# Fiscal and Currency Union with Default and Exit*

Alessandro Ferrari[†] Ramon Marimon[‡] Chima Simpson-Bell[§]

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**Abstract**

Countries which share a common currency potentially have strong incentives to share macroeconomic risks through a system of transfers to compensate for the loss of national monetary policy. However, the option to leave the currency union and regain national monetary policy can place severe limits on the size and persistence of transfers which are feasible inside the union. In this paper, we derive the optimal transfer policy for a currency union as a dynamic contract subject to enforcement constraints, whereby each country has the option to unpeg from the common currency, with or without default on existing obligations. Our analysis shows that the lack of independent monetary policy, or an equivalent independent policy instrument, limits the extent of risk-sharing within a currency union; nevertheless, in the latter, the optimal state-contingent transfer policy take as given the optimal monetary policy as to implement a constrained efficient allocation that minimises the losses of the monetary union. At the steady state welfare is lower than in a fiscal union with independent monetary policies. Neverhtless, in our simulations, the macroeconomic stabilisation effects and the social values achieved, under the two different union regimes, are quantitatively almost the same.

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[†]European University Institute
[‡]European University Institute, UPF - Barcelona GSE, CEPR and NBER
[§]European University Institute

# 1 Introduction

In a federal state, with a single currency, states share risks through the federal budget (automatic stabilisers) and other risk-sharing fiscal policies. Furthermore, well-functioning, and integrated, markets – in particular, financial markets – also provide insurance against local shocks and can help to circumvent local nominal rigidities, leaving little role for an independent monetary policy. However, in a monetary union – such as the European EMU – the federal fiscal risk-sharing instruments are missing and markets may not be developed and integrated enough to provide the necessary private risk-sharing. Therefore, independent monetary policy may still have potential value. This point is made formally in Auclert and Rognlie (2014) and Farhi and Werning (2017) who derive optimal risk-sharing policies in a setting with nominal rigidities. Nevertheless, they do not account for two characteristic aspects of unions: in a union of sovereign countries there is limited enforcement. – exit is always an option even if, as in the case of Brexit, it can be a costly option –, but a union is a long-term partnership where mutually beneficial policies bind countries together, deterring them from exiting. Risk-sharing policies can play this role.

In fact, in the euro crisis the threat of exit from the Euro Area, and defaulting on payment obligations, has triggered sovereign debt spreads of Greece and other countries. Bayer et al. (2018) provide evidence that market participants even attached positive probability to Germany and France's exit from the common currency during the crisis. Any risk-sharing within a currency union is therefore subject to participation constraints, for both borrowing and lending countries. This point has been made by Abraham et al. (2019) who characterise constrained-efficient risk-sharing contracts as a self-enforcing mechanism within a union which is subject to limited enforcement constraints. Nevertheless, since they model a fiscal – not a monetary – union, the loss (or possible gain, if exiting) of an independent monetary policy plays no role in their analysis.

Our paper integrates these two earlier approaches by analysing long-term risk-sharing contracts as self-enforcing mechanisms in currency unions, taking into account that in monetary unions exit can take two forms. Union members can exit the union to regain control of monetary policy. For example, Sweden or Poland are full members of the European Union who persist in keeping their currencies. Alternatively, union members can exit the union to renege on their obligations; in particular, default on their debts. Default, or partial default, does not necessarily imply exit from the union (e.g. defaulting states in United States, Greece in 2012) but, as a union's 'participation constraint', the relevant case is when the possibility of default is associated with exit.

We model the union as two identical countries facing a simple nominal rigidity which creates a stabilisation role for monetary policy. There is no aggregate risk, meaning that country-risks are fully negatively correlated. We then derive the optimal history dependent transfer policies as a long-term dynamic contract subject to participation constraints. Under these constraints, the contract must improve upon an outside option in which each country has independent monetary policy, allowing it to eliminate the distortion caused by the nominal rigidity, and can borrow and lend using defaultable one-period bonds. Due to the forward looking nature of the participation constraints, we are able to characterize the constrained efficient allocation using the recursive contract solution techniques developed in Marcet and Marimon (2019). Effectively, the contract, as a social contract, gives more weight to a country whenever its participation constraint binds.

We compare the performance of the currency union against two benchmarks: an optimal fiscal union in which the nominal rigidity does is fully eliminated by independent monetary policy, and a two good version of the defaultable debt economy in Arellano (2008). We start by characterizing a number of results regarding the comparison between the fiscal union and the currency union with fiscal transfers. We show that if the fiscal transfers are able to achieve full risk-sharing then the two unions are identical. This result is a version of what the literature has labeled the "risk-sharing miracle". This result stems from the ability of a common currency to stabilize both economies when full risk-sharing is achieved on the fiscal side.

We show that when the planner cannot attain full risk-sharing the fiscal union is strictly better than the currency union. This comes from the deadweight loss that the common monetary policy entails. Such loss shrinks the production possibility frontier, thereby reducing the maximum value attainable by a planner in a currency union.

We also show that optimal common monetary policy is designed to minimize the deadweight loss and that the planner allocation in this economy is still constrained efficient.

We then simulate our economy to study three main features of our model. First we ask whether these kind of contracts are feasible. Secondly, if they are, we ask what is the optimal design of fiscal transfers in terms of size and cyclicality. Thirdly, we investigate how costly is the deadweight loss stemming from the common monetary policy.

In our simulations we find that the fiscal and the currency union are close to identical. We attribute this result to the ability of the optimal policies in the currency union to produce very small deadweight losses. Secondly, in most of our parametrizations, the steady states feature partially state dependent consumption, meaning that the limited enforcement constraints prevent the central planner from achieving full risk-sharing.

As we have mentioned, our work is close to Auclert and Rognlie (2014) and Farhi and Werning (2017) and, therefore, to Hoddenbagh and Dmitriev (2017) who takes a similar approach. We build on this work by considering the participation constraints implied by the option of unpegging from the common currency and , taking it a step forward, by deriving constrained-efficient recursive policies in a monetary union with equally patient countries. We also build on Abraham et al. (2019), although, in contrast with our work, they assume – as the sovereign literature does, to match observed levels of debt – that the 'debtor country' is impatient while the 'lender country' is risk-neutral, in fact, the latter acts as a *financial stability fund*[1]. In this respect, our work is also related to the extensive sovereign debt literature; for example, Gourinchas et al. (2019) solve for the

[1]Their contracts also account for moral-hazard constraints: the 'debtor country' can reduce its risk profile with non-contractable effort. We abstract from this feature.

optimal application of a no-bailout rule, finding that less than full enforcement can be sufficient to prevent a fiscally weak country from engaging in risky borrowing.

In our analysis we take the entry or formation decision for the union as given, and focus on the possibility of a breakup, but there is also a literature which considers union formation incentives. In particular, Cooper and Kempf (2003) investigate the conditions under which countries will be able to cooperate to realize the gains from entering a monetary union. Cooper et al. (2008) examines the conditions under which a central authority in a multi-region economy will find it optimal to take on the obligations of regional governments.

After solving for the constrained efficient allocation in our framework, we also propose an approach to implementing the net payments within the union through trading of state contingent debt contracts. For this we rely on Kehoe and Perri (2004) and Alvarez and Jermann (2000) which demonstrate how constrained efficient allocations with limited commitment can be decentralised using trading of securities. A technical contribution of our paper is that it features a unique feedback between the constrained efficient allocation and its decentralisation. The asset positions calculated in the decentralisation also determine the liabilities which each country carries into its outside option, and these outside options in turn determine the constrained-efficient allocation (recall that in our framework, exit does not imply default). This creates an interesting fixed point problem which we are able to solve numerically.

The rest of the paper is organised as follows. In Section 2 we describe the basic two-goods open economy with monopolistic competition in the non-tradeable goods sector and a nominal friction. We describe the contracts which make up the fiscal and currency unions, and the outside options available to each country. We then characterize the constrained efficient allocations of the union contracts and the associated implementation using state contingent debt. In this section we provide the main theoretical results. In Section 3 we display the policy functions in the different economies and simulate their responses to a debt crisis. We also study the behaviour of the stochastic steady states.Section 4 shows discusses the results under different parametrization of the model. Section 5 offers concluding comments.

## 2 Model

We start by modelling a fiscal union between two countries. The two countries have endowments of tradable goods and produce non-tradeables. Differently from the existing literature, we model the two countries as symmetric in terms of risk aversion and patience. Agents can partake in risk-sharing through a long term contract subject to participation constraints, where the outside option is defined by an Arellano economy. Namely, countries can opt out of the risk-sharing contract and borrow through non-state contingent bonds from a risk neutral lender. When in the Arellano economy agents can default on their debt subject to an output cost and temporary exclusion from financial markets.

A key feature of this economy is that if a country leaves the contract but does not default, it starts with a stock of liabilities equal to the present discounted value of the promised transfers in the fiscal union.

Secondly, we extend the setup to accommodate a currency union as a long term risk-sharing contract. In this context nontradables producers are subject to staggered prices nominal rigidities. In this setup the outside option allows countries to move to an Arellano economy with independent monetary policy. When countries leave the currency area they can again pay their previous obligations or default on them. Previous work focuses on optimal risk-sharing schemes within a monetary union without accounting for the incentives of the countries to leave the union.

### 2.1 Environment

Two infinitely lived countries $i \in \{1, 2\}$. Time is discrete. Each period a country receives a random endowment of an identical, freely tradeable good $Y_T^i$. This is the only source of uncertainty in the model. As there is no aggregate uncertainty the country specific endowments are fully negatively correlated; i.e. for all $t \geq 0$, $Y_{T,t}^1 + Y_{T,t}^2 = Y_T$. Uncertainty is described by a finite state Markov process $\{s_t\}$ with elements $s_t \in \mathcal{S}$ and transition matrix $\Pi$ – in fact, $s_t = Y_{T,t}^1$. Given this Markov structure, the relevant exogenous state in this environment will be the vector $(s_{t-1}, s_t)$[2].

Preferences over consumption of tradeable goods $C_T$, non-tradeable goods $C_{NT}$ and labour supply $N$ are given by the utility function:

$$U_i = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left( \frac{(C_{T,t+k})^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{(C_{NT,t+k})^{1-\gamma}}{1-\gamma} - \frac{N_{t+k}^{1+\phi}}{1+\phi} \right) \right) \tag{1}$$

All goods are perishable, non-tradeable goods must be consumed in the country in which they are produced, and labour is immobile between countries.

Non-tradeable goods are produced by each country using a technology which is linear in labour input. In each country there is a continuum of firms $j \in [0, 1]$ which produce output according to:

$$Y_{NT}^{ij} = N_{ij} \tag{2}$$

The consumer's utility value from consuming each of the varieties $j$ is given by the CES aggregator:

$$C_{NT}^i = \left( \int_{j=0}^{1} (C_{NT}^{ij})^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

[2]With an abuse of notation, we also denote $(s_{t-1}, s_t)$ by $s$, making this explicit when needed.

where $\epsilon > 1$ is the elasticity of substitution between varieties. Consumption of each variety must equal output, i.e. $C^{ij}_{NT} = Y^{ij}_{NT}$, and labour market clearing implies that $N_i = \int_j N_{ij}$.

We assume that production of non tradeables is subsidised at $1/\epsilon$ to erase the monopoly quantity friction. Given its price $P^{ij}_{NT}$, the producer of variety $j$ satisfies demand $C^{ij}$ by hiring $N^{ij}$ workers and earns profits

$$\Pi^{ij} = (P^{ij}_{NT} - (1 + \tau^i_L)W^i)N^{ij} \tag{4}$$

Where $\tau^i_L$ is the government labor subsidy. Profits are distributed to households.

## 2.2 Fiscal Union

We model the fiscal union as a long term contract. In this setup countries receive state contingent net transfers of the tradeable good $\tau^i(s)$ from each other to absorb the risk associated with the realization of the tradeable goods shock. In this sense, the contract can also be interpreted as a fiscal union. Country $i$'s consumption of the tradeable good is then

$$C^i_T(s) = Y^i_T(s) + \tau^i(s) \tag{5}$$

Countries remain in the contract as long as they do not choose to leave the fiscal union. Leaving the union results in the loss of the state contingent transfers.

### 2.2.1 Planner's problem

The planner arranges transfers within the union subject to each country's outside option:

$$\max_{\{C_{T,i},C_{NT,i},N_i\}} \sum_{i=1,2} \mu_i E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_{T,t})^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{(C_{NT,t})^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \right) \tag{6}$$

s.t.

$$C^i_T(s) = Y^i_T(s) + \tau^i(s) \tag{7}$$

$$\sum_{i=1,2} \tau^i(s) = 0 \tag{8}$$

$$\sum_{k=0}^{\infty} \beta^k \left( \frac{(C_{T,t+k})^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{(C_{NT,t+k})^{1-\gamma}}{1-\gamma} - \frac{N_{t+k}^{1+\phi}}{1+\phi} \right) \right) \geq V_i^o(s,B) \tag{9}$$

$$Y_{NT}^{ij} = N_{ij} \tag{10}$$

$$C_{NT}^i = \left( \int_{j=0}^1 (C_{NT}^{ij})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{11}$$

$$N_i = \int_j N_{ij} \tag{12}$$

Debt $B$ is defined as the stock of liabilities that a country would have outside the contract. We define this stock as the net present value of the promised transfers inside the contract. The decision of leaving the contract is irreversible. We assume that if a country leaves the contract it refinances its debt with a competitive outside lender borrowing at a risk free rate $r$. In other words, the outside option to the fiscal union is an Arellano type economy. Formally, the stock of liabilities that a country carries outside the contract is defined as

$$B_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} q_{t,s}(Y_{i,s} - c_{i,s}) \tag{13}$$

Where $q_{t,s} = \prod_{s=t}^{k} q_{s,s+1}$ $q_{t-1,t} \equiv \max_j \left\{ \beta \frac{U_j'^{t}}{U_j'^{t-1}} \right\}$. This assignment of liabilities will be made clearer when we outline the decentralization of the contract which describes the union. Note that the planner does not internalize the effect of within contract transfers on the value of the outside option.

### 2.2.2 Outside options

In each period, each country has the option of defaulting on its payments within the union, and choosing to leave the fiscal union. Thus, when it is inside the contract, country $i$ faces a choice over the actions $\{SR, LR, LD\}$, i.e. stay in the fiscal union and repay transfer commitments, leave the contract and honour payments, and leave and default. We assume that defaulting on payments triggers temporary exclusion from financial markets so that the country can no longer trade bonds for a stochastic number of periods. Following Arellano (2008), a country in autarky also suffers an output cost on its endowment of the tradeable good, $\chi(Y_T^i)$; this output cost is chosen to ensure that a country is more likely to consider default when its endowment of the tradeable good is low. We can write the decision problem of each country outside the contract in a recursive form. The value of leaving the contract takes the following form:

$$V_i^o(s,B) = \max_{LR,LD}\{V_i^{LR}(s,B), V_{LD}^i(s)\} \tag{14}$$

Namely, the agent can choose whether to repay the promised transfers or default on its obligations. If the country defaults it is temporarily relegated to financial autarky and faces a proportional output cost in terms of tradable endowment.

$$V_i^{LD}(s) = \max_{C_{NT,i},N_i} \frac{(Y_{T,i}-\chi(Y_{T,i}))^{1-\gamma}}{1-\gamma} + \alpha\left(\frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}\right) + \beta\mathbb{E}\left[\theta V_i^o(s',0) + (1-\theta)V_i^{LD}(s')\right] \tag{15}$$

Where $\theta$ is the probability with which the country financial markets exclusion is terminated and $\chi(E_T)$ is the output cost of financial autarky which, for a constant parameter $\psi$, takes the form:

$$\chi(Y_T) = \begin{cases} Y_T - \bar{Y}_T, & \text{for } Y_T \geq \bar{Y} \\ 0, & \text{for } Y_T < \bar{Y}_T \end{cases}, \text{ where } \bar{Y}_T = \psi\mathbb{E}Y_T$$

If the country regains access to financial markets, it does so with zero outstanding liabilities. If the country has left the risk-sharing agreement but opted not to default on its obligations, then the value of the problem is

$$V_i^{LR}(s,B) = \max_{C_{T,i},C_{NT,i},N_i,B_i'} \frac{C_{T,i}^{1-\gamma}}{1-\gamma} + \alpha\left(\frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}\right) + \beta\mathbb{E}V_i^o(s',B_i') \tag{16}$$

The budget constraints in each case are below. Where the price of tradables is the numeraire.
If the country leaves and repays then it has access to the non contingent one period bond as saving technology:

$$C_T^i(s) + P_{NT}^i(s)C_{NT}^i(s) + B_i \leq Y_T^i(s) + W^i(s)N^i(s) + \Pi^i(s) + B_i'Q(s,B_i') \tag{17}$$

We omit the production subsidy since it is rebated to households and cancels out with increased profits. The subsidy is such that output reaches its efficient level. One can think of a government taxing profits and subsidising production. This will exactly cancel out in the household budget constraint as profits are rebated. Where $Q(s, B_i')$ is the bond price set between the country outside the contract and the competitive lender. The bond price is given by

$$Q(s, B_i') = \frac{1}{r}\mathbb{E}_t(1 - D(s', B_i'))$$

Where $D(s', B_i')$ is the decision to default on debt in the next period.
If the country leaves and default on past liabilities then it has no saving technology and is subject to a per period output cost:

$$C_T^i(s) + P_{NT}^i(s)C_{NT}^i(s) \leq Y_T^i(s) - \chi(Y_T^i(s)) + W^i(s)N^i(s) + \Pi^i(s) \tag{18}$$

### 2.3 Currency Union

Next we study a nominal version of the model to evaluate how the to design a transfer system in a currency union. In order to account for the money in this economy we assume that non-tradeable producers are subject to staggered prices.

#### 2.3.1 Price setting for non-tradeables

Producers of non-tradeable goods face a rigidity in their price setting decisions. We incorporate this by assuming that at the beginning of each period firms must make their pricing decision before the realisation of the tradeable goods endowments $Y_{T,i}$, and wages cannot be conditional on this realisation[3].
Given its price $P_{NT}^{ij}$, the producer of variety $j$ satisfies demand $C^{ij}$ by hiring $N^{ij}$ workers and earns profits

$$\Pi^{ij} = (P_{NT}^{ij} - (1 + \tau_L^i)W^i)N^{ij} \tag{19}$$

which are distributed to households.
For convenience, define the labour wedge $\kappa(s)$ as

$$\kappa^i(s) = 1 - \frac{U_N^i(s)}{U_{NT}^i(s)} = 1 - {C_{NT}^i}^{\gamma+\phi}(s) \tag{20}$$

Such wedge arises due to the lack of state-contingent non-tradeables prices.
The non-tradeable good price is predetermined, hence producers maximize the expected profits across states. The optimal price setting rule is characterized in the following Lemma:

[3] Note that in our framework having independent *state-contingent labour taxes* will play the same role as having an independent monetary policy.

**Lemma 1** (Optimal Price Setting)**.** *The optimal price for non-tradeables producers is:*

$$P_{NT} = \frac{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma} W(s)}{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}}$$

*Such pricing implies zero expected labor wedge*

$$\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}\kappa(s) = 0. \tag{21}$$

*Proof.* See Appendix A. ■

Lemma 1 shows that, in the absence of uncertainty, the optimal price is equal to the wage. Recall that this result is obtained by levying a labor subsidy that undoes the monopoly markup. Similarly, in absence of uncertainty, the labor wedge is equal to zero. This result can be understood as the lack of state-contingency being inconsequential when the state is constant.

We model the currency union as a long term contract. Countries within the currency union face the same price for tradeable goods, so that:

$$P_T^1(s) = P_T^2(s) \tag{22}$$

The implicit assumption is that the fixed nominal exchange rate within the currency union is 1. Countries remain in the contract as long as they do not choose to unpeg from the common currency, or default on the net payments specified by the contract.

In this case the Ramsey planner is subject to pricing frictions and different outside options.

### 2.3.2 Outside options

In each period, each country has the option of defaulting on its payments within the union, and choosing to unpeg from the common currency and regain control of its monetary policy. We assume that either defaulting or unpegging implies abandoning the common currency. Thus, when it is inside the contract, country $i$ faces a choice over the actions $\{PR, UR, UD\}$, i.e. maintain the peg and repay transfer commitments, unpeg and honour payments, and unpeg and default. We assume that defaulting on payments triggers temporary exclusion from financial markets so that the country can no longer trade bonds.

We also assume that the cost of unpegging from the common currency is that the country can only trade non-state contingent bonds $B$, limiting its consumption smoothing ability. The repayment commitments which remain are still denominated in the common currency. Unpegging is also an irreversible decision.

We can write the decision problem of each country outside the contract in a recursive form. Suppose that country $i$ has already both defaulted on its debt and unpegged from the common currency. Its value function $V_i^{UD}(s)$ can then be written as:

$$V_i^{UD}(s) = \max_{C_{NT,i},N_i} \frac{(Y_{T,i} - \chi(Y_{T,i}))^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi} \right) + \beta \mathbb{E} \left[ \theta V_U^i(s', 0) + (1-\theta) V_i^{UD}(s') \right] \tag{23}$$

Suppose now that country $i$ has unpegged but not defaulted yet. Its value function $V_i^U(s)$ can be written as:

$$V_i^U(s, B) = \max_{P,U} \{ V_i^{UR}(s, B), V_i^{UD}(s) \} \tag{24}$$

where $V_{UR}$, the value of maintaining repaying the contractual obligations once the country has unpegged, is given by

$$V_i^{UR}(s, B) = \max_{C_{T,i},C_{NT,i},N_i,B_i'} \frac{(C_{T,i})^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{(C_{NT})^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi} \right) + \beta \mathbb{E} V_U^i(s', B') \tag{25}$$

Finally, the outside option of a country that is still inside the currency union contract is given by the option value of, just unpegging, or defaulting and unpegging at the same time:

$$V_i^o(s, B) = \max\{ V_i^{UR}(s, B), V_i^{UD}(s) \} \tag{26}$$

The budget constraints in each case are below. In each case the constraint is written in the common currency of the union (Euros). If the country unpegs from the common currency, $\epsilon^i(s)$ is the number of units of $i$'s currency per Euro. Note that the price of tradables is always in Euros.
Unpeg without defaulting:

$$P_T^i(s) C_T^i(s) + \frac{P_{NT}^i(s) C_{NT}^i(s)}{\epsilon^i(s)} + B_i' \leq P_T^i(s) Y_T^i(s) + \frac{W^i(s) N^i(s)}{\epsilon^i(s)} + \frac{\Pi^i(s)}{\epsilon^i(s)} + B_i' \frac{Q(s, B_i')}{\epsilon^i(s)} \tag{27}$$

Unpeg and default:

$$P_T^i(s) C_T^i(s) + \frac{P_{NT}^i(s) C_{NT}^i(s)}{\epsilon^i(s)} \leq P_T^i(s)(Y_T^i(s) - \chi(Y_T^i(s))) + \frac{W^i(s) N^i(s)}{\epsilon^i(s)} + \frac{\Pi^i(s)}{\epsilon^i(s)} \tag{28}$$

#### 2.3.3 Monetary Policy

We follow Farhi and Werning (2017) and Auclert and Rognlie (2014) in the definition of monetary policy. Within the currency union, due to the underlying price rigidity, demand externalities arise, generating a wedge between the private and social value of risk-sharing. Monetary policy optimally sets the union wide weighted wedge to zero.
Outside the monetary union, monetary policy is independent and country specific wedges are optimally equal to zero. This implies a relatively increased value of the outside option, compared to the fiscal union setup due to the independent monetary policy outside the currency area.
By the intratemporal first order condition the labour wedge is equal to zero in absence of nominal rigidities (given the subsidy to production). In presence of nominal rigidities the price of non tradeables is not equal to the wage implying that $\kappa^i(s) \neq 0$. Independent monetary policy equates the country specific labour wedge to zero.
We assume that monetary policy inside the currency union equates the weighted average labour wedge to zero. The weights are symmetric and equal to $1/2$.
The following Lemma characterizes optimal monetary policy.

**Lemma 2** (Optimal Monetary Policy)**.** *Optimal independent monetary policy implies*

$$\kappa^i(s) = 0, \quad \forall s$$

*Optimal monetary policy in a currency union implies*

$$\sum_{i=1,2} {C_{NT}^i}^{1-\gamma} \kappa^i(s) = 0, \quad \forall s$$

*Proof.* See Appendix A ■

This implies that the $\kappa^i(s) \neq 0$, for $i = 1, 2$, due to asymmetry of the shock process.

### 2.4 The Union

In this section we describe how to rewrite the problem as a saddle point. We characterize the setup for the currency union since it is more general. The problem for the fiscal union with two independent monetary authorities is identical up to the presence of pricing frictions. We model the currency union with optimal transfers as a long term contract. This contract is subject to two sided limited commitment, whereby both countries can renege on the contract and switch to one of the outside options. The optimal contract is the solution to the following problem:

$$\max_{\{C_{T,i}(s^t),C_{NT,i}(s^t),N_i(s^t)\}_{i=1,2}} \sum_{i=1,2} \mu_{i,0} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{T,i}(s^t)^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{C_{NT,i}(s^t)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^t)^{1+\phi}}{1+\phi} \right) \right)$$

s.t.

$$\sum_{i=1,2} (P_T(s^t)C_T^i(s^t) + P_{NT,i}C_{NT}^i(s^t)) \leq \sum_{i=1,2} (P_T(s)Y_T^i(s) + W_i(s)N_i(s) + \Pi^i(s)) \tag{29}$$

$$\sum_{i=1,2} C_T^i(s^t) = \sum_{i=1,2} Y_T^i(s^t) \tag{30}$$

$$\mathbb{E}_t \sum_{r=t}^{\infty} \beta^r \left( \frac{C_{T,i}(s^r)^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{C_{NT,i}(s^r)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^r)^{1+\phi}}{1+\phi} \right) \right) \geq V_i^o(s_t, B) \tag{31}$$

It is known from Marcet and Marimon (2019) that this problem can be rewritten as the saddle point problem:

$$\mathcal{SP} \min_{\{\lambda_{i,t}\}_{i=1,2}} \max_{\{C_{T,i},C_{NT,i},N_i\}_{i=1,2}} \sum_{i=1,2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \mu_{i,t} \left( \frac{C_{T,i}(s^t)^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{C_{NT,i}(s^t)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^t)^{1+\phi}}{1+\phi} \right) \right) + \lambda_{i,t} \left( \frac{C_{T,i}(s^t)^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{C_{NT,i}(s^t)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^t)^{1+\phi}}{1+\phi} \right) - V_i^o(s_t, B_t) \right) \right] \tag{32}$$

$$\mu_{i,t+1} = \mu_{i,t} + \lambda_{i,t} \tag{33}$$

Here $\lambda_{i,t}$ is the Lagrange multiplier of country $i$'s participation constraint (PC). We now also have a *co-state* variable $\mu_{i,t}$ which effectively keeps track of the cost of keeping each agent inside the contract. We can then make use of a normalization which will reduce the dimension of the state space in the final problem. First we define the *relative weight* $z_t$ of country 1 as

$$z_t = \frac{\mu_{1,t}}{\mu_{2,t}} \tag{34}$$

Then we rescale each country's Lagrange multiplier as follows:

$$\nu_{i,t} = \frac{\gamma_{i,t}}{\mu_{i,t}} \tag{35}$$

We can now derive a new equation of motion for the relative weight $z_{t+1}$:

$$z_{t+1} = z_t \frac{1+\nu_{1,t}}{1+\nu_{2,t}} \tag{36}$$

After this normalization, the state/co-state vector is $(s, z)$ and the **saddle point Bellman equation** can be written as

$$\Omega(s,z) = \mathcal{SP} \min_{\{\nu_i\}_{i=1,2}} \max_{\{C_{T,i},C_{NT,i},N_i\}_{i=1,2}} z\left((1+\nu_1)(\frac{C_{T,1}(s,z)^{1-\gamma}}{1-\gamma} + \alpha\left(\frac{C_{NT,1}(s,z)^{1-\gamma}}{1-\gamma} - \frac{N_1^{1+\phi}}{1+\phi}\right) - \nu_1 V_1^o(s,B)\right)$$
$$+(1+\nu_2)\left(\frac{C_{T,1}(s,z)^{1-\gamma}}{1-\gamma} + \alpha(\frac{C_{NT,2}(s,z)^{1-\gamma}}{1-\gamma} - \frac{N_2(s,z)^{1+\phi}}{1+\phi}\right) - \nu_2 V_2^o(s,B) + (1+\nu_2)\beta E\Omega(s',z') \tag{37}$$

$$z' = z\frac{1+\nu_1}{1+\nu_2} \tag{38}$$

$$\sum_{i=1,2}(P_T(s,z)C_T^i(s,z) + P_{NT,i}C_{NT}^i(s,z)) \leq \sum_{i=1,2}(P_T(s,z)Y_T^i(s,z) + W_i(s,z)N_i(s,z) + \Pi^i(s,z)) \tag{39}$$

$$\sum_{i=1,2} C_T^i(s^t) = \sum_{i=1,2} Y_T^i(s^t) \tag{40}$$

The policies in the union are given by the first order conditions of this problem. For tradeable goods consumption these are:

$$\frac{z(1+\nu_1)}{C_{T,1}(s,z)^{\gamma}} = \zeta(s,z)P_T(s,z) \tag{41}$$

$$\frac{1+\nu_2}{C_{T,2}(s,z)^\gamma} = \zeta(s,z) P_T(s,z) \tag{42}$$

Where $\zeta(s)$ is the multiplier on the resource constraint. From this we can derive the relative tradeables consumption of the two countries as:

$$\frac{C_{T,1}}{C_{T,2}} = \left(\frac{z(1+\nu_1)}{1+\nu_2}\right)^{\frac{1}{\gamma}} = (z')^{\frac{1}{\gamma}} \tag{43}$$

It follows that each country's consumption of the tradeable good is:

$$C_{T,1}(s,z) = \frac{(z')^{\frac{1}{\gamma}}}{1+(z')^{\frac{1}{\gamma}}} \sum_{i=1,2} Y_T^i(s^t) \tag{44}$$

and

$$C_{T,2}(s,z) = \frac{1}{1+(z')^{\frac{1}{\gamma}}} \sum_{i=1,2} Y_T^i(s^t) \tag{45}$$

The conditions for the non-tradeables consumption and labour supply of country i are then:

$$C_{NT,i}(s,z) = \left(\alpha \frac{P_T(s,z)}{P_{NT,i}(s,z)}\right)^{\frac{1}{\gamma}} C_{T,i}(s,z) \tag{46}$$

and

$$N_i = \left(\alpha \frac{C_{T,i}(s,z) P_T(s,z)}{P_{NT,i}(s,z)}\right)^{-\frac{1}{\phi}} \tag{47}$$

Furthermore the Union's value function takes the form:

$$\Omega^U(s,z) = z V_1^U(s,z) + V_2^U(s,z) \tag{48}$$

for $U = F, M$, depending on whether it refers to a Fiscal Union with two independent monetary authorities or to a Monetary Union[4].

[4]In $\Omega^M(s,z), s$ denotes $(s_{-1}, s)$.

### 2.4.1 Decentralization with Endogenous Borrowing Limits

We now show how the contract allocation can be decentralized as a competitive equilibrium with trading of state contingent debt contracts and borrowing constraints.
We will be interested in union allocations for which the present value, at the correctly defined prices, is finite. We say that an allocation has *high implied interest rates* if

$$E_0 \sum_{t=0}^{\infty} q(s^t, z_t \mid s_0, z_0)(Y_{1,t} + Y_{2,t}) < \infty \tag{49}$$

where

$$q(s_{t+1}, z_{t+1} \mid s_t, z_t) = \max_i \beta \left( \frac{C_{T,i}(s_{t+1}, z_{t+1})}{C_{T,i}(s_t, z_t)} \right)^{-\gamma} \tag{50}$$

and $q(s^{t+k}, z_{t+k} \mid s_t, z_t) = \prod_{n=0}^{k-1} q(s_{t+n+1}, z_{t+n+1} \mid s_{t+n}, z_{t+n})$.

**Country Problem** Each country $i$ has access to a one period state contingent debt contract $B_i(s) = \{b_i(s' \mid s)\}_{s'}$, which denotes the amount of the tradeable good which country i promises to deliver in the state $s'$. In addition, let the price of a unit of an Arrow security which pays in state $s'$ be $q(s' \mid s)$. Then the value of the debt contract is $\sum_{s'} q(s' \mid s) b_i(s' \mid s)$. Country i solves the following problem:

$$\omega(b_i, s) = \max_{\{C_T, C_{NT}, N, B_i(s)\}} \frac{C_{T,i}^{1-\gamma}}{1-\gamma} + \alpha \left( \frac{C_{NT,i}^{1-\gamma}}{1-\gamma} - \frac{N_i^{1+\phi}}{1+\phi} \right) + \beta E[\omega(b_i', s') \mid s]$$

subject to

$$\begin{aligned} C_T^i(s) + P_{NT,i} C_{NT}^i(s) + b_i(s) \leq \\ Y_T^i(s) + W_i(s) N_i(s) + \Pi_i(s) + \sum_{s' \mid s} q(s' \mid s) b_i(s' \mid s)) \end{aligned} \tag{51}$$

and

$$b_i(s' \mid s) \leq \bar{B}_i(s') \tag{52}$$

Where $\bar{B}(s')$ is a state contingent **endogenous borrowing limit**.

**Definition 1** (Equilibrium)**.** *A competitive equilibrium with borrowing limits is a collection of borrowing limits* $\{\bar{B}(s)\}$ *and initial debt positions* $\{b_i(s_0)\}$*, together with an allocation* $\{C_{T,i}(s), C_{NT,i}(s), N_i(s)\}$*,* *state contingent debt contracts* $\{B_i'(s)\}$*, goods prices* $\{P_T(s), P_{NT}(s), W_i(s)\}$ *and asset prices* $q(s' \mid s)$ *such that* $\{C_{T,i}(s), C_{NT,i}(s), N_i(s)\}$ *solves country i's decision problem, markets clear and the resource constraint holds.*

The consumption and asset choice decisions give us the Euler equation:

$$q(s' \mid s) \geq \beta\pi(s' \mid s) \left(\frac{C_T^i(s', b')}{C_T^i(s, b)}\right)^{-\gamma} \tag{53}$$

A competitive equilibrium with borrowing limits therefore satisfies this equation and the transversality conditions:

$$\lim_{t\to\infty} E_t \beta^t q(s^{t+1} \mid s_t) C_T^i(s_t, b_i(s_t))^{-\gamma} b_i(s_{t+1}) = 0 \tag{54}$$

**Proposition 1** (Decentralized Equilibrium)**.** *Any union allocation with high implied interest rates can be decentralized as a competitive equilibrium with endogenous borrowing limits.*

*Proof.* See Appendix A ■

With this implementation of the union allocations, we are now able to specify the liabilities generated by each country's participation in the union. In any given state, these same liability levels would also need to be financed outside of the union if one of the countries chose to exit (which, in equilibrium, never happens). Given that in the outside option, each country can decide to default completely on its debt, an obvious question is whether there is any case in which a participation constraint binds and the constrained country's preferred outside option is to continue repaying its debts. In the full currency union, this involves a complex comparison of the value of independent monetary policy with the value of enhanced risk-sharing in the contract. In the fiscal union, however, where there is no nominal friction, we are able to show that there is no case in which a country is indifferent between remaining in the union and the alternative of leaving and continuing to repay its debt.

**Proposition 2** (Optimal Exit in Fiscal Unions)**.** *In the fiscal union with two independent monetary authorities, whenever the participation constraint is binding for country i,* $V_i^{LD}(s) > V_i^{LR}(s, B)$.

*Proof.* See Appendix A ■

The next proposition formalises a different aspect of the two problems: if the currency union is able to achieve full risk-sharing, then it will attain the same value as the fiscal union.

**Proposition 3** (Risk-sharing Miracle)**.** *If in the steady state the currency union attains full risk-sharing, i.e.* $(C_T^1(s)/C_T^2(s))^{-\gamma} = \bar{c}$, $\forall s$, *then the common monetary policy is able to stabilize both economies at once.*

*Proof.* See Appendix A ■

Similarly to Auclert and Rognlie (2014) when countries attain full risk-sharing, stabilizing one economy through common monetary policy also puts the other country's labour wedge to zero. This result carries important consequences for the type of steady state that may arise in this model, and, in particular, for the comparison between fiscal and currency unions in full risk-sharing steady states.

The following definition describes the two types of steady state which can emerge in the monetary and fiscal unions.

**Definition 2** (Steady States)**.**

*a) A steady state with perfect risk-sharing is a path in which for some $k$, the relative weight $z_t$ is constant for all $t > k$.*

*b) A steady state with imperfect risk-sharing is a path in which for some $k$, the relative weight $z_t \in \{\underline{z}, \dots, \bar{z}\}$ (i.e. it is in the discrete support of the ergodic distribution), for all $t > k$, where $\bar{z} > \underline{z}$, $\underline{z} = \min_{s \in S \times S}\{z : V_1^U(s, z) = V_1^o(s, B)\}$ and $\bar{z} = \max_{s \in S \times S}\{z : V_2^U(s, z) = V_2^o(s, B)\}$.*

**Corollary 1** (Values in Full Risk-sharing Steady States)**.** *In a constant weight steady state*

$$V_i^M(s, z) = V_i^F(s, z), \quad i = 1, 2.$$

*Proof.* See Appendix A ■

In this economy full risk-sharing always characterizes the constrained efficient allocation. The implication of Corollary 1 is that if such allocation can be attained, then the currency union delivers the same level of utility as the fiscal union. This is a direct consequence of the risk-sharing miracle. Another equivalence result can be obtained by focusing on periods in which any participation constraint binds in an imperfect risk-sharing steady steady state.

**Proposition 4** (Values with Binding Constraints)**.** *If the optimal choice in the outside option economy is to leave and default on outstanding liabilities, then, whenever the participation constraint binds for country $i$,*

$$V_i^F(s, z) = V_i^M(s, z) = V_i^o(s, B) = V_{UD}^i(s).$$

*Proof.* See Appendix A ■

**Theorem 1** (Steady States with Imperfect Risk-sharing)**.** *In steady states with imperfect risk-sharing*

$$V^F(s, z) > V^M(s, z).$$

*Proof.* See Appendix A

■

**Proposition 5** (Constrained Efficient Currency Unions)**.** *The optimal allocation in a currency union is constrained efficient.*

*Proof.* See Appendix A

■

**Corollary 2** (Monetary Policy with Planner Weights)**.** *If the central bank of a currency area adopts the relative Pareto weights of the planner then risk sharing decreases. This is paired with an increased inefficiency of the non-tradeables consumption.*

*Proof.* See Appendix A

■

In the next section we solve the model numerically to study whether the contracts are feasible (positive surplus), what is the structure of the optimal transfers and whether the optimal policies in a currency union can make up for the deadweight loss due to the lack of independent monetary policy.

# 3 Quantitative Results

In this section we describe the algorithm used to solve the model, the parameterisation and the numerical results for both the real and the nominal setup.

## 3.1 Solution Algorithm and Parameters

The solution algorithm first requires solving for the value functions and policy functions of the outside option, which is an Arellano economy with two goods. The Arellano economy is solved by value function iteration, following the algorithm in Arellano (2008), adjusted to allow updating of the bond pricing schedule in each iteration. This gives us the consumption of tradeables and the borrowing choices in terms of tradeables. Since monetary policy is independent in the outside option, so that non-tradeable production is always at the first best level, we simply set $C_{NT} = 1$ and $N = 1$ for all states; this is also true for the non-tradeable allocation in the fiscal union contract, where there is no nominal rigidity.

The contract allocations are solved for using policy function iteration. We start with an initial guess for the value functions of the contract, and the liabilities. At each iteration $k$, for a given assignment of liabilities $B_k(y, z)$ and a guess for the value functions $V_k(y, z)$, we find the value of relative weight $z$ at which the participation constraint binds in each endowment state $y$. Using the symmetry of the environment, we can then calculate an interval $(\underline{z}(y), \bar{z}(y))$ within which the

| Description | Parameter | Value |
|---|---|---|
| Openness parameter | $\alpha$ | 1 |
| Discount Factor | $\beta$ | 0.95 |
| Utility Curvature | $\gamma$ | 2 |
| Labour Elasticity | $\phi$ | 3 |
| Risk Free Rate | $r$ | 1.02 |
| Reinclusion Probability | $\theta$ | 0.17 |
| Default Output Cost | $\psi$ | 0.96 |
| Endowment AR1 parameter | $\rho$ | 0.9 |
| Endowment Shock Variance | $\sigma_y^2$ | 0.01 |

Table 1: Baseline parameter Values

participation constraints do not bind; outside of this range, the allocations are constant due to the binding participation constraints.

Once we have updated the allocation, we can update the implied liabilities using a recursive version of the budget constraint, as in Equation 74. This allows us to update the assignment of the outside option values $V_i^o(y, B(y,z))$; any values of $B(y,z)$ which lie outside the grid used to solve the Arellano economy are calculated using cubic spline interpolation (or extrapolation if required). We then continue onto the next iteration $k+1$, by again finding the binding values of z. We continue iterating until the changes in the value function for the contract and the liabilities function $B(y,z)$ are sufficiently small.

This completes the solution algorithm for the fiscal union. For the currency union, there are two additional steps in each iteration, to calculate the relative prices $\frac{P_{1,NT}}{P_{2,NT}}$ and $\frac{\epsilon(s)}{P_{1,NT}}$. The exact expressions needed for calculating these relative prices can be found in Auclert and Rognlie (2014), although we have adjusted them to allow for a more flexible specification of risk aversion. Both of these prices are required to calculate the non-tradeable allocation variables $C_{NT}$ and $N$.

The parameter values used for all exercises are shown in Table 1. These values have not been calibrated but have been chosen to lie within ranges which are common in the macroeconomic literature. An exception in this regard is the relative risk aversion parameter $\gamma$, which at 5 would be considered high. Under our current solution algorithm, it becomes extremely difficult to achieve convergence of the currency union contract solution for a lower value of $\gamma$, without introducing aggregate risk into the model. For the sake of simplicity, we have chosen to retain the higher value of $\gamma$, rather than introduce an additional state variable into the model.

The Markov process for the tradeable endowment $y$ is produced by discretizing an AR1 process with persistence and volatility parameters $\rho$ and $\sigma_y^2$, as given in Table 1. We discretize the process using the Rouwenhorst method, which achieves better performance for near unit root persistence. For all of the economies we use a 5 state Markov chain. We report the full transition matrix in equation 79 in the Appendix.

### 3.2 The Outside Option

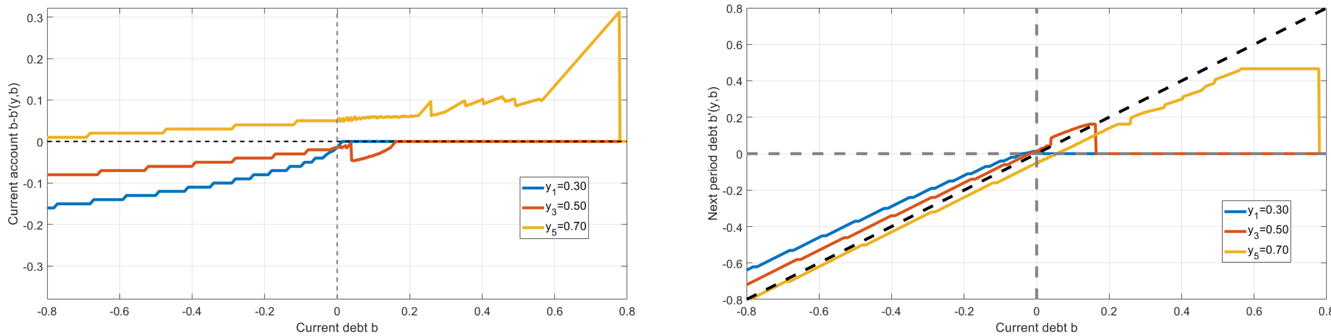
We begin our discussion of the results by first describing the behaviour of the defaultable debt economy which is the outside option to remaining in the union contract.

Figure 1 displays the behaviour of financial variables in the outside option economy. We plot the current account and stock of assets against different levels of current liabilities. Figure 1a shows the evolution of net borrowing. To the left of the zero on the horizontal axis countries have stocks of assets. The optimal policy here suggests that countries run a current account deficit when they are in the lowest endowment realization to smooth consumption. When they are in the highest endowment realization, countries run a current account surplus and increase the stock of assets. Moving rightward, countries have positive debt. Since default is costly and priced in by the lender, countries tend to run current account surpluses to deleverage and reduce the cost of borrowing. The steep declines in debt correspond to default episodes. The general deleveraging pattern is evident in Figure 1b since the lines slopes are less than 1, meaning that tomorrow's debt is lower than the current liability stock.

In Section 2.3.2 we outlined the full set of choices available to each country when considering whether or not to remain in the fiscal or currency union. Figure 1 also gives some information about the preference ordering of these outside options. Suppose the country is in the 3rd highest endowment state $y_3 = 0.5$, represented by the red lines in Figure 1. In Figure 1b, we see that for a current debt level below 0.135, the debt choice $B'$ is non-zero (with an exception around $B = 0.02$ where the country optimally chooses to deleverage slightly to $B' = 0$), meaning that for these debt levels the country continues to participate in financial markets. However, for any current debt level above 0.135, the country's debt in the next period collapses to zero because it defaults. Using the notation of Section 2.3.2, this tells us that when $y = 0.5$ and $B \leq 0.135$, $V_i^{LR}(s,B) \geq V_i^{LD}(s,B)$, so the country chooses to continue servicing its debt; conversely, when $y = 0.5$ and $B > 0.135$, $V_i^{LR}(s,B) < V_i^{LD}(s,B)$, and so the country chooses to default on its existing liabilities.

Furthermore, these preference orderings tell us about the off-equilibrium behaviour of each country in the case that it decides to leave the union (recall that in equilibrium this will never happen because the contracts are designed so that the participation constraints are always satisfied). Consider again the case where country $i$'s current endowment is $y_3 = 0.5$, but now assume that is inside the fiscal union. If its current liabilities inside the union are 0.1, for example, then it considers the choice between remaining in the fiscal union, and leaving the union but continuing to service the liabilities which it has accumulated. On the other hand, if it has liabilities of 0.2 (or any amount greater than 0.135), then it instead considers the choice between remaining inside the fiscal union and leaving the union and immediately defaulting on these liabilities.

In the results which follow, we will see that in the former case, where liabilities are relatively low, the country always *strictly* prefers to remain in the union. This result holds *a fortiori* for the case where the country has accumulated assets within the union. Importantly, we also find that whenever the




(a) Current Accounts Outside Option Economy

(b) Debt Law of Motion Outside Option Economy

Figure 1: Outside Option Economy policis

participation constraint binds, so that the country is indifferent between staying in the union and leaving, its liabilities inside the union are always so large that if it were to leave the union, it would immediately default. These findings hold for both the fiscal and the currency union.

### 3.3 Fiscal Union

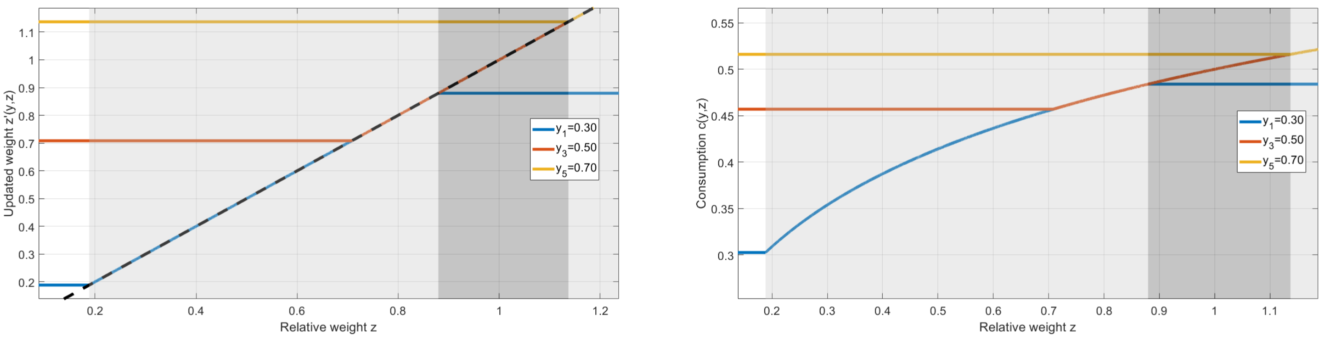
We start the description of the quantitative results of the fiscal union model by characterizing the optimal policies inside the risk-sharing contract.

In what follows we plot the policy functions inside the dynamic contract as a function of the relative weight $z$. We plot all policies for different endowment realizations[5].

Figure 2 shows the relative weights and consumption policies. The dark grey shaded area represents the set of weights that characterize the ergodic distribution of the contract. This is the set in which the weights will lie and fluctuate in the steady state. The lighter grey shaded area represents the basins of attraction of the ergodic set. All graphs contain the lowest, the median and the highest realization of the tradeable endowment.

Figure 2a shows the optimal relative weight policy. Every line corresponds to a specific realization of the endowment. In every line flat regions represent areas in which one of the participation constraints is binding. The flat region to the left is where the participation constraint of country 1 is binding, the flat region on the right shows where the participation constraint of country 2 binds. Since the relative weight describes the consumption allocation of country 1 relative to country 2, in the left area of the graph the relative weight is too low, meaning that country 1 is receiving too little consumption, hence the country is against its participation constraint. Conversely, as one moves rightward, there is a flat portion of the line where the relative weight is too high and country 2's participation constraint is binding. The regions in which the optimal weight coincide with the

[5]Recall that in this setting there is no aggregate risk, meaning that when country 1 is in a high endowment state, country 2 is in a low endowment one.




(a) $z'$ (b) $c$

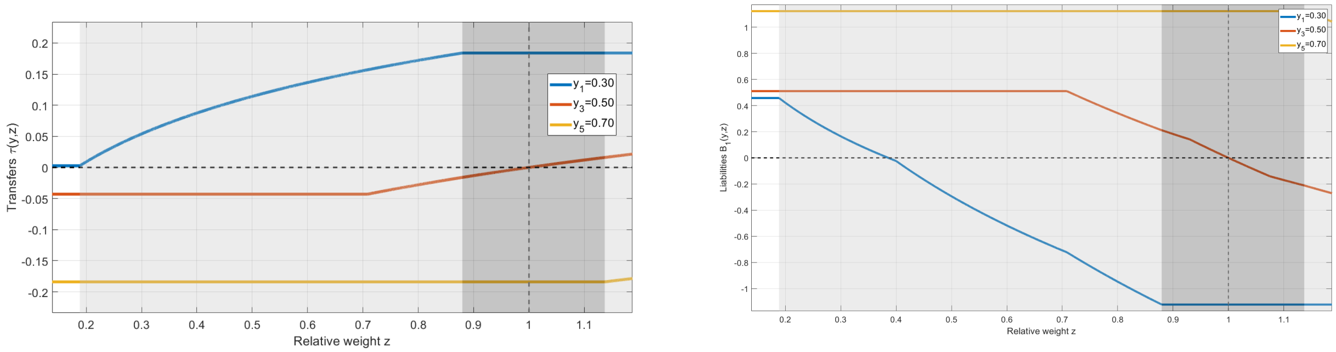
Figure 2: Outside Option Economy policies

45 degree line are areas where neither participation constraint binds, hence the weight is constant across periods.

Figure 2b displays the consumption allocation. When neither country wants to leave the contract the future relative weight is equal to the ratio of marginal utility of tradeable consumption between the two countries. Hence the graph shows that consumption tracks the current relative weight in the same areas where the weight is not updated. Concavity is inherited by preferences.

Figure 2 already shows that this contract features an imperfect risk-sharing steady state. Using Definition 2 it is visible that as there is no set of weights in which neither the particiation constraint of the high endowment country nor the one of the low is binding, the weights must fluctuate as the state changes. Graphically this can be seen by observing, in Figure 2a, that the flat region to the right on the lowest endowment relative weight lies to the left of the region where the PC stops binding for the high endowment. This steady state will then not be able to attain full risk-sharing. In particular in the steady state the following path will occur: suppose we start with both countries at the middle endowment and a relative weight of 1. At this level of weight, given the state, neither participation constrain binds. Suppose now that country 1 moves to the highest endowment level (hence country 2 moves to the lowest). At a relative weight of 1, in the highest endowment state, country 1's participation constraint binds (this can be seen on the yellow line). The planner will then increase the relative weight next period to the minimum level to make country 1 indifferent between the contract and the outside option. Such weight is rightmost point on the dark grey area, namely where the PC is barely binding in the highest endowment state. As long as the states do not change both countries' PCs are slack. Suppose now that the state changes and country 1 moves to the lowest endowment state (hence country 2 moves to the highest). At these relative weights country 2's participation constraint binds and the planner will increase the weight till the PC is slack again. These dynamics define the imperfect risk-sharing steady state.

The next two graphs in Figure 3 show the key policies inside the contract. Figure 3a displays the optimal transfer policy. The contract features optimally large countercyclical transfers between




(a) $\tau$

(b) $b$

Figure 3: Fiscal Union policies

the countries. When countries are at symmetric endowment realizations and neither participation constraint binds, the transfers range between -18% and 18% of the total tradeable endowment. They are as large as 2/3 of the endowment for lower realizations.

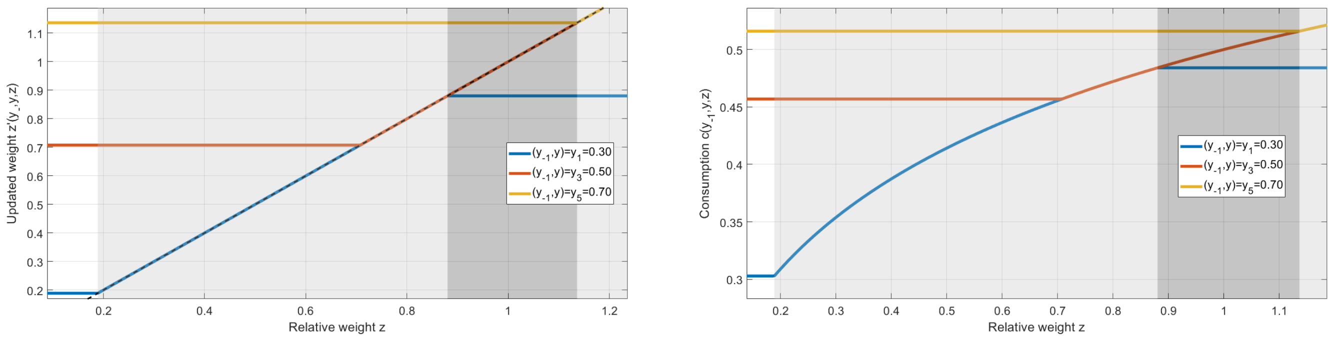
Figure 3b shows the liabilities positions. Countries have higher stocks of debt whenever they have a low relative weight. One feature of the contract is that, since the countries are symmetric and given the persistence of the endowment, a high realization today implies future surpluses in expected terms. This, in turn, generates positive stocks of liabilities today. This feature however is not true for any level of the relative Pareto weight. This result is common to other similar models of dynamic contracts (see Abraham et al., 2019). The key difference is our setting is that the debt position can take both signs (i.e. assets or liabilities) for both countries. This feature stems from the symmetry of risk aversion and impatience in our model. This result can also be interpreted as countries with better endowment realizations being able to absorb larger stocks of debt.

Finally we discuss the steady states of this economy. The fiscal union features an imperfect risk-sharing steady states. The dark shaded area represents the ergodic set. This range of relative weights has the property that if an economy starts (say, in Period 0) with a relative weight inside this set it will always stay there. This set has a basin of attraction, both from the left and from the right, such that if an economy starts inside the basin it will eventually converge to the ergodic set. This basin of attraction is represented by the light grey areas in the graphs.

## 3.4 Monetary Union

In this section we describe the results for the currency union model. In this setup non-tradeables producers face staggered prices friction. In the outside option economy countries have independent monetary policy and close the labour wedge.

One important feature of this model is that the outside option value is identical to the one in the real version of the model since monetary policy eliminates nominal rigidities entirely. However,




(a) $z'$ (b) $c$

Figure 4: Monetary Union policies

inside the contract, countries face lower surplus since the economy is not producing at the efficient level. This is a direct consequence of Corollary 1 and Theorem 1. From Theorem 1 the currency union can never yield a higher value than the fiscal union in imperfect risk-sharing steady states. From Corollary 1 they can be at most equal in constant weights steady states. As the fiscal union features imperfect risk-sharing it is never possible for the currency union to attain the same value of the problem.

Surprisingly, the contract is qualitatively identical to the one described in the previous section. In Figure 4 we plot the future relative weight and consumption as a function of the current relative weight $z$.

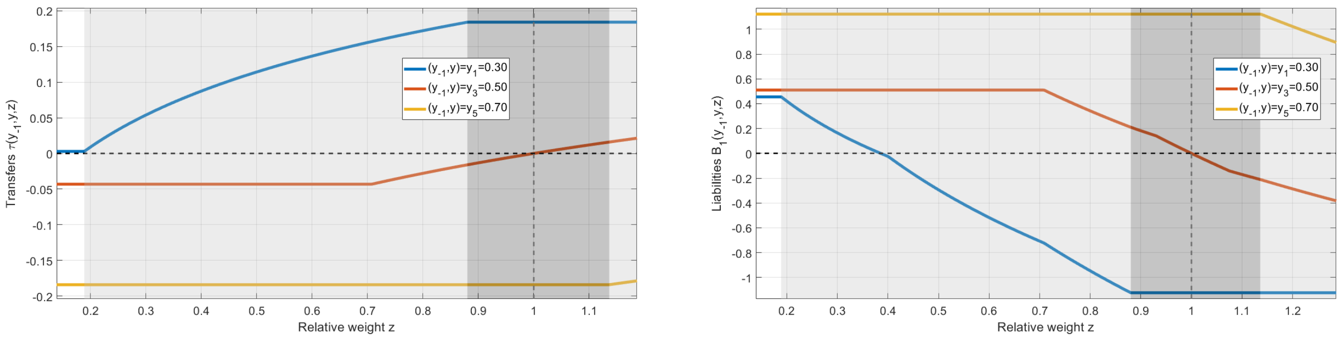
Figure 4a displays the law of motion of the relative weight. All lines feature a flat region on the left where the country's participation constraint binds, a sloped part where it coincides with the 45 degree line and flat region on the right where the other country's participation constraint is binding. The weights are updated upward whenever the country's PC binds, downward when the other country threatens to leave the contract and they remain constant when neither is against the outside option. At first inspection, the path in the ergodic set resembles the one in the fiscal union. This feature will be extensively discussed later in the paper.

Figure 4b plots the consumption allocations for different levels of endowments. Consumption closely tracks the relative weight behaviour, as in the fiscal union setting.

The economy features countercyclical optimal transfer of the tradeable endowment. Their size is numerical identical to the ones in the fiscal union. The same holds for the stock of liabilities, displayed in Figure 5b.

The outside option economy behaves identically to the real model outside the fiscal union. Hence the behaviour of current accounts and the debt law of motion can be seen in Figure 1.

It is important to notice that while the optimal policies in the two economies representing the outside options of the contract are the same the starting levels outside are not. To see this, recall that the stock of liabilities of a country leaving the union is given by the outstanding set of promises




(a) $\tau$

(b) $b$

Figure 5: Monetary Union policies

to the other country. As the transfer policies inside the fiscal and currency unions could be different, so would be the liabilities inside the contract. Hence the starting stock of debt upon breakup could be different. In other words, conditional on a given level of $b$ the policy for $b'$ is the same in the two economies. However, in the same state inside the contract, upon leaving, the countries could start with different levels of $b$.

Exactly as in the fiscal union, this economy features an imperfect risk-sharing steady state. Hence the planner is unable to attain full risk-sharing.

### 3.5 Comparison of the Contracts

In this section we compare the optimal policies in the two contracts. From the previous discussion, we see that the two contracts seem to have very similar values and policies despite the fact that the currency union cannot achieve the optimal allocation of non-tradeable goods. In Section 2, we showed formally that in some special cases (for example, when the participation constraint is binding), the currency union attains the same value as the fiscal union; the numerical results, however, seem to suggest that the similarity is more general. How is this possible? The answer lies in a difference in the behaviour of the optimal transfers in the currency union which enables the planner to compensate partially for the labour wedge.

We start by recalling that the full state space for the economy is $(y_{t-1}, y_t, z_t)$. We can then distinguish between two cases. In the first, between period $t-1$ and $t$, the endowment of tradeables goods does not change, i.e. $y_{t-1} = y_t$. In the second, the endowment does change between periods so that $y_{t-1} \neq y_t$. If the endowment is quite persistent, as we tend to assume, then in period $t-1$ agents' expectations will place a large probability mass on the first case, in which the endowment does not change. In particular, the pricing decisions of the non-tradeable good producers will put a large weight on this outcome. As a result, if the endowment does not change between periods, the labour wedge in the currency union will be relatively small, whereas if does change, it will be

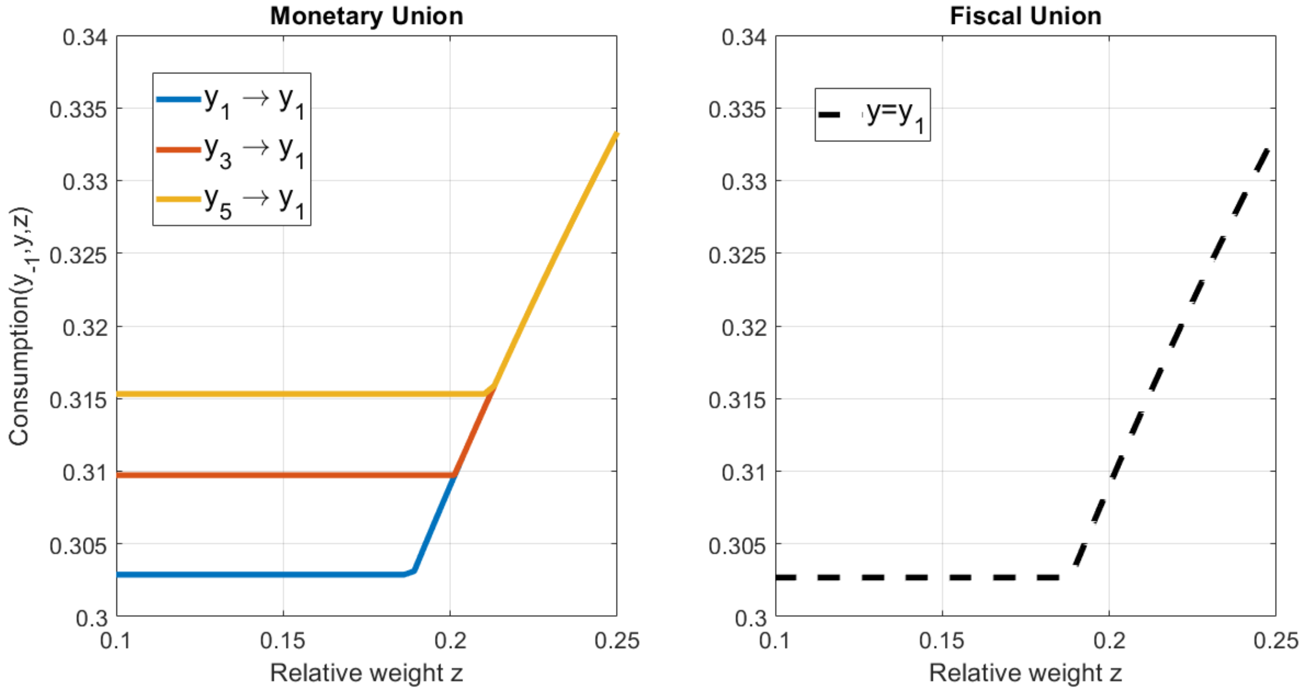


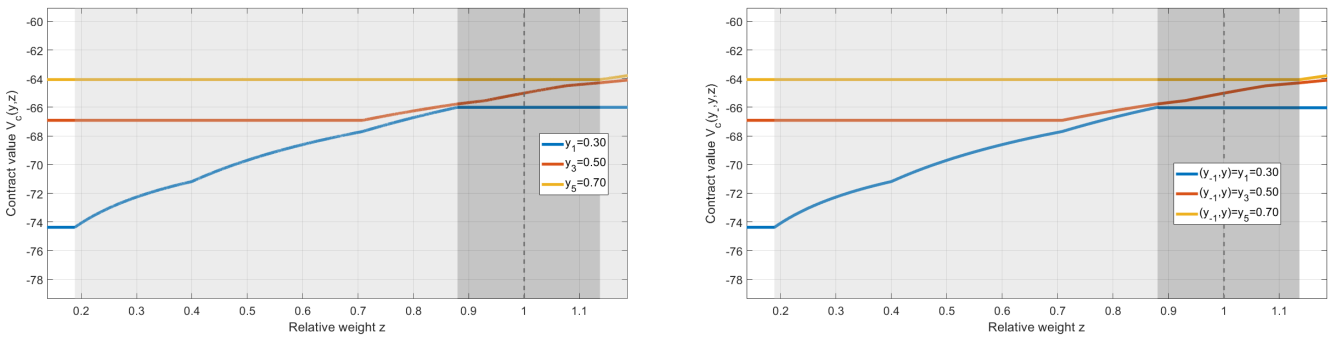

Figure 6: Consumption adjustment in the contracts

larger; in fact, the larger the transition $y_{t-1} \to y_t$, the large the labour wedge in period $t$.

This explains why in the comparisons considered so far, where the realization of $y$ is held constant, the currency union behaves similarly to the fiscal union. If instead we consider transitions where the endowment changes between periods, we see that the transfer policy in the currency union is more complex that that of the fiscal union.

Figure 6 shows consumption in the two contracts when $y_t = y_1$ and the country's participation constraint is binding. We see that in the monetary union, when the constraint binds, the level of consumption depends not only on $y_t$ but also on $y_{t-1}$. Moreover, when the economy arrives at $y_1$ from a higher previous endowment, is receives high current tradeables consumption. This higher consumption compensates for the fact that when the transition is large (say $y_5 \to y_1$), the labour wedge is also large; the additional consumption is therefore needed to keep the country in the monetary union. In contrast, in the fiscal union, monetary policy completely eliminates the wedge; as a consequence consumption does not need to be conditioned on $y_{t-1}$ in this way.

We should reiterate at this point that the extra adjustment of transfers in the monetary union is not enough to completely undo the deadweight loss from having joint monetary policy. We showed formally that the value of the fiscal union is always weakly higher than that of the currency union. However, under certain conditions, the transfer policy in currency union can make the overall loss very small.




(a) Fiscal Union

(b) Monetary Union

Figure 7: Values of the Contracts

### 3.5.1 Steady States

Before discuss the features of the steady states in detail, it is worth describing in more depth the set of weights defining the basin of attraction of the ergodic sets of the two contracts.

In Figures 15 and 16 we plot, for every endowment realization, the set of weights in which the participation constraints do not bind. The ergodic sets are defined by the upper bound of the lowest realization of output and the lower bound of the highest realization. This area is shaded in grey.

The reciprocal bounds, namely the lowerbound for the lowest endowment and the upperbound of the highest endowment, define the the basin of attraction. In order words one can read off the graphs the set of starting weights that will produce convergence to the ergodic set.

We start the discussion on the features of the stochastic steady states by providing one simulation to exemplify the dynamics. We start by simulating one history of endowments for 100 periods. We then plot the optimal policies of a country in the fiscal union, one in the currency union and one in the defaultable debt economy.

We start the contracts with a relative weight of 1 in the median endowment state. As $z = 1$ is the center of the ergodic set in both unions the economy will permanently remain in such set. We then sample 100 period and plot the simulation policies.

The path of the endowment and consumption is plotted in Figure 8. In the left panel, showing the endowment history, the red line shows the path for the contracts economies, while the black line for the defaultable debt one. The vertical black lines show episodes of default and financial market exclusion for the outside option economy. The endowment history is the same, though recall that when output is sufficiently large and the defaultable debt economy is in a period of exclusion from financial markets, it pays a fraction of endowment as a default cost. Hence the small deviations between the two paths when the outside option economy has default episodes. We denote periods of financial autarky, following a default, as a dot at the top of the graph, while periods of financial

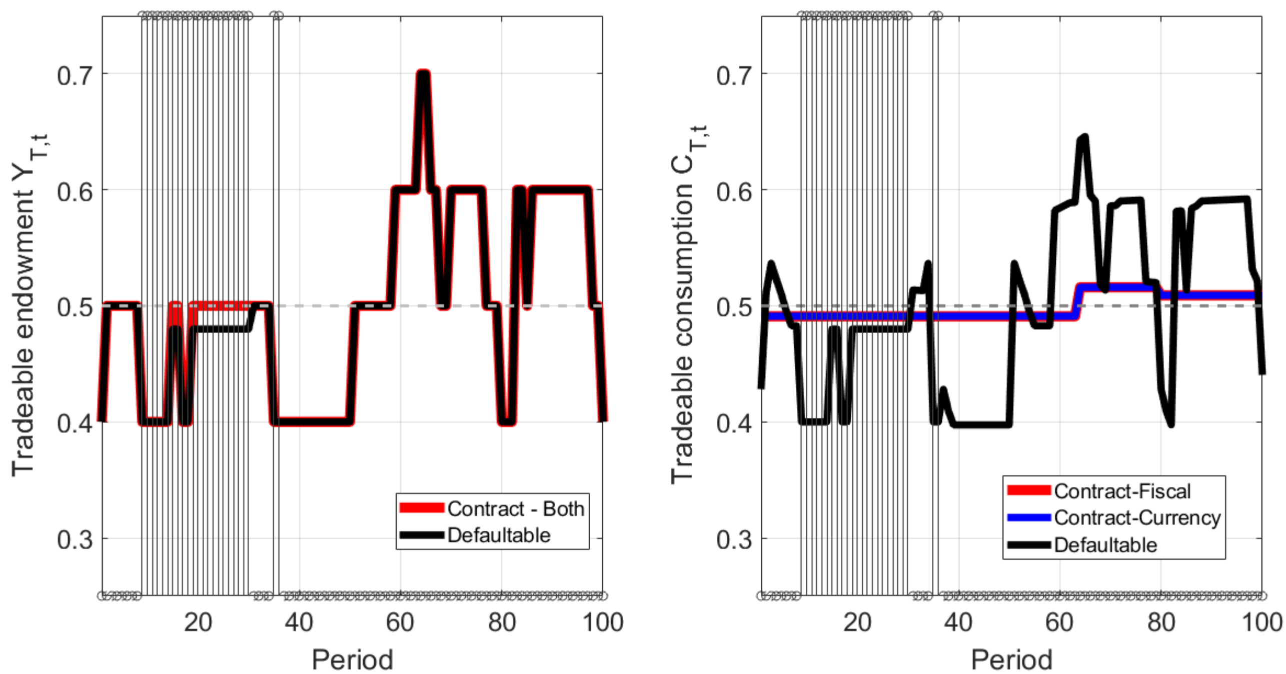



Figure 8: Steady State Endowment and Consumption

market access as dots at the bottom of the graph.

The left panel of Figure 8 shows the behaviour of the consumption of tradeables. The two contracts provide the same level of consumption (red and blue lines). The planner is able to smooth close to all of the fluctuations in the endowment. The jumps in consumption are given by updating in the relative weight, following a binding participation constraint for one of the two countries. Finally the defaultable debt economy shows high volatility in consumption as there is limited possibility to insure against the idiosyncratic risk.

Figure 9 shows the behaviour of the relative weight and some financial variables of these economies. The top left panel shows the behaviour of the relative weight, which is numerically identical for the two contracts. The transfer policy, in the top left panel, shows that the transfers are countercyclical and large, relatively to the endowment. Secondly it shows that, as the relative weight is stable in the first half of this history, consumption of tradeables does not change. This implies that the transfers absorb the entire difference between the constant consumption and the varying endowment. Together with the contract transfers we plot the current account balance of the outside option economy. The defaultable debt economy behaves very differently. At the beginning of this history the endowment realization is low and the economy has some assets. Once these assets are used to smooth consumption and the country starts accumulating debt, as the endowment drop, the country defaults. The country is excluded from financial markets for an extended period of time, hence the

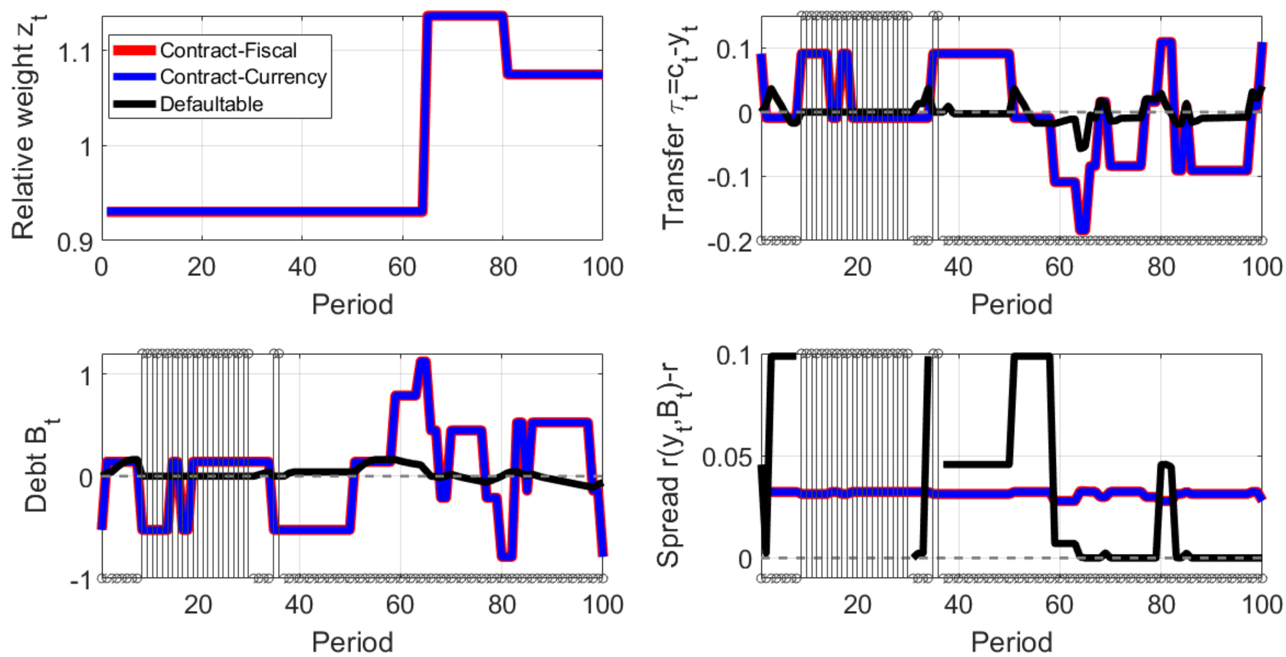



Figure 9: Steady State Optimal Policies

zero debt and current accounts. As the economy is reincluded in financial markets and borrows the endowment drop again and the country defaults again. Subsequently the country enjoys of sustainable borrowing and high endowment though it is still unable to absorb the large variations in the endowment and consumption is quite volatile. Lastly, the bottom right panel, shows the behaviour of the risk spread. In this graph is clear how the external lender prices in default before it happens by increasing the interest rate charge on the defaultable debt.

Table 2 shows some key moments of the economies in steady state. These moments are computed by averaging 50000 simulations in the steady state.

As expected the defaultable debt economy provides less consumption smoothing than the two contracts with a a consumption volatility 7 times higher. Secondly, consumption is lower in the outside option than in the contract due to default episodes in which the endowment is reduced. Thirdly, the two contracts deliver the same policies, which large (10% of GDP) countercyclical fiscal transfers and approximately the same values.

The risk-sharing value of the agreements is evidenced by the correlation between consumption and the endowment. The two contracts significantly reduce this comovement though such correlation is still positive. This stems from periods in which the weights change procyclically, for example when a country moves to high endowment and this makes the PC bind, implying an upward revision of its relative weight.

| | Outside-Defaultable Debt | Contract-Fiscal | Contract-Currency |
|---|---|---|---|
| **Mean** | | | |
| $Y_t$ | 0.4975 | 0.4996 | 0.4996 |
| $C_{T,t}$ | 0.4973 | 0.5 | 0.5 |
| $GDP_t$ | 0.754 | 0.75 | 0.75 |
| $\mid \tau_t \mid$ | 0.013 | 0.075 | 0.075 |
| $B_t$ | -0.001 | -0.001 | -0.002 |
| $z_t$ | - | 1.005 | 1.005 |
| $V(y, b/z)$ | -66.832 | -65.031 | -65.032 |
| $Pr(PCbinding)$ | - | 0.029 | 0.029 |
| **Standard deviation** | | | |
| $\sigma(c_{T,t})$ | 0.094 | 0.013 | 0.013 |
| $\sigma(Y_t)$ | 0.099 | 0.1 | 0.1 |
| **Correlation** | | | |
| $\rho(C_{T,t}, Y_t)$ | 0.983 | 0.5 | 0.501 |
| $\rho(\tau_t, Y_t)$ | -0.379 | -0.993 | -0.993 |

Table 2: Steady State Moments

An important point of comparison between the fiscal and the currency union is in the row labelled $Pr(PCbdinding)$. This represents the fraction of periods in which any participation constraint is binding in this agreement. As discussed above, the fiscal planner in a currency union is implementing transfers that depend on $y$ and $y_{-1}$. Particularly the planner is rewarding the country with the larger transition through higher tradeables consumption. This implies that the steady state path is fluctuates in narrower bands in the currency union. As such, there exists a set of pairs of endowment transitions in which a participation constraint would be binding in the fiscal union but it is not in the monetary union. This yields a lower probability of a binding PC in the currency union. In this case, however, we find that the difference is negligible numerically.

The higher risk-sharing capacity of the contracts is also visible in the maximum amount of liabilities that countries can have inside the agreements.

In the defaultable debt economy countries are unable to borrow due to the high likelihood of default. Inside the unions can accumulate liabilities. .

The graphs in Figure 10 carry one additional set of information. Looking at the red lines in the top graphs, the maximum amount of liabilities describe when the country would optimally default. A country leaving the risk-sharing contract with some stock of liabilities in some given endowment state would default on its obligations if debt was higher than the red line. The line depicts the maximum debt that the country would optimally repay.

In summary, the two contracts behave identically numerically. They both yield higher risk-sharing than the outside option, thereby producing higher values for the problem.

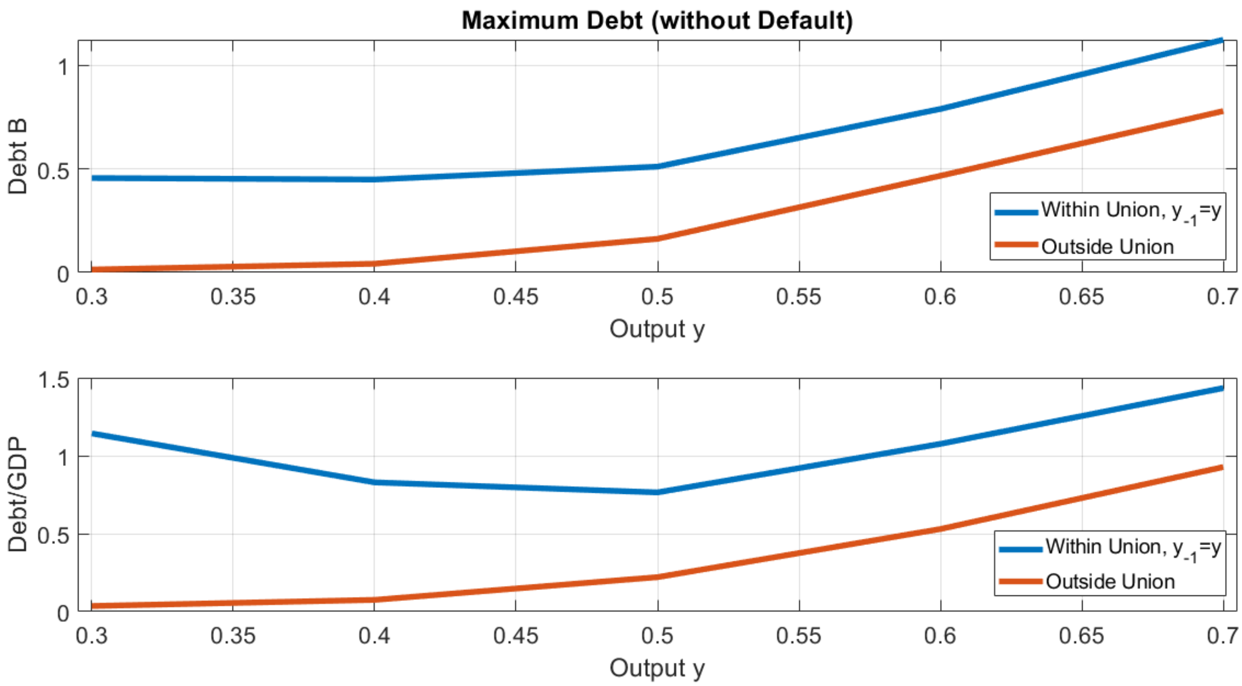



Figure 10: Maximum Debt

### 3.5.2 Crisis Simulation

In this section we describe the economies after a crisis event. Inside the contract we define a crisis state as a country having the lowest endowment realization and having a binding participation constraint. Outside we define the crisis as the lowest endowment realization and having a stock of debt such that the country is indifferent between repaying and defaulting.

We start by showing the result for a single simulation over 100 periods and comparing the behaviour of the fiscal union, the currency union and the defaultable debt economy. Recall that in the outside option the nominal and real economy coincide since monetary policy eliminates pricing frictions.

Figure 11 shows the history of endowment realizations and the consumption of tradeables in the three scenarios: in the defaultable debt economy, in the fiscal union and in the currency union.

The right panel shows the consumption of tradeables over the first 100 periods after the crisis. The black line displays the path for the economy in the outside option. Consumption closely tracks the endowment state, showing limited scope for consumption smoothing through defaultable debt. In the outside option economy default occurs a number of times after the initial crisis before the economy manages to stabilize during a period of above average output realizations, before defaulting again towards the end of the simulation. We also see that the country pays the default cost when the endowment is high enough during exclusion, as evidenced by the difference in the endowments between the outside option and contract economies in the first panel. The two contracts behave

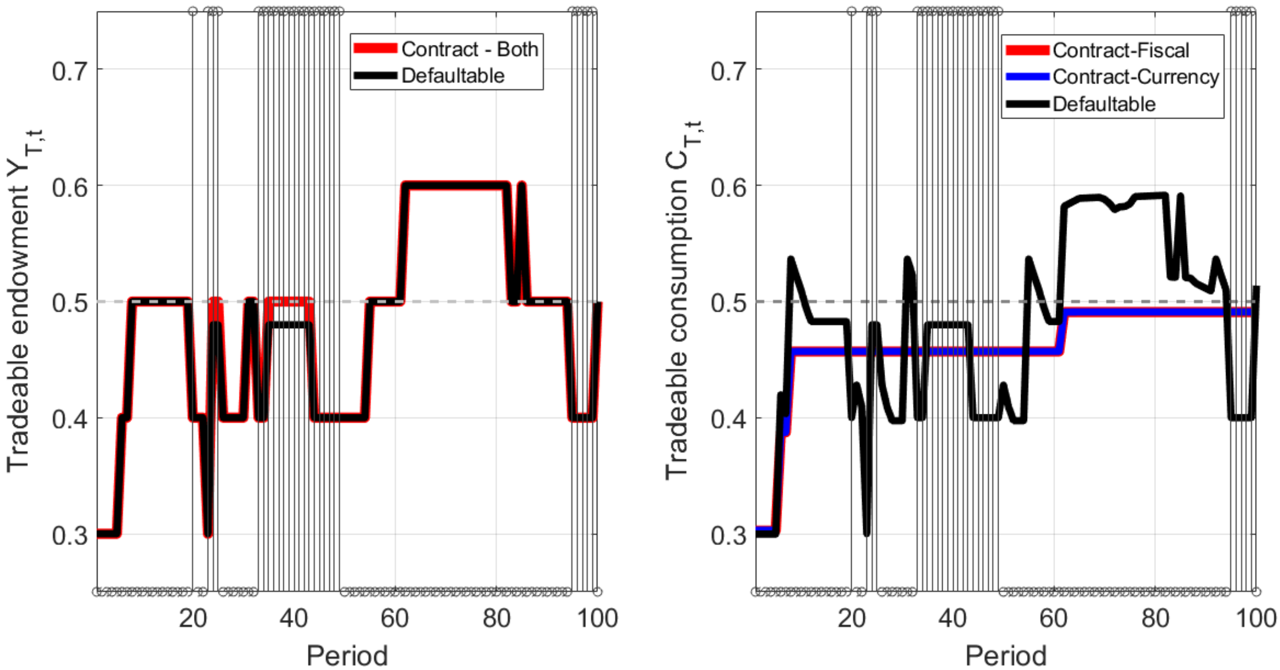



Figure 11: Endowment and Consumption

identically, as we would expect from the policy functions: consumption increases relatively soon after the crisis and remains flat for many periods, before increasing again in response to high endowments. Since the simulation starts with the lowest endowment and relative weight, we only observe increases in consumption as the endowment reverts to its mean and the relative weight moves towards one. In Figure 12 we plot the endowment, transfers, debt and the interest rate spread. The defaultable debt economy shows that when the country defaults and is temporarily excluded from financial markets, it has zero debt and zero net borrowing (which we compare with transfers in the contract). In the path of the interest rate spreads, default periods can be seen by noting that there is no spread (i.e. no debt is being traded and so there is no bond price to quote). As we would expect, spreads rise as the country approaches a default episode, reflecting the increase probability of default. After a long period of exclusion, the country regains access to financial markets in the middle of the simulation, and begins to accumulate a small amount of debt before starting to save, through current account surplus, as it experiences high endowment realizations. During this saving period the interest rate spread is zero. At the end of the simulation output falls again, spreads rise as the country borrows in an attempt to smooth consumption, and eventually the economy defaults again. Inside the risk-sharing contracts the paths of debt and transfers are the same. At the beginning of the crisis, the country is so indebted that it can no longer borrow and so it receives zero transfers; however, the stock of debt which it is able to accumulate is much higher than in the defaultable debt

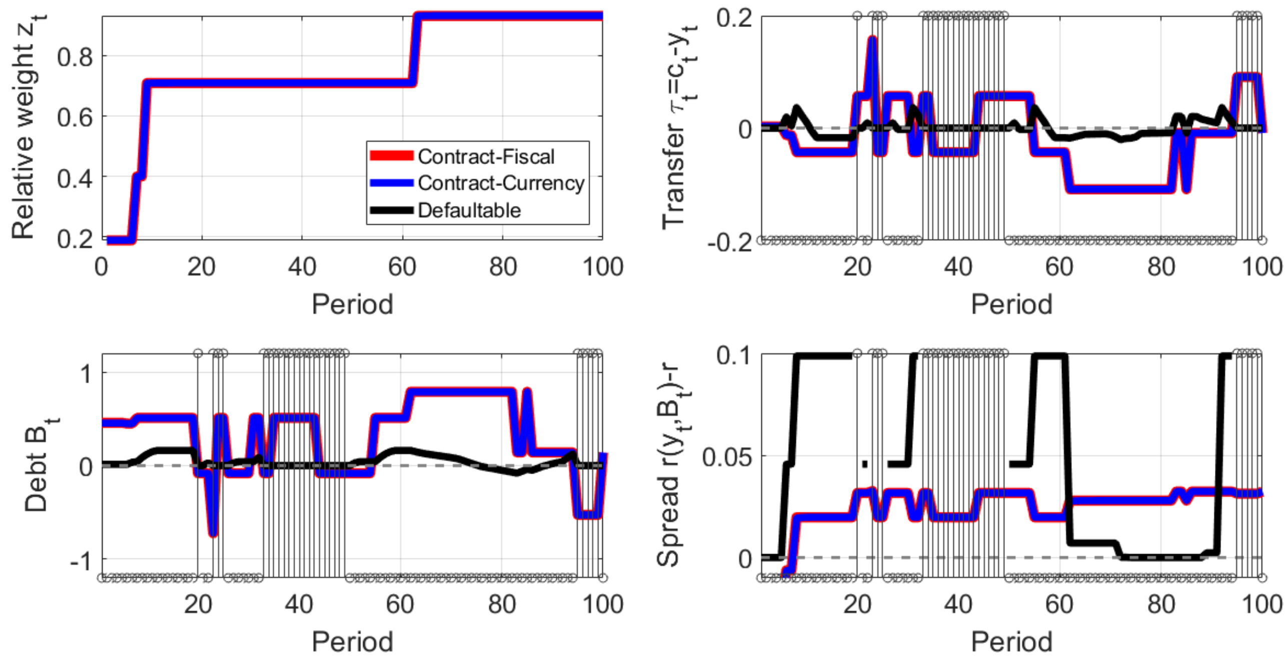



Figure 12: Financial Variables

economy. The liabilities of the country are then reduced in response to a sharp fall in output. In this case, the fall in output it so large that the country expects to receive transfers in the near future; this corresponds to a net asset position. For the rest of the simulation, the country accumulates debt when output falls, and repays it when it rises, in order to smooth consumption. The interest spreads are lower and more stable inside the contract, and are actually negative immediately after the crisis.

Next we look at the average behaviour of the contracts in response to a crisis episode. We do so by averaging across 25000 crisis simulations. Figure 13 shows the average response of consumption in the fiscal and currency union. The dashed lines represent the interquartile range.

Table 3 shows the main moments of some key outcomes of the simulations from the perspective of the country in crisis (recall that if one country is in crisis, the other must be experiencing a boom). As the economy starts in a recession and the endowment state is persistent the average endowment is .48, lower than its unconditional average of 0.5. In the economy with defaultable debt the average endowment is further decreased by the default cost. Consumption, conversely, is at its highest in the defaultable debt economy. The same ranking however holds for consumption volatility and its correlation with the endowment state. The average absolute value of transfers (current accounts) is much smaller in the defaultable debt economy compared to the unions, reflecting the reduced borrowing capacity outside the contract. The stocks of liabilities are quite different inside and

| | Outside-Defaultable Debt | Contract-Fiscal | Contract-Currency |
|---|---|---|---|
| **Mean** | | | |
| $Y_t$ | 0.478 | 0.480 | 0.480 |
| $C_{T,t}$ | 0.478 | 0.471 | 0.471 |
| GDP | 0.716 | 0.696 | 0.696 |
| $\mid \tau_t \mid$ | 0.011 | 0.0652 | 0.0652 |
| $B_t$ | 0.023 | 0.177 | 0.176 |
| $z_t$ | - | 0.839 | 0.840 |
| $V(y, b/z)$ | -67.541 | -66.392 | -66.391 |
| $Pr(PCbinding)$ | - | 0.057 | 0.066 |
| **Standard deviation** | | | |
| $\sigma(c_{T,t})$ | 0.1 | 0.052 | 0.052 |
| $\sigma(Y_t)$ | 0.105 | 0.0105 | 0.105 |
| **Correlation** | | | |
| $\rho(C_{T,t}, Y_t)$ | 0.987 | 0.617 | 0.617 |
| $\rho(\tau_t, Y_t)$ | -0.351 | -0.872 | -0.872 |

Table 3: Crisis Moments

outside the contract. In the outside option, the country on average has a small amount of debt, roughly one eighth of the level inside the contracts.

Finally, transfers are largely countercyclical, particularly inside the risk-sharing contracts. Countercyclicality is stronger in the union, which explains the much greater stabilization of output displayed in the simulations.

In the next two figures we plot the impulse responses of the tradeable goods in the three economies, as well as the relative weight of the crisis country in each of the contracts. The solid lines represent the average paths of the variables whereas the dashed lines represent paths one standard deviation away from the mean. In the right hand panel of Figure 13, we see that after 100 periods, the average level of consumption is roughly the same in all three economies, and close to the mean level of the tradeable goods endowment. After the crisis, consumption also tends to recover faster in the defaultable debt economy. However, the dashed lines tell us that consumption is much more volatile outside the union than it is inside.

Figure 14 shows the average path of the transfers, the stock of debt and interest rate spreads in the fiscal and currency unions compared to the defaultable debt economy outside the contract. We see that on average transfers are close to zero in the defaultable debt economy, reflecting an inability to borrow, whereas in the union the crisis country initially makes net payments to the other country. The fact that the country in the union makes net payments in the aftermath of the crisis may be counterintuitive. However, as we can see in the top left panel of 14, the country begins the crisis with a very low relative weight, which corresponds to low consumption. Along any history

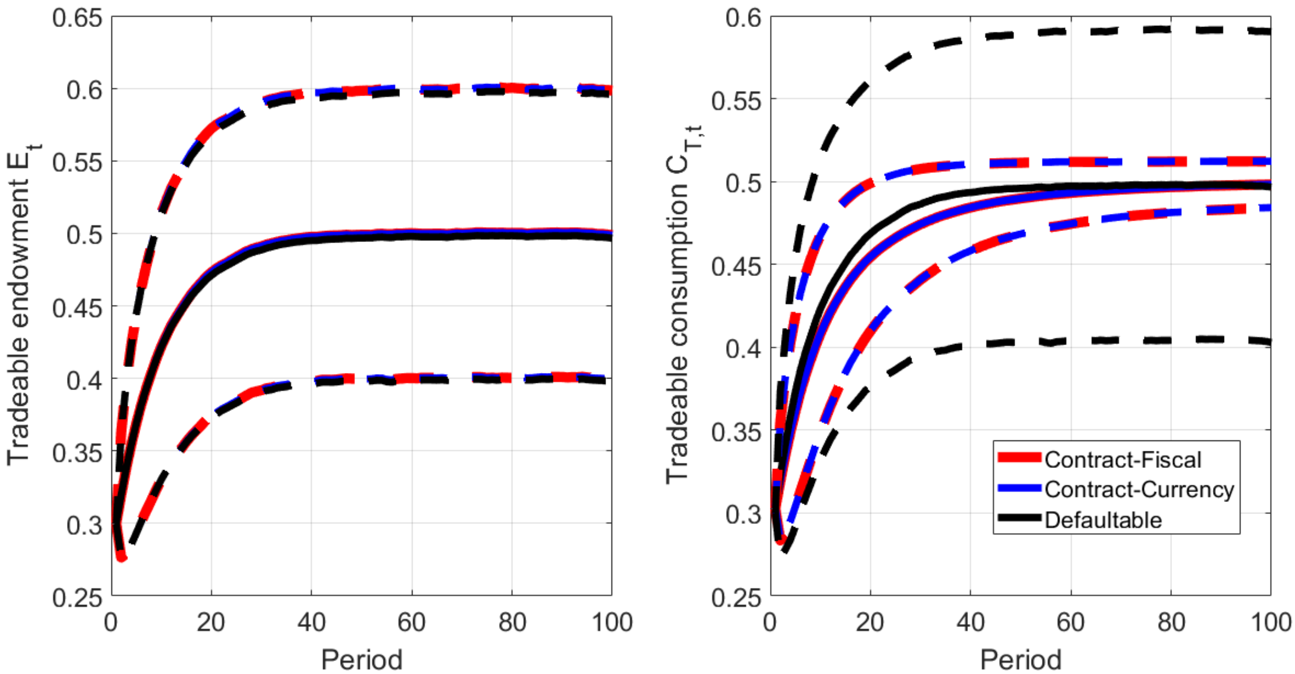



Figure 13: Tradeables Impulse Response After Crisis

which leads to this crisis state, the country will have been able to borrow large amounts to smooth consumption, an option which would not have been available outside the union.

The paths of liabilities are also very different for in the contracts, compared to the outside option. In the contracts, the economy begins the crisis with a large stock of debt, which it gradually repays over the course of the simulation. The defaultable debt economy, on the other hand, tends to spend the periods after crisis with zero net liabilities because it frequently defaults when it enters a crisis and subsequently spends some periods in financial autarky.

In the bottom right panel of Figure 14 we see the average path of the interest rate spreads which correspond to these movements in liabilities. The bold black line, which shows the median spreads for the defaultable debt economy [6], is calculated only for those states in which the economy does not default, since if it does default no debt is traded and there is no interest rate. We therefore see that if the economy does not default immediately during the crisis, it faces elevated interest spreads due to the high probability of default in the future. After this, the tradeables endowment reverts to its mean level, where default is less likely, and we see that spreads are volatile but typically close to zero. In contrast, in the union contracts we see *negative* spreads before the country gradually

[6] For the paths of interest rate spreads, we face the problem that in some states spreads in the defaultable debt economy jump to extremely high levels, which inhibits the convergence of the standard deviation and the average paths across simulation. We therefore plot the median and interquartile range for the defaultable debt economy, since these statistics are more robust to outliers.

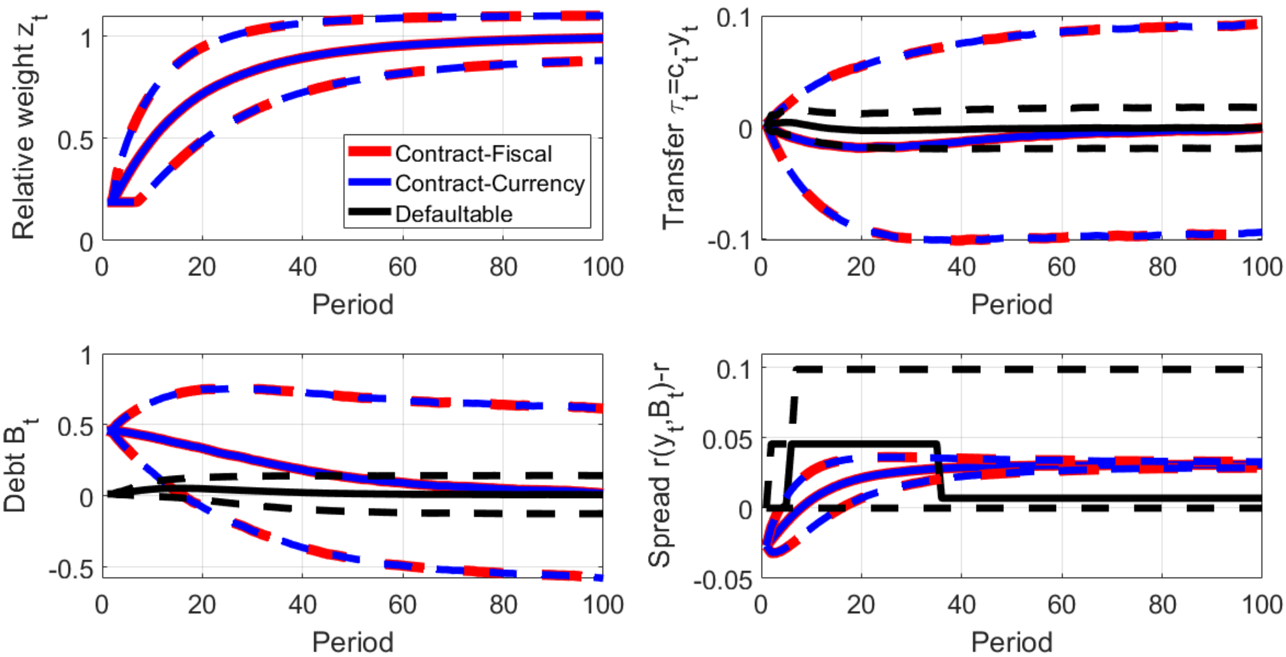



Figure 14: Financial Variables

recovers from the crisis and its outstanding liabilities trade with a stable positive spread. The initial negative spreads are an artefact of the lack of aggregate risk in the union. While one country is in crisis, the other is experiencing a boom, and is therefore extremely willing to hold assets against the likelihood that its endowment (and tradeable consumption) will fall in the near future [7].

Finally, in the top left panel of Figures 14 we see the impulse responses of the relative weight of the crisis country in the two contracts. During the crisis, the country receives the lowest level of tradeables endowment, and is therefore willing to accept a very low relative weight because its outside option is also very unattractive. As the country's endowment reverts to its mean however, the initial level of $z$ is too low to satisfy the country's participation constraint, and so the relative weight is driven upwards to keep the country inside the contract, until the weight reaches one. We should recall that, due to the imperfect risk-sharing in the steady state, while the impulse response for $z$ exhibits a smooth path, actual changes in the relative weight take place through discrete jumps (as seen, for example in Figure 12), due to the discrete set of values at which the participation constraint binds for different realizations of the endowment.

[7] See Appendix A for the relationship between the (implicit) interest rates on the liabilities with the contracts and the marginal rates of substitutions for tradebles consumption in the two countries

| | Outside-Defaultable Debt | Contract-Fiscal | Contract-Currency |
|---|---|---|---|
| **Mean** | | | |
| $Y_t$ | 0.498 | 0.5 | 0.5 |
| $C_{T,t}$ | 0.502 | 0.5 | 0.5 |
| $GDP_t$ | 0.637 | 0.625 | 0.625 |
| $\mid \tau_t \mid$ | 0.021 | 0.076 | 0.076 |
| $B_t$ | -0.201 | -0.002 | -0.002 |
| $z_t$ | - | 1.001 | 1.001 |
| $V(y, b/z)$ | -59.303 | -55.018 | -55.021 |
| $Pr(PCbinding)$ | - | 0.0093 | 0.0093 |
| **Standard deviation** | | | |
| $\sigma(c_{T,t})$ | 0.088 | 0.005 | 0.005 |
| $\sigma(Y_t)$ | 0.1 | 0.1 | 0.1 |
| **Correlation** | | | |
| $\rho(C_{T,t}, Y_t)$ | 0.954 | 0.364 | 0.364 |
| $\rho(\tau_t, Y_t)$ | -0.508 | -0.999 | -0.999 |

Table 4: Moments: $\gamma = 3$

# 4 Robustness Checks

In this section we provide the steady state moments for three alternative calibrations of our model. In the first two we change the parameter governing the risk aversion, which allows us to alter how agents value risk-sharing. In the final one we revert to the risk aversion of the baseline parameter set ($\gamma = 2$), and instead reduce the persistence of the endowment process.

Tables 4 and 5 show the steady state moments for the same model discussed above but with the risk aversion parameter equal to 3 and 4, respectively.

Starting from Table 4, in the outside option economy agents show a higher level of steady state assets. This driven by the higher precautionary motif, which also, through positive interest rate on the assets, allows the country to consume more than the endowment on average.

The contracts show again similar values, with a marginally bigger difference in values between the fiscal and the currency union. As agents value smooth consumption more than in the previous simulation, the planner optimally reduces the variance of consumption by increasing the counter-cyclicality of transfers and increasing their average size by about 1.5%.

Table 5 provides a very different picture. The outside option economy increases the steady state level of assets compared to the previous economies, which significantly increases the average consumption of tradeables due to returns on the stock of assets.

The contracts are now very different from before. The risk aversion is large enough that the limited enforcement friction has little bite, allowing the planner to achieve full risk-sharing. In these economies the steady states feature a constant relative weight. As full risk-sharing is achieved

| | Outside-Defaultable Debt | Contract-Fiscal | Contract-Currency |
|---|---|---|---|
| **Mean** | | | |
| $Y_t$ | 0.5 | 0.5 | 0.5 |
| $C_{T,t}$ | 0.51 | 0.5 | 0.5 |
| $GDP_t$ | 0.577 | 0.563 | 0.563 |
| $\mid \tau_t \mid$ | 0.031 | 0.075 | 0.075 |
| $B_t$ | -0.509 | -0.002 | -0.002 |
| $z_t$ | - | 1 | 1 |
| $V(y, b/z)$ | -73.654 | -65 | -65 |
| $Pr(PCbinding)$ | - | 0 | 0 |
| **Standard deviation** | | | |
| $\sigma(c_{T,t})$ | 0.083 | 0 | 0 |
| $\sigma(Y_t)$ | 0.1 | 0.1 | 0.1 |
| **Correlation** | | | |
| $\rho(C_{T,t}, Y_t)$ | 0.916 | 0 | 0. |
| $\rho(\tau_t, Y_t)$ | -0.583 | -1 | -1 |

Table 5: Moments: $\gamma = 4$

this economy falls into the case described in Proposition 3. We therefore observe that, as there is no deadweight loss, the fiscal and currency union can attain exactly the same allocation.

Finally we consider economies with much lower persistence in the endowment process, where the AR1 parameter $\rho$ is reduced from 0.9 to 0.5. As shown in Table 6, this parameter choice also delivers a constant weight steady state. However, the mechanism is slightly different. Since output reverts to the mean more quickly with lower persistence, a country currently receiving a high endowment faces a more similar future endowment stream to a country with a low endowment. The sets of relative weights which will satisfy both countries is therefore more similar, and actually overlaps. Compared to the baseline parameter set, the experience of the defaultable debt economy is much improved when the persistence of output is lower. In particular, since periods of low output are shorter on average, the economy is more able to borrow against higher future income, and the consumption smoothing which it can achieve is higher; the volatility of consumption is reduced by about one third compared to the baseline.

## 5 Conclusions

In this paper we develop a model of fiscal and currency unions as recursive contracts. We lay down a framework in which two symmetric, equally patient, risk-averse countries face idiosyncratic risk on their tradeables endowment. There is no aggregate risk since the risks are fully negatively correlated. They partake in a risk-sharing agreement subject to a participation constraint. In this

| | Outside-Defaultable Debt | Contract-Fiscal | Contract-Currency |
|---|---|---|---|
| **Mean** | | | |
| $Y_t$ | 0.5 | 0.5 | 0.5 |
| $C_{T,t}$ | 0.502 | 0.5 | 0.5 |
| $GDP_t$ | 0.755 | 0.75 | 0.75 |
| $\mid \tau_t \mid$ | 0.048 | 0.077 | 0.077 |
| $B_t$ | 0 | -0.001 | -0.001 |
| $z_t$ | - | 1 | 1 |
| $V(y, b/z)$ | -65.594 | -65 | -65 |
| $Pr(PCbinding)$ | - | 0 | 0 |
| **Standard deviation** | | | |
| $\sigma(c_{T,t})$ | 0.061 | 0 | 0 |
| $\sigma(Y_t)$ | 0.1 | 0.1 | 0.1 |
| **Correlation** | | | |
| $\rho(C_{T,t}, Y_t)$ | 0.807 | 0 | 0 |
| $\rho(\tau_t, Y_t)$ | -0.813 | -1 | -1 |

Table 6: Moments: $\rho = 0.5$

constraint the outside option is defined by an Arellano (2008) type economy, in which countries can borrow and default on a risk-neutral lender. Inside the agreement they are able to set up state contingent transfers to reduce consumption volatility. We show that a fiscal union with two independent monetary authorities manages to achieve considerable consumption smoothing.

The fiscal union with two independent monetary authorities has one more policy instrument than the currency union and, therefore, it achieves a higher value. The role of independent monetary policy is to close the labor wedge resulting from the pricing rigidities faced by non-tradeables producers. In a currency union the lack of independent monetary policy implies that the economy is producing at a suboptimal level since a single monetary policy cannot simultaneously close the wedges of both countries. Therefore, the possibility of having an independent monetary policy outside the union makes this institutional design relatively more attractive. We show that, indeed, at the steady state, the monetary union cannot do better than the fiscal union with independent monetary policies. Nevertheless, we quantitatively find that an optimal design of state-dependent transfers, taking as given the optimal monetary policy of the currency union, can compensate almost all the losses of loosing monetary independence.

We provide a characterization of the optimal cross country transfers. We show that the optimal policy requires large countercyclical transfers as a device to smooth consumption. In addition, since in the currency union larger changes in the endowment of tradeables result in large labour wedges, the optimal transfers should be higher after large transitions. It is this extra adjustment in transfers which partially closes the gap between the currency union and the fiscal union. In our simulations, where idiosyncratic risk is significant, the monetary union risk-sharing agreement also allows a significantly higher debt capacity than the defaultable debt economy. Neither the fiscal union nor the monetary union achieves full risk-sharing but they are both able to reduce the volatility of consumption by about 4/5, compared to the defaultable debt economy. However, significant cyclicality of consumption remains.

A number of extensions of this paper would be of interest. An extension of this model in which countries are not symmetric, particularly with respect to the average size of their output, would allow us to analyse situations closer to real world experiences such as the Euro Area. In the same spirit, an extension allowing for aggregate uncertainty would be of interest. Lastly, a further interesting addition would be that of a fiscal externality. This would allow the analysis of cases like the Greek debt crisis, where Greek debt was being held by German banks.

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# Appendix

## A Proofs

*Proof of Lemma 1.* Non-tradeable goods producers maximize the expected profits across states, inheriting the households' nominal discount factor $1/\epsilon(s)C_T(s)^{-\gamma}$. Firms maximize

$$\Pi(p) = \sum_s \pi(s|s_{-1}) \frac{1}{\epsilon(s)C_T(s)^{-\gamma}} \left[ (p-(1+\tau_L)W(s)) \left(\frac{p}{P_{NT}(s)}\right)^{-\frac{\varepsilon}{\gamma}} \left(\frac{\alpha\epsilon(s)}{P_{NT}(s)}\right)^{\frac{1}{\gamma}} C_T(s) \right] \quad (55)$$

The first order condition with respect to the price $p$ is

$$\frac{\partial \Pi(p)}{\partial p} : \alpha^{\frac{1}{\gamma}} \sum_s \pi(s|s_{-1}) p^{-\frac{\varepsilon}{\gamma}} \epsilon(s)^{\frac{1-\gamma}{\gamma}} P_{NT}(s)^{\frac{\varepsilon-1}{\gamma}} C_T(s)^{1-\gamma} \left[ 1 - \frac{\varepsilon}{\gamma}(p-(1+\tau_L)W(s))p^{-1} \right] = 0 \quad (56)$$

Using $p = P_{NT}(s) = P_{NT}$, $\forall s$, this condition becomes

$$\sum_s \pi(s|s_{-1}) \epsilon(s)^{\frac{1-\gamma}{\gamma}} P_{NT}^{-\frac{1}{\gamma}} C_T(s)^{1-\gamma} \left[ 1 - \frac{\varepsilon}{\gamma} \left( 1 - (1+\tau_L) \frac{W(s)}{P_{NT}} \right) \right] = 0 \quad (57)$$

Which yields

$$P_{NT} = \frac{\varepsilon}{\varepsilon - \gamma} (1+\tau_L) \frac{\sum_s \pi(s|s_{-1}) \epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma} W(s)}{\sum_s \pi(s|s_{-1}) \epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}} \quad (58)$$

Using the labor subsidy $(1+\tau_L) = \frac{\varepsilon-\gamma}{\varepsilon}$, it simplifies to the first statement in Lemma 1

$$P_{NT} = \frac{\sum_s \pi(s|s_{-1}) \epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma} W(s)}{\sum_s \pi(s|s_{-1}) \epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}} \quad (59)$$

To obtain the second statement, notice that, using the definition of the labor wedge and the household first order condition, one has

$$\frac{W(s)}{P_{NT}} = 1 - \kappa(s) \quad (60)$$

Hence, the optimal non-tradeable good price implies

$$\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}\kappa(s) = 0 \tag{61}$$

Which completes the proof.

∎

*Proof of Lemma 2.* The goal of the central bank is to maximize agents' welfare by means of the exchange rate $\epsilon$. The exchange rate in this setting is equivalent to the price of the tradeable good $P_T$ . Recall the following relationships from the household's first order conditions: $C_T^{-\gamma} = \frac{\alpha P_T}{P_{NT}} C_{NT}^{-\gamma}$. Using $\epsilon = P_T$ and inverting the previous relationship one gets $\frac{\partial C_{NT}}{\partial \epsilon} = \frac{1}{\gamma}\frac{C_{NT}}{\epsilon}$. Finally, recall that by labor market clearing $C_{NT} = N$. As monetary policy does not carry intertemporal effects, the central banks maximizes the contemporenous stream of utility:

$$v(\epsilon) = \frac{C_T^{1-\gamma}}{1-\gamma} + \frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}. \tag{62}$$

Maximizing with respect to the exchange rate

$$\frac{\partial v(\epsilon)}{\partial \epsilon} : \frac{C_{NT}}{\gamma\epsilon}[C_{NT}^{-\gamma} - C_{NT}^{\phi}] = 0 \tag{63}$$

Recalling the definition of the labor wedge

$$\kappa^i(s) = 1 - \frac{U_N^i(s)}{U_{NT}^i(s)} = 1 - C_{NT}^{\gamma+\phi}(s) \tag{64}$$

Then optimal monetary policy implies setting

$$k^i(s) = 0 \tag{65}$$

Which proves the first part of the lemma.

In a currency union, the monetary authority maximizes the weighted sum of the welfare of member states. Assuming equal weighing implies maximizing

$$v(\epsilon) = \frac{1}{2}v^1(\epsilon) + \frac{1}{2}v^2(\epsilon) \tag{66}$$

Maximizing with respect to the exchange rates yields

$$\frac{\partial v(\epsilon)}{\partial \epsilon} : C^1_{NT}[{C^1_{NT}}^{-\gamma} - {C^1_{NT}}^{\phi}] + C^2_{NT}[{C^2_{NT}}^{-\gamma} - {C^2_{NT}}^{\phi}] = 0 \tag{67}$$

Using the definition of the labor wedge, optimal monetary policy implies

$$\sum_{i=1,2} {C^i_{NT}}^{1-\gamma} \kappa^i(s) = 0, \quad \forall s$$

■

*Proof of Proposition 1.* We prove the proposition by constructing the competitive equilibrium which corresponds to the union allocation.

It will be convenient to have the following notation for the marginal rates of substitution of tradeable goods:

$$q(s', z' \mid s, z) = \max_i \beta \left( \frac{C_{T,i}(s', z')}{C_{T,i}(s, z)} \right)^{-\gamma} \tag{68}$$

We can now set the price of an Arrow security in this economy as

$$Q(s' \mid s) = \pi(s' \mid s) q(s', z' \mid s, z) \tag{69}$$

These Arrow prices clearly satisfy the Euler equation, with equality for the country which has the highest marginal rate of substitution. The value of the state contingent debt contract in state s is then

$$\sum_{s' \mid s} Q(s' \mid s) d(s' \mid s) = E q(s', z' \mid s, z) d(s' \mid s) \tag{70}$$

We can derive from the equation of motion for $z'$ that

$$\frac{z''}{z'} = \frac{1 + \nu_1(s', z')}{1 + \nu_2(s', z')} \tag{71}$$

And from the solution to the union contract we know that

$$\left( \frac{C_{T,2}(s, z)}{C_{T,1}(s, z)} \right)^{-\gamma} = z' \tag{72}$$

Thus we can write

$$\frac{z''}{z'} = \frac{1+\nu_1(s',z')}{1+\nu_2(s',z')}$$
$$= \left(\frac{C_{T,2}(s',z')}{C_{T,2}(s,z)}\right)^{-\gamma} \Bigg/ \left(\frac{C_{T,1}(s',z')}{C_{T,1}(s,z)}\right)^{-\gamma} \tag{73}$$

From this expression we can see that the maximum marginal rate of substitution will be attained by the country which is unconstrained ($\nu_i = 0$) in state $(s', z')$.
For the current debt position of each country, we write the budget constraint of country i as

$$b_i(s) = Y_T^i(s) - C_T^i(s,b) + \sum_{s'|s} Q(s' \mid s) b_i(s' \mid s) \tag{74}$$

and iterate forward on this equation and apply the transversality condition to obtain

$$b_{i,t} = \mathbb{E}_t \sum_{k=0}^{\infty} q(s^{t+k} \mid s_t)(Y_{i,t+k} - c_{i,t+k}) \tag{75}$$

where

$$q(s^{t+k} \mid s_t) = \prod_{n=0}^{k-1} q(s_{t+n+1} \mid s_{t+n}) \tag{76}$$

It should be clear from this definition of the debt position and the resource constraint that

$$B_1(s) = -B_2(s) \tag{77}$$

so that asset markets clear in every state. We set the initial debt positions as $b_{i,0} = \mathbb{E}_0 \sum_t^{\infty} q_{0,t}(Y_{i,t} - c_{i,t})$. We then choose borrowing constraints which are *not too tight* in the sense of Alvarez and Jermann (2000) so that

$$\omega(s, \bar{B}_i(s)) = V_o^i(s, \bar{B}_i(s)) \tag{78}$$

By definition, we will then have $b_i(s) = \bar{B}_i(s)$ whenever country $i$'s participation constraint is binding.

To complete the proof we must show that an allocation which has a high implied interest rate also satisfies the transversality condition:

$$\lim_{t\to\infty} \mathbb{E}_t \beta^t q(s^{t+1} \mid s_t) C_T^i(s_t, b_i(s_t))^{-\gamma} b_i(s_{t+1})$$

$$= \lim_{t\to\infty} \mathbb{E}_t \left[ \beta^t C_T^i(s_t, b_i(s_t))^{-\gamma} \max_i \beta \left( \frac{C_{T,i}(s_{t+1,} b_i(s_{t+1}))}{C_{T,i}(s_t, b_i(s_t))} \right)^{-\gamma} \right.$$

$$\left. \times \mathbb{E}_{t+1} \sum_{k=0}^{\infty} q(s^{t+k+1} \mid s_{t+1})(Y_{i,t+k+1} - C_{T,i}(s_{t+k+1,} b_i(s_{t+k+1}))) \right]$$

$$= \lim_{t\to\infty} \sum_{s_{t+1}|s_t} \pi(s_{t+1} \mid s_t) \left[ \beta^t C_T^i(s_t, b_i(s_t))^{-\gamma} \max_i \beta \left( \frac{C_{T,i}(s_{t+1,} b_i(s_{t+1}))}{C_{T,i}(s_t, b_i(s_t))} \right)^{-\gamma} \right.$$

$$\left. \times \mathbb{E}_{t+1} \sum_{k=0}^{\infty} q(s^{t+k+1} \mid s_{t+1})(Y_{i,t+k+1} - C_{T,i}(s_{t+k+1,} b_i(s_{t+k+1}))) \right]$$

$$= \lim_{t\to\infty} \sum_{s_{t+1}|s_t} \beta^t C_T^i(s_t, b_i(s_t))^{-\gamma} \mathbb{E}_{t+1} \sum_{k=0}^{\infty} q(s^{t+k+1} \mid s_t)(Y_{i,t+k+1} - C_{T,i}(s_{t+k+1,} b_i(s_{t+k+1})))$$

$$= \lim_{t\to\infty} \sum_{s_{t+1}|s_t} \beta^t C_T^i(s_0, b_i(s_0))^{-\gamma} \frac{C_T^i(s_t, b_i(s_t))^{-\gamma}}{C_T^i(s_0, b_i(s_0))^{-\gamma}} \mathbb{E}_{t+1} \sum_{k=0}^{\infty} q(s^{t+k+1} \mid s_t)(Y_{i,t+k+1} - C_{T,i}(s_{t+k+1,} b_i(s_{t+k+1})))$$

$$\leq C_T^i(s_0, b_i(s_0))^{-\gamma} \lim_{t\to\infty} \mathbb{E}_{t+1} \sum_{k=0}^{\infty} q(s^{t+k+1} \mid s_t)(Y_{i,t+k+1} - C_{T,i}(s_{t+k+1,} b_i(s_{t+k+1})))$$

$$\leq C_T^i(s_0, b_i(s_0))^{-\gamma} \lim_{t\to\infty} \mathbb{E}_{t+1} \sum_{k=0}^{\infty} q(s^{t+k+1} \mid s_0)(Y_{1,t+k+1} + Y_{2,t+k+1})$$

$$= 0$$

Where the last equality follows from the high implied interest rate condition in Equation 49. ■

*Proof of Proposition 2.* We prove this by contradiction. Assume that country $i$ has a binding participation constraint, so that $\lambda_i > 0$.
Recall that

$$V_i^o(s, B) = \max_{LR,LD} \{V_i^{LR}(s, B), V_{LD}^i(s)\}$$

We have shown in Proposition 1 that the union allocation can be decentralized as a competitive equilibrium with state contingent debt and endogenous borrowing constraints. Recall that $\omega(b_i, s)$ is the value of the problem in the decentralized equilibrium. If the participation constraint binds ($\lambda_i > 0$), it must be that

$$V_i^o(s,B) = \omega(b_i,s)$$

Recall that

$$B_{it} = \mathbb{E}_t \sum_{s=t}^{\infty} q_{t,s}(Y_{i,s} - c_{i,s}) = \mathbb{E}_t \sum_{k=0}^{\infty} q(s^{t+k} \mid s_t)(Y_{i,t+k} - c_{i,t+k}) = b_{i,t}$$

i.e. the face value of the debt in the outside option is the appropriately discounted value of the net payments in the decentralized economy.

In the outside option and in the decentralized economy, the agents maximize the same objective function under different constraints. The budget constraint in the case of exiting and repaying the liabilities is

$$C_T^i(s) + P_{NT}^i(s)C_{NT}^i(s) + B_i \leq Y_T^i(s) + W^i(s)N^i(s) + \Pi^i(s) + B_i'Q(s,B_i')$$

whereas in the decentralization of the union allocation it is

$$C_T^i(s) + P_{NT,i}C_{NT}^i(s) + b_i(s) \leq Y_T^i(s) + W_i(s)N_i(s) + \Pi_i(s) + \sum_{s'|s} q(s' \mid s)b_i(s' \mid s)$$

In the latter, the country is also subject to an endogenous borrowing limit, which we have specified in such a way that it is never binding if the participation constraint is slack. In addition, when the participation constraint binds, the country's liabilities are exactly equal to the borrowing limit. The borrowing limit therefore does not change the allocation.

Comparing the two budget constraints above, it is clear that the allocation in the outside option in case of repayment can always be exactly replicated in the decentralized fiscal union, since the state contingent debt can replicate any payments delivered by non state contingent bonds.

Hence, by optimality, it can never be that the value of the problem is higher in the case of leaving and repaying than in the decentralized fiscal union. Formally, $V_i^{LR}(s,B) < \omega(b_i,s)\ \forall s,B$.

This implies that if the participation constraint binds, it must be that $V_i^o(s,B) = V_{UD}^i(s) > \omega(b_i,s) \geq V_i^{LR}(s,B)$. In other words, it can never be that the participation constraint binds and the country would like to exit and *not* default. ■

*Proof of Proposition 3.* Using the definition of optimal non-tradeable prices, imposing full risk-sharing and taking the ratio of the non-tradeable prices in the two countries, we obtain

$$\frac{P_{NT}^1}{P_{NT}^2} = \left(\frac{C_T^1}{C_T^2}\right)^{\gamma} = \bar{c}^{-\gamma}$$

We show that imposing a zero wedge condition in one country immediately implies a zero wedge in the other. If country 1 has no labor wedge, $\kappa^1(s) = 0$, then

$$1 = \left( \frac{\alpha \epsilon}{P_{NT}^1} \right)^{\frac{1}{\gamma}} C_T^1$$

Substituting in the relative prices and the relative consumption as a function of the constant $\bar{c}$

$$1 = (\alpha \epsilon)^{\frac{1}{\gamma}} \, \bar{c}^{\frac{1}{\gamma}} {P_{NT}^2}^{-\frac{1}{\gamma}} \bar{c}^{-\frac{1}{\gamma}} C_T^2 = (\alpha \epsilon)^{\frac{1}{P_{NT}^2}^{-\frac{1}{\gamma}\gamma}} C_T^2$$

Which implies $\kappa^2(s) = 0$ and completes the proof ■

*Proof of Corollary 1.* A full risk-sharing steady state is a steady state in which relative weights are constant. As a consequence tradeable consumption is constant and marginal utilities are equal to some constant number. In this case Proposition 3 applies and the common monetary policy has no cost as countries attain the optimum on the non-tradeables side of the economy. Conditioning on the current states $s, z$ the economy has the same level of tradeable consumption and the (optimal) non-tradeable and labour supply. As this is a steady state the continuation values are also identical, which proves that the value of the two programs coincide. ■

*Proof of Proposition 4.* Conditional on the optimal choice in the outside option being default, the value of the outside option is independent of the current relative weight as it is independent of the stock of liabilities.
Furthermore, whenever a participation constraint binds, the country's relative weight is increased exactly of the amount that makes it indifference between the contract and the outside option. This implies $V_i^F(s,z) = V_i^M(s,z) = V_i^o(s,B)$. As the outside option value $V_i^o(s,B) = V_{UD}^i(s)$ is independent of the relative weight it must be the same for the fiscal and the currency union. Hence the statement of the proposition

$$V_i^F(s,z) = V_i^M(s,z) = V_i^o(s,B) = V_{UD}^i(s).$$

■

*Proof of Theorem 1.* We start by showing that in a monetary union consumption fluctuates in narrower bands whenever the steady state features non constant consumption. By Lemma 2 in a fiscal union the wedge is zero for both countries in every period, while it is non-zero for both countries in a currency union. We also know that, other things equal, the value of the problem decreases as the wedge moves away from zero

$$\frac{\partial \Omega^M(s,z)}{\partial |\kappa|} < 0, \; \frac{\partial \Omega^F(s,z)}{\partial \kappa}|_{\kappa=0} = 0,$$

in fact, when $\kappa \neq 0$, $\frac{\partial V_i^M(s,z)}{\partial|\kappa|} < 0$ for $i = 1$ and 2, since domestic labor and consumption of non-tradeables is distorted in both economies. However, regarding tradeables, what is important is how the wedge affects limited enforcement constraints. Recall that if $\nu_i > 0$ then $\nu_j = 0$, $j \neq i$. Without loss of generality assume that $\nu_2 = 0$ and $\nu_1 > 0$. The value of the Lagrange multiplier $\nu_1$ is given by

$$\frac{\partial\Omega(s,z)}{\partial V_1^M}|_{V_1^M=V_1^0} = \nu(s,z)$$

By concavity of $\Omega(s,z)$ it must be that

$$\nu_1(s,z)|_{\kappa(s)\neq 0} > \nu_1(s,z)|_{\kappa(s)=0}$$

Recall that $z' = z\frac{1+\nu_1}{1+\nu_2}$, therefore, it must be that

$$z'(s,z)|_{\kappa(s)\neq 0} > z'(s,z)|_{\kappa(s)=0}$$

This implies that for any given $z$, if a PC binds, next period $z'$ will be larger in currency unions than in fiscal unions. By the definition of steady states with imperfect risk-sharing it must be that consumption fluctuates in *broader bands* in currency unions than in fiscal unions.
Per se such higher volatility of consumption decreases the value of the problem. Furthermore, in currency unions, this is always paired with suboptimal non-tradeables. Hence in a steady state $(s,z)$ the value of the currency union is lower than the value of the fiscal union with independent monetary policies.

■

*Proof of Proposition 5.* The common currency monetary policy objective function is such that it minimizes the deadweight loss. Such minimized deadweight loss defines the set of feasible allocations in a monetary union. Conditioning on this restricted feasible allocation set the transfer policy solves the planner problem, thereby picking the efficient allocation in the constrained set. ■

*Proof of Corollary 2.* A central bank using the planner's relative Pareto weight maximizes

$$v(\epsilon) = zv^1(\epsilon) + v^2(\epsilon)$$

This results in the following first order condition:

$$z{C_{NT}^1}^{1-\gamma}\kappa^1(s) + {C_{NT}^2}^{1-\gamma}\kappa^2(s) = 0, \quad \forall s$$

Without loss of generality, assume that $z < 1$. This also implies that $C_{NT}^1 < C_{NT}^2$. Comparing this monetary policy rule with the one of a central banks that weighs equally the two countries:

$$C_{NT}^{1}{}^{1-\gamma}\kappa^{1}(s) + C_{NT}^{2}{}^{1-\gamma}\kappa^{2}(s) = 0, \quad \forall s,$$

country 1 will have a larger wedge as it carries less weight in the first order condition.
Following similar lines as the proof of the previous theorem, as country 1 has a larger wedge, if there is surplus in the contract, it will be rewarded with a larger $z'$ for all current $z$ in which the PC binds.
As in the theorem this implies a higher level of consumption fluctuations and a higher wedge, particularly so for the agent with high marginal utility.

■

# B Quantitative Model

In section 3, we produce a 5 state Markov process for the stochastic endowment $y$ of the tradeable good in each country. We do this by discretizing an AR1 process with persistence parameter $\rho = 0.9$ and shock variance $\sigma_y^2 = 0.01$, using the Rouwenhorst method. The transition matrix for this Markov process is:

$$\pi = \begin{pmatrix} 0.8145 & 0.1715 & 0.0135 & 0.0005 & 0 \\ 0.0429 & 0.8213 & 0.1290 & 0.0068 & 0.0001 \\ 0.0023 & 0.0860 & 0.8235 & 0.0860 & 0.0023 \\ 0.0001 & 0.0068 & 0.1290 & 0.8213 & 0.0429 \\ 0 & 0.0005 & 0.0135 & 0.1715 & 0.8145 \end{pmatrix} \tag{79}$$

The following graphs show, for each level of the tradeable endowment $y$, the interval $[\underline{z}(y), \bar{z}(y)]$ within which the participation constraints are satisfied. They therefore accompany the discussions in Section 3 on the ergodic sets for $z$ in each contract and the basins of attraction for these ergodic sets.

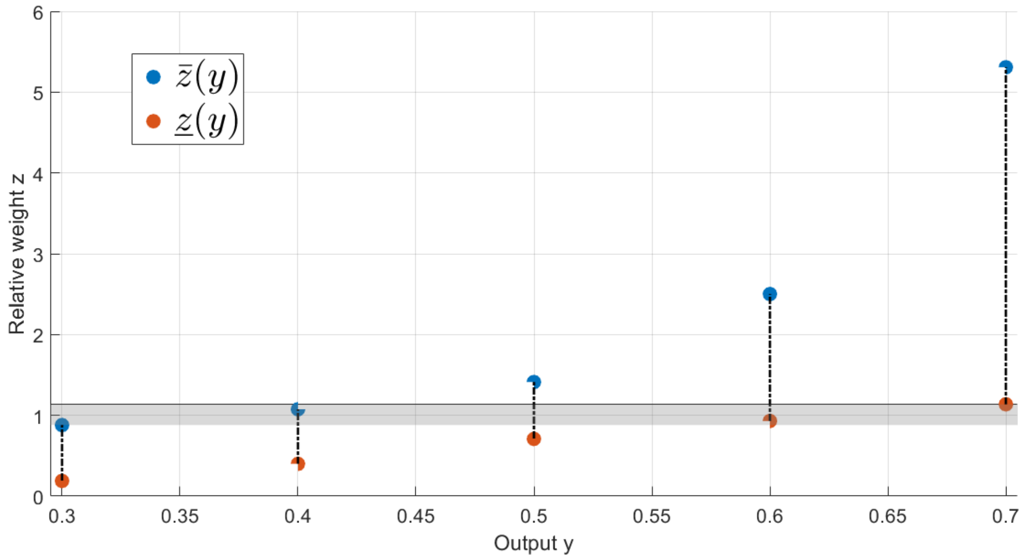


Figure 15: Relative Weights Bounds in Fiscal Unions

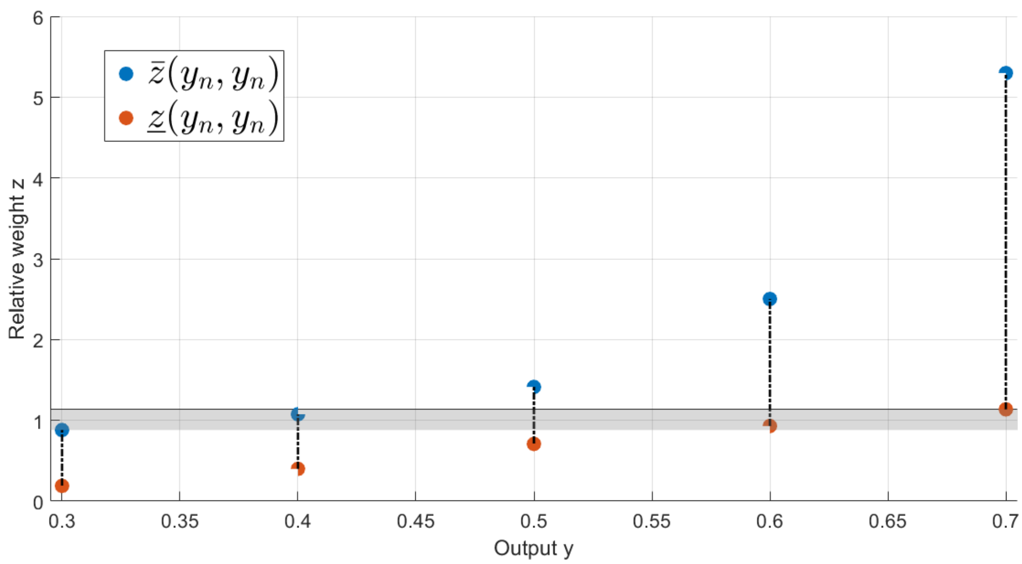


Figure 16: Relative Weights Bounds in Currency Unions