Exchange rate regime and firm dynamics*

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Abstract

This paper explores whether the desire of "currency manipulation" is determined by the dispersion of firm productivity and the selection of firms into exporting markets. For that purpose, we build a two-country model with firm heterogeneity and nominal wage rigidity. Since monetary intervention influences entry and exit decision of firms in exporting markets, it features inevitable characteristics of currency manipulation. We show that the fixed exchange rate regime not only realizes a better congruence between preference and the variety consumed but also reduces uncertainty in labor demand that arises from entry and exit of exporters. In our setting, the fixed exchange rate regime dominates the flexible exchange rate regime when firm productivity is less dispersed. We also show that regulation policy in trade sector that aims at stabilizing the firm turnover in exporting markets does not remove the temptation of currency manipulation.

JEL Classification Codes: F32, F41, E40.

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1 Introduction

Monetary policy as a substitute of trade policy which is often blamed as “currency manipulation” has been widely recognized in today’s globalized world. It’s intervention influences the nominal exchange rate fluctuations and thus limits those in the terms of trade. As a result, it reduces variations in profits from trade, hence regulating the turnover of exporters. Contrary, flexible exchange rate regime makes the entry and exit of exporters and importers adjusted freely creating a high turnover rate among them. A casual look at data suggests a potential causal relationship between trade and the choice of exchange rate regime. Figure 1 plots the volatility of exports and the average “exchange rate regime” overtime for a subset of developed countries. The volatility of exports is adjusted by the volatility of the nominal exchange rate so that the variable takes into account potential endogeneity among them. The average exchange regime is computed from Ilzetzki et al. (2018). This is an index that varies from the least flexible to the most flexible regime. In the figure we observe a surge in flexible regime after the collapse of the Bretton woods system at the beginning of 1970’s, which is followed by a huge drop of the index in the mid 90’s and continuous it’s stagnation. What is striking to see is a negative co-movement between the exchange rate regime and the volatility of exports. Given the recent episodes of the trade war and the debate around the currency manipulation, one might think that a highly volatile trade can be a driver of less flexible exchange rate regime.

In this paper, we explore the pros and cons of the exchange rate regime design that inevitability features an aspect of trade policy, i.e., the characteristics of “currency manipulation”.[1] While we provide a two-country full-fledged DSGE model with selection into exporting markets based on heterogeneous firms and wage nominal rigidity, our theoretical model can be considered as a caricature of the real world. In our model, following a demand shift, flexible exchange rate regime induces entry and exit of exporters and

[1]The adjustment of trade imbalance occurs in both prices and extensive margins of trade. In line of our setting, several papers emphasize the trade adjustment through the changes in the number of exported and imported varieties across countries (Corsetti et al. (2007)Corsetti et al. (2013)Pappadà (2011),Hamano (2014)). By introducing a nominal rigidity, we discuss the choice of exchange rate regime.

importers because of fluctuations in nominal profits while it realizes relatively small adjustment both in domestic investments as well as domestic production. In contrast, under fixed exchange rate regime, the turnover of exporters and importers is sterilized and a better congruence with preference shift is achieved. However, as a drawback, the fixed regime induces a highly volatile adjustment in both domestic investments as well as domestic production. As a result, uncertainty about future labor demand substantially rises. Monetary authority faces thus the above trade-off in designing the exchange rate regime. In our model, nominal exchange rate no longer plays the role of "shock absorber" with which the real side of the economy is stabilized (Friedman (1953) and Mundell (1961)). The view is found to be naif by ignoring the inevitable adjustments in trade sector which is small in size but may large in terms of their political influence. In our setting, the temptation of "currency manipulation" is present depending on the parameters' value in the economy.

Specifically, we emphasize the role played by firm heterogeneity and the selection into exporting market. When firms are less dispersed in terms of their productivities and thus become less efficient, labor demand increases among exporters. In such a situation, monetary intervention that limits the turnover of firms and thus simultaneously mitigates uncertainty about future labor demand in trade sector is welcomed (although it creates a higher economy wide uncertainty per se). With less firm dispersion and thus less efficient exporters, fixed exchange rate regime is more likely to be supported. We also derive optimal monetary policy under demand uncertainty. Accordingly to the above discussion, it is shown that optimal variability in nominal exchange rate is smaller in an economy where the firm productivities are less dispersed. Other than the firm dispersion, it is shown that the fix the exchange is more supported with higher value of the elasticity of labor supply as in the preceding literature, together with a higher elasticity of substitution among goods. In the following section, we compare the outcome under the regulation policy in trade sector and that obtained under the currency manipulation and show that the temptation to fix the exchange rate cannot be removed.

The practice of pricing to market and dollar pricing by exporters has been emphasized in the literature (Betts and Devereux (1996), Devereux and Engel (2003), Corsetti et al. (2010) and Gopinath et al. (2010) among others). Importantly this price rigidity in exporting market breaks down the "expenditure switching effect" of the nominal exchange rate, and allows deviations from the "divine coincidence" where flexible exchange rate regime dominates. We do not introduce this type of distortion related to the pricing behavior of exporters. Instead, they adjust prices in exporting market accordingly with the exchange rate fluctuations (producer currency pricing). However, the fixed exchange rate regime may dominate because of financial market incompleteness. Put differently, in our model, the flexible price allocation is not Pareto efficient as Devereux (2004) and Hamano and Picard (2017). While Devereux (2004) highlights the role of the elasticity of labor supply, and Hamano and Picard (2017) the elasticity of substitution among goods, we therefore study how the heterogeneity in firm productivity shapes the choice of the exchange rate regime.

The paper is structured as follows. In the next section, we introduce a two country DSGE model where countries are subject to external demand shocks and provide an analytical solution of our model. In section 3, we show how the monetary policy responds to external demand shocks when the exchange rate regime is fixed or flexible. Section 4 reports the welfare analysis and shows the optimal exchange rate regime as a function of the fundamentals of the economy. Section 5 provides discussion and extension. We document some supportive evidence in Section 6. Section 7 concludes.

2 The Model

There are two countries, Home and Foreign. Foreign variables are denoted with an asterisk (*). Both countries are inhabited by a unit mass of households which provide imperfectly-substituted labor. Expecting future labor demand, households set wages in advance. There are tradable sectors in which only a fraction of monopolistically competitive firms do export. The number of exporters is determined endogenously. There are demand shock to each countries' goods. How these shocks transmit depends on the conduct of monetary policy, thus resulting exchange rate regime.

2.1 Households

The representative household maximizes her life time utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j)$, where β $(0 < \beta < 1)$ is exogenous discount factor. Utility of individual household j at time t depends on consumption $C_t(j)$ and labor supply $L_t(j)$ as follows

$$U_t(j) = \ln C_t(j) + \chi \ln \frac{M_t(j)}{P_t} - \eta \frac{[L_t(j)]^{1+\varphi}}{1+\varphi},$$

where χ and η represent the degree of satisfaction (unsatisfaction) from real money holdings and labor supply, respectively while the parameter φ measures the inverse of the Frisch elasticity of labor supply.

The basket of goods $C_t(j)$ is defined as

$$C_t(j) = \left(\frac{C_{H,t}(j)}{\alpha_t}\right)^{\alpha_t} \left(\frac{C_{F,t}(j)}{\alpha_t^*}\right)^{\alpha_t^*},$$

where α_t and α_t^* are the preference attached to the bundle of goods produced in Home $C_{H,t}(j)$ and imported goods $(C_{F,t}(j))$, respectively. The process of these demand sifter is discussed below. Furthermore, these baskets are defined over a continuum of goods Ω as

$$C_{H,t}(j) = \left(\int_{\varsigma\in\Omega} c_{D,t}(j,\varsigma)^{1-\frac{1}{\sigma}} d\varsigma\right)^{\frac{1}{1-\frac{1}{\sigma}}},\ C_{F,t}(j) = \left(\int_{\varsigma^*\in\Omega} c_{X,t}(j,\varsigma^*)^{1-\frac{1}{\sigma}} d\varsigma^*\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

In each time period, only a subset of variety of goods is available from the total universe of variety of goods Ω. We denote $N_{D,t}$ and $N_{X,t}^*$ as the number of domestic and imported product varieties, respectively. $c_{D,t}(j,\varsigma)$ and $c_{X,t}(j,\varsigma^*)$ represent the demand addressed for individual product variety indexed by ς and ς^*. $\sigma\,(>1)$ denotes the elasticity of substitution among differentiated goods.

The optimal consumption for each domestic basket, imported basket and individual product variety are found to be

$$C_{H,t}(j) = \left(\frac{P_{H,t}}{P_t}\right)^{-1} \alpha_t C_t(j), \quad C_{F,t}(j) = \left(\frac{P_{F,t}}{P_t}\right)^{-1} \alpha_t^* C_t(j),$$

$$c_{D,t}(j,\varsigma) = \left(\frac{p_{D,t}(\varsigma)}{P_{H,t}}\right)^{-\sigma} C_{H,t}(j), \quad c_{X,t}(j,\varsigma^*) = \left(\frac{p^*_{X,t}(\varsigma^*)}{P_{F,t}}\right)^{-\sigma} C_{F,t}(j).$$

$p_{D,t}(\varsigma)$ stands for the price of product variety ς which is domestically produced. In particular, $p^*_{X,t}(\varsigma^*)$ denotes the price of imported product variety ς^*, denominated in currency unit in Home. $P_{H,t}$ and $P_{F,t}$ are the price of basket of goods produced in Home and that of imported, respectively. P_t is the price of aggregated basket.Price indices that minimize expenditures on each consumption basket are given by

$$P_t = P_{H,t}^{\alpha_t} P_{F,t}^{\alpha_t^*},$$

$$P_{H,t} = \left(\int_{\varsigma\in\Omega} p_{D,t}(\varsigma)^{1-\sigma} d\varsigma\right)^{\frac{1}{1-\sigma}}, \quad P_{F,t} = \left(\int_{\varsigma^*\in\Omega} p^*_{X,t}(\varsigma^*)^{1-\sigma} d\varsigma^*\right)^{\frac{1}{1-\sigma}}.$$

Similar expressions hold for Foreign. Crucially, the subset of goods available to Foreign during period t, $\Omega_t^* \in \Omega$, can be different from the subset of goods available to Home $\Omega_t \in \Omega$.

2.2 Production, Pricing and the Export Decision

There is a mass of $N_{D,t}$ number of firms in Home. Upon entry, firms draw their productivity level z from a distribution $G(z)$ on $[z_{\min}, \infty)$. Since there are no fixed production costs and hence no selection into domestic market, $G(z)$ also represents the productivity distribution of all producing firms. Prior to entry, however, these firms are identical and face sunk entry cost of $f_{E,t} = l_{E,t}$ amounts of labor. The sunk cost is composed of imperfectly differentiated labor services provided by households (indexed by i) such that

$$l_{E,t} = \left(\int_0^1 l_{E,t}(j)^{1-\frac{1}{\theta}} dj\right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{1}$$

where θ represents the elasticity of substitution among different labor services. We consider $f_{E,t}$ to be exogenous. By defining the nominal wage for type j labor as $W_t(j)$,total cost for a firm to setup is thus $\int_0^1 l_{E,t}(j) W_t(j) dj$. The cost minimization yields the following labor demand for type j labor service:

$$l_{E,t}(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} l_{E,t}, \tag{2}$$

where W_t denotes the corresponding wage index, which is found to be

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta} dj\right)^{\frac{1}{}}.$$

Exporting requires an operational fixed cost of $f_{X,t} = l_{f_X,t}$ amount of labor defined in a similar way as (1). The cost minimization provides a similar demand for each specific labor service as (2).[2] Only a subset of firms whose productivity level z is above the cutoff level $z_{X,t}$ exports by charging sufficiently lower prices and earning positive profits despite the existence of fixed export cost $f_{X,t}$. Thus, non-tradeness in the economy arises endogenously with changes in the productivity cutoff $z_{X,t}$.

For production of each product variety, only composite labor basket is required as input. Thus the production function of firm with productivity z is given by$y_t(z) = zl_t(z)$ where

$$l_t(z) = \left(\int_0^1 l_t(z,j)^{1-\frac{1}{\theta}} dj\right)^{\frac{1}{1-\frac{1}{\theta}}}.$$

The cost minimization yields the demand for type j labor for production as

$$l_t(z,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} l_t(z).$$

The firm faces a residual demand curve with constant elasticity σ. The production scale is thus determined by the demand addressed to the firm under monopolistic competition. Profit maximization yields the following optimal price $p_{D,t}(z)$ by firm with productivity z:

$$p_{D,t}(z) = \frac{\sigma}{\sigma-1}\frac{W_t}{z}.$$

[2]These are specifically,

$$l_{f_X,t} = \left(\int_0^1 l_{f_X,t}(j)^{1-\frac{1}{\theta}} dj\right)^{\frac{1}{1-\frac{1}{\theta}}}, \qquad l_{f_X,t}(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} l_{f_X,t}.$$

If the firm exports, its price of export is $p_{X,t}(z) = \tau p_{D,t}(z)\,\varepsilon_t^{-1}$ where ε_t is the nominal exchange rate defined as the price of one unit of foreign currency in terms of home currency units. $\tau > 1$ is iceberg trade cost. In our definition, $p_{X,t}(z)$ is thus denominated in terms of foreign currency units.

Total firm profits $D_t(z)$ can be decomposed into those from domestic sales $D_{D,t}(z)$ and those from exporting sales $D_{X,t}(z)$ (if the firm exports) as $D_t(z) = D_{D,t}(z) + D_{X,t}(z)$. Using the demand functions found previously and aggregate consumption defined as $C_t = \left(\int_0^1 C_t^{1-\frac{1}{\sigma}}(j)\,dj\right)^{\frac{1}{1-\frac{1}{\sigma}}}$, we can write the profits from each market as

$$D_{D,t}(z) = \frac{1}{\sigma}\left(\frac{p_{D,t}(z)}{P_{H,t}}\right)^{1-\sigma}\alpha_t P_t C_t, \tag{3}$$

$$D_{X,t}(z) = \frac{\varepsilon_t}{\sigma}\left(\frac{p_{X,t}(z)}{P_{H,t}^*}\right)^{1-\sigma}\alpha_t P_t^* C_t^* - W_t f_X, \qquad \text{if the firm } z \text{ exports} \tag{4}$$

2.3 Firm Averages

Given a distribution $G(z)$, the productivity level of a mass of $N_{D,t}$ domestically producing firms is distributed over $[z_{\min}, \infty)$. Among these firms, there are $N_{X,t} = [1 - G(z_{X,t})]\,N_{D,t}$ exporters in Home. Following Melitz (2003) and Ghironi and Melitz (2005), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters as follows

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\sigma-1} dG(z)\right]^{\frac{1}{\sigma-1}}, \quad \widetilde{z}_{X,t} \equiv \left[\frac{1}{1-G(z_{X,t})}\int_{z_{X,t}}^{\infty} z^{\sigma-1} dG(z)\right]^{\frac{1}{\sigma-1}}.$$

These average productivity levels summarize all the information about the distribution of productivities. Given these averages, we define the average real domestic and exporting price as $\widetilde{p}_{D,t} \equiv p_{D,t}(\widetilde{z}_D)$ and $\widetilde{p}_{X,t} \equiv p_{X,t}(\widetilde{z}_{X,t})$, respectively. We also define average profits from domestic sales and exporting sales as $\widetilde{D}_{D,t} \equiv D_{D,t}(\widetilde{z}_D)$ and $\widetilde{D}_{X,t} \equiv D_{X,t}(\widetilde{z}_{X,t})$. Finally, average profits among all firms is given by $\widetilde{D}_t = \widetilde{D}_{D,t} + (N_{X,t}/N_{D,t})\,\widetilde{D}_{X,t}$.

2.4 Firm Entry and Exit

New entrants need one time period to built. Firm entry takes place until the expected value of entry is equalized with entry cost, leading to the following free entry condition:

$$\widetilde{V}_t = f_{E,t} W_t, \tag{5}$$

where $\widetilde{V}_t$ is the expected value of entry which is discussed below. For the tractability of the solution of the model, firms are assumed to be depreciated by 100 % after production.

2.5 Parametrization of Productivity Draws

We assume the following Pareto distribution for $G(z)$:

$$G(z) = 1 - \left(\frac{z_{\min}}{z}\right)^{\kappa},$$

where $z_{\min}$ is the minimum productivity level, and $\kappa\,(> \sigma - 1)$ is a shape parameter. With this parametrization, we have

$$\widetilde{z}_D = z_{\min}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}} \qquad \widetilde{z}_{X,t} = z_{X,t}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}}.$$

The share of exporters in the total number of domestic firms is then given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^{\kappa}\left(\widetilde{z}_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{\frac{\kappa}{\sigma-1}}.$$

Finally, there exists a firm with a specific productivity cutoff $z_{X,t}$ that earns zero profits from exporting, as $D_{X,t}\left(z_{X,t}\right) = 0$. With the above Pareto distribution, this implies that

$$\widetilde{D}_{X,t} = W_t f_{X,t} \frac{\sigma - 1}{\kappa - (\sigma - 1)}.$$

2.6 Household Budget Constraints and Intertemporal Choices

A household j in Home faces the following budget constraint at time period t:

$$P_tC_t(j) + B_t(j) + M_t(j) + x_t(j)N_{D,t+1}\widetilde{V}_t$$
$$= (1+\xi) W_t(j) L_t(j) + (1+i_{t-1})B_{t-1}(j) + M_{t-1}(j) + x_{t-1}(j)N_{D,t}\widetilde{D}_t + T_t^f, \quad (6)$$

where $B_t(j)$ and $x_t(j)$ denote bond holdings and share holdings of mutual funds, respectively. $1+\xi$ is the appropriately designed labor subsidy which aims to eliminate distortions due to monopolistic power in labor markets (see later). i_t represents nominal interest rate between t and $t+1$ and T_t^f represents a transfer from domestic government, which can be positive or negative.

We assume that wages are sticky for one time period. Specifically, the household j sets wages in advance at $t-1$ by maximizing her expected utility at t knowing the following demand for her labor:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} L_t.$$

The first order condition with respect to $W_t(j)$ yields

$$W_t(j) = \frac{\eta\theta}{(\theta-1)(1+\xi)} \frac{\mathrm{E}_{t-1}\left[L_t(j)^{1+\varphi}\right]}{\mathrm{E}_{t-1}\left[\frac{L_t(j)}{P_tC_t(j)}\right]}. \quad (7)$$

Households set the wage so that the expected marginal cost by supplying additional labor services $\eta\theta W_t(j)^{-1}\mathrm{E}_{t-1}\left[L_t(j)^{1+\varphi}\right]$ equals to the expected marginal revenue $(\theta-1)(1+\xi)\mathrm{E}_{t-1}\left[\frac{L_t(j)}{P_tC_t(j)}\right]$

Other choices occur within the same time period. The first order condition with respect to share holdings yields

$$\widetilde{V}_t = E_t\left[Q_{t,t+1}(j)\widetilde{D}_{t+1}\right], \quad (8)$$

where $Q_{t,t+1}$ is stochastic discount factor defied as $Q_{t,t+1}(j) = E_t\left[\frac{\beta P_tC_t(j)}{P_{t+1}C_{t+1}(j)}\right]$.

The first order condition with respect to bond holdings is given by

$$1 = (1+i_t)E_t\left[Q_{t,t+1}(j)\right].$$

Finally, the household maximizes its consumption and real money holdings. As a result, we have

$$P_t C_t(j) = \frac{M_t}{\chi}\left(\frac{i_t}{1+i_t}\right). \tag{9}$$

Nominal spending $P_t C_t(j)$ is tight down to the money supply M_t.

2.7 Balanced Trade and Labor Market Clearings

In equilibrium, there is a symmetry across households so that $C_t(j) = C_t$, $L_t(j) = L_t$, $M_t(j) = M_t$ and $W_t(j) = W_t$. Furthermore, we follow Corsetti et al. (2010) and Bergin and Corsetti (2008) and define monetary stance as

$$\mu_t \equiv P_t C_t.$$

Monetary stance is proportional to nominal expenditure.[3] Trade is assumed to be balanced according to which the value of Home export becomes the same to the value of Home import once they are converted to the same unit of currency: $\varepsilon_t P^*_{H,t} C^*_{H,t} = P_{F,t} C_{F,t}$. Combined with the demand system found previously, this implies

$$\varepsilon_t = \frac{\alpha^*_t}{\alpha_t}\frac{\mu_t}{\mu^*_t}. \tag{10}$$

It is assumed that the government has no power to directly control private lending and borrowing. The balanced budget rule is assumed as

$$M_t - M_{t-1} = T^f_t + \xi W_t L_t.$$

Under nominal wage rigidity, the aggregate labor supply L_t adjusts to its demand and the labor market clears as:

[3] Note that combining with the Euler equation with respect to the bond holdings, it is shown that

$$\frac{1}{\mu_t} = \mathrm{E}_t \lim_{s\to\infty} \beta^s \frac{1}{\mu_{t+s}} \prod_{\tau=0}^{s-1}(1+i_{t+\tau}).$$

Monetary stance μ_t is expressed as a function of future expected path of interest rates. Or it can be expressed as a rule concerning money supply M_t as (9).

$$L_t = N_{D,t}\frac{\widetilde{y}_{D,t}}{\widetilde{z}_D} + N_{X,t}\left(\frac{\widetilde{y}_{X,t}}{\widetilde{z}_{X,t}} + f_{X,t}\right) + N_{D,t+1}f_{E,t} \quad (11)$$

In the above expression, $\widetilde{y}_{D,t}$ and $\widetilde{y}_{X,t}$ stand for production scale of each average domestic firms and average exporters.[4] The labor demand comes from production for domestic markets, production for exports (including fixed costs for exporting) and creation of new firms. The similar expressions hold for Foreign.

Finally we assume the following process for the preference shift as

$$\alpha_t = \frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t, \quad \alpha_t^* = \frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^*,$$

with $\alpha_0 = \alpha_0^* = 1$, $E_{t-1}[\upsilon_t] = E_{t-1}[\upsilon_t^*] = 1$ and $\upsilon_t + \upsilon_t^* = 2$, $0 \le \rho \le 1$. Indeed, υ_t and υ_t^* are defined as the i.i.d. shocks with a specific support. Also we assume that $f_{E,t} = f_{E,t}^* = f_E$ and $f_{X,t} = f_{X,t}^* = f_X$ for all time periods. The closed form solution of the model is summarized in Table 1. We relegate all the derivation of endogenous variables into Appendix.

3 Exchange Rate Regimes and Adjustment Mechanism

Assume that under the flexible exchange rate regimes, $\mu_t = \mu_t^* = \mu_0$ for all time periods. On the other hand, under fixed exchange rate regime, $\mu_t = 2\mu_0\alpha_t$ and $\mu_t^* = 2\mu_0\alpha_t^*$. Here we detail the underlining mechanism of the theoretical model under two different exchange rate regimes.

3.1 Flexible Exchange Rate Regime

Under the flexible regime, following a relative demand shift for Home produced goods (a decrease in α_t^*/α_t), the nominal exchange rate ε_t appreciates for Home so that it closes the trade surplus for Home (trade deficit in Foreign). The adjustment, however, takes

[4]we have $\widetilde{y}_{D,t} = (\sigma-1)\frac{\widetilde{D}_{D,t}\widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma-1)\frac{(\widetilde{D}_{X,t}+f_{X,t}W_t)\widetilde{z}_{X,t}}{W_t}$. See Appendix also.

place not only through the terms of trade fluctuations but also through the extensive margins of trade. Under the producer currency pricing as our setting, that appreciation improves the profitability of Foreign exporters in their currency units relative to those of Home (a decrease in $\widetilde{D}_{X,t}/\widetilde{D}^*_{X,t}$ on impact) and hence induces higher number of Foreign exporters relative to Home exporters (a decrease in $N_{X,t}/N^*_{X,t}$).[5] Note that the change in the relative number of exporters is not welcomed because of a lower preference attached to goods produced in Foreign than those produced in Home. At the same time, Foreign exporters become less efficient compared to Home exporters due to changes in cutoff productivity level for exporting (a rise in $\widetilde{z}_{X,t}/\widetilde{z}^*_{X,t}$). Accordingly, the price of goods produced by Foreign exporters increases (a decrease in $\widetilde{p}_{X,t}/\widetilde{p}^*_{X,t}$) and the quantity of average variety produced by them decreases (a rise in $\widetilde{y}_{X,t}/\widetilde{y}^*_{X,t}$). To sum up, using the equilibrium expressions in Table 1, and denoting implied variable X_t under flexible regime with X_t^{FL}, we have indeed,

$$\frac{N^{FL}_{X,t}}{N^{*FL}_{X,t}} = \frac{\varepsilon_t W_t^{*FL} f_X^*}{W_t^{FL} f_X}, \qquad \frac{\widetilde{z}^{FL}_{X,t}}{\widetilde{z}^{*FL}_{X,t}} = \left(\frac{N^{FL}_{X,t}}{N^{*FL}_{X,t}} \frac{N^{*FL}_{D,t}}{N^{FL}_{D,t}} \right)^{-\frac{1}{\kappa}}. \tag{12}$$

Since wages are rigid and the numbers of domestic firms are state, we have the changes in the number of exporters and cutoff level of productivities under flexible exchange rate regime.

Also the equilibrium wage under flexible exchange rate regime is found to be

$$W_t^{FL} = \Gamma \mu_0 \left\{ \frac{E_{t-1}\left[A_t^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]} \right\}^{\frac{1}{1+\varphi}} \tag{13}$$

where A_t embeds preferences and parameters as[6]

$$A_t \equiv \frac{\sigma-1}{\sigma}\alpha_t + \left(1 - \frac{\sigma-1}{\sigma\kappa}\right)\alpha_t^* + \frac{\beta}{\sigma}\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$$

[5] Note that it is always the case that $\mid \Delta\varepsilon_t \mid > \mid \Delta\alpha_t \mid$ in our setting requiring the adjustment in the extensive margins of trade.

[6] Note that with $\alpha_t = \frac{1}{2}\alpha^{\rho}_{t-1}\upsilon_t$, $\alpha^*_t = \frac{1}{2}\alpha^{*\rho}_{t-1}\upsilon^*_t$ and assuming a symmetric steady state across countries as $\alpha_{t-1} = \alpha^*_{t-1}$, we can express A_t as a function of fundamental shocks as

The equilibrium wage under flexible exchange rate regime inherits the uncertainty about the *future* demand shock because of the above mentioned inevitable adjustment in trade sector. The expression (13) highlights the fact that the nominal exchange rate is not a "shock absorber" in our setting with selection into export market.

What would happen then in domestic market? Plugging the equilibrium expressions in Table 1, we have also

$$\frac{\widetilde{y}_{D,t}^{FL}}{\widetilde{y}_{D,t}^{*FL}} = \frac{\alpha_t \widetilde{z}_D}{\alpha_t^* \widetilde{z}_D^*} \frac{N_{D,t}^{*FL} W_t^{*FL}}{N_{D,t}^{FL} W_t^{FL}}, \qquad \frac{N_{D,t+1}^{FL}}{N_{D,t+1}^{*FL}} = \frac{W_t^{*FL} f_E^* E_t \left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]}{W_t^{FL} f_E E_t \left[\alpha_{t+1}^* + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]}. \tag{14}$$

In domestic market, following a relative demand shift for Home produced goods, the production scale of average domestic firms expands compared to that of Foreign (a rise in $\widetilde{y}_{D,t}/\widetilde{y}_{D,t}^*$) following such a favorable demand shift. With a certain persistence of preference as $0 < \rho \leq 1$, it induces *a modest* increase in domestic investment (a rise in $N_{D,t+1}/N_{D,t+1}^*$).

On the one hand, the trade imbalance triggered by a relative positive demand shift for Home goods induces not only fluctuations in the nominal exchange rate ε_t but also undesirable fluctuations in extensive as well as intensive margins of trade. On the other hand, the flexible exchange rate system triggers an adjustment in domestic market in favor of desirable demand shift. Therefore, households in Home enjoy only marginally their desired consumption allocation in domestic market while suffering from welfare detrimental penetration of less desired imported goods which are expensive. Contrary to the models without firm heterogeneity in the literature (Friedman (1953)Devereux (2004) and Hamano and Picard (2017)), the nominal exchange rate only partially absorbs the shock in our setting creating a substantial burden of adjustment specifically in trade sector.

$$\begin{aligned} A_t &= \frac{\sigma-1}{\sigma}\alpha_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\alpha_t^* + \frac{\beta}{\sigma}\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right]. \\ &= \frac{1}{2}\frac{\sigma-1}{\sigma}\alpha_{t-1}^{\rho}\upsilon_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* + \frac{\beta}{\sigma}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1} + \frac{\sigma-1}{\kappa}\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1}\right] \\ &= \frac{1}{2}\left\{\frac{\sigma-1}{\sigma}\upsilon_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma}\left[\upsilon_t^{\rho}\upsilon_{t+1} + \frac{\sigma-1}{\kappa}\upsilon_t^{*\rho}\upsilon_{t+1}^*\right]\right\} \end{aligned}$$

3.2 Fixed Exchange Rate Regime

When the nominal exchange rate is fixed, the allocation in the economy dramatically changes. A counter acting shift in monetary stance as $\mu_t = 2\mu_0 \alpha_t$ and $\mu_t = 2\mu_0 (1 - \alpha_t)$ in both countries mitigates the profits fluctuations in trade sector. As a result, the number of exporters as well as the production scales hence their prices remain constant in equilibrium following a demand shift. Using the equilibrium expressions in Table 1, and denoting implied variable X_t under flexible regime with X_t^{FX}, we have

$$\frac{N_{X,t}^{FX}}{N_{X,t}^{*FX}} = \frac{W_t^{*FX} f_X^*}{W_t^{FX} f_X}, \qquad \frac{\widetilde{z}_{X,t}^{FX}}{\widetilde{z}_{X,t}^{*FX}} = \left(\frac{N_{X,t}^{FX}}{N_{X,t}^{*FX}} \frac{N_{D,t}^{*FX}}{N_{D,t}^{FX}} \right)^{-\frac{1}{\kappa}}. \tag{15}$$

As is clear from the above expressions, the fixed regime results in a sterilization in extensive and intensive margins of trade. However, it induces abrupt change in domestic market for production scale of domestic firms and investment for future product varieties rise substantially in Home compared to those in Foreign (a strong rise both in $\widetilde{y}_{D,t}/\widetilde{y}_{D,t}^*$ and $N_{D,t+1}/N_{D,t+1}^*$):

$$\frac{\widetilde{y}_{D,t}^{FX}}{\widetilde{y}_{D,t}^{*FX}} = \frac{\alpha_t^2 \widetilde{z}_D}{\alpha_t^{*2} \widetilde{z}_D^*} \frac{N_{D,t}^{*FX} W_t^{*FX}}{N_{D,t}^{FX} W_t^{FX}}, \qquad \frac{N_{D,t+1}^{FX}}{N_{D,t+1}^{*FX}} = \frac{\alpha_t}{\alpha_t^*} \frac{W_t^{*FX} f_E^* E_t \left[\alpha_{t+1} + \frac{\sigma-1}{\kappa} \alpha_{t+1}^* \right]}{W_t^{FX} f_E E_t \left[\alpha_{t+1}^* + \frac{\sigma-1}{\kappa} \alpha_{t+1} \right]}. \tag{16}$$

Comparing the above expressions with (14), it is noticed that volatility in domestic average production and the number of domestic entry (investment) are higher than those under the flexible regime. To sum up, the exchange rate policy that attempts to fix the exchange rate shifts the burden of adjustment away from exporting markets to domestic markets. Thereby, the fixed regime realizes a desirable allocation increasing substantially the production of goods and varieties preferred in domestic market while shutting down completely undesirable allocation in trade sector.

Finally, as one can expect, the above mentioned equilibrium allocation under fixed regime influences wage setting behavior. The wage under fixed exchange rate regime is found to be

$$W_t^{FX} = 2\Gamma\mu_0 \left\{ \frac{E_{t-1}\left[(A_t\alpha_t)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]} \right\}^{\frac{1}{1+\varphi}}.$$

Because of monetary policy intervention in the realization of a shock as $\mu_t = 2\mu_0\alpha_t$, the equilibrium wage under the fixed regime depends on the expected interaction between labor demand fluctuations and monetary shock which is captured by the term $(A_t\alpha_t)^{1+\varphi}$ in the expectation operator. W_t^{FX} is thus ultimately depends on the level of each component (A_t and α_t) and the covariance ($\text{Cov}(A_t, \alpha_t)$) augmented by the elasticity of labor supply, φ. On the one hand, monetary intervention increases wage in level because of a higher aggregate demand, captured by α_t in expectation operator. On the other hand, since labor demand and monetary policy shock can be *negatively* correlated ($\text{Cov}(A_t, \alpha_t) < 0$), monetary policy that attempts to fix the exchange rate simultaneously end up to dampen fluctuations in labor demand and hence uncertainty. Intuitively, under the fixed exchange rate regime, profitability for exporters remain constant realizing sterilized exporting market although domestic production and investment rise more abruptly under fixed regime than flexible regime. The first mitigation impact in sterilized trade sector dominates the second amplification effect of labor demand volatility. As we will see later on, that negative correlation between labor demand and demand shift is a function of underlining parameters' value in the economy and crucial in deriving the welfare ranking between fixed and flexible exchange rate regime.

4 Welfare Analysis

In this section, we explore the welfare implication of policy intervention in the presence of demand shock. Monetary authority can have various policy objectives. They put different priority on these objectives based on the political economic environment. The policymakers target the extent of desired variability of the exchange rate which can be achieved through monetary interventions. In particular we compare the choice between fixed and flexible exchange rate regime.

4.1 Expected Utility

First, we characterize the expected utility of the households as a function of exogenous disturbances and monetary stances. Although the expected discounted sum of utility is defined over infinite horizon of time, policy intervention at time t has impact just for two consecutive time periods due to the assumption of one time to build, one time period of production and one time period of wage stickiness. In deriving welfare metric, we thus express the expected utility only for two consecutive periods without loss of generality. The expected utility of the Home representative household at time t and $t+1$ being at $t-1$ is presented therefore as

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] &\equiv \mathrm{E}_{t-1}[U_t] + \beta \mathrm{E}_{t-1}[U_{t+1}] \\
&= E_{t-1}[\alpha_t \ln C_{H,t} + \alpha_t^* \ln C_{F,t}] + \beta E_{t-1}[\alpha_t \ln C_{H,t+1} + \alpha_t^* \ln C_{F,t+1}] \\
&= \mathrm{E}_{t-1}\left[\alpha_t\left(\frac{\sigma}{\sigma-1}\ln N_{D,t} + \ln\widetilde{y}_{D,t}\right) + \alpha_t^*\left(\frac{\sigma}{\sigma-1}\ln N_{X,t}^* + \ln\widetilde{y}_{X,t}^*\right)\right] \\
&+ \beta \mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\frac{\sigma}{\sigma-1}\ln N_{D,t+1} + \ln\widetilde{y}_{D,t+1}\right) + \alpha_{t+1}^*\left(\frac{\sigma}{\sigma-1}\ln N_{X,t+1}^* + \ln\widetilde{y}_{X,t+1}^*\right)\right] \quad (17)
\end{aligned}
$$

The expected utility is a function of the current number of domestic and imported varieties ($N_{D,t}$ and $N_{X,t}^*$) and their production scales ($\widetilde{y}_{D,t}$ and $\widetilde{y}_{X,t}^*$) at time t and the expected number of them at time $t+1$. Note that the sum of utility in any two consecutive time periods can be expressed as the above expression without loss of generality.

Furthermore, plugging the equilibrium expression in Table 1 and and shock process discussed previously, the equation (17) becomes (see Appendix for derivation.)

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] &= \frac{1}{2}\left\{\mathrm{E}_{t-1}[\upsilon_t \ln\mu_t] - \frac{1}{1+\varphi}\ln E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]\right\} \\
&+ \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}[\upsilon_t^* \ln\mu_t^*] - \frac{1}{1+\varphi}\ln E_{t-1}\left[(A_t^*\mu_t^*)^{1+\varphi}\right]\right\} \\
&+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}[\upsilon_t^{\rho}\ln\mu_t] - \frac{E_{t-1}[\upsilon_t^{\rho}]}{1+\varphi}\ln E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]\right\} \\
&+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}[\upsilon_t^{*\rho}\ln\mu_t^*] - \frac{E_{t-1}[\upsilon_t^{*\rho}]}{1+\varphi}\ln E_{t-1}\left[(A_t^*\mu_t^*)^{1+\varphi}\right]\right\} + \text{cst} \quad (18)
\end{aligned}
$$

The expected utility can be expressed as a function of shocks and monetary stances in Home and Foreign.

4.2 Fixed vs. Flexible Exchange Rate Regimes

Given the above expected utility, the welfare difference between the fixed and flexible exchange rate regime is found as (see Appendix)

$$\begin{aligned}
&\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right] \\
&\qquad=\frac{1}{2}\left(\frac{1}{\sigma-1}+2-\frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t\ln\upsilon_t\right]-\Delta\ln W_t\right\} \\
&\qquad+\left(\frac{1}{2}\right)^{1+\rho}\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t^{\rho}\ln\upsilon_t\right]-E_{t-1}\left[\upsilon_t^{\rho}\right]\Delta\ln W_t\right\} \qquad (19)
\end{aligned}$$

where $\Delta\ln W_t \equiv \ln W_t^{FX}-\ln W_t^{FL}$ represents the wage difference between fixed and flexible regimes:[7]

$$\begin{aligned}
&\Delta\ln W_t\equiv\ln W_t^{FX}-\ln W_t^{FL} \\
&\qquad=\frac{1}{1+\varphi}\left[\ln E_{t-1}\left[\left(A_t\upsilon_t\right)^{1+\varphi}\right]-\ln E_{t-1}\left[A_t^{1+\varphi}\right]\right] \qquad (20)
\end{aligned}$$

In the expression of welfare ranking (19), $E_{t-1}\left[\upsilon_t\ln\upsilon_t\right]>0$ and $E_{t-1}\left[\upsilon_t^{\rho}\ln\upsilon_t\right]>0$ present a congruence between preference and intensive as well as extensive margins in both

[7]To get the expression (20),

$$\begin{aligned}
\Delta\ln W_t\equiv\ln W_t^{FX}-\ln W_t^{FL}&=\ln\Gamma\left\{\frac{E_{t-1}\left[\left(A_t2\mu_0\alpha_t\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}^{\frac{1}{1+\varphi}}-\ln\Gamma\left\{\frac{E_{t-1}\left[\left(A_t\mu_0\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}^{\frac{1}{1+\varphi}} \\
&=\frac{1}{1+\varphi}\ln\left\{\frac{E_{t-1}\left[\left(A_t2\mu_0\alpha_t\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\}-\frac{1}{1+\varphi}\ln\left\{\frac{E_{t-1}\left[\left(A_t\mu_0\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]}\right\} \\
&=\frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t2\mu_0\alpha_t\right)^{1+\varphi}\right]-\frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_0\right)^{1+\varphi}\right] \\
&=\frac{1}{1+\varphi}\left[\ln E_{t-1}\left[\left(A_t\upsilon_t\right)^{1+\varphi}\right]-\ln E_{t-1}\left[A_t^{1+\varphi}\right]\right].
\end{aligned}$$

domestic and imported variety of goods for current and future time period under fixed exchange rate regime. These terms stand for benefit under fixed regime and similar terms can be found in Devereux (2004) and Hamano and Picard (2017). While the preceding literature find volatile intensive or/and extensive margins induced by stochastic monetary policy intervention under fixed regime costly and detrimental to welfare, in our model monetary policy intervention reduces uncertainty for future *in trade sector* and can serve to improve welfare. The result is a direct consequence of firm heterogeneity and selection into exporting market combined with a sluggish wage setting behavior of workers. As in the standard New Keynesian model, the role of monetary policy under fixed regime here is, in part, to reduce the uncertainty about marginal cost specifically in trade sector. Not only from a desired demand congruence but also from this stand,monetary authority has incentive to sterilize selection into exporting markets by fixing the nominal exchange rate.. The sign of $\Delta \ln W_t$ and hence the difference in expected utility ultimately depend on the parameters' value in the economy as we will discuss in the next subsection.

4.3 The role of selection into exporting market

4.3.1 Variety effect with selection into exporting market

Let us now discuss the role played by parameters, specifically the term $\frac{1}{\sigma-1} - \frac{1}{\kappa}$ and $\left(\frac{1}{2}\right)^{1+\rho} \beta \left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)$ in the welfare comparison (19) which are strictly positive.

First, the term $\frac{1}{\sigma-1} - \frac{1}{\kappa}$ indeed captures the balance between preference for the *current* number of imported varieties and the prices of those varieties. Given the size of $E_{t-1}\left[\upsilon_t \ln \upsilon_t\right] - E_{t-1}\left[\upsilon_t^{\rho}\right] \Delta \ln W_t$, the gain under fixed regime that realizes a better congruence between preference and imported number of varieties is higher the higher the preference for variety (lower value of σ) and the lower the firm dispersion (higher value of κ). This is because when the number of imported varieties increases, these varieties, however, are produced by less efficient firms that charge expensive prices on average. Given the love for variety, the welfare gain in consuming a higher number imported varieties is fully acknowledged when exporters are homogeneous ($\kappa = \infty$) and hence no increase in price of import.

Second, the term $\left(\frac{1}{2}\right)^{1+\rho}\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)$ scales the welfare impact on the number of *future* domestic varieties and the *future* cutoff level of imported goods. Given the size of $E_{t-1}\left[\upsilon_t^{\rho}\ln\upsilon_t\right]-E_{t-1}\left[\upsilon_t^{\rho}\right]\Delta\ln W_t$, a rise in the number of domestic products in future period provides a higher utility gain when the love for variety is high (lower value of σ). Note that the impact is amplified by lower discount factor (a higher value of β) with which future varieties are more appreciated and through a high shock persistence (a higher value of ρ) with which a current positive shock will result in a higher number of future varieties. Furthermore, a rise in the number of varieties in the next period increases competition and it makes tougher to survive as exporters. As a result, the future cutoff level increases due to selection and the price of imported varieties become cheaper. That gain from future competition is captured by $\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}$. From this channel, the higher the value of κ, the lower the welfare gain is because of a lower selection and hence the survival of less efficient producers that charge higher prices in future period.

The above welfare improving effect under fixed regime thorough the demand congruence is similar to Devereux (2004) without extensive margins and Hamano and Picard (2017) with extensive margins. Here the mechanism is more elaborated due to the selection into exporting market among heterogeneous firms.

4.3.2 Mitigation impact under fixed regime

The sign of $\Delta\ln W_t$, specifically, the covariance terms in the expression of wage under fixed regime W_t^{FX} is crucial to determine the welfare ranking. As is explained, under the fixed regime, monetary intervention increases the expected labor demand through the first order effect and the second order effect. The intuition can be best described in setting $\varphi=0$ (the case of infinitely elastic labor supply). In such a case, the wage difference equation (20) is expressed as

$$\begin{aligned}\Delta\ln W_t\mid_{\varphi=0} &= \left[\ln E_{t-1}\left[(A_t\upsilon_t)\right]-\ln E_{t-1}\left[A_t\right]\right]\\ &= \ln\left[1+\frac{E_{t-1}\left[\upsilon_t\right]+Cov\left(A_t,\upsilon_t\right)}{E_{t-1}\left[A_t\right]}\right]. \quad (21)\end{aligned}$$

The term $E_{t-1}[\upsilon_t]$ is the first order level effect of the monetary intervention under the fixed regime while $Cov(A_t, \upsilon_t)$ captures the second order effect that stems from the covariance between the labor demand and monetary shock under fixed regime.

Assuming a symmetric steady state across countries as $\alpha_{t-1} = \alpha^*_{t-1}$, by deriving the expression A_t with respect to the monetary shock, we have

$$\frac{\partial A_t}{\partial \upsilon_t} = -\frac{1}{2\sigma}\left(1 - \frac{\sigma - 1}{\kappa}\right)\left[1 - \left(\frac{1}{2}\right)^{\rho} \beta\rho\upsilon_t^{\rho-1}\right] < 0. \tag{22}$$

The expression is strictly negative indicating a negative covariance between labor demand and monetary intervention. The extent of the negative covariance is dependent on the parameters' value. When κ is high, firms are less dispersed and less productive. Labor demand is higher from these less productive exporters. In such a situation, monetary intervention under the fixed regime that stabilizes trade sector and hence potentially volatile labor demand is well acknowledged and welfare improving. Monetary intervention mitigates the first order level effect by the second order covariance effect of it's own. Figure 2 highlights the point with a numerical simulation. In the figure, $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\Delta \ln W_t$ are shown for different value of κ. In computation we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$ (solid line) and $\varphi^{-1} = 1$ (dotted line). As discussed, when κ is high, wage difference is decreasing and fixed exchange rate regime is more likely supported.

Not only κ, but also the elasticity of substitution among goods, σ, determines the size of negative covariance. With a higher value of σ, the covariance is increasing and the welfare improving mitigation effect of monetary intervention is thus weaker. In short, with a higher value of σ labor demand is low in exporting sector due to a tougher competition and the mitigation effect of monetary intervention is less acknowledged. [8]In Figure 3, $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\Delta \ln W_t$ are shown for different value of σ. In computation we

[8]To be precise, by deriving (22) with respect to σ, we get indeed

$$\frac{\partial A_t / \partial \upsilon_t}{\partial \sigma} = \frac{1}{2\sigma}\left(\frac{1}{\kappa} + \frac{1}{\sigma}\left(1 - \frac{\sigma - 1}{\kappa}\right)\right)\left(1 - \left(\frac{1}{2}\right)^{\rho} \beta\rho\upsilon_t^{\rho-1}\right) < 0.$$

set the value of $\kappa = 10.64$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$ (solid line) and $\varphi^{-1} = 1$ (dotted line). As discussed, when σ is high, wage difference is increasing and fixed exchange rate regime is less likely supported.

Similarly, when the shock persistence ρ or the value of discount factor β is high, the covariance increases giving a more support to flexible exchange rate regime. With such a high shock persistence or high discount factor, workers acknowledge less the *current* monetary intervention which does not persist into the future period.[9]

Finally note that a higher value of the elasticity of labor supply makes the fixed regime more supported. The result is consistent with the preceding literature.

4.4 Optimal Monetary Policy

Neither fixed exchange rate regime nor flexible exchange rate regime analyzed above is optimal. We discuss here the optimal monetary policy and implied fluctuations in the nominal exchange rate. Given the welfare metrics derived above (17), the first order condition with respect to μ_t is found as,

$$\frac{1}{2}\left\{\frac{\upsilon_t}{\mu_t} - \frac{1}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}\frac{(A_t\mu_t)^{1+\varphi}}{\mu_t}\right\} + \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\frac{\upsilon_t^{\rho}}{\mu_t} - \frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}\frac{(A_t\mu_t)^{1+\varphi}}{\mu_t}\right\} = 0.$$

Under the above optimal policy, the exchange rate is expressed as

$$\varepsilon_t = \frac{\alpha_t^*}{\alpha_t}\frac{\mu_t}{\mu_t^*} = \frac{\upsilon_t^*}{\upsilon_t}\frac{A_t^*}{A_t}\left[\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}\right]^{\frac{1}{1+\varphi}}.$$

From the expression, it is obvious that following the shock, monetary stance covariates positively hence limiting the fluctuations in the nominal exchange rate. Figure 4 and Figure 5 documents the variability of the nominal exchange rate under the optimal policy with respect to different value of κ and σ together with two values of the elasticity of

[9]The result of the numerical simulations with respect to ρ and β is available upon on request.

labor supply, φ^{-1} (0.8 for the solid line and 1 for the dotted line). As is discussed in the previous section, as κ increases, it reduces the wage difference and the cost related to fixed regime decreases. Accordingly, the optimal fluctuations in the nominal exchange rate decreases as the firm dispersion increases. Also, as σ increases, it increases the wage difference and the cost related to the fixed exchange rate regime. Accordingly, the optimal volatility of the nominal exchange rate increases as σ increases. As a lower value of φ^{-1} amplifies the wage difference and hence increases the cost under fixed regime, for a given value of κ or σ the optimal variability of the nominal exchange rate is high with a lower value of $\varphi^{-1} = 0.8$ (solid line).

5 Discussion

5.1 Regulation Policy

In this section, we explore the case where policymakers target the extent of firm turnover in trade sector and control it either by regulation policy or through the exchange rate policy as previously discussed. Specifically, by relaxing the assumption of stable regulation in trade sector, a counteracting regulation policy following demand shift is now introduced as

$$f_{X,t} = \frac{f_X}{\alpha_t}, \qquad f^*_{X,t} = \frac{f_X}{\alpha^*_t}.$$

The aim of the above policy reaction is to sterilize volatile trade sector which is detrimental to welfare in our setting. By looking the expression of the relative number of exporters, with the implementation of the above policy, we have

$$\frac{N_{X,t}}{N^*_{X,t}} = \frac{\mu_t}{\mu^*_t}\frac{W^*_t}{W_t}.$$

Thanks to the regulation policy, monetary policy is now free from the pressure of stabilizing trade sector and can let the exchange rate fluctuated freely.

We then compare the choice between fixed exchange rate regime as previously argued and the above regulation policy regime. Denoting the expected utility under regulation as $\mathrm{E}_{t-1}\left[\mathcal{U}^{REG}\right]$, the welfare difference is expressed as (see Appendix):

$$\begin{aligned}
&\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{REG}\right] \\
&\qquad = E_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\frac{1}{2}\left(\frac{1}{\sigma-1}+2-\frac{1}{\kappa}\right)\Delta \ln W_t \\
&\qquad + \left(\frac{1}{2}\right)^{1+\rho}\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t^{\rho}\ln \upsilon_t\right]-E_{t-1}\left[\upsilon_t^{\rho}\right]\Delta \ln W_t\right\}
\end{aligned}$$

Note that under regulation policy since the exchange rate float freely we have the same expression for $\Delta \ln W_t$. As a result, fixed exchange rate regime still may dominate as $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] > \mathrm{E}_{t-1}\left[\mathcal{U}^{REG}\right]$. Obviously, we have $\mathrm{E}_{t-1}\left[\mathcal{U}^{REG}\right] > \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ because of sterilized trade sector in case of regulation. The regulation regime is slightly better than flexible exchange rate regime since it realizes a better congruence between preference and intensive as well as extensive margins at least for traded goods. However it is helpless to nail down the economy wide uncertainty stemming from stochastic demand. Figure 6 and Figure 7 show the above welfare ranking with different value of κ, σ and φ^{-1}. The take away of the above exercise is that the temptation of manipulating currency is always present for monetary authority.

6 Empirical evidence

In this section we report some empirical facts concerning the impact of the exchange rate volatility on export volatility, and its relationship with the exchange rate regime.

As is presented in introduction, overall, data suggest a negative relationship between μ_{tc} and the degree of flexibility of the exchange rate regime: a higher ratio of the volatility of exports over the volatility of the exchange rate tends to be associated with more fixed exchange rate regime, suggesting a need for currency manipulation.

This finding is confirmed by the following regression:

$$\text{regime}_{tc} = \alpha \frac{\text{vol(EXP)}_{tc}}{\text{vol(TCEN)}_{tc}} + \delta_t + \gamma_c + \varepsilon_{tc}, \tag{23}$$

where t indexes the period (quarter) and c stands for the country.

- regime_{tc} is de facto exchange rate regime a la Ilzetzki et al. (2018),
- δ_t captures time fixed-effects,
- γ_c captures country fixed-effects,
- ε_{tc} is the error term with standard errors clustered at the country level.

The results are reported in table 2.

A higher level of *regime* refers to higher flexibility of the exchange rate regime. The results confirm that the ratio of volatility of consumption over nominal exchange rate decreases when the regime becomes more flexible.

Then focus on the last column of table 2. When splitting the sample in high heterogeneous and low-heterogeneous countries, we find that high heterogeneous countries are those where the ratio of volatility of consumption over nominal exchange rate decreases further.[10] When the firm distribution is more heterogeneous, a flexible exchange rate reduces the volatility of consumption despite the higher volatility of the nominal effective exchange rate.

7 Conclusion

The paper explores the choice of exchange rate regime with firm heterogeneity and resulting selection into export markets. Fixed regime not only realizes a better congruence between preference and the variety consumed but also substantially reduces uncertainty in labor demand that arises from entry and exit of exporters. In our setting, fixed exchange rate regime can be superior to flexible exchange rate regime depending on the parameters' value. We also show that regulation policy in trade sector that aims to stabilize firm

[10] High heterogeneous countries: Belgium, France, Greece, Ireland, Italy, Japan, Norway, Portugal, Spain, Sweden. Low heterogeneous countries: Australia, Austria, Canada, Finland, Germany, Netherlands, New Zealand, South Korea, Switzerland, United Kingdom, United States.

turnover cannot remove the temptation of currency manipulation. For future research, it would be interesting to think how the view of Friedman, the almighty flexible exchange rate, is reestablished in the model with firm heterogeneity and the selection into export markets as ours.

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Figures and tables

Figure 1: Exchange rate regime and volatility of trade

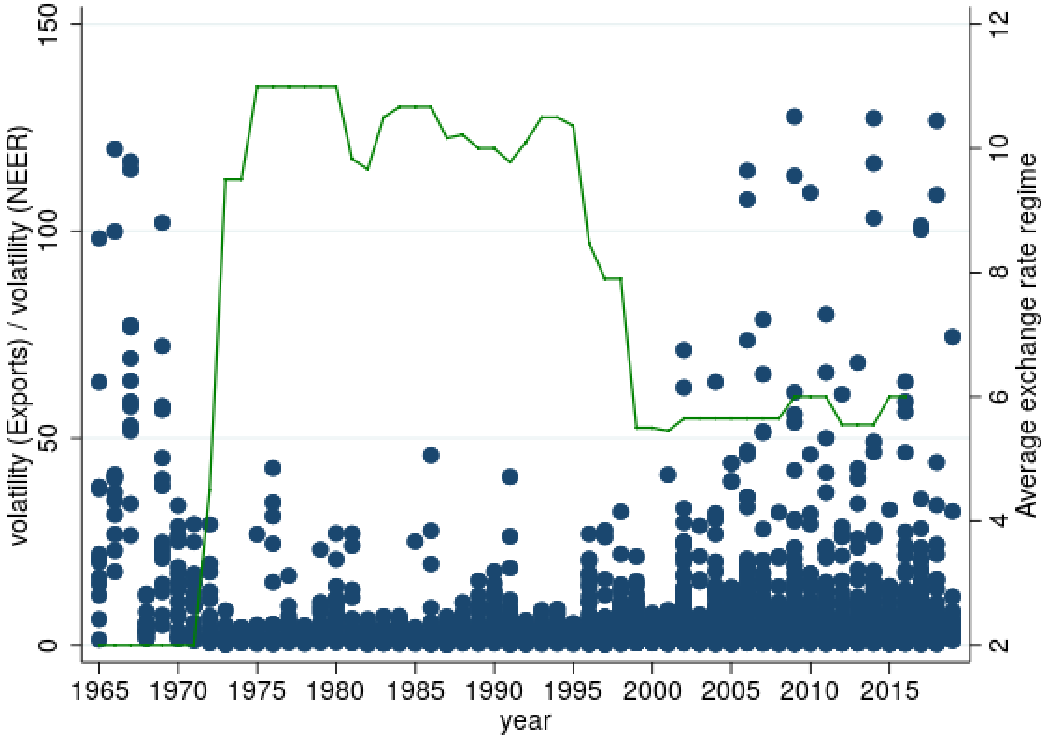


Figure 2: Volatility of exports and exchange rate - fixed exchange rate regime

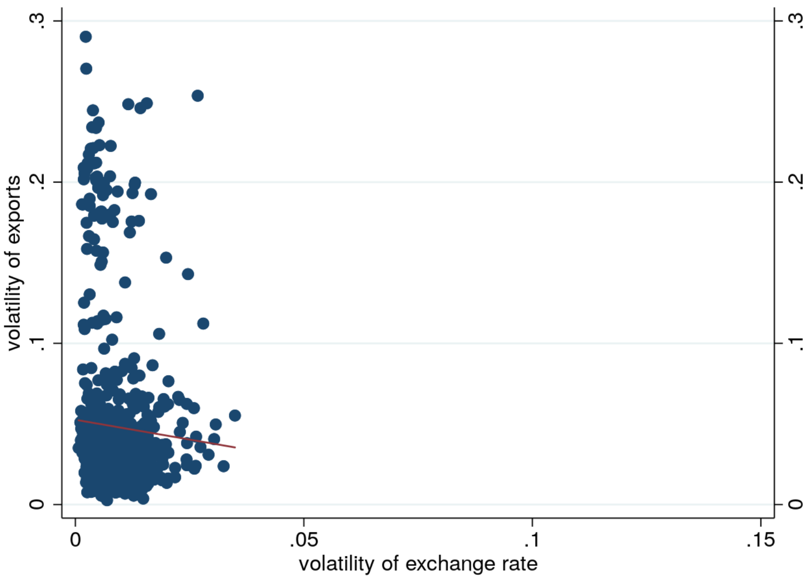


Figure 3: Volatility of exports and exchange rate - flexible exchange rate regime

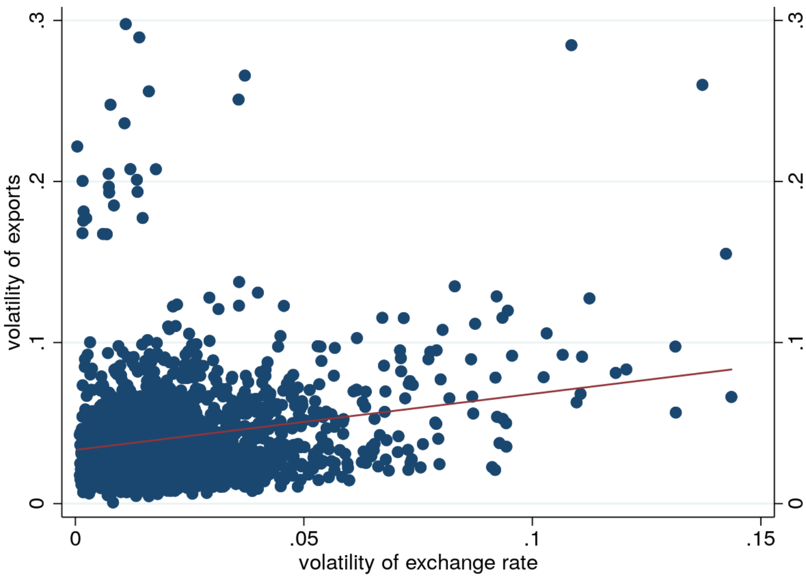
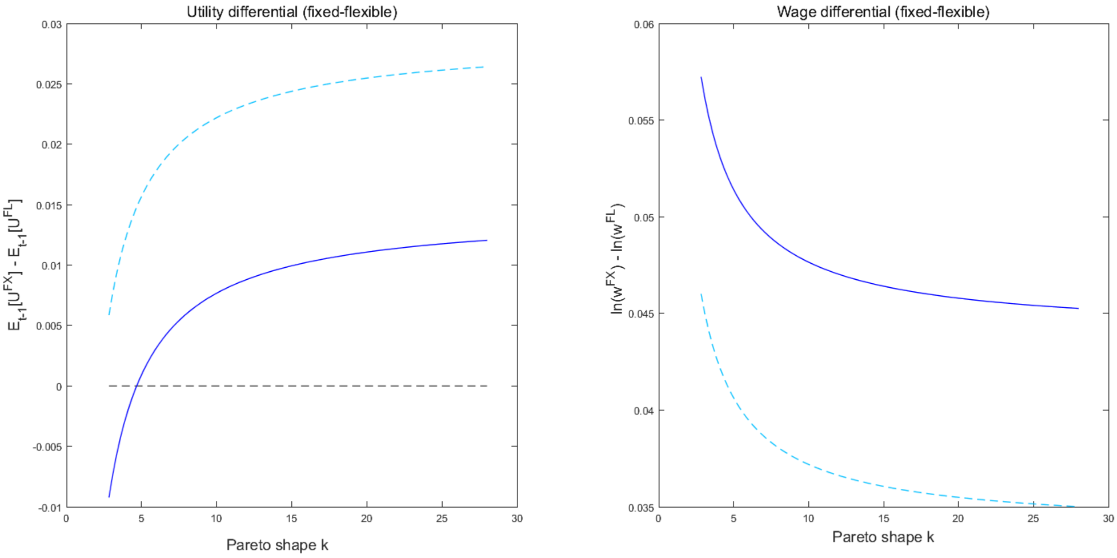
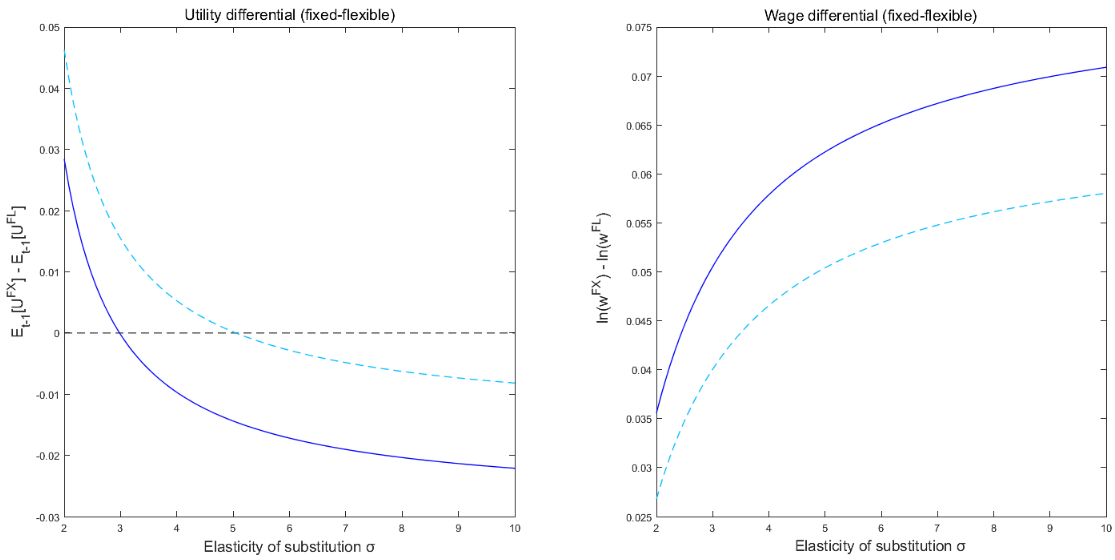


Figure 4: Welfare Ranking: Fixed vs.Flexible Regime



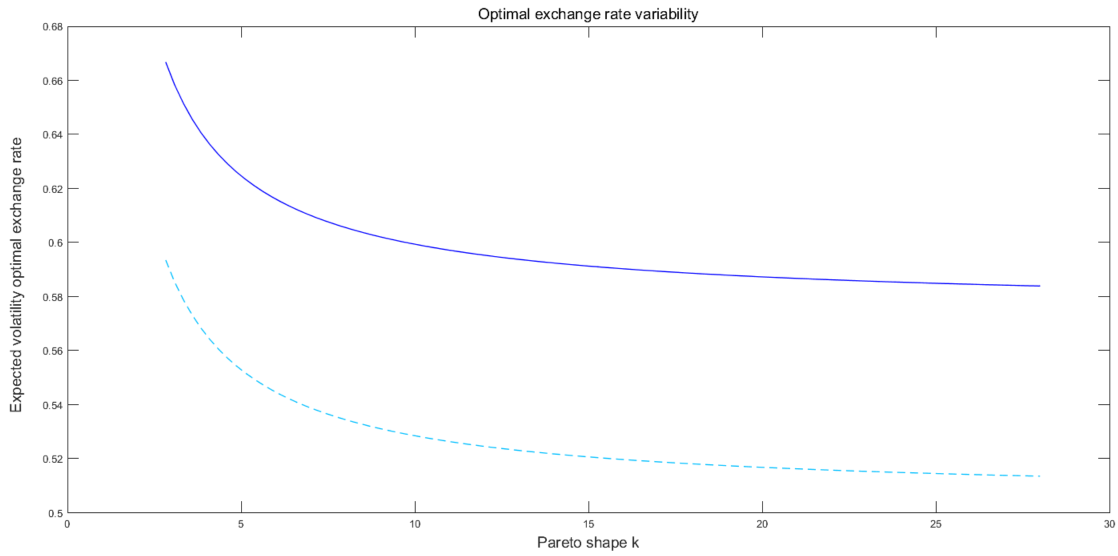
Notes: $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\Delta \ln W_t$ are shown for different value of κ. In computation we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$ (solid line) and $\varphi^{-1} = 1$ (dotted line).

Figure 5: Welfare Ranking: Fixed vs.Flexible Regime



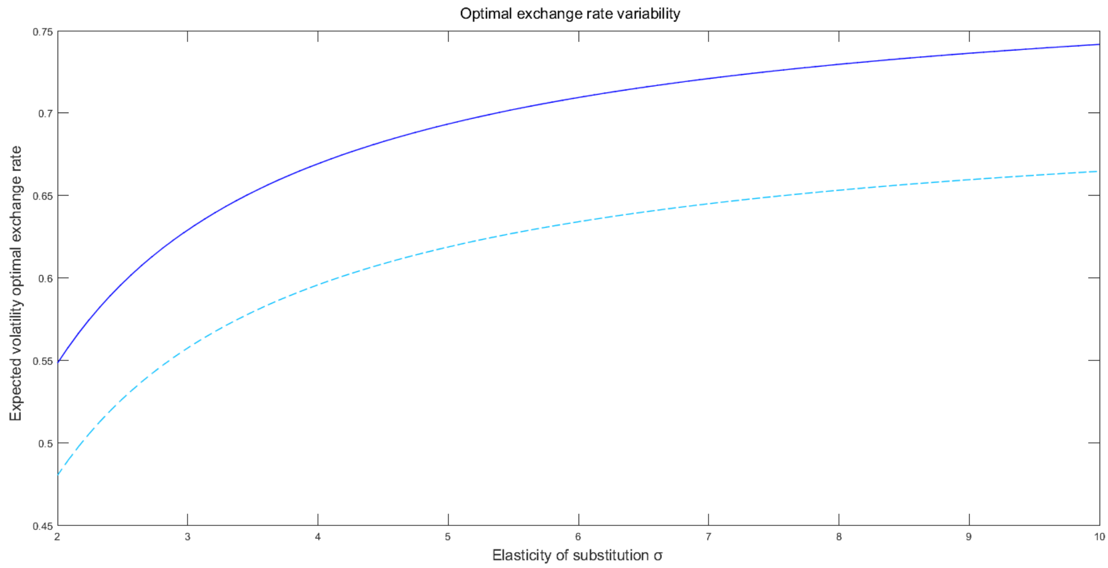
Notes: $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ and $\Delta \ln W_t$ are shown for different value of σ. In computation we set the value of $\kappa = 10.64$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$ (solid line) and $\varphi^{-1} = 1$ (dotted line).

Figure 6: The Optimal Policy and Variance of the Nominal Exchange Rate



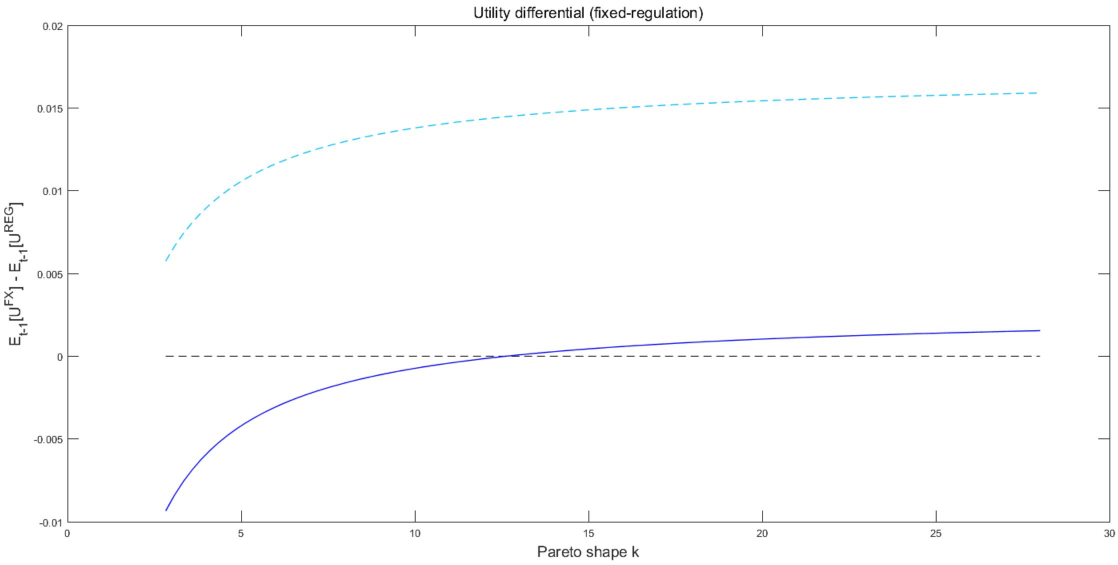
Notes: Optimal volatility of the nominal exchange rate is shown for different value of κ. In computation we set the value of $\sigma = 3.8$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$ (solid line) and $\varphi^{-1} = 1$ (dotted line).

Figure 7: The Optimal Policy and Variance of the Nominal Exchange Rate



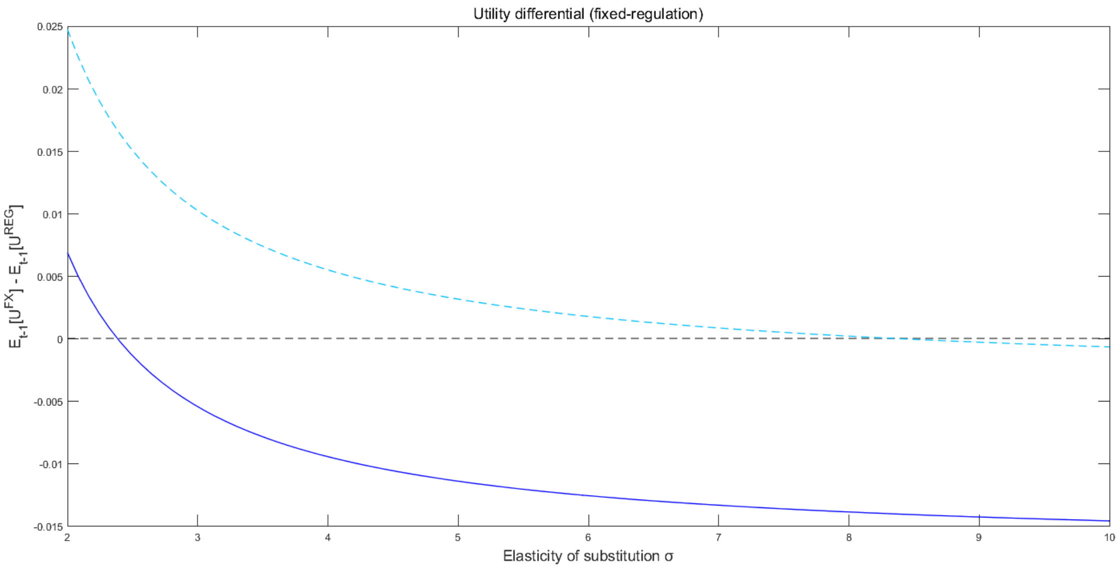
Notes: Optimal volatility of the nominal exchange rate is shown for different value of σ. In computation we set the value of $\kappa = 10.64$, $\rho = 0.9$, $\beta = 0.95$ and $\varphi^{-1} = 0.8$ (solid line) and $\varphi^{-1} = 1$ (dotted line).

Figure 8: Welfare Ranking: Fixed vs. Regulation Regime



Notes: $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ is shown for different value of κ. In computation we set the value of $\sigma=3.8$, $\rho=0.9$, $\beta=0.95$ and $\varphi^{-1}=0.8$ (solid line) and $\varphi^{-1}=1$ (dotted line).

Figure 9: Welfare Ranking: Fixed vs. Regulation Regime



Notes: $\mathrm{E}_{t-1}\left[\mathcal{U}^{FX}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{FL}\right]$ is shown for different value of σ. In computation we set the value of $\kappa=10.64$, $\rho=0.9$, $\beta=0.95$ and $\varphi^{-1}=0.8$ (solid line) and $\varphi^{-1}=1$ (dotted line).

Figure 10: Volatility of exports and exchange rate - heterogeneous firms countries

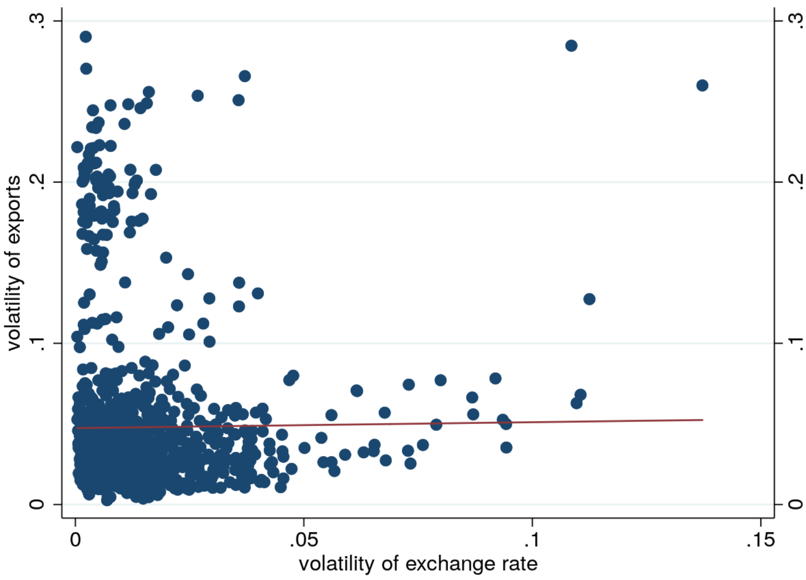


Figure 11: Volatility of exports and exchange rate - homogeneous firms countries

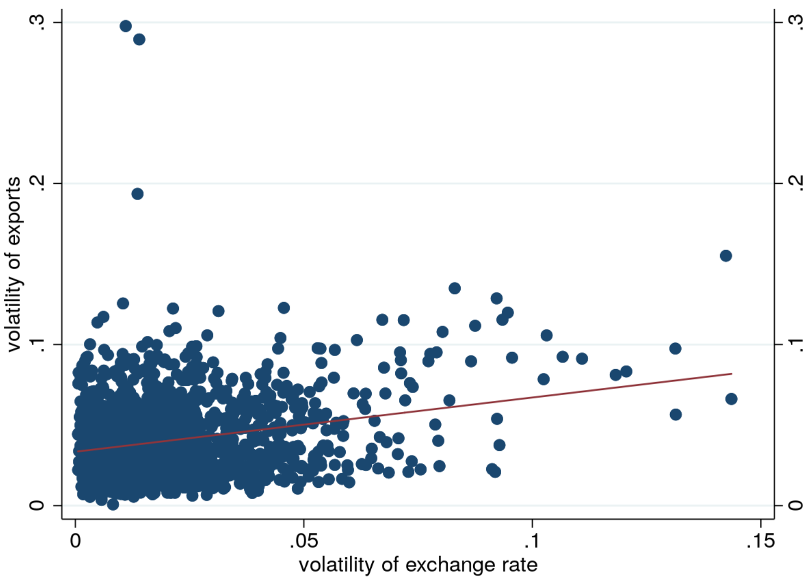


Table 1: The Model's Solution

Nb of Entrants	$N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}} E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$	$N^*_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu^*_t}{W^*_t f^*_{E,t}} E_t\left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]$
Nb of Exporters	$N_{X,t} = \frac{1}{\sigma}(1-\frac{\sigma-1}{\kappa})\frac{\alpha^*_t\mu_t}{W_t f_{X,t}}$	$N^*_{X,t} = \frac{1}{\sigma}(1-\frac{\sigma-1}{\kappa})\frac{\alpha_t\mu^*_t}{W^*_t f^*_{X,t}}$
Av. Exporters	$\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N^*_{X,t}}{N^*_{D,t}}\right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t} = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t\widetilde{z}_D}{N_{D,t}W_t}$, $\quad \widetilde{y}_{X,t} = \frac{\sigma-1}{\sigma}\frac{\alpha^*_t\mu_t\widetilde{z}_{X,t}}{N_{X,t}W_t}$	$\widetilde{y}^*_{D,t} = \frac{\sigma-1}{\sigma}\frac{\alpha^*_t\mu^*_t\widetilde{z}^*_D}{N^*_{D,t}W^*_t}$, $\quad \widetilde{y}^*_{X,t} = \frac{\sigma-1}{\sigma}\frac{\alpha_t\mu^*_t\widetilde{z}^*_{X,t}}{N^*_{X,t}W^*_t}$
Average Price	$\widetilde{p}_{D,t} = \frac{\sigma}{\sigma-1}\frac{W_t}{\widetilde{z}_D}$, $\quad \widetilde{p}_{X,t} = \frac{\sigma}{\sigma-1}\frac{\tau_t\varepsilon_t^{-1}W_t}{\widetilde{z}_{X,t}}$	$\widetilde{p}^*_{D,t} = \frac{\sigma}{\sigma-1}\frac{W^*_t}{\widetilde{z}^*_D}$, $\quad \widetilde{p}^*_{X,t} = \frac{\sigma}{\sigma-1}\frac{\tau_t\varepsilon_t W^*_t}{\widetilde{z}^*_{X,t}}$
Price Indices	$P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{D,t}$, $\quad P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{X,t}$, $\quad P_t = P_{H,t}^{\alpha_t}P_{F,t}^{\alpha^*_t}$	$P^*_{F,t} = N_{D,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{D,t}$, $\quad P^*_{H,t} = N_{X,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{X,t}$, $\quad P^*_t = P_{F,t}^{*\alpha^*_t}P_{H,t}^{*\alpha_t}$
Consumption	$C_t = \left(\frac{C_{H,t}}{\alpha_t}\right)^{\alpha_t}\left(\frac{C_{F,t}}{\alpha^*_t}\right)^{\alpha^*_t}$	$C^*_t = \left(\frac{C^*_{F,t}}{\alpha^*_t}\right)^{\alpha^*_t}\left(\frac{C^*_{H,t}}{\alpha_t}\right)^{\alpha_t}$
Profits	$\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}}$, $\quad \widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha_t}{\sigma}\frac{\varepsilon_t\mu^*_t}{N_{X,t}}$, $\quad \widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}}\widetilde{D}_{X,t}$	$\widetilde{D}^*_{D,t} = \frac{\alpha^*_t}{\sigma}\frac{\mu^*_t}{N^*_{D,t}}$, $\quad \widetilde{D}^*_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha^*_t}{\sigma}\frac{\varepsilon_t^{-1}\mu_t}{N^*_{X,t}}$, $\quad \widetilde{D}^*_t = \widetilde{D}^*_{D,t} + \frac{N^*_{X,t}}{N^*_{D,t}}\widetilde{D}^*_{X,t}$
ZPC	$\widetilde{D}_{X,t} = W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$	$\widetilde{D}^*_{X,t} = W^*_t f^*_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$
Share Price	$\widetilde{V}_t = f_{E,t}W_t$	$\widetilde{V}^*_t = f^*_{E,t}W^*_t$
Labor Supply	$L_t = (\sigma-1)\frac{N_{D,t}\widetilde{D}_t}{W_t} + \sigma N_{X,t}f_{X,t} + N_{D,t+1}f_{E,t}$	$L^*_t = (\sigma-1)\frac{N^*_{D,t}\widetilde{D}^*_t}{W^*_t} + \sigma N^*_{X,t}f^*_{X,t} + N^*_{D,t+1}f^*_{E,t}$
Monetary Stance	$\mu_t = P_tC_t$	$\mu^*_t = P^*_tC^*_t$
Wages	$W_t = \Gamma\left\{\frac{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\}^{\frac{1}{1+\varphi}}$	$W^*_t = \Gamma\left\{\frac{E_{t-1}\left[(A^*_t\mu^*_t)^{1+\varphi}\right]}{E_{t-1}[A^*_t]}\right\}^{\frac{1}{1+\varphi}}$
Exchange Rate	$\varepsilon_t = \frac{\alpha^*_t}{\alpha_t}\frac{\mu_t}{\mu^*_t}$	
Definition of A_t	$A_t = \frac{\sigma-1}{\sigma}\alpha_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\alpha^*_t + \frac{\beta}{\sigma}E_{t-1}\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$	$A^*_t = \frac{\sigma-1}{\sigma}\alpha^*_t + \left(1-\frac{\sigma-1}{\sigma\kappa}\right)\alpha_t + \frac{\beta}{\sigma}E_{t-1}\left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]$
Shock Process	$\alpha_t = \frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t$, $\quad \alpha^*_t = \frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon^*_t$, $\alpha_0 = \alpha^*_0 = 1$, $\quad E_{t-1}[\upsilon_t] = E_{t-1}[\upsilon^*_t] = 1$, $\quad E_{t-1}[\upsilon_t\upsilon_{t+1}] = 1$, $\quad \upsilon_t + \upsilon^*_t = 2$, $\quad 0 < \rho < 1$	

Table 2: Exchange rate regime and volatility of exports to nominal exchange rate.

degree of flexibility exchange rate	(1)	(2)	(3)
$vol(EXP)_{tc}/vol(TCEN)_{tc}$	-.117	-.115	-.018
	(.008)	(.009)	(.004)
Observations	2379	2379	2379
Year FE	No	Yes	Yes
Country FE	No	No	Yes

degree of flexibility exchange rate	(1)	(2)	(3)
$vol(EXP)_{tc}/vol(TCEN)_{tc}$	-.119	-.115	-.012
	(.007)	(.009)	(.004)
$vol(EXP)_{tc}/vol(TCEN)_{tc} \times$ homogeneous	.011	-.003	-.055
	(.016)	(.019)	(.009)
Observations	2379	2379	2379
Year FE	No	Yes	Yes
Country FE	No	No	Yes

Online Appendix

A Solution of the Model

We derive here the closed form solution of the theoretical model presented in Table 1. The similar expressions hold for Foreign. First, note using average prices and the expressions of price indices, we have $P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}} \widetilde{p}_{D,t}$ and $P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}} \widetilde{p}^{*}_{X,t}$. Plugging these expressions in the expression of domestic profits, profits from exporting and total profits on average, we have $\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}}$, $\widetilde{D}_{X,t} = \frac{\alpha_t}{\sigma}\frac{\varepsilon_t \mu_t^*}{N_{X,t}} - f_{X,t}W_t$ and $\widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}}\widetilde{D}_{X,t}$. With zero cutoff profits (ZCP) condition, we have $\widetilde{D}_{X,t} = W_t f_{X,t} \frac{\sigma-1}{\kappa-(\sigma-1)}$. Note that by combining these two expressions of $\widetilde{D}_{X,t}$ we have $\widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha_t}{\sigma}\frac{\varepsilon_t \mu_t^*}{N_{X,t}}$. Also with ZCP and the expression of $\widetilde{D}_{X,t}$ previously found, we have $N_{X,t} = \frac{1}{\sigma}(1-\frac{\sigma-1}{\kappa})\frac{\alpha_t^* \mu_t}{W_t f_{X,t}}$. With the Pareto distribution as in the paper, it implies that $\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$.

We are now ready to derive the number of new entrant, $N_{D,t+1}$. Free entry implies that $\widetilde{V}_t = f_{E,t}W_t$. Combined with the expression of $\widetilde{D}_{t+1}$, the Euler equation about the share holdings, $\widetilde{V}_t = E_t\left[Q_{t,t+1}\widetilde{D}_{t+1}\right]$, is expressed as

$$E_t\left[\frac{\beta P_t C_t}{P_{t+1}C_{t+1}}\left(\widetilde{D}_{D,t+1} + \frac{N_{X,t+1}}{N_{D,t+1}}\widetilde{D}_{X,t+1}\right)\right] = f_{E,t}W_t.$$

Plugging the expression of $\widetilde{D}_{D,t+1}$, $\widetilde{D}_{X,t+1}$ and using the definition of monetary stance, it is rewritten as

$$E_t\left[\frac{\beta\mu_t}{\mu_{t+1}}\left(\frac{\alpha_{t+1}}{\sigma}\frac{\mu_{t+1}}{N_{D,t+1}} + \frac{N_{X,t+1}}{N_{D,t+1}}\frac{\sigma-1}{\kappa}\frac{\alpha_{t+1}}{\sigma}\frac{\varepsilon_{t+1}\mu^*_{t+1}}{N_{X,t+1}}\right)\right] = f_{E,t}W_t$$

Further, by plugging the expression of the equilibrium exchange rate $\varepsilon_t = \frac{\alpha_t^*}{\alpha_t}\frac{\mu_t}{\mu_t^*}$ and rearranging the terms, we have

$$\frac{\beta}{\sigma}\frac{\mu_t}{N_{D,t+1}}E_t\left[\left(\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right)\right] = f_{E,t}W_t$$

which gives $N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$.

Next we derive the labor demand in general equilibrium. Note that $\widetilde{D}_{X,t} = \frac{1}{\sigma}\frac{\varepsilon_t \widetilde{p}_{X,t}}{\tau}\widetilde{y}_{X,t} - f_{X,t}W_t$ and $\widetilde{D}_{D,t} = \frac{1}{\sigma}\widetilde{p}_{D,t}\widetilde{y}_{D,t}$. Also plugging the expression of prices into these profits, we have $\widetilde{y}_{D,t} = (\sigma - 1) \frac{\widetilde{D}_{D,t}\widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma - 1) \frac{\left(\widetilde{D}_{X,t}+f_{X,t}W_t\right)\widetilde{z}_{X,t}}{W_t}$. Putting these expression of intensive margins of average domestic and exporting firms in the labor market clearings (11), we have

$$L_t = N_{D,t} (\sigma - 1) \frac{\widetilde{D}_{D,t}}{W_t} + N_{X,t} \left((\sigma - 1) \frac{\widetilde{D}_{X,t} + f_{X,t}W_t}{W_t} + f_{X,t} \right) + N_{D,t+1} f_{E,t}$$

Plugging the expression of $\widetilde{D}_{D,t}$ and $\widetilde{D}_{X,t}$ found previously, the above expression becomes

$$L_t = \frac{\sigma - 1}{\sigma} \frac{\alpha_t \mu_t}{W_t} + \frac{(\sigma - 1)^2}{\sigma\kappa} \frac{\alpha_t \varepsilon_t \mu_t^*}{W_t} + \sigma N_{X,t} f_{X,t} + N_{D,t+1} f_{E,t}$$

Further, plugging $N_{D,t+1}$, $N_{X,t}$ and the exchange rate found previously, we have

$$L_t = \frac{\sigma - 1}{\sigma} \frac{\alpha_t \mu_t}{W_t} + \frac{(\sigma - 1)^2}{\sigma\kappa} \frac{\alpha_t^* \mu_t}{W_t} + (1 - \frac{\sigma - 1}{\kappa}) \frac{\alpha_t^* \mu_t}{W_t} + \frac{\beta}{\sigma} \frac{\mu_t}{W_t} E_t \left[\alpha_{t+1} + \frac{\sigma - 1}{\kappa} \alpha_{t+1}^* \right]$$

which can be further rewritten as

$$L_t = \frac{\mu_t}{W_t} \left[\frac{\sigma - 1}{\sigma} \alpha_t + (1 - \frac{\sigma - 1}{\sigma\kappa}) \alpha_t^* + \frac{\beta}{\sigma} E_t \left[\alpha_{t+1} + \frac{\sigma - 1}{\kappa} \alpha_{t+1}^* \right] \right]$$

Finally, plugging the expression found in wage setting equation (7), we have

$$W_t = \Gamma \left\{ \frac{E_{t-1} \left[(A_t \mu_t)^{1+\varphi} \right]}{E_{t-1} [A_t]} \right\}^{\frac{1}{1+\varphi}}.$$

A.1 Comparison of the Solution with Hamano and Picard (2017)

Stochastic labor demand followed by demand shock and its mitigation by monetary intervention is the key in deriving the main result of the paper. To see this, it would be useful make a comparison with a model without selection into exporting market as described in Hamano and Picard (2017).

Note that by setting $f_{X,t} = 0$, all firms export despite firm heterogeneity, hence $N_{X,t} = N_{D,t}$ and $\widetilde{z}_{X,t} = \widetilde{z}_D$. In such a specific case, we have $\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma} \frac{\mu_t}{N_{D,t}}$, $\widetilde{D}_{X,t} = \frac{\alpha_t}{\sigma} \frac{\varepsilon_t \mu_t^*}{N_{D,t}}$. Putting these expressions in t

$$E_t\left[\frac{\beta P_t C_t}{P_{t+1}C_{t+1}}\left(\widetilde{D}_{D,t+1}+\widetilde{D}_{X,t+1}\right)\right]=f_{E,t}W_t$$

$N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}E_t\left[\alpha_{t+1}+\alpha^*_{t+1}\right] = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}$ with symmetric process of the shocks across countries.

the labor market clearings becomes

$$L_t = N_{D,t}\left(\frac{\widetilde{y}_{D,t}}{\widetilde{z}_D}+\frac{\widetilde{y}_{X,t}}{\widetilde{z}_D}\right)+N_{D,t+1}f_{E,t}$$

where $\widetilde{y}_{D,t}=(\sigma-1)\frac{\widetilde{D}_{D,t}\widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t}=(\sigma-1)\frac{\widetilde{D}_{X,t}\widetilde{z}_D}{W_t}$. Plugging these expressions and $N_{D,t+1}$, we have

$$L_t=\frac{\mu_t}{W_t}\left[\frac{\sigma-1}{\sigma}+\frac{\beta}{\sigma}\right]$$

This is the labor demand found in Hamano and Picard (2017) for their model called "lagged entry". The equilibrium wage is found to be

$$W_t=\Gamma\left\{E_{t-1}\left[\mu_t^{1+\varphi}\right]\right\}^{\frac{1}{1+\varphi}}.$$

B Expected Utility

The expected utility of Home representative household for any consecutive time period is given by

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] &\equiv \mathrm{E}_{t-1}\left[U_t\right]+\beta\mathrm{E}_{t-1}\left[U_{t+1}\right]\\
&= E_{t-1}\left[\alpha_t\ln C_{H,t}+\alpha^*_t\ln C_{F,t}\right]+\beta E_{t-1}\left[\alpha_t\ln C_{H,t+1}+\alpha^*_t\ln C_{F,t+1}\right]\\
&\quad \mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t}\right)+\alpha^*_t\left(\ln N_{X,t}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}^*_{X,t}}{\tau}\right)\right]\\
&\quad +\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\ln N_{D,t+1}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t+1}\right)+\alpha^*_{t+1}\left(\ln N_{X,t+1}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}^*_{X,t+1}}{\tau_t}\right)\right]
\end{aligned}$$

Plugging the equilibrium expression of $\widetilde{y}_{D,t}$, $\widetilde{y}^*_{X,t}$, $\widetilde{y}_{D,t+1}$ and $\widetilde{y}^*_{X,t+1}$,

$$\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \mathrm{E}_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t\widetilde{z}_D}{W_t}\right) + \alpha_t^*\left(\ln N_{X,t}^{*\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t^*\widetilde{z}_{X,t}^*}{W_t^*\tau}\right)\right]$$
$$+\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\left(\ln N_{D,t+1}^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_{t+1}\mu_{t+1}\widetilde{z}_D}{W_{t+1}}\right) + \alpha_{t+1}^*\left(\ln N_{X,t+1}^{*\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\frac{\alpha_{t+1}\mu_{t+1}^*\widetilde{z}_{X,t+1}^*}{W_{t+1}^*\tau}\right)\right]$$

Developing the expression, we have

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \frac{1}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_t\ln N_{D,t}\right] + \mathrm{E}_{t-1}\left[\alpha_t\ln\alpha_t\right] + \mathrm{E}_{t-1}\left[\alpha_t\ln\mu_t\right] - \mathrm{E}_{t-1}\left[\alpha_t\ln W_t\right] \\
+\frac{1}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_t^*\ln N_{X,t}^*\right] + \mathrm{E}_{t-1}\left[\alpha_t^*\ln\alpha_t\right] \\
+\mathrm{E}_{t-1}\left[\alpha_t^*\ln\mu_t^*\right] + \mathrm{E}_{t-1}\left[\alpha_t^*\ln\widetilde{z}_{X,t}^*\right] - \mathrm{E}_{t-1}\left[\alpha_t^*\ln W_t^*\right] \\
+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}\ln N_{D,t+1}\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\ln\alpha_{t+1}\right] \\
+\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\ln\mu_{t+1}\right] - \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\ln W_{t+1}\right] \\
+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln N_{X,t+1}^*\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\alpha_{t+1}\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\mu_{t+1}^*\right] \\
+\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\widetilde{z}_{X,t+1}^*\right] - \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln W_{t+1}^*\right] + \mathrm{cst}
\end{aligned}$$

Plugging the equilibrium solution of $\widetilde{z}_{X,t}^*$ and $\widetilde{z}_{X,t+1}^*$ and relegating some terms as constant,

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = \mathrm{E}_{t-1}\left[\alpha_t\ln\mu_t\right] - \mathrm{E}_{t-1}\left[\alpha_t\ln W_t\right] \\
+\frac{1}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_t^*\ln N_{X,t}^*\right] + \mathrm{E}_{t-1}\left[\alpha_t^*\ln\mu_t^*\right] \\
+\mathrm{E}_{t-1}\left[\alpha_t^*\ln\left(\frac{N_{X,t}^*}{N_{D,t}^*}\right)^{-\frac{1}{\kappa}}\right] - \mathrm{E}_{t-1}\left[\alpha_t^*\ln W_t^*\right] \\
+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}\ln N_{D,t+1}\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\ln\mu_{t+1}\right] - \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}\ln W_{t+1}\right] \\
+\frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln N_{X,t+1}^*\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\alpha_{t+1}\right] \\
+\beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\mu_{t+1}^*\right] + \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln\left(\frac{N_{X,t+1}^*}{N_{D,t+1}^*}\right)^{-\frac{1}{\kappa}}\right] - \beta\mathrm{E}_{t-1}\left[\alpha_{t+1}^*\ln W_{t+1}^*\right] + \mathrm{cst}
\end{aligned}$$

Relegating the terms for future policies (μ_{t+1} and μ^*_{t+1} and thus variables that depend on these policies, W_{t+1} and W^*_{t+1} $N^*_{X,t+1}$as constant and further rearranging,

$$\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = {} & \mathrm{E}_{t-1}[\alpha_t \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_t \ln W_t] \\
& + \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\alpha^*_t \ln N^*_{X,t}\right] + \mathrm{E}_{t-1}[\alpha^*_t \ln \mu^*_t] \\
& - \mathrm{E}_{t-1}[\alpha^*_t \ln W^*_t] + \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}[\alpha_{t+1} \ln N_{D,t+1}] \\
& + \frac{\beta}{\kappa}\mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln N^*_{D,t+1}\right] + \mathrm{cst}.
\end{aligned}$$

Plugging the equilibrium solution of $N^*_{X,t}$, $N_{D,t+1}$and $N^*_{D,t+1}$, we have

$$\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = {} & \mathrm{E}_{t-1}[\alpha_t \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_t \ln W_t] \\
& + \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\alpha^*_t \ln \frac{\alpha_t \mu^*_t}{W^*_t f^*_{X,t}}\right] + \mathrm{E}_{t-1}[\alpha^*_t \ln \mu^*_t] \\
& - \mathrm{E}_{t-1}[\alpha^*_t \ln W^*_t] + \frac{\beta}{\sigma-1}\mathrm{E}_{t-1}\left[\alpha_{t+1} \ln \frac{\mu_t}{W_t f_E}\right] \\
& + \frac{\beta}{\kappa}\mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln \frac{\mu^*_t}{W^*_t f^*_E}\right] + \mathrm{cst}
\end{aligned}$$

Further rearranging,

$$\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = {} & \mathrm{E}_{t-1}[\alpha_t \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_t \ln W_t] \\
& + \left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}[\alpha^*_t \ln \mu^*_t] - \mathrm{E}_{t-1}[\alpha^*_t \ln W^*_t]\right\} \\
& - \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\alpha^*_t \ln f^*_{X,t}\right] \\
& + \frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}[\alpha_{t+1} \ln \mu_t] - \mathrm{E}_{t-1}[\alpha_{t+1} \ln W_t]\right\} \\
& + \frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln \mu^*_t\right] - \mathrm{E}_{t-1}\left[\alpha^*_{t+1} \ln W^*_t\right]\right\} + \mathrm{cst}
\end{aligned}$$

Rearranging and plugging shock process, the expression becomes

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = &\mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t \mathrm{ln}\mu_t\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t \mathrm{ln}W_t\right] \\
&+\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* \mathrm{ln}\mu_t^*\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* \mathrm{ln}W_t^*\right]\right\} \\
&-\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^* \mathrm{ln}f_{X,t}^*\right] \\
&+\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1}\mathrm{ln}\mu_t\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{\rho}\upsilon_t\right)^{\rho}\upsilon_{t+1}\mathrm{ln}W_t\right]\right\} \\
&+\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^*\right)^{\rho}\upsilon_{t+1}\mathrm{ln}\mu_t^*\right] - \mathrm{E}_{t-1}\left[\frac{1}{2}\left(\frac{1}{2}\alpha_{t-1}^{*\rho}\upsilon_t^*\right)^{\rho}\mathrm{ln}W_t^*\right]\right\} + \mathrm{cst}
\end{aligned}
$$

Note that monetary authority attempt to maximize the expected utility by optimally setting μ_t which has impact on for any two consecutive periods. With a symmetric steady state across countries we assume that $\alpha_{t-1} = \alpha_{t-1}^* = 1$, with which the expression becomes finally

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}\right] = &\frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t \mathrm{ln}\mu_t\right] - \frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t \mathrm{ln}W_t\right] \\
&+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \mathrm{ln}\mu_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^* \mathrm{ln}W_t^*\right]\right\} \\
&-\frac{1}{2}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\upsilon_t^* \mathrm{ln}f_{X,t}^*\right] \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\upsilon_{t+1}\mathrm{ln}\mu_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\upsilon_{t+1}\mathrm{ln}W_t\right]\right\} \\
&+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^*\mathrm{ln}\mu_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^*\mathrm{ln}W_t^*\right]\right\} + \mathrm{cst.}
\end{aligned}
$$

With symmetry of shocks as $\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\upsilon_{t+1}\mathrm{ln}\upsilon_t\right] = \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\upsilon_{t+1}^*\mathrm{ln}\upsilon_t^*\right]$ and with no serial correlation across them such that $\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\mathrm{ln}\upsilon_t\right]\mathrm{E}_{t-1}\left[\upsilon_{t+1}\right] = \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\mathrm{ln}\upsilon_t\right]$, we have

$$
\begin{aligned}
\mathrm{E}_{t-1}[\mathcal{U}] = \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln \mu_t] - \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln W_t] \\
+ \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}[\upsilon_t^* \ln \mu_t^*] - \mathrm{E}_{t-1}[\upsilon_t^* \ln W_t^*]\right\} \\
- \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\mathrm{E}_{t-1}\left[\upsilon_t^* \ln f_{X,t}^*\right] \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}[\upsilon_t^\rho \ln \mu_t] - \mathrm{E}_{t-1}[\upsilon_t^\rho \ln W_t]\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}[\upsilon_t^{*\rho} \ln \mu_t^*] - \mathrm{E}_{t-1}[\upsilon_t^{*\rho} \ln W_t^*]\right\} + \mathrm{cst}
\end{aligned}
$$

Shutting down the fluctuations of fixed cost for exporting and plugging the expression of wages in equilibrium, the expression becomes (18).

C Fixed vs. Flexible Regime

Again with symmetry at the steady state and with $\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL}$, the difference of the expected utility across different regime is

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FL}}\right] = \frac{1}{2}\mathrm{E}_{t-1}[\upsilon_t \ln \upsilon_t] - \frac{1}{2}\Delta \ln W_t \\
+ \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}[\upsilon_t^* \ln \upsilon_t^*] - \Delta \ln W_t\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}[\upsilon_t^\rho \upsilon_{t+1} \ln \upsilon_t] - \mathrm{E}_{t-1}[\upsilon_t^\rho \upsilon_{t+1}]\Delta \ln W_t\right\} \\
+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \upsilon_{t+1}^* \ln \upsilon_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \upsilon_{t+1}^*\right]\Delta \ln W_t\right\}
\end{aligned}
$$

With symmetry of shock $\mathrm{E}_{t-1}[\upsilon_t^\rho \upsilon_{t+1} \ln \upsilon_t] = \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \upsilon_{t+1}^* \ln \upsilon_t^*\right]$ and with no serial correlation of the shock such that $\mathrm{E}_{t-1}[\upsilon_t^\rho \ln \upsilon_t]\,\mathrm{E}_{t-1}[\upsilon_{t+1}] = \mathrm{E}_{t-1}[\upsilon_t^\rho \ln \upsilon_t]$, we have

$$\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FL}}\right]=\frac{1}{2}\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\frac{1}{2}\Delta\ln W_t$$
$$+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\Delta\ln W_t\right\}$$
$$+\beta\left(\frac{1}{2}\right)^{1+\rho}\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln \upsilon_t\right]-\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right]\Delta\ln W_t\right\}.$$

D The Optimal Policy

$$\mathrm{E}_{t-1}\left[\mathcal{U}\right]=\frac{1}{2}\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \mu_t\right]-\frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\}$$
$$+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*}\ln \mu_t^{*}\right]-\frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t^{*}\mu_t^{*}\right)^{1+\varphi}\right]\right\}$$
$$+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln \mu_t\right]-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\}$$
$$+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln \mu_t^{*}\right]-\frac{E_{t-1}\left[\upsilon_t^{*\rho}\right]}{1+\varphi}\ln E_{t-1}\left[\left(A_t^{*}\mu_t^{*}\right)^{1+\varphi}\right]\right\}+\mathrm{cst}$$

$$\frac{1}{2}\left\{\frac{\upsilon_t}{\mu_t}-\frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\}$$
$$+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\frac{\upsilon_t^{\rho}}{\mu_t}-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\}=0$$

$$\left\{\upsilon_t-\frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\left(A_t\mu_t\right)^{1+\varphi}\right\}$$
$$+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\left\{\upsilon_t^{\rho}-\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\left(A_t\mu_t\right)^{1+\varphi}\right\}=0$$

$$\left\{\frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\right\}\left(A_t\mu_t\right)^{1+\varphi}$$
$$=\upsilon_t+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}$$

$$(A_t\mu_t)^{1+\varphi} = \frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\frac{1}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]} + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\frac{E_{t-1}\left[\upsilon_t^{\rho}\right]}{E_{t-1}\left[(A_t\mu_t)^{1+\varphi}\right]}}$$

$$\mu_t = \frac{1}{A_t}\left\{\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}$$

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\frac{\frac{1}{A_t}\left\{\frac{\upsilon_t+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}}{\frac{1}{A_t^*}\left\{\frac{\upsilon_t^*+\left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}{\text{cst}}\right\}^{\frac{1}{1+\varphi}}}$$

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\frac{A_t^*}{A_t}\left[\frac{\upsilon_t + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{\rho}}{\upsilon_t^* + \left(\frac{1}{2}\right)^{\rho}\frac{\beta}{\sigma-1}\upsilon_t^{*\rho}}\right]^{\frac{1}{1+\varphi}}$$

Note that when $\beta = 0$ and $A_t = A_t^* = cst$, we have

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\left[\frac{\upsilon_t}{\upsilon_t^*}\right]^{\frac{1}{1+\varphi}}$$

The above is the expression found in Devereux (2004). When $\varphi = 0$, $\varepsilon_t = 1$.

E Regulation Policy

$$\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{FX}}\right] - \mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{RG}}\right] = \frac{1}{2}\Big\{\Big[\mathrm{E}_{t-1}\left[\upsilon_t \ln\mu_0\upsilon_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t \ln W_t^{FX}\right]\Big] - \Big[\mathrm{E}_{t-1}\left[\upsilon_t \ln\mu_0\right] - \mathrm{E}_{t-1}\left[\upsilon_t \ln W_t^{FL}\right.\\
+\frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\Big\{\mathrm{E}_{t-1}\left[\upsilon_t^*\ln\mu_0^*\upsilon_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^*\ln W_t^{*FX}\right] - \Big[\mathrm{E}_{t-1}\left[\upsilon_t^*\ln\mu_0^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^*\ln W_t^{*FL}\right]\Big]\Big\}\\
-\frac{1}{2}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\Big\{\mathrm{E}_{t-1}\left[\upsilon_t^*\ln f_X^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^*\ln f_X^*\upsilon_t^{*-1}\right]\Big\}\\
+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\Big\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\mu_0\upsilon_t\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln W_t^{FX}\right] - \Big[\Big[\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln\mu_0\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\ln W_t^{FL}\right]\Big]\Big]\Big\}\\
+\left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\Big\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln\mu_0^*\upsilon_t^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln W_t^{*FX}\right] - \Big[\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln\mu_0^*\right] - \mathrm{E}_{t-1}\left[\upsilon_t^{*\rho}\ln W_t^{*FL}\right]\Big]\Big\}
\end{aligned}$$

Further rearranging,

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{F}\mathcal{X}}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{R}\mathcal{G}}\right] &= \frac{1}{2}\left\{\left[\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\Delta \ln W_t\right]\right\} \\
&+ \frac{1}{2}\left(\frac{1}{\sigma-1}+1-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \ln \upsilon_t^*\right]-\Delta \ln W_t^*\right\} \\
&- \frac{1}{2}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^* \ln \upsilon_t^*\right]\right\} \\
&+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\sigma-1}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln \upsilon_t\right]-\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right]\Delta \ln W_t\right\} \\
&+ \left(\frac{1}{2}\right)^{1+\rho}\frac{\beta}{\kappa}\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{*\rho} \ln \upsilon_t^*\right]-\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right]\Delta \ln W_t\right\}
\end{aligned}
$$

With symmetry as argued, we have

$$
\begin{aligned}
&\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{F}\mathcal{X}}\right]-\mathrm{E}_{t-1}\left[\mathcal{U}^{\mathcal{R}\mathcal{G}}\right] = \\
&\quad \frac{1}{2}\left(\frac{1}{\sigma-1}+2-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]-\Delta \ln W_t\right\} \\
&\quad - \frac{1}{2}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t \ln \upsilon_t\right]\right\} \\
&\quad + \left(\frac{1}{2}\right)^{1+\rho}\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\left\{\mathrm{E}_{t-1}\left[\upsilon_t^{\rho} \ln \upsilon_t\right]-\mathrm{E}_{t-1}\left[\upsilon_t^{\rho}\right]\Delta \ln W_t\right\}.
\end{aligned}
$$