Inefficiencies in Globalized Economies with Labor Market Frictions

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Abstract

We study an open-economy model with labor market search frictions that affect both the intensive and the extensive labor margins. We identify two intertwined inefficiencies in the decentralized allocation: The trade externality (TE) and labor market frictions (LMF). While LMF create inefficiently low output, TE generates inefficiently high output and terms of trade. As a result, protectionism succeeds in eliminating the TE, but lowers employment. In a Ramsey problem, the government faces a trade-off: While TE calls for a higher overall tax wedge, LMF at the extensive labor margin calls for a lower tax burden.

Keywords: Labor market search, extensive margin of labor, intensive margin of labor, open economy, inefficiency wedge, trade externality, trade policy, employment subsidy, tax wedge

JEL classification: E27, E62, H21, J38

1 Introduction

Since 2018, the United States has imposed import tariffs ranging from 10 to 50 percent on goods including solar panels, washing machines, steel and aluminium.[1] This new tariff policy

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[1]See Bowden & Kolb (2019) for an overview of the tariff policy.

is part of Trump's "America First" economic policy based on the objective to negotiate "*fair, bilateral trade deals that bring jobs and industry back onto American shores*". President Trump's statement stresses the link between tariff policy and concerns on labor market outcomes. Our paper echoes this issue by studying the relationship between trade and labor market adjustments.

To what extent can an economy with a comparative advantage in a given sector take advantage of this specialization pattern? What are the consequences once the general equilibrium effects are taken into account, in particular regarding employment performances in a context of labor search frictions? Is it achievable through a trade protectionist policy, as advocated by the Trump's proponents? What if the government uses the tax wedge components as fiscal tool instead? Those are the key questions we tackle in the paper.

We address these questions on theoretical grounds. In line with Costinot et al. (2015), we study the optimal trade policy in a model with a Ricardian pattern of trade. In contrast to them, we explicitly take into account how international trade and the associated trade policy interact with labor market outcomes in a general equilibrium framework. To this aim, we incorporate search and matching frictions on the labor market where agents decide upon both the number of jobs and the hours per worker. Modeling both labor margins indeed proves to be key in characterizing the effects of international trade on the labor market performances. We deliberately keep the framework simple enough to derive all results analytically, for the purpose of decomposing economic mechanisms in a transparent way. In particular, we adopt a static search model along the lines of Hungerbuhler et al. (2006) (closed economy) or Helpman & Itskhoki (2010) (open economy). This framework thus offers a convenient starting point to explore the long-run consequences of inefficiencies in open economies featured with labor market frictions.

In this set-up, we characterize the decentralized equilibrium. We also solve for a fictitious planning problem in which the government directly controls consumption, labor and output decisions. This allows us to derive sharp predictions about the structure of the optimal allocation. In particular, we characterize the wedges causing the decentralized allocation to differ from the centralized economy. We identify two inefficiency wedges: The trade externality, as private domestic agents do not take into account the impact of their decisions on international prices and traded quantities; labor market inefficiency, as labor market institutions (i.e., unemployment benefits and the union's bargaining power) are not adequately internalized by the agents in presence of search and matching labor market frictions. The comparison between the decentralized economy and the centralized allocation leads us to establish a first inefficiency result.

Our analysis thus points out the complex interactions between trade externality and labor market frictions, which is at the root of the inefficiency of the decentralized equilibrium. In particular, both dimensions exert opposite pressures on total employment and output. The trade externality pushes towards working and producing too much, along both labor margins. By lowering the price of home goods, this makes foreign imports inefficiently expensive. By contrast, labor market institutions tend to generate inefficiently low output by hampering job creation, which workers compensate by working longer hours when employed. Provided the extensive labor margin effect dominates, Home output is inefficiently low at an excessively high Home price, thereby pushing the price of imports below the first best level. This inefficiency result calls for a careful policy design when trying to correct the effects of international trade on labor market, as (i) the desirable direction of changes for each labor margin might not be the same, (ii) the extensive and the intensive margins might not respond in the same direction to policy reforms.

This has strong policy implications, which can be summarized in two policy results. First, neither a trade protectionist policy (designed to tackle the trade externality) nor an employment subsidy policy (targeted towards the labor market frictions impact on the extensive labor margin) can succeed in reaching the first-best allocation. More importantly, each policy in isolation succeeds in reducing its targeted inefficiency gap, but it has detrimental effects on the other wedge. The introduction of import tariff alone thus succeeds in shifting imports but actually deteriorates the employment target, by pushing job creation even further down. Symmetrically, hiring subsidies achieve higher employment, but, in doing so, excessively boost output. The price of home goods drops further away from its optimal level, which makes foreign imports even more expensive. Only a package combining import taxes and employment subsidies can bring the economy closer to the optimal allocation for employment, worked hours and terms of trade.

Second, when the government can only use indirect consumption and direct labor taxes, the second-best Ramsey tax wedge is a compromise between both inefficiencies. On the one hand, in order to reduce trade externality leading to excessive terms of trade, the government wants to *increase* taxation to lower output and boost the price of home goods, thereby lowering imports and bringing the terms of trade closer to its optimum value. On the other hand, as labor market frictions lower employment, the government needs to *reduce* taxation to boost inefficiently low job creation, thereby increasing output and terms of trade. The Ramsey tax scheme is an illustration of the opposite effects of trade externality and labor market frictions on output.

To sum up, we show that two key dimensions of most contemporaneous economies, the trade specialization pattern and labor market frictions, are at the root of the inefficiency of the decentralized equilibrium. Using one policy tool is not sufficient to bring the economy closer to the first-best allocation. This result stands in line with the Tinbergen rule that requires to match the number of policy tools with the number of policy goals. The originality of our work lies in analyzing the economic mechanisms behind the policy implications. In particular, we stress that trade externality and labor market frictions have opposite effects on output. As a result, Trump's trade policy, in isolation, might actually destroy American jobs, whereas hiring subsidy such as the 2010 US Hiring Incentives to Restore Employment (HIRE) Act, in isolation, might actually make foreign imports much more expensive than their first-best level. Our results call for cautiousness when discussing trade and labor policies, as none of these two policies can be considered as "good" or "bad" per se.

Given the simple framework, one might wonder about the generality of our results. Some assumptions are made for tractability: the separability between consumption and leisure in the utility function, the import of foreign goods in the final demand, the absence of physical capital, the zero-trade balance. However, the key economic mechanisms rely on the presence of search and matching frictions and a foreign trading partner with finite elastic demand, which suggests that our results are relevant for economies with these two characteristics and will remain in more general frameworks.

The paper is organized as follows. We present the related literature in section 2 and the analytical framework in section 3. The macroeconomic impact of trade externality and labor marker is analyzed in section 4. Policy implications are explored in sections 5 and 6. Section 7 concludes.

2 Related literature

If the paper relates to several strands of the literature, its originality lies in proposing to encompass the labor market along both the intensive and extensive margins, with the trade externality. Further, we analyse the policy design not only from a positive perspective but also in normative terms, as we carefully study the effects of the policies in light of the inefficiency gap with respect to the first best. To our knowledge, existing works do not look at the strong interaction between all these dimensions.

The paper relates to the labor literature that explores the two margins of labor adjustment. Quantifying the relative contribution of each element in the understanding of total hours is a debated issue in the literature. The main takeaway from existing works is that both margins of adjustment matter in the understanding of aggregate hours in the economy whether in the long run (Ohanian & Raffo, 2012, Van Rens, 2012, Blundell et al., 2013, Langot & Pizzo, 2019) or along the business cycle (Kudo et al., 2019, Rogerson & Shimer, 2011). While existing works focus on closed-economy models, we extend the literature by taking into account the open-economy setting. We relate in particular to Fang & Rogerson (2009) who investigate the adjustment along the extensive and intensive margins in a search and matching framework, in a long-run perspective. They stress how policy reforms can trigger similar or opposite response along both labor margins. In particular, tax and transfer policies lead to decreases along both margins, whereas regulations that increase the cost of creating or maintaining a job may lead to decreases in employment, but necessarily lead to increases in hours per worker. We extend their result by looking at the interaction between open economy setting and labor market frictions. Our result on the Ramsey policy echoes their finding that lower taxes can boost employment and worked hours. However, unlike their closed-economy setting, this requirement for lower taxes is not necessarily the best reform in an open economy with large trade externality: Higher taxation might be desirable to reduce output and make imports less expensive.

The paper also relates to the search and matching literature in open economy. Existing papers study the business cycle implications of these models using an open economy setting (Gadatsch et al., 2016, Cacciatore et al., 2016a, Cacciatore et al., 2016b, to name a few). With respect to this literature, our paper looks at a long-run perspective, rather than business cycles, and explores the interaction between trade externality and labor market frictions. In addition, we lay stress on the tension between the extensive and intensive labor margins, which is left aside in the papers mentioned above. Last, one originality of our work is to study whether the fiscal tool (and which one) can be effective in reducing inefficiency sustainably, leaving aside the question of structural reforms, e.g. labor market reforms (as the series of papers by Cacciatore et al., 2016b).

The paper also takes part to the vast literature on the macroeconomic impact of protectionism or its opposite, trade liberalization. We can first relate our work to Barattieri et al. (2018), which study the consequences of protectionism for macroeconomic fluctuations.

They develop a small open economy model with firm heterogeneity, endogenous selection into trade, and nominal rigidity to study the channels through which protectionism affects aggregate fluctuations. We complement their work by (i) adopting a long-run perspective, rather than a short-run focus, (ii) considering labor market frictions. In doing so, the policy implications differ from theirs. In particular, in Barattieri et al. (2018)'s paper, as tariff lower output, protectionism appears as detrimental to the economy. Our paper suggests that, when considering both the trade externality and labor market frictions, with intensive and extensive labor margins, a decline in output can actually be desirable in the economy where the trade externality dominates, in order to allow home firms to sell their product at higher prices. Secondly, we relate to Costinot et al. (2015) who study optimal trade policy in a canonical Ricardian model of trade. They find non-zero import tariff as part of an optimal trade policy. If we subscribe to the point of view that protectionism is not detrimental to the economy *per se*, we shed light on the labor market conditions required for this claim to hold. Specifically, it can only be the case in open economies that are specialized enough for the trade externality to dominate. In economies featured with high structural unemployment, trade taxes conversely push employment even further down. In this respect, import tariff can be part of an optimal policy if *combined* with labor tax policy.

Third, our paper can be connected to the growing literature that studies the relation between trade liberalization and unemployment (Helpman & Itskhoki, 2010, Dutt et al., 2009 or Helpman et al., 2010 among others, following the pioneering work of Davidson et al., 1999). In Dutt et al. (2009), when trade is solely driven by Ricardian comparative advantage, then trade liberalization results in a reduction in unemployment. In an incomplete specialization framework, things are less clear-cut as it depends on relative factor endowments (as shown by Dutt et al. (2009) in a HOS framework) or the relative burden of labor market frictions across sectors and countries (Helpman & Itskhoki, 2010). In line with Dutt et al. (2009) in a Ricardian setting, we find that a trade liberalization (protectionist) policy reduces (raises) unemployment by pushing up (down) job creation. Our contribution to this literature is to point out the conditions under which reducing employment through trade taxes might be optimal. In this respect, our results complement the more nuanced picture reached by Helpman & Itskhoki (2010), according to which trade liberalization might foster unemployment. We stress the importance of considering endogenous adjustments at the intensive labor margin to get a complete picture on labor market outcomes, a dimension which is typically left aside in the papers mentioned above.

3 Reforming an open economy: An analytical characterization

We develop an analytical model of a small open economy with labor-market search and matching frictions. Analytical tractability is ensured by retaining some simplifying assumptions. First, we abstract from dynamic aspects. This drives us to exclude capital accumulation and international bond trading, implying the zero-trade balance condition to hold. In this respect, we adopt a long-term view of the tax scheme analysis. For the sake of tractability, we leave aside the question of an endogenous job-search effort from the unemployed. Labor-market frictions, thus, are modeled by adopting the static-matching setting of Hungerbuhler et al. (2006) or Helpman & Itskhoki (2010). While they only model the extensive labor margin (the number of employees), we extend this setting by incorporating the intensive margin of labor (hours worked per employee). Our results demonstrate the importance of incorporating both margins for the optimal policy design. In this static framework, we characterize how optimal reform packages can be implemented in order to (at least partly) correct for inefficiencies. We start by describing the model's main assumptions. For sake of conciseness, we then report the main steps of the model's solving in the decentralized and the centralized cases (the detailed solving is reported in the online Appendix, available on the authors' webpages).

3.1 Model assumptions

Matching frictions on the labor market. Each firm opens a vacancy that can be filled by a searching worker. The cost of posting one vacancy is $\overline{\omega} > 0$. Hirings occur according to a constant returns-to-scale matching function, $M = \chi V^{\psi} U^{1-\psi}$ with $0 < \psi < 1$, where V is the total number of new jobs made available by firms, U is the number of searching workers, and $\chi > 0$ is a scale parameter measuring the efficiency of the matching function. The job-finding rate p, defined by $p \equiv \frac{M}{U} = \chi \left(\frac{V}{U}\right)^{\psi}$, is a function of labor-market tightness $\frac{V}{U}$. The vacancy filling rate q is given by $q \equiv \frac{M}{V} = \chi \left(\frac{V}{U}\right)^{\psi-1}$. The size of the population is normalized to 1. At the beginning of the period, all workers are looking for a job, that is, $U = 1$, implying $M = N = p$. Hence, the matching process in the economy is summarized as follows.

$$N = \chi V^{\psi} \tag{1}$$

The open economy dimension. We model an open economy, which trades goods with the rest of the world (also referred to as the foreign country). Consistent with the Ricardian model of trade, the home country is completely specialized in the production of a homogenous good (Y), implicitly assuming a given comparative advantage pattern *vis-$\tilde{A}$ -vis* the rest of the world. The home good is consumed both domestically (in quantity C_H) and exported abroad (in quantity X) against foreign imports, in quantity C_F. In addition, we normalize prices by considering the domestic good as numéraire. The relative price of the foreign good $\phi \equiv P_F/P_H$ is interpreted as the home terms of trade.

Given the small-open economy setting, we take the import behavior of the rest of the world as given. Precisely, we assume the following functional form for the import function, denoted Z^* (expressed in terms of volume of the Home good):

$$Z^* = \phi^{\sigma^*} \tag{2}$$

with $\sigma^* > 1$ the price elasticity of foreign imports.[2] Notice that home exports X necessarily constitute the volume of imports of the rest of the world Z^*. Further, in the absence of the international trading of financial assets, the home country (as well as the rest of the world) is featured by a zero trade balance, such that $X = \phi C_F$. Given Equation (2), this defines the following expression for domestic imports:

$$\begin{aligned} X = \phi C_F \quad &\Leftrightarrow \quad \phi C_F = \phi^{\sigma^*} \\ &\Leftrightarrow \quad C_F = \phi^{\sigma^*-1} \end{aligned} \tag{3}$$

Preferences. Due to labor-market matching frictions, a fraction N of the labor force is employed, with h denoting the hours worked per employee, while unemployed agents $(1-N)$ spend their time enjoying leisure. Each worker derives utility from leisure and consumption, the consumption bundle made of domestic goods (with index H) and foreign goods (with index F), with respective weights in the expenditure functions ξ and $1-\xi$ $(0 < \xi < 1)$. Denoting $\mathcal{U}_x$ the utility level of agent x, with $x = \{e, u\}$ for employed and unemployed respectively, we retain the following specifications:

[2] We derive the micro-foundations of such trade flows in a two-country model in Appendix A.1. We thank Jean-Pascal Bénassy for helpful input on the functional forms.

$$
\begin{cases}
\mathcal{U}_e = \underbrace{\dfrac{C_{H,e}^{\xi} C_{F,e}^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}}}_{=C_e} - \sigma_L \frac{h^{1+\eta}}{1+\eta} & \text{if employed} \\
\mathcal{U}_u = \underbrace{\dfrac{C_{H,u}^{\xi} C_{F,u}^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}}}_{=C_u} & \text{if unemployed}
\end{cases} \tag{4}
$$

with the inverse of the Frisch labor supply elasticity $\eta > 0$ and $\sigma_L > 0$ a scale parameter of labor disutility.

Technology. Each occupied job yields production using a decreasing production function Ah^{α} with $0 < \alpha < 1$ and h denoting the number of hours worked by an individual. As a result, at the aggregate level, with N the number of workers (i.e., of firms), the aggregate output Y is given as follows.[3]

$$
Y = ANh^{\alpha}, \qquad 0 < \alpha < 1 \tag{5}
$$

3.2 Decentralized Economy

3.2.1 Agents' program

Firms. Firms are in perfect competition in the production of domestic goods. They are subject to direct labor taxation, with τ_f denoting the payroll tax rate. We also introduce a subsidy to job creation c, given for each created jobs. Firms freely enter the goods market as long as the return on the vacancy posting exceeds its cost.

$$
\frac{\overline{\omega}}{q} - c = (Ah^{\alpha} - (1+\tau_f)wh) \;\; \Rightarrow \;\; \frac{\overline{\omega}}{\chi} V^{1-\psi} - c = Ah^{\alpha} - (1+\tau_f)wh \tag{6}
$$

Notice that this condition can also be interpreted as the zero-profit condition, with profits given by $\pi = [Ah^{\alpha} - (1+\tau_f)wh + c]\, N - \overline{\omega} V$ with $N = \chi V^{\psi}$.[4]

[3]Notice that the aggregate production function exhibits increasing returns to scale. However, this does not jeopardize our assumption of perfect competition on the goods market, as each firm decides on the basis of the production function $y = Ah^{\alpha}$.

[4]Note that, even with a linear production function in N, the share of wages in the GDP wNh/Y is smaller than 1 in the presence of a non-zero vacancy cost, as detailed in the online Appendix.

Workers. Worker (employed or unemployed) maximizes her utility function (4) with respect to $C_{H,z}, C_{F,z}$ for $z = e, u$, subject to her budget constraint:

$$\left\{\begin{array}{rcll} (1+\tau_c)\overbrace{[C_{H,e} + (1+\tau_e)\phi C_{F,e}]}^{PC_e} & = & (1-\tau_w)wh + \pi + T & \text{if employed} \\ (1+\tau_c)\underbrace{[C_{H,u} + (1+\tau_e)\phi C_{F,u}]}_{PC_u} & = & (1-\tau_w)\widetilde{b} + \pi + T & \text{if unemployed} \end{array}\right. \tag{7}$$

with $\widetilde{b}$ the net unemployment benefits,[5] and T the lump-sum transfers. Labor revenues are taxed at the employee tax rate τ_w, while consumption expenditures are subject to indirect taxation, with τ_c the indirect tax rate. Further, we allow for trade policy as imports can be taxed at rate $\tau_e > 0$. In Equation (7), P denotes the aggregate price index (in terms of numéraire).

The first-order conditions relative to the consumption of home and foreign goods lead to the following arbitrage condition, for $z = e, u$:

$$\frac{\mathcal{U}'_{C_{F,z}}}{\mathcal{U}'_{C_{H,z}}} = (1+\tau_e)\phi \;\Leftrightarrow\; \frac{1-\xi}{\xi}\frac{C_{H,z}}{C_{F,z}} = (1+\tau_e)\phi, \tag{8}$$

which shows that the sharing rule between domestic and foreign consumption is simply driven by the terms of trade, and identical whatever the agents' employment status. Further, it can be shown that the optimal shares of domestic and foreign consumptions are equal to, for $z = e, u$:

$$C_{H,z} = \xi PC_z \tag{9}$$

$$(1+\tau_e)\phi C_{F,z} = (1-\xi)PC_z \tag{10}$$

$$P = [(1+\tau_e)\phi]^{1-\xi}, \tag{11}$$

As can be inferred from the agents' budget constraints (7), the gap between the aggregate

[5]For simplicity, we make the distinction between "net" unemployment benefits $\tilde{b}$, that is, net of social contributions, perceived by the household, and "gross" unemployment benefits, that is, including social security contributions b, both linked through $b = (1+\tau_f)\widetilde{b}$. If we do not make this assumption, a distortion is introduced in the taxation of work w versus non-work b. Discussing the impact of this distortion is beyond the focus of this paper, while it substantially complicates the model.

consumption bundles between employed and unemployed is equal to:

$$C_e - C_u = \frac{1-\tau_w}{1+\tau_c}\left(\frac{wh-\widetilde{b}}{P}\right).$$

Nash bargaining. We assume that wages and hours worked are determined via generalized Nash bargaining as solutions of the following problem:

$$\max_{w,h} \quad \Omega = \mathcal{V}_e^{1-\epsilon}\mathcal{V}_f^{\epsilon} \tag{12}$$

with the workers' bargaining power $0 < 1-\epsilon < 1$, $\mathcal{V}_e$ and $\mathcal{V}_f$ the marginal values of a match for a worker and a firm respectively (expressed in monetary terms). In the static setting, $\mathcal{V}_e$ and $\mathcal{V}_f$ can be written as:

$$\begin{aligned}
\mathcal{V}_e &= (1-\tau_w)(wh-\widetilde{b}) - P(1+\tau_c)\sigma_L\frac{h^{1+\eta}}{1+\eta} \\
\mathcal{V}_f &= Ah^{\alpha} - (1+\tau_f)wh + c
\end{aligned}$$

Making use of this in the surplus expression (12), solving the bargaining problem leads to the following negotiated values for w and h:

$$wh = \frac{1-\epsilon}{1+\tau_f}\left(Ah^{\alpha}+c\right) + \frac{\epsilon}{1-\tau_w}\left[(1-\tau_w)\widetilde{b} + (1+\tau_c)\sigma_L\frac{h^{1+\eta}}{1+\eta}P\right] \tag{13}$$

$$\sigma_L h^{1+\eta}P = \frac{1-\tau_w}{(1+\tau_c)(1+\tau_f)}\alpha Ah^{\alpha} \tag{14}$$

The negotiated wage is a weighted average of the worker's outside option and marginal product of a match, with the relative weights depending on the relative bargaining power of both players, distorted by the tax rates (Equation (13)). The negotiated number of hours worked equalizes the marginal product of hours with the disutility of work, given the tax scheme.

Government In this static framework, the government budget constraint is necessarily balanced:

$$(1-\tau_w)\widetilde{b}(1-N) + cN = \tau_c\left[C_H + (1+\tau_e)\phi C_F\right] + \tau_e\phi C_F + (\tau_w+\tau_f)wNh + T, \tag{15}$$

where $C_H = NC_{H,e} + (1-N)C_{H,u}$ and $C_F = NC_{F,e} + (1-N)C_{F,u}$ represent the total domestic consumption of Home and Foreign (imported) goods respectively, and T denotes lump-sum taxes taken as exogenous. We assume that net unemployment benefits are proportional to the wage bill, that is, $\widetilde{b} = \rho_b wh$, with $0 \leq \rho_b < 1$; for analytical tractability reasons, we also assume a similar pattern for the employment subsidy ratio: $c = \rho_c(1+\tau_f)wh$, with $0 \leq \rho_c < 1$.

3.2.2 Market equilibria

Given Equation (2), the home-goods equilibrium condition $Y = C_H + Z^* + \bar{\omega}V$ and the zero-trade balance equation $Z^* = \phi C_F$ can be rewritten as:

$$C_H = ANh^\alpha - \phi^{\sigma^*} - \overline{\omega}V, \tag{16}$$

$$C_F = \phi^{\sigma^*-1} \tag{17}$$

3.2.3 Solving the model

In this section, we report the main results of the model's solving. Combining Equations (16), (17) along with the sharing rule (8) allows us to deduce the terms of trade as a function of net output, according to:

$$\phi = \left(\frac{1-\xi}{1+\tau_e}(1+t^e)(Y-\overline{\omega}V)\right)^{\frac{1}{\sigma^*}} \tag{18}$$

with

$$1+t^e = \frac{1}{\xi + \frac{1-\xi}{1+\tau_e}} \tag{19}$$

increasing with the tariff τ^e. From this, we can also express the consumptions of the Home and Foreign goods in function of net output:

$$C_H = \xi(1+t^e)(Y-\overline{\omega}V) \tag{20}$$

$$(1+\tau_e)\phi C_F = (1-\xi)(1+t^e)(Y-\overline{\omega}V) \tag{21}$$

Making use of this, as well as the bargained wage solution (13) and zero-profit condition (6), leads to comprehensive analytical solution of the hours worked, from which the whole set of equilibrium values for the endogenous variables can be derived:

$$h^{dec} = \left[\frac{A\alpha}{\sigma_L} \left[\frac{1}{1+\tau_e} \right]^{1-\xi} \frac{1}{TW} \left(\frac{1}{\frac{1-\xi}{1+\tau_e}(1+t^e)\Theta} \right)^{\frac{(1-\xi)}{\sigma^*}} \right]^{\nu} \tag{22}$$

with the subscript ^{dec} referring to the decentralized allocation, TW the tax wedge defined as

$$TW \equiv \frac{(1+\tau_c)(1+\tau_f)}{1-\tau_w} \tag{23}$$

$\nu \equiv \frac{1-\psi}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}}$ and Θ a combination of structural and policy parameters according to:

$$\Theta = \left(\frac{\chi A}{1+\eta} \right)^{\frac{1}{1-\psi}} \left(\frac{\epsilon}{\bar{\omega}} \frac{(1+\eta)(1-\rho_b)-\alpha(1-\rho_c)}{1-\epsilon\rho_b-\rho_c(1-\epsilon)} \right)^{\frac{\psi}{1-\psi}} (1-\rho_c) \frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\epsilon\rho_b-\rho_c(1-\epsilon)} \tag{24}$$

Ensuring a positive number of hours worked leads us to impose a sufficient condition on Θ, such that $(1+\eta)(1-\rho_b)-\alpha(1-\rho_c) > 0$, i.e.:[6]

$$\begin{aligned} & \rho_b < 1 - \frac{\alpha(1-\rho_c)}{1+\eta} \\ \Leftrightarrow \quad & \rho_b < \overline{\rho} + \frac{\alpha\rho_c}{1+\eta}, \quad \text{with} \quad \overline{\rho} \equiv 1 - \frac{\alpha}{1+\eta} < 1 \end{aligned}$$

This condition imposes an upper bound on the unemployment benefit ratio strictly below 1. From now on, we will assume this condition to be fulfilled.

From the equilibrium value of hours worked (22), the rest of the model can be solved recursively. Vacant jobs can be expressed as a function of hours worked according to:

$$V = \left[\frac{\epsilon\chi}{\bar{\omega}} \left(\frac{1+\eta-\alpha}{1+\eta} Ah^{\alpha} + c - b \right) \right]^{\frac{1}{1-\psi}}$$

Given the endogenous values of c and b at the decentralized equilibrium, vacancies can be

[6] This condition should be completed by the denominator being positive as well, i.e.: $1-\rho_b\epsilon-\rho_c(1-\epsilon) > 0$, which rewrites as: $\rho_b < \frac{1-\rho_c(1-\epsilon)}{\epsilon}$. Since $0 < \epsilon < 1$ and $0 \leq \rho_c < 1$, the term $\frac{1-\rho_c(1-\epsilon)}{\epsilon}$ lies within the range $[1; 1/\epsilon]$. Given the definition of $\rho_b < 1$, then the condition $\rho_b < \frac{1-\rho_c(1-\epsilon)}{\epsilon}$ is always fulfilled. Hence the positivity condition on hours worked resumes to imposing $(1+\eta)(1-\rho_b)-\alpha(1-\rho_c) > 0$. See the online Appendix for more details.

rewritten as a function of hours worked. Applying a similar reasoning for net output and the terms of trade, we get:

$$
\begin{aligned}
V &= \Theta^{\frac{1}{\psi}} \left[\frac{A\chi}{1+\eta}\right]^{-\frac{1}{\psi}} \left[(1-\rho_c)\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\rho_b\epsilon-\rho_c(1-\epsilon)}\right]^{-\frac{1}{\psi}} h^{\frac{\alpha}{1-\psi}} && (25)\\
Y-\bar{\omega}V &= \Theta h^{\frac{\alpha}{1-\psi}} && (26)\\
\phi &= \left(\frac{1-\xi}{1+\tau_e}(1+t_e)\Theta\right)^{\frac{1}{\sigma^*}} h^{\frac{\alpha}{\sigma^*(1-\psi)}} && (27)
\end{aligned}
$$

Combining this with the rest of the model's equations, we can solve for the equilibrium values of the whole set of variables (as detailed in the Online Appendix).

3.3 Centralized Economy

The program of the social planner is to maximize the utility function subject to the set of resource constraints of the small-open economy. The utility function in the planner's objective can be written as:[7]

$$
\mathcal{U}^{sp} = \underbrace{\frac{C_H^{\xi} C_F^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}}}_{C} - N\sigma_L \frac{h^{1+\eta}}{1+\eta} \tag{28}
$$

As goods are imperfect substitutes at the international level, the planner of the home country can compute a "fictitious" allocation by acting as a monopoly vis-à-vis the foreign country. Unlike the decentralized economy, the home planner uses information regarding the finite price elasticity of foreign demand for the domestic good (σ^*) to extract a positive markup. In this respect, we adopt a similar model of the allocation of the centralized small open economy to those in related trade papers (see Costinot et al. (2015)).

The program of the social planner is to maximize the utility function (28) with respect to C_H, C_F, h, and V, subject to the set of resource constraints of the small-open economy. Precisely, using the production function (5) and the matching function (1) as well as the

[7] When allocating consumption to maximize utility, the planner does not make any difference regarding the employment status of the workers, as long as this does not translate in different preferences. In this respect, the planner considers the utility of the "aggregate" family, in which all workers both employed and unemployed, pool their revenues. In other words, the reasoning can be held discarding the employment status, i.e. such that $C_{H,e} = C_{H,u} = C_H$ and $C_{F,e} = C_{F,u} = C_F$.

import function (2), the resource constraint on the domestic goods and trade-balance equilibrium condition that the planner takes into account are still given by Equations (16) and (17), respectively. Accordingly, the planner' problem is equivalent to choosing $\{\phi, h, V\}$ so as to maximize:

$$\max_{\phi,V,h} \mathcal{U}^{sp} = \max \left\{ \frac{(Y(h,V)-Z^*(\phi)-\varpi V)^{\xi}(X^*(\phi))^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} - N(V)\sigma_L \frac{h^{1+\eta}}{1+\eta} \right\}$$

The first-order conditions with respect to ϕ, h and V are respectively:

$$\mathcal{U}^{sp\prime}_{\phi} = 0 \quad \Leftrightarrow \quad \frac{U'_{C_F}}{U'_{C_H}} = \frac{\epsilon_{Z^*/\phi}}{\epsilon_{X^*/\phi}} \frac{Z^*}{X^*} \tag{29}$$

$$\mathcal{U}^{sp\prime}_{h} = 0 \quad \Leftrightarrow \quad -\frac{U'_{C_H}}{U'_h} = Y'_h \tag{30}$$

$$\mathcal{U}^{sp\prime}_{V} = 0 \quad \Leftrightarrow \quad U'_{C_H}\left[Y'_V - \varpi\right] = N'_V \sigma_L \frac{h^{1+\eta}}{1+\eta} \tag{31}$$

with $\epsilon_{Z^*/\phi}$ the elasticity of foreign imports (i.e., home exports $X = Z^*$) and $\epsilon_{X^*/\phi}$ the elasticity of foreign exports (i.e., home imports) with respect to the relative price of Foreign goods ϕ, and

$$\mu^* \equiv \frac{\sigma^*}{\sigma^* - 1} > 1 \tag{32}$$

Equation (29) determines the optimal arbitrage between home and foreign goods. The social planner, in choosing the terms of trade, acts as a monopoly who is able to take into account the impact of her price setting on the relative demand for goods coming from abroad. By doing so, she extracts a part of the surplus of the foreign agents, whose magnitude is scaled by the foreign demand price elasticity. Using our functional forms, Equation (29) rewrites as:

$$\frac{1-\xi}{\xi} \frac{C_H}{C_F} = \mu^* \phi, \tag{33}$$

indicating that the markup is equal to μ^*, decreasing with the price elasticity of foreign demand σ^*. The first-order conditions with respect to consumption can be rewritten as:

$$C_H = \xi(1+t^*)\left(Y - \varpi V\right), \tag{34}$$

$$\mu^* \phi C_F = (1-\xi)(1+t^*)\left(Y - \varpi V\right), \tag{35}$$

with

$$1 + t^* \equiv \frac{1}{\xi + (1-\xi)/\mu^*} \tag{36}$$

which we interpret as a measure of the terms-of-trade externality, increasing with μ^*. All else equal, the markup μ^* reduces the share of foreign goods in the total basket, which is $(1-\xi)/\mu^*$ for the planner, compared to $(1-\xi)$ at the decentralized equilibrium (Equations (35) vs. (21), respectively). For the labor-market aggregates, the planner's allocation is summarized by the two first-order conditions (30)) and (31). Equation (30) equalizes the marginal rate of substitution between hours and consumption of the home good to the marginal product of labor, while Equation (31) determines the optimal value of job vacancies.

Using a similar reasoning as in the decentralized case, one can solve the planner's problem to obtain the equilibrium value for hours worked:

$$h^{sp} = \left[\frac{\alpha A}{\sigma_L} \left(\frac{1}{\mu^*} \right)^{1-\xi} \left(\frac{1}{\frac{1-\xi}{\mu^*}(1+t^*)\Psi} \right)^{\frac{1-\xi}{\sigma^*}} \right]^{\nu} \tag{37}$$

with ν similarly defined as in the decentralized case and Ψ a combination of structural and policy parameters given by:

$$\Psi = \left(\frac{\chi A}{1+\eta} \right)^{\frac{1}{1-\psi}} \left(\frac{\psi}{\varpi}(1+\eta-\alpha) \right)^{\frac{\psi}{1-\psi}} [(1-\psi)(1+\eta)+\psi\alpha] \tag{38}$$

From this, we can deduce the equilibrium values of the whole set of macroeconomic variables at the planner's solution. In particular, the optimal function relating vacancies, net output and the terms of trade to hours worked are given by:

$$V = \Psi^{\frac{1}{\psi}} \left(\frac{A\chi}{1+\eta} \right)^{-\frac{1}{\psi}} [(1+\eta)(1-\psi)+\psi\alpha]^{-\frac{1}{\psi}} h^{\frac{\alpha}{1-\psi}} \tag{39}$$

$$Y - \bar{\omega} V = \Psi h^{\frac{\alpha}{1-\psi}} \tag{40}$$

$$\phi = \left[\frac{1-\xi}{\mu^*}(1+t^*)\left(Y - \varpi V\right) \right]^{\frac{1}{\sigma^*}} \tag{41}$$

4 Two sources of inefficiencies: Trade externality and labor market frictions

4.1 Identifying inefficiencies

The comparison between the social planner's and private agent's allocations allows to identify two inefficiency gaps. Making use of the above results, the inefficiency of the decentralized equilibrium can indeed be summarized by the three following ratios, with $\rho_x \equiv \frac{x^{dec}}{x^{sp}}$, for $x = \{h, V, \phi\}$:

$$\rho_h = \left[\frac{1}{TW} \overbrace{\left(\frac{\mu^*}{1+\tau_e}\right)^{\frac{1-\xi}{\mu^*}} \left(\frac{1+t^*}{1+t_e}\right)^{\frac{1-\xi}{\sigma^*}}}^{\text{Trade Ext. } >1} \overbrace{\left(\frac{\Theta}{\Psi}\right)^{\frac{-(1-\xi)}{\sigma^*}}}^{\text{LMI}^h > 1} \right]^{\nu} \tag{42}$$

$$\rho_V = \left[\frac{1}{TW} \overbrace{\left(\frac{\mu^*}{1+\tau_e}\right)^{\frac{1-\xi}{\mu^*}} \left(\frac{1+t^*}{1+t_e}\right)^{\frac{1-\xi}{\sigma^*}}}^{\text{Trade Ext. } >1} \right]^{\frac{\alpha\nu}{1-\psi}} \overbrace{\Upsilon^{\frac{1}{\psi}} \left(\frac{\Theta}{\Psi}\right)^{\kappa_V}}^{\text{LMI}^V < 1} \tag{43}$$

$$\rho_\phi = \left(\frac{1}{TW}\right)^{\frac{\alpha\nu}{\sigma^*(1-\psi)}} \underbrace{\left(\frac{\Theta}{\Psi}\right)^{\kappa_{\phi_1}}}_{\text{LMI}^\phi < 1} \underbrace{\left[\frac{\mu^*\xi + 1 - \xi}{(1+\tau_e)\xi + 1 - \xi}\right]^{\kappa_{\phi_1}} \left(\frac{\mu^*}{1+\tau_e}\right)^{\kappa_{\phi_2}}}_{\text{Trade Ext. } >1} \tag{44}$$

with t^e, TW, μ^*, t^* defined by equations (19),(23),(32),(36) respectively, and κ_V, κ_{ϕ_1}, κ_{ϕ_2}, $\frac{\Theta}{\Psi}$ and Υ combinations of deep parameters respectively given by::

$$\begin{aligned}
\kappa_V &\equiv \frac{1}{\psi}\left(\frac{1+\eta-\alpha+\frac{\alpha(1-\xi)}{\sigma^*}}{1+\eta-\alpha+\frac{\alpha(1-\xi)}{\sigma^*(1-\psi)}}\right) \\
\kappa_{\phi_1} &\equiv \frac{(1+\eta-\alpha)(1-\psi)}{\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)} \\
\kappa_{\phi_2} &\equiv \frac{\alpha(1-\psi)}{\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)} \\
\frac{\Theta}{\Psi} &= \left[\frac{(1+\eta)(1-\rho_b)-\alpha(1-\rho_c)}{(1+\eta-\alpha)(1-\epsilon\rho_b-\rho_c(1-\epsilon))}\right]^{\frac{\psi}{1-\psi}} \left(\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{(1-\psi)(1+\eta)+\psi\alpha}\right)\left(\frac{1-\rho_c}{1-\rho_b\epsilon-\rho_c(1-\epsilon)}\right) \\
\Upsilon &= \left(\frac{1-\rho_b\epsilon-\rho_c(1-\epsilon)}{1-\rho_c}\right)\frac{(1+\eta)(1-\psi)+\psi\alpha}{(1+\eta)(1-\epsilon)+\epsilon\alpha}
\end{aligned} \tag{45}$$

Notice that all these parameters are positive. From Equations (26) and (40), one can also obtain the ratio of net output at the decentralized equilibrium relative to the first best. Defining $\rho_y \equiv \frac{Y^{dec}-\bar{\omega}V^{dec}}{Y^{sp}-\bar{\omega}V^{sp}}$, we obtain:

$$\rho_y = \left(\frac{1}{TW}\right)^{\frac{\alpha\nu}{1-\psi}} \overbrace{\left(\frac{\Theta}{\Psi}\right)^{\kappa_y}}^{\text{LMI}^y<1} \overbrace{\left[\left(\frac{\mu^*}{1+\tau_e}\right)^{\frac{1-\xi}{\mu^*}} \left(\frac{1+t^*}{1+t_e}\right)^{\frac{1-\xi}{\sigma^*}}\right]^{\frac{\alpha\nu}{1-\psi}}}^{\text{Trade Ext.}>1} \tag{46}$$

with

$$\kappa_y \equiv \frac{(1+\eta-\alpha)(1-\psi)}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}} > 0$$

Inefficiency of the decentralized equilibrium comes from two types of wedges: *i)* due to the trade externality (τ_e that may differ from μ^*, inducing t^e to differ from t^* in Equations (42)-(46), denoted "Trade Ext.") ; *ii)* due to labor-market frictions, through unemployment benefits and inadequate bargaining power ($\rho_b > 0, \epsilon \neq \psi$, leading to a difference in Θ and Ψ in Equations (42)-(46)). We will analyze below the size of inefficiency wedges in the above system (higher or lower than 1). Let us first have an overview of the inefficiencies.

Labor-market frictions. In the context of labor market search frictions, labor market institutions (LMIs) generate inefficiencies in total employment. Specifically, as reported in Equations (42)-(43), labor market institutions have a direct impact on each labor margin (hours and vacancies), identified by LMI^h and LMI^V respectively. Yet, they do not play in the same direction: with search and matching frictions, the decentralized economy faces low employment, workers compensate this lack of earnings by putting in longer hours relative to the planner's values.[8] This, in turn, affects net output (see LMI^y in Equation (46)). By affecting the quantities of goods produced, labor market institutions do affect the equilibrium value of the terms of trade, as displayed by the term $\text{LMI}^\phi > 1$ in Equation (44).

Trade externality. In the centralized case, the optimal arbitrage between home and foreign consumption is given by Equation (33), which compares to Equation (8) in the decentralized economy. The gap between the two depends on μ^*, hence ultimately on the price-elasticity of foreign demand σ^*. The intuition is straightforward. The less-than-infinite price elasticity of foreign demand for domestic goods potentially gives some market power to the home country. Exactly as a monopoly would, the home country might extract some positive

[8] As indicated by $\text{LMI}^h > 1$ and $\text{LMI}^V < 1$, for which we provide a formal demonstration in section 4.2.

rent from this, by exporting a relatively low quantity of domestic goods at a higher relative price. However, in the decentralized economy, private agents do not internalize the effect of their consumption choices on the terms of trade. This lies at the root of the trade externality that affects both quantities and prices as identified by the term "Trade Ext." in the system (42)-(46).[9]

4.2 Consequences on macroeconomic outcomes

Using system (42)-(44), we study the inefficiencies of the decentralized equilibrium focusing on how they affect the three key variables which summarize the model: hours worked, vacancies and the terms of trade. In this section, the analysis of the centralized economy is performed after assuming non-negative hours and no fiscal policy.[10]

4.2.1 Inefficiencies on hours worked

Both the trade externality and labor market frictions induce a non-optimal amount of hours worked at the decentralized equilibrium. The direction of each effect is stated in Propositions 1 (for LMIs) and 2 (for the trade externality).

Proposition 1. LMIs generate longer hours worked: *The ratio of the equilibrium number of hours worked in the decentralized economy (relative to the first best) is an increasing function of the unemployment benefit ratio and the worker's bargaining power under the necessary and sufficient condition:*

$$\epsilon < \overline{\epsilon}, \qquad \textit{with} \;\; \overline{\epsilon} \equiv \frac{\psi}{\overline{\rho} - \rho_b(1-\psi)} > \psi \tag{47}$$

Proof. From Equation (22), it can be shown that hours worked are a decreasing function of Θ. From the definitions of Θ and Ψ (Equations (24) and (38)), having $\epsilon = \psi$ and $\rho_b = 0$ ensures $\Theta = \Psi$ under $\rho_c = 0$. As formally proved in Appendix B.1, under the condition (47),

[9] The consequences of a terms-of-trade externality, arising in an open economy facing a less-than-infinite price elasticity of foreign demand, are well documented in the trade literature; see, for example, Corben (1984) and Costinot et al. (2015).

[10] Non-negative hours means that the unemployment benefit ratio satisfies the condition to ensure a positive number of hours, i.e., such that $\rho_b < \overline{\rho} + \frac{\alpha\rho_c}{1+\eta}$, with $\overline{\rho} \equiv 1 - \frac{\alpha}{1+\eta}$. In addition, there is no fiscal policy such that $\rho_c = \tau_x = 0$ for $x = f, w, e, c$.

$\frac{\partial\Theta}{\partial\rho_b} < 0$ and $\frac{\partial\Theta}{\partial\epsilon} > 0$. In particular, the ratio $\frac{\Theta}{\Psi} < 1$ as long as $\epsilon < \psi$ or $\rho_b > 0$. This, in turn, allows to establish that $\frac{\partial h^{dec}}{\partial\rho_b} > 0$ and $\frac{\partial h^{dec}}{\partial\epsilon} < 0$. □

Proposition 2. Trade externality generates longer hours worked: *The ratio of hours worked at the decentralized equilibrium relative to the first-best is increasing with the market power of the economy relative to the rest of the world, μ^*.*

Proof. The first-best equilibrium value of hours worked decreases with μ^*, while the equilibrium value of hours is insensitive to μ^* in the decentralized case. See formal proof in Appendix B.1. □

Corollary 2.1. *Either an excessive workers' bargaining power with respect to their contribution to the matching technology ($\epsilon < \psi$), a positive unemployment-benefit ratio (up to $\overline{\rho}$) or a less-than-finite price elasticity of foreign demand ($\sigma^* < \infty$) are sufficient conditions for the number of hours worked to be excessively high in the decentralized economy, i.e. higher than at the planner's solution:*

$$h^{dec} > h^{sp} \quad \textit{as long as} \quad \epsilon < \psi, \quad \textit{or} \quad \rho_b > 0, \quad \textit{or} \quad \sigma^* < \infty$$

As stated in Corollary 2.1, the trade externality and labor market frictions complement each other in pushing the number of hours worked too high at the decentralized equilibrium (relative to the first best). By contrast, in the limit case with no labor market frictions ($\epsilon = \psi$ and $\rho_b = 0$, implying $\Theta = \Psi$) and no trade externality ($\sigma^* \to \infty \quad \leftrightarrow \quad \mu^* \to 1$), hours worked reach their first-best value in the decentralized economy ($h^{dec} = h^{sp}$). As we show below, the complementarity between both inefficiencies not longer neither applies to the other quantities, e.g. the labor extensive margin, nor the international relative price ϕ.

4.2.2 Inefficiency wedge on vacancies

Both the trade externality and labor market frictions induce sub-optimal job creation at the decentralized equilibrium. The direction of the effects is stated in Propositions 3 and 4.

Proposition 3. LMIs generate low employment: *Unemployment benefits or a too high bargaining power of workers relative to their contribution to the match ($\rho_b > 0$ or $\epsilon < \psi$) induce a lower amount of vacant jobs at the decentralized equilibrium than at the first-best equilibrium.*

Proof. Straightforward from Equation (43) and the previous result that the ratio $\frac{\Theta}{\Psi}$ falls below 1 under $\rho_b > 0$ or $\epsilon < \psi$, given that $\kappa_V > 0$. $\square$

Proposition 4. Trade externality generates high employment: *The ratio of vacancies at the decentralized equilibrium relative to the first-best is increasing with the market power of the economy relative to the rest of the world, μ^*.*

Proof. From Equation (43), it is straightforward that the derivative of ρ_V relative to μ^* is positive. $\square$

Corollary 4.1. *The equilibrium value of vacant jobs in the decentralized economy can be lower or above the first-best value, as the result of two opposite forces. On the one hand, labor market frictions exert a dampening influence on job creation (Proposition 3). On the other hand, the trade externality pushes towards an excessively high number of vacant jobs (Proposition 4). The final equilibrium value of vacant jobs (relative to the first best) is then* a priori *ambiguous, depending on which channel dominates.*

The intuition of the above result is the following. As can be deduced from Equation (63), labor market frictions exert a dampening influence on the job creation incentive, leading to too few jobs opened at the decentralized equilibrium everything else equal for a given amount of hours worked. By contrast, as private agents do not internalize the trade externality, they work too much along the intensive margin (Proposition 2). Given the increasing relation between vacancies and hours worked, this tends to push vacant jobs upwards as well. In the end, the final effect depends on which inefficiency plays the strongest.

Notice that the same result applies to net output: As can be deduced from Equation (46), LMIs tend to push net output too low relative to the first best ($\text{LMI}^y < 1$ under $\rho_b > 0$ or $\epsilon < \psi$): The dampening effect of LMIs on the labor extensive margin dominates. By contrast, the trade externality pushes both labor margins above their first-best values, hence net output (in Equation (46), the term "Trade ext." (increasing with μ^*) pushes towards $\rho_y > 1$). Again, the final effect of both inefficiencies on net output is ambiguous, depending on which channel dominates.

4.2.3 Inefficiency wedge on terms of trade

Proposition 5. *The equilibrium value of the relative international price ϕ at the decentralized solution relative to the first-best value results of two opposite forces. The trade externality* *leads to an excessively high value of the relative price of imports, whereas labor market imperfections conversely exert a downward pressure on ϕ. The final equilibrium value of ϕ (relative to the first best) is a priori ambiguous, depending on the two forces at work.*

Proof. Straightforward from Equation (44). The trade externality term (the "Trade Ext." in Equation (44)), increasing with μ^*, positively enters Equation (44), hence pushing towards $\rho_\phi > 1$. In contrast, labor market institutions ($\epsilon < \psi$ or $\rho_b > 0$), by inducing $\Theta/\Psi < 1$, exert a dampening effect on the relative price of imports at the decentralized equilibrium (see the term "LMI^ϕ in Equation (44)). □

The intuition behind Proposition (5) can be reformulated through the lens of Equations (18) and (41), which relate the terms of trade to net output. In relative terms, they rewrite as:

$$\frac{\phi^{dec}}{\phi^{dec}} = [1 + \xi(\mu^* - 1)]^{\frac{1}{\sigma^*}} \left[\frac{Y^{dec} - \bar{\omega}V^{dec}}{Y^{sp} - \bar{\omega}V^{sp}}\right]^{\frac{1}{\sigma^*}} \tag{48}$$

In the decentralized setting, as long as the price-elasticity of foreign demand is less than infinite ($\sigma^* < \infty$), the households ask for too many imported goods (the share of domestic goods $C_H/\phi C_F$ is too low). All else equal, this drives the relative price of foreign goods ϕ upward in the decentralized economy, everything else equal for a given amount of output (first bracket of the RHS of Equation (48). However, given that ϕ unambiguously decreases with net output, the possibly excessively low value of net output (relative to the first best) under the dominant pressure of labor market institutions at the extensive margin, exerts a downward pressure on ϕ. In other words, by reducing home production, this pushes the relative price of the home good upwards, ie ϕ downwards (see the second bracket of Equation (48)). As a result, the final equilibrium value of ϕ (relative to the fist best) is ambiguous.

We explore the policy implications of these results in Sections 5 and 6.

5 Trade policy, employment subsidy policy: Does it work?

In this section, we investigate the extent to which two policies, trade taxes and employment subsidy can eliminate the sub-optimality of the decentralized equilibrium. This dwells on two arguments. A first reason comes from the nature of the inefficiencies pointed out in the model. Trade taxes seem appropriate to tackle the trade externality, due to a sub-optimal rent extraction of the country's trade specialization pattern. Similarly, hiring subsidies appear as a relevant tool to eliminate the distortions on employment induced by the labor market institutions in the context of a search and matching labor market. Second, and most importantly, both policies find a strong empirical support in the light of recent policy measures taken by various countries over the last decade. The United States and many European countries have thus implemented hiring subsidies to tackle the employment consequences of the recession started in 2008-2009.[11] Following the increase in import tariffs by the United States since 2008, protectionist measures have been also applied by China or Brazil, leading to the resurgence of a "trade war" that would hit the world economy.[12] It is hence important to have a critical view of these policies, to analyse their consequences at the general macroeconomic scale.

Throughout the section, we will hence study the two policies in the presence of the two trade and labor inefficiencies (with $\rho_b > 0$ or $\epsilon < \psi$ such that $\Theta < \Psi$ and $\Upsilon < 1$; and $\mu^* > 1$ (or equivalently $\sigma^* < \infty$)), along with the absence of indirect consumption or direct labor taxes (such that $TW = 1$). From the system (42)-(46), with $TW = 1$, we know that the decentralized economy starts from a benchmark situation with excessively long hours worked ($\rho_h^b > 1$, with the superscript ^{b} for "Benchmark"), while vacancies, net output and the terms of trade might be above or below their first-best values (i.e., $\rho_x^b \gtreqless 1$ for $x = V, y, \phi$).

We start investigating the effects of each policy separately. Unsurprisingly, in presence of two inefficiencies, we reach the result that one instrument alone is not able to correct for the two and achieve the first-best allocation. More importantly, we show that both inefficiencies are deeply intertwined and play in opposite direction on employment and output. As a result, if one policy is able to remove part of an inefficiency gap, it does not succeed in bringing all elements of the economy closer to its first-best equivalent, due to general equilibrium effects. Only combined trade and labor policies can succeed in replicating the planner's allocation, as we show in a second step.

5.1 Trade policy

Import tariff. Assume that the government sets a tariff on imports such that:

$$\tau_e^{tp} = \mu^* - 1 \quad \leftrightarrow \quad \tau_e^{tp} = \frac{1}{\sigma^* - 1} > 0, \tag{49}$$

along with all taxes and employment subsidy set to 0, ie $\rho_c = \tau_x = 0, \;\; \forall \;\; x = w, c, f$ (with

[11] See OECD (2010) for a detailed review of the measures taken in 2009 and network (2014) for policies implemented in the European countries since 2003, and in particular following 2008.

[12] See https://www.spglobal.com/en/research-insights/articles/the-u-s-china-trade-war-the-global-economi

the superscript ^{tp} for "trade policy"). Equivalently, this amounts having $t_e^{tp} = t^*$. Comparing the equilibrium values for the terms of trade ϕ and aggregate consumption levels C_F, C_H at the decentralized vs planner' equilibrium (Equations (18), (20), (21) vs Equations (41), (34) and (35)) shows that this policy is able to reach the optimal sharing rule between domestic consumption of Home vs Foreign goods, everything else equal conditional on a given value of quantities (i.e., Y and V). Otherwise stated, the trade policy is able to remove the trade externality in System (42)-(46).

Macroeconomic impact. Accordingly, the system under the trade policy now writes:

$$\rho_h^{tp} = \left(\frac{\Theta}{\Psi}\right)^{\frac{-(1-\xi)\nu}{\sigma^*}} > 1, \qquad \rho_V^{tp} = \Upsilon^{\frac{1}{\psi}} \left(\frac{\Theta}{\Psi}\right)^{\kappa_V} < 1$$
$$\rho_\phi^{tp} = \left(\frac{\Theta}{\Psi}\right)^{\kappa_{\phi_1}} < 1, \qquad \rho_y^{tp} = \left(\frac{\Theta}{\Psi}\right)^{\kappa_y} < 1$$

with Θ and Υ equal to their values at the benchmark equilibrium, i.e. given by Equations (24) and (45) for $\rho_c = 0$ and $TW = 1$.

With import tariff, home consumers reduce their demand for imported goods. Consumption of domestic goods increases, which lowers home marginal utility of consumption. Leisure is then a more appealing activity for home workers. In comparison with the initial situation, hours per worker go down, thereby bridging part of the gap with the first best. However, the inefficiency due to labor market institutions remains and the value of worked hours still lies above its first-best counterpart under the trade policy ($\rho_h^b > \rho_h^{tp} > 1$). With reduced hours worked, the marginal gain from a filled vacancy goes down: Firms' incentive to create job is reduced such that employment declines. Without any ambiguity, the trade policy induces too few jobs in the decentralized economy relative to the socially-efficient value. In particular, if the number of jobs was already too low in the benchmark situation (with $\rho_V^b < 1$), the trade policy worsens the situation (with $\rho_V^{tp} < \rho_V^b < 1$).

Reduced employment combined with the drop in worked hours lead to a fall in output, which unambiguously lies below its efficient value. By reducing output, the trade policy drives the price of the Home goods upwards on international markets, and the terms of trade go down below their first-best value ($\rho_\phi^{tp} < 1$). Applying tariffs on imports is (at least partially) offset by the endogenous reduction in the relative price of imports at the general equilibrium, such that the terms of trade is now lower then its optimal level. Would the terms of trade be initially too high, the target of optimal ϕ is missed: The fall in terms of trade is actually too large because of the large output drop (driven by the combined drop in V and h), with ϕ becoming lower than its optimal level.

This result sheds new light on the import tax policy, as the one recently set place by the Trump administration in the US. Firstly, whereas the main argument advocated in support of this policy is to boost American jobs, our result conversely suggests that import tariffs alone can actually miss their employment target. Due to the interaction between the trade externality and labor market frictions, focusing only on trade externality by lowering US imports could actually lead to inefficient labor allocation, especially, in our paper, in terms of jobs. Secondly, this policy goal ignores the intensive margin of labor. The consequences are particularly striking in the benchmark case where decentralized employment and hours are not on the same side of the optimal allocation (employment being too low while hours per worker is too high). In this case, the trade policy is inappropriate as its impact on the two labor margins goes in the same downwards direction, hence compressing total employment and output by too far. This calls for a careful design of economic policy when trying to correct the effects of globalization on labor market, which the trade policy cannot address alone.

5.2 Employment subsidy policy

Employment subsidy. Consider now that the employment subsidy is implemented, along with all taxes set to 0, ie $\tau_x = 0, \quad \forall \quad x = w, c, f, e$. One "natural" assignment to the employment subsidy is to correct for labor market inefficiencies. Specifically, as identified in Section 4, labor market institutions generate inefficiencies at the intensive/extensive labor margins that go in opposite directions (pushing hours upwards and vacancies downwards relative to the first best). One might then wonder whether one instrument (the employment subsidy) will succeed reach efficiency on both labor margins simultaneously. As we show below, the answer is positive.

Comparing the expressions of Θ and Ψ (Equations (24) and (38)), one can show that the optimal employment subsidy policy that allows to ensure $\Theta = \Psi$ should be such as:[13]

$$\rho_c^{es} = \frac{(1+\eta-\alpha)(\psi-\epsilon)+\epsilon\rho_b\left[(1+\eta)(1-\psi)+\psi\alpha\right]}{(1+\eta-\alpha)(psi-\epsilon)+\epsilon\left[(1+\eta)(1-\psi)+\psi\alpha\right]} \tag{50}$$

[13]Details of the demonstration are reported in Appendix C.

As straightforward from Equation (42), under such a policy, hours worked at the decentralized equilibrium are closer to their first-best values, up to the terms-of trade inefficiency. The question is then, does this employment subsidy also removes the inefficiency at the extensive margin (LMI^V in Equation (43))? For this to be the case, it should be that ρ_c^{es} as defined above, not only sets $\Theta = \Psi$, but also $\Upsilon = 1$. As shown in Appendix C, this is indeed the case. If labor market institutions distort the two labor margins in opposite directions (pushing hours upwards and vacant jobs downwards), they yet constitute one deep source of inefficiency on total employment, that can be removed using one instrument, the employment subsidy.

Macroeconomic effects. The system under the employment subsidy policy (identified by the superscript ^{es}), in the absence of any distortive taxes ($\tau_x = 0 \;\; \forall \;\; x = c, f, w, e$) now writes:

$$\rho_h^{es} = \left[(\mu^*)^{\frac{1-\xi}{\mu^*}} (1+t^*)^{\frac{1-\xi}{\sigma^*}} \right]^{\nu} > 1, \qquad \rho_V^{es} = \left[(\mu^*)^{\frac{1-\xi}{\mu^*}} (1+t^*)^{\frac{1-\xi}{\sigma^*}} \right]^{\frac{\nu\alpha}{1-\psi}} > 1$$
$$\rho_\phi^{es} = (\mu^*)^{\kappa_{\phi_2}} (\xi\mu^* + 1 - \xi)^{\kappa_{\phi_1}} > 1, \qquad \rho_y^{es} = \rho_V^{es} > 1$$

With the employment subsidy, firms face a stronger incentive to create jobs, and vacant jobs increase relative to the benchmark situation. Further, the sole remaining inefficiency being the trade externality, vacancies are now excessively high in the decentralized economy. Would the initial situation be characterized by underemployment, the optimal target is missed as the Home country now features too many jobs posted.[14] In this environment with more jobs, workers do not have to work long hours: hours per worker decline relative to the benchmark situation, partially reducing the gap with respect to the first best ($1 < \rho_h^{es} < \rho_h^b$). With more employed workers, even though each working less hours, home output increases, again to lie above its efficient value. By pushing Home production upwards, the employment subsidy policy pushes the relative price of Home goods down on international markets, such that the price of imports increases, with Home agents buying imports at an excessively high price (with $\rho_\phi^{es} > 1$, while it was possibly too low initally).

The employment subsidy is actually very effective on the labor market: It does boot employment (V increases) so much that the price of home goods goes even further down,

[14] Under the dominant role of LMIs initially, we get the following ranking: $\rho_V^b < 1 < \rho_v^{es}$. In the opposite case of trade externality dominating initially, it would be: $1 < \rho_V^b < \rho_V^{es}$.

thereby making foreign imports even more expensive. In line with Fang & Rogerson (2009), we find that the employment subsidy boosts employment while lowering hours worked per worker. Yet, our result sheds light on the importance of having a more global picture on the effects of the employment policy. With the open-economy dimension, it does not deliver desirable effects on the terms of trade. As for the trade policy, the need to think about the impact of employment policies on labor adjustments as well as terms of trade. In this respect, our results call for a comprehensive analysis of labor market adjustments in open economies.

5.3 Reaching the first-best solution: Combining employment and trade policy

Fiscal package. For given values of labor market institutions $\epsilon < \psi$, $\rho_b > 0$, the above results suggest that combining a protectionist trade policy ($\tau_e > 0$) and an employment subsidy policy ($c > 0$) can be effective in reaching the first-best equilibrium. Precisely, this is ensured if the government sets $\tau_e = \tau_e^*$ and $\rho_c = \rho_c^{es}$ as defined in Equations (49) and (50). In this case, no other policy intervention is required, implying the optimal values for the indirect tax rate, labor taxes and lump-sum transfers should be 0.

To summarize, for given values of the LMIs ($\epsilon \neq \psi$, $\rho_b > 0$), the optimal policy (denoted ^{fb} for "first-best" policy) that allows the decentralized economy to reach the first-best level should be such that:

$$
\begin{aligned}
\tau_e^{fb} &= \mu^* - 1 \\
\rho_c^{fb} &= \frac{(1+\eta-\alpha)(\psi-\epsilon)+\epsilon\rho_b\left[(1+\eta)(1-\psi)+\epsilon\alpha\right]}{(1+\eta-\alpha)(\psi-\epsilon)+\epsilon\left[(1+\eta)(1-\psi)+\psi\alpha\right]} \\
TW^{fb} &= 1
\end{aligned}
$$

The equilibrium level of transfers adjusts to balance the government budget (15). In this case, both wedges tied to the terms of trade externality and labor market inefficiencies are eliminated ($t^e = t^*$, $LMI^h = LMI^V = 1$) such that simultaneously $h^{sp} = h^{dec}$, $V^{sp} = V^{dec}$ and $\phi^{sp} = \phi^{dec}$. This result generalizes to the whole set of variables, and the decentralized economy reaches the first-best equilibrium, both regarding quantities and prices.

Discussion. Trade externality and labor market frictions are deeply intertwined such that any policy focusing on one dimension only can actually drive the economy further away from the optimal allocation. Focusing only on trade externality by taxing imports could actually lead to inefficient labor allocation, especially, in our paper, in terms of jobs. The result that import tariffs lower output is consistent with Barattieri et al. (2018)'s paper. Yet, In Barattieri et al. (2018)'s paper, as tariffs lower output, protectionism appears as detrimental to the economy. We conversely reach a more nuanced picture: It might be efficient to do so, provided the home country initially features over-employment and excessive production. In this respect, our paper echoes Costinot et al. (2015)'s view that positive import tariff can be optimal. Protectionism *per se* is not necessarily detrimental to economic efficiency, as it can bring the economy closer to the first-best allocation. This might be notably relevant in economies facing over-employment and excessive production, which would gain from working less, producing less to allow home firms to sell their product at higher prices. Taxing imports can yet be detrimental to economic activity and welfare if the economy initially features under-employment and under-production, by aggravating the output and employment gap. In sharp contrast with the argument usually put forward by the promoters of protectionist policies, e.g. the Trump administration, trade policy might be relevant only in countries with over-employment and over-production. Further, unlike Costinot et al. (2015), trade policy alone can never reach the first best allocation, due to the presence of labor market frictions which it cannot correct for. Import tariff can bring the economy to its optimal allocation if the policy is implemented along with a labor policy.

Symmetrically, the employment subsidy alone is actually very effective on the labor market. Yet, taking the general equilibrium effects of the policy leads to a more nuanced conclusion. By boosting employment and output so much, the price of home goods goes even further down, thereby making foreign imports even more expensive than in the decentralized allocation. Complementing Fang & Rogerson (2009), we thus show that the employment subsidy alone, if very effective in boosting job creation, misses its target by leaving aside the open-economy dimension of our economies.[15] Our results thus call for a comprehensive analysis of labor market adjustments in open economies, as they condition the desirability of trade or employment policy.

[15]Brown (2015) also underlines the importance of taking into account the indirect effects of hiring subsidies when empirically evaluating their impact.

6 Designing the optimal tax scheme in a second-best environment

The above analysis shows that a combination of trade taxes and employment subsidies can bring the decentralized economy to the first best. Yet, one can be doubtful about the possibility of the government to apply these measures as credible and permanent policies. Given the importance of WTO agreements at the world level, or regional treaties at a more regional level (notably Europe), an aggressive trade policy by one country might face severe political constraints that make it unlikely to happen.[16] As for the employment policy, the related literature agrees that the effectiveness of hiring subsidies in boosting job creation is very dependent on the policy design (conditionality, targeted workers, administrative monitoring to avoid windfalls), in particular the unexpected and temporary nature of hiring subsidies (Cahuc et al., 2019, Kaas & Kircher, 2015). This casts doubt about the effectiveness of permanent hiring subsidies, all the more in a context of flexible wages (Cahuc et al., 2019).

Given the long-run perspective adopted in the paper, this drives us to leave aside both policies to focus on the effectiveness of the tax wedge components as fiscal tool. The fiscal tool comes as a natural candidate for the policy maker as standard in the labor literature (Fang & Rogerson, 2009). Specifically, we study how the government should manipulate the tax wedge components in the decentralized economy to reach the social planner's allocation, in a context where trade taxes and employment subsidies are out of the government's hands, and for the existing social norms that shape labor market institutions (i.e, for given values of ϵ and ρ_b, and assuming $\tau_e = \rho_c = 0$). We thus characterize the optimal fiscal design that can be achieved through changes either in direct labor taxation or indirect consumption taxation. As the three tax rates (τ_c, τ_f, τ_w) affect the decentralized equilibrium only in a joint manner through the tax wedge $TW = \frac{(1+\tau_c)(1+\tau_f)}{1-\tau_w}$, the question of closing the gap between the decentralized and centralized economies requires the determination of an optimal tax wedge.

6.1 Ramsey problem

In this section, we characterize the allocation when the government implements a tax reform through changing the tax wedge TW (defined in equation (23)), for the given values of labor-

[16] One may invoke the high import tariffs policy set in place by the President Trump in the United States since 2016 as counter-argument. If true, this argument cannot necessarily apply to other countries outside the US. Further, the intense negotiations surrounding the political meetings to attenuate this policy illustrate the difficulty of making use of trade taxes as long-lasting policy tool, even for the leading countries.

market institutions (ϵ, ρ_b). Further, it operates in a second-best environment, as trade taxes and employment subsidies are not available policy tools for the government ($\tau^e = \rho_c = 0$). Precisely, we solve for the Ramsey problem of the government, which chooses the tax wedge TW so as to maximize the welfare function of the economy according to the utilitarian view (as specified below), subject to technological constraints (Equations (1) and (5)) and the optimal behaviors of the agents and market equilibria (Equations (22) and (25)).

The objective of the utilitarian government can be written as:

$$\begin{aligned} & \max_{TW} \mathcal{U}^g = N\mathcal{U}_e + (1-N)\mathcal{U}_u \\ \Leftrightarrow \quad & \max_{TW} \mathcal{U}^{gov} = NC_e + (1-N)C_u - N\sigma_L \frac{h^{1+\eta}}{1+\eta} \end{aligned}$$

Making use of the optimal decisions of the agents and the market equilibrium conditions (see Online appendix for details), the problem of the government can be rewritten so as to maximize $\mathcal{U}^g(V,h)$ with respect to TW, under the constraints (52) and (53) that relate V, h and TW as specified below:

$$\max_{TW} \ \mathcal{U}^g = [1-\xi]^{-\frac{1-\xi}{\sigma^*}} \left[A\chi V^\psi h^\alpha - \bar{\omega}V\right]^{\frac{\sigma^*-1+\xi}{\sigma^*}} - \chi V^\psi \sigma_L \frac{h^{1+\eta}}{1+\eta} \quad , \tag{51}$$

$$\text{s.t.} \quad \frac{\overline{\omega}}{\chi} V^{1-\psi} = \epsilon \left[\frac{1+\eta-\alpha}{1+\eta} A h^\alpha - b\right], \tag{52}$$

$$\sigma_L h^{1+\eta} \left[(1-\xi)(A\chi V^\psi h^\alpha - \bar{\omega}V)\right]^{\frac{1-\xi}{\sigma^*}} = \alpha A h^\alpha \frac{1}{TW} \tag{53}$$

Characterizing the optimal tax scheme The constraint (52) implicitly defines $V = \mathcal{V}(h)$. The constraint (53) is such that $h = \widetilde{\mathcal{H}}(TW, V)$. Given $V = \mathcal{V}(h)$, this implicitly defines a link between h and TW which is denoted by $h = \mathcal{H}(TW)$. The government problem then becomes:

$$\max_{TW} \ \mathcal{U}^g(\mathcal{V}(\mathcal{H}(TW)), \mathcal{H}(TW))$$

The associated first-order condition is:

$$\mathcal{H}'(TW^*) \times \left[\mathcal{V}'(h)\mathcal{U}_V^{g\prime} + \mathcal{U}_h^{g\prime}\right] = 0 \tag{54}$$

Two differences with the planner's solution appear. First, the marginal utilities of vacancies and hours worked differ across the Ramsey and the planner's solutions, as $\mathcal{U}^g$ differs from $\mathcal{U}^{sp}$. Specifically, the discrepancy between the planner's and the government's marginal utilities of labor margins is related to the open-economy dimension, decreasing with σ^*.[17] When σ^* tends to infinity (ie, the Home country has no market power on foreign demand), then the discrepancy between the planner's and the government's marginal utilities of each labor margin vanishes. By contrast, the larger the monopoly power of the Home economy, the larger the gap ($\mathcal{U}_x^{sp\prime} - \mathcal{U}_x^{g\prime} > 0$, for $x = h, V$ as long as $\sigma^* < \infty$). As the Ramsey problem starts from the private agents' decision rules, the government cannot adequately take into account the role of the open-economy dimension on both labor margins when determining the optimal tax scheme.

Second, the best the government can do is to set a linear combination of marginal utilities with respect to the extensive and the intensive labor margins equal to 0, while the planner's solution allows to set them both to 0:

$$\begin{cases} \text{Ramsey problem:} & \mathcal{H}'(TW^*) \times \left[\mathcal{V}'(h)\mathcal{U}_V^{g\prime} + \mathcal{U}_h^{g\prime}\right] = 0 \\ \\ \text{Planner:} & \mathcal{U}_h^{sp\prime} = 0 \quad \text{and} \quad \mathcal{U}_V^{sp\prime} = 0 \end{cases} \tag{55}$$

6.2 Determining the second-best Ramsey tax scheme

While the planner sets both marginal utilities of vacancies and hours worked to 0, the best the government policy can achieve is to set a combination of the two marginal utilities to 0 (see Equation (55)). Beyond its inability to manage the terms of trade externality (see above), the government can only reduce the employment and hour gaps, without being able to eliminate both of them. The second-best tax wedge is then a compromise between both objectives, as characterized in Proposition 6.

Proposition 6. *The second-best optimal tax wedge that solves the Ramsey problem of the government is equal to:*

$$TW^{sb} = (1-\rho_b\epsilon)\frac{1-\psi}{1-\epsilon}\left(\frac{1+\eta+\alpha\frac{\psi}{1-\psi}}{1+\eta+\alpha\frac{\epsilon}{1-\epsilon}}\right) \times (1+t^*) \tag{56}$$

It is i) increasing in the trade externality $(1+t^*)$ and ii) decreasing in the unemployment benefit ratio. The worker's bargaining power exerts an ambiguous impact on TW^{sb}.

Proof. See Appendix D. □

[17] See the online Appendix for a formal demonstration.

The government's arbitrage for the second-best tax wedge goes hand-by-hand with the contrasting effects of the two inefficiencies at the macro level. On the one hand, focusing on the labor intensive margin dimension of the government's problem, the trade externality and an excessively high workers' bargaining power call for an *increase* in the tax wedge. Let us consider the extreme case of no labor market frictions ($b = 0$ or $\epsilon = \psi$). The optimal tax scheme is $TW^{sb} = 1 + t^* > 1$: In order to reduce excessive terms of trade, the government increases taxation to lower output, so that the price of home goods goes up, thereby making foreign imports less expensive. The magnitude of the tax increase depends on the extent of trade externality t^*, which depends on the openness of the economy ξ and trade elasticity μ^*.

On the other hand, labor market frictions (either $b > 0$ or $\epsilon < \psi$) tend to reduce vacancies (everything else equal for a given h), thereby calling for a *reduced* tax wedge. This result echoes Fang & Rogerson (2009)'s finding that, in a search and matching model with intensive and extensive margins, tax policies lead to decreases along both margins. With $b > 0$ and $\epsilon < \psi$, the benchmark economy suffers from low employment. Focusing only on the employment target, the government needs to lower taxes to boost employment and bring the economy closer to its optimal employment level. Our paper extend Fang & Rogerson (2009)'s result in an open-economy environment. In contrast to them, we show that increased taxation can actually make sense in an open economy setting, even in the presence of labor market frictions.[18] Finally, if the effect of unemployment benefits on the labor extensive margin dominate (always calling for a reduced tax wedge *ceteris paribus*), the effect of the workers' bargaining power is more ambiguous.

This solution shows the constraints faced by the government, as the optimal tax scheme lies between the fiscal policy that could cancel the hours gap (hours worked, $\mathcal{U}_h^{g\prime} = 0$) and the one that could cancel the employment gap (through vacancies, $\mathcal{U}_V^{g\prime} = 0$). While labor market inefficiencies that burden job creation call for reducing the tax wedge, the trade externality and labor market inefficiency along the labor intensive margin conversely call for increased taxation.

[18] This point is another illustration of the fact that labor market frictions, by reducing employment and output, somehow alleviates the trade externality which is linked to excessively high output and terms of trade.

6.3 Financing the Reforms: Indirect Versus Direct Taxation

After dealing with the level of the optimal tax wedge, we determine the optimal mix of distortive tax rates $\{\tau^c, \tau^f\}$ that lies behind the fiscal reform. We then reformulate the government problem in terms of optimal labor taxation, conditional to a given set of institutions (i.e., for given values of ρ_b and ϵ), with τ_c adjusting to balance the government's budget constraint. Investigating this point first requires us to make an assumption about the behavior of transfers. Specifically, we will assume the following rule for the lump-sum transfers: $T = \rho_T(Y - \underline{\omega}V)$, with $0 < \rho_T < 1$ exogenously set by the government. In this framework, the ability of reducing the overall tax wedge by a switch from direct to indirect taxation depends on the relative tax bases. This is formally stated in Proposition 7, which we refer to as the "tax base" effect. [19]

Proposition 7. *For given values of the unemployment policy rule (ρ_b) and the other dimensions of the budget rule (ρ_T, τ^w), the government can implement a reduction in the overall tax wedge TW by simultaneously reducing the payroll tax rate τ^f and increasing the indirect tax rate τ^c if the wage share of output is lower than the consumption share of output.*

Proof. See Appendix D.3.1. □

Corollary 7.1. *When the decentralized economy initially features under-employment due to the dominant role of stringent labor market institutions on the labor extensive margin, it is optimal to switch from direct labor taxation to indirect taxation if the wage share of output is lower than the consumption share of output.*

The tax base condition stated in Proposition 7 is a sufficient condition for a decrease in τ_f to be compensated for by a less than proportional increase in τ_c. As stated in Corollary 7.1, it is optimal to do so if labor market inefficiencies dominate in the economy. Conversely, TW must increase with the size of the other distortion, ie the terms-of trade externality. In this case, the direct taxation will be the more efficient provided the tax base condition holds. Importantly, one has to note that the tax base condition is satisfied empirically:[20] When households have other sources of revenues than labor incomes, our results demonstrate the

[19] We here study the switch from direct payroll taxation to indirect consumption taxation. We would obtain virtually the same results if considering a reduction in the employee's labor tax rate τ^w (rather than in τ^f) as long as the tax base condition holds, since what ultimately matters in changing the overall tax wedge TW.

[20] We have verified that this holds for a large number of countries over the recent decades, using OECD data on national accounts. Results are available upon request.

relevance of switching from direct to indirect taxation as long as implementing the reforms requires alleviating the overall tax burden in the economy.

7 Conclusion

We develop an open-economy model with search and matching on the labor market, with intensive and extensive labor margins. We characterize the key inefficiencies that distort the decentralized equilibrium: The trade externality (inherent to the open-economy feature) and the inefficiencies induced by labor market frictions.

Three main results emerge. First, trade externality and labor market frictions create conflicting effects on total employment and output. While labor market frictions creates inefficiently low output by hampering job creation, trade externality conversely pushes towards inefficiently high output, which lowers the price of home goods and make foreign imports expensive. Two main policy results follow. First, if a trade protectionist policy or an employment subsidy alone meets some success in reducing its targeted inefficiency gap, it is with detrimental effects on the other wedge. In particular, the introduction of import tariffs succeeds in shifting imports but actually deteriorates the employment target, by pushing job creation even further down. Only a package combining import taxes and employment subsidies can bring the economy closer to the optimal allocation for employment, worked hours and terms of trade. Second, if the sole instrument is the tax wdege, the Ramsey problem of the government faces an arbitrage between both externalities. While the trade externality calls for an increase in the overall tax wedge, the labor inefficiency at the extensive labor margin calls for a decrease in the tax burden.

These results have been obtained in a framework deliberately kept simple enough to preserve analytical tractability. In a very transparent way, we show that the key economic mechanisms thus rely on the presence of search and matching frictions and a foreign trading partner with finite elastic demand. This suggests that our results are relevant for economies with these two characteristics and will remain in more general frameworks. These results open the route to further research on the optimal fiscal design in open economies. In particular, we have assumed that the foreign country is passive in front of the fiscal reform implemented at home, whereas it may suffer from the deterioration of its price competitiveness. This suggests the endogenization of the foreign country's fiscal response, as well as the opportunity to coordinate fiscal reform. This is left for future research.

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Appendix

A Complements on the model's solving

A.1 Trade flows: Some microfoundations

We detail here a rationale for the specifications of domestic trade flows vis-à-vis the rest of the world. We relate the flow of foreign imports (for the Home good) to optimal demand for goods from abroad, according to the following program.

Let us thus assume that the rest of the world is endowed with a quantity Y^* of a tradable good, none of the home good. The equilibrium market condition for the foreign good is such that foreign private consumption is given by

$$C_F^* = Y^* - X^* \tag{57}$$

where X^* refer to exports from the foreign country to the home country. The foreign country also imports Z^* of the home good, which she consumes totally. Given our assumption of fixed production, there is no leisure choice hence the foreign households derive utility from the consumption of national good (C_F^*) and the imports of goods from abroad (Z^*). Given the absence of international trading of financial assets, both countries are characterized by a zero trade balance:

$$\phi X^* = Z^* \tag{58}$$

The foreign household's maximization program can be written:

$$\max_{C_F^*, Z^*} \mathcal{U}^*(C_F^*, Z^*) \quad = \quad \max_{C_F^*, Z^*} \left\{ C_F^* + \frac{(Z^*)^{\frac{\sigma^*-1}{\sigma^*}}}{(\sigma^*-1)/\sigma^*} \right\}$$

with $\sigma^* > 1$, the price elasticity of foreign imports, subject to two constraints, the equilibrium condition for the foreign good (Equation (57)) and the zero-trade balance condition (Equation (58). Integrating these two conditions, the problem simplifies to be written as:

$$\max_{Z^*} \mathcal{U}^*(C_F^*, Z^*) \quad = \quad \max_{Z^*} \left\{ Y^* - \frac{Z^*}{\phi} + \frac{(Z^*)^{\frac{\sigma^*-1}{\sigma^*}}}{(\sigma^*-1)/\sigma^*} \right\} \tag{59}$$

The first-order condition with respect to Z^* therefore leads to the import function of the foreign country (expressed in terms of the Home good):

$$Z^* \;=\; \phi^{\sigma^*} = X, \tag{60}$$

with X the volume of the Home good exported abroad.

From the zero-trade balance condition (58), we deduce the export function of the rest of the world:

$$X^* = \phi^{\sigma^*-1} = C_F, \tag{61}$$

with C_F the volume of Foreign goods imported at Home, necessarily equal to Foreign exports.

A.2 Obtaining the equilibrium values of hours worked, vacancies and the terms of trade

As reported with more details in the Online Appendix, the inefficiency of the decentralized equilibrium can be summarized by the following expressions:

$$\frac{h^{dec}}{h^{sp}} = \left[\frac{1}{TW}\left(\frac{\mu^*}{1+\tau_e}\right)^{\frac{1-\xi}{\mu^*}}\left(\frac{1+t^*}{1+t_e}\right)^{\frac{1-\xi}{\sigma^*}}\left(\frac{\Theta}{\Psi}\right)^{\frac{-(1-\xi)}{\sigma^*}}\right]^{\nu} \tag{62}$$

$$\frac{V^{dec}}{V^{sp}} = \left[\frac{\Theta}{\Psi}\left(\frac{1-\rho_b\epsilon-\rho_c(1-\epsilon)}{1-\rho_c}\right)\frac{(1+\eta)(1-\psi)+\psi\alpha}{(1+\eta)(1-\epsilon)+\epsilon\alpha}\right]^{\frac{1}{\psi}}\left(\frac{h^{dec}}{h^{sp}}\right)^{\frac{\alpha}{1-\psi}} \tag{63}$$

$$\frac{\phi^{dec}}{\phi^{sp}} = \left[\left(\frac{\mu^*}{1+\tau_e}\right)\frac{1+t_e}{1+t^*}\right]^{\frac{1}{\sigma^*}}\left[\frac{\Theta}{\Psi}\right]^{\frac{1}{\sigma^*}}\left(\frac{h^{dec}}{h^{sp}}\right)^{\frac{\alpha}{\sigma^*(1-\psi)}} \tag{64}$$

Integrating Equation (62) in Equations (63) and (64) respectively, allows to obtain the ratio of vacancies and the terms of trade relative to the fist-best solution as functions of the deep parameters (Equations (43) and (44)), as we show now.

A.2.1 Obtaining the equilibrium value of vacancies

The objective here is to obtain the general equilibrium value of vacant jobs (in relative terms to the first best). To do so, start from Equation (63):

$$\rho_V = \left[\frac{\Theta}{\Psi}\Upsilon\right]^{\frac{1}{\psi}} (\rho_h)^{\frac{\alpha}{1-\psi}},$$

with

$$\Upsilon = \left(\frac{1-\rho_b\epsilon-\rho_c(1-\epsilon)}{1-\rho_c}\right)\frac{(1+\eta)(1-\psi)+\psi\alpha}{(1+\eta)(1-\epsilon)+\epsilon\alpha}$$

Plugging ρ_h as given by Equation (42) into the above equation, we get:

$$\rho_V = \Upsilon^{\frac{1}{\psi}}\left[\frac{\Theta}{\Psi}\right]^{\frac{1}{\psi}-\frac{(1-\xi)\alpha\nu}{\sigma^*(1-\psi)}}\left[\frac{1}{TW}\left(\frac{\mu^*}{1+\tau_e}\right)^{\frac{1-\xi}{\mu^*}}\left(\frac{1+t^*}{1+t_e}\right)^{\frac{1-\xi}{\sigma^*}}\right]^{\frac{\alpha\nu}{1-\psi}},$$

Consider the power of the term $\frac{\Theta}{\Psi}$:

$$\frac{1}{\psi}-\frac{(1-\xi)\alpha\nu}{\sigma^*(1-\psi)} = \frac{\sigma^*(1-\psi)-\psi(1-\xi)\alpha\nu}{\psi\sigma^*(1-\psi)} \tag{65}$$

Given the definition of $\nu = \frac{1-\psi}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}}$, the numerator of the above equation rewrites as:

$$\begin{aligned}
\sigma^*(1-\psi)-\psi(1-\xi)\alpha\nu &= \sigma^*(1-\psi)-\frac{(1-\psi)\psi(1-\xi)\alpha}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}}\\
&= (1-\psi)\left[\sigma^*-\frac{\psi\alpha(1-\xi)}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}}\right]\\
&= \frac{1-\psi}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}}\left[\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)(1-\psi)\right]\\
&= \frac{1-\psi}{1+\eta-\alpha+\alpha\frac{1-\xi}{\sigma^*(1-\psi)}}\left[\sigma^*(1+\eta-\alpha)+\alpha(1-\xi)\right],
\end{aligned}$$

such that the numerator can be written as:

$$\sigma^*(1-\psi)-\psi(1-\xi)\alpha\nu = (1-\psi)\sigma^*\left[\frac{1+\eta-\alpha+\alpha\frac{1-\xi}{\sigma^*}}{1+\eta-\alpha+\alpha\frac{1-\xi}{\sigma^*(1-\psi)}}\right]$$

Incorporating this in Equation (65), we obtain:

$$\frac{1}{\psi} - \frac{(1-\xi)\alpha\nu}{\sigma^*(1-\psi)} = \frac{1}{\psi}\left[\frac{1+\eta-\alpha+\alpha\frac{1-\xi}{\sigma^*}}{1+\eta-\alpha+\alpha\frac{1-\xi}{\sigma^*(1-\psi)}}\right],$$

such that the general equilibrium value of ρ_y becomes:

$$\rho_V = \left[\frac{1}{TW}\overbrace{\left(\frac{\mu^*}{1+\tau_e}\right)^{\frac{1-\xi}{\mu^*}}\left(\frac{1+t^*}{1+t_e}\right)^{\frac{1-\xi}{\sigma^*}}}^{\text{Trade Ext. }>1}\right]^{\frac{\alpha\nu}{1-\psi}} \overbrace{\Upsilon^{\frac{1}{\psi}}\left(\frac{\Theta}{\Psi}\right)^{\kappa_V}}^{\text{LMI}^V<1}$$

with

$$\kappa_V \equiv \frac{1}{\psi}\left(\frac{1+\eta-\alpha+\frac{\alpha(1-\xi)}{\sigma^*}}{1+\eta-\alpha+\frac{\alpha(1-\xi)}{\sigma^*(1-\psi)}}\right) > 0$$

that is, Equation (43).

As for the effect of labor market institutions on vacancies, one might think that their impact on the equilibrium value of vacancies in the decentralized economy relative to the planner's solutions is ambiguous. On the one hand, from Equation (63), vacant jobs tend to be lower at the decentralized equilibrium than at the first-best equilibrium *ceteris paribus* for a given number of hours worked as long as $\epsilon < \psi$ or $\rho_b > 0$ (such that $V^{dec}(h) < V^{sp}(h) \;\; \forall h$ under $\rho_b > 0$ or $\epsilon < \psi$, see formal proof in Appendix B.2. On the other hand, under Proposition 1, the same conditions $\epsilon < \psi$ or $\rho_b > 0$ imply an excessively high amount of hours worked at the decentralized equilibrium. This exerts an upward pressure on vacancies, as they are an increasing function of hours worked. However, at the general equilibrium the first effect always dominates, as displayed in Equation (43), and as formally stated in Proposition 3.

A.2.2 Obtaining the equilibrium value of the terms of trade

Integrating Equation (62) in Equation (64), we obtain that the terms of trade at the decentralized equilibrium (relative to the first best) can be written as:

$$\rho_\phi = \left[\frac{\mu^*}{1+\tau_e}\frac{1+t_e}{1+t^*}\right]^{\frac{1}{\sigma^*}}\left(\frac{\Theta}{\Psi}\right)^{\frac{1}{\sigma^*}\left(1-\frac{\alpha(1-\xi)\nu}{\sigma^*(1-\psi)}\right)}\left(\frac{1}{TW}\right)^{\frac{\alpha\nu}{\sigma^*(1-\psi)}}\left[\frac{\mu^*}{1+\tau_e}\right]^{\frac{1-\xi}{\mu^*}\frac{\alpha\nu}{\sigma^*(1-\psi)}}\left[\frac{1+t^*}{1+t_e}\right]^{\frac{1-\xi}{\sigma^*}\frac{\alpha\nu}{\sigma^*(1-\psi)}}$$

Reorganizing terms, we get:

$$\rho_\phi \;=\; \left(\frac{1}{TW}\right)^{\frac{\alpha\nu}{\sigma^*(1-\psi)}} \left(\frac{\Theta}{\Psi}\right)^{\frac{1}{\sigma^*}\left(1-\frac{\alpha(1-\xi)\nu}{\sigma^*(1-\psi)}\right)} \left[\frac{\mu^*}{1+\tau_e}\right]^{\frac{1}{\sigma^*}\left(1+\frac{\alpha(1-\xi)\nu}{\mu^*(1-\psi)}\right)} \left[\frac{1+t_e}{1+t^*}\right]^{\frac{1}{\sigma^*}\left(1-\frac{\alpha(1-\xi)\nu}{\sigma^*(1-\psi)}\right)}$$

Consider first the power that is common to the terms $\frac{\Theta}{\Psi}$ and $\frac{1+t_e}{1+t^*}$:

$$\frac{1}{\sigma^*}\left(1-\frac{\alpha(1-\xi)\nu}{\sigma^*(1-\psi)}\right) = \frac{1}{\sigma^*}\left(\frac{\sigma^*(1-\psi)-\alpha(1-\xi)\nu}{\sigma^*(1-\psi)}\right)$$

Given the definition of ν, the above term becomes:

$$\begin{aligned}\frac{1}{\sigma^*}\left(1-\frac{\alpha(1-\xi)\nu}{\sigma^*(1-\psi)}\right) &= \frac{1}{(\sigma^*)^2}\left[\sigma^* - \frac{\alpha(1-\xi)}{(1+\eta-\alpha)(1-\psi)+\frac{\alpha(1-\xi)}{\sigma^*}}\right] \\ &= \frac{1}{\sigma^*}\left[\frac{(1+\eta-\alpha)(1-\psi)}{(1+\eta-\alpha)(1-\psi)+\frac{\alpha(1-\xi)}{\sigma^*}}\right] \\ &= \frac{(1+\eta-\alpha)(1-\psi)}{\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)}, > 0\end{aligned}$$

which we identify as κ_{ϕ_1}.

Consider now the power relative to the term $\frac{\mu^*}{1+\tau_e}$:

$$\frac{1}{\sigma^*}\left(1+\frac{\alpha(1-\xi)\nu}{\mu^*(1-\psi)}\right)$$

Given the definitions of ν and μ^*, it comes:

$$\begin{aligned}\frac{1}{\sigma^*}\left(1+\frac{\alpha(1-\xi)\nu}{\mu^*(1-\psi)}\right) &= \frac{1}{\sigma^*}\left[1+\frac{(1-\xi)\nu\alpha}{\frac{\sigma^*}{\sigma^*-1}(1-\psi)}\right] \\ &= \frac{1}{\sigma^*}\left[\frac{\frac{\sigma^*}{\sigma^*-1}(1-\psi)+(1-\xi)\alpha\nu}{\frac{\sigma^*}{\sigma^*-1}(1-\psi)}\right] \\ &= \frac{\sigma^*-1}{(\sigma^*)^2}\left[\frac{\sigma^*}{\sigma^*-1}+\frac{(1-\xi)\alpha}{(1+\eta-\alpha)(1-\psi)+\frac{\alpha(1-\xi)}{\sigma^*}}\right] \\ &= \frac{1}{(\sigma^*)^2}\left[\frac{\sigma^*(1+\eta-\alpha)(-\psi)+\alpha(1-\xi)+\sigma^*-1)(1-\xi)\alpha}{(1+\eta-\alpha)(1-\psi)+\frac{\alpha(1-\xi)}{\sigma^*}}\right] \\ &= \frac{1}{\sigma^*}\left[\frac{(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)}{(1+\eta-\alpha)(1-\psi)+\frac{\alpha(1-\xi)}{\sigma^*}}\right],\end{aligned}$$

which we identify as κ_{ϕ_3}:

$$\kappa_{\phi_3} \equiv \frac{(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)}{\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)} > 0$$

At this stage, we have obtained:

$$\begin{aligned}
\rho_\phi &= \left(\frac{1}{TW}\right)^{\frac{\alpha\nu}{\sigma^*(1-\psi)}} \left(\left(\frac{1+t_e}{1+t^*}\right)\frac{\Theta}{\Psi}\right)^{\kappa_{\phi_1}} \left[\frac{\mu^*}{1+\tau_e}\right]^{\kappa_{\phi_3}} \\
\text{with} & \\
\kappa_{\phi_1} &\equiv \frac{(1+\eta-\alpha)(1-\psi)}{\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)}, \\
\kappa_{\phi_3} &\equiv \frac{(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)}{\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)}
\end{aligned}$$

Next step is to simplify the terms related to the trade externality, given the link between $1+t^*$ and μ^*, and t_e and τ_e. To do so, rewrite the above equation as:

$$\rho_\phi = \left(\frac{1}{TW}\right)^{\frac{\alpha\nu}{\sigma^*(1-\psi)}} \left(\frac{\Theta}{\Psi}\right)^{\kappa_{\phi_1}} \frac{(\mu^*)^{\kappa_{\phi_3}}}{(1+t^*)^{\kappa_{\phi_1}}} \frac{(1+t_e)^{\kappa_{\phi_1}}}{(1+\tau_e)^{\kappa_{\phi_3}}}$$

Given the definition of $1+t^* = \frac{\mu^*}{\mu^*\xi+1-\xi}$, and $1+t_e = \frac{1+\tau_e}{(1+\tau_e)\xi+1-\xi}$, we can rewrite:

$$\begin{aligned}
\frac{(\mu^*)^{\kappa_{\phi_3}}}{(1+t^*)^{\kappa_{\phi_1}}} &= (\mu^*)^{\kappa_{\phi_3}-\kappa_{\phi_1}} \left[\mu^*\xi+1-\xi\right]^{\kappa_{\phi_1}}, \\
\frac{(1+t_e)^{\kappa_{\phi_1}}}{(1+\tau_e)^{\kappa_{\phi_3}}} &= (1+\tau_e)^{-(\kappa_{\phi_3}-\kappa_{\phi_1})} \left[(1+\tau_e)\xi+1-\xi\right]^{-\kappa_{\phi_1}}
\end{aligned}$$

such that we obtain:

$$\begin{aligned}
\rho_\phi &= \left(\frac{1}{TW}\right)^{\frac{\alpha\nu}{\sigma^*(1-\psi)}} \left(\frac{\Theta}{\Psi}\right)^{\kappa_{\phi_1}} \left(\frac{\mu^*}{1+\tau_e}\right)^{\kappa_{\phi_2}} \left(\frac{\mu^*\xi+1-\xi}{(1+\tau_e)\xi+1-\xi}\right)^{\kappa_{\phi_1}}, \\
\text{with} & \\
\kappa_{\phi_2} &= \kappa_{\phi_3} - \kappa_{\phi_2} \\
&= \frac{\alpha(1-\xi)}{\sigma^*(1+\eta-\alpha)(1-\psi)+\alpha(1-\xi)} > 0
\end{aligned}$$

that is, Equation (44).

A.2.3 Equilibrium value of output

From Equations (26) and (40), the ratio of net output at the decentralized equilibrium relative to the first best, can be written as a function of hours worked (in ratio) according to:

$$\frac{(Y - \bar{\omega}V)^{dec}}{(Y - \bar{\omega}V)^{sp}} \equiv \rho_y = \frac{\Theta}{\Phi}\left(\frac{h^{dec}}{h^{sp}}\right)^{\frac{\alpha}{1-\psi}}$$

Making use of Equation (42), this rewrites as:

$$\rho_y = \left[\frac{\Theta}{\Phi}\right]^{1-\frac{(1-\xi)\alpha\nu}{\sigma^*(1-\psi)}} \left[\frac{1}{TW}\left(\frac{\mu^*}{1+\tau_e}\right)^{\frac{1-\xi}{\mu^*}}\left(\frac{1+t^*}{1+t_e}\right)^{\frac{1-\xi}{\sigma^*}}\right]^{\frac{\alpha\nu}{1-\psi}}$$

Considering the power term on $\frac{\Theta}{\Phi}$:

$$\begin{aligned}
1 - \frac{(1-\xi)\alpha\nu}{\sigma^*(1-\psi)} &= \frac{\sigma^*(1-\psi) - (1-\xi)\alpha\nu}{\sigma^*(1-\psi)} \\
&= \frac{1}{\sigma^*}\left[\sigma^* - \frac{(1-\xi)\alpha}{(1+\eta-\alpha)(1-\psi) + \alpha\frac{1-\xi}{\sigma^*}}\right] \\
&= \frac{(1+\eta-\alpha)(1-\psi)}{(1+\eta-\alpha)(1-\psi) + \alpha\frac{1-\xi}{\sigma^*}}
\end{aligned}$$

which we identify as $\kappa_y > 0$.

B Characterizing inefficiencies

B.1 Inefficiencies on hours worked: Proof of Propositions 1 and 2

The rationale between the proofs of Propositions 1 and 2 can be stated starting from the comparison between the social planner's and private agent's allocations (Equation (42)). In the absence of fiscal policy, it becomes:

$$\frac{h^{dec}}{h^{sp}} = \left[(\mu^*)^{1-\xi}\left[\left(\frac{1+t^*}{\mu^*}\right)\frac{\Psi}{\Theta}\right]^{\frac{1-\xi}{\sigma^*}}\right]^{\nu}$$

Given the definition of $1 + t^*$, the above expression can be simplified to yield:

$$\frac{h^{dec}}{h^{sp}} = \left[\underbrace{(\mu^*)^{1-\xi} \left(\frac{1}{\xi\mu^* + 1 - \xi} \right)^{\frac{1-\xi}{\sigma^*}}}_{\text{Trade ext.}} \underbrace{\left(\frac{\Psi}{\Theta} \right)^{\frac{1-\xi}{\sigma^*}}}_{\text{LMIs}} \right)^{\nu} \tag{66}$$

with

$$\mu^* = \frac{\sigma^*}{\sigma^* - 1} \geq 1, \quad \text{and} \quad 1 + t^* = \frac{1}{\xi + \frac{1-\xi}{\mu^*}} \geq 1$$

and, under $\rho_c = 0$:

$$\begin{aligned}
\Theta &= \left[\frac{A\chi}{1+\eta} \right]^{\frac{1}{1-\psi}} \left[\frac{\epsilon}{\bar{\omega}} \frac{(1+\eta)(1-\rho_b) - \alpha}{1 - \epsilon\rho_b} \right]^{\frac{\psi}{1-\psi}} \left(\frac{(1-\epsilon)(1+\eta) + \epsilon\alpha}{1 - \epsilon\rho_b} \right) \\
\Psi &= \left[\frac{\chi A}{1+\eta} \right]^{\frac{1}{1-\psi}} \left(\frac{\psi}{\bar{\omega}} (1 + \eta - \alpha) \right)^{\frac{\psi}{1-\psi}} \left[(1-\psi)(1+\eta) + \psi\alpha \right]
\end{aligned}$$

Also recall the condition on the unemployment benefit ratio we impose to ensure a positive amount of hours worked at the decentralized equilibrium:

$$\rho_b < \overline{\rho} \equiv 1 - \frac{\alpha}{1+\eta}$$

B.1.1 Proof of Proposition 1: Labor market frictions and hours worked

To determine the role of labor market frictions on worked hours, it is sufficient to study how they affect $\Theta \neq \Psi$, as h is a decreasing function of Θ (Equation (22)). In this section, we establish the sign of the derivative of Θ with respect to the unemployment benefit ratio ρ_b and the employer's bargaining power ϵ alternatively.

Establishing the derivative with respect to ρ_b Deriving the above expression (67) with respect to ρ_b leads to:

$$\begin{aligned}
\frac{\partial \Theta}{\partial \rho_b} &= \varsigma \left(\frac{(1-\epsilon)(1+\eta) + \epsilon\alpha}{1 - \epsilon\rho_b} \right) \left[\frac{\psi}{1-\psi} \frac{1 - \epsilon\rho_b}{(1+\eta)(1-\rho_b) - \alpha} \right] \left(\frac{-(1+\eta)(1-\epsilon\rho_b) + \epsilon((1+\eta)(1-\rho_b) - \alpha)}{(1-\epsilon\rho_b)^2} \right) \\
&\quad + \varsigma \left((1-\epsilon)(1+\eta) + \epsilon\alpha \right) \left(\frac{\epsilon}{(1-\epsilon\rho_b)^2} \right)
\end{aligned}$$

with

$$\varsigma \equiv \left[\frac{A\chi}{1+\eta}\right]^{\frac{1}{1-\psi}} \left[\frac{\epsilon}{\bar{\omega}} \frac{(1+\eta)(1-\rho_b)-\alpha}{1-\epsilon\rho_b}\right]^{\frac{\psi}{1-\psi}} \lesseqgtr 0$$

.

As shown with more details in the Online Appendix, this can be rewritten as:

$$\frac{\partial\Theta}{\partial\rho_b} = -\underline{\Theta}\left[\underbrace{(1+\eta)(1-\rho_b)-\alpha}_{\gamma_1}\right]^{-\frac{1-2\psi}{1-\psi}} \underbrace{[(\psi-\epsilon)(1+\eta)+\epsilon(1-\psi)\rho_b(1+\eta)+\epsilon\alpha]}_{\gamma_2}$$

with

$$\underline{\Theta} = \frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1-\psi)(1-\epsilon\rho_b)^2}\left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}\left[\frac{\epsilon}{\bar{\omega}(1(\epsilon\rho_b)}\right]^{\frac{\psi}{1+\psi}} > 0$$

From this, the sign $\frac{\partial\Theta}{\partial\rho_b} < 0$ is ensured under two cases: ($\gamma_1 > 0$ and $\gamma_2 > 0$), or ($\gamma_1 < 0$ and $\gamma_2 < 0$). As detailed in the Online Appendix, the second case ($\gamma_1 < 0$ and $\gamma_2 < 0$) violates the condition ensuring positivity of hours worked, ie $\rho_b < \overline{\rho}$. Accordingly, the only relevant case is ($\gamma_1 > 0$ and $\gamma_2 > 0$). From the expression of γ_1, we can state that $\gamma_1 > 0$ if and only if $\rho_b < \overline{\rho}$, which is assumed by assumption. Under this condition, the positivity condition on γ_2 is obtained putting an upward threshold value on ϵ:

$$\epsilon < \bar{\epsilon} \equiv \frac{\psi}{\overline{\rho} - \rho_b(1-\psi)}$$

Importantly, we can establish that $\bar{\epsilon} > \psi$, such that the condition $\epsilon < \psi$ is a sufficient condition to ensure γ_2 >0, hence (conditional on $\rho_b < \overline{\rho}$), having $\frac{\partial\Theta}{\partial\rho_b} < 0$.

Establishing the derivative with respect to ϵ Deriving the expression for Θ given by Equation (67) with respect to ϵ leads to:

$$\frac{\partial\Theta}{\partial\epsilon} = \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}\left[\frac{\psi}{1-\psi}\widetilde{\varsigma}^{\frac{\psi}{1-\psi}}\frac{\gamma_3}{\widetilde{\varsigma}}\frac{\partial\widetilde{\varsigma}}{\partial\epsilon} + \widetilde{\varsigma}^{\frac{\psi}{1-\psi}}\frac{\partial\gamma_3}{\partial\epsilon}\right]$$

with:

$$\widetilde{\varsigma} = \frac{\epsilon}{\bar{\omega}}\left(\frac{(1+\eta)(1-\rho_b)-\alpha}{1-\epsilon\rho_b}\right) \lesseqgtr 0$$

$$\gamma_3 = \frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\epsilon\rho_b} > 0$$

As shown with more details in the Online Appendix, this can be simplified to become:

$$\frac{\partial\Theta}{\partial\epsilon} = \bar{\Theta}\gamma_1^{\frac{\psi}{1-\psi}}\gamma_2$$

$$\text{with} \quad \begin{cases} \bar{\Theta} & = \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}} \frac{1}{(1-\epsilon\rho_b)^2}\frac{1}{\epsilon(1-\psi)}\left(\frac{\epsilon}{\bar{\omega}}\right)^{\frac{\psi}{1-\psi}} > 0 \\ \gamma_1 & = (1+\eta)(1-\rho_b)-\alpha \\ \gamma_2 & = (\psi-\epsilon)(1+\eta)+\epsilon\left[\rho_b(1-\psi)(1+\eta)+\alpha\right] \end{cases}$$

where we recognize in γ_1 and γ_2 the combinations of parameters already defined above. Consequently, the condition for $\frac{\partial\Theta}{\partial\epsilon} > 0$ is the same as for $\frac{\partial\Theta}{\partial\rho_b} < 0$: ($\gamma_1 > 0$ and $\gamma_2 > 0$). The same results apply accordingly.

B.1.2 Proof of Proposition 2: Trade externality

As displayed in Equation (22), hours worked at the decentralized equilibrium are not function of the trade externality as measured by μ^*. As a result, establishing how the ratio h^{dec}/h^{sp} varies with μ^* is equivalent to evaluate how the first-best value of hours worked changes with μ^*. To this aim, starting from Equation (66), it is sufficient to consider the term referring to the trade externality, $f(\mu^*)$, where:

$$f(\mu^*) = (\mu^*)^{1-\xi}\left(\frac{1}{\xi\mu^*+1-\xi}\right)^{\frac{1-\xi}{\sigma^*}}$$

The objective is to determine the sign of the derivative of $f(\mu^*)$ relative to μ^*. It comes that:

$$\begin{aligned} f'_{\mu^*} &= \left[\frac{1}{\xi\mu^*+1-\xi}\right]^{\frac{1-\xi}{\sigma^*}}(1-\xi)(\mu^*)^{-\xi} + (\mu^*)^{1-\xi}\left(\frac{1-\xi}{\sigma^*}\right)\left(\frac{1}{\xi\mu^*+1-\xi}\right)^{\frac{1-\xi}{\sigma^*}-1}\left(\frac{-\xi}{\xi\mu^*+1-\xi}\right) \\ &= \left(\frac{1}{\xi\mu^*+1-\xi}\right)^{\frac{1-\xi}{\sigma^*}}(\mu^*)^{1-\xi}\underbrace{\left[\frac{1-\xi}{\mu^*} - \frac{\xi}{\xi\mu^*+1-\xi}\frac{1-\xi}{\sigma^*}\right]}_{(a)} \end{aligned}$$

Given that $\left(\frac{1}{\xi\mu^*+1-\xi}\right)^{\frac{1-\xi}{\sigma^*}}(\mu^*)^{1-\xi} > 0$, this amounts establishing the sign of Term (a), that is

$$
\begin{aligned}
(a) &= \frac{1-\xi}{\mu^*} - \frac{\xi}{\xi\mu^* + 1 - \xi}\frac{1-\xi}{\sigma^*} \\
&= \frac{(1-\xi)\sigma^*(\xi\mu^* + 1 - \xi) - \xi(1-\xi)\mu^*}{\mu^*\sigma^*(\xi\mu^* + 1 - \xi)} \\
&= \frac{(1-\xi)\xi\mu^*(\sigma^* - 1) + (1-\xi)(\sigma^* - 1)}{\mu^*\sigma^*(\xi\mu^* + 1 - \xi)} \\
\Rightarrow \quad (a) &= (\sigma^* - 1)(1-\xi)(\xi\mu^* + 1) > 0
\end{aligned}
$$

which implies $f'_{\mu^*} > 0$. Accordingly, hours worked at the first-best solution are decreasing with the degree of market power of the Home economy vis-à-vis the rest of the world. Stated differently, the ratio h^{dec}/h^{sp} is increasing with μ^*: The larger the market power of the Home country, the more excessive the hours worked at the decentralized equilibrium in comparison with the first best.

B.2 Inefficiencies on vacant jobs: Proof of Propositions 3 and 4

B.2.1 Characterizing the effects of LMIs on the slope of the function $V(h)$

In this first sub-section, we establish how labor market frictions affect the relation linking vacant jobs to hours worked, i.e. the slope of the function $V(h)$. For the sake of consistency with previous reasoning though, we run the analysis contemplating the ratio of vacant jobs at the decentralized equilibrium relative to the first-best (keeping in mind that neither ϵ nor ρ_b does affect the function $V(h)$ at the planner's solution).

Let us recall the gap between vacancies at the decentralized equilibrium and the planner's case, as a function of hours worked and deep parameters:

$$
\frac{V^{dec}}{V^{sp}} = \left[\frac{\Theta}{\Psi}\right]^{\frac{1}{\psi}} \left[\left(\frac{1-\rho_b\epsilon - \rho_c(1-\epsilon)}{1-\rho_c}\right)\frac{(1+\eta)(1-\psi)+\psi\alpha}{(1+\eta)(1-\epsilon)+\epsilon\alpha}\right]^{\frac{1}{\psi}} \left(\frac{h^{dec}}{h^{sp}}\right)^{\frac{\alpha}{1-\psi}} \tag{67}
$$

Our objective is to characterize how labor market institutions affect the slope of the above function. From our above results (Proof of Proposition 1), we know that the ratio Ψ/Θ is equal to 1 under $\epsilon = \psi$ and $\rho_b = \rho_c = 0$, increasing (above 1) with $\rho_b > 0$ and decreasing with ϵ. In particular, having $\epsilon < \psi$ or $\rho_b > 0$ is a sufficient condition for $\Psi/\Theta > 1$. In the terms of Equation (67), this implies that the ratio $\frac{\Theta}{\Psi}$ falls below 1, i.e. reducing the slope and pushing vacancies downwards everything else equal for a given number of hours worked.

Next step is to determine how LMIs affect the second term of the slope of Equation (67), that is:

$$\Gamma(\epsilon, \rho_b) = \left(\frac{1 - \rho_b\epsilon - \rho_c(1-\epsilon)}{1-\rho_c}\right) \frac{(1+\eta)(1-\psi) + \psi\alpha}{(1+\eta)(1-\epsilon) + \epsilon\alpha}$$

Derivative with respect to ρ_b Let us determine the sign of the derivative $\Gamma'_{\rho_b} \equiv \frac{\partial \Gamma(\epsilon,\rho_b)}{\partial \rho_b}$.

It is straightforward that:

$$\Gamma'_{\rho_b} = \left[\frac{(1+\eta)(1-\psi) + \psi\alpha}{(1+\eta)(1-\epsilon) + \epsilon\alpha}\right] \left[\frac{-\epsilon(1-\rho_c)}{(1-\rho_c)^2}\right] < 0$$

This establishes that the term $\Gamma(\epsilon, \rho_b)$ is decreasing with ρ_b. Combined to our previous results on the ratio $\frac{Theta}{\Psi}$, this establishes that the slope of the function relating vacancies to hours worked (relative to the first best) in Equation (67) is decreasing with the unemployment benefit ratio ρ_b.

Derivative with respect to ϵ Let us determine the sign of the derivative $\Gamma'_{\epsilon} \equiv \frac{\partial \Gamma(\epsilon,\rho_b)}{\partial \epsilon}$. We have:

$$\begin{aligned}
\Gamma'_{\epsilon} &= \left[\frac{(1+\eta)(1-\psi) + \psi\alpha}{(1+\eta)(1-\epsilon) + \epsilon\alpha}\right] \left(\frac{(\rho_c - \rho_b)}{(1-\rho_c)^2}\right) \\
&\quad + \frac{1 - \rho_b\epsilon - \rho_c(1-\epsilon)}{1-\rho_c} \left[\frac{-((1+\eta)(1-\psi) + \psi\alpha)(-(1+\eta) + \alpha)}{((1+\eta)(1-\epsilon) + \epsilon\alpha)}\right]
\end{aligned}$$

That is:

$$\Gamma'_{\epsilon} = \overbrace{\left[\frac{(1+\eta)(1-\psi) + \psi\alpha}{(1+\eta)(1-\epsilon) + \epsilon\alpha}\right] \frac{1}{1-\rho_c}}^{>0} \left[\rho_c - \rho_b + \frac{1 - \rho_b\epsilon - \rho_c(1-\epsilon)}{(1+\eta)(1-\epsilon) + \epsilon\alpha}(1 + \eta - \alpha)\right]$$

Considering the term into bracket, denoted $\widetilde{\Gamma}'_{\epsilon}$ for reading convenience:

$$\begin{aligned}
\widetilde{\Gamma}'_{\epsilon} &= \frac{(\rho_c - \rho_b)\left[(1+\eta)(1-\epsilon) + \epsilon\alpha\right] + (1 + eta - \alpha)(1 - \rho_b\epsilon - \rho_c(1-\epsilon))}{(1+\eta)(1-\epsilon) + \epsilon\alpha} \\
&= \frac{1}{(1+\eta)(1-\epsilon) + \epsilon\alpha}\left[(1+\eta)(1-\rho_b) - \alpha(1-\rho_c)\right]
\end{aligned}$$

It comes that $\Gamma'_\epsilon > 0$ iif $(1+\eta)(1-\rho_b) - \alpha(1-\rho_c)$. Rewriting this condition:

$$\begin{aligned} & (1+\eta)(1-\rho_b) - \alpha(1-\rho_c) > 0 \\ \Leftrightarrow \quad & 1+\eta - \alpha(1-\rho_c) > \rho_b(1+\eta) \\ \Leftrightarrow \quad & \rho_b < 1 - \frac{\alpha(1-\rho_c)}{1+\eta}, \end{aligned}$$

which we recognize as the positivity condition on hours worked. This establishes that, under this positivity condition, $\widetilde{\Gamma}'_\epsilon > 0$. The slope of the function relating vacancies to hours worked (relative to the first best) in Equation (67) is increasing with ϵ.

Summary Both results go in the same direction: Unemployment benefits ($\rho_b > 0$) or an excessively high workers' bargaining power ($\epsilon < \psi$) reduce the slope of the function relating vacancies to hours worked at the decentralized equilibrium (relative to the first-best). Everything else equal, for a given amount of hours worked, firms are less enticed to open vacant jobs under stringent labor market institutions.

B.2.2 Proof of Propositions 3 and 4

Proof of Proposition 3 From the proof of Proposition 1, we know that $\Theta < \Psi$ under $\rho_b > 0$ or $\epsilon < \psi$, with the ratio $\frac{\Theta}{\Psi}$ decreasing with ρ_b and increasing with ϵ. To sign the effect of LMIs on the equilibrium value of vacancies (in absolute as in relative terms), it is hence sufficient to establish the sign of the derivative of ρ_V with respect to the ratio Θ/Ψ. Given that $\kappa_V > 0$, it is straightforward from Equation (43), that more stringent labor market institutions (pushing $\Theta/\Psi < 1$) also induces a lower ratio of vacancies relative to the first best, i.e. pushing $\rho_V < 1$.

Proof of Proposition 4 Given the definition of $1+t^*$, this term is increasing in μ^*, that measures the market power of the Home economy, hence the strength of the trade externality. From Equation (43), it is straightforward that ρ_V is an increasing function of both μ^* and $1+t^*$. This proves Proposition 4.

B.3 Inefficiencies on the terms of trade: Proof of Proposition 5

From Equation (44), it is straightforward that the derivative $\frac{\partial \rho_\phi}{\partial \mu^*}$ is positive, as both κ_{ϕ_1} and κ_{ϕ_2} ar positive coefficients. Put it in plain words, the trade externality exerts an upward pressure on the terms of trade, relative to the first-best value, i.e. pushing towards $\rho_\phi > 1$.

As for the LMIs, given $\kappa_{\phi_1} > 0$ and the previous results that $\frac{\partial\Theta}{\partial\rho_b} < 0$ and $\frac{\partial\Theta}{\partial\epsilon} > 0$ under the sufficient condition $\epsilon < \psi$, then more stringent LMIs (i.e., an increase in the unemployment benefit ratio ρ_b or a increase in the bargaining power of unions $(1-\epsilon)$) exert a dampening impact of the terms of trade, i.e. pushing towards $\rho_\phi < 1$.

The equilibrium value of the terms of trade then results of these two opposite forces, the final effect depending on which channel dominates.

C Reaching the first-best solution

C.1 Employment subsidy policy

In this section, we focus on the employment subsidy policy that is needed to remove the inefficiency due to labor market institutions. We proceed in two steps. First, we determine the size of the employment subsidy that is needed to offset the inefficiency due to to labor market institutions at the intensive margin (on hours worked, see equation (42)). Second, we show that this subsidy is also able to eliminate the inefficiency gap due to LMIs on vacancies (see equation (43)).

Targeting labor market inefficiencies at the intensive margin Here, the objective is to determine ρ_c that removes the labor market inefficiencies that dwell on the labor intensive margin, i.e. such that $\Theta = \Psi$. From Equation (67), one can rewrite the definition of Θ according to:

$$\begin{aligned} \Theta &= \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}} \left[\frac{1}{\omega}1+\eta-\mathcal{A}\right]^{\frac{\psi}{1-\psi}} \mathcal{A} \\ \text{with} \qquad \mathcal{A} &= \left[(1-\epsilon)(1+\eta)+\epsilon\alpha\right] \frac{1-\rho_c}{1-\rho_b\epsilon-\rho_c(1-\epsilon)} \end{aligned} \tag{68}$$

Hence, given the expression of Ψ in Equation (38)), the ratio $\frac{\Theta}{\Psi}$ can be written as:

$$\frac{\Theta}{\Psi} = \underbrace{\left[\frac{1+\eta-\mathcal{A}}{\psi(1+\eta-\alpha)}\right]^{\frac{\psi}{1-\psi}}}_{(a)} \underbrace{\left[\frac{\mathcal{A}}{(1+\eta)(1-\psi)+\psi\alpha}\right]}_{(b)} \tag{69}$$

If $\frac{1+\eta-\mathcal{A}}{\psi(1+\eta-\alpha)} = 1$, then Term (a) is equal to 1 in Equation (69). Provided this holds, it is straightforward that Term (b) is also equal to 1 in Equation (69). That is, having $\frac{1+\eta-\mathcal{A}}{\psi(1+\eta-\alpha)} = 1$ is the necessary and sufficient condition to establish $\Theta = \Psi$.

Starting from this condition:

$$\frac{1+\eta-\mathcal{A}}{\psi(1+\eta-\alpha)} = 1 \quad \leftrightarrow \quad 1+\eta-\mathcal{A} = \psi(1+\eta-\alpha)$$

Making use of Equation (68) for $\mathcal{A}$:

$$\begin{aligned} & 1+\eta-[(1-\epsilon)(1+\eta)+\epsilon\alpha]\,\frac{1-\rho_c}{1-\rho_b\epsilon-\rho_c(1-\epsilon)} = \psi(1+\eta-\alpha) \\ \Rightarrow \quad & \underbrace{\frac{(1+\eta)(1-\psi)+\psi\alpha}{(1+\eta)(1-\epsilon)+\epsilon\alpha}}_{\kappa}[1-(1-\epsilon)\rho_c-\epsilon\rho_b] = 1-\rho_c \end{aligned} \tag{70}$$

From which we deduce:

$$\begin{aligned} & \rho_c = \frac{1-\kappa+\kappa\epsilon\rho_b}{1-\kappa(1-\epsilon)} \\ \Rightarrow \quad & \rho_c = \frac{(1+\eta)(1-\epsilon)+\epsilon\alpha-[(1+\eta)(1-\psi)+\psi\alpha]+\epsilon\rho_b[(1+\eta)(1-\psi)+\psi\alpha]}{(1-\epsilon)\,[1+\eta-(1+\eta)(1-\psi)-\psi\alpha]} \end{aligned}$$

Simplifying the numerator of Equation (71) leads to:

$$1-\kappa(1-\rho_b\epsilon) = \frac{(1+\eta-\alpha)(\psi-\epsilon)+\epsilon\rho_b\,[(1+\eta)(1-\psi)+\psi\alpha]}{(1+\eta)(1-\epsilon)+\epsilon\alpha}$$

Simplifying the denominator of Equation (71) leads to:

$$1-\kappa(1-\epsilon) = \frac{(1+\eta-\alpha)(\psi-\epsilon)+\epsilon\,[(1+\eta)(1-\psi)+\psi\alpha]}{(1-\epsilon)\psi(1+\eta-\alpha)+\epsilon\alpha}$$

Replacing these two expressions in Equation (71) finally gives the optimal value for ρ_c that eliminates the inefficiency gap due to LMIs, ie ensuring $\Theta = \Psi$, denoted ρ_c^{es}:

$$\rho_c^{es} = \frac{(1+\eta-\alpha)(\psi-\epsilon)+\epsilon\rho_b\,[(1+\eta)(1-\psi)+\psi\alpha]}{(1+\eta-\alpha)(\psi-\epsilon)+\epsilon\,[(1+\eta)(1-\psi)+\psi\alpha]}$$

Targeting labor market inefficiencies at the extensive margin The objective is now to determine whether ρ_c^{es} (determined above) also eliminates the LMIs inefficiency that affects the extensive labor margin, ie such that $LMI^V = 1$ in System (42)-(44). As established above, we have determined ρ_c such that $\Theta = \Psi$, which is ensured under condition (70). Alternatively, this condition writes down as:

$$
\begin{aligned}
& \frac{(1+\eta)(1-\psi)+\psi\alpha}{(1+\eta)(1-\epsilon)+\epsilon\alpha} \frac{[1-(1-\epsilon)\rho_c-\epsilon\rho_b]}{1-\rho_c} = 1 \\
\Leftrightarrow \quad & \frac{1-\rho_c}{[1-(1-\epsilon)\rho_c-\epsilon\rho_b]} \frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1+\eta)(1-\psi)+\psi\alpha} = 1 \\
\Leftrightarrow \quad & LMI^V = 1
\end{aligned}
$$

This proves that subsidizing employment by an amount ρ_c^{ES} (in proportion of the real wage per worker) allows to eliminate the inefficiency due to labor market institutions on both margins, such that $\Theta = \Psi$ and $\Upsilon = 1$, implying $LMI^h = LMI^V = 1$. Notice, however, that the trade externality might remain, implying some inefficiency gap with respect to the first best.

C.2 Combining employment and trade policy

As shown above, neither the trade policy nor the employment subsidy is able to eliminate inefficiencies when implemented alone. This also suggests that implementing both should constitute the first-best policy to reach the social planner's allocation. Assume the employment subsidy that eliminates the LMIs inefficiency is implemented (ie, by setting $\rho_c = \rho_c^{ES}$). From System (42)-(44), it is straightforward that completing this with a trade tax such that $\tau_e = \mu^* - 1$ allows to eliminate both the labor market inefficiency ("LMI^h") and the trade externality ("Trade Ext.") that dwell on hours worked. Under such a policy rule τ_e, ρ_c^{ES} as defined in Equations (49) and (50), hours worked and the terms of trade reach their first-best values for $TW = 1 \;\leftrightarrow\; \tau_x = 0, \;\; \forall \;\; x = c, f, w$. Given that the function relating vacancies to hours worked also matches the efficient one, this also establishes that vacancies are also equal to their first-best value. This generalizes to the whole set of macroeconomic variables.

D The second-best policy

In this section, we provide the elements of proof regarding the second-best tax policy in the absence of trade taxes and employment subsidy.

D.1 Difference in marginal utilities

In solving the Ramsey problem, the government takes into account the decision rules of the private agents. In presence of trade and labor market externalities, this is likely to induce a difference in the government's utility function and the planner's one ($\mathcal{U}^g \neq \mathcal{U}^{sp}$. This, in turn, induces different expressions for marginal utilities. Given our functional forms, the first-order conditions with respect to vacancies (V) and hours worked (h) respectively given by Equations (31) and (30) can be written as:

$$\mathcal{U}_h^{sp\prime} = 0 \quad \Leftrightarrow \quad \alpha A \frac{C_H^{\xi-1} C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} - \sigma_L h^{1+\eta-\alpha} = 0 \tag{71}$$

$$\begin{aligned} \mathcal{U}_V^{sp\prime} = 0 \quad &\Leftrightarrow \quad \left[\psi A \chi V^{\psi-1} h^{\alpha} - \bar{\omega}\right] \frac{C_H^{\xi-1} C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} - \sigma_L \frac{h^{1+\eta}}{1+\eta} \chi \psi V^{\psi-1} = 0 \\ &\Leftrightarrow \quad \frac{\psi}{V}\left(\left[A\chi V^{\psi} h^{\alpha} - \frac{\bar{\omega} V}{\psi}\right] \frac{C_H^{\xi-1} C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} - \sigma_L \frac{h^{1+\eta}}{1+\eta} \chi V^{\psi}\right) = 0 \end{aligned} \tag{72}$$

From this, replacing C_H, C_F and ϕ from the planner's solving, we can express the discrepancy through the following system (details of the calculus are reported in the online Appendix):

$$\begin{cases} \mathcal{U}_V^{g\prime} = \frac{\psi}{V}\left\{\left(\frac{\sigma^*-1+\xi}{\sigma^*}\right)\left[(1-\xi)(Y-\bar{\omega}V)\right]^{-\frac{1-\xi}{\sigma^*}}\left(\chi A V^{\psi} h^{\alpha} - \bar{\omega}\frac{V}{\psi}\right) - \sigma_L \frac{h^{1+\eta}}{1+\eta}\chi V^{\psi}\right\} \\ \mathcal{U}_V^{sp\prime} = \frac{\psi}{V}\left\{(\mu^*)^{-\frac{1-\xi}{\mu^*}}(1+t^*)^{-\frac{1-\xi}{\sigma^*}}\left[(1-\xi)(Y-\bar{\omega}V)\right]^{-\frac{1-\xi}{\sigma^*}}\left[A\chi V^{\psi} h^{\alpha} - \frac{\bar{\omega}V}{\psi}\right] - \sigma_L \frac{h^{1+\eta}}{1+\eta}\chi V^{\psi}\right\} \end{cases} \tag{73}$$

A similar discrepancy appears regarding the marginal utility of the intensive labor margin:

$$\begin{cases} \mathcal{U}_h^{g\prime} = \left[(1-\xi)(Y-\bar{\omega}V)\right]^{-\frac{1-\xi}{\sigma^*}}\left(\frac{\sigma^*-1+\xi}{\sigma^*}\right) A\alpha\chi V^{\psi} h^{\alpha-1} - \chi V^{\psi}\sigma_L h^{\eta} \\ \mathcal{U}_h^{sp\prime} = \left[(1-\xi)(Y-\bar{\omega}V)\right]^{-\frac{1-\xi}{\sigma^*}}(\mu^*)^{-\frac{1-\xi}{\mu^*}}(1+t^*)^{-\frac{1-\xi}{\sigma^*}}\alpha A\chi V^{\psi} h^{\alpha-1} - \chi V^{\psi}\sigma_L h^{\eta} \end{cases} \tag{74}$$

D.2 Obtaining the second-best tax scheme

Assuming that $\tau_e = \rho_c = 0$, hours worked at the decentralized equilibrium are equal to:

$$
\begin{aligned}
h^{dec} &= \left[\frac{A\alpha}{\sigma_L}\frac{1}{TW}\left(\frac{1}{(1-\xi)\Theta}\right)^{\frac{(1-\xi)}{\sigma^*}}\right]^{\nu} \\
\text{with} & \quad \begin{cases} TW \equiv \frac{(1+\tau_c)(1+\tau_f)}{1-\tau_w} \\ \nu \equiv \frac{1-\psi}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}} \\ \Theta = \left(\frac{\chi A}{1+\eta}\right)^{\frac{1}{1-\psi}}\left(\frac{\epsilon}{\bar{\omega}}\frac{(1+\eta)(1-\rho_b)-\alpha}{1-\epsilon\rho_b}\right)^{\frac{\psi}{1-\psi}}\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\epsilon\rho_b} \end{cases}
\end{aligned}
$$

Vacant jobs, output and the terms of trade are then given by:

$$
\begin{aligned}
V &= \underbrace{\Theta^{\frac{1}{\psi}}\left[\frac{A\chi}{1+\eta}\right]^{-\frac{1}{\psi}}\left[\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\rho_b\epsilon}\right]^{-\frac{1}{\psi}}}_{\Omega_v} h^{\frac{\alpha}{1-\psi}} \\
Y-\bar{\omega}V &= \Theta h^{\frac{\alpha}{1-\psi}} \qquad (75)\\
\phi &= ((1-\xi)\Theta)^{\frac{1}{\sigma^*}}\, h^{\frac{\alpha}{\sigma^*(1-\psi)}}
\end{aligned}
$$

Such that:

$$
V = \Omega_v h^{\frac{\alpha}{1-\psi}} \qquad (76)
$$

Coming back to the government's problem:

$$
\begin{aligned}
\max_{TW}\ \mathcal{U}^g &= (1-\xi)^{-\frac{1-\xi}{\sigma^*}}\left[\overbrace{A\chi V^{\psi}h^{\alpha}-\bar{\omega}V}^{Y-\bar{\omega}V}\right]^{\frac{\sigma^*-1+\xi}{\sigma^*}} - \chi V^{\psi}\sigma_L\frac{h^{1+\eta}}{1+\eta} \quad , \\
\text{s.t.} & \quad \begin{cases} \frac{\bar{\omega}}{\chi}V^{1-\psi} = \epsilon\left[\frac{1+\eta-\alpha}{1+\eta}Ah^{\alpha}-b\right], \\ \sigma_L h^{1+\eta}\left[(1-\xi)(A\chi V^{\psi}h^{\alpha}-\bar{\omega}V)\right]^{\frac{1-\xi}{\sigma^*}} = \alpha A h^{\alpha}\frac{1}{TW} \end{cases}
\end{aligned}
$$

Replacing net output and vacancies via Equations (75) and (76), the government's program can be rewritten as:

$$\begin{aligned}
\max_{TW}\ \mathcal{U}^g &= (1-\xi)^{-\frac{1-\xi}{\sigma^*}}\,\Theta^{\frac{\sigma^*-1+\xi}{\sigma^*}} h^{\frac{\alpha}{1-\psi}\frac{\sigma^*-1+\xi}{\sigma^*}} - \frac{\chi\sigma_L\Omega_v^{\psi}}{1+\eta} h^{1+\eta+\frac{\alpha\psi}{1-\psi}} \\
\text{s.t.} \qquad & h = \left[\frac{A\alpha}{\sigma_L}\frac{1}{TW}\left(\frac{1}{(1-\xi)\Theta}\right)^{\frac{(1-\xi)}{\sigma^*}}\right]^{\nu}
\end{aligned} \tag{77}$$

Put it differently:

$$\begin{aligned}
\max_{TW}\ \mathcal{U}^g &= \mathcal{A}_1 h^{\frac{\alpha}{1-\psi}\frac{\sigma^*-1+\xi}{\sigma^*}} - \mathcal{A}_2 h^{1+\eta+\frac{\alpha\psi}{1-\psi}} \\
\text{with} \qquad & \begin{cases} \mathcal{A}_1 = (1-\xi)^{-\frac{1-\xi}{\sigma^*}}\,\Theta^{\frac{\sigma^*-1+\xi}{\sigma^*}} \\ \mathcal{A}_2 = \frac{\sigma_L}{A}\Theta\frac{1-\rho_b\epsilon}{(1-\epsilon)(1+\eta)+\epsilon\alpha} \end{cases} \\
\text{and} \qquad & h = \left[\frac{A\alpha}{\sigma_L}\frac{1}{TW}\left(\frac{1}{(1-\xi)\Theta}\right)^{\frac{(1-\xi)}{\sigma^*}}\right]^{\nu} = \mathcal{H}(TW)
\end{aligned}$$

From this, we deduce the first-order condition (with $h = \mathcal{H}(TW)$):

$$0 = \left(\frac{\alpha}{1-\psi}\frac{\sigma^*-1+\xi}{\sigma^*}\right)\mathcal{A}_1 h^{\frac{\alpha}{1-\psi}\frac{\sigma^*-1+\xi}{\sigma^*}} - \left(1+\eta+\frac{\alpha\psi}{1-\psi}\right)\mathcal{A}_2 h^{1+\eta+\frac{\alpha\psi}{1-\psi}}$$

Give the definitions of $\mathcal{A}_1$ and $\mathcal{A}_2$, this gives the solution for the value of hours worked at the second-best equilibrium:

$$h^{\varsigma} = \frac{\alpha A}{\sigma_L}\frac{\mathcal{A}_1}{\Theta}\left(\frac{\sigma^*-1+\xi}{\sigma^*}\right)\frac{1}{1-\epsilon\rho_b}\left(\frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1+\eta)(1-\psi)+\psi\alpha}\right),$$

with $1+t^* = \frac{\sigma^*}{\sigma^*-1+\xi}$ and ς such as:

$$\begin{aligned}
\varsigma &= \frac{\alpha\psi-\eta(1-\psi)}{1-\psi} - \frac{\alpha(\sigma^*-1+\xi)-\sigma^*(1-\psi)}{\sigma^*(1-\psi)} \\
&= \frac{1}{1-\psi}\left[(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}\right] \\
&= \frac{1}{\nu}
\end{aligned}$$

with ν as defined above. Plugging the definitions of $\mathcal{A}_1$ and $\mathcal{A}_2$ and simplifying then leads us to the second-best equilibrium value of hours worked:

$$h^{sb} = \left[\frac{\alpha A}{\sigma_L}\left((1-\xi)\Theta\right)^{-\frac{1-\xi}{\sigma^*}}\left(\frac{1}{1+t^*}\right)\left(\frac{1}{1-\epsilon\rho_b}\right)\frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1+\eta)(1-\psi)+\psi\alpha}\right]^{\nu} \quad (78)$$

Last step consists in finding the second-best value of the tax wedge (TW^{sb}) such that $\mathcal{H}(TW^{sb}) = h^{sb}$, with $\mathcal{H}(TW)$ as given by Equation (77):

$$h^{sb} = \left[\frac{A\alpha}{\sigma_L}\frac{1}{TW}\left(\frac{1}{(1-\xi)\Theta}\right)^{\frac{(1-\xi)}{\sigma^*}}\right]^{\nu}$$

$$\Leftrightarrow \quad \left[\frac{\alpha A}{\sigma_L}\left(\frac{1}{(1-\xi)\Theta}\right)^{\frac{1-\xi}{\sigma^*}}\left(\frac{1}{1+t^*}\right)\left(\frac{1}{1-\epsilon\rho_b}\right)\frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1+\eta)(1-\psi)+\psi\alpha}\right]^{\nu} = \left[\frac{A\alpha}{\sigma_L}\frac{1}{TW}\left(\frac{1}{(1-\xi)\Theta}\right)^{\frac{(1-\xi)}{\sigma^*}}\right]^{\nu}$$

This leads us to the second-best value of the tax wedge TW^{sb}:

$$TW^{sb} = (1-\rho_b\epsilon)\frac{1-\psi}{1-\epsilon}\left(\frac{1+\eta+\alpha\frac{\psi}{1-\psi}}{1+\eta+\alpha\frac{\epsilon}{1-\epsilon}}\right)(1+t^*)$$

D.3 Properties of the second-best tax scheme

Let us decompose the second-best optimal tax wedge as follows:

$$TW^{sb} = \underbrace{(1-\rho_b\epsilon)}_{(a)}\underbrace{\frac{1-\psi}{1-\epsilon}}_{(b)}\underbrace{\left(\frac{1+\eta+\alpha\frac{\psi}{1-\psi}}{1+\eta+\alpha\frac{\epsilon}{1-\epsilon}}\right)}_{(c)}\underbrace{(1+t^*)}_{\text{Trade Ext.}} \quad (79)$$

In this section, we investigate the effects of the two inefficiencies on the optimal tax wedge.

Trade externality As straightforward from Equation (79), the trade externality pushes towards an increase in the optimal tax wedge, as $\frac{\partial TW}{\partial t^*} > 0$.

Labor market inefficiencies To characterize how LMIs affect the second-best tax wedge, we rely on the signs of $\frac{\partial TW}{\partial \rho_b} > 0$ and $\frac{\partial TW}{\partial \epsilon} > 0$.

Consider first the role of the unemployment benefit system. It comes that:

$$\frac{\partial TW}{\partial \rho_b} = -\epsilon \left[\frac{1-\psi}{1-\epsilon}\right] \left(\frac{1+\eta+\alpha\frac{\psi}{1-\psi}}{1+\eta+\alpha\frac{\epsilon}{1-\epsilon}}\right)(1+t^*) > 0$$

That is, an increase in the unemployment benefit ratio calls for a reduced tax wedge. Given our above analysis, this suggests that the dampening effect of unemployment benefits on the extensive labor margin dominates. To cope with the inefficiently low number of vacant jobs due to $\rho_b > 0$, it is necessary to reduce the tax burden. This contributes to restore efficiency along the extensive margin by raising the hours worked per employee, as $\mathcal{H}(TW)' > 0$ and $V = \mathcal{V}(h)$ is increasing in h.

Consider now the effects of the firms bargaining power (ϵ). Unlike ρ_b, the impact of ϵ on the optimal tax wedge channels to all three elements (a), (b) and (c) of TW^{sb} (see Equation (79)). We thus determine the derivative of each term separately. It comes that:

$$\frac{\partial(a)}{\partial\epsilon} = -\rho_b < 0 \tag{80}$$

$$\frac{\partial(b)}{\partial\epsilon} = \frac{1-\psi}{(1-\epsilon)^2} > 0 \tag{81}$$

$$\frac{\partial(c)}{\partial\epsilon} = -\left(1+\eta+\alpha\frac{\psi}{1-\psi}\right)\frac{\alpha}{(1-\epsilon)^2\left(1+\eta+\alpha\frac{\epsilon}{1-\epsilon}\right)^2} < 0 \tag{82}$$

From effects (a) and (c), an excessive workers' bargaining power relative to its weight in the matching function ($\epsilon < \psi$) calls for an increase in the overall tax wedge (see Equations (80) and (82)). This illustrates the influence of LMIs at the labor intensive margin, which push hours worked above their first-best value every thing else equal (see Proposition 1). In contrast, through Effect (b), having $\epsilon < \psi$ rather calls for a reduced tax wedge (see Equation (81)), due to the dampening effect of LMIs on the extensive labor margin everything else equal for a given h.

D.3.1 Proposition 7. Financing the Reforms: Indirect Versus Direct Taxation

In this section, we prove the "tax base" condition stated in Proposition 7. From Equation (55), the first-order condition of the problem with respect to τ^f yields:

$$\mathcal{H}'(TW) \times \left[\frac{\partial TW}{\partial \tau^f} + \frac{\partial TW}{\partial \tau^c}\frac{\partial \tau^c}{\partial \tau^f}\right] \times \left[\mathcal{V}'(h)\mathcal{U}_V^{g'} + \mathcal{U}_h^{g'}\right] = 0 \tag{83}$$

In view of Equation (83), two cases should be considered. First, if $\left[\frac{\partial TW}{\partial \tau_f} + \frac{\partial TW}{\partial \tau_c}\frac{\partial \tau_c}{\partial \tau_f}\right] = 0$, any change in the payroll tax is offset by the opposite change in the indirect tax, such that it does not affect the tax wedge. Consequently, changing the payroll tax rate has no impact on hours worked or vacancies and more broadly on the decentralised equilibrium allocation. Secondly, if $\left[\frac{\partial TW}{\partial \tau_f} + \frac{\partial TW}{\partial \tau_c}\frac{\partial \tau_c}{\partial \tau_f}\right] \neq 0$, the government is able to manipulate the payroll tax rate such that it improves the decentralised allocation. It is the government budget constraint that determines the condition under which changes in direct taxation are offset or not by the opposite change in indirect taxation.

Proof. Using the decision rules, the budgetary constraint of the government is:

$$\tau_c + \frac{\tau^f + \tau^w}{1+\tau^f} + \rho_T = \rho_g + (1-\tau^w)\rho_b \frac{1-N}{N}$$

where we assume that $\frac{b}{1+\tau^f} = \rho_b wh \Rightarrow \frac{b}{1+\tau^f}N = \rho_b(Y-\omega V)$. We thus deduce that a sufficient condition for $\left|\frac{d\tau^c}{d\tau^f}\right| < 1$ is $\frac{1-\tau^w}{1+\tau^f}\frac{whN}{Y-\omega V} < \frac{C_H+\phi C_F}{Y-\omega V} + \rho_b \varepsilon \frac{\tau^f-\tau^c}{TW}$, where $\varepsilon \equiv -N'(TW)\frac{TW}{N}$ stands for the elasticity of employment to the tax burden. If $\frac{whN}{Y-\omega V} < \frac{C_H+\phi C_F}{Y-\omega V} \equiv \frac{C}{Y-\omega V}$, then $\left|\frac{d\tau^c}{d\tau^f}\right| < 1$ because $\frac{1-\tau^w}{1+\tau^f} < 1$ and $\rho_b \varepsilon \frac{\tau^f-\tau^c}{TW} > 0$. In this case $\frac{dTW}{d\tau^f} > 0$ given the required adjustment in τ^c. $\square$