ_t is isomorphic to a shock to the stock of endowment in Z.

Consumption Sector

The numeraire consumption good is traded freely across islands and is produced by a continuum of identical competitive firms. The representative consumption producer combines labor from all sectors to maximizes profits

$$\Pi_t^c \equiv Y_t - \int W_{it} N_{it}^c$$

subject to the production function,

$$Y_t = \left(\int e^{\tilde{\mu}_{it}/\eta} {N_{it}^c}^{1-\frac{1}{\eta}} di \right)^{\frac{1}{1-\frac{1}{\eta}}}. \tag{7}$$

The quantity of local labor used is denoted by N_{it}^c, and labor types can be substituted with elasticity $\eta > 0$. Island-specific labor productivity is a random walk, and evolves according to $\tilde{\mu}_{it} = \tilde{\mu}_{it-1} + \hat{\mu}_{it}$, where $\hat{\mu}_{it}$ is i.i.d. and drawn from the normal distribution $N\left(0, \hat{\sigma}_\mu\right)$. We consider an extension with an aggregate shock to consumption productivity in Section 5.

Market Clearing

Clearing in the local land and labor markets requires

$$L_{it} = 1 \quad \text{and} \quad N_{it} = N_{it}^c + N_{it}^h. \tag{8}$$

interpreted as a unspecific type of labor that is supplied inelastically and is free to be traded across islands.

[8] These assumptions do not imply that land supply grows over time. Provided an appropriate transformation of the depreciation rate, this formulation is equivalent to a model in which structures are placed on a fixed stock of land and existing land becomes free as those structures depreciate. See Davis and Heathcote (2005) for details.

Per the discussion above, we omit market clearing conditions for all past housing vintages, since their trade is irrelevant at the island level.

Finally, clearing in the aggregate markets for bonds, consumption, and the commodity good requires

$$Y_t = \int C_{it} di, \quad 0 = \int B_{it} di, \quad \text{and} \quad Z = \int Z_{it} di. \tag{9}$$

2.2 Timing and equilibrium

To introduce our information friction, we use the family metaphor initially discussed by Lucas (1980) and Woodford (2003, pp. 144-145), and more recently adopted by Angeletos and La'O (2010) and Amador and Weill (2010). The household is composed of two types of agents: a shopper, who uses household resources to buy consumption and housing, and a worker-saver, who decides on the number of hours to supply on the labor market and also on the quantity of bonds to buy.

Both family member types act in the interest of the household, but their actions are decentralized and they cannot pool their information within a time period. This means that the shopper commits the household to buy Δ_{it} and C_{it} having only partial information about its problem. In other words, whereas Δ_{it} and C_{it} are optimal choices conditional on the information set of shoppers, N_{it} and B_{it} are conditioned on the full information set of workers.

Each period is composed of four stages:

1. The household splits into shoppers and worker-savers.

2. Shocks realize, namely future local productivity innovations, $\{\hat{\mu}_{i,t+1}\}_{i\in(0,1)}$, and the current aggregate shock, ζ_t. The "best available" information set, $\Omega_t \equiv \{\{\hat{\mu}_{i,\tau}\}_0^{t+1}, \{\zeta_\tau\}_0^t\}$, is observed by firms and worker-savers, but not shoppers.

3. Production and trade take place. Shoppers and workers make their choices based on the information they have, which includes the competitive equilibrium prices in the markets in which they trade. Firms make production choices in light of realized productivity and input prices; and all markets clear.

4. Family members share information, revealing Ω_t to the shoppers.

Because shoppers do not immediately observe Ω_t, they make choices under uncertainty. However, they do observe the local price of housing in their island, P_{it}, which they use to make inference; shoppers' information set is therefore $\{P_{it}, \Omega_{t-1}\}$. We derive the information about current conditions contained in P_{it} shortly.

The family fiction is a convenient modelling tool, but it is not essential to our mechanism. What is important is that some agents have access to information about realized shocks, since prices cannot reveal information unless that information is already available, perhaps noisily, to some agents in the economy (Hellwig, 1980). We could have achieved the same effect without separation between shoppers and workers by assuming that only a fraction of households on each island are informed, in the spirit of Grossman and Stiglitz (1980). Nothing crucial about our results would change if we followed this alternative, though the algebra becomes more cumbersome.[9]

The formal definition of equilibrium is the following.

Definition 1 (Equilibrium). *Given initial conditions* $\left\{\{B_{i-1}, \mathcal{H}_{i-1}, \tilde{\mu}_{i0}\}_{i\in(0,1)}, \tilde{\zeta}_{i-1}\right\}$, *a rational expectations equilibrium is a set of prices,* $\{\{P_{it}, V_{it}, W_{it}\}_{i\in(0,1)}, Q_t, R_t\}_{t=0}^{\infty}$, *and quantities,* $\{\{B_{it}, N_{it}^c, N_{it}^h, N_{it}, C_{it}, H_{it}, \Delta_{it}, X_{it}, L_{it}, Z_{it}\}_{i\in(0,1)}, Y_t\}_{t=0}^{\infty}$, *which are contingent on the realization of the stochastic processes* $\{\{\tilde{\mu}_{it}\}_{i\in(0,1)}\}_{t=0}^{\infty}$ *and* $\{\tilde{\zeta}_t\}_{t=0}^{\infty}$, *such that for each* $t \geq 0$ *and* $i \in (0,1)$*:*

(a) Shoppers optimize, i.e. $\{C_{it}, \Delta_{it}\}$ *are solutions to* $\max_{C_{it},\Delta_{it}} E[U_{it}|P_{it}, \Omega_{t-1}]$ *subject to* $E[\mathcal{B}_{it}|P_{it}, \Omega_{t-1}] \leq 0$*;*

(b) Workers optimize, i.e. $\{N_{it}, B_{it}\}$ *are solutions to* $\max_{N_{it},B_{it}} E[U_{it}|\Omega_t]$ *subject to* $\mathcal{B}_{it} \leq 0$*;*

(c) Housing producers optimize, i.e. $\{N_{it}^h, Z_{it}, L_{it}, \Delta_{it}\}$ *are solutions to* $\max_{N_{it}^h,Z_{it},L_{it},\Delta_{it}} \Pi_{it}^h$ *subject to (5) and (6);*

[9] We followed this track in our earlier working paper, Chahrour and Gaballo (2017). Earlier drafts also showed that our mechanism could arise on the supply side of the economy, more like Lucas (1972). This version of the model leads to demand-driven fluctuations in the market for intermediate inputs rather than in the final market. Our choice to place the main friction with households is consistent with the evidence of Chahrour and Ulbricht (2017) that household expectation errors are needed to match aggregate data.

(d) *Consumption producers optimize, i.e.* $\{N^c_{it}\}_{i\in(0,1)}$ *are solutions to* $\max_{\{N_{it}\}_{i\in(0,1)}} \Pi^c_t$ *subject to (7);*

(e) *Markets clear, i.e. equations (8) - (9) hold.*

Let Λ^c_{it} be the Lagrangian associated with the expected budget constraint showing up in the problem of the shopper, and Λ_{it} be the Lagrangian associated with the problem of the worker. As the worker is perfectly informed, only the latter is the actual shadow value of relaxing the household budget constraint. Therefore, optimality on the side of the shopper requires $\phi C^{-1}_{it} = \Lambda^c_{it} = E[\Lambda_{it}|P_{it}]$. In the Appendix, we derive the full set of optimality conditions describing equilibrium.

2.3 Linearized Model

We now derive the model in terms of log-deviations from an initial point. We choose as initial point the steady state of the deterministic economy and, to economize notation, solve for equilibrium at time $t = 0$ (i.e. when all states are initialized to zero). Going forward, we then omit time indexes, denote future variables with an apostrophe, and call the shoppers' information set simply p_i. Given our assumption that past shocks are common knowledge, nothing in the description of equilibrium changes when $t \neq 0$.

Shoppers demand consumption and housing goods according to the following:

$$c_i = -E[\lambda_i|p_i] \tag{10}$$

$$\delta_i = -E[\lambda_i|p_i] - p_i \tag{11}$$

where the operator $E[\cdot|p_i]$ represents the expectation of the shopper conditional on the market housing price p_i and λ_i is the actual marginal value of the household i's resources (known by the worker but not by the shopper). In fact, p_i, is the only piece of information (jointly with common prior and past shock realizations) that a shopper has to infer the marginal value of consumption. The higher the perceived marginal valuation value of budget resources the lower the demand for consumption and housing by the shopper.

The solution to the worker-shopper problem is given by:

$$w_i = -\lambda_i \tag{12}$$

$$\lambda_i = E[\lambda_i'|\Omega] + r. \tag{13}$$

The worker provides any quantity of labor demanded, so long as the offered wage equals the household Lagrangian, and purchases bonds until the interest rate reflects the difference between the current and the expected future marginal value of budget resources, which the worker-saver forecasts based on Ω, the full current information set.

Housing firm optimality conditions are standard:

$$z_i + q = p_i + \delta_i, \tag{14}$$

$$n_i^h + w_i = p_i + \delta_i \tag{15}$$

$$l_i + v_i = p_i + \delta_i \tag{16}$$

with production technology given by

$$\delta_i = \alpha\gamma n_i^h + \alpha(1-\gamma)\left(z_i - \tilde{\zeta}\right), \tag{17}$$

after imposing the fact that $l_i = 0$.

Consumption producer's optimal choices imply:

$$n_i^c = \tilde{\mu}_i - \eta(w_i - w) + n^c \tag{18}$$

$$y = n^c \tag{19}$$

$$w = 0 \tag{20}$$

where w_t denotes the average log-wage in the economy. Condition (18) captures firms' demand for island-specific labor. Firms demand more of a type of labor whenever its productivity is high or its wage is low compared to the average, or if they demand more labor overall. Nevertheless, the wage for the optimized bundle of labor type is invariant as there is no change in aggregate productivity in the consumption sector; we relax this condition in Appendix.

All relations above obtain as exact log transformation. Only the island resource constraint needs to be log-linearized as follows:[10]

$$C(c_i - c) = C(w_i - w) + C(n_i - n) - Qz_i - \beta b_{it}. \tag{21}$$

In equation (21), C and Q represent the deterministic steady state values of C and Q used in the linearization, and we have set past bond holdings to zero. The equation states that higher than average consumption in one island must be financed by a higher local wage, by higher local labor supply, by selling the commodity good (i.e. using less z_i in production), or by decreased savings. As noted before, an important implication of market clearing in the local housing market is that the existing housing stock cannot be used to raise island level consumption. Other market clearing conditions $0 = \int z_i di = \int b_i di$ and $n = \int n_i^c di + \int n_i^h di$ complete the description of equilibrium relations.

3 Learning from Prices

This section presents the main theoretical results regarding the inference problem of shoppers. We first derive the marginal value of household resources as a function of exogenous shocks and then characterize the price signal seen by shoppers. Finally, we derive the implications for equilibrium inference.

3.1 The marginal value of budget resources

The only friction in the economy is shoppers' uncertainty regarding the marginal value of household budget resources. Without this friction, the model behaves like a standard real business cycle economy. Lemma 1 expresses the value of resources, λ_{it}, as a function of fundamentals.

Lemma 1. *In equilibrium,*

$$\lambda_i = E[\lambda_i' | \Omega] = -\omega_\mu \hat{\mu}_i' - \omega_b b_i \qquad \textit{and} \qquad r = 0 \tag{22}$$

for any $\tau \geq 0$ *and any* $i \in (0, 1)$*. In addition,* $\omega_\mu > 0$ *and* $\omega_b > 0$*, with* $\lim_{\beta \to 1} \omega_b = 0$.

Proof. Proofed in Appendix. □

[10]We linearize bond holdings in levels because B_{it} can take negative values.

Intuitively, the intertemporal arbitrage carried out by worker-savers allows them to equalize the marginal value of budget resources across time. One important implication is that the real interest rate does not react to the aggregate productivity shock in housing production. This result is a consequence of the fact that housing wealth cannot be sold across islands and therefore cannot be used to increase consumption of the tradable good.

While "housing wealth is not wealth" in the sense of Buiter (2010), bonds and local labor income are, since they can be sold across islands in exchange for consumption. Therefore, islands with more productive labor (or higher savings) have better consumption prospects and a lower marginal value of budget resources. For this reason, the value of the Lagrangian multiplier depends on the realization of future local labor productivity shock, $\hat{\mu}_i'$, and bond holdings at the end of the first period, b_i.

As β tends to unity, however, λ_i becomes independent of bond holdings since, in this case, bond wealth generates no interest earnings and is rolled over indefinitely without affecting consumption. To simplify our exposition going forward, we focus our analytical results on the case of β approaching one so that λ_i can be treated as entirely exogenous. However, our propositions hold generically. We also provide a code for solving the general model numerically, which we we use to produce the numerical simulations in the next section.

We conclude this section with a remark on the distinction between local and aggregate productivity in consumption sector. Here, as in the standard real business cycle model, an *aggregate* shock to future productivity in the consumption sector would drive the real interest rate and the future marginal value of resources in opposite directions, leaving their current marginal value unchanged. This is why papers looking for business cycle effects of productivity news typically require nominal frictions along with suboptimal monetary policy, e.g. Lorenzoni (2009). In our environment, however, *local* news has an equilibrium effect on λ_{it} because the real interest rate can only neutralize the aggregate components of news. Transforming the effects of local news into an aggregate demand shock requires a friction, however, such as the information friction we describe below.

3.2 The local housing price

We now derive the signal that shoppers use to make their inferences about $\hat{\mu}_i'$. We focus on the limit case $\beta \to 1$. However, our propositions are proven in the Appendix for any $\beta \in (0,1)$.

Rearranging first order conditions from the housing sector, we recover the standard Cobb-Douglass result that the price is a linear combination of input costs weighted by their elasticity:

$$p_{it} = (1-\alpha)v_i + \alpha\gamma w_i + \alpha(1-\gamma)(\zeta_{it} + q_{it}). \tag{23}$$

We wish to re-write (23) in terms of the exogenous variables (and expectations thereof.) Taking $\beta \to 1$ so that $\omega_b = 0$, we substitute (22) into the local wage in (12) to conclude

$$w_i = \omega_\mu \hat{\mu}_i' \equiv \mu_i,$$

where the rescaled local shock μ_i has variance σ_μ^2. Equations (11), (16) and (22) can be combined to get $v_{it} = E[\mu_{it}|p_{it}]$, implying that the shoppers' observations of the house price already entail all the information they might glean from observing the land price independently. Finally, the market clearing condition for the commodity good and (14) implies that

$$q = \int v_i di = \int E[\mu_i|p_i]di.$$

In sum, when workers expect higher future local productivity, they demand higher current wages which increases the price of housing. On the other hand, the price of land and of the commodity good respond to fluctuations in demand, so shoppers' increased desire for housing pushes up the price of these two goods.

Importantly, the price of local land only reflects local housing demand, but the price of the commodity good, which is traded globally, varies with the aggregate appetite for housing across islands. The shopper is able to infer the land component of the house price because it varies with her own actions. By contrast, the remaining component of house prices confounds the price of the commodity good and the local wage, so the shopper cannot be sure which is driving their observation.

Because shoppers can infer v_i, observing the housing price is informationally equivalent to observing the following price-signal:

$$s_i = \gamma\mu_i + (1-\gamma)\left(\zeta + \int E[\mu_i|p_i]di\right). \tag{24}$$

Upon observing an increase in the price signal, the shopper will therefore revise their expected Lagrangian down, increasing her demand for consumption and housing. Yet, the same increase in price could be driven by aggregate factors, either an increase in the commodity price or a decrease in aggregate housing productivity, that are not related to the local conditions that shopper would like to infer.

In this context, a shoppers' optimal response to a price change depends on the reason that the price has changed. Yet, shoppers cannot directly observe why prices are changing, and they attribute only a (small) part of of every observed price change to aggregate conditions. Because of this, a price increase driven by a fall in aggregate productivity maybe interpreted *on every island* as a positive local shock. This common mistake triggers an increase in demand on each island for consumption and local housing. Through this learning channel, a current aggregate shock to housing productivity can generate a change in aggregate demand, for both consumption and houses.

Equation (24) also shows the potential for feedback effects that lead to amplification. In particular, an economy-wide increase in demand for housing leads to a higher demand for the inelastically supplied commodity good, raising its price, q. Higher q, in turn, pushes all houses prices up, driving more optimism on every island. In this way, an initial mistake in inference — driven in this case by ζ — leads to an effect on the signal that exceeds the $(1-\gamma)$ on the fundamental shock itself.

3.3 Equilibrium

We now solve the shoppers' inference problem. A key feature is that the precision of the price-signal about μ_i depends on the equilibrium volatility of the commodity price, which is a general equilibrium object. Following the related literature, we focus our analysis on linear equilibria.

We begin by conjecturing that the optimal individual expectation is linear in p_i and can be written as

$$E[\mu_i|p_i] = as_i = a\left(\gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|p_i]di + \zeta\right)\right),$$

where a is a weight measuring the strength of the reaction of shopper i's beliefs to the signal she receives. Since the signal is *ex ante* identical for all shoppers, each uses a similar strategy. By integrating across the population we get:

$$\int E[\mu_i|p_i]di = a\,(1-\gamma)\,(E[\mu_i|p_i]di + \zeta)\,.$$

Solving for the average expectation then yields

$$c = \int E[\mu_i|p_i]di = \frac{a\,(1-\gamma)}{1-a\,(1-\gamma)}\zeta, \tag{25}$$

which is a nonlinear function of the average weight, a. The fact that the average expectation is normally distributed confirms the conjectured form of the individual forecast.

In the (25), we have used (10) to equate consumption with the average expectation. Hence, as long as a is different form zero, i.e. the local housing price is informative about μ_i, aggregate consumption moves with the aggregate shock to housing productivity. Using (25), the variance of consumption is given by

$$\sigma_c^2(a) = \left(\frac{a\,(1-\gamma)}{1-a\,(1-\gamma)}\right)^2\sigma^2, \tag{26}$$

where $\sigma_c^2 \equiv var(\int E[\mu_i|p_i]di)/\sigma_\mu^2$ and $\sigma^2 \equiv \sigma_\zeta^2/\sigma_\mu^2$ are the variances of the average expectation and the aggregate shock, respectively, once each is normalized by the variance of the idiosyncratic fundamental.

Substituting the average expectation in (25) into the price signal described in equation (24), we get an expression for the local signal exclusively in terms of exogenous shocks:

$$s_i(a) = \gamma\mu_i + \frac{1-\gamma}{1-a\,(1-\gamma)}\zeta, \tag{27}$$

whose precision with regard to μ_i is given by

$$\tau(a) = \left(\frac{\gamma\,(1-a\,(1-\gamma))}{(1-\gamma)\sigma}\right)^2.$$

Note that as a approaches $(1-\gamma)^{-1}$, precision is nul. This is because for that value there is a one-to-one feedback from individual to average expectation, which can be satisfied only if the size of the exogenous shock get infinitesimally smaller than the fluctuations of expectations. Economically this corresponds to having the average demand in the housing market that approaches the average supply, so that any aggregate perturbation gets indefinitely amplified. See page XX for more details. We are now ready to compute a shopper's optimal inference, taking the average weight of other households as given. We seek an a^* such that $\mathrm{E}[s_i(a)(\mu_i -$

$a^* s_i(a))] = 0$, i.e., the covariance between the signal and forecast error is zero in expectation. This condition implies that information is used optimally. The best individual weight is given by

$$a^*(a) = \frac{1}{\gamma}\left(\frac{\tau(a)}{1+\tau(a)}\right). \tag{28}$$

Given the linear-quadratic environment, we can interpret $a^*(a)$ in a game-theoretic fashion as an individual's best reply to the profile of others' actions summarized by the sufficient statistic a. To be precise, every a^* is associated with one and only one contingent strategy that describes the conditional expectation $E[\mu_i|p_i] = a^* s_i(a)$ of the shopper i, where $s_i(a)$ identifies a set of states of the world indistinguishable to the shopper i.

An equilibrium of the model corresponds to a fixed point of the individual best-weight mapping given by equation (28), which describes a cubic equation. In practice, there are as many equilibria as intersections between $a^*(a)$ and 45% line. In the two top panels of Figure 2 we plot the best weight function for $\beta \to 1$ as a function of the actual weight against the 45% line in the case $\gamma > 1/2$ in panel (a) and $\gamma < 1/2$ for panel (b) for two different values of σ. Irrespective of the number of intersections, the best weight function exhibit some invariant properties.[11] Let us start by looking to the case of a unique equilibrium first.

Unique Equilibrium

When local housing prices respond more strongly to local conditions, i.e. $\gamma > 1/2$ the model exhibit a unique equilibrium.

Proposition 1. *For $\gamma \geq 1/2$ and any $\beta \in (0,1)$, there exists a unique REE equilibrium for $\hat{a} = a^*(\hat{a}) \in (0, \gamma^{-1})$. Moreover, $\lim_{\sigma\to\infty} \hat{a} = 0$ and $\lim_{\sigma\to 0} \hat{a} = \gamma^{-1}$ with $\partial\hat{a}/\partial\sigma < 0$.*

Proof. Given in Appendix A.5. □

[11] First, an equilibrium weight has to be positive, $a^*(0) > 0$, since the local housing price always positively correlate with μ_i. Second, the individual best weight is increasing in the average weight, $a_i'(a) < 0$, when $a \in (0, (1-\gamma)^{-1})$ as larger a magnify noise into the price signal, and increasing, $a_i'(a) > 0$, when $a \in ((1-\gamma)^{-1}, \gamma^{-1})$ as larger a shrinks noise into the price signal. In particular, note that at the limit $a \to (1-\gamma)^{-1}$ the price signal is uninformative so that $a^*(a) = 0$. Third, the best weight function is decreasing in the size of the variance of aggregate productivity shocks $\partial a^*(a)/\partial\sigma \geq 0$ as more noise into the price signal decreases the optimal weight.

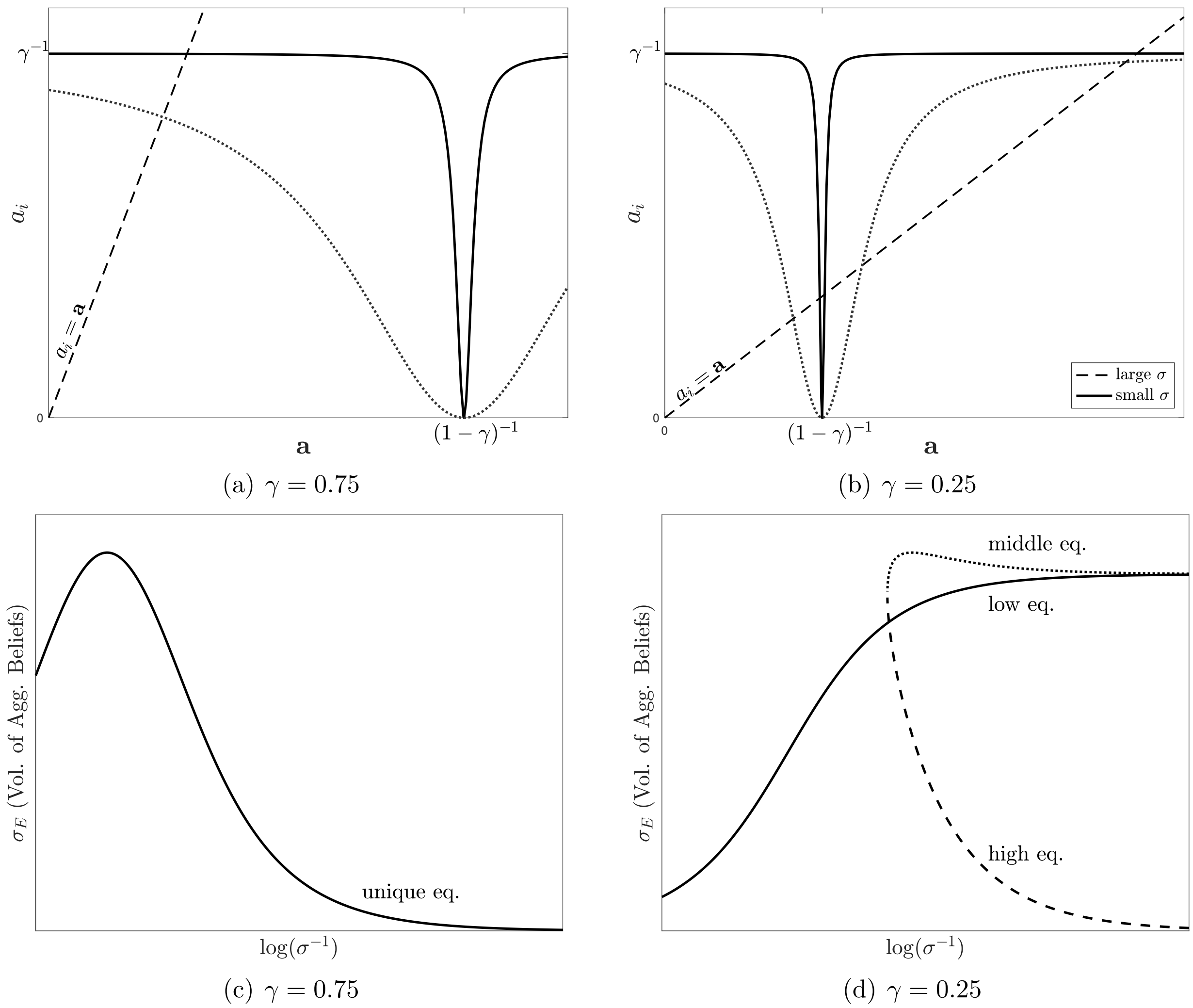


Figure 2: Top panels illustrate the best weight function $a^*(a)$ in a case with unique equilibrium (a) and with multiplicity (b) for two different values of σ. Bottom panels show the evolution of aggregate consumption volatility in a case with a unique equilibrium (c) and with multiplicity (d) as a function of an inverse measure of σ.

Proposition 1 states that when the aggregate component receives relatively low weight in the price signal, the model exhibits uniqueness of equilibrium. This is evident from the top-left panel of Figure 2. It shows that the best weight function is a decreasing function of the actual weight in the relevant range implying that a unique equilibrium exists. We demonstrate in appendix that a lower β monotonically decreases the best weight function, so uniqueness is preserved for any $\beta < 1$.

Note that as the average reaction approaches that value from below the precision of the price signal decreases. i.e. there is a *substitutability* between average weight and precision $\tau'(a) < 0$, which is peculiar of our model. In words, as the average weight increases, any initial aggregate shock has a higher impact on expectations because of the expectational feedback, making the signal noisier. In other examples of endogenous signals in the literature, e.g. Amador and Weill (2010), expectational feedback amplifies the common fundamental instead that the common noise, so that there is complementarity between average weight and precision. The presence of substitutability also explains why we can establish uniqueness of the equilibrium irrespective of the variance of the exogenous noise, which is typically not a property of setting that exhibit complementarity.

Moreover, an important feature of our unique equilibrium is that an amplification of the noise term obtains as it variance shrinks. Intuitively, for any given a, the smaller noise term receives the same amplification and the signal is less noisy. Hence $a^*(a)$ must rise. But as a increases, the impact of the smaller zeta on expectation rises: i) directly because of the higher weight and ii) indirectly via the expectational feedback, as a increases.

Panel (c) of figure 2 plots the variance of aggregate beliefs as a function σ and shows that in the case of the unique equilibrium amplification , although substantial, does not grow faster than the shrinking of the shock size, so that in the limit $\sigma \to 0$ average beliefs exhibits no fluctuations.

Multiple Equilibria

When local housing prices respond strongly to aggregate conditions, i.e. $\gamma < 1/2$, the feedback loop between demand and commodity input prices can be so strong that multiple equilibria exist. Proposition 2 summarizes this result.

Proposition 2. *For $\gamma < 1/2$ there always exists a low REE equilibrium for $a_- = a^*(a_-) \in (0, (1-\gamma)^{-1})$; in addition, there exists a threshold $\bar{\sigma}^2(\beta)$ with $\partial\bar{\sigma}^2(\beta)/\partial\beta \geq 0$ such that, for any $\sigma^2 \in (0, \bar{\sigma}^2(\beta))$, a middle and a high REE equilibrium also exist for $a^*(a_\circ) = a_\circ$ and $a^*(a_+) = a_+$, respectively, both lying in the range $((1-\gamma)^{-1}, \gamma^{-1})$. In the limit $\sigma^2 \to 0$:*

i. the high equilibrium converges to a point with no aggregate volatility:

$$\lim_{\sigma^2 \to 0} a_+ = \min\left(\frac{1}{\gamma}, \frac{1}{1-\gamma}\right) \qquad \lim_{\sigma^2 \to 0} \sigma_c^2(a_+) = 0.$$

ii. the low and middle equilibria get the same value and exhibit non-trivial aggregate volatility:

$$\lim_{\sigma^2 \to 0} a_{\circ,-} = \frac{1}{1-\gamma} \qquad \lim_{\sigma^2 \to 0} \sigma_c^2(a_{\circ,-}) = \frac{\gamma(1-2\gamma)}{(1-\gamma)^2}. \tag{29}$$

Proof. Given in Appendix A.5. □

The best weight function in this case is plotted in panel (b) of Figure 2. It shows that the best weight function yields three intersections with the 45% line provided the variance of productivity shocks σ is sufficiently low, otherwise a unique equilibrium exists. We demonstrate in the proof that a lower β is equivalent to consider a larger σ, so $\beta \to 1$ turns out to be the case most favorable towards multiplicity.

The key difference with the previous case is that now the value for which the price signal gets completely uninformative, i.e. $a = (1-\gamma)^{-1}$, is strictly lower than the perfect information value $a = (1-\gamma)^{-1}$. Everything we discussed for the unique equilibrium equally holds now relative to the range $[0, (1-\gamma)^{-1}]$: i) there is substitutability between reaction and precision ii) the equilibrium features larger amplification of the exogenous shocks as they get smaller in variance, iii) a unique equilibrium exist no matter how small is σ. However, it is clear that at the limit $\sigma \to 0$ this equilibrium is distinct from the perfect information equilibrium, which obtains at the strictly higher value $a = \gamma^{-1}$: a multiplicity obtains at the limit!

The question in this case is whether responsiveness grows fast enough to offset shrinking shock. But the variance of the average expectation cannot go to zero, or else we have perfectly-revealing signal and $a = \gamma^{-1}$; since this value is not feasible in the rand $0, (1-gamma)^{-1}$, it must be that signal is not perfectly revealing in the limit, which can only happen with infinite amplification. Panel (d) of figure 2 illustrates the statement. Consumption volatility in the “high” equilibrium case converges to zero as σ^{-1} goes to infinity. By contrast, consumption volatility in the “middle” and “low” equilibria converges to a positive, finite number in the limit of zero variance of productivity shocks.

The nature of our multiplicity is therefore different with the more familiar complementarity mechanism studied in the literature (see literature review in the introduction for more references). Note that, also in our case in the limit of vanishing exogenous noise the signal is fully informative even when agents do not react to it. However with complementarity, i.e. precision strictly increases with average reaction, this is a sufficient condition to the the perfect information equilibrium being the unique equilibrium in the limit (as in Amador and Weill (2010)).

Surprisingly, the low and middle limit equilibria have the same stochastic properties as the extrinsic sentiment equilibrium described by Benhabib et al. (2015). In our case, however, fluctuations are driven by infinitesimally-small fundamental shocks, whose realizations coordinate sizable fluctuations in agents’ expectations. We elaborate on the connection with Benhabib et al. (2015) in Section 5.3.

4 Business cycle fluctuations

In this section, we show that many qualitative features of the business cycle can be explained by our model. Our analysis also suggests that the learning-from-prices mechanism can qualitative change the comovement properties of fundamental shocks, implying that many common strategies for disentangling shocks may give misleading results when learning from prices is important.

4.1 Public News

Before proceeding to our analysis, we introduce an anticipated (common-knowledge) component of aggregate housing productivity. The decomposition of productivity into a forecastable and surprise component serves two purposes. First, it allows us to isolate the effects of the learning channel in our model, as the forecasted component of productivity transmits as standard supply-side shock. Second, by combining the responses of the economy to forecasted and surprise productivity shocks, the model can generate a rich and realistic correlation structure among business cycle variables.

Formally, we assume that the housing productivity innovation is composed of two independently distributed components

$$\zeta = \zeta^n + \zeta^s;$$

with $\zeta^n \sim (N, \sigma^2_{\zeta^n})$, $\zeta^s \sim (N, \sigma^2_{\zeta^s})$ and $\sigma^2_{\zeta^n} + \sigma^2_{\zeta^s} = \sigma^2_\zeta$. The first term ($\zeta^n$) is a "news" component; it corresponds to the forecastable component of productivity, and is commonly known to all agents before their consumption choices are made. The second term (ζ^s) is the "surprise" component; it is unknown to shoppers and they seek to forecast it using their observation of prices.[12] For future reference, let $\sigma^2_n \equiv \sigma^2_{\zeta^n}/\sigma^2_\mu$, and $\sigma^2_s \equiv \sigma^2_{\zeta^s}/\sigma^2_\mu$ be the normalized variances of the news and surprise components of productivity respectively.

Only modest modifications are necessary to characterize equilibrium in this general case. Shoppers use the forecasted component to refine the information contained in the price signal by "partialing-out" the known portion of productivity. In particular, we can rewrite households' expectations as

$$E[\mu_i|p_i] = a(s_i - (1-\gamma)\zeta^n), \tag{30}$$

where $s_i - (1-\gamma)\zeta^n$ represents a new signal embodying the information available to the individual household, after she has controlled for the effect of ζ^n. It follows that the equilibrium values $\hat{a}$ and the conditions for their existence are isomorphic to the ones in the baseline economy once σ^2_s takes the place of σ^2. For a given total variance of productivity, σ^2, we can now span the space between two polar cases, one in which productivity occurs as a pure "surprise" to the case where the productivity shock is common-knowledge "news". Thus, overall co-movements in the economy will depend on the balance of forecastable and surprise productivity, as well as the overall size of these shocks relative to local conditions.

[12] Chahrour and Jurado (2018) show that this information structure is equivalent to assuming that agents observe a noisy aggregate signal, $s = \zeta + \vartheta$.

Table 1: Business Cycle Comovements

	GDP	Cons	Hours	Res. Inv.	House Pr	Constr. Pr	Constr. TFP
GDP	1.00	0.93	0.88	0.64	0.51	0.53	-0.17
Cons		1.00	0.80	0.65	0.47	0.47	-0.02
Hours			1.00	0.50	0.54	0.66	-0.35
Res. Inv.				1.00	0.62	0.37	-0.11
House Pr					1.00	0.81	-0.37
Constr. Pr						1.00	-0.43
Constr. TFP							1.00

Note: Data are real per-capita output, real per-capita consumption, per-capita hours in the non-farm business sector, real per-capita residential investment, Case-Schiller real house price index, real price of residential investment, and relative TFP in the construction sector from the World KLEMS databse (http://www.worldklems.net/data.htm). All data are annual log-levels, HP-detrended using smoothing parameter $\lambda = 10$. Date range: 1960 to 2018, except for construction TFP which ends in 2010.

4.2 Demand-driven Fluctuations

Table 1 summarizes simple unconditional correlations between business cycle variables in US data. Although this exercise is very simple, the table summarizes several facts that have been documented by more sophisticated empirical analysis. The table indicates that business cycles are dominated by demand-driven fluctuations with business cycle variables, housing market prices, and residential investment all substantially co-moving. Meanwhile, structures productivity is at best weakly related to any of these variables.

The emergence of demand-driven fluctuations in the model can be seen intuitively by analyzing the aggregate demand and aggregate supply schedules in our economy. Using the aggregate version of equations (10), (11), (15) and (17), we can express the aggregate demand for consumption and aggregate demand and supply in the housing market as

$$c = \int E[\mu_i | p_i] di \tag{AD:C}$$

$$\delta = c - p, \tag{AD:H}$$

$$\delta = \frac{\alpha\gamma}{1-\alpha\gamma} p - \frac{\alpha(1-\gamma)}{1-\alpha\gamma}\zeta. \tag{AS:H}$$

respectively. Because of the learning channel we know that aggregate consumption shifts upwards in response to a correlated increase in price signals across island, i.e.

$$c = \int E[\mu_i | p_i] di = a(s - (1-\gamma)\zeta^n).$$

Bear in mind that the expression above means that c does not move with the news component of housing productivity as it does not enter into the inference (it is in fact removed from the price-signal), but only with the surprise.

To derive the implications shopper inference for housing demand, use $p = (1-\alpha)v + \alpha s$ and $v = c$ to express $s = (p + (1-\alpha)a(1-\gamma)\zeta^n)/((1-\alpha)a + \alpha)$. Substituting the expression for c into (AD:H) we get

$$\delta = \frac{\alpha(a-1)}{(1-\alpha)a+\alpha}p + \frac{\alpha a(1-\gamma)}{(1-\alpha)a+\alpha}\zeta^n. \tag{31}$$

When aggregate conditions do need feed into shoppers' beliefs ($a = 0$), equation (31) entails a standard downward-sloping aggregate demand relation in the housing market, and consumption and working hours that are invariant to housing sector productivity. In contrast, when learning from prices is sufficiently important—i.e. whenever a is larger than one—equation (31) shows that δ and p must comove in response to surprise shocks.

Moreover, substituting (11) and (13) into (15) we get

$$\int n_i^h di = \int \lambda_{it} - E[\lambda_i|p_i]di = c = \int n_i^c di. \tag{32}$$

Equation (32) implies that an increase in consumption corresponds to an increase in working hours in both sectors. This occurs because shoppers have correlated perceptions of the marginal value of consumption. In times of optimism, shoppers' spending increases but wages do not, so production increases.

To derive the model's implications for empirical measures of house prices, which typically include new and existing homes, we need derive the connection between p and the price of the total housing stock, p_t^H. In the Appendix, we show that the price of each vintage moves with shoppers expected Lagrangian, $p_{it|k} = -E[\lambda_{it}|p_{it}]$. This relationship is intuitive because the supply of past vintages cannot adjust, so that prices must absorb any change in expectations. We therefore find that $p_t^H = \kappa p_t + (1-\kappa)E[-\lambda_{it}|p_{it}]$ where $\kappa \in (0,1)$ is the steady state fraction of new houses in the total housing stock.

It is then straightforward to demonstrate the following.

Proposition 3. *For σ_s^2 sufficiently small, surprise aggregate productivity shocks drive positive comovement of consumption, employment (in both sectors), residential investment, prices for* *new and existing housing, commodity prices, and the price of land.*

Proof. The results follows from continuity of the best-response function, and the observation that $\lim_{\sigma_s \to 0} a > 1$ in the case of uniqueness ($a \to 1/\gamma$) or multiple equilibria ($a \to 1/(1-\gamma)$ or $1/\gamma$). □

In sum, our model exhibits comovements of aggregate business cycle variables in response to sufficiently small productivity shocks, *in any equilibrium* and for *any configuration of parameters*. To an outside observer, the economy would appear to be buffeted by recurrent shocks to aggregate demand. The reason productivity shocks need to be sufficiently small is that, as aggregate productivity shocks shrink, the informational value of the price signal rises, leading agents' beliefs about their local conditions to respond more strongly to it. Stronger aggregate effects on beliefs eventually lead the informational channel of prices to dominate, so that consumption increases in response to higher house prices. In this way, learning from prices provides a new mechanism for generating expectations-driven demand shocks in an economy hit only by fundamental shocks to productivity.

With a few additional lines of algebra, it is possible to solve for consumption, residential investment, and the price of new housing as functions of shocks and the equilibrium inference coefficient:

$$c = \frac{a(1-\gamma)}{1-a(1-\gamma)}\zeta^s \tag{33}$$

$$p = \alpha(1-\gamma)\zeta + (1-\alpha\gamma)c \tag{34}$$

$$\delta = -\alpha(1-\gamma)\zeta + \alpha\gamma c. \tag{35}$$

The expressions above are useful for disentangling the direct effects of productivity from the learning channel. Equation (33) shows that a correlated mistake due to a surprise in aggregate productivity moves consumption. Equations (34) and (35) show how this change in beliefs transmits into the housing market, moving prices for new housing and residential investment in the same direction. The same equations also show that productivity shocks affect the housing market through a neoclassical channel, driving prices and quantities in opposite directions. As a result, consumption is correlated with the housing market only via the surprise component of productivity, whereas prices and quantities in the housing market itself are correlated via the news component.

We note at this point that unconditional variance and conditional variance are not the same, since ... For example...

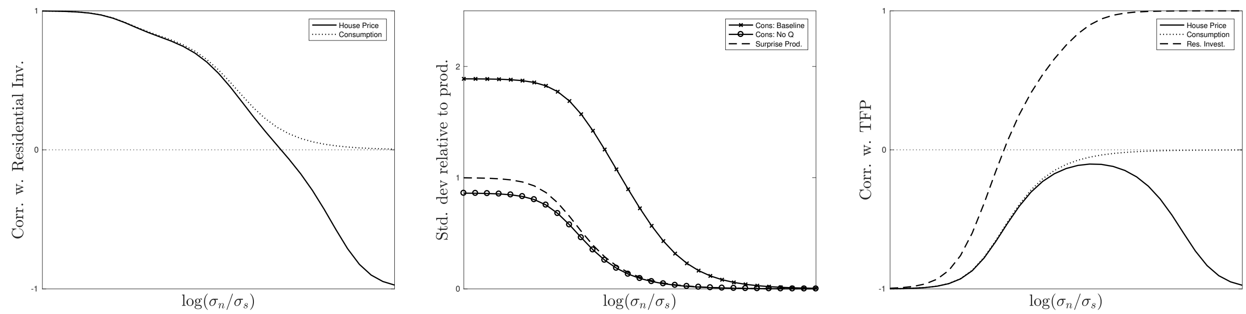
4.3 Business cycles under unique equilibrium

In this section, we discuss the model's business cycle properties when it has a unique equilibrium. We organize the discussion around three main pictures illustrating the business cycle co-movements, amplification and the relation with productivity shocks implied by the model. The objective is to show the potential of our simple model to qualitatively account for several empirical patterns that typically require more cumbersome frictions to match.

While we do not undertake a full quantitative evaluation of the model, we wish to demonstrate the mechanism can be very powerful for reasonable parameterizations of the model. To this end, we calibrate a set of parameters to standard values and/or long run targets in the data. We set the model period to year. We set $\beta = 0.96$ consistent with an annual real interest rate of roughly 4%. We set $\phi = 0.66$, to be consistent with 2013-2014 CPI relative importance weight placed on shelter. Estimates of η, the elasticity of local labor demand, range in the literature from below one (Lichter et al., 2015) to above twenty (Christiano et al., 2005). We use $\eta = 2$ as a baseline, and note that the aggregate effects of changing η can be offset one-for-one by changing the volatility of local productivity.

For the housing sector, we follow Davis and Heathcote (2005) in fixing $\alpha = 0.89$ to match the evidence of that land accounts about 11% of new home prices.[13] We pick the residential investment labor share parameter $\gamma = .526$ by computing the ratio of labor input costs to materials and energy costs in the construction sector, using the Bureau of Labor Statistics KLEMS data from 1997-2014. Finally, we select the volatility of local productivity shocks relative to aggregate shocks to housing productivity $\text{std}(\hat{\mu}_i)/\text{std}(\zeta) = 10$, implying $\sigma = .238$.

[13] For existing homes, Davis and Heathcote (2007) find that land prices accounts for a larger portion of home prices.



(a) Comovements (b) Amplification (c) Relation with productivity

Figure 3: Panels illustrate correlation and the unconditional volatility of business cycle variables as a function of the ratio between volatility of the forecastable and non forecastable component for the baseline case of $\gamma = 0.526$.

Comovement in business cycle variables

Figure 3 plots the correlations and volatilities of several variables in the economy. On the horizontal axis of each panel we vary the ratio between of the forecastable and non-forecastable component of productivity, going from pure "surprise" on the left to pure "news" on the right, holding the total variance of the shock constant.

Panel (a) of the figures plots the correlation of hours worked and house prices with residential investment. Towards the left of the panel, when productivity is mostly unanticipated, our learning channel dominates: residential investment, house prices and consumption all perfectly comove. Given the results derived above, this also implies comovement in hours in both sectors, the average price of land, and the price of commodities.

By contrast, when productivity is largely commonly knowledge, prices and quantities in the housing market exhibit the negative correlation associated with supply-fluctuations, while consumption does not move. Therefore, the more housing productivity is anticipated, the more the economy behaves like a standard real business cycle model. In between these two extremes, the model generates positive but imperfect correlations, consistent with the data report in Table 1.

Amplification

What is the role of the endogeneity of the signal in generating amplification in the model? Panel (b) of Figure 3 plots the standard deviation of consumption relative to that of aggregate productivity, as a function of the share of productivity that is forecastable. The panel contrasts two cases (i) the baseline model and (ii) the counter-factual case in which the price signal, $\tilde{s}_i = \gamma\mu_i + (1-\gamma)\zeta^s$, excludes its dependence on q. This comparison is useful to evaluate the role of q in amplifying the impact of surprise shocks. To highlight this aspect we also draw the standard deviation of the surprise component of productivity, which by construction falls from one to zero going from left to right.

The comparison is striking. With a completely exogenous price signal, the volatility of consumption, while positive, would be strictly less than the volatility of the surprise component of productivity. This is not the case for our baseline calibration, when the signal is endogenous. The surprise component is amplified substantially, such that consumption remains more volatile than aggregate productivity even when more than 90% of productivity fluctuations are anticipated (near the middle of the horizontal axis.)

The source of amplification can also be seen in our analytical results via equation (33). That equation shows there is a range of parameters where aggregate consumption responds more than one-to-one to productivity shocks.[14] Note that this is a peculiar feature of having the price signal with endogenous precision and, in particular, of having the commodity price q entering in local housing prices. One can easy verify that, with a constant q, the reaction of expectations to productivity shocks cannot exceed the unity, provided $\gamma > 1/2$.[15]

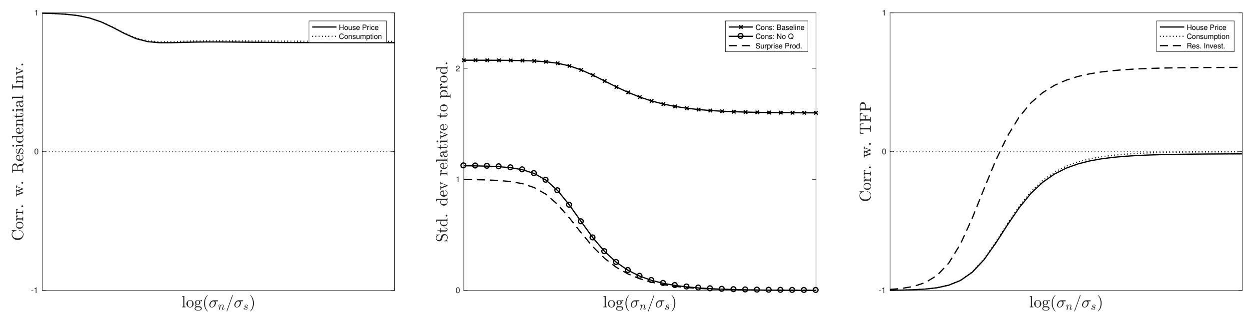
Relationship with construction TFP

In our model, the noise in peoples' inference comes from a fundamental shock: productivity in the housing sector. One major advantage of our approach to microfounding information is that it provides testable implications about how beliefs fluctuations should relate to this, in principle, measurable economic fundamental. In this section, we explore whether the data are indeed consistent with these implications. Our goal is not to argue that construction productivity is the only driver of beliefs in practice, but instead to show that the model's implications are generally consistent with one direct measure of construction productivity.

To this end, the last column of Table 1 compares productivity in the construction sector

[14] This occurs when $\hat{a} \in (1/2(1-\gamma), 1/\gamma)$ with $\gamma \in (1/2, 2/3)$ then $\partial c/\partial\zeta^s > 1$.

[15] To see, suppose that q is fixed, so that the price signal corresponds to $s_i(0)$ in (27) having a precision $\tau(0)$. Then $E[\mu_i|s_i(0)] = \frac{1}{\gamma}\frac{\tau(0)}{1+\tau(0)}s_i(0)$, so that $\frac{\partial E[\mu_i|s_i(0)]}{\partial\zeta} = \frac{1-\gamma}{\gamma}\frac{\tau(0)}{1+\tau(0)} < 1$.



(a) Comovements (b) Amplification (c) Relation with productivity

Figure 4: Panels illustrate correlation and the unconditional volatility of business cycle variables as a function of the ratio between volatility of the forecastable and non forecastable component for the case of $\gamma = 0.45$.

relative to aggregate productivity — the data analogue to ζ — using the USA KLEMS productivity data assembled by Jorgensen et al. (2012). Overall, the column shows that this measure of housing-sector productivity is negatively, but weakly, correlated with business cycle variables. Most notably, residential investment is somewhat negatively correlated with this measure of productivity, a result that would be difficult to reproduce in a full information neoclassical environment.[16]

Panel (c) of Figure 3 illustrates the correlations of residential investment, the price of housing, and consumption as a function of the ratio between the volatilities of the forecastable and non forecastable component for the same calibration of the other panels. As in the data, the equilibrium correlation between business cycle variables depend on the fraction of anticipated productivity, and is in general not perfect. Correlations with total productivity are imperfect because the two components of productivity – surprise and news – are transmitted very differently in the economy. In particular, so long as a sufficient portion of productivity is unanticipated, all of these variables are negatively correlated with productivity. When instead productivity is mostly common knowledge, business variables exhibit the classical supply-side fluctuations, consumption does not move, and residential investment and housing prices move in opposite directions.

4.4 Multiple equilibria: supply shocks or animal spirits?

In this section, we explore the properties of one of the equilibria when $\gamma < 1/2$ as an illustration of the virtually unbounded amplification power of our mechanism. We focus on the low equilibrium, characterized by a_{-} in Proposition 2. As the proposition states, this equilibrium generates sizable fluctuations in expectations even in the limit of $\sigma^s \to 0$. We focus on this equilibrium because it turn out to be learnable in the sense of the adaptive learning literature (see the proof in Section 5.4.)

In Figure 4 we present correlations and amplification plots for the case of the low equilibrium, changing only $\gamma = 0.45$ with respect to our baseline calibration. Panel (b) shows that, in contrast to our original calibration, consumption remains roughly twice as volatile as realized productivity even as the variance of its surprise component goes to zero. This happens because even infinitesimal surprises drive large fluctuations in beliefs. Note also that the endogeneity of the price signal is crucial to this result: if inference were based on the counter-factual signal $\tilde{s}_i$ that excludes q, the model would deliver some fluctuations in consumption, but these would disappear as the size of the surprise component in productivity shrinks.

To understand the economic reasons behind this powerful amplification, note that for a approaching $1/(1-\gamma)$, the slope of the aggregate demand in the market for new housing (31) approaches the one for the aggregate supply (AS:H). This configuration delivers extreme price-quantity comovements even after vanishing shocks to either curve.

Since belief fluctuations do not disappear with σ_s in this parameterization of the model, it has very different implications for the correlation of consumption and house prices with residential investment. In particular, these variables all remain positively correlated even when nearly all of realized productivity is anticipated. ~~The relation of these variables with productivity is also affected. As more of productivity is anticipated, the correlation of consumption and the price of existing housing, which driven by the expectational component, approaches zero.~~ I don't think I understand these lines yet. On the other hand residential investment does not go full way to negative correlation as the expectation component continues to explain fraction of its volatility.

[16] Fernald et al. (2014) also find evidence that investment-specific productivity has contractionary effects.

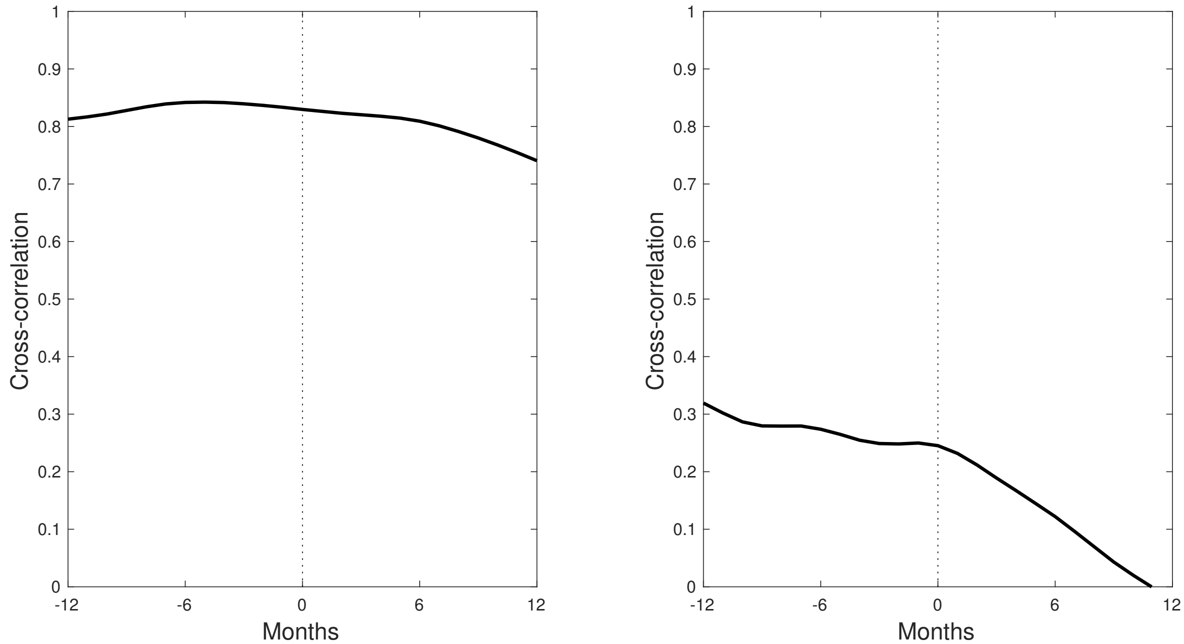
In the limit of a small surprise component, housing prices and residential investment are moved by infinitesimal productivity surprises. An econometrician looking at the data generated by our model would be unable to measure such small changes in fundamentals and would probably conclude that the housing market is moved by animal spirits in the vein of Burnside et al. (2016); Shiller (2007) or sentiments as in Benhabib et al. (2013). Our model shows how demand-driven waves can be the result of extreme amplification of small fundamental shocks sustained by the feedback loop of learning from prices. The difference, which from an empirical point of view may seem irrelevant, is instead sharp from a theoretical point of view: the degree of optimism or pessimism in the economy in our model is actually determined by (potentially small) fundamental changes rather being totally erratic or “animal”.

4.5 Evidence from survey data

The essential feature of our model is that people’s expectations about their future prospects depend on their own market experiences, particularly in the housing market. We provide here one piece of evidence from survey data that suggests this mechanism may be important in the data.

To this end, we used evidence from the Michigan Survey of Consumer Expectations. Survey participants are asked each month about (i) the perceptions of local house price growth over the last year (ii) what they expect regarding their real income growth over the coming year and (iii) whether they have heard good or bad news about overall economic conditions in the economy. The survey then produces index numbers from the answers to these questions, essentially subtracting those who experienced/expect/heard about negative outcomes from those who have experienced/expect/heard positive ones.

Panel (a) of Figure 5 plots the autocorrelation structure of people’s current expectations about future income, with respect to their recent experiences in the housing market. Negative numbers on the horizontal axis reflect past responses to the housing experience question, while positive numbers reflect future response to the housing experience question. The figure shows that their two series are *extremely* strongly correlated, with past housing experiences leading income expectations by roughly half a year (as measured by the peak correlation.) This result suggests a strong connection between peoples’ past experiences in the housing market and



(a) Income expectations at time t vs house price experiences at time $t+h$.

(b) Income expectations at time t vs economic news heard $t+h$.

Figure 5: Auto-correlations of survey measure of own income expectations with respect to own house price experience (panel a) and with respect to news heard about the economy (panel b).

their expectations about their own income, exactly as our model predicts.

By contrast, Panel (b) of the figure plots the autocorrelation structure of peoples' current expectations of their own income with respect to what they report having heard about aggregate economic developments. The correlation in this picture is *much* smaller than in Panel (a), suggesting that whatever people have heard about the aggregate economy (if they've heard anything) plays a much smaller role in forming peoples' expectations about their own prospect.

While these results are far from dispositive on the merits of our mechanism, we think they provide some initial evidence that the learning from price mechanism is plausible in the context of housing.

5 Extensions

This section presents several extensions to the basic setup, showing that the insights of the main mechanism are robust to various modeling details. In Section 5.1, we explore the impact of contemporaneous and future aggregate shocks to consumption production, which in our model is equivalent to a shock in housing spending. In Section 5.2, we allow households to observe additional private information about local conditions and show that our results do not rely on excluding exogenous sources of information. In Section 5.3, we explore whether extrinsic noise may drive fluctuations jointly with aggregate productivity and conclude that this is never the case. Finally, to address concerns about the plausibility of learning from prices equilibria, Section 5.4 studies the issue of stability under adaptive learning for the various equilibria of the baseline model.

5.1 Aggregate shocks in consumption production

For this extension, we modify the production function of the consumption sector to allow for aggregate shocks to labor productivity,

$$Y_t = \tilde{\zeta}_t^c \left(\int e^{\tilde{\mu}_{it}/\eta} {N_{it}^c}^{1-\frac{1}{\eta}} di \right)^{\frac{1}{1-\frac{1}{\eta}}} . \tag{36}$$

The consumption productivity shock is $\zeta_t^c \sim N(0, \sigma_\zeta)$ and is an i.i.d. disturbance. To simplify our exposition, we focus on time t and assume that workers in island i, but not shoppers, know $\{\zeta_{t+1}^c, \zeta_t^c, \mu_{it+1}\}$ and abstract from the presence of other aggregate shocks. A few lines of algebra shows that

$$\lambda_{it} = -\omega_\mu \mu_{it+1} - \omega_b b_{it} - \zeta_t^c \tag{37}$$

We note immediately that a contemporaneous productivity shock in consumption is equivalent to an increase in consumption spending (measured in consumption units). Given the properties of log utility, an increase in consumption spending induces an increase in housing spending as well. In other words, a productivity shock to consumption production is equivalent to an exogenous demand shock in the housing sector.

Including the future realization of aggregate productivity helps to clarify that the model cannot generate demand shocks in the form of news about aggregate productivity as in Lorenzoni (2009). To see that notice that in this case,

$$r_t = \lambda_t - \lambda_{t+1} = -\zeta_t^c + \zeta_{t+1}^c. \tag{38}$$

Thus, the real interest rate adjusts to equalize the return on savings in the two periods. Therefore, the anticipation of higher productivity in the future has no effect on consumption choices today. This is a feature that our model shares with frictionless real economies, as Angeletos (2018) clarifies. In contrast to our approach, in Lorenzoni (2009) news about future productivity create a demand shock because of the presence of nominal rigidities and monetary policy is suboptimal. A corollary to this result is that no current variable in the economy, other than the real interest rate moves with anticipated aggregate consumption shocks, so shopper will not be able to learn them in advance.

Contemporaneous consumption productivity shocks, by contrast, influence the marginal value of current budget resources. In particular, higher current productivity decreases the marginal value of households resources pushing up the real wages demanded by workers. In appendix we show that the price signal in this case is:

$$s_{it} = \gamma(\mu_{it+1} + \zeta_t^c) + (1-\gamma)\int E[\mu_{it+1} + \zeta_t^c | s_{it}]di, \tag{39}$$

where, as in the main text, we have presented the case $\lim_{\beta \to 1} \omega_\mu = 0$ and normalized $\tilde{\mu}_{it+1}$ by ω_μ.

One again, correlated fundamentals generate confusion between the idiosyncratic and common components of the signal. As before, the individual expectation of a household of type i is formed according to the linear rule $E[\mu_{it+1} + \zeta_t^c | s_{it}] = a^* s_i$. Hence, the signal embeds the average expectation, which again causes the precision of the signal to depend on the average weight a. Following the analysis of the earlier section, the realization of the price signal can be rewritten as

$$s_i = \gamma\mu_{it+1} + \frac{\gamma}{1-a(1-\gamma)}\zeta_t^c, \tag{40}$$

where a represents the average weight placed on the signal by other shoppers. The average expectation is given by

$$\int E[\mu_{it+1} + \zeta_t^c | s_{it}]di = \frac{\gamma a}{1-a(1-\gamma)}\zeta_t^c, \tag{41}$$

which is slightly different from (26). The shopper's best response function is now given by

$$a^*(a) = \frac{1}{\gamma}\left(\frac{(1-a(1-\gamma))^2 + (1-a(1-\gamma))\,\sigma^2}{(1-a(1-\gamma))^2+\sigma^2}\right). \tag{42}$$

While the best-response function in equation (42) is slightly different than in (28), the characterization of the limit equilibria is identical.

Proposition 4. *In the limit $\sigma_\mu^2 \to 0$, the equilibria of the economy converge to the same points as the baseline economy. For $\gamma > 1/2$: there exists a unique equilibrium $\hat{a}$ such that $\lim_{\sigma^2_{\zeta^c}\to 0} a^\mu = \gamma^{-1}$ with $\lim_{\sigma^2_{\zeta^c}\to 0} \sigma_c^2 = 0$. For $\gamma < 1/2$ instead three equilibria exist such that*

$$\lim_{\sigma^2_{\zeta^c}\to 0} \hat{a} \in \{a_-, a_\circ, a_+\} \qquad \textit{with} \lim_{\sigma^2_{\zeta^c}\to 0} \sigma_c^2(\hat{a}) \in \{\sigma_c^2(a_-), \sigma_c^2(a_\circ), \sigma_c^2(a_+)\}.$$

Proof. Postponed to appendix. □

The proposition has a straightforward intuition. In the limit of small productivity shocks, it does not matter if those perturbations emerge from the consumption or housing sector. Hence, Proposition 3 follows identically, and the proof proceeds in parallel with only obvious algebraic substitutions.

The important difference with respect to our baseline model is that, in this case, our mechanism is amplifying an otherwise smaller demand driven fluctuation. In other words, under perfect information a shock to consumption productivity would already translate into a smaller, but still correlated, movement in business cycle variables. To see this, rewrite aggregate consumption of residential investment and consumption in the case of perfect information: $c = \zeta_t^c$ and $h_t = -\lambda_t - p = (1-\gamma)\zeta_t^c$, which says that residential investment, the price of new housing and consumption move together even under perfect information. Therefore, having focused our main discussion on the case of aggregate productivity shocks in the housing markets has the merits of showing that, not only our mechanism is able to generate high amplification of fundamental shocks, but also can dramatically affect the transmission of shocks in the economy.

5.2 Signal extraction problem with private signals

Here we show that the signal extraction problem, and corresponding equilibria, are not qualitatively affected by the availability of a private signal about the local shock. Instead, the addition of private information maps into our analysis of Section 3.3 as an increase in the relative variance of aggregate shocks.

Let us assume that a household $j \in (0,1)$ in island i has a private signal

$$\omega_{ij} = \mu_i + \eta_{ij} \tag{43}$$

where $\eta_{ij} \sim N(0, \sigma_\eta)$ is identically and independently distributed across households and islands. In this case, households form expectations according to

$$E[\mu_i | p_i, \omega_{ij}] \quad = \quad a\left(\gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|p_i,\omega_{ij}]di - \zeta\right)\right) + b\left(\mu_i + \eta_{ij}\right),$$

where b measures the weight given to the additional private signal. Averaging out the relation above and solving for the aggregate expectation gives

$$\int E[\mu_i|p_i,\omega_{ij}]di \quad = \quad -\frac{a\left(1-\gamma\right)}{1-a\left(1-\gamma\right)}\zeta,$$

which is identical to (25). However, now we need two optimality restrictions to determine a and b. These are

$$E[p_i(\mu_i - E[\mu_i|p_i,\omega_{ij}])] = 0 \quad \Rightarrow \quad \gamma\sigma_\mu - a\left(\gamma^2\sigma_\mu + \frac{(1-\gamma)^2}{\left(1-a\left(1-\gamma\right)\right)^2}\sigma_\zeta\right) - b\gamma\sigma_\mu = 0,$$
$$E[\omega_{ij}(\mu_i - E[\mu_i|p_i,\omega_{ij}])] = 0 \quad \Rightarrow \quad \sigma_\mu - a\gamma\sigma_\epsilon - b\left(\sigma_\mu + \sigma_\eta\right) = 0,$$

which identify the equilibrium a and b such that each piece of information is orthogonal with the forecast error. Solving the system for a, we get a fix point equation written as

$$a = \frac{\gamma}{\gamma^2 + \frac{(1-\gamma)^2}{(1-a(1-\gamma))^2}\frac{\sigma_\mu+\sigma_\eta}{\sigma_\eta}\frac{\sigma_\zeta}{\sigma_\mu}}. \tag{44}$$

For $\sigma_\eta \to \infty$, the right-hand side of the relation above matches (28). In particular, it follows that a lower σ_η in (44) is equivalent to considering a larger σ_ζ in (28). The analysis of the baseline model thus applies directly to this generalization, and small amounts of exogenous private information do not qualitatively change any of our earlier results.

5.3 Relation with Sentiments

A natural question, given the results in Proposition 2, is whether errors driven by extrinsic shocks can coexist with the fundamental-driven fluctuations in aggregate beliefs captured by our model. The next proposition demonstrates that, in fact, *extrinsic* sentiments are always crowded-out by common shocks to productivity.

Proposition 5. *Suppose that*

$$\int E[\mu_i|p_i]di = \phi_\zeta \zeta + \phi_\varepsilon \varepsilon,$$

where ϕ_ε is the equilibrium effect of an extrinsic sentiment shock, $\varepsilon \sim N(0, \sigma_{\varepsilon}^2)$, not related to fundamentals. Then, $\phi_\varepsilon = 0$ for any $\sigma^2 > 0$.

Proof. Suppose not, i.e. suppose that

$$\int E[\mu_i|p_i]di = \phi_\zeta \zeta + \phi_\varepsilon \varepsilon,$$

where ϕ_ε is the equilibrium effect of an extrinsic sentiment shock, ε, not related to fundamentals. Then, the price signal is equivalent to

$$p_i = \gamma \mu_i + (1-\gamma)\left((\phi_\zeta + 1)\zeta + \phi_\varepsilon \varepsilon\right)$$

Using the conjectured weights a^*, we have

$$\int a^* p_i di = a(1-\gamma)(\phi_\zeta + 1)\zeta + a(1-\gamma)\phi_\varepsilon \varepsilon$$

implying that

$$\phi_\zeta = a(1-\gamma)(\phi_\zeta + 1)$$
$$\phi_\varepsilon = a(1-\gamma)\phi_\varepsilon$$

which cannot both be true unless $\phi_\varepsilon = 0$. Notice that, differently from the case with multiple sources of signals studied by Benhabib et al. (2015) (section 2.8 page 565), in our case an aggregate shock (our productivity shock) shows up directly in the signal, which ensures determinacy of the average expectation. This is equivalent to say that the analysis in Benhabib et al. (2015) is not robust to the introduction of correlation (no matter how small) in the v_{jt} shocks appearing in their endogenous signals. □

The fundamental shock always dominates the extrinsic shock because its fundamental nature gives it two channels — one endogenous and one exogenous — through which it influences people's information and, therefore, their actions. Intuitively, conjecture that the average action reflects a response to both fundamental and extrinsic shocks. In equilibrium, agents respond to the average expectation, and therefore proportionally to the conjectured endogenous coefficients for each shock. But agents also respond to the exogenous component of the fundamental that appears in the price signal. Thus, any equilibrium must depend somewhat more-than-conjectured on the fundamental relative to the extrinsic shock. This guess and update procedure cannot converge unless the weight on the extrinsic shock is exactly zero.

This logic highlights the fragility of the extrinsic version of sentiments, which are coordinated by endogenous signal structures. For, any shock which tends to coordinate actions for exogenous reasons will also benefit from the self-reinforcing nature of learning, thereby absorbing the role of belief shock for itself. Indeed, as we have shown above, the same equilibria emerge if *local* shocks μ_i have any common component.

5.4 Stability analysis

Here, we examine the issue of out-of-equilibrium convergence, that is, whether or not an equilibrium is a rest point of a process of revision of beliefs in a repeated version of the static economy. We suppose that agents behave like econometricians. At time t they set a weight $a_{i,t}$ that is estimated from the sample distribution of observables collected from past repetitions of the economy.

Agents learn about the optimal weight according to an optimal adaptive learning scheme:

$$a_{i,t} = a_{i,t-1} + \gamma_t \; S_{i,t-1}^{-1} \; p_{i,t} \left(\mu_{i,t} - a_{i,t-1} p_{i,t}\right) \tag{45}$$

$$S_{i,t} = S_{i,t-1} + \gamma_{t+1} \left(p_{i,t}^2 - S_{i,t-1}\right), \tag{46}$$

where γ_t is a decreasing gain with $\sum \gamma_t = \infty$ and $\sum \gamma_t^2 = 0$, and matrix $S_{i,t}$ is the estimated variance of the signal. A rational expectations equilibrium $\hat{a}$ is a locally learnable equilibrium if and only if there exists a neighborhood $\digamma(\hat{a})$ of $\hat{a}$ such that, given an initial estimate $a_{i,0} \in \digamma(\hat{a})$, then $\lim_{t\to\infty} a_{i,t} \overset{a.s}{=} \hat{a}$; it is a globally learnable equilibrium if convergence happens for any $a_{i,0} \in \mathbb{R}$.

The asymptotic behavior of statistical learning algorithms can be analyzed by stochastic approximation techniques (see Marcet and Sargent, 1989a,b; Evans and Honkapohja, 2001, for details.) Below we show that the relevant condition for stability is $a_i'(a) < 1$, which can easily checked by inspection of Figure 2.

It turns out that the unique equilibrium is globally learnable, that is, no matter the initial estimate, revisions will lead agents to coordinate on the equilibrium. In case of multiplicity, the high and low equilibrium are locally learnable, whereas the middle equilibrium is not. Hence the middle equilibrium works as a frontier between the basins of attraction of the two equilibria.

To check local learnability of the rational expectations equilibrium, suppose we are already close to the resting point of the system. That is, consider the case $\int \lim_{t\to\infty} a_{i,t}\, di = \hat{a}$, where $\hat{a}$ is one of the equilibrium points $\{a_-, a_\circ, a_+\}$, and so

$$\lim_{t\to\infty} S_{i,t} = \sigma_s^2\left(\hat{a}\right) = \gamma^2\sigma_\mu^2 + \frac{\left(1-\gamma\right)^2}{\left(1-\hat{a}\left(1-\gamma\right)\right)^2}\sigma_\zeta^2. \tag{47}$$

According to stochastic approximation theory, we can write the associated ODE governing the stability around the equilibria as

$$\begin{aligned}
\frac{\mathrm{d}a}{\mathrm{d}t} &= \int \lim_{t\to\infty} \mathrm{E}\left[S_{i,t-1}^{-1} p_{i,t}\left(\mu_{i,t} - a_{i,t-1}p_{i,t}\right)\right] di \\
&= \sigma_s^2\left(\hat{a}\right)^{-1} \int \mathrm{E}\left[p_{i,t}\left(\mu_{i,t} - a_{i,t-1}p_{i,t}\right)\right] di \\
&= \sigma_s^2\left(\hat{a}\right)^{-1}\left(\gamma\sigma_\mu^2 - a_{i,t-1}\left(\gamma^2\sigma_\mu^2 + \frac{\left(1-\gamma\right)^2}{\left(1-a_{t-1}\left(1-\gamma\right)\right)^2}\sigma_\zeta^2\right)\right) \\
&= a_i\left(a\right) - a.
\end{aligned} \tag{48}$$

For asymptotic local stability to hold, the Jacobian of the differential equation in (48) must be less than zero at the conjectured equilibrium. The derivative of $a_i(a)$ with respect to a is given by:

$$a_i'(a) = -\frac{2\gamma\left(1-\gamma\right)^3\left(1-\left(1-\gamma\right)a\right)\sigma^2}{\left(\left(1-\gamma\right)^2\sigma^2 + \left(1-\left(1-\gamma\right)a\right)^2\gamma^2\right)^2}, \tag{49}$$

which is positive whenever $a > (1-\gamma)^{-1}$. Then, necessarily, $a_i'(a_\circ) > 1$, $a_i'(a_+) \in (0,1)$, $a_i'(a_-) < 0$ and $a_i'(a_u) < 0$. This proves that the low and unique equilibrium are respectively locally and globally learnable.

6 Conclusion

Learning from prices has played an important role in our understanding of financial markets since at least Grossman and Stiglitz (1980). Yet, learning from prices appeared even earlier in the macroeconomics literature, including in the seminal paper of Lucas (1972). Nevertheless, that channel gradually disappeared from models of the business cycle, in large part because people concluded that fundamental shocks would be almost completely revealed before incomplete knowledge about them could influence relatively slow-moving macroeconomic aggregates.

In this paper we have shown that, even if aggregate shocks are *nearly* common knowledge, learning from prices may still play a crucial role driving fluctuations in beliefs. In fact, the feedback mechanism we described is strongest precisely when the aggregate shock is almost, but not-quite-fully, revealed. Endogenous information structures can deliver strong multipliers on small common disturbances, and thus offer a foundation for coordinated, expectations-driven economic fluctuations. Such fluctuations are completely consistent with rational expectations. Moreover, the key feature of our theory is also a feature of reality: agents observe and draw inference from prices that are, themselves, influenced by aggregate conditions.

We have applied this idea to house prices, because these are among the most salient prices in the economy. Even if the economy is driven only by productivity shocks, we have shown that this mechanism captures several salient features of business cycles and its close correlation with the evolution of the housing market. Our approach is consistent both with the evidence that productivity and endogenous outcomes are weakly correlated and our results suggest that the relationship between supply and demand shocks is more subtle than typically assumed in the empirical literature. Future empirical work may wish to take in account the implications of price-based learning.

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A Model

A.1 The extended version

In this section, we introduce and solve the model in its extended version. The representative Household living in island i has the following utility function

$$\sum \beta^t \Theta_t \left(\log \left(C_{it}^{\phi} \mathcal{H}_{it}^{1-\phi} \right) - v^c \frac{(N_{it}^c)^{1+\chi_c}}{1+\chi_c} - v^h \frac{\left(N_{it}^h\right)^{1+\chi_h}}{1+\chi_h} \right)$$

and faces the budget constraint,

$$\frac{1}{R_t} B_{it} + C_{it} + P_{it}\Delta_{it} = W_{it}^c N_{it}^c + W_{it}^h N_{it}^h + B_{it-1} + \Pi_{it}^c + \Pi_t^h.$$

In this new version we consider the case the household may be hot by a "demand" shock Θ_t such that $\log \Theta_t \equiv \theta \sim N(0, \sigma_\theta)$. We also consider convex disutility of labor with potential different curvatures for working hours supplied in the consumption sector and the housing sector, parametrised respectively by χ_c and χ_h. We need therefore to differentiate between wages in the the two sectors, which we do by introducing W_{it}^c and W_{it}^h, and dropping the labor market clearing condition.

In the competitive consumption sector we introduce the possibility of an aggregate productivity shock and decreasing return to scale. The new technological constraint is given by

$$Y_t = e^{\tilde{\zeta}_t^c} (N_t^c)^{\alpha_c},$$

and

$$N_t^c \equiv \left(\int e^{\tilde{\mu}_{it}/\eta} N_{it}^{c\,1-\frac{1}{\eta}} di \right)^{\frac{1}{1-\frac{1}{\eta}}}$$

where $\tilde{\zeta}_t^c = \tilde{\zeta}_{t-1}^c + \zeta_t^c$ where ζ_t^c is an iid innovation drawn from a normal distribution $N(0, \sigma_{\zeta^c})$, and $\alpha_c \in (0,1)$ measures economies of scale. We denote by W_t^c the price of N_t^c such that $W_t^c N_t^c = \int W_{it}^c N_{it}^c di$. The rest of the model is as in the main text. The version presented in the main text obtains fixing $\chi_c = \chi_h = 0$, $v^c = v^h$, $\sigma_{\zeta^c} = 0$ and $\alpha_c = 1$.

A.2 Complete list of equilibrium conditions

We list here all the equilibrium conditions under full information at a given time t. The first order conditions for household are:

$$\begin{aligned} \Lambda_{it} &= \beta \Lambda_{i,t+1} R_t \\ \Theta_t \phi C_{it}^{-1} &= \Lambda_{it}, \\ \Theta_t W_{it}^c &= \Lambda_{it}^{-1} N_{it}^{\chi_c} \end{aligned}$$

Optimality in the production of non-durable consumption requires

$$\begin{aligned} N_{it}^c &= e^{\tilde{\mu}_{it}} \left(\frac{W_{it}^c}{W_t^c} \right)^{-\eta} N_t^c \\ N_t^c W_t^c &= \alpha_c Y_t \\ Y_t &= e^{\zeta_t^c} (N_t^c)^{\alpha_c}. \end{aligned}$$

One can easily check that

$$\frac{\partial U_{i0}}{\partial \Delta_{it}} = (1-\psi)(1-\phi)\sum_{\tau=t}^{\infty}((1-d)\beta\psi)^{\tau-t}\Delta_{it}^{-1} = \frac{(1-\psi)(1-\phi)}{1-(1-d)\beta\psi}\Delta_{it}^{-1}. \tag{1}$$

The first order conditions for the housing market are then

$$\begin{aligned}
(1-\psi)(1-\phi)(1-(1-d)\beta\psi)^{-1}\Delta_{it}^{-1} &= \Lambda_{it}P_{it}, \\
\left(N_{it}^{h}\right)^{\chi_h} &= \Lambda_{it}W_{it}^{h} \\
Z_{it}Q_t &= \alpha(1-\gamma)P_{it}\Delta_{it}, \\
N_{it}^{h}W_{it}^{h} &= \gamma\alpha P_{it}\Delta_{it} \\
V_{it}L_{it} &= (1-\alpha)P_{it}\Delta_{it}
\end{aligned}$$

where technology is given by

$$\Delta_{it} = L_{it}^{1-\alpha}\left(\left(N_{it}^{h}\right)^{\phi}\left(e^{-\tilde{\zeta}_t}Z_{it}\right)^{1-\phi}\right)^{\alpha}$$

with a market clearing condition $\int Z_{it}di = Z_t$. The budget constraint

$$\frac{1}{R_t}B_{it} + C_{it} + P_{it}\Delta_{it} = W_{it}^{c}N_{it}^{c} + W_{it}^{h}N_{it}^{h} + B_{it-1} + \Pi_t^{c} + \Pi_{it}^{h}$$

must hold as an equality and the transversality condition

$$\lim_{\tau\to\infty} R^{-1-\tau}B_{it+\tau} = 0,$$

must hold at the individual level. Finally market clearing for the endowment reads as:

$$Z_t = \int Z_{it}di, \tag{2}$$

where keeping track of Z_t will help us making clear that productivity shock in housing production could be interpreted equivalently in changes in the supply of raw capital. Market clearing conditions (8) - (9) complete the list of equilibrium conditions.

A.3 Linearized Model

In the following subsection we will introduce log-linear relations. At any time t, we will keep distinct the expectations of shoppers – denoted by $E_t^{i,c}[\cdot]$ – and the ones of workers – denoted by $E_t^{i,w}[\cdot]$ – to demonstrate some interesting properties of the model. Let us list first the equations at the island level. The first order conditions for the consumption sector and bond holdings are:

$$E_t^{i,w}[\lambda_{it}] = E_t^{i,w}[\lambda_{it+1}] + r_t \tag{3}$$

$$-c_{it} = E_t^{i,c}[\lambda_{it}] \tag{4}$$

$$\chi_c n_{it}^{c} = E_t^{i,w}[\lambda_{it}] + w_{it}^{c} \tag{5}$$

$$n_{it}^{c} = \tilde{\mu}_{it} - \eta\left(w_{it}^{c} - w_t^{c}\right) + n_t^{c} \tag{6}$$

$$n_t^{c} + w_t^{c} = y_t \tag{7}$$

$$y_t = \tilde{\zeta}_t^{c} + \alpha_c n_t^{c}. \tag{8}$$

The first order conditions for the housing market are

$$\begin{aligned}
-\delta_{it} &= E_t^{i,c}[\lambda_{it}] + p_{it}, && (9)\\
\chi_h n_{it} &= E_t^{i,w}[\lambda_{it}] + w_{it}^h && (10)\\
z_{it} + q_t &= p_{it} + \delta_{it}, && (11)\\
n_{it}^h + w_{it}^h &= p_{it} + \delta_{it} && (12)\\
v_{it} = &= p_{it} + \delta_{it} && (13)
\end{aligned}$$

where technology is given by

$$\delta_{it} = (1-\alpha)l_i + (\alpha\gamma)n_{it}^h + \alpha\,(1-\gamma)\left(-\tilde{\zeta}_t + z_{it}\right). \tag{14}$$

We log-linearize the budget constraint here. At the individual level we have

$$\frac{1}{R_t}B_{it} + C_{it} + P_{it}\Delta_{it} + P_{it}H_{it-1} =$$

$$= W_{it}^c N_{it}^c + W_{it}^h N_{it}^h + \underbrace{Y_t - W_t^c N_t^c}_{\Pi_t^c} + \underbrace{P_{it}H_{it} - W_{it}^h N_{it}^h - Q_t\,(Z_{it} - Z_t) + V_{it}}_{\Pi_{it}^h} + P_{it}H_{it-1} + B_{it-1}$$

$$\frac{1}{R_t}B_{it} + (C_{it} - Y_t) - (W_{it}^c N_{it}^c - W_t^c N_t^c) = -Q_t\,(Z_{it} - Z_t) + B_{it-1}$$

We consider a linearization computed from the situation of the economy at time $t = 0$ before shock realize, in which $B_{it} = 0$ for all i, hence we linearize around B_{it} and log-linearize for other variables.

In such a steady-state, all of the terms in parenthesis above are zero, so that the linearization is

$$\beta b_{it} + C(c_{it} - c_t) = C(w_{it}^c - w_t^c) + C(n_{it}^c - n_t^c) - Q(z_{it} - z_t) + b_{it-1}, \tag{15}$$

where capital letters denote steady states values. Finally market clearing conditions read as: $z_t = \int z_{it}di$, $\int b_{it}di = 0$ and $c_t = y_t$.

A.4 Solution

This section shows the analytical solution of the model in this extended version. We also generalise our shock structure by introducing a news about future aggregate productivity. To demonstrate some properties of our model, we focus on time t and we assume that in the second stage the worker-saver i knows the current housing productivity, current and future consumption productivity and local productivity, i.e. $\Omega_t = \{\zeta_t, \zeta_t^c, \zeta_{t+1}^c, \mu_{it+1}\}$. We also assume that shoppers only observe p_{it} at time t and share the information set of the worker-saver at time $t + 1$.

A.4.1 Solution from $t+1$ onwards

Derivation of λ_{it+1}. Manipulating first order conditions, one finds that:

$$\begin{aligned} p_{i,t}+\delta_{it} &= -E_t^{i,c}[\lambda_{it}] = c_{it}, \\ q_t &= -\int E_t^{i,c}[\lambda_{it}]di - z_t = c_t - z_t, \\ z_{it}-z_t &= c_{it}-c_t, \end{aligned}$$

for any t, which we will use in the following. The transversality condition at the individual level requires that we focus on the stationary solution $b_{it+1}=b_{it}$, with b_{it} being predetermined in the t period. We first use the budget constraint to characterise the solution as follows:

$$(C+Q)(c_{it+1}-c_{t+1})-(1-\beta)b_{it} = C(\mu_{it+1}-\eta\left(w^c_{it+1}-w^c_{t+1}\right))+C(w^c_{it+1}-w^c_{t+1})$$

or

$$(C+Q)(c_{it+1}-c_{t+1})-(1-\beta)b_{it} = C\mu_{it+1}+(1-\eta)C(w^c_{it+1}-w^c_{t+1}).$$

We use relations at the aggregate level to get $c_{t+1}=w^c_{t+1}=\zeta^c_{t+1}$, $E^{i,w}_{t+1}[\lambda_{it+1}]=E^{i,c}_{t+1}[\lambda_{it+1}]=-c_{it+1}$ and $n_{t+1}=0$ to establish

$$(C+Q)(c_{it+1}-\zeta^c_{t+1})-(1-\beta)b_{it} =$$

$$= C\mu_{it+1}+(1-\eta)C\left(\underbrace{\frac{\chi_c}{1+\eta\chi_c}\mu_{it+1}+\frac{1}{1+\eta\chi_c}c_{it+1}+\frac{\eta\chi_c}{1+\eta\chi_c}\zeta^c_{t+1}}_{w_{it+1}}-\zeta^c_{t+1}\right)$$

that becomes

$$\left(C\frac{\eta(1+\chi_c)}{1+\eta\chi_c}+Q\right)(c_{it+1}-\zeta^c_{t+1}) = C\left(1+\frac{(1-\eta)\chi_c}{1+\eta\chi_c}\right)\mu_{it+1}+(1-\beta)b_{it}.$$

So that we finally get (remember $b_{it+1}=b_{it}$)

$$\lambda_{it+1}=-c_{it+1}=-\omega_\mu\mu_{it+1}-\omega_b b_{it}-\zeta^c_{t+1},$$

where

$$\omega_\mu=\frac{C\left(1+\frac{(1-\eta)\chi_c}{1+\eta\chi_c}\right)}{C\frac{\eta(1+\chi_c)}{1+\eta\chi_c}+Q}>0, \quad \text{and} \quad \omega_b=\frac{1-\beta}{C\frac{\eta(1+\chi_c)}{1+\eta\chi_c}+Q}>0.$$

As stated in the main text, notice that $\lim_{\beta\to 1}\omega_b=0$.

A.4.2 Solution at time t

Derivation of λ_{it}. The first step is finding out an expression for λ_t. One can use: $\chi_c n_t=\lambda_t+w_t$ and $w_t=\zeta^c_t+(\alpha_c-1)n_t$ to get

$$(1-\alpha_c+\chi_c)n_t = \lambda_t+\zeta^c_t$$

and then $c_t=\zeta^c_t+\alpha_c n_t$ to get a relation between the actual aggregate lambda and shoppers' expectations

$$\lambda_t=-\frac{1-\alpha_c+\chi_c}{\alpha_c}\int E^{i,c}_t[\lambda_{it}]di-\frac{1+\chi_c}{\alpha_c}\zeta^c_t.$$

Note that this expression is valid also for future times. In fact, under the assumption that uncertainty vanishes after the first period, i.e. $\int E^{i,c}_{t+1}[\lambda_{it+1}]di = -\zeta^c_{t+1}$, we have that $\lambda_{t+1} = -\zeta^c_{t+1}$ which is consistent with what we have found above. In this case, the Euler equation implies,

$$r_t = \lambda_t - \lambda_{t+1} = -\frac{1-\alpha_c+\chi_c}{\alpha_c}\int E^{i,c}_t[\lambda_{it}]di - \frac{1+\chi_c}{\alpha_c}\zeta^c_t + \zeta^c_{t+1}.$$

Note that in the quasi linear case ($\chi_c = 0$ and $\alpha_c = 1$) actual lambda is independent of consumers' expectations, and the above equation reduces to (22).

Given the Euler equation must hold at the local level, we have the following

$$\lambda_{it} = \underbrace{-\omega_\mu \mu_{it+1} - \omega_b b_{it} - \zeta^c_{t+1}}_{=\lambda_{it+1}} + r_t = -\omega_\mu \mu_{it+1} - \omega_b b_{it} + \frac{1-\alpha_c+\chi_c}{\alpha_c}c_t - \frac{1+\chi_c}{\alpha_c}\zeta^c_t \tag{16}$$

The equation above shows that the anticipation of future aggregate productivity does not affect the marginal valuation of current consumption. This is a standard finding in real business cycle model, where real interest rates neutralize the effect of anticipated aggregate productivity news. On the other hand current productivity moves the current marginal valuation of current consumption. The case explored in the main text obtains in the quasi-linear case $\alpha_c = 1$ and $\chi_c = 0$.

Derivation of price for new housing. Here we derive the expression for the equilibrium price of new housing. By simple algebra we get

$$\begin{aligned} p_{it} + \delta_{it} &= -E^{i,c}_t[\lambda_{it}], \\ z_{it} &= -E^{i,c}_t[\lambda_{it}] - q_t, \\ n^h_{it} &= \frac{1}{1+\chi_h}\left(\lambda_{it} - E^{i,c}_t[\lambda_{it}]\right) \end{aligned}$$

And the housing price gets

$$\begin{aligned} p_{it} &= -E^{i,c}_t[\lambda_{it}] - \alpha\gamma n_{it} - \alpha(1-\gamma)(-\zeta_t + z_{it}), \\ p_{it} &= -E^{i,c}_t[\lambda_{it}] - \alpha\gamma\left(\frac{1}{1+\chi_h}\left(\lambda_{it} - E^{i,c}_t[\lambda_{it}]\right)\right) - \alpha(1-\gamma)\left(-\zeta_t - q_t - E^{i,c}_t[\lambda_{it}]\right), \\ p_{it} &= \left(1 - \alpha(1-\gamma) - \frac{\alpha\gamma}{1+\chi_h}\right)E^{i,c}_t[-\lambda_{it}] + \alpha\underbrace{\left(\gamma\left(\frac{1}{1+\chi_h}(-\lambda_{it})\right) + (1-\gamma)(\zeta_t + q_t)\right)}_{=s_i}. \end{aligned}$$

The final step is substituting $q_t = -\int E^{i,c}_t[\lambda_{it}]di - z_t$ in it to clearly see that an increase in productivity (negative ζ_t) in the housing sector is isomorphic to an increase in the endowment in raw capital (positive z_t). The case in the text obtains for $\chi_h = 0$.

Derivation of price of the stock of housing. In analogy with (1) we can derive the price in consumption units $P^h_{it|v}$ of housing vintage $\Delta_{it|v}$ as

$$P^h_{it|v} = E[\Lambda_{it}|p_{it}]^{-1}\frac{\partial U_{i0}}{\partial \Delta_{it|v}} = \frac{(1-\psi)(1-\phi)\psi^{t-v}}{1-(1-d)\beta\psi}\Delta^{-1}_{it|v}E[\Lambda_{it}|p_{it}]^{-1}, \tag{17}$$

for any $v \leq t$. Therefore the price of the total stock of housing is given as

$$P_{it}^h = \sum_{v=-\infty}^{t} \frac{P_{it|v}^h \Delta_{it|v}}{H_{it}} = \frac{1-\phi}{1-(1-d)\psi\beta} E[\Lambda_{it}|p_{it}]^{-1} H_{it}^{-1} \tag{18}$$

which in log-terms gives

$$p_{it}^h = -E[\lambda_{it}|p_{it}] - \kappa\delta_{it} = (1-\kappa)E[-\lambda_{it}|p_{it}] + \kappa p_{it} \tag{19}$$

where $\kappa = \bar{\Delta}/\bar{H}$ defines the steady state share of residential investment over existing housing stock.

Derivation of b_{it}. To compute b_{it} (bear in mind that we are assuming $b_{it-1} = 0$ and $\mu_{it} = 0$ here) we re-consider the same step leading to (16) at time t where now $\mu_{it} = b_{it-1} = 0$ to get:

$$(C+Q)(c_{it} - c_t) + \beta b_{it} = (1-\eta)C(w_{it}^c - w_t^c)$$

or

$$(C+Q)\left(-E_t^{i,c}[\lambda_{it}] + \int E_t^{i,c}[\lambda_{it}]di\right) + \beta b_{it} = (1-\eta)C(-\lambda_{it} + \lambda_t).$$

Use the fact $-E_t^{i,c}[\lambda_{it}] = as_i$, where s_i is defined as above, to get

$$\begin{aligned}(C+Q)\left(a\frac{\gamma}{1+\chi_h}(\omega_\mu \mu_{it+1} + \omega_b b_{it})\right) + \beta b_{it} &= (1-\eta)C(\omega_\mu \mu_{it+1} + \omega_b b_{it}) \\ \left((C+Q)a\frac{\gamma}{1+\chi_h}\omega_b - (1-\eta)C\omega_b + \beta\right) b_{it} &= \left((1-\eta)C - (C+Q)a\frac{\gamma}{1+\chi_h}\right)(\omega_\mu \mu_{it+1})\end{aligned}$$

and finally

$$b_{it} = \frac{-(1+\chi_h)(\eta-1)C - (C+Q)a\gamma}{(C+Q)a\gamma\omega_b + (1+\chi_h)(\eta-1)C\omega_b + (1+\chi_h)\beta}(\omega_\mu \mu_{it+1}),$$

so that

$$\begin{aligned}\lambda_t - \lambda_{it} &= \omega_\mu \mu_{it+1} + \omega_b b_{it} = \\ &= \omega_\mu \mu_{it+1} + \omega_b \frac{-(1+\chi_h)(\eta-1)C\omega_\mu - (C+Q)a\gamma\omega_\mu}{(C+Q)a\gamma\omega_b + (\eta-1)(1+\chi_h)C\omega_b + (1+\chi_h)\beta}\omega_\mu \mu_{it+1} \\ &= \frac{(1+\chi_h)\beta}{\underbrace{(C+Q)a\gamma\omega_b + (1+\chi_h)(\eta-1)C\omega_b}_{\equiv f(a,\beta)} + (1+\chi_h)\beta}\omega_\mu \mu_{it+1}.\end{aligned} \tag{20}$$

Remark: Given that ω_b is a decreasing function of β, we can conclude that a higher a or lower β strictly increases $f(a,\beta)$, and so it strictly decreases the volatility of the idiosyncratic component of λ_{it}, namely $Var(\lambda_t - \lambda_{it})$. This remark will be useful in the following proof.

A.5 Proofs of Propositions

Proof of Proposition 1. Let us first solve the case for which σ is exogenous and fixed which corresponds to the limit case $\beta \to 1$. The fix point equation reads as

$$a^*(a) = \frac{1}{\gamma}\frac{\tau(a)}{1+\tau(a)} = \frac{1}{\gamma}\frac{1}{1+\left(\frac{(1-\gamma)\sigma}{\gamma(1-a(1-\gamma))}\right)^2} = \frac{\gamma\left(1-a\left(1-\gamma\right)\right)^2}{\gamma^2\left(1-a\left(1-\gamma\right)\right)^2+(1-\gamma)^2\sigma^2} \tag{21}$$

To prove uniqueness for $\gamma \geq 1/2$, observe that the function $a^*(a)$ is continuous, bounded above by γ^{-1}, and monotonically decreasing in the range $(0,(1-\gamma)^{-1})$. From $\gamma \geq 1/2$, we have $(1-\gamma)^{-1} > \gamma^{-1}$. Thus $a^*(a)$ intersects the 45-degree line a single time.

To prove the existence of a_-, notice that $\lim_{a\to-\infty} a^* = \gamma^{-1}$ and $a^*((1-\gamma)^{-1}) = 0$. By continuity, an equilibrium $a_- \in (0,(1-\gamma)^{-1})$ must always exist. Moreover a_- must be monotonically decreasing in σ^2 as a^* is monotonically decreasing in σ^2.

We now assess the conditions under which additional equilibria may also exist. Because $\lim_{a\to\infty} a^* = \gamma^{-1}$, the existence of a second equilibrium (crossing the 45-degree line in Figure 2) implies the existence of a third. Thus, we must determine whether the difference $a^*(a)-a$ is positive anywhere in the range $a > (1-\gamma)^{-1}$. Such a difference is positive if and only if

$$\Phi\left(\sigma\right) \equiv \gamma\left(1-a\left(1-\gamma\right)\right)^2\left(1-\gamma a\right) - a\left(1-\gamma\right)^2\sigma^2 > 0, \tag{22}$$

which requires $a < \gamma^{-1}$ as a necessary condition. Therefore, if two other equilibria exist they must lie in $((1-\gamma)^{-1},\gamma^{-1})$. Fixing $a \in ((1-\gamma)^{-1},\gamma^{-1})$, $\lim_{\sigma\to 0}\Phi\left(\sigma\right)$ is positive, implying that there always exists a threshold $\bar{\sigma}$, and so a threshold $\bar{\sigma_\zeta}$, such that two equilibria $a_+, a_\circ \in ((1-\gamma)^{-1},\gamma^{-1})$ exist with $a_+ \geq a_\circ$ for $\sigma^2 \in (0,\bar{\sigma}^2)$.

Let us now consider β less than one. In this case, $\omega_b \neq 0$ and the variance of the idiosyncratic portion of λ_{it} is also endogenous to a, as captured by the function $f(a,\beta)$ in equation (20). Since $\eta > 1$ and $\omega_b > 0$, it follows that $f(a,\beta)$ is strictly positive and increasing in a for all $\beta < 1$. In this case, we must replace σ with the endogenous variance $\sigma(a,\beta)$ in the fixed-point equation (A.5). Since the $\sigma(a,\beta) > \sigma$ and is increasing in a, $a^*(a,\beta)$ is weakly below $a^*(a)$ and any intersection (fixed point) $a^*(\beta)$ must lie strictly to the left of the value a^* for the model with $\beta \to 1$. Hence, if the economy has a unique equilibrium when $\beta \to 1$ it must also have a unique equilibrium $\beta < 1$. Moreover, since $\sigma(a,\beta)$ increases with β, it must be true that the threshold $\bar{\sigma}$ for a multiplicity falls along with β.

□

Proof of Proposition 2. To prove the limiting statement for $\gamma \geq 1/2$, consider any point $a_\delta = \frac{1-\delta}{1-\gamma}$ such that $\delta > 0$. We then have

$$a^*(a_\delta) = \frac{\gamma\delta^2}{\gamma^2\delta^2+\sigma^2(1-\gamma)^2.} \tag{23}$$

Since $\lim_{\sigma^2\to 0} a^*(a_\delta) = \frac{1}{\gamma}$ for any δ, the unique equilibrium must converge to the same point. That the variance of this equilibrium approaches zero follows from equation (25).

To prove the limiting statement for $\gamma < 1/2$, recall the monotonicity of $a^*(a)$ on the range $(0,(1-\gamma)^{-1})$. Following the logic of Proposition 1, for any point a_δ in that range, $\lim_{\sigma^2\to 0} a^*(a_\delta) = \gamma^{-1}$, while $a^*((1-\gamma)^{-1}) = 0$. Thus, the intersection defining a_- must approach $(1-\gamma)^{-1}$. An analogous argument for the point just to the right of $(1-\gamma)^{-1}$

establishes that a_- converges to the same value. Finally, the bounded monotonic behavior of $a^*(a)$ establishes that $\lim_{\sigma^2 \to 0} a_+ = \gamma^{-1}$ for the high equilibrium.

That the output variance of the high equilibrium in the limit $\sigma \to 0$ is zero follows from equation (26). The limiting variance of the two other limit equilibria can be established by noticing that () implies

$$\frac{(1-\gamma)^2 a^2 \sigma^2}{(1-a(1-\gamma))^2} = a\gamma(1-a\gamma) \tag{24}$$

which gives (29) for $a \to (1-\gamma)^{-1}$. □

Proof of Proposition 4. We can prove that an equilibrium with no aggregate variance exists for $a = \gamma^{-1}$ by simple substitution in (42). The limiting variance of the other limit equilibrium at the singularity $a \to (1-\gamma)^{-1}$ can be established by noticing that (42) implies that

$$\frac{\sigma^2}{(1-a(1-\gamma))^2} = \frac{1-a\gamma}{a\gamma} + \frac{1-a(1-\gamma)}{a\gamma}\frac{\sigma^2}{(1-a(1-\gamma))^2},$$

which gives

$$\frac{\sigma^2}{(1-a(1-\gamma))^2} = -\frac{1-a\gamma}{1-a}.$$

Substituted into (41), this gives the value of σ_c^2 in (29) for $a \to (1-\gamma)^{-1}$. □

A.6 Data Definitions