Endogenous trends*

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Abstract

Conventional business cycle analysis interprets economic fluctuations as high frequency variations around an exogenous trend. In contrast to this approach, we include two sources of growth (ideas and knowledge) to determine the endogenous trend of an economy, and examine its quantitative potential in a standard medium scale New Keynesian model. We estimate this model on the US data between 1950q1-2018q4 with an occasionnally binding constraint on the nominal rate. We find that the endogenous trend has been sharply declining since 1970, thus corroborating the secular stagnation theory. This dynamic is captured by a slowdown in the accumulation technology reflecting the low productivity of the R&D sector. While the contribution of human capital has been remarkably stable, the financial crisis deteriorated its contribution over the last decade.

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1 Introduction

In modern models of the business cycle, economic fluctuations are interpreted as high frequency fluctuations around a trend growing at an exogenous rate (either deterministic or stochastic). This conception of business cycles is questionable given the strong body of evidence in empirical macroeconomics showing that the trend of the US economy is time-varying (Nelson and Plosser (1982)) and reducing over time.[1] Despite this evidence, most of recent medium scale macroeconomic models assume either a fixed slope of growth (*e.g.*, Smets and Wouters (2007)) or exogenous drifts to productivity (*e.g.*, Christiano et al. (2014)). The resulting interpretation of business cycles is at odd with the evidence of structural changes in the long run growth of an economy observed over the last decades. In particular, the underlying factors that are jointly driving low frequency changes in macroeconomic time series are usually swept out by business cycle filters, or erroneously captured by exogenous disturbances.

The main goal of this paper is therefore to develop a quantitative model that features an endogenous slope of growth, referred to as an *endogenous trend*.[2] Guided by the endogenous growth theory, the trend at which the economy is growing at a low frequency is determined by two growth engines based on the accumulation of ideas and knowledge. For the first engine of growth based on the accumulation of ideas, the endogenous productivity mechanism we develop is based on Comin and Gertler (2006), which uses the approach to connect business cycles to growth. This model of Comin and Gertler (2006) is itself a variant of Romer (1990)'s expanding variety model of technological change, modified to include a friction on the endogenous probability of technology adoption. We include a sticky rate of adoption to capture a congestion externality in the diffusion of new technologies. The second engine of growth is based on the accumulation of knowledge (i.e., "experience" or "skill"), through a model of human capital à la Lucas Jr (1988). Each period, firms engage a fraction of their labor inputs into vocational training in order to produce human capital. We modify the Lucas framework to allow for an endogenous rate of adoption of new skills, along with endogenous human capital formation. By doing so, we are able to allow for empirically reasonable diffusion lags but still generate endogenous medium-term swings in productivity.

We then estimate the model with endogenous trend on a sample spanning from

[1] For a long run perspective on growth, see Antolin-Diaz et al. (2017) For recent papers after the Great Recession documenting the slowdown of the US economy, see Fernald and Jones (2014) and (Gordon, 2012, 2017).

[2] These cyclical movement are interpreted by Comin and Gertler (2006) as medium term fluctuations. In this paper, I interpret these fluctuations as persistent changes in the growth rate of the economy that affects key macroeconomic variables.

1950q1 up to 2018q4 using Bayesian techniques. The solution method employed to estimate the model features an occasionnally binding constraint on the nominal rate. We then use the model to assess the slowdown of long term growth, in particular following the onset of the Great Recession. Based on the estimated model, our key result is that we corroborate the thesis of a strong decline in the long term trend of the US economy. Among the two sources of growth examined in the paper, the slowdown mainly is induced by the technology engine reflecting a decline in the productivity of creation of new technologies since 1960. This finding tends to favor the Gordon (2012) theory stating that the US growth has strongly declined since 1970. In addition, we find that a standard macro-model with exogenous growth erroneously captures low frequency changes in economic growth by highly persistent macroeconomic shocks. In contrast, the model featuring an endogenous trend successfully captures this low frequency fluctuations. This endogenous persistence is key, as it allows the model to outperform the forecasting performance of a DSGE model with an exogenous trend.

In addition to the literature cited above, there are several other papers related to our analysis. Anzoategui et al. (2019) estimates a macroeconomic model with one source of growth for the US economy. They evaluate the role of R&D in the productivity slowdown following the financial crisis, they find that the reduction in productivity is induced by a reduction in the adoption rate of technology. Moran and Queralto (2017) complete this analysis by including the role of monetary policy, in particular when the ZLB is binding. Both Queralto (2019) and Bianchi et al. (2019) inspect the role of financial frictions on knowledge accumulation to capture the recent slowdown in economic growth for the US. An alternate approach of Garcia-Macia (2017) stresses misallocation between tangible and intangible capital following a financial crisis. Annicchiarico and Pelloni (2016) inspect the implications of the endogenous growth on the optimal conduct of monetary policy.

This paper is also related to a litterature that puts endogenous growth mechanism into real business cycles models. Hercowitz and Sampson (1991) is probably the first paper that attempts to connect the business cycles and endogenous growth. They estimate their model as a VAR process and find that endogenous growth successfully accounts for the persistence of output growth. Boileau (1996) evaluates how in an open economy context the endogenous growth mechanism helps the model in replicating salient business cycle statistics. Barlevy (2004) revisited the welfare cost of business under endogenous growth and finds that the presence of endogenous growth exacerbates the welfare cost of business cycles. Similarly, Wu and Zhang (1998) revisited the welfare cost of inflation under endogenous growth. While most of this literature adresses the question of growth as an expanding variety effect, Jones et al.

(2005) originally consider growth as an accumulation of human capital.

The rest of the paper is organized as follows. section 2 presents a New Keynesian Model with two sources of endogenous growth. section 3 is devoted to the estimation of the model using Bayesian econometrics. section 4 evaluates the consequences an endogenous rate of growth on the transmission of TFP shock, and the role of a time-varying trend on the cross-correlation of observable variables. section 5 studies the contribution of the accumulation of technologies and knowledge in the historical evolution of the long run growth rate of the US economy since 1950. section 6 evaluates the role of the zero lower bound on the economic contraction during the financial crisis. section 8 concludes.

2 A New Keynesian Model with Two Sources of Endogenous Growth

This section describes the theoretical framework, consisting of a standard medium-sized New Keynesian model augmented to include endogenous creation and adoption of new technologies and human capital, respectively denoted A_t and H_t. The slope of growth of the economy, denoted Γ_t, is thus a function of these two engines of growth, with $\Gamma_t = f(A_t, H_t)$. Sub-sections 2.2.3 and 2.4 constitute the main departures from other medium-sized DSGE models found in the literature. The rest of the subsection provides the conventional ingredients of the a New Keynesian model similar to Smets and Wouters (2007).

2.1 Households

The preferences of the j^{th} family are given by:

$$E_t\left\{\sum_{\tau=0}^{\infty}\beta^{\tau}\left[\frac{\left(c_{jt+\tau}-hc_{jt-1+\tau}\right)^{1-\sigma_C}}{1-\sigma_C}\exp\left(\left(\frac{\sigma_C-1}{1+\sigma_L}\right)\chi_L l_{jt+\tau}^{1+\sigma_L}\right)\right]\right\} \tag{1}$$

where E_t denotes the expectation operator and $\beta\in(0,1)$ is the discount factor. The consumption index c_{jt} is subject to external habits governed by parameter $h\in[0;1)$ while $\sigma_C>0$ is the risk aversion parameter on consumption. Parameter $\sigma_L>0$ shapes the consumption-leisure trade-off, while $\chi_L>0$ is a shift parameter pinning down the steady state amount of hours worked.

Household j face a budget constraint:

$$c_{jt}+b_{jt}=b_{jt-1}\underline{r}_{t-1}/\pi_t+w_t h_{jt} l_{jt}+\Pi_{jt}-t_{jt}, \tag{2}$$

The income of the representative household is made of labor income with real wage w_t combined with human capital h_{jt} (or skills) and hours worked l_{jt}, total firm profits Π_{jt}, real interest payments $\underline{r}_{t-1}/\pi_t$ from riskless bonds b_{jt}, with inflation rate $\pi_t = P_t/P_{t-1}$.

2.2 Intermediate Firms

2.2.1 Intermediate goods composite

There exists a continuum of measure A_t of monopolistically competitive intermediate goods firms that each make a differentiated product. The endogenous variable A_t is the stock of types of intermediate goods adopted in production, i.e., the stock of adopted technologies. We assume that one firm produced one type of variety such that $i \in [0, A_t]$ both refers to a good or an intermediate firm. Each firms produces produces output x_{it} at a selling price p^x_{it}. The intermediate goods composite is the following CES aggregate of individual intermediate goods:

$$X_t = \left[\int_0^{A_t} x_{it}^{(\vartheta-1)/\vartheta} \mathrm{d}i\right]^{\vartheta/(\vartheta-1)}, \tag{3}$$

where parameter $\vartheta > 1$ is the degree of imperfect substitution between varieties allowing intermediate firms to make profits. The aggregate price index is given by: $P^x_t = [\int_0^{A_t} (p^x_{it})^{1-\vartheta} \mathrm{d}i]^{1/(1-\vartheta)}$. The optimal demand for the i-th varieties is given by:

$$x_{it} = \left(p^x_{it}/P^x_t\right)^{-\vartheta} X_t. \tag{4}$$

2.2.2 Production technology

There is a continuum of i firms that produces an homogenous good by combining labor inputs, capital inputs and technology. The i^{th} firm has the following Cobb-Douglas technology:

$$x_{it} = \varepsilon^A_t \left[(1-e_{it})\, l^d_{it} h^{\omega}_{it}\right]^{\alpha} \left[u_{it} k_{it-1}\right]^{1-\alpha} \tag{5}$$

where (exogenous) AR(1) technology is ε^A_t, hours worked demand l^d_{it}, human capital h_{it}, e_{it} the fraction of the labor supply involved in the accumulation of knowledge, u_{it} is the utilization rate of physical capital and k_{it-1} is the physical capital. Parameter $\alpha \in [0, 1]$ measures the labor intensity in the firms technology. Workers may spend a fraction e_{it} of their time acquiring skills. That is, they can learn to use more advanced capital goods. Parameter $\omega \geq 0$ is the internal effect of human capital which benefits to the overall economy. According to Mincer (1974), an additional year of schooling or an additional year of experience should increase wages proportionally. To incorporate this mechanism in the model, we assume increasing returns on human capital, with elasticity $\omega > 1$.

Real profits are given by:

$$d_{it}^{x} = \frac{P_{it}^{x}}{P_t} x_{it} - w_t h_{it} l_{it}^{d} - \varepsilon_t^{I} \left(1 + S_I \left(\frac{i_{it}}{\gamma_t i_{it-1}}\right)\right) i_{it} - \frac{P_t^{I}}{P_t} \left(1 + S_H \left(\frac{z_{it}^{H}}{z_{it-1}^{H}}\right)\right) z_{it}^{H} - \frac{P_t^{I}}{P_t} s_{it}^{H} h_{it}^{u}, \tag{6}$$

where $\varepsilon_{i,t}^{I}$ is a stochastic process which captures exogenous changes in the value of physical capital, regarding adjustment cost functions $S_a(x_t)$=$\chi_a(x_t - \bar{x})^2$ with $\chi_a \geq 0$ is the adjustment cost parameter.

For clarity purpose, we separate production and labor decisions in the following subsections.

2.2.3 Production and adoption of skills

As in Lucas Jr (1988), we assume that there firms can spent a fraction e_{it} of working to the accumulation of human capital while $(1 - e_{it})\, l_{it}^{d}$ is the skill-weighted man-hours devoted to current production. The rise in more skilled worker does not necessary translate into immediate growth of output. We capture this pattern by assuming that all skills in the economy are not necessarily adopted by firms, this can interpreted as an "education inflation". Let us assume there is a stock of unadopted human capital, denoted h_{it}^{u}, given by:

$$h_{it}^{u} = (1 - \delta_H) \left[F_H \left(e_{it-1}, z_{it-1}^{H}\right) + \left(1 - p_{it-1}^{H}\right) h_{it-1}^{u}\right] \tag{7}$$

where δ_H is the obsolescence rate of a skill, $F_H\,(.)$ is the production function of new human capital and p_{t-1}^{H} is the endogenous probability of adoption of a skill by the i-th firm. Regarding the adoption probability of a skill, our goal is to capture the notion that adoption takes time on average, but allow for adoption intensities to vary pro-cyclically. These considerations lead us to the following formulation for the functional form: $p_t^{H} = \varsigma_t^{H} \left(s_{it}^{H}\right)^{\varkappa_H}$, where ς_t^{H} is a scaling factor that pins down the steady state in the balanced growth path, s_{it}^{H} are the adoption expenditures in units of final goods.

As in Jones et al. (1993), human capital creation is a Cobb-Doublas function that combines education hours e_{it} and education expenditures z_{it}^{H} :

$$F_H \left(e_{it}, z_{it}^{H}\right) = \xi_t^{H} \left(e_{it}\right)^{1-\upsilon} \left(z_{it}^{H}\right)^{\upsilon} \tag{8}$$

where ξ_t^{H} is a productivity parameter that pins down the steady state in the balanced growth path, υ is a technology parameter determining the intensity of education expenditures in the production of knowledge. For $\upsilon = 0$, the model reads as in Lucas, while for $\upsilon > 0$, the model is similar to the setup of Jones et al. (1993).

The law of motion of adopted skills, or effective human capital, is given by:

$$h_{it} = (1 - \delta_H) \left[p_{t-1}^H h_{it-1}^u + h_{it-1} \right] \tag{9}$$

Intermediate firms maximize their profits under Equation 6, the supply constraint 5, the demand constraint 4 and law of motions 7 and 9. Letting v_t^U and v_t^H denote the Lagrangian multipliers associated with laws of motion of unadopted and adopted human capital respectively. They represent the current marginal value of unadopted and adopted skills, respectively.

The optimal fraction of hours worked spent in education e_{it} is given by:

$$\frac{p_t^x}{\mu_\vartheta} \alpha \frac{x_{it}}{(1 - e_{it})} = (1 - \delta_H) \, E_t \left\{ m_{t,t+1} V_{t+1}^U \right\} F_{H,t}^{\prime e}, \tag{10}$$

where $F_{H,t}^{\prime e}$ is the derivative in education of the production function of knowledge and V_t^U is the value of unadopted skills. The left hand side of Equation 10 is the productivity loss of increasing e_{it}, while the right hand side denotes the expected marginal product of unadopted skills. Parameter $\mu_\vartheta = \vartheta/(\vartheta - 1)$ is the markup over the marginal cost of producing intermediate goods.

The optimal education spending z_{it}^H reads as:

$$1 + \frac{\partial S_{H,t} z_{it-1}^H}{\partial z_{it}^H} + E_t \left\{ m_{t,t+1} \frac{\partial S_{H,t+1} z_{it}^H}{\partial z_{it}^H} \right\} = (1 - \delta_H) \, E_t \left\{ m_{t,t+1} V_{t+1}^U \right\} F_{H,t}^{\prime z}, \tag{11}$$

Similarly to the optimal education, the left hand side denotes the marginal cost of education spending and the right hand side is the expected marginal product of unadopted skills.

In addition, the optimal amount of adopted human capital h_{it} is given by:

$$V_t^H = \frac{p_t^x}{\mu_\vartheta} \omega \alpha \frac{y_{it}}{h_{it}} - w_t l_{it}^d + (1 - \delta_H) \, E_t \left\{ m_{t,t+1} V_{t+1}^H \right\} \tag{12}$$

The current value of human capital V_t^H is determined by its marginal productivity, net of wage payments, and the expected value of the adopted skill if the human capital does not depreciate.

The value of unadopted skills:

$$V_t^U = (1-\delta_H)\, E_t \left\{ m_{t,t+1} \left[p_t^H V_{t+1}^H + \left(1-p_t^H\right) V_{t+1}^U \right] \right\} \tag{13}$$

Finally the optimal demand for s_{it}^H is given by:

$$(1-\delta_H)\, E_t \left\{ m_{t,t+1} \left[V_{t+1}^H - V_{t+1}^U \right] \right\} p_t^{H\prime} = 1 \tag{14}$$

where $p_t^{H\prime}$ is the derivative of the probability of adoption with respect to the quantity of goods s_{it}^H spent in adoption of skills. The term on the right is the marginal gain from adoption expenditures: the increase in the adoption probability p_t^H times the discounted difference between the value of an adopted versus an unadopted skill. The right side is the marginal cost. The term $V_{t+1}^H - V_{t+1}^U$ is pro-cyclical, given the greater influence of near term profits on the value of adopted skills relative to unadopted ones.

2.2.4 Capital decisions

Intermediate firms maximize their profits under Equation 6 under the supply constraint 5 and the demand constraint 4 and the following law of motion of capital:

$$k_{it} = i_{it} + (1-\delta(u_{it}))\, k_{it-1}, \tag{15}$$

where $\delta(u_{it}) = \delta_c + \frac{b}{1+\psi} u_{it}^{1+\psi}$. In this function $\delta_c > 0$ is the fixed part of the depreciation, while the time-varying part is a function of the utilization rate of capital. $\psi \geq 0$ is the elasticity of the depreciation with respect to utilization. Parameter $b \geq 0$ is a shift parameter which allows to pin down the steady state utilization rate.

The first order condition determining the shadow value of investment goods is given by:

$$q_t = \varepsilon_t^I \left(1 + \frac{\partial i_{it} S\left(i_{it}/i_{it-1}\right)}{\partial i_{it}} \right) + m_{t,t+1} \varepsilon_{t+1}^I \frac{\partial S\left(i_{it+1}/i_{it}\right) i_{it+1}}{\partial i_{it}}, \tag{16}$$

where q_t is the Lagrangian multiplier associated with the law of motion of physical capital.

The optimal demand for physical capital is given by:

$$q_t = E_t \left\{ m_{t,t+1} \left[\frac{p_{t+1}^x}{\mu_\vartheta} (1-\alpha) \frac{y_{it+1}}{k_{it}} - (1-\delta(u_{it+1}))\, q_{t+1} \right] \right\}. \tag{17}$$

The optimal utilization rate is given by:

$$(1-\alpha)\frac{y_{it}}{u_{it}}\frac{p_t^x}{\mu_\vartheta}=\delta'(u_{it})\,q_t \tag{18}$$

2.3 Final firms

A continuum of mass unity of final firms produce final output using intermediate output as input. Each producer simply purchases intermediate output, differentiate and sell them to final output consumers. Final output is a CES composite of differentiated varieties

$$Y_t=\left[\int_0^1 y_{jt}^{(\epsilon-1)/\epsilon}\mathrm{d}z\right]^{\epsilon/(\epsilon-1)}. \tag{19}$$

where y_{it} is the output by final firm $z\in[0,1]$ with an associated price denoted p_{jt}. The parameter ϵ, satisfying $\epsilon>1$, governs the extent of imperfect substitutability accross final goods varieties. Cost minimization by final goods consumers implies a downward sloping demand curve for each variety of final good

$$y_{jt}=(p_{jt}/P_t)^{-\epsilon}\,Y_t, \tag{20}$$

where P_t is the aggregate price index determined by the zero profit condition in this market: $P_t=[p_{jt}^{(1-\epsilon)/\epsilon}\mathrm{d}z]^{1/(1-\epsilon)}$. To introduce nominal rigidities, we employ a Calvo pricing scheme. In particular, a fraction of final firms is not allowed to re-optimize its selling price with probability θ but price increases by $\xi\in[0;1)$ with respect to the previous period's rate of price inflation, $p_{jt}=\pi_{t-1}^{\xi}\bar{\pi}^{1-\xi}p_{jt-1}$. The z^{th} firm allowed to update its selling price p_{jt}^{*} with a probability $1-\theta$ maximizes the following discounted sum of profits

$$\max_{\{p_{jt}^*\}} E_t\left\{\sum_{s=0}^{\infty}\theta^s m_{t,t+s}\left[\frac{p_{jt}^*}{P_{t+s}}\Xi_{t+s}-\varepsilon_{t+s}^P\frac{P_{t+s}^x}{P_{t+s}}\right]y_{zt+s}\right\},$$
$$s.t.\ \ y_{zt+s}=(p_{jt}^*\Xi_{t+s}/P_{t+s})^{-\epsilon}Y_{t+s}$$

where ε_t^P is an *ad hoc* cost-push shock to the inflation equation following an AR(1) process which captures exogenous changes in input costs of final firms. Variable Ξ_t captures the contribution of the indexation rule to the firm's future profits, $\Xi_{t+s}=\prod_{j=1}^{s}\pi_{t-1+j}^{\xi}\bar{\pi}^{1-\xi}$ for $s>0$, while $\Xi_{t+s}=1$ for $s=0$.

2.4 Innovators

We model technology following Comin and Gertler (2006), which is in turn based on the expanding-variety framework due to Romer (1990). Innovations in the model take the form of new patents Z_t which are discovered endogenously as a result from private R&D spending. As Comin and Gertler (2006), patents are subject to a "time-to-adop" friction: a new technology does not necessarily give birth immediately to a new variety of intermediate goods. Converting a patent into a new variety is costly for innovators and create a lag between the creation of a new technology and its translation into an stronger rate of growth for the economy.

Assuming that among the family members of each household, there is a fraction $j \in [0; \eta]$ of innovators that creates new technologies.[3] Each innovator owns a stock of existing patents, denoted Z_{jt}, representing the technological frontier in the economy. These technologies are subject to exogenous obsolescence, which occurs with probability δ_A. Letting x_{jt}^A denotes R&D expenditures (in units of growth-oriented investment goods) devoted to the creation of a new patent, denoted $v(x_{jt}^A)$, the law of motion of patents (or the "technological frontier") are given by:

$$Z_{jt} = (1 - \delta_A) \left(Z_{jt-1} + F_A \left(x_{jt-1}^A \right) \right). \tag{21}$$

Here, both existing and new patents are subject to the obsolescence shock, this implies that some new technologies are abandoned and never translate into intermediate varieties. Regarding the production of a new technology, we assume $F_A(x_{jt}^A) = \varepsilon_t^A \xi_t^A (x_{jt}^A)^{\alpha_A}$ where α_A is a technology parameter, and ξ_t^A pins down the growth rate of technology in the balanced growth path. As suggested by Griliches (1990), the production of new patent has decreasing return to scale (*i.e.* $\alpha_A < 1$) that captures a congestion effect that raises the cost of developing new products as the aggregate level of R&D intensity increases. This effects is usually referred to as the "stepping on toes": *i.e.* the obvious new ideas are discovered first and it gets increasingly difficult to find the next new one (see Jones (2005) for a discussion).

Recall that A_{jt} is the number of varieties of intermediate goods, thus any point on the real line between 0 and A_{jt} represents a distinct variety of intermediate goods. With a time-to-adopt assumption, there is a gap between numbers of available and adopted technologies, $Z_{jt} - A_{jt} > 0$. This gap, denoted A_{jt}^u, is referred to as the stock of unadopted technologies and has the following law of motion:

$$A_{jt}^u = (1 - \delta_A) \left(F_A \left(x_{jt-1}^A \right) + \left(1 - p_A \left(s_{jt-1}^A \right) \right) A_{jt-1}^u \right). \tag{22}$$

[3] The number of innovator pins down the steady state of R&D spending-to-GDP ratio.

In this expression, $p_A(.)$ denotes the speed of adoption of an unadopted technology, that is an increasing function of R&D spending s^A_{jt-1} in units of growth-oriented investment goods.[4] If the adopter is not successful, he may try again in the next period. Thus, under our formulation there is slow diffusion of technologies on average that varies positively with the intensity of adoption expenditures. This endogenous mechanism of adoption reproduces the cyclicality of technology diffusion that is observed in the micro data, as shown by Anzoategui et al. (2016).

The remaining set of technologies A_{jt} that are effectively converted into an intermediate goods are given by the following law of motion:

$$A_{jt} = (1-\delta_A)\left(p_A\left(s^A_{jt-1}\right)\left(1-S_A\left(p^A_{jt}/p^A_{jt-1}\right)\right)A^u_{jt-1} + A_{jt-1}\right) \tag{23}$$

where $S_A\left(p^A_{jt}/p^A_{jt-1}\right)$ denotes an adjustment cost on rising the probability of adoption with $S_A(x_t) = 0.5\chi_A(x_t-\bar{x})^2$ similar to Christiano et al. (2005). This cost function is new with respect to the literature and has two goals. First, it captures another congestion externality *à la* Romer (1990) on the adoption of a new technology: firms trying to get a new product to market face a lower probability success. Second, this cost aims at capturing the low frequency nature of A_{jt}: an higher value for χ_A implies a lower frequency for the growth of technology χ_A. We are thus free to estimate this cost parameter to match the evidence by setting a diffuse prior distribution on this parameter. The fit exercise of Moran and Queralto (2017) shows that adjustment cost on R&D expenditures are much larger than for investment goods.

The real profit of the innovator is given by:

$$\Pi^A_{jt} = A_{jt}\frac{\Pi^X_{jt}}{P_t} - \frac{P^I_t}{P_t}x^A_{jt} - \frac{P^I_t}{P_t}A^u_{jt}s^A_{jt} \tag{24}$$

where Π^X_{jt} is the monopoly rent that the innovator obtain from selling an amount A_{jt} of varieties of intermediate goods. At every stage of the innovation process, the innovator successfully adopting a new technology exploits the competitive advantage and monopolize the market as in Aghion and Howitt (1996). The innovator must pay cost of adoption $A^u_{jt}s^A_{jt}$ and R&D expenditures x^A_{jt} in units of growth-oriented investment goods at market price P^I_t/P_t.

Each period maximizes the discounted sum of profits Equation 24 using control variables x^A_{jt}, s^A_{jt}, A_{jt}, A^u_{jt} and p^A_{jt} under technology law of motions Equation 22 and Equation 23. Anticipating symmetry, and letting J^U_t and J^A_t denotes the real shadow

[4]The functional form for $p_A\left(s^A_{jt-1}\right) = \varsigma^A_t\left(s^A_{jt-1}\right)^{\varkappa_A}$ is taken from Comin and Gertler (2006), parameter $\varkappa_A$ is the elasticity of adoption with R&D spending s^A_{jt-1} while ς^A_t is a scaling factor pining down the steady state in the balanced growth path of the model.

values of unadopted and adopted technologies respectively, the value of adopted technologies is the present discounted value of profits from producing the good:

$$J_t^A = \Pi_t^X / P_t + (1 - \delta_A) E_t \left\{ m_{t,t+1} J_{t+1}^A \right\}. \tag{25}$$

While the value of unadopted technologies is determined by:

$$J_t^U = -\frac{P_t^I}{P_t} s_t^A + (1 - \delta_A) E_t \left\{ m_{t,t+1} \left[J_{t+1}^U \left(1 - p_t^A\right) + J_{t+1}^A p_t^A \left(1 - S\left(p_{t+1}^A / p_t^A\right)\right)\right]\right\} \tag{26}$$

Firm invest x_t^A units of growth-oriented investment goods in R&D until the expected marginal product of discovering a new patent reaches the marginal cost of production:

$$\frac{P_t^I}{P_t} = (1 - \delta_A) \Phi_A' \left(x_t^A\right) E_t \left\{ m_{t,t+1} J_{t+1}^U \right\}. \tag{27}$$

The marginal cost of rising the adoption rate, denoted q_t^A, reads as follows:

$$\frac{P_t^I}{P_t} A_t^u = q_t^A p_A' \left(x_t^A\right). \tag{28}$$

Finally, optimal adoption rate is given by:

$$\frac{q_t^A}{A_t^u} + \Psi_t = (1 - \delta_A) E_t \left\{ m_{t,t+1} \left[J_{t+1}^A - J_{t+1}^U \right] \right\} \tag{29}$$

The left hand side of this equation reflects the current marginal cost of adopting a technology,[5] while the right hand side is the discounted benefits in the next period. Innovators increases their adoption expenditures until the marginal cost of adopting is equal to the expected marginal gain. As Comin and Gertler (2006), this marginal gain is $J_{t+1}^A - J_{t+1}^U$ is pro-cyclical, given the greater influence of near term profits on the value of adopted technologies relative to unadopted ones

2.5 Authorities

Concerning federal monetary policy, the general expression of the central bank's rate follows a standard Taylor rule:

$$r_t = r_{t-1}^{\rho} \left[\bar{r} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \right]^{(1-\rho)} \left(\frac{Y_t}{Y_{t-1} \bar{\gamma}} \right)^{\phi_G} \varepsilon_t^R, \tag{30}$$

[5]The term Ψ_t denotes the adjustment cost that must be paid by the innovator that makes the adoption rate sluggish : $\Psi_t = v_t^A (1 - \delta_A) p_{t-1}^A \frac{\partial S\left(p_t^A / p_{t-1}^A\right)}{\partial p_t^A} \frac{A_{t-1}^u}{A_t^u} + (1 - \delta_A) v_{t+1}^A m_{t,t+1} \frac{\partial p_t^A S\left(p_{t+1}^A / p_t^A\right)}{\partial p_t^A}$.

where ε_t^R is a monetary policy shock, $\phi_\pi \geq 1$ is the inflation stance, ϕ_G is another stance on deviations of production growth from its steady state $\bar{\gamma}$. Recall that here, changes in the medium term component, denoted γ_t, is affecting the nominal rate as long as $\phi_G \neq 0$. Following Gust et al. (2017), the smoothing of the rule is based on the shadow rate rather than the effective interest rate, this allows the shadow rate to go far beyond zero as suggested by Wu Miao. A REMPLIR

However, a ZLB constraint on the nominal rate generates a wedge between the desirable interest rate for the economy and the effective one. The effective rate, denoted $\underline{r}_t$, determining the rate of return of government bonds reads as:

$$\underline{r}_t = \max\left(r_t, 1\right) \tag{31}$$

Regarding the government, it consumes G_t units of final goods. The government supports these expenditures by issuing one-period debt securities, b_t, and charging a lump-sum tax to household, T_t. The government budget balance reads as: $G_t + b_{t-1}r_{t-1}/\pi_t = b_t + T_t$. We assume that along the balance growth path, the share of government purchases in output, denoted s_g, is constant over time. To this end, we impose $G_t = \Gamma_t s_g \bar{Y} \varepsilon_t^G$, where Γ_t is the time-varing trend of output, $s_g\bar{Y}$ is the detrended steady state of public spending and ε_t^G is an exogenous AR(1) capturing exogenous changes in aggregate demand. The presence of Γ_t maintains the balanced growth path by making the share of public spending stable as the economy grows.

2.6 Market clearing conditions

The aggregate constraint on final goods market is given by:

$$\frac{Y_t}{\Delta_t^P} = C_t + I_t\varepsilon_t^I\left(1 + S_I\left(\cdot\right)\right) + \frac{P_t^I}{P_t}\left(I_t^H + I_t^A\right) + G_t \tag{32a}$$

where Δ_t^P is the price dispersion term induced by the Calvo pricing scheme.

Aggregate expenditures in R&D and educations are given by:

$$I_t^H = Z_t^H\left(1 + S_H\left(\cdot\right)\right) + H_t^u S_t^H \text{ and } I_t^A = \eta\left(Z_t^A + A_t^u S_t^A\right).$$

The equilibrium on the intermediate market is given by the demand function:

$$Y_t = A_t^{\vartheta-1}\varepsilon_t^A\left(\left(1-e_t\right)H_t^\omega L_{it}\right)^\alpha\left(u_t K_{t-1}\right)^{1-\alpha}$$

where $K_{t-1} = \int_0^{A_t} K_{it-1}\mathrm{d}i$ and $L_t = \int_0^{A_t} L_{it}\mathrm{d}i$. Here, Y_t is interpreted as an average firm given by $Y_t = X_t/A_{t.}$

2.7 Balanced growth path

Empirically, the growth of education and R&D expenditures have both been secularly increasing twice faster than output. To capture this upward trend in the expenditure side of the GDP without structurally modifying key supply side ratios in output, we introduce a common investment-specific trend, denoted, Υ_t, which grows at a fixed gross rate $\bar{\gamma}_X = \Upsilon_t/\Upsilon_{t-1}$. These investment goods I_t^H and I_t^A are produced from final goods by means of a linear technology whereby $1/\Upsilon_t$ units of final goods yield one unit of investment goods. The slope of this investment-specific trend crucially appears in the measurement equation of the model and is estimated in the fit exercise.

This economy features three sources of permanent growth: two are endogenous (A_t and H_t) and one is exogenous Υ_t. As a result, a number of variables, such as output, are not stationary. We therefore perform a change of variable in order to obtain a set of equilibrium conditions that involves only stationary variables. Along the balanced growth path, per capita output $\{Y_t\}$, *per capita* expenditure categories $\{Z_t^H, Z_t^A, A_t^U X_t^A, I_t, C_t\}$ and per capita capital stocks $\{K_{t-1}\}$, per capita income categories $\{W_t\}$ and government expenditures and lump sump transfers $\{T_t\}$ grow at the same rate. This growth rate is equal to:

$$\gamma_t = \Gamma_t/\Gamma_{t-1} \text{ with } \Gamma_t = \left[A_t^{\vartheta-1} H_t^{\mu\alpha}\right]^{1/\alpha}. \tag{33}$$

The growth rate given in Equation 33 depends on technology parameters μ, α and γ; competition parameter in intermediate markets ϑ; and stocks of adopted technologies A_t and skills H_t. These stocks grow both at rates $g_{H,t} = H_t/H_{t-1}$ and $g_{A,t} = A_t/A_{t-1}$.

3 Estimation

3.1 Solution method

To take into account the zero lower bound constraint on the nominal rate, we employ the solution method developed by Guerrieri and Iacoviello (2015). It applies a first order perturbation approach in a piecewise fashion in order to handle occasionally binding constraints. In this model, the presence of the ZLB is treated as a second regime that occasionally binds when the state variable in Equation 30 is below zero, otherwise the constraint is slack. The piecewise linear solution method maps these two different regimes in the same model by using first order approximation of each regime around the same steady state. The solution of the model is non-linear as decision rules parameters depend on the value of the nominal rate. Unlike global methods, this piecewise solution is fast enough to allow the estimation with full information methods of models with many state variables.

Because the solution is state-dependent, the Kalman filter cannot be employed to compute the smoothed sequence of shocks. We follow the estimation method of Guerrieri and Iacoviello (2017) by replacing the Kalman filter by an inversion filter in order to construct the log-likelihood function. Pionneered by Kollmann (2013), this filter extracts the sequence of innovations recursively by inverting the observation equation. One of the drawbacks of this approach lies in the number of shocks that has to be exactly the same as the number of innovations to allow the recursive inversion of the observation equation.[6] Given this limitation, the model is estimated on 8 observable macroeconomic time series and are jointly replicated by the model through the joint realization of 8 corresponding innovations.

3.2 Data

The model is estimated with Bayesian methods on US quarterly data over the sample time period 1950Q1 to 2018Q4 and are all taken from FRED. Our sample spans an extended period of time to capture US growth patterns.

Concerning the transformation of series, the point is to map non-stationary data to a stationary model (namely, the GDP, consumption, investment, R&D and educations expenditures). Following Smets and Wouters (2007), data which exhibit a trend or unit root are made stationary in two steps. First, we divide the sample by the working age population. Second, data are taken in logs and we use a first difference filtering to obtain growth rates. Real variables are deflated by GDP deflator price index. Following **?**, who underline the limited coverage of the nonfarm business sector compared to GDP, we multiply the index of average hours for the nonfarm business sector (all persons) by civilian employment. The inflation rate is computed from the log variations of the GDP deflator, while the nominal rate is measured by the effective fund rate. The latter is divided by 4 to be in a quarterly basis. Interest rate data prior 1955 are taken from Olson and Enders (2012). The effective FF rate is not the central bank target, but an average interest rate charged by depository institutions on money market.

[6]Another drawback concerns the accuracy of this solution method, in particular to capture the precautionary effect that emerges when the ZLB is expected to bind. Atkinson et al. (2019) compare the estimation accuracy of the Occbin solution method with the inversion filter versus a fully non-linear model with a particle filter. They naturally find that the non-linear model is more accurate, but the overall gain does not compensate the computational burden induced by the solution method and the filter. Given the large number of state variables and observables, the Occbin solution with inversion provides enough tractability to deal with the ZLB.

The use of this series with no prior transformation rules out the ZLB, as the FF rate never exactly reached zero but remained slightly above. In addition, the piece-wise solution method does not capture the precautionary effects, so the likelihood that the ZLB binds in the future would have no effects on consumption. To portray more accurately the ZLB, we set the nominal rate data to zero when the lower limit of the federal funds target established by the Federal Open Market Committee reached zero.

To measure the empirical contribution of endogenous growth, we use a cost-based approach by including two new time series with respect to the benchmark model of Smets and Wouters (2007). First, R&D expenditures are observable which allows to characterize the unobserved growth of technology. We use the nonresidential gross fixed private domestic investment in intellectual property products. Second, we measure investment in education through personal consumption expenditures in education services. However this series is in an annual basis, so we apply the temporal disaggregation method of Fernandez (1981). This method makes the use of the information obtained from related indicators observed at the desired higher frequency. We use health expenditure as the latter is the most correlated time series with education expenditures among all sub-elements constituting personal consumption expenditures. Finally, these two new time series are transformed using the same scheme as output.

Measurement equations are given by:

$$\begin{bmatrix} \text{Output Growth} \\ \text{Hours} \\ \text{Consumption Growth} \\ \text{Investment Growth} \\ \text{Inflation} \\ \text{Interest Rate} \\ \text{R\&D Investment} \\ \text{Education Expenditures} \end{bmatrix} = \begin{bmatrix} 100 \times \log \bar{\gamma} \\ 0 \\ 100 \times \log \bar{\gamma} \\ 100 \times \log \bar{\gamma} \\ 100 \times \log \bar{\pi} \\ 100 \times \log \bar{r} \\ 100 \times \log (\bar{\gamma}.\bar{\gamma}_X) \\ 100 \times \log (\bar{\gamma}.\bar{\gamma}_X) \end{bmatrix} + \begin{bmatrix} \hat{\gamma}_t \\ 0 \\ \hat{\gamma}_t \\ \hat{\gamma}_t \\ 0 \\ 0 \\ \hat{\gamma}_t \\ \hat{\gamma}_t \end{bmatrix} + \begin{bmatrix} \Delta \hat{y}_t \\ \hat{l}_t \\ \Delta \hat{c}_t \\ \Delta \hat{\imath}_t \\ \hat{\pi}_t \\ \hat{r}_t \\ \Delta \hat{\imath}_t^A \\ \Delta \hat{\imath}_t^H \end{bmatrix}, \quad (34)$$

where the hat over the variables' names denotes the percentage deviations of these variables from their steady state, while those with a bar denotes the steady state. A striking feature of this model with respect to other estimated macroeconomic models is the existence of a common endogenous trend. We note that $\Delta \hat{y}_t$, $\Delta \hat{c}_t$, $\Delta \hat{\imath}_t$, $\Delta \hat{\imath}_t^A$ and $\Delta \hat{\imath}_t^H$ are cointegrated with $\hat{\gamma}_t$, thus the endogenous determination of $\hat{\gamma}_t$ is key as it jointly affects most of observed variables.

3.3 Calibration and Prior Distributions

Calibrated parameters are reported in Table 1. As Christiano et al. (2014), the discount factor is set as to 0.9989, the depreciation rate of physical capital is 2.5% and the government spending to GDP ratio is 20%. As in most real business cycles models, steady state working hours are given a value of 1/3. Given the high value of the discount factor, we impose $\alpha = 0.8$ for the labor intensity parameter in the production function to obtain an investment to GDP ratio close to 20%. Substitution on final goods market is set to 10 as in Smets and Wouters (2007) thus implying a 11% percent steady state markup. For intermediate goods, the elasticity of substitution is set to 3.85 as Anzoategui et al. (2016) to be in line with the estimate of Broda and Weinstein (2006). Steady state adoption rate for technology is set to 0.2/4 as Anzoategui et al. (2016) to get an average time lag to adopt of five years. The calibration of human capital adoption rate is more problematic as human capital is an unobservable variable. We impose an adoption rate of 0.33/4 in order to mimic the graduation of a bachelor degree in 3 years. Regarding the elasticity of patents creation to R&D expenditures, we follow the calibration strategy of Comin and Gertler (2006) by borrowing the lower bound interval value estimated by Griliches (1990). R&D expenditures in GDP are set to 1.31% to match postwar US data. Finally, regarding the skill premium ω, Alon et al. (2018) finds that this parameter lies at 95% in the interval [1;2] for the US economy. Consistently with this estimate, we assign a value of 1.15 to match the education spending to GDP over the same sample period.

Table 2 and 3 report prior distributions of shock and structural parameters, respectively. Common parameters with Smets and Wouters (2007) are given prior distributions similar or close to this benchmark paper. Regarding the adoption elasticity to final goods inputs, papers featuring an endogenous technology such as Comin and Gertler (2006) typically calibrate this parameter to 0.95. To get an estimated parameter in the same range, we impose a beta distribution with prior mean of 0.8 and standard deviation of 0.05. We impose the same prior distribution for human capital. For congestion costs on adopting new technologies, Moran and Queralto (2017) argue that adjustment cost of R&D are higher than those of investment, unlike these authors we do not make any strong prior assumption on this cost by setting the same prior information as investment adjustment costs. This prior is not informative and will let the data be informative about their posterior values. For the percentage growth rate of human capital $\bar{\gamma}_H$, Lucas Jr (1988) calibrates this parameter to 0.014 using the estimation of Denison Edward (1962). This would correspond to a 0.35% quarterly growth rate that would abnormally drive all the contribution to the growth in the model.[7] We thus impose on $\bar{\gamma}_H$ a gamma distribution with mean of 0.2 and standard deviation of 0.15: this prior is diffuse enough to allow the data to decide whether one engine drives all the observed growth of output. The remaining set of parameters are which are not estimated nor calibrated are determined endogenously in the deterministic steady state of the model.

3.4 Posterior Distributions

In addition to priors distributions, Table 2, 3 and Figure 10 also report posterior distributions drawn from four parallel chains of 100,000 iterations of the Metropolis-Hasting algorithm, with an acceptance ratio close to 25%. To contrast the result with the fixed trend assumption of Smets and Wouters (2007), an alternative version of the model was estimated with the same prior distribution but with a fixed trend. Two time series and shocks related to the two engines of growth are thus discarded from the estimation, while the ZLB is preserved. This difference in the number of observable time series between the two models does not allow us to compare likelihood ratios. Figure 10 shows that data were all informative as the posterior distribution of each parameter is fairly different its posterior distribution.

Regarding the model with endogenous trends, standard parameters from the workhorse New Keynesian model are rather consistent with previous findings such as Christiano et al. (2005) and Smets and Wouters (2007).

Regarding parameters specific to the two endogenous growth engines, I find that shocks which are the most persistent are those related to the accumulation of technologies and knowledge, these shocks are probably the main source of persistence in the model with endogenous trends, and generate desired low frequency variations for the endogenous trend.

For parameters related to technology, the obsolescence rate of technology is 0.75% in a quarterly basis, which is consistent with the 3% annual obsolescence rate of Comin and Gertler (2006). In the same vein, the adoption rate elasticity is strikingly the close to the one of Comin and Gertler (2006). Regarding the sluggishness of the adoption rate, the cost parameter is much higher than for investment goods as suggested by Moran and Queralto (2017).

Next we turn to the parameter related to the accumulation of knowledge. First, we find a quarterly obsolescence rate of knowledge of 0.4% that lies in the ballpark of the 1.5% annual rate of Jones et al. (1993), while the technology of skill creation is more intensive in goods compared to the same benchmark paper. In addition, the external

[7] This result is not surprising as Lucas' model only include one source of growth.

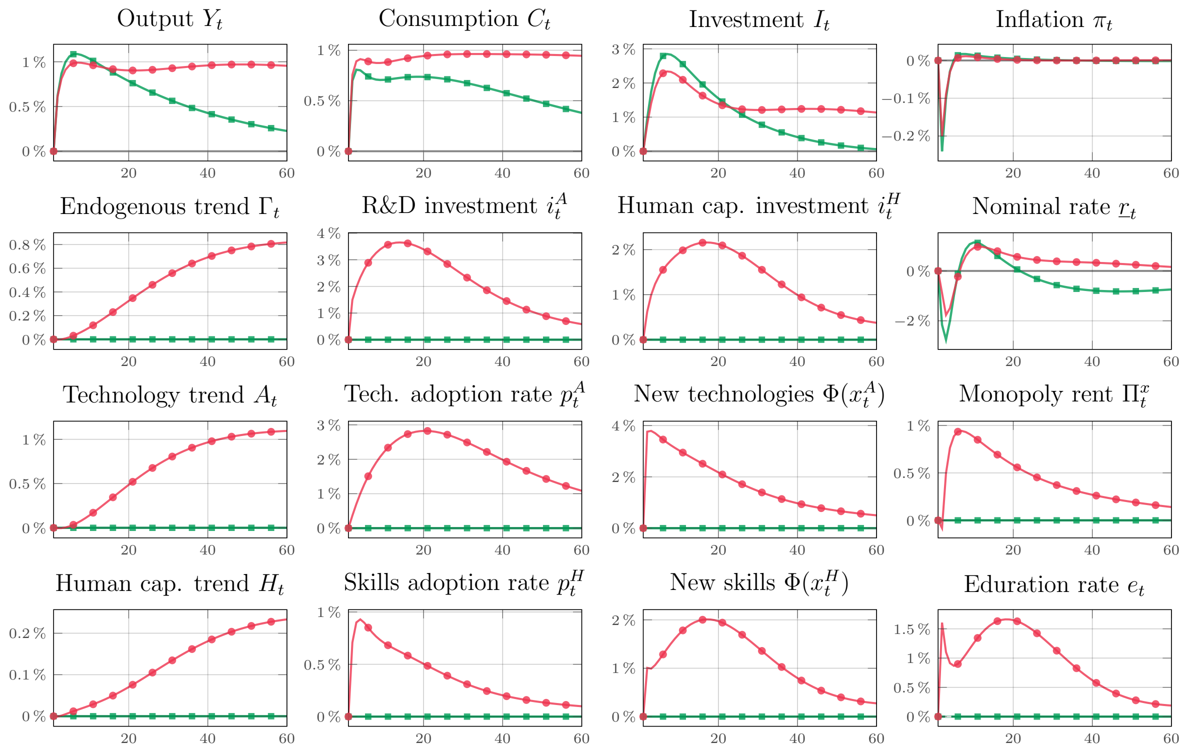


Figure 1: Propagation of an unit productivity shock in the endogenous and exogenous trends models.

Notes: Variables with a trend are detrended using a linear trend. Both models are calibrated using the posterior mean of the endogenous trends model.

effect of knowledge is twice lower than the one computed by Lucas Jr (1988).

Finally by comparing the models with endogenous versus exogenous growths, we find that low frequency fluctuations are not correctly accounted by the exogenous growth model, and are thus captured by more persistence in the shocks processes.

4 Macroeconomic implications of endogenous trends

4.1 Inspecting the propagation mechanism

To understand how the two endogenous trends affect the propagation mechanism, we contrast the impulse response functions of our model with those obtained with the exogenous trend model. We use the same calibration for the two models based on the posterior mean of the endogenous growth model. We thus examine the propagation following a standard productivity shock and a cost-push shock. We consider a cost-push shock, given his importance in shaping the monetary policy trade-off.

4.1.1 A productivity shock

Figure ?? reports the impulse response functions of the model following a standard productivity shock in the production function of firms. In the short term, the IRFs between the two models are remarkably the same: the rise in productivity makes both labor and physical capital more productive, leading to a decline of the inflation rate combined with a rise in the rate of growth of output. As in the workhorse New Keynesian model, monetary reacts to the decline in inflation by lowering the nominal rate. The decline in the real rate lowers the incentive to save for households, and thus rises in turn consumption expenditures. In the meantime, the cost of physical is lower and allows intermediate firms to investment more.

However after about 10 periods, the IRFs between the two models seriously diverge. This divergence originates from the higher persistence featured by the endogenous trends. The rise in productivity increases the marginal product of human capital, and in turn enhances the value of unadopted skills. Firms thus engage their employees into vocational training which rises the share of the labor force into education. Accumulating one effective unit of human capital takes on average 3 years which makes the adoption of a new skill very sticky. The resulting consequence of this persistence mechanism lies in the fact that education efforts takes time to translate into effective units of human capital. The trend of adopted skills gradually rises which drives the endogenous persistence of output, consumption and investment above their linear trends for an extended period of time.

For the growth of technology engine, the propagation of a TFP shock features similar dynamics with respect to the knowledge engine of growth. Following a TFP shock, firms are more profitable as they produce more with less inputs. Higher profits increases the monopoly rent for innovators which through a Schumpterian effect drives upward the shadow value of adopted technologies. Innovators have more incentive to innovate and adopt new technologies to monopolize the rent, in turn they rise their R&D spending which enhances the demand for final goods. As for human capital, this engine of growth features important delays in the propagation of a TFP shock. The trend of technologies thus requires up to 60 quarters to peak drives output growth.

4.1.2 A cost push shock

Figure ?? reports the response following a cost push shock to the marginal cost of production of intermediate firms. This shock typically increases inflation and reduces real production, which creates a trade-off for monetary policy between output and prices stabilization. As in the standard New Keynesian model with exogenous growth, a cost push shocks reduces output, consumption and investment, while monetary policy rise the interest rates to dampen inflationary pressures in the economy.

However, the presence of endogenous trends affects the persistence of the cost push shock. This shock deteriorates the monopoly rent of intermediate firms, innovators thus have in turn less incentive to engage into R&D spending as prospects of future profits sinks. Innovators thus reduce their R&D spending, which in turn exhacerbates the recession under endogenous growth. The production of new patent and the adoption rate of technologies both declines, as a consequence the endogenous trend of technology is below its linear trend level for an extended period of time. As output, consumption and investment are co-integrated with the endogenous trend.

In contrast, the endogenous trend of knowledge features a different dynamic with respect to the technology trend. The cost push shock deteriorates the marginal product of labor, the opportunity cost of being in education (rather than employment) declines sharply. As a result, during a recession firms cope with the crisis by increasing their efforts in education as its opportunity cost is lower.[8] However, these education efforts don't materialize immediately into adopted units of human capital, as higher inflation increases the adoption costs. The decline in the adoption rate of skills dominates the positive effect of more education effort. However after 15 periods, the accumulation of new skills are finally adopted which drives the human capital above its linear trend.

4.2 Business and medium cycle moments analysis

In this subsection, we inspect how the model with endogenous trend model is able to capture salient features of the data at a business and medium term frequencies. Table **??** presents the set of statistics of observable variables common to the endogenous and exogenous trends models..

Regarding standard deviations, the endogenous trends model clearly does a better job at a business and medium term cycles frequencies. In contrast, when the endogenous trends model overshoots the volatility of some variables, the exogenous trends model does even worse.

For auto-correlations, the standard model does a slightly better job than the endogenous trends model at a business cycle frequency. Auto-correlations at a medium frequency are rather uninformative as the correlation coefficients are closed to one at

[8]This result is not new in the literature. Opportunity cost models of growth have argued that recessions are times when firms engage in productivity-improving activities because of intertemporal substitution. See Saint-Paul (1993) for an empirical evaluation of these models.

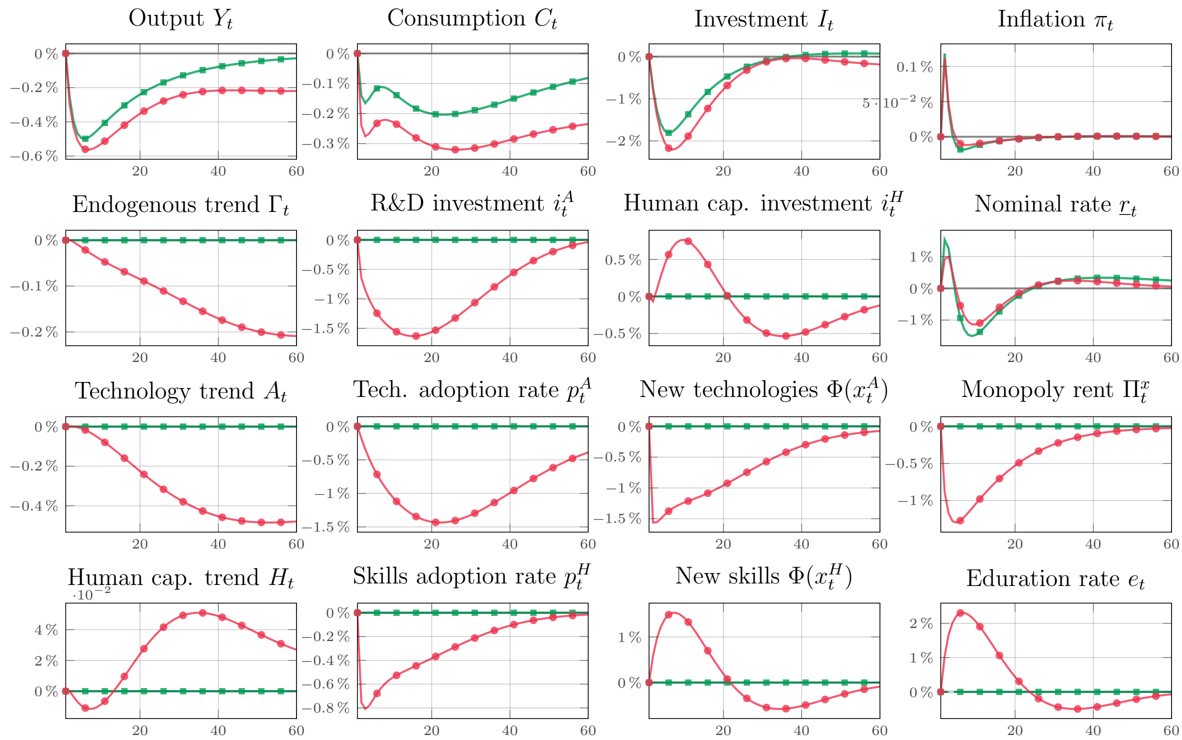


Figure 2: Propagation of an unit cost-push shock in the endogenous and exogenous trends models.

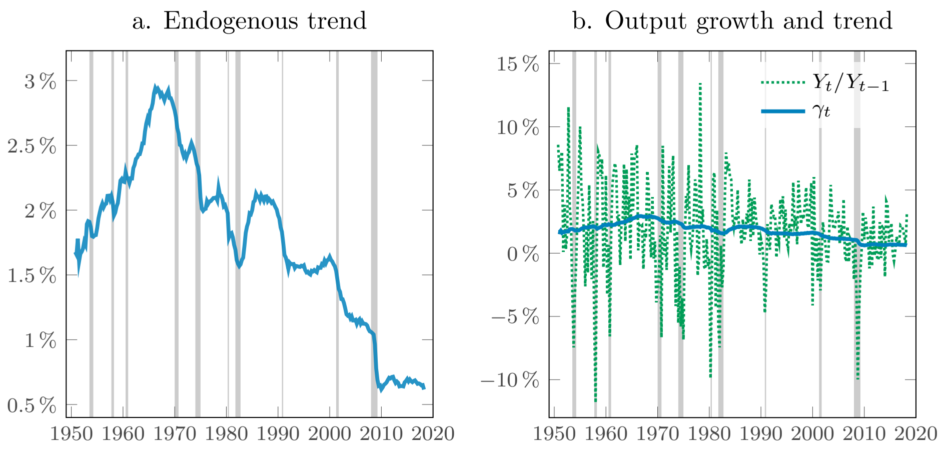
Notes: Variables with a trend are detrended using a linear trend. Both models are calibrated using the posterior mean of the endogenous trends model.

a low frequency.

5 Why economic growth has declined since WWII?

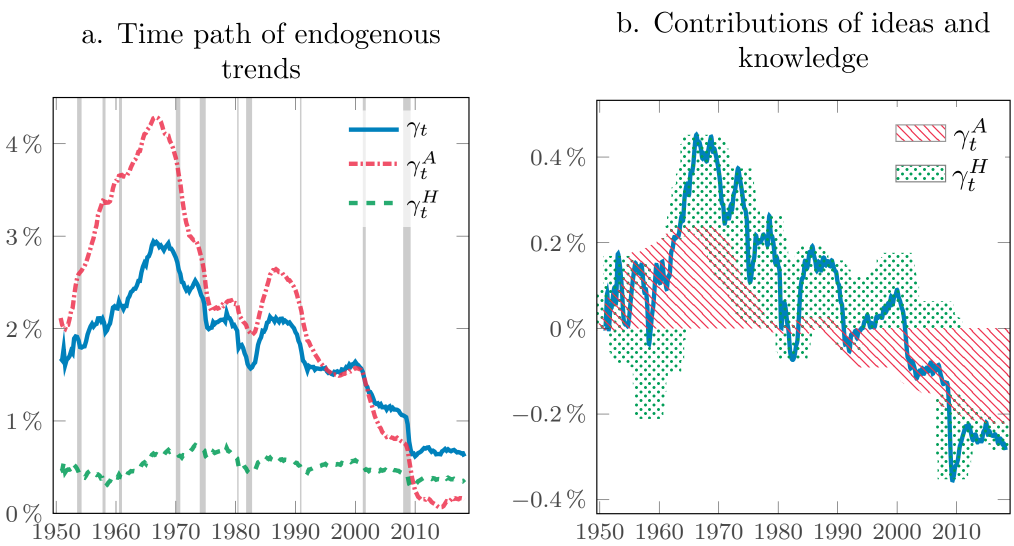
5.1 The role of endogenous trends

Figure 3 reports the time-path of this medium term component measured by the estimated model. Recall that this component jointly rise growth rates of key macroeconomic aggregates such as consumption, investment and GDP. Over the postwar, the US economy has experienced sizable medium frequency oscillations. From 1950 up to 1970, the endogenous trend has been continually increasing upward despite small recessionary episodes. This period is characterized by a persistent increase in R&D expenditures, thus leading the trend to peak up to 3% at the end of the 60s. In decades following the 70s, the trend growth rate have been declining synchronously with the different recessions hitting the US economy. Recessions induced by oil price shocks in the 70s and the Great Recession clearly damaged the engine of growth. If at first sight the trend to be volatile, Figure 3.b shows that these fluctuations are less volatile com-



Notes: The shaded areas represent the recessions as dated by the NBER.

Figure 3: Historical path of the endogenous trend between 1950q1 to 2018q4.



Notes: The shaded areas represent the recessions as dated by the NBER.

Figure 4: Historical path of the endogenous trend between 1950q1 to 2018q4.

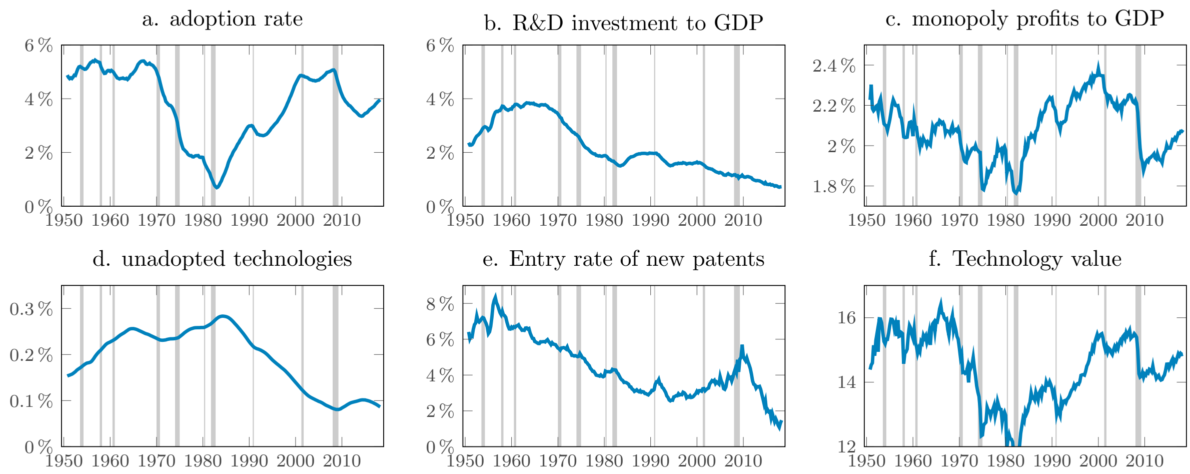
pared to the annual fluctuations of real output. Thus the endogenous trend clearly replicates a fraction of the low frequency volatility in macroeconomic time series. Antolin-Diaz et al. (2017) employs a dynamic factor model to track changes in the long run growth rate of GDP, by separating them from their cyclical counterpart. Their sample span a period as long as the one used in the fit exercise and thus allows us to examine any similarity between their estimates of the long run growth with the endogenous trend. Both models seems to generate close estimates of the long run growth, which confirms that the endogenous growth model is able to successfully capture low frequency variations.

Why has the trend reduced over time? Unlike Anzoategui et al. (2016) who consider only one source of growth, the present framework allows to disentangle contributions induced by the growth of knowledge (human capital) from those induced by the growth of ideas (technology). Figure 4 reports on the left the common endogenous trend of the economy γ_t, that is a non-linear function of γ_t^A and γ_t^H (see Equation 33). Over the sample period, it's striking to notice that the growth of skills has been remarkably stable over time while the main source of variations of the endogenous trend since 1950 has been the growth of technology. From 1950, the growth rate of technology peaked up to 4.5% but started to decline prior to the two coming recessions induced by rising oil prices.

We next explore the relative importance of the two sources of growth on the common trend. Equation 33.b reports the percentage contribution of each source of growth using a linear approximation of Equation 33. This figure confirms that the R&D engine accounts for much of the cyclical variation in the endogenous trend, as it has contributed on average up to 60% of the variation of the trend. The downward pressure on the trend has clearly been driven by variations in technology since 1970. While before the financial crisis, the growth of knowledge was driving up the trend, the financial crisis worryingly reversed the contribution of human capital.

How does the model account for the decline in the growth rate of technology? Figure 5 plots the detrended evolution of main state variables determining the aggregate evolution of technology. During the first twenty years of the sample, the growth of R&D investment has been high enough (figure b) to fuel an high rate of entry of new patents (figure e), the latter were mostly effectively adopted and thus converted into new intermediate goods (figure a). In the meantime, the monopoly was declining but did not translated into lower expected technology value (figure f) as expectations about future monopoly rents were high. However the 70s recessions irrevocably damaged the main engine of growth and announce the beginning of a slowdown.

For the post 70s period, (Gordon, 2012, 2017) argues that technological advancement has been slowing and translates into slower growth over time. The model tends provides a theoretical formulation of Gordon's narrative that explains this reduction in the rate of growth of the US economy. The model captures this decline in the growth rate by a reduction in the entry rate of new patents, that measures the productivity of innovators during 1950. This result is corroborated by the estimated model of Anzoategui et al. (2016) that finds a similar path for the R&D productivity. Recessions in the 70s reduced the monopoly rents (figure c), and thus reduced the value of adopted technologies (figure f). Thus, the incentive for the innovator to adopt a technology became low (figure a). This reduction in the adoption rate of technology rose the stock of unadopted technologies until 1985. After this date, the stock of unadopted technologies has been critically falling, mainly because the creation rate of new patent



Notes: The shaded areas represent the recessions as dated by the NBER. These figures are generated by feeding the smoothed errors into the model's policy function to obtain unobservable state variables.

Figure 5: Historical path of the endogenous trend between 1950q1 to 2018q4.

and the R&D expenditures were declining. Despite an improvement of the situation in the 2000s, the financial crisis broken this recovery in the engine of growth through a large contraction of the monopoly rent.

5.2 The role of labor productivity

The model is able to disantangle the driving forces of productivity growth. Growth accounting provides further perspective on the forces driving labor productivity growth over the sample period. The expression of labor productivity growth, defined as growth in real output per hour, is directly obtained by dividing the production function by hours worked:

$$\frac{Y_t}{L_t} = \underbrace{\left[A_t^{(\vartheta-1)} H_t^{\mu}\right]^{1/\alpha}}_{\text{endogenous TFP}} \times \underbrace{\varepsilon_t^Z}_{\text{Exogenous TFP}} \times \underbrace{(1-e_t)^{\alpha}(u_t)^{1-\alpha}}_{\text{inputs utilization}} \times \underbrace{\left(\hat{K}_{t-1}/L_t\right)^{1-\alpha}}_{\text{capital deepening}}. \tag{35}$$

Our model offers four different sources of productivity growth. In a similar growth accounting exercise as Fernald (2015), labor productivity is explained by TFP, variable inputs utilization and capital deepening. A few differences with Fernald (2015) are worth to be discussed. First, Fernald interprets inputs utilization as variations in capital's workweek and labor effort. In our model there is no labor effort but education effort e_t, the latter behaves very similarly through its countercylical aspects: in a recession firms increase their education efforts as the opportunity cost of being in vocational training rather than working reduces. As a result, our measure of inputs utilization includes both education efforts and capital utilization rate. Secondly, unlike Fernald who considers TFP as a Solow residual from a growth accounting ex-

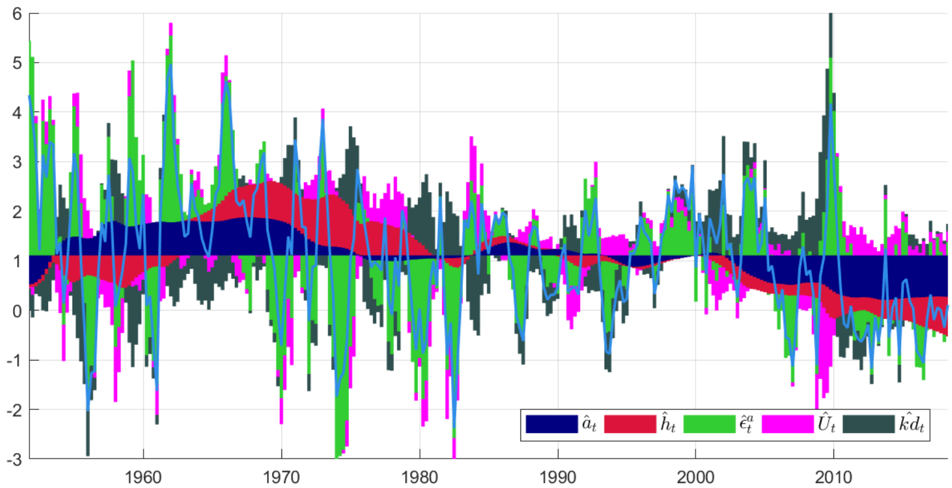


Figure 6: Growth accounting of US labor productivity (y-o-y basis) between 1951Q3 to 2019Q3.

ercise, our model provides three different sources of TFP growth. TFP is determined by the standard TFP shock from the real business cycle theory, denoted ε_t^Z, and by two endogenous sources based on the accumulation of ideas A_t and knowledge H_t. Our approach slightly differs from Anzoategui et al. (2019) as our measure of TFP includes the role of human capital. Human capital is likely to be important, and can be interpreted as the labor quality of Fernald's growth accounting.

Differentiating logarithmically (where hats are log-changes) yields the expression of labor productivity growth:

$$\hat{y}_t - \hat{l}_t = \frac{(\vartheta - 1)}{\alpha}\hat{a}_t + \frac{\mu}{\alpha}\hat{h}_t + \hat{\varepsilon}_t^A + \hat{U}_t + (1-\alpha)\, kd_{t-1}. \tag{36}$$

Figure 6 reports the contribution of ideas, knowledge, exogenous TFP, inputs utilization and capital deepening on the annual labor productivity growth of the US economy. Strikingly, the endogenous components of TFP plays a non-trivial role on the observed fluctuations of productivity. Labor productivity has been largely fuelled by the accumulation of technologies and knowledge during the 60s. At their peak in 1970, endogenous trends increased by 1.5% of the growth of labor productivity. However, oil price shocks in the 70s reduced the role of endogenous trends in driving labor productivity. During the Great Moderation period, the role of endogenous trends was modest. In the 2000s, the contribution has become strongly negative, thus corroborating the findings of Anzoategui et al. (2019) who found that technical change accounted for an important slowdown of the labor productivity. In our setup, this slowdown is explained by both the decline in the accumulation of ideas and knowledge. Regarding the financial crisis, we find that the decline in productivity is fueled by the drop in capital intensity as underlined by Hall (2015) and Anzoategui et al. (2019).

6 Quantifying the effect of the zero lower bound

compare results with Gust et al. (2017)

compute the difference between pice-wise and observed time series with uncertainty on MH

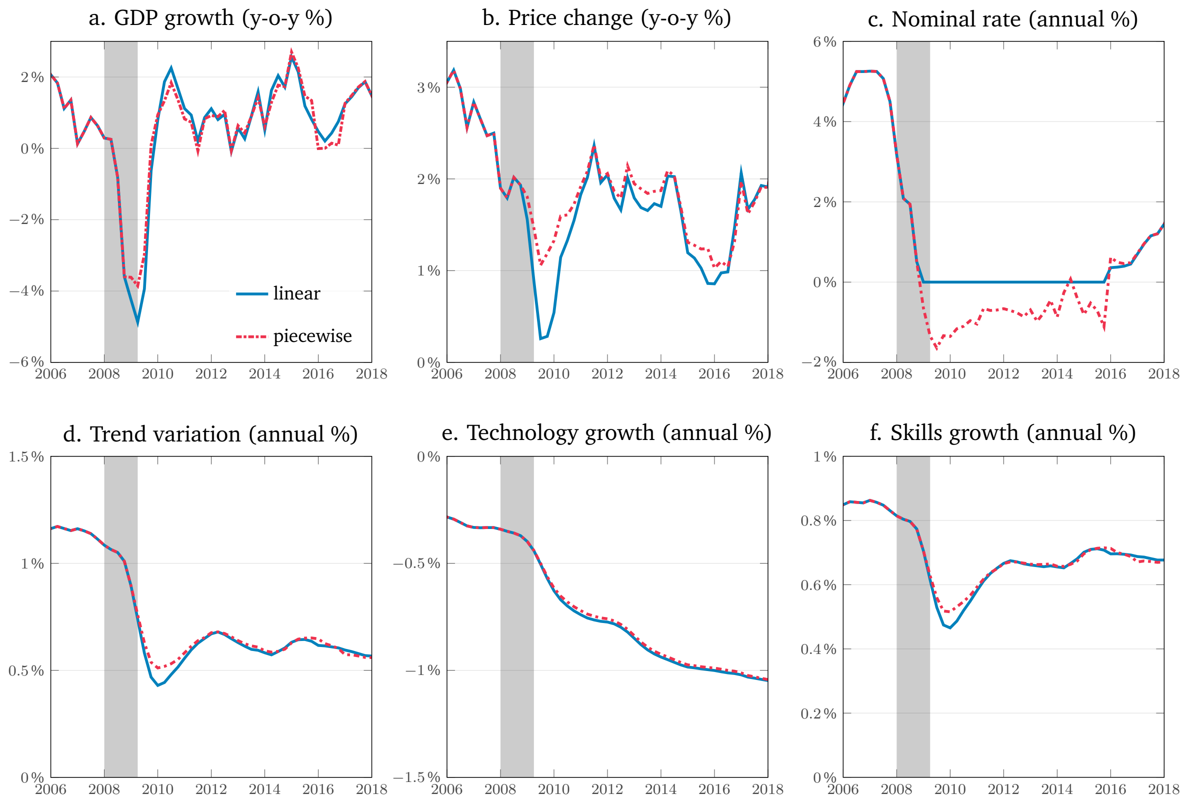
compare with model with exogenous trend

We now explore how important was the presence of the zero lower bound on the economic contraction of the US economy during the Great Recession. Figure 7 compares the observed data against the outcome from the same model without the constraint on the nominal rate.

When the ZLB started to bind in 2009, monetary policy could not accommodate further the nominal rate to dampen the recession. As a consequence, real interest rates were abnormally high i which, through the Euler equation, artificially increased both the marginal utility of consumption and the cost of capital renting, and in turn it weakened aggregate demand. Our results show that without the ZLB the annual growth rate of output would have been 1.5% higher in 2009. In addition, the ZLB has amplified the deflation mechanism, this translates into year-on-year inflation differential in 2009 of 1%, and 0.3% in 2014 and 2016.

Using the estimated model, we can also gauge the effect of the zero lower bound on the two engines of growth in the economy. Figure 7 provides the annualized growth rate of the medium term component in Equation 33. This component is itself a combination of adopted technologies and adopted skills depicted in subfigures e and d. According to the model, the role of the ZLB on growth is trivial as the trend decline was quick and negligible: the trend reduced of 0.1 pp in 2009 before recovering quickly with no persistent effect. The main contributor to this modest drop is the accumulation process of human capital that is temporary damaged by the high rates.

A natural question at hand is to the possible causal relation between the slowdown in economic growth and the zero lower bound. As Orphanides (2003) emphasized, real-time misperceptions about the long-run growth of the economy can play a large role in monetary policy mistakes. In the standard workhorse New Keynesian, monetary policy stabilizes short run fluctuations of output and inflation without having any concerns about possible long-term changes in the growth patterns of the economy. Here, we perform a counterfactual exercise to examine whether the monetary policy



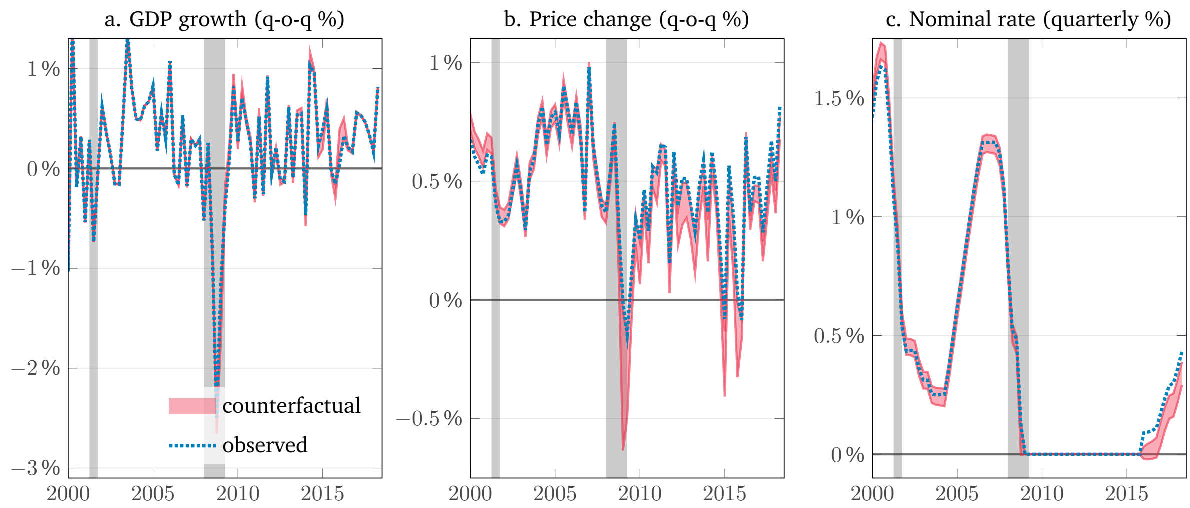
Notes: The shaded areas represent the recessions as dated by the NBER. The simulation shows the filtered series for 6 variables from the estimated model. The dashed line show their paths feeding in the same shocks but in absence of the zero lower bound on the nominal rate.

Figure 7: Macroeconomic implications of the zero lower bound during the Great Recession.

reaction to long term growth strongly has lead the nominal rate to reach the zero lower bound. Figure 8 reports a counterfactual interest rate that does not respond to the endogenous trend. When monetary policy does not respond to long term change in the growth rate, the nominal rate is higher. An higher interest rate induces a reduction of inflation and actually increases the zero lower bound probability. Thus the macroeconomic situation is worse when monetary policy does not respond to long term growth as a ZLB binds for more quarters.

7 Forecasting performance

DSGE models has been criticized for not being able to anticipate the slow growth in output after the financial crisis. In this section, we investigate whether our model with endogenous growth is the missing ingredient of current state-of-art forecasting models. We compute out-of-sample forecasts between 2003Q1 up to 2018Q2 based on model parameters estimated only on revised data available at the date of the forecast. We assess the accuracy of each forecast through the root the mean square errors



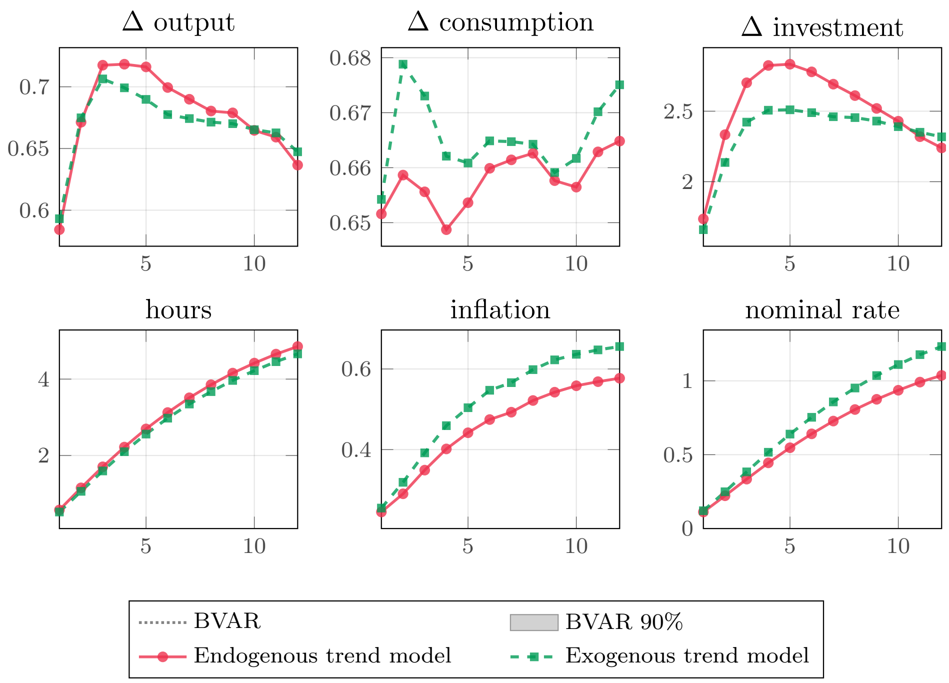
Notes: The shaded areas represent the recessions as dated by the NBER. The simulation shows the filtered series for 6 variables from the estimated model. The dashed line show their paths feeding in the same shocks but in absence of monetary policy reaction to long term changes in output. Uncertainty in red is drawn from by randomly picking 500 draws from the posterior distribution.

Figure 8: Counterfactual path in a model with no reaction to the endogenous trend

(RMSE). Figure 9 display RMSE at forecast horizon lying between 1 up to 12 quarters. We compare the RMSEs of our baseline model with those implied by the exogenous trend model as well as RMSEs implied by a Bayesian Vector Autoregression.[9] Forecasts of the BVAR are based on the posterior means of the parameters updated each quarter while those of the DSGE are based on the posterior modes. The grey area is uncertainty on the RMSE of the BVAR. As Christiano et al. (2014), this uncertainty is constructed so that if the RMSE of our baseline model lies in the grey area for a particular variable and forecast horizon, then the classical null hypothesis that the two RMSE are actually the same fails to be rejected at the 90 percent level.

With the exception of inflation and the nominal rate, we find that the DSGE models are performing better than the BVAR model. This corroborates the findings of Smets and Wouters (2007) that theoretical models are serious alternatives to atheoretical models for forecasting exercises. Strikingly, our model compares very well against the two alternatives. Except for inflation, RMSE of the endogenous trends model is statistically different than the BVAR model. This shows that the endogenous growth mechanism is able to improve the forecasting performance of the model, even during the financial crisis episode.

[9] As Smets and Wouters (2007), our BVAR includes four lags and its parameters follow the Minnesota priors.



Notes: The shaded areas and red dashed lines represent 90% intervals of the RMSE from the BVAR model assuming that $N * RMSE/\sigma^2$ follows a chi-squared distribution with N degrees of freedom.

Figure 9: RMSE comparison for three forecasting models

8 Conclusion

We have estimated a non-linear DSGE model that originally features a time varying trend driven by two sources of endogenous growth. We then used the model to assess the slowdown of long term growth, in particular following the onset of the Great Recession. Based on the estimated model, our key result is that we corroborate the thesis of a strong decline in the long term trend of the US economy. Among the two sources of growth examined in the paper, the slowdown mainly is induced by the technology engine reflecting a decline in the productivity of creation of new technologies since 1960. This finding tends to favor the Gordon (2012) theory stating that the US growth has strongly declined since 1970. In addition, we find that a standard macro-model with exogenous growth erroneously captures low frequency changes in economic growth by highly persistent macroeconomic shocks. In contrast, the model featuring an endogenous trend successfully captures this low frequency fluctuations.

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	Calibrated parameters	Values
β	Discount factor	0.9989
$\bar{l}$	Labor supply	1/3
s_A	Public spending share in output	0.20
$\bar{u}$	Capital utilization rate	1
α	Labor intensity	0.80
ε_A	Patent production function	0.60
ϑ	Substitution intermediate goods	3.85
$\delta\left(\bar{U}\right)$	Capital depreciation rate	0.025
$\bar{U}$	Utilization rate in steady state	1
$p_A\left(\bar{x}^A\right)$	Technology adoption rate	0.20/4
$p_H\left(\bar{x}^H\right)$	Skill adoption rate	0.33/4
$\bar{I}^A/\bar{Y}$	R&D expenditures to GDP	0.0131
ω	Skill premium	1.15
ϵ	Substitution final goods	10

Table 1: Calibrated parameter values (quarterly basis)

		Prior distributions			Posterior distributions mean Mean [0.050;0.950]	
		Shape	Mean	Std.	Endogenous Trend	Exogenous Trend
Std. productivity	$100 \times \sigma_Z$	IG_2	0.1	0.5	0.702 [0.651;0.756]	0.807 [0.715;0.902]
Std. premium	$100 \times \sigma_B$	IG_2	0.1	0.5	0.193 [0.171;0.218]	0.242 [0.207;0.286]
Std. markup	$100 \times \sigma_P$	IG_2	0.1	0.5	1.807 [1.520;2.127]	4.380 [2.928;4.944]
Std. investment	$100 \times \sigma_I$	IG_2	0.1	0.5	1.327 [1.172;1.515]	1.194 [1.050;1.367]
Std. spending	$100 \times \sigma_G$	IG_2	0.1	0.5	3.033 [2.833;3.268]	3.208 [2.957;3.485]
Std. monetary policy	$100 \times \sigma_R$	IG_2	0.1	0.5	0.281 [0.255;0.312]	0.247 [0.226;0.274]
Std. patent	$100 \times \sigma_A$	IG_2	0.1	0.5	3.640 [3.051;4.862]	-
Std. human capital	$100 \times \sigma_H$	IG_2	0.1	0.5	1.953 [1.653;2.290]	-
AR(1) productivity	ρ_Z	B	0.5	0.2	0.964 [0.951;0.975]	0.977 [0.969;0.984]
AR(1) premium	ρ_B	B	0.5	0.2	0.916 [0.892;0.940]	0.926 [0.901;0.948]
AR(1) markup	ρ_P	B	0.5	0.2	0.928 [0.909;0.946]	0.978 [0.908;0.993]
AR(1) investment	ρ_I	B	0.5	0.2	0.963 [0.942;0.978]	0.945 [0.926;0.960]
AR(1) spending	ρ_G	B	0.5	0.2	0.986 [0.980;0.991]	0.987 [0.979;0.992]
AR(1) patent	ρ_A	B	0.5	0.2	0.982 [0.966;0.992]	-
AR(1) human capital	ρ_H	B	0.5	0.2	0.918 [0.881;0.949]	-
Marginal log-likelihood					-800.2968	-176.1236

Table 2: Prior and Posterior distributions of shocks

		Prior distributions			Posterior distributions mean Mean [0.050;0.950]	
		Shape	Mean	Std.	Endogenous Trend	Exogenous Trend
Consumption aversion	σ_C	G	1	0.35	0.981 [0.905;1.084]	1.425 [1.244;1.584]
Labor Disutility	σ_L	G	2	0.5	0.973 [0.709;1.247]	1.829 [1.357;2.282]
Consumption habits	h	B	0.5	0.2	0.074 [0.030;0.124]	0.240 [0.169;0.314]
Calvo price lotery	θ	B	0.5	0.1	0.663 [0.627;0.697]	0.863 [0.810;0.882]
Price indexation rate	ξ	B	0.5	0.2	0.041 [0.012;0.096]	0.033 [0.009;0.076]
Capital utilization elasticity	ψ	B	4	1	3.995 [3.245;4.815]	0.082 [0.024;0.205]
Investment cost	χ_I	N	4	1	0.928 [0.743;1.156]	1.246 [0.956;1.626]
MPR smoothing	ρ	B	0.75	0.1	0.822 [0.800;0.841]	0.867 [0.847;0.885]
MPR inflation	ϕ_π	N	1.5	0.25	2.693 [2.529;2.863]	2.746 [2.562;2.951]
MPR output growth gap	$\phi_{\Delta y}$	G	0.5	0.25	0.136 [0.106;0.167]	0.146 [0.122;0.176]
Patents obsolescence rate	$\delta_A \times 100$	G	2.5	0.4	0.876 [0.682;1.168]	-
Skills obsolescence rate	$\delta_H \times 100$	G	2.5	0.4	0.366 [0.252;0.522]	-
Adoption rate elasticity	$\varkappa_A$	B	0.7	0.07	0.944 [0.929;0.958]	-
Adoption rate elasticity	$\varkappa_H$	B	0.7	0.07	0.665 [0.630;0.703]	-
Adoption congestion cost	χ_A	N	4	1.5	8.608 [6.837;10.34]	-
Adoption congestion cost	χ_H	N	4	1.5	8.605 [7.085;9.794]	-
Goods intensity in skills	ν	B	0.2	0.05	0.548 [0.429;0.616]	-
Trend slope	$100 \times \log \bar{\gamma}$	G	0.4	0.15	0.277 [0.196;0.329]	0.474 [0.456;0.491]
Human capital trend slope	$100 \times \log \bar{\gamma}_H$	G	0.2	0.05	0.104 [0.040;0.145]	-
Investment specific slope	$100 \times \log \bar{\gamma}_X$	G	0.4	0.15	0.661 [0.625;0.696]	-
Nominal rate	$100 \times \log \bar{r}$	G	1	0.10	1.449 [1.393;1.524]	1.947 [1.815;1.996]
Marginal log-likelihood					-800.2968	-176.1236

Table 3: Prior and Posterior distributions of structural parameters.

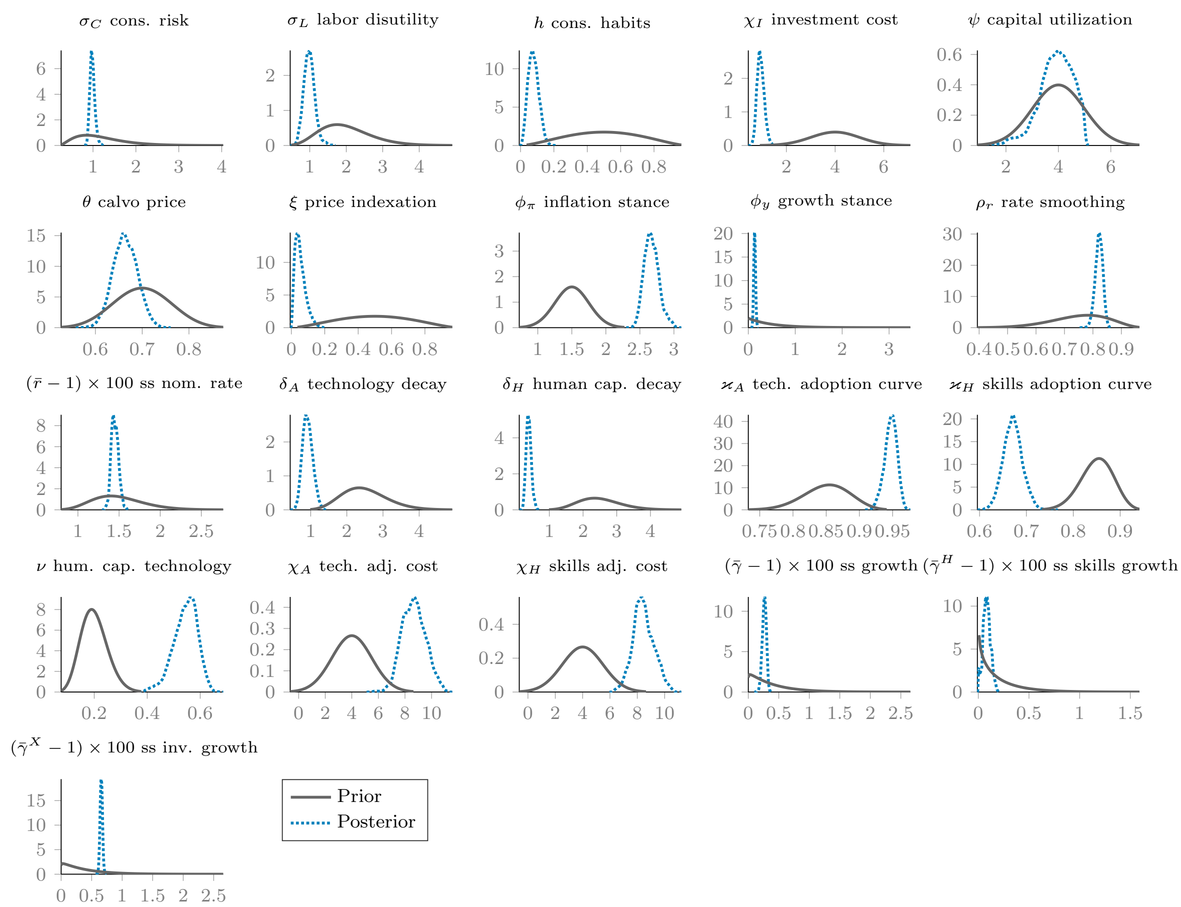

Figure 10: Prior and posterior distributions of the model with endogenous growth

	High Frequency 2q-32q			Medium Frequency 32q-200q		
	Data	$\mathcal{M}_X$	$\mathcal{M}_E$	Data	$\mathcal{M}_X$	$\mathcal{M}_E$
standand deviations:						
$sd\,(\log Y_t)$	[1.32;1.64]	**1.89**	1.93	[3.27;4.08]	**5.10**	5.20
$sd\,(\log C_t)$	[1.00;1.25]	**1.24**	1.41	[3.10;3.87]	**3.49**	**3.53**
$sd\,(\log I_t)$	[4.11;5.14]	**6.00**	7.54	[6.76;8.44]	**14.83**	19.52
$sd\,(\log P_t)$	[0.70;0.87]	1.04	**1.01**	[6.21;7.76]	4.75	**4.45**
$sd\,(\log H_t)$	[1.16;1.45]	1.67	**1.50**	[2.24;2.79]	3.74	**3.49**
$sd\,(\underline{r}_t)$	[0.31;0.38]	**0.37**	**0.34**	[0.34;0.43]	0.51	**0.43**
auto-correlations:						
$\rho\,(\log Y_t, \log Y_{t-1})$	[0.80;0.87]	**0.85**	**0.86**	[1.00;1.00]	**1.00**	**1.00**
$\rho\,(\log C_t, \log C_{t-1})$	[0.76;0.84]	**0.73**	**0.73**	[1.00;1.00]	**1.00**	**1.00**
$\rho\,(\log I_t, \log I_{t-1})$	[0.87;0.92]	**0.90**	**0.92**	[0.99;1.00]	**1.00**	**0.99**
$\rho\,(\log P_t, \log P_{t-1})$	[0.89;0.93]	**0.90**	**0.90**	[1.00;1.00]	**1.00**	**1.00**
$\rho\,(\log H_t, \log H_{t-1})$	[0.85;0.90]	**0.81**	**0.81**	[0.99;1.00]	**1.00**	**1.00**
$\rho\,(\underline{r}_t, \underline{r}_{t-1})$	[0.78;0.86]	**0.79**	**0.78**	[0.99;0.99]	**0.99**	**0.99**
correlation w/ output:						
$\rho\,(\log C_t, \log Y_t)$	[0.79;0.88]	0.38	**0.54**	[0.98;0.99]	0.54	**0.60**
$\rho\,(\log I_t, \log Y_t)$	[0.78;0.87]	**0.86**	**0.85**	[0.49;0.69]	**0.90**	**0.90**
$\rho\,(\log P_t, \log Y_t)$	[-0.45;-0.17]	**-0.36**	-0.47	[-0.78;-0.62]	-0.31	**-0.50**
$\rho\,(\log H_t, \log Y_t)$	[0.80;0.89]	**0.86**	**0.84**	[0.61;0.77]	**0.84**	**0.84**
$\rho\,(\underline{r}_t, \log Y_t)$	[0.25;0.52]	**0.06**	0.00	[-0.33;-0.03]	0.13	0.03

Notes: Model moments are averages over 1000 simulations of a sample size corresponding to the data. For output, investment, consumption, prices, we reverse the first difference filter and apply the bandpass filter on the transformed series.

Table 4: Second moment statistics comparison at a business and medium term cycle frequencies for the endogenous trends (M_E) and exogenous trend (M_X) models.