The role of labor market frictions on mortgage debt dynamics*

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Abstract

According to the SCE housing survey, a decline in home value accounts for 17.1% of all foreclosure decisions while 36% are due to job loss. However, this evidence conflicts with the collateral literature constraint (*e.g.* Kiyotaki and Moore (1997), Iacoviello (2005)) where only the housing price and the interest rate are drivers of borrowing dynamics.

Then, this paper proposes to link the collateral constraint with the inflows and outflows of employment. As a result of our new microfounded borrowing constraint, we obtain three main results. First, the Loan-To-Value ratio becomes endogenous and dependent of the finding probability for a job seeker. Second, by estimating our model on US data with different collateral constraints, we find that our approach is more able to catch the dynamics between mortgage debts and employment fluctuations and outperforms the other considered models. Third, we find important implications for policy-makers where a deregulation labor market conduces to an increase of the housing price and debt while a macroprudential tightening helps to reduce these negative externalities.

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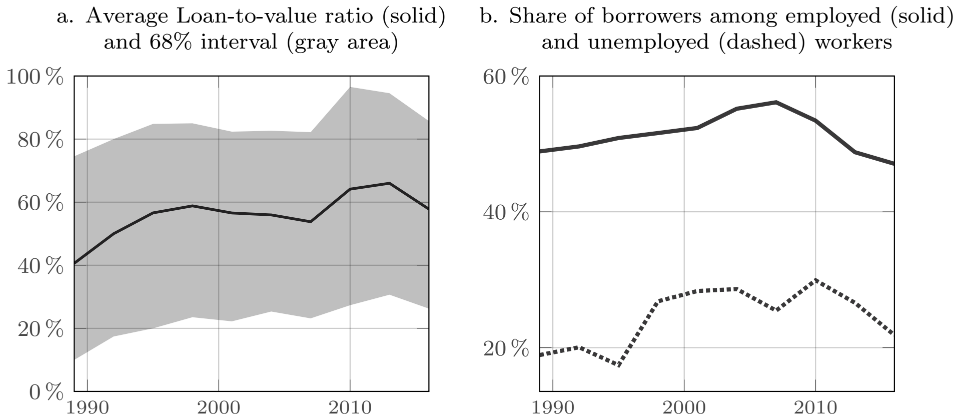
Introduction

Conventional business cycle models featuring a housing market exemplified by Iacoviello (2005) interpret mortgage debt cycles as a macroeconomic response to changes in the future value of durable goods. The underlying debt contract, referred to as collateral constraints, limits the borrowing capacity of an agent to its next period collateral value. Despite its success in policymaking institutions, this conception of mortgage cycles is questionable on two main aspects.

The first aspect is theoretical and concerns the lack of micro-foundation of the financial contract with respect to the borrower's employment situation. In real life situations, financial intermediaries typically review the borrower's ability to make the payments on the loan and avoid potential losses from a future default.[1] Financials intermediary naturally consider employment as an important criterion in their decision to grant a mortgage, which *de facto* excludes jobseekers from the mortgage market. To illustrate this phenomenon in ths US, Figure 1.b shows that the share of borrowers is three times higher among employed workers than jobseekers. In addition, the 2019 SCE housing survey suggests that a decline in home values accounts for 17.1% of all foreclosure decisions, while a rise in mortgage rates accounts for 8.3%. Conversely, job loss and income reduction accounts for 36% and 48% of foreclosure decisions. This evidence conflicts with the current micro-foundation of the collateral constraint model as the latter only consider house price value and interest rates as drivers of borrowing dynamics.

[1] In the literature on the empirical determinants of mortgage default, there is a broad agreemement that employment plays a critical role in causing default (*e.g.* Case et al. (1995), Elul et al. (2010) and Gerardi et al. (2013)) among other factors such as house price drop or equity balance.

Figure 1 – Housing market characteristics (sources: Survey of Consumer Finance)



The second aspect is empirical: this class of financial frictions typically exhibits poor performances in replicating both (i) Loan-to-Value (LTV) dynamics;[2] (ii) and salient business cycle features of mortgage data.[3] These failures have important implications for the estimation of these models with full information methods as they typically fail at replicating the joint dynamics of house prices and mortgage debt. As a consequence, the usual practice in current state-of-art DSGE models is to: (i) discard mortgage debt liabilities as an observable variable (*e.g.* Neri and Iacoviello (2010) or Guerrieri and Iacoviello (2017)); (ii) arbitrary sweep out the low frequency component of mortgage data using business cycle filters (*e.g.* Gerali et al. (2010)); (iii) include some *ad hoc* persistence mechanism as Iacoviello (2015) that captures a reduced form for some contract persistence as loans are typically not renegotiated on a quarterly basis. The failure of standard collateral constraint models calls for an alternative friction that seriously tackles the empirical relevance of housing models.

As a tractable solution to these concerns, we propose to link the borrowing capacity of households to their employment situation on the labor market. To do so, we enrich the

[2]This debt contract typically imposes a time-invariant loan-to-value ratio, which conflicts with the cyclical change of the LTV ratio in Figure 1.a.

[3]To illustrate this limitation, let us consider a simple collateral constraint $d = m.Eq.h$, where the real amount of credit d is limited by a fixed fraction $0 < m < 1$ of future house value, denoted $Eq.h$. Assuming fixed housing stock h, applying logs and differentiating the collateral constraint, then second moment statistics between housing debt and expected house price are theoretically the same. However empirically US data suggest that the autocorrelation of growth real debt is 0.85 vs -0.12 for house price, while 1.09 vs 1.7 for the standard deviations.

collateral constraint by limiting mortgage granting solely to households in employment. Given the presence of inflows and outflows in employment, the collateral constraint originally depends on employment flows balance. This new collateral constraint referred in this article to as the labor-adjusted collateral constraint, introduces a new propagation channel: new matches on the labor market translate into more mortgages (where classical collateral requirements apply), while separation induces an exclusion from financial markets for jobseekers. As a result, the LTV becomes endogenous by responding pro-cyclically to employment cycles.

Our *labor-adjusted* constraint successfully exhibits appealing business cycle features with respect to the canonical setup of Iacoviello (2005). On empirical grounds, our model is able to (*i*) better account for salient features of financial business cycles, (*ii*) significantly improve the forecasting performances for most of macroeconomic time series, (*iii*) be favored by the data according to likelihood ratios. On theoretical grounds, we find that labor market frictions are a key determinant of housing debt dynamics. As a consequence, we show that leakages from the labor to the housing market poses important policy implications for structural reforms and macroprudential policy. In particular, we find that a labor market reform aimed at lowering structural unemployment also leaks to the mortgage market through a surge in mortgages and house prices. As employment rises, the borrowing limit mechanically eases through our labor-adjusted collateral constraint. For macroprudential policy, we find that a loan-to-value tightening affects the labor market through a temporary rise in employment.

Our article is connected to the literature that examine the link between labor market fluctuations and mortgage cycles. Andrés et al. (2013) is the closest approach in terms of the theoretical framework except for the collateral constraint which is simply the expected value of the real estate holdings for borrowers. They find that the response of labor market variables have been substantially affected by the slackening of the LTV ratio in the US in the last twenty years. More precisely, they find that the unemployment is less responsive following a technological shock with lower LTV ratios. Sterk (2015) study the role of house prices on geographical mobility. The author introduces a collateral constraint that depends on the mobility rate and the expected value of real estate holdings. With this setup, he finds that housing price affects the unemployment negatively via the geographical mobility channel. Finally, Liu et al. (2016) documents the relationship between land prices and unemployment. They find an important role of housing shocks in driving unemployment fluctuations. However, in their model constrained household is not present and only firms faced collateral constraint which is the exact opposite of our approach.

Our article is organised as follows. The section 1 presents the theoretical framework with our collateral constraint. In the section 2, we present the data that we used and the estimation of the three different models i.e. one for each specific collateral constraint using Bayesian econometrics. The section 3 is dedicated to the empirical performance (RMSE, marginal density, business cycles statistics) of each model and the propagation mechanism implied by each of them. The section 4 discuss the role of different calibration for labor market variables which differs in the literature. Finally, the section 5 investigate how the presence of labor in the collateral constraint affects the obtained from a labor market reform and a macroprudential policy tightening.

1 Theoretical framework

The economy is populated by a continuum of households of unit mass. As in Kiyotaki and Moore (1997), this continuum is composed by patient and impatient households. Impatient households are characterised by a lower discount factor than patient ones such that in equilibrium impatient are net borrowers and patient net lenders. Variables with the superscript P (I) refers to (im)patient households. Following Andolfatto (1996) and Merz (1995), the family for both types of households provides perfect consumption insurance for its members which allows the latter to have the same consumption level between employed and unemployed family members. Patient households work, consume and accumulate housing and physical capital. Impatient households work, consume and accumulate housing. Due to some underlying frictions in financial markets, borrowers face a binding constraint in the amount of credit they can take.

A key innovation of the model is that collateral requirements depends on the employment status of impatient households. New mortgages are contracted when an impatient household family member finds a job and then classical collateral requirement such as the expected real value of their real estate holdings are applied. For existing mortgage, debt is simply limited not to exceed the amount of the previous period and conditionally to keep the job. This new modeling device establishes a direct link between housing debt and the labor market.

1.1 Labor market

The labor market is subject to matching frictions *à la* Mortensen and Pissarides (1994). For each type of household $j = \{P, I\}$, the hiring process led by firms first starts by a vacancy posting, denoted v_t^j. A vacant position is matched with a job seeker u_t^j through a constant return to the scale matching technology $e_t^j = \psi(v_t^j)^{\varsigma}(u_t^j)^{1-\varsigma}$ with $\psi \in [0,1]$ the efficiency degree of this function, $\zeta \in [0,1]$ the elasticity of matches with respect to vacancies. As in Gertler et al. (2008), we suppose that unemployed workers who find a match immediately go to work within the period. Regarding the outflow from employment, old matches are destroyed at a constant rate δ^L.

Normalising to one the size of the active population, the pool of unemployed workers searching for a job at t is given by the difference between unity and the number of unemployed workers at the end of period $t-1$:

$$u_t^j = 1 - \left(1 - \delta^L\right) n_{t-1}^j. \tag{1}$$

Thus, the law of motion of employment is given by:

$$n_t^j = \left(1 - \delta^L\right) n_{t-1}^j + e_t^j, \tag{2}$$

where e_t^j is the net inflow of employment. This inflow can be expressed in two different ways for the firm or the household. Then for an individual firm, the inflow of new gross hires in t is represented by $e_t^j = q_t^j v_t^j$ while for households by $e_t^j = f_t^j(1 - (1 - \delta^L)n_{t-1}^j)$. The probability that both a firm fills a vacancy and an unemployed worker finds a job are respectively $q_t^j \equiv e_t^j / v_t^j$ and $f_t^j \equiv e_t^j / u_t^j$.

The evolution of employment considering by the firm evolves according to:

$$n_t^j = \left(1 - \delta^L\right) n_{t-1}^j + q_t^j v_t^j, \tag{3}$$

and for each type of household by:

$$n_t^j = \left(1 - \delta^L\right) n_{t-1}^j + f_t^j \left(1 - \left(1 - \delta^L\right) n_{t-1}^j\right). \tag{4}$$

1.2 Households

There is a continuum of measure 1 of agents in each of the two groups of patient and impatient households. As Neri and Iacoviello (2010), the relative size of each group is measured by its wage share, which is assumed to be constant through a unit elasticity of substitution production function. Recall that variables and parameters indexed by I and P denote respectively impatient and patient households, non-indexed variables apply indistinctly to both types of households.

1.2.1 Impatient households

The impatient households maximise the following welfare index:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^I)^t \left\{ \left(1 - h^C\right) \log\left(c_t^I - bc_{t-1}^I\right) + \varepsilon_t^H j \log\left(h_t^I\right) \right\}, \tag{5}$$

where β^I is their discount factor, c_t^I is consumption subject to habits $h^C \in [0,1]$, h_t^I the holdings of housing and j is the consumption weight in life time utility. The term ε_t^H is a shock to housing preferences. This shock can be interpreted as a reduced form source of fluctuations emanating from the productivity changes in the housing sector, or social and institutional changes that shift the demand toward dwellings. Each period, borrowers decide on the optimal amount of nondurable consumption, housing, debt and labor subject to the following budget constraint:

$$c_t^I + q_t^H \Delta h_t^I + r_{t-1} d_{t-1}^I = w_t^I n_t^I + (1 - n_t^I) b^I + d_t^I. \tag{6}$$

where Δ is the first difference operator. The left side of the budget constraint is composed by nondurable consumption c_t^I, housing spending Δh_t^I with q_t^H the housing price and one-period housing loan payment d_{t-1}^I at an interest rate r_{t-1}. The right side consists of income with w_t^I the wage per employed worker, b^I the unemployment benefit per unemployed family members and the amount of newly issued loans d_t^I.

In the literature on business cycle models with collateral constraints exemplified by Kiyotaki and Moore (1997) and Iacoviello (2005), impatient households face a borrowing constraint that limits the amount they can borrow d_t^I to a fraction m^I of the expected housing value $\mathbb{E}_t\left\{q_{t+1}^H\right\}h_t^I$. The remaining fraction $1-m^I$ can be interpreted as a down-payment requirements from financial intermediaries. Thus, the collateral constraints read as $d_t^I \leq m^I\mathbb{E}_t\left\{q_{t+1}^H\right\}h_t^I$. As a consequence, loans are mainly driven by future house prices. This conception of housing debt cycle is actually at odds with the data as loans typically exhibit more persistence than house prices. As a consequence of this simplistic setup, these models are poorly relevant when they are estimated through full information methods with housing debt as an observable variable. As an alternative to these specifications, we propose to link the borrowing capacity of households to both their collateral and their situation on the labor market. As underlined by Elul et al. (2010), unemployment is an important driver of default on the mortgage market.[4] To consider these effects into a full-fledged macroeconomic model, we assume that only family members of the households who are in employment obtain mortgages from financial intermediaries. Given the presence of inflows and outflows in employment, the collateral constraint directly depends of this employment flows:[5]

$$d_t^I \leq \left(1-\delta^L\right)n_{t-1}^I d_{t-1}^I + e_t^I m^I \varepsilon_t^M \mathbb{E}_t\left\{q_{t+1}^H\right\}h_t^I. \tag{7}$$

For the fraction of family members experiencing the separation shock on the labor market, denoted $\delta^L n_{t-1}^I$, they simply cannot pursue the existing mortgage contract. In contrast

[4] In particular, these authors find that when unemployment is high, the mortgage default probability rises simultaneously.

[5] We have not included the real interest rate as a determinant of the collateral constraint. However its inclusion has very modest effects on the transmission channels of the model and does not statistically improve the fit of the model.

for the remaining fraction of family members in employment, denoted $\left(1-\delta^{L}\right) n_{t-1}^{I}$, they simply roll over their existing mortgage. Regarding inflows in employment, only jobseekers filling a vacancy - denoted e_t^I - are granted new loans by patient households. We also include a structural disturbance, denoted ε_t^M, that captures some exogenous changes in down-payment requirements from financial intermediaries. This shock can be interpreted as reduced form for financial frictions from the supply of assets from banks.

The representative borrower chooses the optimal amount of consumption, housing, debt and labor by maximising his utility (Eq.5) subject to his budget constraint (Eq.6), his collateral constraint (Eq.7) and the flow of labor (Eq.4). Thus, the optimal consumption choice gives the marginal utility of consumption denoted by λ_t^I:

$$\lambda_t^I = \left(1-h^C\right)\left(c_t^I - h^C c_{t-1}^I\right)^{-1}. \tag{8}$$

Letting $\mathbb{E}_t \Lambda_{t,t+1}^I = \beta^I \mathbb{E}_t \left\{\lambda_{t+1}^I\right\}/\lambda_t^I$ denote the borrower's stochastic discount factor and ϕ_t^I the Lagrangian multiplier on the collateral constraint normalised by the marginal utility of consumption, the Euler condition for borrowers is given by:

$$1-\phi_t^I = \mathbb{E}_t \Lambda_{t,t+1}^I \left[r_t - \phi_{t+1}^I \left(1-\delta^L\right) n_t^I\right]. \tag{9}$$

In this expression, variable ϕ_t^I can be interpreted as the lifetime utility stemming from borrowing for a home purchase. The borrowing constraint introduces a wedge with respect to the patient Euler equation. A rise in borrowing - captured in the Euler equation through $\Delta\phi_t^I > 0$ - implies that the impatient household increases the fraction of his income spent for a home purchase, to the detriment of his current consumption. As a result in log-linearised form of the Euler equation, variations in consumption are negatively linked to changes in the current shadow value of borrowing, ϕ_t^I. This effect of borrowing on consumption is standard in the collateral literature. In contrast, the labor-adjusted collateral constraint also offers a second original effect on the Euler equation that is directly connected to the worker's employment situation. Recall that if the borrower remains employed, he simply rolls over his existing debt contract without further renegotiating with his creditors. The opportunity cost of investing in housing in turn reduces through a rise in current consumption. The magnitude of this effect is positively driven by the borrower's employment rate n_t^I and implies that a rise in the employment rate drives current consumption upward. This positive effect of employment over consumption is usually featured by non-separable utility function such as King et al. (1988).

The first order condition for housing reads as follows:

$$q_t^H = \frac{j\varepsilon_t^H}{h_t^I}\frac{1}{\lambda_t^I} + \mathbb{E}_t\left\{\Lambda_{t,t+1}^I q_{t+1}^H + \varepsilon_t^M \phi_t^I f_t^I (1 - \left(1-\delta^L\right) n_{t-1}^I) m^I q_{t+1}^H\right\}. \tag{10}$$

This equation determines the housing price q_t^H. The hand right side of this equation is composed of three terms. The first term captures the lifetime utility gain from a marginal unit of housing. The second term is the future gain from reselling the house at the next period, while the third one denotes the lifetime utility gain for matched jobseekers allowed to borrow on financial markets.

Finally, the first-order condition with respect to labor is given by:

$$\mu_t^I = \begin{array}{c} w_t^I - b^I + \left(1-\delta^L\right)\mathbb{E}_t\left\{\Lambda_{t,t+1}^I \mu_{t+1}^I\left(1-f_{t+1}^I\right)\right\} \\ + \left(1-\delta^L\right)\mathbb{E}_t\left\{\Lambda_{t,t+1}^I \phi_{t+1}^I\left(d_t^I - d_{t+1}^I\right)/u_{t+1}^I\right\} \end{array}, \tag{11}$$

where μ_t^I stands for the marginal utility of a match.[6] The marginal utility of a match is determined by three terms, two are standard with respect to the matching literature and one is new. The first term is net pecuniary gain of being in employment rather than being unemployed. The second term is the continuation value if the worker remains in employment. In contrast, the third term results from the presence of employment in the collateral constraint. The continuation value of match now includes the roll over of the mortgage contract which increases employment value.

[6] μ_t^I correspond to the Lagrangian multiplier associated with the labor market law of motion normalised by the marginal utility of consumption.

1.2.2 Patient household

Patient household discounts the future more weakly than impatient ones so their discount factor satisfies $\beta^P > \beta^I$.[7] They maximise the following welfare index:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{ \left(1 - h^C\right) \log\left(c_t^P - b c_{t-1}^P\right) + \varepsilon_t^H j \log\left(h_t^P\right) \right\}, \tag{12}$$

subject to the budget constraint:

$$c_t^P + q_t^H \Delta h_t^P + I_t + d_t^P + T_t^P + \Phi_K\left(\Delta K_t\right) = w_t^P n_t^P + (1 - n_t^P) b^P + r_{t-1} d_{t-1}^P + z_t \upsilon_t K_{t-1}, \tag{13}$$

where the left side displays spending with consumption (c_t^P), the holdings of housing (h_t^P), investment in physical capital (I_t) subject to some adjustment costs ($\Phi_K\left(\Delta K_t\right)$), deposits ($d_t^P$) and taxes ($T_t^P$). The right side gathers different sources of income from labor activities (w_t^P if they are employed and b^P otherwise), interest payments on deposits ($r_{t-1} d_{t-1}^P$) and physical capital remuneration z_t at some utilisation rate υ_t. Our functional form for physical capital adjustment costs is taken from Iacoviello (2015) and reads as $\Phi_K\left(\Delta K_t\right) = \frac{\phi^K}{2\bar{K}} \left(K_t - K_{t-1}\right)^2$. The latter allows the model to replicate a hump shape response of investment as suggested by VAR models.

The law of motion of investment is given by:

$$I_t = \frac{K_t - (1 - \delta_t^K) K_{t-1}}{\varepsilon_t^I}, \tag{14}$$

where ε_t^I is an investment shock to the efficiency of investment as in Smets and Wouters (2007) and δ_t^K is the time-varying depreciation of physical capital. As in Greenwood et al. (1988), when the cost of installing new units of physical capital rises, firms prefer to postpone investment and raise the utilisation rate of existing physical capital at the cost of more depreciation.

[7] This restriction on discount factors implies that the Lagrangian multipliers ϕ_t^I on the collateral constraint Eq.7 is always positive and thus the constraint holds to equality in the neighborhood of the steady state. Our calibration for the gap between discount factors ensures that $\phi_t^I > 0$ which allows a linear approximation to be accurate.

The representative lenders maximise the welfare index (Eq.11) to choose the optimal amount of consumption, housing, deposits, labor and capital subject to his budget constraint (Eq.13) and the flow of labor (Eq.4). Then the First Order Condition with respect to c_t^P gives the marginal utility of consumption denoted by (λ_t^P) :

$$\lambda_t^P = \left(1 - h^C\right) (c_t^P - h^C c_{t-1}^P)^{-1}. \tag{15}$$

Letting $\Lambda_{t,t+1}^P = \beta^P \mathbb{E}_t \left\{\lambda_{t+1}^P\right\} / \lambda_t^P$ denote the lender's stochastic discount factor, the optimal choice for deposit provides a standard Euler condition for the patient household:

$$\mathbb{E}_t \left\{\Lambda_{t,t+1}^P\right\} r_t = 1. \tag{16}$$

The optimal stock of housing is given by:

$$\frac{1}{\lambda_t^P} \frac{\varepsilon_t^H j}{h_t^P} - q_t^H = -\mathbb{E}_t \left\{\Lambda_{t,t+1}^P q_{t+1}^H\right\}, \tag{17}$$

where the left hand side denotes the current net gain in consumption equivalents from housing purchase.

Letting μ_t^P denote the Lagrangian multiplier associated to the employment law of motion normalised by the marginal utility of consumption, then the marginal utility to have a new match for patient household is:

$$\mu_t^P = w_t^P - b^P + \mathbb{E}_t \left\{\Lambda_{t,t+1}^P \mu_{t+1}^P \left(1 - \delta^L\right) \left(1 - f_{t+1}^P\right)\right\}, \tag{18}$$

compared with the marginal utility of a match for an impatient household (Eq.10), this equation is standard in the matching literature with the net pecuniary gain of being in employment rather than being unemployed and the expected future discounted pecuniary gains if the worker remains in employment.

The optimal condition for lenders to provide capital services is:

$$\frac{1}{\varepsilon_t^I} + \Phi_K' \left(\Delta k_t\right) = \mathbb{E}_t \left\{\Lambda_{t,t+1}^P \left(\frac{(1 - \delta_t^K)}{\varepsilon_{t+1}^I} + z_{t+1} \upsilon_{t+1} + \Phi_K' \left(\Delta k_{t+1}\right)\right)\right\}. \tag{19}$$

The choice for utilisation rate depends on the functional form for the depreciation rate δ_t^K is the same than Iacoviello (2015) and reads as:

$$\delta_t^K = \delta^K + \left(\frac{1}{\beta_P} + 1 - \delta^K\right)\left(\frac{1}{2}\left(\frac{\psi}{1-\psi}\right)(\upsilon_t)^2 + \frac{(1-2\psi)}{1-\psi}\upsilon_t + \frac{1}{2}\left(\frac{\psi}{1-\psi}\right) - 1\right). \quad (20)$$

$\psi \in [0,1]$ measures the curvature of the utilisation rate function. When $\psi = 1$, δ_t^K stays constant over time i.e. $\delta_t^K = \delta^K$ and with ψ approaching 0 the depreciation rate is very sensitive to the utilisation rate. Considering the budget constraint (Eq.13) with the definition of investment (Eq.14) and the functional form of the depreciation rate (Eq.20), the optimal utilisation rate is:

$$z_t = \left(\frac{1}{\beta_P} + 1 - \delta^K\right)\left(\left(\frac{\psi}{1-\psi}\right)\upsilon_t + \frac{(1-2\psi)}{1-\psi}\right). \quad (21)$$

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1.3 Firms

Each competitive producer produce an intermediate good Y_t according to:

$$Y_t = \varepsilon_t^Z N_t^{1-\alpha}\left(K_t^U\right)^\alpha, \quad (22)$$

where K_t^U is the utilised capital stock ($K_t^U = \upsilon_t K_{t-1}$), $N_t = \left(n_t^P\right)^\lambda \left(n_t^I\right)^{1-\lambda}$ is the total input of labor input used by the firms where λ measures the relative size of lenders, $\alpha \in [0;1]$ is the part of labor capital in the production and ε_t^Z is the Total Factor Productivity (TFP) shock.[8]

Following Gertler et al. (2008), to hire new workers the firm has to pay quadratic hiring cost $\varepsilon_t^L \kappa^j \left(x_t^j\right) n_{t-1}^j$ where $x_t^j = \frac{q_t^j \upsilon_t^j}{n_{t-1}^j}$ is the hiring rate and ε_t^L an exogenous shock on hiring new workers.

[8] As experimented by Iacoviello and Neri (2010), a formulation in which labor supply across different households are substitutes are analytically less tractable, since it implies that labor supply by one group will affect total wage income received by the other group, thus creating a complex interplay between borrowing constraints and labor supply decisions.

Then, using the definition of output (Eq.22) and the definition of labor composite the problem faced by the representative firm is to choose the optimal amount of labor and vacancies for both type (lenders and borrowers) and the optimal amount of capital to maximise his profit given by:

$$\max_{\left\{v_t^j, n_t^j, K_t^U\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{ Y_t - z_t K_t^U - \sum_{j=P,I} \left(w_t^j n_t^j + \varepsilon_t^L \frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{n_{t-1}^j} \right)^2 n_{t-1}^j \right) \right\}, \tag{23}$$

subject to the labor market law of motion (Eq.3) from the firm's perspective.

To obtain the job creation condition for both types of household ($j = \{P, I\}$), we combine the optimal choice of vacancies and labor. Then the job creation condition for patient and impatient households read respectively:

$$\varepsilon_t^L \kappa^P x_t^P = \frac{(1-\alpha)\,\lambda Y_t}{n_t^P} - w_t^P + \kappa^P \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \Lambda_{t,t+1}^P \left(\frac{\left(x_{t+1}^P\right)^2}{2} + \left(1-\delta^L\right) x_{t+1}^P \right) \right\}, \tag{24}$$

$$\varepsilon_t^L \kappa^I x_t^I = \frac{(1-\alpha)\,(1-\lambda)\,Y_t}{n_t^I} - w_t^I + \kappa^I \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \Lambda_{t,t+1}^P \left(\frac{\left(x_{t+1}^I\right)^2}{2} + \left(1-\delta^L\right) x_{t+1}^I \right) \right\}. \tag{25}$$

These two equations state that the job creation condition occurs until the marginal cost of hiring reaches the net marginal profit per worker and the expected continuation value.

For installed capital, the first order condition is simply given by:

$$z_t = \alpha Y_t / K_t^U, \tag{26}$$

where the rental rate equates the marginal cost of using capital.

1.4 Wage setting

The wage is set according to a Nash bargaining scheme which splits the surplus between workers and employers. For each type j, the Nash bargaining solution is determined by the following program,

$$w_t^j = \arg\max_{\{w_t^j\}} \left(\mu_t^j\right)^{\eta} \left(\mu_{j,t}^L\right)^{1-\eta}, \tag{27}$$

where $\eta \in [0,1]$ is the exogenous bargaining power of the worker of type j and $\mu_{j,t}^L$ the marginal value of adding a new worker of type j to the firm's workforce equal to $\mu_{j,t}^L = \kappa^j \varepsilon_t^L x_t^j$.[9]

Solving this program for the patient household, we have the following wage:

$$w_t^P = \begin{array}{c} \eta\left(\frac{(1-\alpha)\lambda Y_t}{n_t^P} + \mathbb{E}_t\left\{\Lambda_{t,t+1}^P \varepsilon_{t+1}^L \frac{\kappa^P}{2}\left(x_{t+1}^P\right)^2\right\}\right) + (1-\eta)\, b^P \\ +\eta\left(1-\delta^L\right)\mathbb{E}_t\left\{\Lambda_{t,t+1}^P \varepsilon_{t+1}^L \kappa^P x_{t+1}^P f_{t+1}^P\right\} \end{array}, \tag{28}$$

and for impatient :

$$w_t^I = \begin{array}{c} \eta\left(\frac{(1-\alpha)(1-\lambda)Y_t}{n_t^I} + \mathbb{E}_t\left\{\varepsilon_{t+1}^L \Lambda_{t,t+1}^P \frac{\kappa^I}{2}\left(x_{t+1}^I\right)^2\right\}\right) + (1-\eta)\, b^I \\ +\eta\left(1-\delta^L\right)\kappa^I \mathbb{E}_t\left\{\varepsilon_{t+1}^L x_{t+1}^I \left(\Lambda_{t,t+1}^P - \Lambda_{t,t+1}^I\left(1-f_{t+1}^I\right)\right)\right\} \\ +(1-\eta)\left(1-\delta^L\right)\mathbb{E}_t\left\{\Lambda_{t,t+1}^I \phi_{t+1}^I \frac{\Delta d_{t+1}^I}{u_{t+1}^I}\right\} \end{array}. \tag{29}$$

The patient wage corresponds to the definition of wages in the search equilibrium when impatient households are not present in the model. This corresponds to a weighted average between the worker value for a firm (i.e. the marginal productivity of labor plus the saving cost of hiring) and the outside option (i.e. the transfer b^P).

For the impatient wage, the first line is classic as for the patient wage. The second line take into account that impatient household discount the future more heavily than patients. Thus, we have a utility gap between lenders and borrowers (i.e. $\Lambda_{t,t+1}^P - \Lambda_{t,t+1}^I$). Finally, the last line is due to the integration of the labor market into the borrowing constraint (Eq.7). Thus, this term depends positively on the growth of debt and negatively on the anticipation of future unemployment.

[9] This equation comes from the FOC with respect to vacancies. See appendix B for more details.

1.5 General equilibrium

Market clearing is implied by Walras's law by aggregating the budget constraints of lenders and borrowers. In absence of explicit residential production sector the supply of dwellings is normalised to one as Iacoviello (2005), the market clearing for housing just reads as:

$$h_t^P + h_t^I = 1. \tag{30}$$

As a consequence, housing cycle boils down to reallocation effect between impatient and patient households driven by the borrowing constraint. An easing on the borrowing constraint through higher a housing price rises the demand for durable goods h_t^I and mechanically reduces the housing detention for patient households as long as Eq.30 holds.

In absence of explicit financial frictions on financial markets, we assume that patient households grant loans to impatient ones at no cost using their own deposits as liabilities, which implies the following equilibrium condition on the mortgage loans:

$$d_t^P = d_t^I. \tag{31}$$

Taxes finance unemployment insurance for both types of workers, denoted $\left(1 - n_t^P\right) b^P + \left(1 - n_t^I\right) b^P$ and government spending G_t. Following the usual practice in modern macroeconomic models, public spending are exogenous, $G_t = g^Y \varepsilon_t^G$ where g^Y is the fixed spending relative to GDP ratio and ε_t^G is an exogenous process allowing the government to transitory deviate from this fixed ratio. The balance for the government reads as:

$$T_t^P = \left(1 - n_t^P\right) b^P + \left(1 - n_t^I\right) b^I + g^Y \varepsilon_t^G, \tag{32}$$

where T_t^P is the lump-sum tax from patient households.

Then using these relationships, the aggregate GDP is defined as:

$$Y_t = C_t + g^Y \varepsilon_t^G + I_t + \Phi_K\left(\Delta K_t\right) + \sum_{j=P,I} \varepsilon_t^L \left(x_t^j\right)^2 \left(n_{t-1}^j\right)^{-1}, \tag{33}$$

where aggregate consumption is given by $C_t = c_t^P + c_t^I$.

Table 1 – Description of the estimated models featuring different collateral constraints

Model Symbol	Collateral constraint type	Collateral constraint equation
$\mathcal{M}_1$	Simple	$d_t^I = \varepsilon_t^M m^I h_t^I \mathbb{E}_t \left\{ q_{t+1}^H \right\}$
$\mathcal{M}_2$	Exogenous persistence	$d_t^I = \chi d_{t-1}^I + \varepsilon_t^M (1-\chi) m^I \mathbb{E}_t h_t^I \mathbb{E}_t \left\{ q_{t+1}^H \right\}$
$\mathcal{M}_3$	Labor-adjusted	$d_t^I = \left(1-\delta^L\right) n_{t-1}^I d_{t-1}^I + \varepsilon_t^M m^I e_t^I \mathbb{E}_t \left\{ q_{t+1}^H \right\} h_t^I$

2 Data and estimation

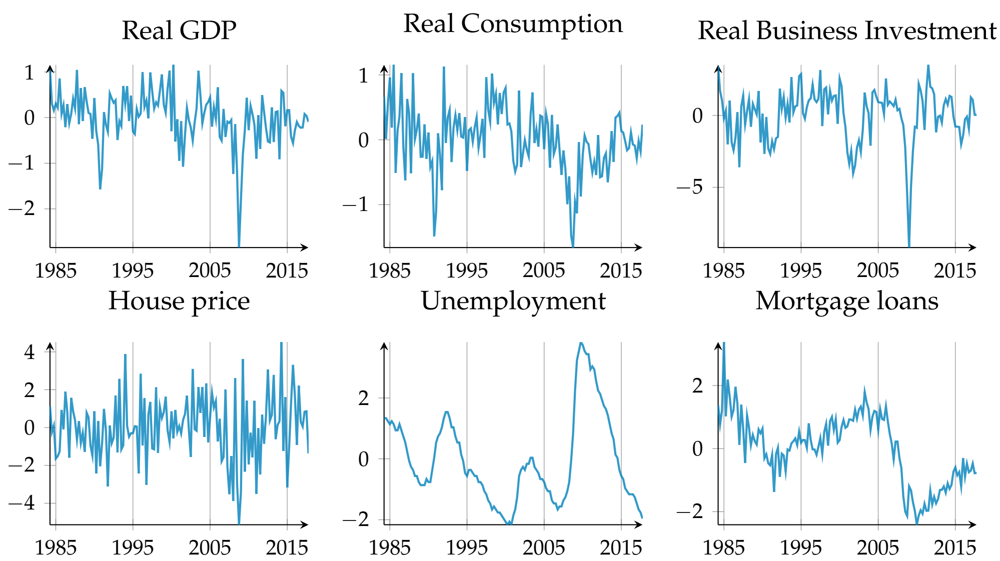
The model is estimated using Bayesian methods and quarterly data for the US economy. We estimate the structural parameters and the sequence of shocks following the seminal contributions of Smets and Wouters (2007) and An and Schorfheide (2007). In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method. We solve the model using a linear approximation to the model's policy function, and employ the Kalman filter to form the likelihood function and compute the sequence of errors. For a detailed description, we refer the reader to the original papers.

In this article, we compare our labor-sensititive constraint to two different benchmark models that are typically employed in the literature of collateral constraints. Tab.1 summarises the three models estimated using the same sample. The first model, referred to as 'Simple' in the first row of Tab.1, is a model in which the borrowing capacity is bounded by market expectations about future prices of dwellings in the same way as Iacoviello (2005) or Kiyotaki and Moore (1997). Alternatively, we also consider the collateral constraint of Iacoviello (2015) characterised by an *ad hoc* persistence mechanism. In this setup, parameter $\chi \in [0,1]$ captures the difference between existing mortgage and new mortgage (in proportion $(1-\chi)$). Given its *ad hoc* nature, we simply call this setup as the 'Exogenous persistence' model. Finally our last model explained in the model section is referred to as the 'Labor-adjusted' model. Since we perform a linear approximation to the policy function of each model, we assume that each constraint holds to equality and we select a calibration which allows the Lagrangian multiplier associated to any of these collateral constraints always to remain positive.

2.1 Data

We fit the DSGE model to US time series data from 1984$Q2$ to 2017$Q4$. Following the usual practice, we keep the number of exogenous disturbances (ε_t^Z, ε_t^L, ε_t^H, ε_t^I, ε_t^G and ε_t^M) the same as the number of observable variables in order to obtain the smoothing of filtered disturbances. Our sample includes housing price, gross domestic product, consumption, investment, unemployment rate and loans. Appendix A describes the data sources.

Figure 2 – Data used in the estimation



Notes: The veritcal axis plots the percentage deviation of the data sample from their average.

Concerning the transformation of series, the point is to map non-stationary data to a stationary model. Except for the unemployment rate, all other data exhibit a trend and are made stationary in two steps. First, we deflate nominal variables using the GDP deflator as in Iacoviello (2015). Second, data are taken in logs to use a first difference filtering to obtain growth rates. Since we do not consider trends, we demean our sample to make the sample consistent with the measurement equations of our model. The Fig.2 plot the transformed series.

Table 2 – Calibrated parameter

	Parameter	Value		Parameter	Value
α	Capital share	0.33	β^P	Discount factor lenders	0.9925
δ^K	Capital depreciation rate	0.035	β^I	Discount factor borrowers	0.97
g^Y	Spending-to-GDP ratio	0.21	λ	Share of lenders in technology	0.75
j	Housing preference	0.09	d^I/Y	Mortgage debt-to-GDP	0.50
f	Finding rate	0.70	ξ	Elasticity matching function	0.50
q	Filling rate	0.73	δ^L	Labor separation	0.10
b^j/w^j	SS unemployment benefit	0.75	η	Worker negotiation power	0.50

2.2 Calibration and prior distributions

Our calibration is reported in Tab.2. Parameters which are calibrated are typically those are weakly identified by the data. As a simplifying assumption, most of parameters common between patient and impatient households are symmetric (an exception for discount factors). For the labor market, we fix the steady-state of the finding rate to 70% ($f^j = 0.7$) as in Shimer (2005) and the exogenous separation by δ^L to 0.1 matching the average job duration of two and a half years in the US. Together they lead to a steady-state value of unemployment to 5% as in Blanchard and Galí (2010) and which is close to the average 6% unemployment rate observed in our sample. The filling rate is fixed to 73% ($q^j = 0.73$) in order to have a steady-state tightness in the labor market to be below one. Consistently with labor matching models, we fix the negotiation power of households η to 0.5 and impose the so-called Hosios condition by imposing $1 - \zeta = \eta$, i.e. the elasticity with respect to unemployment in the matching function is equal to the negotiation power of workers. For the replacement rate $\tau_j^B = b^j/w^j$, we use the same calibration as Christiano et al. (2016) with $\tau_j^B = 0.75$.

Turning to the calibration of discount factor, we use the same calibration as Iacoviello and Neri (2010) with $\beta^P = 0.9925$ and $\beta^I = 0.97$. According to the estimation of Iacoviello (2015) we set the share of lenders to 75% i.e. $\lambda = 0.75$ and the share of capital in the production function to $\alpha = 0.33$.[10] As common practice in the literature, we calibrate

[10]The calibration diverges in the literature concerning the discount factor parameter for impatient household, typically between 0.94 and 0.98. However, we choose the same calibration than Iacoviello and Neri (2010) since in our knowledge it is the only paper that estimates a model with lenders and borrowers without considering entrepreneurs as in our model.

the depreciation rate of capital to $\delta^K = 0.035$ and the ratio of public spending to GDP $g^Y = .21$.

The housing wealth is 123 percent of annual output ($q^H(h^P+h^I)/(4Y) = 1.23$) following Iacoviello (2005) and lead to a housing preference of $j = 0.09$. However, in other versions of our model, this ratio can vary in function of the value of χ. For $\chi = 0$ (the simple version $\mathcal{M}_1$) we have a housing wealth of 126% of annual output against 120% for $\chi = 0.95$. Finally, the only parameter which differ between models is the LTV ratio m^I. In the baseline model, the steady-state of debt is $d^I = m^I f^I q^H h^I$ against $d^I = m^I q^H h^I$ for other models. Since the steady-state value of the finding rate is already calibrated, we fix $m^I = 0.9$ in the baseline model leading to a global LTV ($m^I f^I$) of 0.63. As a consequence, we calibrate m^I in other models to 0.63 to have the same debt to output ratio which is 50% closer from the average of mortgage loans to GDP ratio observed in the data.

For the estimation of our structural parameters, we choose the same prior distribution for the utilisation curvature (ψ), the capital adjustment cost (ϕ^K) and the habit consumption (h^C) than Iacoviello (2015). For the exogenous component in the collateral constraint (χ), in our knowledge two articles estimate it. Iacoviello (2015) choose a beta distribution with a mean of 0.25 and a standard deviation of 0.1 and Guerrieri and Iacoviello (2017) a beta distribution with a mean of 0.75 and a standard deviation of 0.1. Then, we thus consider an average between these two papers by imposing a prior means of 0.5 and a standard deviation of 0.1.

Concerning the prior of shock process we follow Smets and Wouters (2007) where the standard deviations of the shocks are assumed to follow an Inverted Gamma distribution with a means of 0.5 percent and two degrees of freedom and for the persitence of the shock (AR(1)) a Beta distribution with means 0.5 and a standard deviation 0.15.

2.3 Posterior distributions

In this subsection, we discuss our posterior results and contrast them with the results obtained from previous estimates in the literature. The Tab.3 summarises means and the 5% and 95% of the posterior distributions for the structural parameters as well as for shock processes.

The estimated degree of habit formation is rather lower compared to other studies (Smets and Wouters (2007) and Gertler et al. (2008) estimate this parameter around 0.7). Traditionnally, higher degree of habits formation is a necessary device to account for consumption persistence. However, models featuring a housing market such as Iacoviello and Neri (2010), Iacoviello (2015) and our models generate the desired persistence with a lower habits degree. The intuition behind this result is that the wedge in the Euler equation of impatient households is affected sufficiently to account for this business cycle pattern. Regarding the adjustment costs on investment (ϕ^K) and capital utilisation elasticity (ψ), these are close to the findings of Iacoviello (2015). We also find that there is no clear difference between estimated parameters across the three models.

The estimated value of the *ad hoc* persistence parameter in the second model is higher than in the literature. We guess that this result is obtained by imposing a looser prior to χ, the data are thus more informative and predicts a very high persistence. This highlights the underlying problem of collateral constraint models that fails at capturing the persistence of loans.

Except for the LTV shock ε_t^M, all other sources of disturbance exhibit quite similar persistence and standard deviations across models. The difference regarding the standard deviation of the LTV shock is simply induced by a scale effect because $\mathcal{M}_1$ the steady state pre-multiplying the shock is much higher than in $\mathcal{M}_2$ and $\mathcal{M}_3$.

3 Empirical implications

The labor-sensitive collateral constraint introduces a new propagation channel from the labor to the housing market. In this section, we investigate the empirical relevance of this channel in a four-step analysis. First, we compare the fit of our three models through a likelihood comparison between models. Second, we compare the business cycle moments to see how models are able to account for salient features of the data. Third, we examine the forecasting performances of each model. Finally, we compare the propagation mechanism by comparing the IRFs between models.

Table 3 – Prior and posterior distributions for structural parameters

		Prior(P1,P2)	Posterior Mean [5%, 95%] $\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
Shock processes:					
ρ_Z	AR - Productivity	$B(0.50,\ 0.15)$	0.98 [0.97;0.98]	0.97 [0.97;0.98]	0.97 [0.97;0.98]
ρ_L	AR - Hiring	$B(0.50,\ 0.15)$	0.89 [0.86;0.92]	0.89 [0.86;0.92]	0.91 [0.89;0.94]
ρ_H	AR - Housing preference	$B(0.50,\ 0.15)$	0.97 [0.96;0.99]	0.97 [0.95;0.98]	0.96 [0.95;0.98]
ρ_G	AR - Public Spending	$B(0.50,\ 0.15)$	0.91 [0.89;0.94]	0.91 [0.89;0.94]	0.90 [0.88;0.93]
ρ_I	AR - Investment	$B(0.50,\ 0.15)$	0.93 [0.89;0.97]	0.93 [0.88;0.97]	0.93 [0.89;0.97]
ρ_M	AR - Loan To Value	$B(0.50,\ 0.15)$	0.97 [0.95;0.98]	0.82 [0.75;0.90]	0.90 [0.86;0.95]
σ_Z	Std.Dev Productivity	$IG(0.1,\ 2)$	0.39 [0.34;0.44]	0.39 [0.34;0.44]	0.40 [0.35;0.45]
σ_L	Std.Dev Hiring	$IG(0.1,\ 2)$	4.89 [4.40;5.38]	4.90 [4.41;5.40]	6.13 [5.51;6.73]
σ_H	Std.Dev Housing preference	$IG(0.1,\ 2)$	0.07 [0.04;0.09]	0.08 [0.05;0.11]	0.08 [0.05;0.11]
σ_G	Std.Dev Public Spending	$IG(0.1,\ 2)$	2.37 [2.13;2.61]	2.36 [2.11;2.59]	2.38 [2.14;2.63]
σ_I	Std.Dev Investment	$IG(0.1,\ 2)$	0.66 [0.52;0.81]	0.64 [0.50;0.77]	0.61 [0.48;0.73]
σ_M	Std.Dev Loan To Value	$IG(0.1,\ 2)$	2.05 [1.85;2.27]	6.39 [4.52;8.30]	5.45 [4.92;6.01]
Structural parameters:					
h^C	consumption habits	$B(0.50,\ 0.15)$	0.27 [0.15;0.39]	0.26 [0.14;0.37]	0.24 [0.13;0.34]
ϕ^K	capital adj. cost	$G(1.00,\ 0.50)$	0.85 [0.21;1.46]	0.89 [0.22;1.51]	0.79 [0.20;1.34]
ψ	utilisation elasticity	$B(0.50,\ 0.10)$	0.34 [0.20;0.48]	0.33 [0.19;0.47]	0.38 [0.23;0.52]
χ	Collateral persistence	$B(0.50,\ 0.10)$	-	0.91 [0.89;0.94]	-

Notes: $\mathcal{B}$, beta; $\mathcal{G}$, gamma; $\mathcal{N}$, normal; $\mathcal{IG}$, inverse gamma type 1; $P1$, prior mean and $P2$ prior standard deviation for all distributions.

3.1 Fit comparison

To gauge the empirical relevance of employment in directly shaping debt dynamics, Tab.4 reports the (Laplace-approximated) marginal data densities, the posterior odd ratio and probability for each of the three models considered.[11] Since the simple collateral model is the most popular model in this literature, we consider this model as the benchmark to compute the posterior odds ratios and probabilities. To compute the latter, we impose an uninformative prior distribution over models (*i.e.* 1/3 prior probability for each model). In a nutshell, one should favor a model whose data density, posterior odds ratio and model probability are the highest compared to any other model.

Which model best explains the behaviour of the sample?

Our model with labor-adjusted collateral constraints appears to be favored by the data as its marginal data density is the highest. This model is next followed by the exogenous persistence model while the simple model is naturally the last in the ranking. The difference in marginal data densities across models is large enough to validate this ranking.[12] This is therefore confirmed by posterior odds ratios and model probabilities. Given this evidence, we conclude that our model with labor-adjusted constraints outperforms the two other models.

Table 4 – Models fit comparison

	$\mathcal{M}_1$ Simple	$\mathcal{M}_2$ Exo. Persistence	$\mathcal{M}_3$ Labor-adjusted
Prior probability	1/3	1/3	1/3
Log marginal density	-921.81	-899.87	-849.11
Bayes ratio	1.000000	3.388×10^{9}	3.735×10^{31}
Posterior probability	0.000000	0.000000	1.000000

The likelihood ratio does not allow to clearly understand how one model is able to outperform the other. The next subsections provide further investigations through business cycle moments and forecasting performance comparisons between models.

[11]We refer to Rabanal and Rubio-Ramírez (2005) for a formal description and discussion of these criteria to compare estimated DSGE models.

[12]The results are statistically strong as the marginal data density difference between $\mathcal{M}_3$ and $\mathcal{M}_2$ is 72.69, thus we would need a prior probability ratio (currently this ratio is unity) to be higher than $\exp(72.69) = 3.73 \times 10^{31}$ to alter the ranking.

3.2 Business cycle analysis

To assess the empirical relevance of the model, Tab.5 report's key business cycle statistics for observable variables generated by the three models considered, taking for each model the parameters at their posterior means in Tab.3. The aims of this exercise is to assess whether these models are able to capture salient features of the data. Observed moments are expressed in terms of a 90% confidence interval to highlight whether a moment generated by the model is not statistically different from its empirical counterpart.

We first start by examining the standard deviations generated by each model. The three models considered exhibit similar quantitative patterns: they all overpredict the volatility of unemployment, while they are doing a good job in replicating the other standard deviations. The only exception lies in the growth rate of mortgages which is best replicated by the *ad hoc* persistence model $\mathcal{M}_2$, followed by the labor-adjusted constraint.[13] This result is rather expected as models $\mathcal{M}_2$ has one additional estimated parameter with respect to the two other competitors, which clearly helps in capturing the dynamic patterns of mortgages.

Regarding the persistence, all the models are well able to replicate the observed correlation of each observable variable, except for mortgage and investment growth rates. For investment, all the models fail at replicating the observed persistence of investment. This puzzling result is explained by the capital cost function which does not generate the desired autocorrelation. For mortgage dynamics, only the simple collateral constraint models fail at capturing the observed persistence of housing debt growth.

[13]This result contrasts with respect to the literature of estimated models with matching. For instance, Lubik (2009) estimates a simple labor matching model on US data and finds that his model with matching over shoot the variance of unemployment.

Table 5 – Business cycle statistics comparison between the three different collateral constraint models

	U_t	ΔY_t	ΔC_t	ΔI_t	Δd_t^I	Δq_t^H
	Standard deviations					
Data	[1.30;1.66]	[0.52;0.66]	[0.46;0.58]	[1.67;2.13]	[0.97;1.23]	[1.52;1.93]
$\mathcal{M}_1$ - Simple	2.33	0.62	0.67	2.15	1.99	1.84
$\mathcal{M}_2$ - Exo. persistence	2.30	0.62	0.66	2.16	1.15	1.82
$\mathcal{M}_3$ - Labor-adjusted	2.28	0.61	0.68	2.12	1.67	1.81
	Auto-correlation					
Data	[0.97;0.99]	[0.17;0.55]	[0.16;0.54]	[0.44;0.72]	[0.80;0.91]	[-0.33;0.10]
$\mathcal{M}_1$ - Simple	0.99	0.17	0.26	0.04	0.69	-0.01
$\mathcal{M}_2$ - Exo. persistence	0.99	0.18	0.25	0.04	0.84	-0.01
$\mathcal{M}_3$ - Labor-adjusted	0.99	0.17	0.25	0.02	0.90	-0.01
	Correlation w/ unemployment					
Data	[1.00;1.00]	[-0.34;0.10]	[-0.42;-0.00]	[-0.33;0.11]	[-0.62;-0.28]	[-0.28;0.16]
$\mathcal{M}_1$ - Simple	1.00	-0.02	-0.04	0.01	-0.00	-0.02
$\mathcal{M}_2$ - Exo. persistence	1.00	-0.02	-0.04	0.01	-0.05	-0.02
$\mathcal{M}_3$ - Labor-adjusted	1.00	0.06	0.03	0.05	-0.28	0.01

Finally, concerning the correlation coefficient with unemployment, all the models succeed in replicating the co-movement with other observed variables except for housing debt. Our model $\mathcal{M}_3$ with labor-adjusted borrowing capacity is the only one that successfully captures the important negative correlation link between mortgage and unemployment. A rise in unemployment mechanically terminate the mortgage contracts for workers experiencing the separation. As a consequence, this constraint naturally generate the appropriate correlation magnitude and sign for these two observed variables.

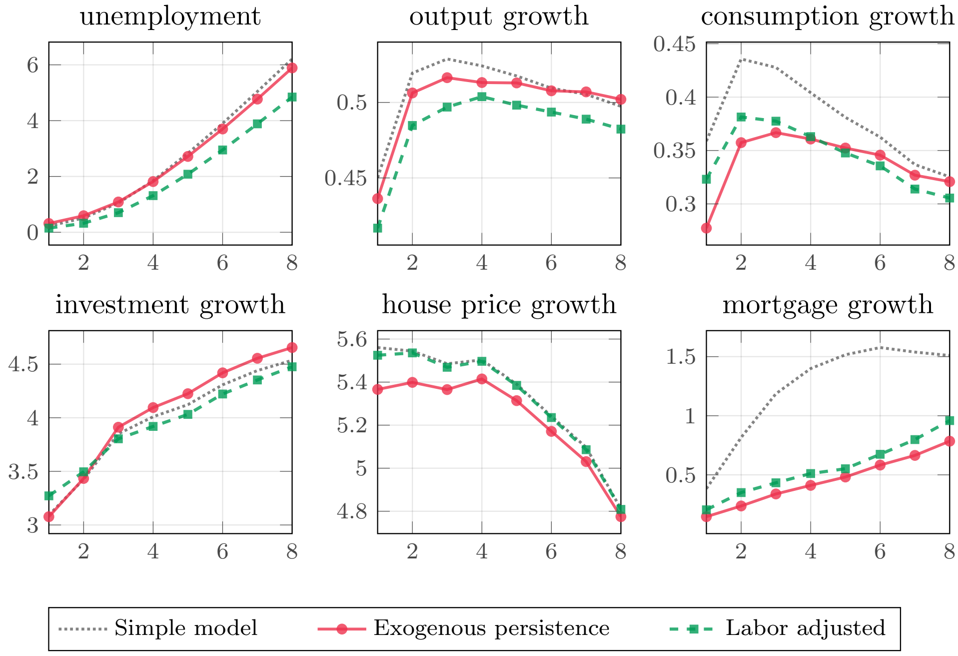
3.3 Forecasting performance

Turning to the forecasting performances, Fig.3 displays the out-of-sample root means square errors (RMSE, hereafter) at different forecast horizons (1 to 8 periods) for each observable variable. Our RMSE's are computed on the out-of-sample forecasting on ten years, spanning the period from 2004Q4 to 2014Q4.[14] Our models are estimated each

[14]Our forecasts stops at 2014Q4 (*i.e.*, 8 periods before the end our sample) because after this date we would not have the corresponding observable to compute the error distance between the forecast and its realised value.

quarter, forecasts are performed using the posterior mode. Therefore, the best model at forecasting is the one that obtains the lowest RMSE.

Figure 3 – Root mean square errors (RMSE) comparison between models featuring different collateral constraints



Our model with labor in the collateral constraint remarkably outperforms the other models for unemployment, output, investment. The results are rather unclear for consumption, as our model $\mathcal{M}_3$ is doing better than $\mathcal{M}_2$ in the short run, however, after 5 periods, the forecasting performance between these two models reverses in favor of $\mathcal{M}_2$. However for house price growth prediction, the model with exogenous persistence $\mathcal{M}_2$ clearly outperforms other models, in particular in the very short-run. Mortgage growth clearly illustrates a clear weakness of simple collateral constraints which clearly does not generate the desired level of persistence. In contrast, both models $\mathcal{M}_2$ and $\mathcal{M}_3$ are able to capture this persistence either through an ad hoc device in $\mathcal{M}_2$ or through employment in $\mathcal{M}_3$.

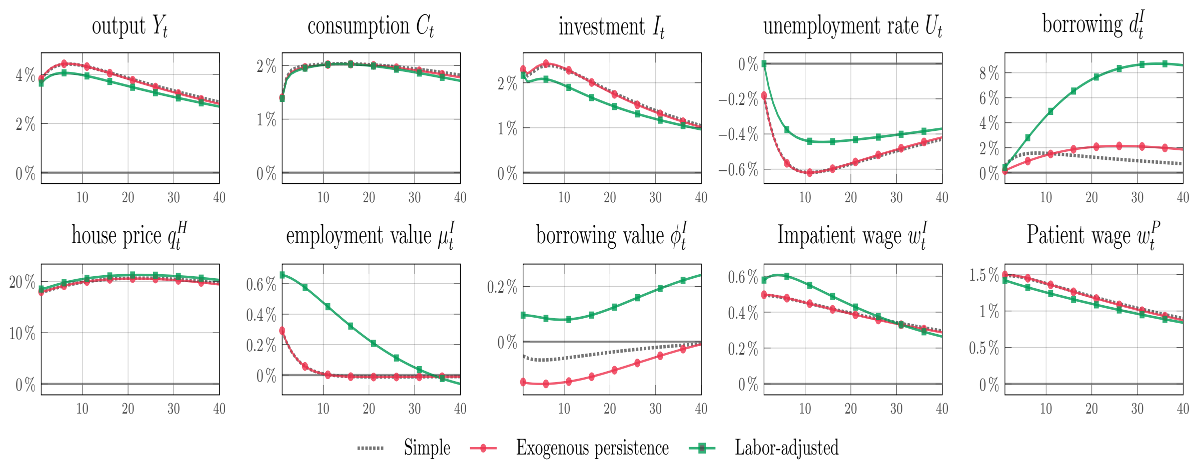
3.4 Inspecting the propagation mechanism

The three models provide different representations of the data, these differences are implied by the propagation mechanisms reshaped according to the type of collateral constraint considered. To illustrate how conditioning mortgage on employment affects the propagation, we contrast the response of our model with the two others. We examine the response following a productivity shock, this shock is particularly relevant as it is the main driving force of unemployment fluctuations in the RBC literature (*e.g.* Mortensen and Pissarides (1994); Shimer (2005)), and we complement the analysis with a hiring shock to highlight how changes in hiring affect mortgage dynamics.[15] Each model parameters are set to their posterior mean.

A productivity shock

Fig.4 displays the response of the three models following an increase in the productivity of firms. The response obtained by the three models are rather in line with matching model literature: a rise in TFP makes labor and capital more productive, which raise the marginal profit from hiring a new worker and then encourage firms to hire more.

Figure 4 – System response to a 1% productivity shock η_t^Z for each estimated models.



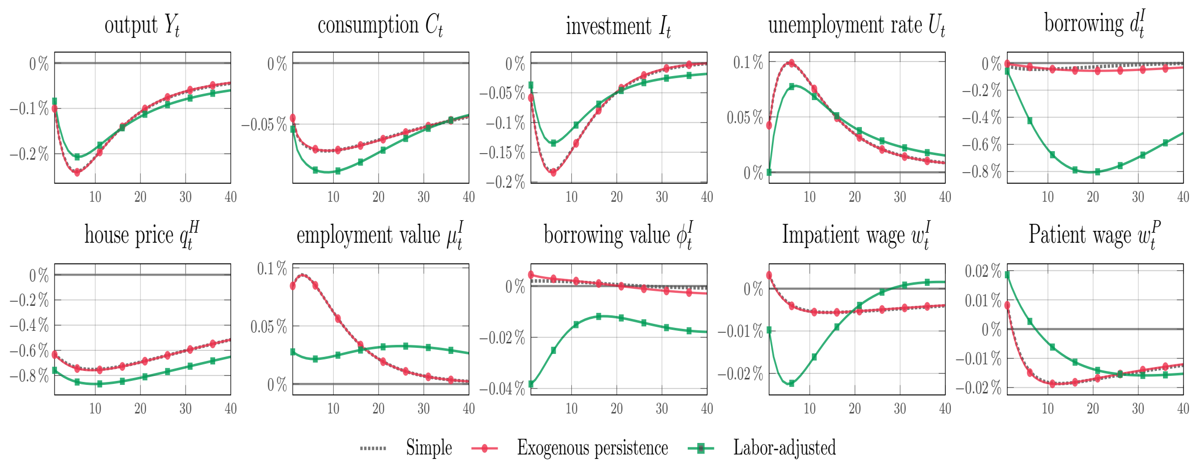
Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

[15]In the model $\mathcal{M}_3$, hiring and productivity shocks respectively drive 62% and 34% of unemployment fluctuations.

Therefore employment slowly rises as a result of the sluggishness of the matching process. Thus there are no clear difference on output, consumption, investment and housing price. Our model differs in the response of debt and unemployment compared to the two others. For the debt, in our model is four times more responsive than the others through two complementary reasons. The first reason lies in the direct easing of our collateral constraint (Eq.7) when there are more matches on the labor market. The value of borrowing rises as a response from employment, which naturally encourages impatient households to borrow more. Then impatient households buy more durable goods by contracting new mortgages, these fuels a temporary re-allocation of housing goods from patient to impatient households. The second reason is indirect and concerns how the labor market respond to housing fluctuations. Our collateral constraint also affects households labor decisions. As a result, the rise in the value of borrowing also increases the value of employment (μ_t^I) as more workers are allowed to borrow. The equilibrium wage also responds positively (Eq.29) as a result of the Nash bargaining process.

Hiring shock Fig.5 displays the response of the three models following an exogenous increase in the cost of hiring.

Figure 5 – System response to a 1% hiring cost shock η_t^L for each estimated models.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

On the labor market, there are more outflows from separation than inflows from new matching, so employment reduces. For most of real variables, all models exhibit similar responses: when hiring gets costlier, the number of matching decreases, so the workforce also reduces which drives down aggregate production, investment and consumption. However, propagation predictions across models become different on housing and borrowing aspects. For standard models, disturbance within the labor market has no implication for mortgages, as the stock of borrowing remains almost unchanged. In contrast with a labor-sensitive borrowing, there are fewer family members of the impatient who can get mortgages, so the stock of loans decreases, which further depresses house prices. A remarkable feature of our constraint is the endogenous persistence of borrowing, that is directly connected to the sluggishness of the labor market. This allows our setup to endogenously replicate the desired persistence observed in the data, which is typically overstated by standard models.

4 Discussing the role of labor frictions

Since we have shown that our model is favored by the data through the presence of an endogenous persistence of mortgages determined by the labor market, in this section, we examine how labor market frictions actually shape the propagation. We thus check how the propagation is sensitive to the bargaining power, the separation rate and the share of impatient workers.

Bargaining power. We first start examining the role of the bargaining power of households on sharing the surplus between the firm and the worker. We have not estimated the bargaining power η, but rather have calibrated it as in Mortensen and Pissarides (1994).[16] We thus examine how this parameter affects the housing market by contrasting a situation with a low bargaining power for households versus a high-bargaining power.

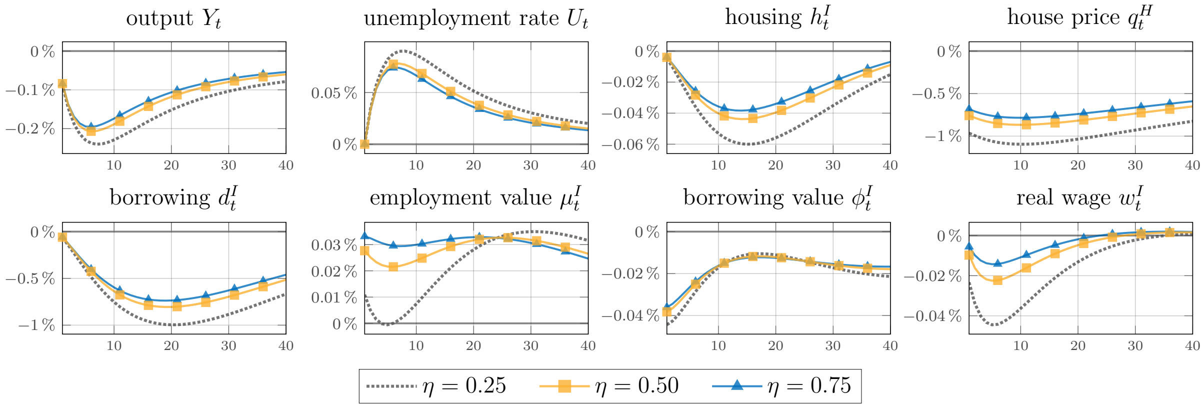
First considering $\eta = 0.75$ which is a rather widespread calibration in the labor match-

[16] Only a model featuring wage rigidities (e.g. Gertler et al. (2008)) can provide an estimation of the bargaining power, as using real wages as an observable variable provide enough information on η to identify it accurately.

ing literature (*e.g.* Christiano et al. (2016); Gertler et al. (2008)). In this case, the employment value is less dependant on the growth of debt and unemployment as described by the wage setting (Eq.29). Then, the real wage for impatient household as for the employment value is less affected than the other calibration in their fall. However compared with our calibration i.e. $\eta = 0.50$, for all the other variables they have a similar behaviour.

Turning to the case of a low negotiation power calibrated by Liu et al. (2016) which study the relationship between unemployment and land price. The impact on the wage for impatient household is more important. This seems counterintuitive since the impatient wage becomes less dependent on the growth debt. However, since the unemployment rate rises more the effect of the debt is more important and compensate the less impact of growth debt in the impatient wage.

Figure 6 – System response to a 1% hiring cost shock η_t^L under three different values for households' bargaining power η.



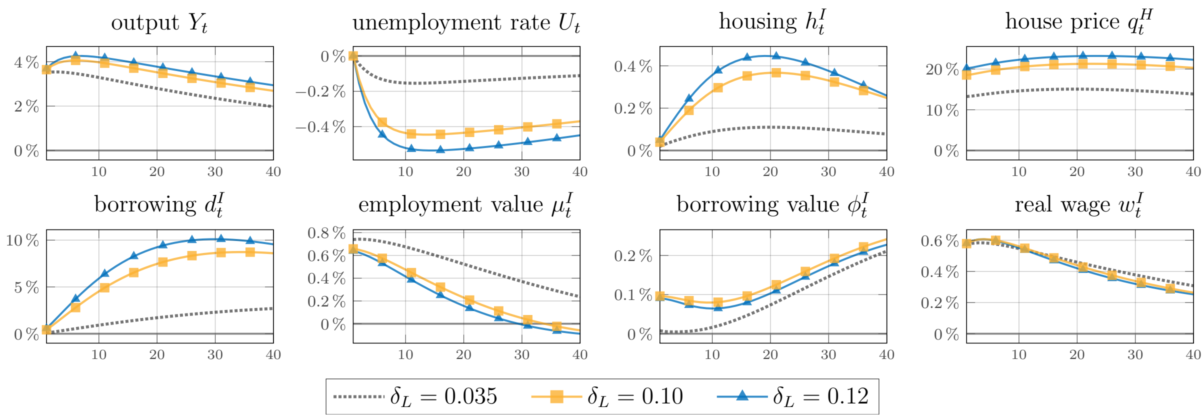
Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

Separation rate. In the literature of matching frictions in the labor market, there are two conflicting approaches for the calibration of the separation rate. The first approach, pioneered by Shimer (2005), is motivated by the microeconomic evidence to imposes a low value for the job separation rate with *i.e.* $\delta^L = 0.035$, The second approach aims at matching the macroeconomic unemployment rate by setting a higher value for this parameter, *i.e.* $\delta^L = 0.1$.[17] This calibration features a mean duration for a job in US

[17]See for instance Christiano et al. (2016).

that is two years and a half.[18] We also consider an even higher value with $\delta^L = 0.12$ as in Liu et al. (2016). Then the Fig.7 displays the response of the model to a 1% productivity shock under this three different values of δ^L.

Figure 7 – System response to a 1% productivity shock under three different values for the separation rate δ^L.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

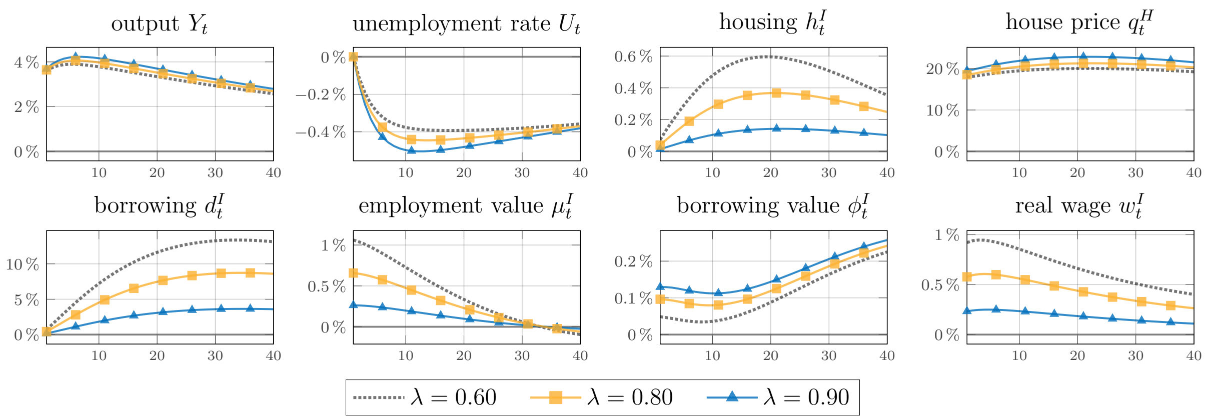
Unemployment is becoming more responsive with the increase in value of the separation rate. Recall that in the deterministic steady-state, the hiring rate is equal to the separation rate, so a higher separation rate induces mechanically a higher hiring rate. Following a productivity shock, firms have less effort to hire new workers when the hiring rate is higher. Since there are more matches when the separation rate is high, borrowing responds accordingly with more mortgage contracted by successful job seekers. This fuels the demand for housing and the price of housing clears by increasing the market value of housing. Thus the separation rate clearly shapes the dynamics of borrowing and housing prices.

The share of impatient workers. We assess how the relative number of borrowers shapes the dynamics of employment and housing. The Fig.8 displays the response of the three models after a 1% technological shock with different shares of patient households. With a large proportion of impatient households $\lambda = 0.60$, the unemployment rate is less

[18] See for example Den Haan et al. (2000), Blanchard and Galí (2010).

responsive, as already documented by Andrés et al. (2013). This result is driven here by our borrowing constraint: a higher share of borrowers mechanically induces more borrowing, as borrowing increases the value of employment, the contraction of unemployment is slightly mitigated. Since unemployment is less responsive with a higher fraction of the borrower, the equilibrium wage rises to compensate for the higher value of employment.

Figure 8 – System response to a 1% technological shock η_t^Z under three different shares of borrowers.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

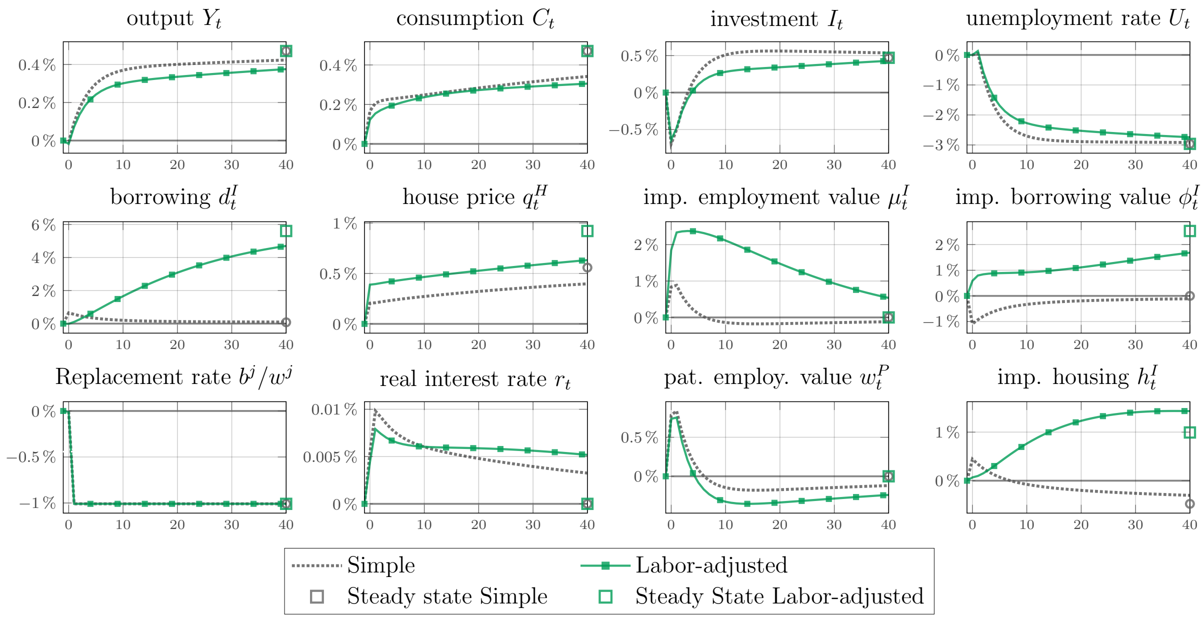
5 Policy implications

In this section, we investigate how the presence of labor in the collateral constraint affects the obtained from a labor market reform and a macroprudential policy tightening. Since our model with labor-adjusted constraint is favored by the data, then the results obtained from our models are more plausible than those obtained from the standard collateral model.

5.1 Labor market reforms

There is an extensive empirical literature on the macroeconomic effect of labor market reforms in advanced economies which typically focuses on long-term effects of labor market reforms (*e.g.* Blanchard and Wolfers (2000), Nicoletti and Scarpetta (2003), Bassanini and Duval (2009)). This literature has been completed by papers who also evaluate the dynamics effects from reforming the labor market using empirical models (*e.g.* Bouis et al. (2012), Cette et al. (2016) and Duval and Furceri (2018)) or using theoretical ones (*e.g.* Arpaia et al. (2007) and Cacciatore et al. (2016), Cacciatore and Fiori (2016)). There is a broad consensus in this literature about the high gains from reforming the labor market, but none of these papers examine (i) the role of financial frictions in the housing market on the effect of labor market reforms, (ii) the spillover effect of labor market reforms on mortgages. To fill this gap, we investigate the consequences of structural reforms when the housing market matters. Fig.9 plots the transition dynamics between two steady states characterised by a permanent decrease by 1% in the replacement rate ($^b/_w$). We also report on the right side the terminal steady state toward which the economy will converge in the long run for each model.

Figure 9 – System response to a 1% decrease in unemployment insurance b^j/w^j in both the simple and the labor-ajdusted collateral constraint models.



Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

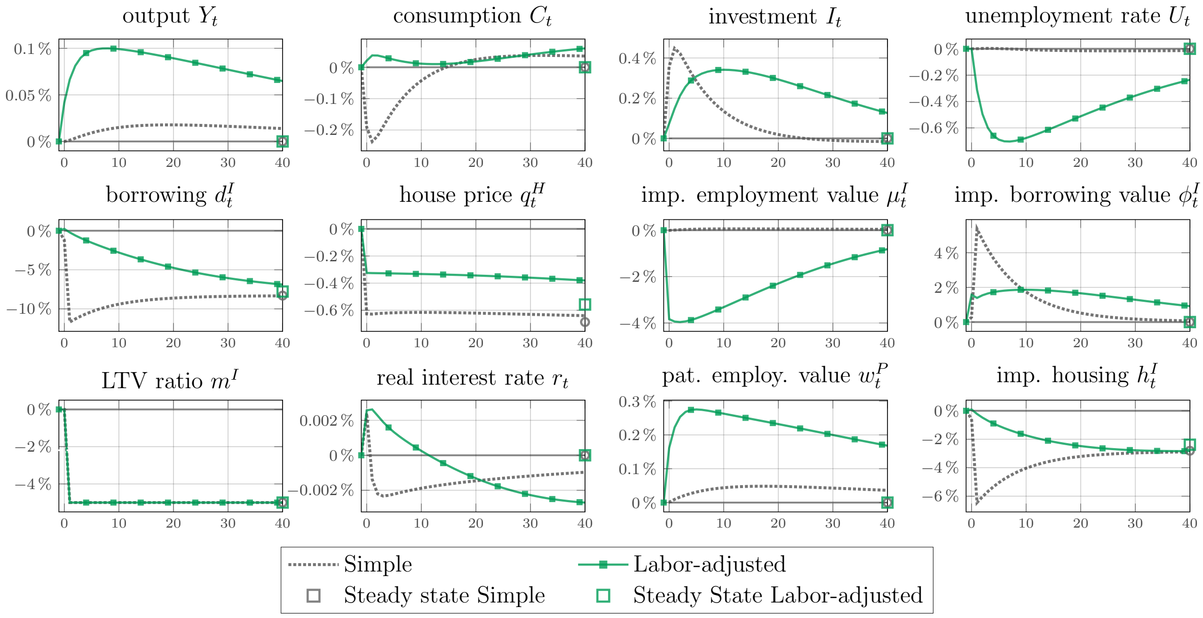
As already documented in the literature, this reform implies a permanent rise in output and employment. A reduction in employment insurance rises the relative lifetime utility from being employed rather than unemployed, so the worker's employment value μ_t^j increases in response to this structural change in economic fundamentals. As a result, workers are more willing to find a job, there are more the matches as more vacancies are filled. The equilibrium wage clears the labor market through an increase in real wages for both types of workers.

Regarding financial aspects, the reform also leaks to the housing market but in a very different fashion between our two models. Under standard collateral constraints, the effect is rather modest as house prices increase by 0.5% while borrowing exhibits a negligible response. The value of borrowing surprisingly decreases, which translates into a reduction of housing goods after the reform. As a result, patient households are the main winner following the reform. In absence of explicit demand channels, long-term changes in the unemployment rate have no important real effects on the mortgage market. Conversely under labor adjusted collateral constraints, the rise in employment allows more family members of the patient household to get mortgages from financial intermediaries, the borrowing constraint eases which materialises through an increase in the borrowing value. Since they can borrow more, the demand for housing goods rises which makes house prices increase up to 1% in the long run. Because more resources are devoted to housing, investment and consumption take more time to reach the terminal steady state. Unlike the simple collateral constraint setup, the impatient household becomes the winner from implementing the reform as the number of dwellings substantially increases by 1% in the terminal steady state.

5.2 Macroprudential policy

The model is also amenable for the analysis of macroprudential policy by changing the amount a household can borrow against his housing collateral when a mortgage is granted. Assuming that the Loan-to-Value ratio is determined by US institutional factors (*e.g.* a prudential authority), then we simulate a permanent change in the LTV ratio as in Chen and Columba (2016). Fig.10 displays the transition dynamics from permanently reducing the LTV ratio by 5% under the two models of housing.

Figure 10 – System response to a 5% LTV tightening m^I in both the simple and the labor-ajdusted collateral constraint models.



Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

Both models find similar transition dynamics in line with Chen and Columba (2016): a tightening in mortgage origination reduces house prices, borrowing and the number of houses purchased by the impatient households. Patient households re-allocate their saving toward capital goods which boosts in turn investment. However for the rest of the variables, both models have rather different predictions. Concerning borrowing, macroprudential policy has irrealistic detrimental effects in the simple model as all contracts are renegotiated each quarter. In contrast in our setup, only workers newly matched in the labor market face tighter credit conditions. As a result, borrowing slowly adjusts to these tighter credit conditions at the same speed as inflows and outflows in employment.

Recall that under a labor-adjusted collateral constraint, house prices positively affect the value of employment. Thus the decline in house prices reduces both the utility gain of being in employment and the wage for impatient households. The labor demand for impatient rises and fuels the rise in employment of impatient households. As new matches are created, our constraint implies a higher demand in housing, which partially dampens the contractionary effect of macroprudential on house prices. To summarise, our model suggests the existence of sizable leakages of macroprudential policy on the labor market. In our setup, macroprudential policy deserts the value of employment and induces a rise in labor demand.

Conclusion

In this article, we have proposed to link the borrowing capacity of households to their employment situation on the labor market. Under this setup, new matches on the labor market translate into more mortgages while separation induces an exclusion from financial markets for jobseekers. As a result, the LTV becomes endogenous by responding pro-cyclically to employment fluctuations. We have shown that this device is empirically relevant and solves the anomalies of the standard collateral constraint model. We have also shown that this constraint poses important implications for economic policy. Structural reforms in the labor market induce more borrowing in the economy, while macropruential policy tightening induces a pro-cyclical response of output and employment.

In this appendix, we describe the main features of the article with the data sources used in the estimation, the complete set of First Order Condition (FOC hereafter) and the steady-state.

A Data sources

In this appendix, we provide the data sources of all chapters.

- Nominal GDP : Gross Domestic Product, Billions of Dollars, Seasonally Adjusted Annual Rate from the FRED database https://fred.stlouisfed.org/series/GDP.

- Unemployment: Civilian Unemployment Rate, in Percent, Seasonally Adjusted from the Fred database https://fred.stlouisfed.org/series/UNRATE/.

- Vacancies: data from Barnichon (2010) which combine job openings from the JOLTS data set, the Help-Wanted Online Advertisement Index published by the Conference Board , and the Help-Wanted Print Advertising Index that was discontinued in October 2008 and it was also constructed by the Conference Board. (https://sites.google.com/site/regisbarnichon/research, Composite Help-Wanted Index)

- Job Finding Probability: we apply the methodology of Shimer (2007) by defining the probability to find a job for an unemployed worker as:

$$f_t = 1 - \frac{u_{t+1} - u^S_{t+1}}{u_t}$$

where u^S_{t+1} correspond to unemployed workers less than 5 weeks (from the Bureau of Labor Statistic, BLS hereafter) and u_{t+1} the number of unemployed people. We convert the obtained serie in a quarterly basis, (f_t in all chapters).

- Labor market tightness : Ratio of vacancies to unemployment defined below.

- Consumption: Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonnaly Adjusted Annual Rate from the FRED database https://fred.stlouisfed.org/series/PCE.

- Investment: the Fixed Private Investment, Billions of Dollars, Seasonally Adjusted Annual Rate from the FRED database, (https://fred.stlouisfed.org/series/FPI).

- House price : Census Bureau House Price Index (new one-family houses sold including value of lot).

- Loans to household : Household and nonprofit organizations; home mortgages; liability https://fred.stlouisfed.org/series/HHMSDODNS plus household and nonprofit organizations; consumer credit, liability https://fred.stlouisfed.org/series/HCCSDODNS.

Data in nominal terms are converted using the GDP deflator.

B Non-linear model

In this section, we present the model in such a way that the collateral constraint for borrowers appears in different forms i.e. our version of the collateral constraint, the exogenous component as in Iacoviello (2015) and the classic version without exogenous components.

B.1 Households

B.1.1 Impatient households

The problem faced by the borrowers can be summarised as:

$$
\begin{aligned}
&\max_{\left\{c_t^I,h_t^I,d_t^I,n_t^I\right\}} \mathbb{E}_0 \textstyle\sum_{t=0}^{\infty}(\beta^I)^t \left\{\left(1-h^C\right)\log\left(c_t^I-h^C c_{t-1}^I\right)+\varepsilon_t^H j\log\left(h_t^I\right)\right\}\\
&s.t \quad c_t^I+q_t^H\left(h_t^I-h_{t-1}^I\right)+r_{t-1}d_{t-1}^I=w_t^I n_t^I+(1-n_t^I)b_I+d_t^I\\
&s.t \quad n_t^I=\left(1-\delta^L\right)n_{t-1}^I+f_t^I\left(1-\left(1-\delta^L\right)n_{t-1}^I\right).\\
&s.t \quad d_t^I=\begin{cases} a\left\{\left(1-\delta^L\right)n_{t-1}^I d_{t-1}^I+\varepsilon_t^M f_t^I(1-\left(1-\delta^L\right)n_{t-1}^I)m^I\mathbb{E}_t\left\{q_{t+1}^H\right\}h_t^I\right\}\\ +(1-a)\left\{\chi d_{t-1}^I+(1-\chi)\,\varepsilon_t^M m^I\mathbb{E}_t\left\{q_{t+1}^H\right\}h_t^I\right\}\end{cases}
\end{aligned},
$$

using the utility function (Eq.5), the budget constraint (Eq.6) and the labor market law of motion (Eq.4). We also use a version of the collateral constraint such that if $a=0$, we have the exogenous version of the collateral constraints. Moreover, if $a=0$ and $\chi=0$ we have the original version of collateral constraint used in Kiyotaki and Moore (1997). We note by λ_t^I the lagrangian mutliplier associated with the budget constraint i.e. the marginal utility of consumption, μ_t^I the Lagrangian multiplier associated to the labor market law of motion normalised by λ_t^I and ϕ_t^I the Lagrangian multiplier associated with the collateral constraint and normalised by λ_t^I. Thus, the FOC with respect to consumption is:

$$
\lambda_t^I=\frac{\left(1-h^C\right)}{\left(c_t^I-h^C c_{t-1}^I\right)}.
$$

As in the text we note by $\Lambda_{t,t+1}^I=\beta^I\mathbb{E}_t\left(\frac{c_t^I-h^C c_{t-1}^I}{c_{I,t+1}-h^C c_t^I}\right)$ the stochastic discount factor of borrowers. The FOC for housing is:

$$
\varepsilon_t^H j\frac{(c_t^I-h^C c_{t-1}^I)}{\left(1-h^C\right)h_t^I}=q_t^H-\mathbb{E}_t\left\{\Lambda_{t,t+1}^I q_{t+1}^H\right\}-\mathbb{E}_t\left\{q_{t+1}^H\right\}\varepsilon_t^M\phi_t^I m^I\begin{cases} a f_t^I(1-\left(1-\delta^L\right)n_{t-1}^I)\\ +(1-a)\left(1-\chi\right)\end{cases}. \tag{34}
$$

For the Euler condition we have:

$$\mathbb{E}_t\left\{\Lambda^I_{t,t+1}\right\} r_t = 1-\phi^I_t + \mathbb{E}_t\left\{\Lambda^I_{t,t+1}\phi_{I,t+1}\right\}\begin{cases} a\left(1-\delta^L\right)n^I_t \\ +(1-a)\chi \end{cases}. \tag{35}$$

Finally, the FOC with respect to n^I_t is:

$$\mu^I_t = \begin{array}{c} w^I_t - b^I + \mathbb{E}_t\left\{\Lambda^I_{t,t+1}\mu^I_{t+1}\left(1-\delta^L\right)\left(1-f^I_{t+1}\right)\right\} \\ +\mathbb{E}_t a\Lambda^I_{t,t+1}\phi^I_{t+1}\left(1-\delta^L\right)\left(d^I_t - \varepsilon^M_{t+1}f^I_{t+1}m^I q^H_{t+2}h^I_{t+1}\right) \end{array}. \tag{36}$$

Since the second line depends only on our collateral constraint i.e. when $a=1$, we can rewrite this using the definition of the debt in $t+1$:

$$d^I_{t+1} = \left(1-\delta^L\right)n^I_t d^I_t + \varepsilon^M_{t+1}f^I_{t+1}(1-\left(1-\delta^L\right)n^I_t)m^I q^H_{t+2}h^I_{t+1},$$

$$\Leftrightarrow \frac{d^I_{t+1}-\left(1-\delta^L\right)n^I_t d^I_t}{\left(1-\left(1-\delta^L\right)n^I_t\right)} = \varepsilon^M_{t+1}f^I_{t+1}m^I q^H_{t+2}h^I_{t+1}.$$

Then replace it in the marginal value of being employed:

$$\mu^I_t = \begin{array}{c} w^I_t - b^I + \mathbb{E}_t\left\{\Lambda^I_{t,t+1}\mu^I_{t+1}\left(1-\delta^L\right)\left(1-f^I_{t+1}\right)\right\} \\ +\mathbb{E}_t\left\{a\Lambda^I_{t,t+1}\phi^I_{t+1}\left(1-\delta^L\right)\left(\frac{d^I_t-d^I_{t+1}}{(1-(1-\delta^L)n^I_t)}\right)\right\} \end{array}.$$

Note by $\Delta d^I_{t+1} = d^I_{t+1} - d^I_t$ and using the definition of unemployment we have the same equation as in the text (Eq.11):

$$\mu^I_t = \begin{array}{c} w^I_t - b^I + \mathbb{E}_t\left\{\Lambda^I_{t,t+1}\mu^I_{t+1}\left(1-\delta^L\right)\left(1-f^I_{t+1}\right)\right\} \\ -\mathbb{E}_t\left\{a\Lambda^I_{t,t+1}\phi^I_{t+1}\left(1-\delta^L\right)\frac{\Delta d^I_{t+1}}{u^I_{t+1}}\right\} \end{array}.$$

Then, it is more obvious than our collateral constraint acting on the choice of consumption, housing and labor since in the case of exogenous collateral constraints they only act as a persistence mechanism and not a link between markets.

B.1.2 Patient household

The problem faced by the lenders can be summarised as:

$$\max_{\left\{c_t^P, h_t^P, d_t^P, n_t^P, K_t, \upsilon_t\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{\left(1-h^C\right)\log\left(c_t^P - h^C c^P_{t-1}\right) + \varepsilon_t^H j \log\left(h_t^P\right)\right\}$$

$$\begin{aligned} s.t. \quad & c_t^P + q_t^H\left(h_t^P - h_{t-1}^P\right) + d_t^P + T_t^P + \left(\frac{K_t - (1-\delta_t^K)K_{t-1}}{\varepsilon_t^I}\right) + \frac{\phi^K}{2}\frac{(K_t - K_{t-1})^2}{K} \\ & = w_t^P n_t^P + (1-n_t^P)b^P + r_{t-1}d_{t-1}^P + z_t K_{t-1}\upsilon_t + \Pi_t \\ s.t. \quad & n_t^P = \left(1-\delta^L\right)n_{t-1}^P + f_t^P\left(1-\left(1-\delta^L\right)n_{t-1}^P\right) \\ \delta_t^K = \delta^K + & \left(\frac{1}{\beta^P} + 1 - \delta^K\right)\left(\frac{1}{2}\left(\frac{\psi}{1-\psi}\right)(\upsilon_t)^2 + \frac{(1-2\psi)}{1-\psi}\upsilon_t + \frac{1}{2}\left(\frac{\psi}{1-\psi}\right) - 1\right) \end{aligned},$$

using the budget constraint (Eq.13), the labor market law of motion (Eq.4) and the functional form of the capital depreciation rate (Eq.20). Let λ_t^P the Lagrangian multiplier associated with the budget constraint i.e. the marginal utility of consumption and μ_t^P the Lagrangian multiplier associated to the labor market law of motion normalised by λ_t^P. The FOC for consumption is:

$$\lambda_t^P = \frac{\left(1-h^C\right)}{\left(c_t^P - h^C c_{t-1}^P\right)}.$$

As in the text, the stochastic discount factor for lenders is denoted by $\Lambda_{t,t+1}^P = \beta^P \mathbb{E}_t\left(\frac{c_t^P - h^C c_{t-1}^P}{c_{P,t+1} - h^C c_t^P}\right)$. Then the FOC for housing is given by:

$$\varepsilon_t^H j \frac{(c_t^P - h^C c_{t-1}^P)}{\left(1-h^C\right)h_t^P} = q_t^H - \mathbb{E}_t\left\{\Lambda_{t,t+1}^P q_{t+1}^H\right\}.$$

The Euler condition is obtained with the FOC for deposits:

$$\mathbb{E}_t\left\{\Lambda_{t,t+1}^P\right\} r_t = 1. \tag{37}$$

For labor, we have:

$$\mu_t^P = w_t^P - b^P + \mathbb{E}_t\left\{\mu_{t+1}^P \Lambda_{t,t+1}^P \left(1-\delta^L\right)\left(1-f_{t+1}^P\right)\right\}. \tag{38}$$

Concerning capital services, the FOC with respect to K_t and υ_t are respectively:

$$\frac{1}{\varepsilon_t^I} + \phi^K \frac{(K_t - K_{t-1})}{K} = \mathbb{E}_t \left\{ \Lambda_{t,t+1}^P \left(\frac{(1-\delta_{t+1}^K)}{\varepsilon_{t+1}^I} + z_{t+1}\upsilon_{t+1} + \phi^K \frac{(K_{t+1} - K_t)}{K} \right) \right\}, \quad (39)$$

$$z_t = \left(\frac{1}{\beta^P} + 1 - \delta^K \right) \left(\left(\frac{\psi}{1-\psi} \right) \upsilon_t + \frac{(1-2\psi)}{1-\psi} \right). \quad (40)$$

B.2 Firms

The representative firm maximise their dividends subject to the labor market law of motion with both types of household:

$$\begin{cases} \max_{\left\{v_t^j, n_t^j, K_t^U\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{ Y_t - z_t K_t^U - \sum_{j=P,I} \left(w_t^j n_t^j + \varepsilon_t^L \frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{n_{t-1}^j} \right)^2 n_{t-1}^j \right) \right\} \\ s.t \quad n_t^j = \left(1-\delta^L\right) n_{t-1}^j + q_t^j v_t^j \end{cases},$$

We start with the labor force provides by patient household i.e. with $j = P$. Let $x_t^j = \frac{q_t^j v_t^j}{n_{t-1}^j}$ be the hiring rate, the FOC for posting a vacancy is:

$$\mu_{P,t}^L = \varepsilon_t^L \kappa^P x_t^P, \quad (41)$$

and the FOC with respect to n_t^P:

$$\mu_{P,t}^L = \frac{(1-\alpha)\lambda Y_t}{n_t^P} - w_t^P + \mathbb{E}_t \left\{ \Lambda_{t,t+1}^P \left(\varepsilon_{t+1}^L \frac{\kappa^P}{2} \left(x_{t+1}^P\right)^2 + \left(1-\delta^L\right) \mu_{P,t+1}^L \right) \right\}. \quad (42)$$

By combining the two FOCs we have the same job creation as in the text (Eq.24). For the impatient household i.e. with $j = I$, the FOC for posting a vacancy is:

$$\mu_{I,t}^L = \varepsilon_t^L \kappa^I x_t^I \quad (43)$$

and for n_t^I :

$$\mu_{I,t}^{L}=\frac{(1-\alpha)(1-\lambda)Y_{t}}{n_{t}^{I}}-w_{t}^{I}+\mathbb{E}_{t}\left\{\Lambda_{t,t+1}^{P}\left(\varepsilon_{t+1}^{L}\frac{\kappa^{I}}{2}\left(x_{t+1}^{I}\right)^{2}+\left(1-\delta^{L}\right)\mu_{I,t+1}^{L}\right)\right\}. \tag{44}$$

As for lenders, combines the two FOC, we get the job creation as in the text (Eq.25). Turning to the choice of capital, the FOC with respect to K_t^U is :

$$z_{t}=\alpha\frac{Y_{t}}{K_{t}^{U}}. \tag{45}$$

B.3 Wage setting

The period-by-period Nash bargaining implies that the firm and each of its workers determine the wage in period t by solving the following problem:

$$\arg\max_{\{w_t^j\}}\left(\mu_t^j\right)^{\eta}\left(\mu_{j,t}^{L}\right)^{1-\eta},$$

for $j=\{I,P\}$ and $\eta\in[0,1]$ the power of negotiation for the worker. Whatever the type of worker, the solution of this problem gives:

$$\eta\mu_{j,t}^{L}=(1-\eta)\,\mu_t^j.$$

For the patient household, we use the marginal value of a new match from the firm's perspective i.e. $\mu_{P,t}^L$ (Eq.42) and from the household's perspective i.e. μ_t^P (Eq.38). Replace it and rearrange to make appear w_t^P:

$$w_{t}^{P}=\begin{array}{c}\eta\left(\frac{(1-\alpha)\lambda Y_{t}}{n_{t}^{P}}+\mathbb{E}_{t}\left\{\Lambda_{t,t+1}^{P}\left[\varepsilon_{t+1}^{L}\frac{\kappa^{P}}{2}\left(x_{t+1}^{P}\right)^{2}+\left(1-\delta^{L}\right)\mu_{P,t+1}^{L}\right]\right\}\right)\\+(1-\eta)\left(b^{P}-\mathbb{E}_{t}\left\{\Lambda_{t,t+1}^{P}\mu_{t+1}^{P}\left(1-\delta^{L}\right)\left(1-f_{t+1}^{P}\right)\right\}\right)\end{array}.$$

Using the fact that the Nash Bargaining hold in $t+1$ we can replace μ_{t+1}^P by $\mu_{t+1}^P=\frac{\eta}{(1-\eta)}\mu_{P,t+1}^L$.

$$w_{t}^{P}=\begin{array}{c}\eta\left(\frac{(1-\alpha)\lambda Y_{t}}{n_{t}^{P}}+\varepsilon_{t+1}^{L}\mathbb{E}_{t}\Lambda_{t,t+1}^{P}\frac{\kappa^{P}}{2}\left(x_{t+1}^{P}\right)^{2}\right)+(1-\eta)\,b^{P}\\+\eta\left(1-\delta^{L}\right)\Lambda_{t,t+1}^{P}\mu_{P,t+1}^{L}f_{t+1}^{P}\end{array}.$$

and using the FOC with respect to vacancies i.e.$\mu^L_{P,t} = \varepsilon^L_t \kappa^P \frac{q^P_t v^P_t}{n^P_{t-1}}$ (Eq.41) and the definition of the hiring rate, we have the same equation as in the text :

$$w^P_t = \begin{array}{c} \eta\left(\frac{(1-\alpha)\lambda Y_t}{n^P_t} + \mathbb{E}_t\left\{\varepsilon^L_{t+1}\Lambda^P_{t,t+1}\frac{\kappa^P}{2}\left(x^P_{t+1}\right)^2\right\}\right) + (1-\eta)\, b^P \\ +\mathbb{E}_t\left\{\eta\left(1-\delta^L\right)\Lambda^P_{t,t+1}\varepsilon^L_{t+1}\kappa^P x^P_{t+1} f^P_{t+1}\right\} \end{array}.$$

Concerning the impatient household, we follow the same step to have the definition of wages. Using the definition of μ^I_t in Eq.36 and $\mu^L_{I,t}$ in Eq.44, we have:

$$w^I_t = \begin{array}{c} \eta\left(\frac{(1-\alpha)(1-\lambda)Y_t}{n^I_t} + \mathbb{E}_t\left\{\varepsilon^L_{t+1}\Lambda^P_{t,t+1}\frac{\kappa^I}{2}\left(x^I_{t+1}\right)^2 + \Lambda^P_{t,t+1}\left(1-\delta^L\right)\mu^L_{I,t+1}\right\}\right) \\ +(1-\eta)\left(b^I - \mathbb{E}_t\left\{\Lambda^I_{t,t+1t}\mu^I_{t+1}\left(1-\delta^L\right)\left(1-f^I_{t+1}\right)\right\}\right) \\ +(1-\eta)\,\mathbb{E}_t\left\{a\Lambda^I_{t,t+1}\phi^I_{t+1}\left(1-\delta^L\right)\frac{\Delta d^I_{t+1}}{u^I_{t+1}}\right\} \end{array}.$$

Using the $t+1$ relationship, we can replace μ^I_{t+1} by $\mu^I_{t+1} = \frac{\eta}{(1-\eta)}\mu^L_{I,t+1}$:

$$w^I_t = \begin{array}{c} \eta\left(\frac{(1-\alpha)(1-\lambda)Y_t}{n^I_t} + \mathbb{E}_t\left\{\varepsilon^L_{t+1}\Lambda^P_{t,t+1}\frac{\kappa^I}{2}\left(x^I_{t+1}\right)^2\right\}\right) + (1-\eta)\, b^I \\ +\mathbb{E}_t\left\{\eta\mu^L_{I,t+1}\left(1-\delta^L\right)\left(\Lambda^P_{t,t+1} - \Lambda^I_{t,t+1}\left(1-f^I_{t+1}\right)\right)\right\} \\ +(1-\eta)\,\mathbb{E}_t a\Lambda^I_{t,t+1}\phi^I_{t+1}\left(1-\delta^L\right)\frac{\Delta d^I_{t+1}}{u^I_{t+1}} \end{array}.$$

Using $\mu^L_{I,t} = \varepsilon^L_t\kappa^I\frac{q^I_t v^I_t}{n^I_{t-1}}$ we obtain the same equation as in the text:

$$w^I_t = \begin{array}{c} \eta\left(\frac{(1-\alpha)(1-\lambda)Y_t}{n^I_t} + \mathbb{E}_t\left\{\varepsilon^L_{t+1}\Lambda^P_{t,t+1}\frac{\kappa^I}{2}\left(x^I_{t+1}\right)^2\right\}\right) + (1-\eta)\, b^I \\ +\mathbb{E}_t\left\{\eta\varepsilon^L_{t+1}\kappa^I x^I_{t+1}\left(1-\delta^L\right)\left(\Lambda^P_{t,t+1} - \Lambda^I_{t,t+1}\left(1-f^I_{t+1}\right)\right)\right\} \\ +(1-\eta)\,\mathbb{E}_t\left\{a\Lambda^I_{t,t+1}\phi^I_{t+1}\left(1-\delta^L\right)\frac{\Delta d^I_{t+1}}{u^I_{t+1}}\right\} \end{array}.$$

B.4 General Equilibrium

The total stock of housing is fixed and normalised to one such that $h^P_t + h^I_t = 1$, deposits equals loans i.e. $d^I_t = d^P_t$ and the taxes collected by the government is used to cover the transfer to unemployed people and public spending i.e. $T^P_t = g^Y\varepsilon^G_t + \left(1-n^I_t\right)b^I + \left(1-n^P_t\right)b^P$. Using these three relationship, we can obtain the same GDP aggregate as in the text.

We start with the budget constraint of patient household:

$$\begin{aligned} &c_t^P + q_t^H \Delta h_t^P + d_t^P + T_t^P + \left(\tfrac{K_t-(1-\delta_t^K)K_{t-1}}{\varepsilon_t^I}\right) + \tfrac{\phi^K}{2}\tfrac{(K_t-K_{t-1})^2}{K} \\ &= w_t^P n_t^P + (1-n_t^P)b^P + r_{t-1}d_{t-1}^P + z_t K_{t-1}\upsilon_t + \Pi_t \end{aligned} .$$

Using the definition of dividends $\Pi_t = Y_t - z_t K_t^U - \sum_{j=P,I}\left(w_t^j n_t^j + \varepsilon_t^L \frac{\kappa_j}{2}\left(\frac{q_t^j v_t^j}{n_{t-1}^j}\right)^2 n_{t-1}^j\right)$ and the definition of taxes we have:

$$\begin{aligned} &c_t^P + q_t^H \Delta h_t^P + d_t^P + g^Y \varepsilon_t^G + \left(1-n_t^I\right) b^I + \left(\tfrac{K_t-(1-\delta_t^K)K_{t-1}}{\varepsilon_t^I}\right) + \tfrac{\phi^K}{2}\tfrac{(K_t-K_{t-1})^2}{K} \\ &= r_{t-1}d_{t-1}^P + Y_t - w_t^I n_t^I - \textstyle\sum_{j=P,I}\left(\varepsilon_t^L \frac{\kappa^j}{2}\left(\frac{q_t^j v_t^j}{n_{t-1}^j}\right)^2 n_{t-1}^j\right) \end{aligned} .$$

Turning to the budget constraint of impatient household and using the relationship between deposits and loans :

$$d_t^P - r_{t-1}d_{t-1}^P = c_t^I + q_t^H \Delta h_t^I - w_t^I n_t^I - (1-n_t^I)b^I.$$

We can use this equation into the previous :

$$\begin{aligned} &c_t^P + c_t^I + q_t^H \Delta h_t^I + q_t^H \Delta h_t^P + g^Y \varepsilon_t^G + \left(\tfrac{K_t-(1-\delta_t^K)K_{t-1}}{\varepsilon_t^I}\right) + \tfrac{\phi^K}{2}\tfrac{(K_t-K_{t-1})^2}{K} \\ &= Y_t - \textstyle\sum_{j=P,I} \frac{\kappa^j}{2}\varepsilon_t^L \left(x_t^j\right)^2 \left(n_{t-1}^j\right)^{-1} \end{aligned} .$$

Now using the fact that $h_t^P + h_t^I = 1$, the definition of total consumption $C_t = c_t^P + c_t^I$ and the definition of investment : $I_t = \frac{K_t-(1-\delta_t^K)K_{t-1}}{\varepsilon_t^I}$ we obtain the same expression:

$$Y_t = C_t + g^Y \varepsilon_t^G + I_t + \Phi_K\left(\Delta K_t\right) + \sum_{j=P,I} \frac{\kappa^j}{2}\varepsilon_t^L \left(x_t^j\right)^2 \left(n_{t-1}^j\right)^{-1}.$$

C Steady-State

We distinguish between two steady states. First, there is the estimated steady state that results from the set of estimated parameters that determines the long run value of endogenous variables. Some estimated parameters pins down structural parameters, such as preference parameters, the cost of hiring, etc. However, these parameters are expected not to adjust following a structural reform. We thus define a second steady state, referred to as the policy steady state, which is affecting the key ratios of the economy without affecting preferences and technology parameters.

C.1 Estimated Steady-state

We start the steady-state by the labor market. Since we have set the exogenous rate of destruction (δ^L) and the finding rate (f) for both types, using the law of motion from the household's perspective (Eq.4), we get the steady-state value of labor:

$$n^j = \frac{f}{(\delta^L + f\,(1-\delta^L))}.$$

Then we have the same level of employment for each type of household $n^P = n^I$ and by extension the total labor used in the production function : $N = \left(n^P\right)^{\lambda}\left(n^I\right)^{1-\lambda}$. With the law of motion from the firm's perspective (Eq.3) and the probability to fill a vacancy (q) we have $v = \frac{n\delta^L}{q}$ and $x = \frac{qv}{n}$. The matching efficiency parameter is determined using the definition of the finding rate ($f = \frac{\psi(v)^{\varsigma}\left(1-\left(1-\delta^L\right)N\right)^{1-\varsigma}}{(1-(1-\delta^L)N)}$) such that $\psi = \frac{f}{(v)^{\varsigma}(1-(1-\delta^L)N)^{-\varsigma}}$. After that we can obtain the total amount of capital and by extension the total output. Recall that the utilisation rate is equal to one at the steady-state ($\upsilon = 1$ and $K^U = K$), using the FOC with respect to capital for household (Eq.39) and for firms (Eq.45) and the definition of output we get:

$$K = \left(\frac{\left(1-\beta^P\left(1-\delta^K\right)\right)}{\beta^P\alpha N^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}.$$

Thus, we have $Y = K^{\alpha}N^{1-\alpha}$, $z = \alpha\frac{Y}{K}$ and $I = \delta^K K$.

After that, we can have the steady-state value of wages (w^j) and the cost to hiring (κ^j) for both types using the job creation condition and the wage setting. Start with the patient household using the definition of the replacement rate: $\tau_P^B = \frac{b^P}{w^P}$ and rearrange:

$$\begin{cases} \kappa^P \left(x - \beta^P \frac{1}{2}(x)^2 - \beta^P x \left(1-\delta^L\right)\right) = \frac{(1-\alpha)\lambda Y}{n^P} - w^P \\ w^P = \frac{\eta}{\left(1-(1-\eta)\tau_P^B\right)} \left[\frac{(1-\alpha)\lambda Y}{n^P} + \kappa^P \left(\beta^P \frac{1}{2}(x)^2 + \left(1-\delta^L\right)\beta^P x f\right)\right] \end{cases},$$

using $\phi_1 = x - \beta^P \frac{1}{2}(x)^2 - \beta^P x \left(1-\delta^L\right)$ and $\phi_2 = \left(\beta^P \frac{1}{2}(x)^2 + \left(1-\delta^L\right)\beta^P x f\right) \frac{\eta}{\left(1-(1-\eta)\tau_P^B\right)}$ and put the first equation into the second we have the SS of the hiring cost:

$$\kappa^P = \frac{(1-\alpha)\lambda Y}{(\phi_1+\phi_2)\, n^P}\left(1 - \frac{\eta}{\left(1-(1-\eta)\,\tau_P^B\right)}\right),$$

and by extension the SS of the wage.

For impatient household, we proceed in the same way by using the Job creation condition and the wage setting:

$$\begin{cases} \kappa^I x = \frac{(1-\alpha)(1-\lambda)Y}{n^I} - w + \beta^P \frac{\kappa^I}{2}(x)^2 + \beta^P \kappa^I x \left(1-\delta^L\right) \\ \qquad\qquad \eta\left(\frac{(1-\alpha)(1-\lambda)Y}{n^I} + \beta^P \frac{\kappa^I}{2}(x)^2\right) + (1-\eta)\, b^I \\ w^I = \qquad +\eta\kappa^I x \left(1-\delta^L\right)\left(\beta^P - \beta^I(1-f)\right) \\ \qquad\qquad +(1-\eta)\, a\beta^I \phi^I \left(1-\delta^L\right)\frac{\Delta d^I}{u^I} \end{cases}.$$

Note that whatever our endogenous collateral constraint is at work i.e. if $a=1$, the last line of the equation for wage disappears since $\Delta d^I = 0$. Then, using $\phi_3 = x - \beta^P \frac{1}{2}(x)^2 - \beta^P x_, \left(1-\delta^L\right)$ and $\phi_4 = \left(\beta^P \frac{1}{2}(x)^2 + x\left(1-\delta^L\right)\left(\beta^P - \beta^I(1-f)\right)\right)\frac{\eta}{\left(1-(1-\eta)\tau_I^B\right)}$ we get the steady-state value of hiring costs and wages for the impatient household:

$$\begin{cases} \kappa^I = \frac{(1-\alpha)(1-\lambda)Y}{n(\phi_3+\phi_4)}\left(1 - \frac{\eta}{\left(1-(1-\eta)\tau_I^B\right)}\right) \\ w^I = \frac{\eta}{\left(1-(1-\eta)\tau_I^B\right)}\frac{(1-\alpha)(1-\lambda)Y}{n} + \kappa^I \phi_4 \end{cases}.$$

As we have seen, the collateral constraint choice does not affect the steady-state of labor, capital and product market. However, they can have an impact on two important ratios namely the ratio housing stock to annualised GDP i.e. $g_H = \frac{q^H\left(h^P+h^I\right)}{4Y}$ and debt to gdp i.e. $g^D = \frac{d^I}{Y}$.

Using the Euler condition for patient and impatient household (respectively Eq.37 and

Eq.35), we deduce the steady-state value of the interest rate $r = \frac{1}{\beta^P}$ and the Lagrangian multiplier associated to the borrowing constraint :

$$\phi_I = \frac{\left(1 - \beta^I r\right)}{\left(1 - \beta^I \left(a \left(1 - \delta^L\right) n^I + (1 - a)\chi\right)\right)}.$$

To obtain the level of consumption for impatient household, we start with their budget constraint:

$$c^I = w^I n^I + \left(1 - n^I\right) b^I + (1 - r)\, d^I.$$

Using the steady-state value of d^I:

$$d^I = m^I q^H h^I \left(\frac{af(1 - \left(1 - \delta^L\right) n^I) + (1 - a)\,(1 - \chi)}{\left(1 - a \left(1 - \delta^L\right) n^I - (1 - a)\chi\right)} \right),$$

and the steady-state value of the housing price from the housing demand for impatient household (Eq.34):

$$q^H = j \frac{c^I}{h^I \left(1 - \beta^I - \phi^I m^I \left(af(1 - \left(1 - \delta^L\right) n^I) + (1 - a)\,(1 - \chi)\right)\right)}.$$

Note by $\phi^5 = \left(\frac{j(1-r)m^I\left(af(1-\left(1-\delta^L\right)n^I)+(1-a)(1-\chi)\right)}{(1-\beta^I-\phi^I m^I(af(1-(1-\delta^L)n^I)+(1-a)(1-\chi)))(1-a(1-\delta^L)n^I-(1-a)\chi)} \right)$ and replace the definition of debt and housing price in the budget constraint we have:

$$c^I = w^I n^I + \left(1 - n^I\right) b^I + c^I \phi^5.$$

Then, $c^I = \frac{w^I n^I + \left(1 - n^I\right) b^I}{(1 - \phi^5)}$. The consumption of patient household is obtained with his budget constraint

$$c^P = \left(1 - g^Y\right) Y - c^I - \frac{\kappa_I}{2} (x)^2 \left(n^I\right)^{-1} - \frac{\kappa^P}{2} (x)^2 \left(n^P\right)^{-1}.$$

To obtain the steady-state value of h^I we use the relationship between the two stocks of housing : $h^P = 1 - h^I$, the value of housing price and the optimal housing demand for lenders:

$$h^I = \frac{1}{1 + \frac{c^P(1-\beta^I-\phi^I m^I(af(1-(1-\delta^L)n^I)+(1-a)(1-\chi)))}{(1-\beta^P)c^I}}.$$

Then we have h^P, q^H and d^I which closed the steady-state.

C.2 Policy steady-state

To introduce the policy steady-state, we take the value of the hiring cost κ^I and κ^P and the parameter for the matching efficiency ψ from the estimated steady-state and relax the calibration of the finding rate for both type of households. We note by $\left(\tau_j^B\right)^*$ and $\left(m^I\right)^*$ respectively the implementation of the labor market reform and the macroprudential reform. For the lack of simplicity, we only consider the case of the collateral constraint with labor market flows i.e. $a = 1$ in this appendix.

We know the hiring rate for both type of households since by definition it is equal to the exogenous destruction rate i.e. $x^j = \delta^L$, the SS for the interest rate $r = \frac{1}{\beta^P}$ and the SS for the utilisation rate normalised to one $\upsilon = 1$. Using the FOC with respect to capital for household (Eq.39) and for firms (Eq.45) and the definition of output we get the SS value of capital depending on the level of labor :

$$K = \left(\frac{\left(1-\beta^P\left(1-\delta^K\right)\right)}{\beta^P\alpha}\right)^{\frac{1}{\alpha-1}} N,$$

we note by $\phi^K = \left(\frac{\left(1-\beta^P\left(1-\delta^K\right)\right)}{\beta^P\alpha}\right)^{\frac{1}{\alpha-1}}$ to have $K = \phi^K N$ and by extension the level of output becomes : $Y = N^{-\alpha}\phi^K$ with $N = \left(n^P\right)^\lambda \left(n^I\right)^{1-\lambda}$.

We start with the job creation condition for both type of households and we use $x^j = \delta^L$ and the previous expression for output:

$$\begin{cases} \kappa^I\delta^L = (1-\alpha)(1-\lambda)\left(n^P\right)^{-\alpha\lambda}\left(n^I\right)^{-\alpha(1-\lambda)-1}\phi^K - w^I + \kappa^I\beta^P\left(\frac{1}{2}\left(\delta^L\right)^2 + \left(1-\delta^L\right)\delta^L\right) \\ \kappa^P\delta^L = (1-\alpha)\lambda\left(n^P\right)^{-\alpha\lambda-1}\left(n^I\right)^{-\alpha(1-\lambda)}\phi^K - w^P + \kappa^P\beta^P\left(\frac{1}{2}\left(\delta^L\right)^2 + \left(1-\delta^L\right)\delta^L\right) \end{cases}.$$

We rearrange these two expressions with the parameters that we know and note by $\phi^{I1} =$

$\kappa^I\left(\delta^L-\beta^P\left(\frac{1}{2}\left(\delta^L\right)^2+\left(1-\delta^L\right)\delta^L\right)\right)$ and $\phi^{P1}=\left(\delta^L-\beta^P\left(\frac{1}{2}\left(\delta^L\right)^2+\left(1-\delta^L\right)\delta^L\right)\right)$. Then we have:

$$\Leftrightarrow\begin{cases}\phi^{I1}=(1-\alpha)(1-\lambda)\left(n^P\right)^{-\alpha\lambda}\left(n^I\right)^{-\alpha(1-\lambda)-1}\phi^K-w^I\\ \phi^{P1}=(1-\alpha)\lambda\left(n^P\right)^{-\alpha\lambda-1}\left(n^I\right)^{-\alpha(1-\lambda)}\phi^K-w^P\end{cases}. \tag{46}$$

Now, we work with the expression of wages for both types of household:

$$\Leftrightarrow\begin{cases}w^I= & \eta\left(\frac{(1-\alpha)(1-\lambda)Y}{n^I}+\beta^P\frac{\kappa^I}{2}\left(\delta^L\right)^2\right)+(1-\eta)\left(\tau_I^B\right)^*w^I\\ & +\eta\kappa^I\delta^L\left(1-\delta^L\right)\left(\beta^P-\beta^I\left(1-f^I\right)\right)\\ w^P= & \eta\left(\frac{(1-\alpha)\lambda Y}{n^P}+\beta^P\frac{\kappa^P}{2}\left(\delta^L\right)^2\right)+(1-\eta)\left(\tau_P^B\right)^*w^P\\ & +\eta\left(1-\delta^L\right)\beta^P\kappa^P\delta^Lf^P\end{cases}.$$

Rearrange these expressions :

$$\Leftrightarrow\begin{cases}w^I= & \frac{\eta}{\left(1-(1-\eta)\left(\tau_I^B\right)^*\right)}\left(\frac{(1-\alpha)(1-\lambda)Y}{n^I}-\kappa^I\delta^L\left(1-\delta^L\right)\beta^I\left(1-f^I\right)\right)\\ & +\frac{\eta\kappa^I\delta^L\left(1-\delta^L\right)\beta^P+\eta\beta^P\frac{\kappa^I}{2}\left(\delta^L\right)^2}{\left(1-(1-\eta)\left(\tau_I^B\right)^*\right)}\\ w^P= & \frac{\eta}{\left(1-(1-\eta)\left(\tau_P^B\right)^*\right)}\left(\frac{(1-\alpha)\lambda Y}{n^P}+\left(1-\delta^L\right)\beta^P\kappa^P\delta^Lf^P\right)\\ & +\frac{\beta^P\frac{\kappa^P}{2}\left(\delta^L\right)^2}{\left(1-(1-\eta)\left(\tau_P^B\right)^*\right)}\end{cases}.$$

Note by $\phi^{I2}=\frac{\eta\kappa^I\delta^L\left(1-\delta^L\right)\beta^P+\eta\beta^P\frac{\kappa^I}{2}\left(\delta^L\right)^2}{\left(1-(1-\eta)\left(\tau_I^B\right)^*\right)}$, $\phi^{I3}=\frac{\eta}{\left(1-(1-\eta)\left(\tau_I^B\right)^*\right)}$, $\phi^{P2}=\frac{\beta^P\frac{\kappa^P}{2}\left(\delta^L\right)^2}{\left(1-(1-\eta)\left(\tau_P^B\right)^*\right)}$ and $\phi^{P3}=\frac{\eta}{\left(1-(1-\eta)\left(\tau_P^B\right)^*\right)}$, then we have:

$$\Leftrightarrow\begin{cases}w^I= & \phi^{I3}\left(\frac{(1-\alpha)(1-\lambda)Y}{n^I}-\kappa^I\delta^L\left(1-\delta^L\right)\beta^I\left(1-f^I\right)\right)+\phi^{I2}\\ w^P= & \phi^{P3}\left(\frac{(1-\alpha)\lambda Y}{n^P}+\left(1-\delta^L\right)\beta^P\kappa^P\delta^Lf^P\right)+\phi^{P2}\end{cases}.$$

Integrating these expressions into JC conditions (46), using $f^j=\frac{\delta^Ll^j}{(1-(1-\delta^L)l^j)}$ from labor market law of motion and rearrange with the level of output, we have:

$$\Leftrightarrow \begin{cases} \phi^{I1} = \left(1-\phi^{I3}\right)(1-\alpha)(1-\lambda)\left(n^P\right)^{-\alpha\lambda}\left(n^I\right)^{-\alpha(1-\lambda)-1}\phi^K \\ +\phi^{I3}\kappa^I\delta^L\left(1-\delta^L\right)\beta^I\left(1-\frac{\delta^L n^I}{(1-(1-\delta^L)n^I)}\right)-\phi^{I2} \\ \phi^{P1} = \left(1-\phi^{P3}\right)(1-\alpha)\lambda\left(n^P\right)^{-\alpha\lambda-1}\left(n^I\right)^{-\alpha(1-\lambda)}\phi^K \\ -\phi^{P3}\left(1-\delta^L\right)\beta^P\kappa^P\delta^L\frac{\delta^L n^P}{(1-(1-\delta^L)n^P)}-\phi^{P2} \end{cases}.$$

Since we have a system of non-linear system of equation, we use a solver[19] to find the exact value of n^P and n^I from these two equations. The labor for patient and impatient are approximately equal. For example, without implementing the labor market reform we have: $n^P = 0.958904$ and $n^I = 0.958908$ implying a difference of $-4*10^{-6}$ between them. In the same way, after the reform we find the same order of difference.

Then, we have the value of the finding rate for both type of households, the unemployment rate, the level of capital (and by extension the investment) and output. To obtain the SS value of the total vacancy posted, we used the definition of the matching function and the labor market law of motion :

$$v^j = \left(\frac{\delta^L n^j}{\psi\left(1-\left(1-\delta^L\right)\right)^{1-\zeta}}\right)^{\frac{1}{\zeta}},$$

and thus the filling rate $q^j = \frac{n^j\delta^L}{v^j}$.

Thus, we can have the new SS for the borrowing value of impatient household ϕ^I:

$$\phi_I = \frac{\left(1-\beta^I r\right)}{\left(1-\beta^I\left(1-\delta^L\right)n^I\right)}.$$

To obtain the level of consumption for impatient household we start with their budget constraint:

$$c^I = w^I n^I + \left(1-n^I\right)b^I + (1-r)\,d^I.$$

Using the steady-state value of d^I:

[19] We use the solver optimset from Matlab with 100000 iterations.

$$d^I = \left(m^I\right)^* q^H h^I f^I,$$

and the steady-state value of the housing price from the housing demand for impatient household (Eq.34):

$$q^H = j \frac{c^I}{h^I \left(1 - \beta^I - \phi^I \left(m^I\right)^* \left(f^I(1 - \left(1 - \delta^L\right) n^I\right)\right))}.$$

Note by $\phi^5 = \left(\frac{j(1-r)\left(m^I\right)^* \left(f(1-\left(1-\delta^L\right)n^I)\right)}{\left(1-\beta^I-\phi^I(m^I)^* f^I(1-(1-\delta^L)l^I\right)(1-(1-\delta^L)n^I)} \right)$ and replace the definition of debt and housing price in the budget constraint we have:

$$c^I = w^I n^I + \left(1 - n^I\right) b^I + c^I \phi^5.$$

Then, $c^I = \frac{w^I n^I + \left(1-n^I\right) b^I}{(1-\phi^5)}$. The consumption of patient household is obtained with his budget constraint,

$$c^P = \left(1 - g^Y\right) Y - c^I - \frac{\kappa_I}{2} (x)^2 \left(n^I\right)^{-1} - \frac{\kappa^P}{2} (x)^2 \left(n^P\right)^{-1}.$$

To obtain the steady-state value of h^I we use the relationship between the two stocks of housing : $h^P = 1 - h^I$, the value of housing price and the optimal housing demand for lenders:

$$h^I = \frac{1}{1 + \frac{c^P\left(1-\beta^I-\phi^I(m^I)^*(f(1-(1-\delta^L)n^I))\right)}{(1-\beta^P)c^I}}.$$

Then we get h^P, q^H and d^I which closed the steady-state.

Thus, a labor market deregulation conduces to a higher level of output, employment (lower wages), debt, housing price, borrowing value and a reallocation effect in the housing market between patient and impatient households (more housing for impatient).

A tightening macroprudential policy affects negatively the housing price, the level of debt in the economy and a reallocation in the housing market between patient and impatient households (more housing for patient).

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