

THE UNDERLYING MECHANISMS OF ENTRY AND EXIT IN THE SMALL BUSINESS SECTOR*

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Abstract

This paper gathers new evidences on small business owners' entry and exit behaviors and emphasizes the importance of business transfers. It introduces a new theoretical framework designed to account for the underlying mechanisms of entry and exit with endogenous option to buy, found, sell or liquidate business assets. The model embeds a business for sale market with asset transfers and an equilibrium price designed to capture both the intertemporal and the intangible value of a firm. We use the model to characterize the important mismatches occurring on the business for sale market and recount the current episode of aging entrepreneurs and its consequences.

Keywords: Entrepreneurship, Business Selling and Buying Frictions, Aging

JEL classification: E21, J11, D24

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1 Introduction

Entry and exit constitute a fundamental aspect of the entrepreneurial activity and as such has been the subject of a substantial literature. But surprisingly, not much has been said about the underlying mechanisms supporting entry and exit despite their importance in the entrepreneurial lifecycle: a small and medium firm is either acquired (founded or bought mainly) by the entering entrepreneur and either transferred or liquidated by the exiting one. Instead, the existing literature has either treated the entrepreneur as an individual, discarding the details of how business assets are acquired and gotten rid of or focused solely on the firm aspects discarding the business owner. Interestingly, the empirical evidences on how businesses are founded, bought and sold tend to show that accounting for both the entrepreneur and the business assets is necessary. One such evidence is the existence of important mismatches on the business for sale market, barring transfers. Depending on the survey, between 11% and 17% of owners declare selling their business upon exit. However, this number is dwarfed by the number of entrepreneurs who were planning on selling: in the Annual Survey of Entrepreneurs (ASE) 2016, between 50% to 60% declared planning to sell upon exit.¹ It is thus illusory to consider that, for instance, an entrepreneur can exit immediately and without much financial harm and misguided to believe that productive business assets might survive independently of the capacity of the owner to sell. At the other end, consistently across a number of US surveys, 1 in 5 entries into entrepreneurship is due to a business purchase. Thus accordingly, the transfer of existing productive assets is dependent on the ability of entrants to buy and on the trade-off between purchasing and founding a business.

The observations above are reinforced by well known demographic trends showing that over the next decades the peak of the retiring baby-boomers will be reached, with a pivotal implication: the aging of entrepreneurs.² This evolution might be significant, both for entrepreneurial entry and exit and since entrepreneurs hold very undiversified portfolios ([Moskowitz and Vissing-Jørgensen \(2002\)](#)), which is of crucial importance for retirement planning. Thus the underlying mechanisms of entry and exit, with a business for sale market as the main means of asset transfers, are decisive in understanding the recent transformations of the entrepreneurial sector. However, the agenda of assessing these underlying mechanisms presents a few challenges. First, data on small and medium-sized enterprises (SME) transfers is scarce. Second, there is no theoretical framework in the literature to properly consider transfers in a standard entrepreneurial setting. Thus, the main contribution of this paper is to provide such a theo-

¹See [European Commission \(2011\)](#) for an European account: in Europe up to 60% of businesses are failing to transfer, including a large number for reasons unrelated to economic performances.

²In 2016, 30% of entrepreneurs were aged 60 years and more compared to only 22% in 2004 and 18% in 1989. Quite predictably the aging of the entrepreneurial population will only intensify in the upcoming years.

retical framework with, notably, endogenous options to buy or found on the entry side and sell and liquidate on the exit side. The model embeds a business for sale market to properly consider business transfers and is designed to capture the mismatches appearing on this market. We further use the model to provide a quantitative assessment of entrepreneurial aging on entry and exit and business transfers. We support our model by reporting a number of micro-data based new empirical facts on business creation and acquisition, the SMEs for sale market and on the aging and life-cycle aspects of entrepreneurship. In addition, we supplement these empirical evidences by collecting and building our own dataset. To the best of our knowledge, our paper is the first to tackle this agenda.

Our baseline economy is a stylized life-cycle incomplete markets model with heterogeneous agents and occupational choices. We introduce endogenous business buying, founding, selling and liquidating decisions into this framework as a decisive and novel mechanism for entrepreneurial entry and exit. Each period an incumbent entrepreneur might need to sell (voluntarily or involuntarily) her business and will face an endogenous selling price as well as a selling probability. Absent a selling opportunity, the incumbent will be forced to either dismantle the business and liquidate the entrepreneurial assets or continue the activity. Conversely, a non-entrepreneur might enter an entrepreneurial activity by endogenously choosing a firm size and either finding an available business to buy or founding a new business, these decisions being subject to credit constraints. A small and medium-sized enterprises for sale market (SMESM) aggregates selling and buying decisions. Its equilibrium price is designed as an abstract object to account for both the intertemporal and intangible value of a business. Outside this market, illiquid capital adjustments are subject to adjustment costs. We argue that all the above ingredients are key in reproducing the business selling, buying/founding and age related frictions appearing in the empirical data.

We mainly use data from the Annual Survey of Entrepreneurs (ASE), the National Longitudinal Survey of Youth 1979 (NLYS79), the Survey of Small Business Finance (SSBF) and the Panel Study of Income Dynamics (PSID) to document entrepreneurial acquisition and ceasing characteristics. We complement the available data by collecting our own dataset from transactions on the online business selling marketplace *Bizbuysell.com* (BBS) while comparing our sample to private business valuation datasets BizComps and ValuSource. Interestingly, our BBS dataset confirms the magnitude of the frictions on the business selling market: depending on the period, we find only 10% to 20% successful business selling transactions within a year, corroborating the ASE evidence on the gap between intentions to sell and actual transactions.

We argue and show that not capturing the fundamental evolution of the economy would affect our results in a significant way and that therefore, the standard stationary calibration

used in most of the literature is not appropriate for a number of questions. Instead, we develop a general calibration strategy for evolving economies that here takes into account the demographic shock and the aging of the population. Our results show that the model is able to capture both the cross-sectional properties and more importantly the effects of the changing demographic structure on the underlying mechanisms of entry and exit and the aggregates. For instance, our model reproduces the general evolution of the interest rate with its increase in the 1980's with the flow of arrival of baby-boomers and then its steady reduction with the aging of this population. Similar properties are also observed on the price of SMEs where we reproduce the hump-shaped price profile over the last 40 years. The model is also able to match the distribution of business sellers by age and the changing dynamics of retirement. Importantly, our results can be related to the ongoing debate on the declining US business dynamism.³ The model consistently produces the flat business exit rate and the declining entry rate of the last 40 years.

Finally, we conduct a number of experiments that displays the importance of the existence of a business for sale market. When we shut down this market, we find a significative decrease in entrepreneurial production and capital as well as of the number entrepreneurs. We also find that the interest rate increases. This is explained by the fact that reputable businesses, those that are the more productive, more likely to expand and have access to lower interest rates, are forced to liquidate instead of being transferred. Models with only capital adjustment costs as the sole means of exit are unable to capture this aspect of SMEs dynamics and we show in a decomposition that the maturity of a business is an important driver of the underlying mechanisms of entry and exit. Policy wise, we show that the tax menu imposed on business selling and buying has a remarkable impact on entrepreneurial production and on the fraction of entrepreneurs in the economy, with lower taxes fostering both.

Related Literature This paper is related to the extensive literature on SMEs and entrepreneurship. Many papers, both quantitative and theoretical, investigate entrepreneurial behavior and the selection into entrepreneurship. These models generally depict entrepreneurs as agents adjusting physical capital and hiring employees subject to idiosyncratic business shocks, entrepreneurial abilities or unexpected capital destruction and subject to financial frictions. Seminal papers in this literature are [Quadrini \(2000\)](#) and [Cagetti and De Nardi \(2006\)](#). In this framework, a number of studies focus on issues related to misallocation ([Buera and Shin \(2013\)](#) or [Dyrda and Pugsley \(2017\)](#)), tax policies (for instance, [Kitao \(2008\)](#) on capital taxation and [Cagetti and De Nardi \(2009\)](#) on estate taxation), or policies that aim to promote entrepreneurship ([Fairlie et al. \(2011\)](#); [Mankart and Rodano \(2015\)](#); [Gaillard and Kankanamge \(2018\)](#)). Also related to our contribution are [Liang et al. \(2014\)](#) and [Engbom \(2017\)](#) that show the link between

³See for instance [Decker et al. \(2016\)](#) or [Pugsley et al. \(2016\)](#) among many others.

entry into entrepreneurship and age. Compared to the above literature, our paper introduces an empirically realistic theoretical framework that accounts for the life-cycle property of entrepreneurship and the underlying mechanisms of entry and exit while explicitly modeling the market frictions arising upon the transfer of business capital. As in [Bhandari and McGrattan \(2018\)](#), our model distinguishes intangible and tangible assets. In particular, liquidating a firm allows entrepreneurs to recover tangible business assets, while selling a (or part of a) business reproduces the transfer of both tangible and intangible assets.

Concerning the macroeconomic effects of the demographic change, [Pugsley et al. \(2016\)](#) argue that the lower population growth rate and the aging of the population has led to a lower start-up rate in the US. Since entrepreneurship and the demographic evolution are closely related, our calibration strategy takes into account the change that occurred in the 80s by targeting the (possibly) non-stationary 2007 US economy. Our paper also contributes to the growing literature trying to understand the implications of illiquid assets versus liquid assets. In our model, entrepreneurs face adjustment costs when investing and disinvesting in business assets, which is crucial in generating the opportunity to buy and sell an existing business.

Finally, we use the recent *DC-EGM* computational techniques in [Iskhakov et al. \(2017a\)](#); [Druedahl and Jørgensen \(2017\)](#) that extend the Endogenous Grid Method (EGM) initially introduced by [Carroll \(2006\)](#) to the combination of multiple assets with discrete and continuous endogenous variables.⁴ In addition, to achieve some notable computational advantages, this method also allows for *IID extreme value taste shocks* which is of particular interest since, as argued in [Hurst and Pugsley \(2011, 2015\)](#), unobservable heterogeneity in non-pecuniary benefits is of first order importance for the decision to become an entrepreneur.⁵

The remaining of the paper is organized as follows. Section 2 documents empirical facts on business acquisition and transfers, the business for sale market and the entrepreneurial life-cycle. In Section 3 we present our baseline model and Section 4 describes how we take the model to the data. We discuss the results of the model in Section 5. In Section 6, we evaluate the importance of the business for sale market. Finally, section 7 concludes.

⁴Applying these methods make the model tractable. The two assets structure, the presence of occupational choices in addition to occasionally binding constraints make the problem difficult to solve notably due to the existence of kinks. Thus, first order conditions are no longer sufficient, while still necessary.

⁵The authors have shown that a significant fraction of entrepreneurs choose to own businesses due to non-pecuniary reasons and never intend to grow their firms. In line with this, our model allow for non-pecuniary self-employment benefits reflected by idiosyncratic taste shocks over households' occupational choices.

2 Business transfers and the SME for sale market

In this section we detail the empirical evidences on business transfers and the small and medium-sized enterprises (SME) for sale market. We rationalize disparate information from the following US survey micro datasets: the 2007 Survey of Business Owners (SBO), the Annual Survey of Entrepreneurs (ASE), the National Longitudinal Survey of Youth 1979 (NLYS79), the Panel Study of Income Dynamics (PSID), the Survey of Consumer Finances, and the Survey of Small Business Finances (SSBF). These datasets provide broad pictures of the characteristics of business buyers and sellers and of the key differences between purchased businesses with respect to founded ones. To better characterize the SME for sale market, we also collected and built a novel dataset with unique business transaction observations. This dataset originates from the gathering of business for sale transactions data from 2011 to 2019 on the online business selling platform *Bizbuysell.com* (hereafter BBS).⁶ This dataset contains key statistics on the time needed to sell and the probability of selling a business during a month, a quarter or a year. It also contains information on the main reason a business is being sold, its sector, size, cash-flows, EBITDA, fixed capital, etc.⁷

Finally, for consistency reasons among the datasets and the model specification, we define an entrepreneur as an active self-employed business owner whenever it is possible and as a business owner otherwise.⁸

2.1 Business acquisition and exit

The literature on entrepreneurship has long been interested in the behavior of incumbent entrepreneurs but has been somewhat silent on how businesses came to be in the first place. Throughout this paper, we argue that purchasing and selling a business are important components of entrepreneurship, as evidenced by the behavior of a non negligible fractions of entrepreneurs in the data. Survey questions often define as *acquisition* the way the entrepreneur became the owner of the business: founding a new business or purchasing an existing business are two common alternative types of acquisition. Using several survey data, [Table 1](#) provides estimates on the type of acquisition. Excluding inheritance and gifts, 20% of entries into entrepreneurship are the result of the purchase of an existing business, consistently across surveys

⁶This platform is among the oldest and largest marketplaces dedicated to facilitate business selling transactions in the US either directly or through brokers. The available data correspond to 92,900 observations of for sale and sold business records obtained from continuously gathering public data available on this website over a number of years and complementing them by gathering various broker and archived information.

⁷In contrast to other business for sale transactions data, such as *ValuSource* and *BizComps*, BBS lets us infer the probability of actually selling a business in a year. [subsection A.1](#) shows the summary statistics.

⁸Unfortunately, we cannot control for active self-employment in BBS.

in the US.^{9,10}

Table 1. Business acquisition by type

Survey	Year	Sample selection ^a	Acquisition (%)		Transmission (%)	
			Founded	Purchased	Inherited ^b	Other/Gift ^b
SCF	2016	all entrepreneurs	74.4	18.2	3.5	3.9
ACF	2016	only employers	68.1	20.8	4.0	7.1
SSBF	2003	all entrepreneurs	79.8	16.7		– 3.4 –
SSBF	2003	entrepreneurs (< 5y)	77.4	20.8		– 1.8 –
SBO	2007	all entrepreneurs	74.8	18.2	4.8	2.2
SBO	2007	entrepreneurs (< 5y)	74.0	19.3	5.5	1.2

^a The estimates are based on self-employed entrepreneurs defining themselves as business owners. Early-stage entrepreneurs are those who acquired their businesses within the last 5 years.

^b When possible, we distinguish the acquisition type between gift/other and inheritance.

For new entrants into entrepreneurship, purchasing or founding a business are two options with different implications. Using the SSBF, we show that purchasing new self-employed business owners obtain a 1.6 percentage points lower interest rate financing on their debt as compared to founders.¹¹ Moreover, using the SBO, we find that from the date of acquisition, early-stage founded firms (less than 3 years of operation here) are almost twice as more likely to close for economic reasons (due to low sales or inadequate credit conditions) than purchased ones, with respective failure rates of 9.1% and 5.5%. We therefore infer that the operational *maturity* of an existing business is an essential component to evaluate when deciding either to found or purchase a business. On the other hand, purchasing a business is costly as necessary funds to purchase the business capital, including intangible assets, have to be provided.

Concerning the exit from entrepreneurship, we find little detailed evidence in the literature despite an important number of papers focusing on exit and its link to life-cycle aspects. We find that a non-negligible fraction of entrepreneurs sell their businesses upon exit: 7.5% according to the SBO (2007) (11% if we take into account business owners with paid employee(s)), 15% in the NLSY79 (2002-2016) and 17.1% in the 2016 ASE for business owners with paid employee(s).¹² This selling behavior is largely related to the entrepreneurial age profile and the decision to retire. Using the PSID data, we show on [Figure 1](#) that the selling of business assets

⁹We also provide estimates for other advanced economies, such as France and Spain, in the Online Appendix. Results show that business transfers through inheritance are slightly more frequent in Europe.

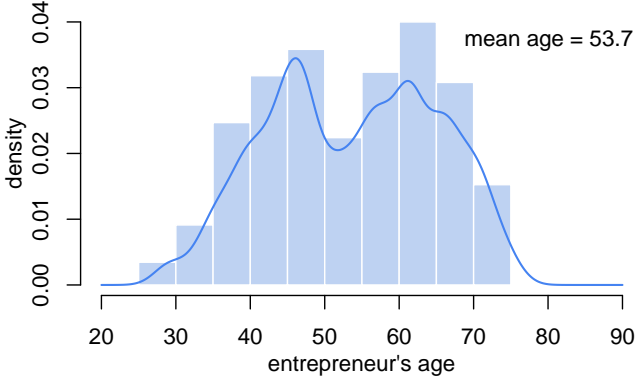
¹⁰It is worth noting that individuals who are purchasing an existing business are not more likely to have experienced a previous self-employment situation than those starting a new business: 28.6% of purchasers already had one, against 38.6% for founders (SBO, 2007). We find similar evidence using the NLSY79.

¹¹See [Appendix B](#) for the detail of our regression on financing conditions between purchasers and founders.

¹²Note that we collect this information from different years using the NLSY79, due to the lack of observations.

peaks at two age brackets: the 45-50 and the 60-65, close to the typical US average retirement age. This is corroborated in the SBO and the ASE: the frequency of businesses being sold is particularly high for age brackets 55-64 and 65 or over.

Figure 1. Distribution of business assets sold by age in PSID (1990 - 2015).



Finally, we find evidences suggesting substantial difficulties for transferring businesses in the SME for sale market. According to the ASE 2016, among the business owners with paid employee(s) reporting how they planned to exit entrepreneurship, 50% were thinking of selling their businesses to a third-party and 10% planned to sell to a member of their family. This is in contrast to the much lower number of businesses actually being sold that we report above. Moreover, among the business owners holding a firm of 16 years of age and more, i.e. a business much less likely to close due to economic reasons, the main exit strategy is consistently the sale of the entire business (53%). However, even in this population, only 26% declared effectively selling their business while 43% declared just retiring and 14% declared failing due to business conditions in the *ex post* realized exits.

2.2 Small and middle-sized businesses for sale market

Given the remarkably low number of entrepreneurs reporting having sold their business, it seems that there are potentially important failures when trying to sell. In particular, we are interested in whether there are mismatches in the businesses for sale market, with some SME owners not able to make any deal, resulting in liquidations (i.e. the sale at a very low price and usually restricted to tangible assets). We use the BBS data to infer the probability of selling a business as well as the time needed to sell.

We first compare the BBS dataset to other available sources. Our estimates shows that among sold businesses, 23% to 25% were sold because the owner(s) retired, against 19% in the ASE (2016). Concerning the selling prices distribution, we find a mean value of 579K USD and a median value of 190K USD in BBS against respectively 682K USD and 95K USD in the

PSID.¹³ Therefore, the BBS price distribution is somewhat shifted to the right, with fewer very small businesses for sale.¹⁴ Overall, we believe the BBS dataset offers better quality data and most importantly provides the first characterization of the potential mismatches and frictions on the SME for sale market.

Using the BBS data on sold businesses from 2011 to 2017, we find an average time on market (TOM) of about 232 days (7-8 months). This is substantially longer than on the housing market, already characterized by a long process with observed TOM around 4-5 months. However, the fraction of sold businesses account only for a small fraction of the total number of firms for sale and importantly the TOM is only indicative of the time needed to sell a business that was eventually sold at some point. It is not indicative of the likelihood that any businesses for sale are sold. Therefore, we use the BBS data to infer the probability that a business is sold within a year by constructing a daily panel of businesses for sale between 2018 and 2019. We then construct cohorts of businesses for sale of 2 months and compute the total number of sold businesses in time. We exclude from the cohorts all the businesses that disappeared, without resulting in a sale.¹⁵ Then, for a given number of periods after the listing dates, we compute the fraction of sold businesses relative to the total initial number of businesses for sale within the cohort. The resulting indicator provides the fraction of businesses for sale that is actually sold after a given number of periods.

Figure 2 displays the probability to sell a business after a number of months from the listing date and by listing price brackets. After a full year, only around 25% of businesses for sales are actually sold. While this number is fairly similar for any business size (as proxied by the listing price), it seems to be slightly easier to sell a smaller business.

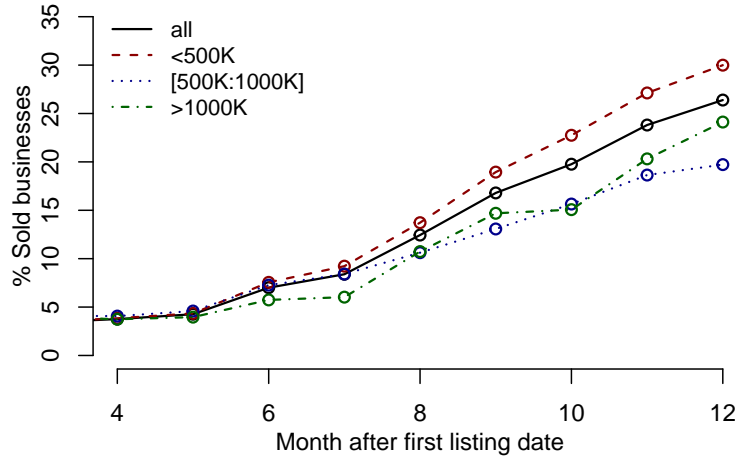
In Table 2, we further document the probability to sell disaggregating by sectors, employment size and main reason to sell. Overall, the probability to sell a business within a year is quite low, around 20% to 30%. Whether it is related to underlying business performances or to potential mismatches in finding a buyer, not transferring a business appears as a first-order concern for entrepreneurs planning to exit, and, to a larger extend, influences employment, payrolls and production, leading to potential economic losses. By not transferring businesses, tangible (machinery, property, equipment, furniture, inventory, cash) and intangible (customer lists, good credit conditions, business methods, patents and copyrights, trademarks, etc.) assets will be lost, a fact that has so far attracted only very little attention in the academic literature.

¹³This comparison is indicative: there are only 357 observations concerning sold businesses in the PSID (1990 to 2015) against more than 90,000 in BBS. Notice that we can not control for self-employment in BBS, we therefore use business owners in the PSID.

¹⁴Many listings on the platform are broker mediated and announcers have to pay a monthly premium membership to list their entry. This might be constraining enough for very small businesses and could explain the shift.

¹⁵Results are qualitatively similar if we include those businesses, while the magnitude of the probabilities are lowered by around 10%.

Figure 2. Probability to sell with listing price as a proxy for size.



To be read as follows: *after a year, 25% of all businesses have been sold.* The listing price corresponds to that at the date of the. *Source:* Authors' own computation using collected BBS data.

3 Model

The economy consists of a corporate sector and a unit measure of heterogeneous agents. A fraction of the latter, called entrepreneurs, hold small and medium sized businesses while the remaining, called workers, occupy a wage paying job in the corporate sector. Entry and exit in and out of the small business sector is subject to specific conditions. On the one hand, individuals entering self-employment have to either found a new business or purchase an existing one. On the other hand, upon exit, entrepreneurs can either sell their business or liquidate the physical business assets. Therefore, a *small and medium-sized enterprises for sale market (SMESM)* constitutes a pivotal piece of our model. Finally, a government levies a menu of taxes to cover for old age pensions and other public spendings.

3.1 Aggregate Variables and Corporate Sector

The corporate sector output Y is produced by a single competitive representative firm using a Cobb-Douglas technology with capital share $\alpha \in (0, 1)$ and total factor productivity A , capital level K_c and labor L_c , such that: $Y = F(K_c, L_c) = AK_c^\alpha L_c^{1-\alpha}$. Capital depreciates at rate δ in both the corporate and the SME sectors.

The model is characterized by a distribution of agents Φ and a set of prices. Factor prices are the interest rate $r = F'_{K_c}(K_c, L_c)$ and the wage rate $w = F'_{L_c}(K_c, L_c)$. Finally q is the private business price clearing the *SMESM* market. Because we are interested in the impact of demographic evolutions and thus will not employ a stationary calibration, model variables can be subject to specific aggregate changes. We regroup these changes in the set of aggregate variables Ω . Examples of aggregate variables include demographic changes or aggregate pro-

Table 2. Probability to sell and various characteristics

Characteristic	Subsample	Probability (in %) to sell after	
		6 months	1 year
All businesses		8.4	26.4
US State	Pennsylvania	7.5	23.8
	California	7.2	31.9
	Texas	15.8	28.1
Listing price	< 500K	9.2	30.0
	[500K : 1000K]	8.4	19.7
	> 1000K	6.0	24.1
Number of employees	≤ 5	10.7	30.0
	[6 : 49]	13.3	30.9
	> 49	3.3	17.4
Reason to sell	Retirement	6.2	25.4
	Manufacturing	3.2	17.4
	Food and Restaurants	9.2	26.1
Sector	Wholesale and Distributors	10.2	36.8

Source: Authors' own computation using collected BBS data.

ductivity changes. In essence, these aggregate variables have an effect of the distribution of agents and prices over time.¹⁶ However, for the sake of clarity, we drop in our notations this dependence and write $\Phi = \Phi(\Omega)$, $r = r(\Phi, \Omega)$, $w = w(\Phi, \Omega)$ and $q = q(\Phi, \Omega)$. A stationary equilibrium is attained by assuming that the variables in Ω are constant.

3.2 Agents

We use a stylized life-cycle setup with aging and probabilistic dying in the last age bracket. Households live through J stages of life and the total population, of unit mass, is divided among J generations indexed with $j \in [1; J]$. Groups 1 through $J - 1$ are called Juniors and have access to the labor market. The J th group, called Seniors, is comprised of individuals beyond the retirement age. We assume that a fraction p_{die} of the Seniors deacease and exit the model.¹⁷ Over the life-cycle, households belong in an occupation $o \in \{o_e, o_w, o_r\}$. Junior households can be entrepreneurs (o_e) or occupied in the workforce (o_w) whereas Senior households are either retired (o_r) or are old age entrepreneurs.

Households have preferences over consumption c described by utility:

$$\mathcal{U}(c, j, o) = u(c) - \mathbb{1}_{j=J, o=o_e} u_R$$

¹⁶Our calibration strategy consists in replicating the demographic transition starting in the 1970's and 1980's.

¹⁷This assumption is widely used in the literature, see [Sommer and Sullivan \(2017\)](#) for an application in housing models or [Cagetti and De Nardi \(2006\)](#) in a related literature.

Senior households also face an additional utility cost u_R when operating a business, in order to translate the difficulty of still being active in old age.

Depending on its occupation, a household can possess liquid and/or illiquid assets. Liquid assets are akin to savings and are noted a . Illiquid (business) assets, noted k , are used to produce with the entrepreneurial technology. Both the corporate and the entrepreneurial sector produce a homogenous consumption good. The liquid asset can be freely used to purchase it but not the illiquid asset. Our setup explicitly defines conditions to convert illiquid capital into liquid assets and conversely. To obtain liquid assets from illiquid assets, individual have to either sell their firm contingent on finding a buyer or liquidate partly or totally subject to an adjustment cost. Conversely, acquiring illiquid capital using liquid capital is subject to an adjustment cost but can be also achieved by buying a firm with a specific illiquid capital amount.

The state space for an entrepreneur are savings a , business capital k , and $\mathbf{x}_e = \{j, m\}$, where $m = \{0, 1\}$ indicates whether the business is mature. Entrepreneurs are not permitted to possess multiple firms. Similarly, the state space for a worker is a , and $\mathbf{x}_w = \{j, y, \iota\}$, with y the worker's productivity and ι her entrepreneurial ability to manage a business. We note $\mathcal{Y}(j, y)$ the worker's income. The entrepreneurial income comes from entrepreneurial production using production function $f(k)$.

Note that for the sake of parsimony, we abstract from entrepreneurs hiring workers but it remains a straightforward extension. We also abstract from the explicit representation of unemployment dynamics in the economy, mainly for the sake of simplicity.

3.3 Dynamic Problem

We decompose an agent's intra-period decision process into a sequence of three subperiods. In the last subperiod, the consumption-saving and entrepreneurial investment problems are tackled. In the middle subperiod, the buying and selling problems are addressed contingent on occupational changes and the maturity of a business.¹⁸ Finally, in the first subperiod occupational choices are made. Given that $\mathcal{W}(a, \mathbf{x}_w)$ and $E(a, k, \mathbf{x}_e)$ are respectively the general value function of a worker and an entrepreneur, [Figure 3](#) summarizes this decomposition. Following this decomposition, the problem is solved backwards and we detail below the problem in each subperiods.

3.3.1 The last subperiod: Consumption-Saving problem

Depending on choices made in the previous subperiods, consumption and saving decisions in the last subperiod can be distinguished into those of workers either continuing or exiting

¹⁸Only mature firms ($m = 1$) can be sold on the *SMESM*. This is to reflect the fact that the average age of sold businesses is much higher than the average age of all firms.

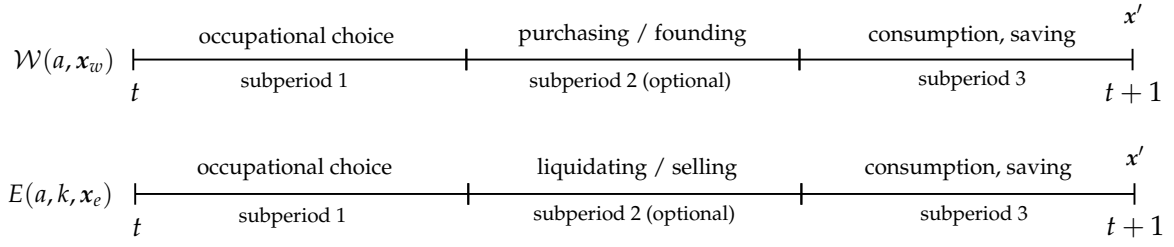


Figure 3. Intra-period timing of an individual.

their activity and those of entrepreneurs continuing or exiting theirs. For the sake of simplicity, continuing workers are subject to a no-borrowing constraint. Similarly to an incumbent entrepreneur, an exiting worker entering an entrepreneurial activity can borrow in order to invest in a level of business assets k , as long as a minimum amount is pledged using their own wealth. Thus those individuals are subject to the following borrowing constraint:¹⁹

$$a' \geq -\psi(k) \quad (1)$$

$$\psi(k) = (1 - \theta)qf(k) \quad (2)$$

An indebted entrepreneur faces an interest $r_b(m)$ that depends on the maturity m of her business such that $\tilde{r}(m) = \mathbb{1}_{a' \geq 0}r - \mathbb{1}_{a' < 0}r_b(m)$.

Continuing entrepreneurs An incumbent entrepreneur continuing her activity chooses next period's illiquid capital k' and saving a' given her current income $f(k)$. Investing or disinvesting in the illiquid capital are subject to a capital adjustment cost noted $\mathcal{C}(k, k', \phi)$. ϕ an asymmetric adjustment cost parameter with $\phi = \phi_u$ when investing and $\phi = \phi_d$ when liquidating. The consumption-saving problem of this entrepreneur is thus:

$$E^c(a, k, \mathbf{x}_e) = \max_{c > 0, a' \geq -\psi(k'), k' \geq 0} \left\{ \mathcal{U}(c, j, o_e) + \beta \mathbb{E}_{j', m' | j, m} E(a', k', \mathbf{x}'_e) \right\} \quad (3)$$

$$s.t. \quad c + a' + k' = (1 + \tilde{r}(m))a + f(k)(1 - \tau_y) + k(1 - \delta) - \mathcal{C}(k(1 - \delta), k', \phi) \quad (4)$$

with E^c the subperiod specific value function of this entrepreneur and τ_y the income tax rate.

Exiting entrepreneurs When exiting, an entrepreneur has to choose savings a' subject to the no-borrowing constraint. Depending on whether the entrepreneur is exiting entrepreneurship by voluntarily liquidating or is forced to liquidate because of business failure ($z = 0$) or is

¹⁹This kind of borrowing constraint specification is widely used in the literature. See for instance the papers by [Kitao \(2008\)](#) or [Buera and Shin \(2013\)](#).

selling the business ($z = 1$), we have:

$$E^e(a, k, \mathbf{x}_e, z) = \max_{c > 0, a' \geq 0} \left\{ \mathcal{U}(c, j, o_e) + \beta \mathbb{E}_{j', a' | j} \mathcal{W}(a', \tilde{\mathbf{x}}'_w) \right\} \quad (5)$$

$$\begin{aligned} \text{s.t. } c + a' &= (1 + \tilde{r}(m))a + f(k)(1 - \tau_y) + (1 - z)(k(1 - \delta) - \mathcal{C}(k(1 - \delta), 0, \phi)) \\ &\quad + zq(k(1 - \delta))(1 - \tau_s) \end{aligned} \quad (6)$$

with E^e the subperiod specific value function of this entrepreneur and $\tilde{\mathbf{x}}'_w$ the specific exogenous worker state of an exiting entrepreneur.²⁰ Liquidating is identical to adjusting the business capital to zero by fully paying the corresponding adjustment cost $\mathcal{C}(k(1 - \delta), 0, \phi)$. Alternatively, by successfully selling the business the entrepreneur recovers the total amount $q(k)$, subject to the sales tax τ_s .

Continuing workers Such a worker has to choose savings a' subject to the no-borrowing constraint and solves:

$$\mathcal{W}^c(a, \mathbf{x}_w) = \max_{c > 0, a' \geq 0} \left\{ \mathcal{U}(c, j, o_w) + \beta \mathbb{E}_{j', y', a' | j, y, a} \mathcal{W}(a', \mathbf{x}'_w) \right\} \quad (7)$$

$$\text{s.t. } c + a' = (1 + r)a + (1 - \tau_y)\mathcal{Y}(j, y) \quad (8)$$

with \mathcal{W}^c the subperiod specific value function of this worker.

Exiting workers An exiting worker enters entrepreneurship with business assets k' in the next period, either by purchasing an existing mature business ($d = 1$ and $m' = 1$) and paying the total amount $q(k')$ or by founding a new business ($d = 0$ and $m' = 0$) and paying the adjustment cost $\mathcal{C}(0, k', \phi)$ with d the index for the type of acquisition. This worker solves:

$$\mathcal{W}^e(a, k', \mathbf{x}_w, d) = \max_{c > 0, a' \geq -(1 - \theta)k'} \left\{ \mathcal{U}(c, j, o_w) + \beta \mathbb{E}_{j' | j} E(a', k', \mathbf{x}'_e) \right\} \quad (9)$$

$$\text{s.t. } c + a' = (1 + r)a + (1 - \tau_w)\mathcal{Y}(j, y) - dq(k')(1 + \tau_b) - (1 - d)(k' + \mathcal{C}(0, k', \phi)) \quad (10)$$

with \mathcal{W}^e the subperiod specific value function and τ_b the business purchase tax.

3.3.2 The middle subperiod: acquisition and selling problems

In the middle subperiod the buying/founding and selling/liquidating problems are solved.

²⁰The main specificity is the assumption that any new worker coming from the entrepreneurial sector starts with the lowest level of worker productivity. The argument is that the productivity state y is strongly related to a worker's experience in a specific corporate job. This seniority on a job can not be randomly obtained but has to be *earned*. Recall, however, that there is an age-component in the determination of the wage process. Moreover, in this case the entrepreneurial ability is determined using the invariant distribution of this process.

The selling problem An entrepreneur with a mature business ($m = 1$) can try to sell it on the SMESM. A buyer is found with probability $h_s(q)$. Otherwise, the entrepreneur chooses whether to liquidate or continue the business. In the end, when the entrepreneur exits endogenously, the following problem is solved :

$$\mathcal{S}(a, k, \mathbf{x}_e) = h_s(q) \underbrace{E^e(a, k, \mathbf{x}_e, 2)}_{\text{Selling}} + (1 - h_s(q)) \max \left\{ \underbrace{E^e(a, k, \mathbf{x}_e, 0)}_{\text{Liquidating}}, \underbrace{E^c(a, k, \mathbf{x}_e)}_{\text{Continuing}} \right\} \quad (11)$$

and the following alternative problem when the entrepreneur is forced to exit:

$$\tilde{\mathcal{S}}(a, k, \mathbf{x}_e) = h_s(q) \underbrace{E^e(a, k, \mathbf{x}_e, 2)}_{\text{Selling}} + (1 - h_s(q)) \underbrace{E^e(a, k, \mathbf{x}_e, 1)}_{\text{Liquidating(forced)}} \quad (12)$$

where \mathcal{S} and $\tilde{\mathcal{S}}$ are the subperiod specific value functions for this problem.

The founding and purchasing problem Depending on whether a worker is currently buying ($d = 1$) or founding ($d = 0$) a business, the problem of the future entrepreneur is to choose the optimal capital size k' :

$$\tilde{\mathcal{W}}(a, \mathbf{x}_w, d = 0) = \max_{k'} \mathcal{W}^e(a, k', \mathbf{x}_w, d = 0) \quad (13)$$

$$\tilde{\mathcal{W}}(a, \mathbf{x}_w, d = 1) = \max_{k' \geq \bar{k}} \mathcal{W}^e(a, k', \mathbf{x}_w, d = 1) \quad (14)$$

We assume that businesses have to be larger than a minimum size \bar{k} in order to be bought and we parameterize this minimum size on the empirical data. A worker trying to buy a business has a probability $h_b(q)$ of finding a seller. Otherwise, she chooses whether to found a new business or to continue being a worker. In the end, the following problem is solved:

$$\mathcal{B}(a, \mathbf{x}_w) = h_b(q) \underbrace{\tilde{\mathcal{W}}(a, \mathbf{x}_w, 1)}_{\text{Buying}} + (1 - h_b(q)) \max \left\{ \underbrace{\tilde{\mathcal{W}}(a, \mathbf{x}_w, 0)}_{\text{Founding}}, \underbrace{\mathcal{W}^c(a, \mathbf{x}_w)}_{\text{Continuing}} \right\} \quad (15)$$

with $\mathcal{B}(a, \mathbf{x}_w)$ the subperiod specific value function for this problem.

3.3.3 The first subperiod: occupational choice and exit strategy

Worker A worker starts the period with states $\{a, \mathbf{x}_w\}$ and, provided she has an entrepreneurial ability (i.e. $\iota = 1$), chooses whether to try to purchase an existing business, to found a new business or to remain a worker.²¹ In the end, the following problem is solved:

$$\mathcal{W}(a, \mathbf{x}_w) = (1 - \iota) \mathcal{W}^c(a, \mathbf{x}_w) + \iota \max \left\{ \mathcal{B}(a, \mathbf{x}_w) - u_b, \tilde{\mathcal{W}}(a, \mathbf{x}_w, 0), \mathcal{W}^c(a, \mathbf{x}_w) \right\} \quad (16)$$

with u_b the implied small utility cost of searching for a seller.

²¹The entrepreneurial ability follows a first order Markov process with two states: possessing ($\iota = 1$) or not possessing ($\iota = 0$) the ability. An alternative specification would be to set an exogenous probability of drawing an entrepreneurial ability each period. However, our specification allows for a persistent entrepreneurial ability that generates higher saving rates and brings the model closer to the data.

Entrepreneur An entrepreneur starts the period with states $\{a, k, \mathbf{x}_e\}$ and decides whether to sell, liquidate or continue her business endogenously unless she is forced to exit. In the end, the following problem is solved:

$$\begin{aligned}
E(a, k, \mathbf{x}_e) = & (1 - \chi(m)) \left[\zeta \left(m \max\{\mathcal{S}(a, k, \mathbf{x}_e) - u_s, E^e(a, k, \mathbf{x}_e, 0)\} + (1 - m)E^e(a, k, \mathbf{x}_e, 0) \right) \right. \\
& + (1 - \zeta) \left(m \max\{\mathcal{S}(a, k, \mathbf{x}_e) - u_s, E^e(a, k, \mathbf{x}_e, 0), E^c(a, k, \mathbf{x}_e)\} \right. \\
& \left. \left. + (1 - m) \max\{E^e(a, k, \mathbf{x}_e, 0), E^c(a, k, \mathbf{x}_e)\} \right) \right] + \chi(m)E^e(a, k, \mathbf{x}_e, 0) \quad (17)
\end{aligned}$$

with u_s the implied small utility cost of searching for a buyer. ζ is the unconditional probability of entrepreneurial exit whereas $\chi(m)$ is the conditional probability of entrepreneurial exit due to business failure, assuming that the maturity of the business is an important determinant of the latter probability. Only businesses that does not fail can be sold.

3.4 The small and medium-sized enterprises for sale market (SMESM)

On the *SMESM*, businesses sellers and buyers meet. For the sake of tractability, we make a number of assumptions:

- (i) The market is intermediated by passive brokers on both the selling and the buying side. With this assumption, we avoid the complex problem of the direct matching between heterogenous buyers and sellers. Moreover, the brokers provide liquidity to the market.
- (ii) We assume that firms are valued and exchanged in this market as units of production. A firm generating a production $f(k)$ will have each unit of production valued at the equilibrium price q (i.e. the price of a business of size k is $qf(k)$). Selling a firm means providing all the units of production owned by an entrepreneur at the same time to the market. But, as units of production are undistinguishable, buying a firm is equivalent to collecting available units of production until the endogenously decided production size is attained. This is a key assumption and we argue that a number of reasons supports it. First, it lets us recover in a stylized way that entire businesses are exchanged without loosing global production value.²² Second, we find empirical evidence that in many instances SMEs are bought not by a single individual but by several individuals grouped together. Our assumption lets us recover this fact: a sold business can be bought by many individuals as bundles of units of production. In the aggregate, we thus have a seamless transfer of the value of a business and when it disappears we do not need to track each of its profit units.

²²If for instance units of illiquid capital were exchanged instead, the aggregate total production value, bought as a sum of smaller units, would diminish due to the (generally decreasing) returns to scale of the entrepreneurial production function.

Finally, it lets us capture that selling a business cannot be reduced to selling only its tangible assets. Instead, the value recovered after a transaction should cover the discounted value of future profits and as argued for instance by [Bhandari and McGrattan \(2018\)](#), intangible assets constitute a large part of this value.²³ We convey this idea here through the fact that the price q is an abstract object. It is determined at the global equilibrium between the amount of units of production sold and bought translating at the same time the intertemporal and intangible value of a business.²⁴

- (iii) Sellers, buyers and brokers are price takers. We assume that a seller upon meeting a broker agrees on a selling price that is $q(1 - \mu_s) \leq q$. Symmetrically, a buyer upon meeting a broker agrees a buying price $q(1 + \mu_b) \geq q$. where μ_s and μ_b represents commissions that have to be paid by the seller and the buyer to brokers. Brokers exchange units of production at equilibrium price q among themselves. The equilibrium of the market also solves the matching problem as shown below.

We call sellers side tightness θ_s the brokers to sellers ratio. We use a standard matching technology with constant returns to scale with parameter γ . The probability that a broker matches a seller is $\alpha_s(\theta_s) = (\frac{1}{\theta_s})^{1-\gamma}$, computed as the ratio of matches to brokers. Conversely, the probability that a seller meets a broker is $h_s = \theta_s^\gamma$, computed as the ratio of matches to sellers. γ is a matching function parameter. The brokers' free entry condition states that they have to equalize the per unit entry cost κ_s with net revenues from buying a business given the probability of a match. The buyers' side is symmetric with tightness θ_b . The probability α_b that a broker matches a buyer is $\alpha_b = \alpha_b(\theta_b) = (\frac{1}{\theta_b})^{1-\gamma}$. The probability that a buyer meets a broker is $h_b = \theta_b^\gamma$. The free entry condition on the two sides of the market are:

$$\begin{aligned} \kappa_s \pi(k) &= \underbrace{\alpha_s(\theta_s)}_{\text{Probability of a match}} \underbrace{q\mu_s f(k)}_{\text{Net revenue}} \\ \kappa_b \pi(k) &= \underbrace{\alpha_b(\theta_b)}_{\text{Probability of a match}} \underbrace{q\mu_b f(k)}_{\text{Net revenue}} \end{aligned}$$

From the above broker condition we can extract the tightness as $\theta_s = \left(\frac{q\mu_s}{\kappa_s}\right)^{\frac{1}{1-\gamma}}$ and $\theta_b = \left(\frac{q\mu_b}{\kappa_b}\right)^{\frac{1}{1-\gamma}}$. The seller's selling probability is thus: $h_s(q) = \left(\frac{q\mu_s}{\kappa_s}\right)^{\frac{\gamma}{1-\gamma}}$ and the buyer's buying probability is thus: $h_b(q) = \left(\frac{q\mu_b}{\kappa_b}\right)^{\frac{\gamma}{1-\gamma}}$. Note that there is an implicit assumption that $q > 1$.²⁵ Moreover these conditions create a link between the price of a unit of private business production and the probabilities of finding a buyer and a seller.

²³Using the VS data, we estimate a ratio of intangible assets over the business price of about 38% for the median and 54% for the mean.

²⁴[Bhandari and McGrattan \(2018\)](#) find that there is little cross-sectional dispersion in intangible assets valuation, supporting our choice of a single price q for all production units.

²⁵This is likely to be the case since a business can be kept several periods. Therefore, an entrepreneur enjoy the

3.5 Demography and bequest

We assume that individuals in the Senior group value leaving a bequest and that this valuation is obtained with a warm-glow utility $\hat{u}((1 - \tau_a)a)$, with τ_a the estate tax rate. Seniors have a probability p_{die} of dying such that their continuation value $V(\mathbf{x})$ is defined as:

$$V(\mathbf{x}) = (1 - p_{die})V(\mathbf{x}') + p_{die}\Lambda\hat{u}((1 - \tau_a)a) \quad (18)$$

with Λ a parameter controlling their degree of altruism. We assume for each such death, a new worker with a financial wealth equal to the bequest minus the estate tax appears in the economy as a Junior from the 1st group.

3.6 Government

The government collects revenues from income taxes on labor and entrepreneurial earnings and pensions as well as from the product of estate taxation and revenues from taxes on the sale and the purchase of a business. Government expenditures comprises an exogenous government spending proportional to aggregate output, $G = \bar{G}Y$ and pensions. The government budget constraint is:

$$\int_{\mathbf{x}_w} \left(\mathcal{Y}(y, j)\tau_y + \mathbb{1}_{d=1}qf(k)\tau_b \right) d\Phi(\mathbf{x}_w) + \int_{\mathbf{x}_e} \left(f(k)\tau_y + \mathbb{1}_{z=2}qf(k)\tau_s \right) d\Phi(\mathbf{x}_e) + \int_{\mathbf{x}} \mathbb{1}_{j=J}p_{die}\tau_a a d\Phi(\mathbf{x}) = \bar{G}Y + \int_{\mathbf{x}_w} \mathcal{Y}(y, J) d\Phi(\mathbf{x}_w) \quad (19)$$

with $\Phi(\cdot)$ a measure over agents of the specified type.

3.7 Equilibrium

A Recursive Equilibrium in this economy consists of a set of agent's decision rules, a distribution $\Phi(\mathbf{x}, \Omega)$ of agents, factor prices $\{w(\Omega, \Phi), r(\Omega, \Phi)\}$, a price $q(\Omega, \Phi)$ for a unit of business production and government spending G such that:

- The decision rules $a'(\mathbf{x}, \Omega), k'(\mathbf{x}, \Omega), d(\mathbf{x}_w, \Omega), z(\mathbf{x}_e, \Omega)$ solve the agent's problem.
- The distribution of agents $\Phi(\mathbf{x}, \Omega)$ is induced by the transition matrix of the system $M(x', \Phi', \Omega' | x, \Phi, \Omega)$. A steady state is implied by a constant aggregate behavior $\Omega' \equiv \Omega$ such that $\Phi(\mathbf{x}, \Omega)$ is stationary.
- The government budget constraint in (19) is balanced with \bar{G} .

profit of a business of size k during the current period and the next ones. In a model with infinitely lived firms discounting the future at rate r and pricing the business according to the stream of future profits, with no exit probabilities and constant capital size, the price would be $q\pi(k)$, with $q = \frac{1-r}{r}$ reflecting the discount rate.

- The labor market clears and total labor demand by the corporate sector equals household labor supply. The wage is determined by the marginal productivity of labor in the corporate sector, such that $L_c = \int_{\mathbf{x}} \mathbb{1}_{\{o=ow\}} d\Phi(\mathbf{x}, \Omega)$.
- The capital market clears. Corporate capital and the total entrepreneurial capital equate total agent's net worth in the economy. The interest rate is determined by the marginal productivity of capital in the corporate sector.²⁶

$$K_c + \int_{\mathbf{x}_e} k(\mathbf{x}_e) d\Phi(\mathbf{x}_e, \Omega) = \int_{\mathbf{x}} a(\mathbf{x}) d\Phi(\mathbf{x}, \Omega) \quad (20)$$

- The *SMESM* clears. The price q clears the market such that: $\int_{\mathbf{x}} \mathbb{1}_{z(\mathbf{x})=2} h_s(q) f(\mathbf{x}) d\Phi(\mathbf{x}, \Omega) = \int_{\mathbf{x}} \mathbb{1}_{d(\mathbf{x})=1} h_b(q) f(\mathbf{x}) d\Phi(\mathbf{x}, \Omega)$ and $\{\theta_b(q(\Omega, \Phi)), \theta_s(q(\Omega, \Phi))\}$ adjust accordingly.

This problem has no analytical solution and has to be solved numerically. Two major problems arise in our setup. First, the dimensionality of the problem with two-assets is itself a challenge and one would refer to fast optimization method in order to solve the model. Second, due to the presence of both discrete (occupational choice) and continuous choices, FOCs are no longer sufficient (while still necessary). To encompass these issues, we modified the recent implementation of *DC-EGM* as introduced in [Iskhakov et al. \(2017b\)](#) and extended it to multiple assets as in [Druedahl and Jørgensen \(2017\)](#). While making the model tractable, this also substantially increases the speed of the whole algorithm.²⁷ We provide details on the pertubated solution we use to solve the model using the *DC-EGM* procedure in appendix ??.

4 Parameterization

Our parameterization has two general objectives. First we need to replicate important cross-sectional features observed in the data, including occupational choice decisions and life-cycle patterns. Second, we want to account for the demographic change observed in the data since the 1970s and to that end, we adopt and detail a non-stationary calibration.

4.1 Fixed parameters

Demography and preferences We assume that $J = 41$, with 40 stages to represent adult working periods and a last bracket to capture all ages beyond the retirement age. We use the CRRA utility function $u(c) = \frac{(c^{1-\sigma}-1)}{1-\sigma}$ with relative risk aversion $\sigma = 1.5$. In the benchmark case, we set the altruism parameter Λ to 0.²⁸

²⁶Profit maximization implies: $r = A\alpha \left(\frac{L_c}{K_c}\right)^{1-\alpha} - \delta$ and $w = A(1-\alpha) \left(\frac{K_c}{L_c}\right)^\alpha$, with w and r the wage and interest rates, which by a no arbitrage condition are identical in the entrepreneurial sector.

²⁷Our model equilibrium is solved on average in ten minutes for reasonably fine grids for both illiquid capital and liquid net worth using a reasonably fast desktop computer.

²⁸We do not specify the functional form of the warm-glow utility function for the benchmark calibration, as $\Lambda = 0$.

Labor income and labor frictions The labor income allows workers to accumulate savings at different rates. This is especially important for the decision to become an entrepreneur.²⁹ As it is standard in the literature (see, among others, ? and ?), we define labor earnings as a function of the wage level w , an age-dependent component $h(j)$ and a persistent stochastic process for labor productivity y such that:

$$\log(\mathcal{Y}_{i,t}(j, y)) = \log(w_t) + \log(y_{i,t}) + \log(h_{i,t}(j)) \quad (21)$$

$$\log(y_{i,t}) = \rho_y \log(y_{i,t-1}) + \epsilon_{i,t}^y; \quad \epsilon_{i,t}^y \sim \mathcal{N}(0, \sigma_y) \quad (22)$$

When $j = J$, $h(j)$ defines the retirement pension that we set to 40% of the average income. Once retired, an individual keeps the same component y forever and her offsprings' productivity is drawn from the invariant distribution $F(y)$. Otherwise, the components $h(j)$ for $j \in \{1, \dots, J-1\}$ is chosen in order to replicate the average lifetime earning profile within each earning percentile as observed in the PSID between age 25 and age 65, such that $h(j) = \alpha_0^j + \alpha_1^j(j+24) + \alpha_2^j(j+24)^2$. We estimate that $\alpha_0^j = 0.413$, $\alpha_1^j = 0.045$ and $\alpha_2^j = -0.001$.

The probability of dying, p_{die} , is set to 0.091 (corresponding to an expected retirement period of 11 years as in [Cagetti and De Nardi \(2009\)](#)). The logarithm of productivity y follows an AR(1) process with autocorrelation ρ_y and standard deviation σ_y . We discretize the process by setting $\rho_y = 0.96$ and adjusting the variance to $\sigma_y = 0.2$ to generate an earnings Gini of 0.36.

Business maturity We distinguish mature and immature (early-stage) businesses in the model. We set the probability to switch from early-stage to mature to 20% (about 5 years in operation). Only a mature business can be sold but we also assume that its owner pays a lower interest rate on financing, translating the higher amount of information that a creditor has access to (i.e. history of past transactions, lists of customers etc.). We therefore define the debtor interest rate as $r_b(m) = r + v_s + v_m \mathbb{1}_{m=0}$, where v_s is a wedge common to all businesses while v_m is the additional interest rate premium charged on early-stage businesses. We set $v_s = 2\%$, the usual value used in the literature. Concerning the early-stage premium, we use a regression over financing conditions and maturity detailed in [Appendix B](#). We find a premium of about 1.6% between recently purchased firms and founded ones and set $v_m = 1.6\%$ accordingly.³⁰

Finally, we also assume that mature and immature businesses display different probabilities $\chi(m)$ to fail. In the ASE (2016), [Table 4](#), the fraction of early-stage business owners exiting for reasons related only to business conditions account for 70% of total exits while it is 26%

²⁹Three saving motives arise in the model. A precautionary one due to the inherent productivity risk, a life-cycle one and an accumulation motive in order to become an entrepreneur.

³⁰In the baseline model, we do not take into account different credit limits. However, extending the model to account for this could generate additional misallocation effects concomitantly to the destruction of older mature businesses.

for established business owners with over 5 years of operation. We therefore let immature businesses be 3 times more likely to exit due to business failure and set $\chi(0) = 3 \times \chi(1)$. We set the value for $\chi(0)$ to 70% of the average entrepreneurial exit rate in the economy, which is 25% in the data, corresponding to $\chi(0) = 18\%$ and $\chi(1) = 6\%$. We then adjust ζ , the unconditional probability of entrepreneurial exit (independently of the business stage), in order to match a realized entrepreneurial exit rate of 25% in the model.

Technology Private businesses produce according to the technology $f(k) = k^\nu$ with $\nu < 1$. The value of ν is part of the joint calibration explained below. The corporate sector features a constant returns to scale Cobb-Douglas production function with capital share $\alpha = 0.33$, total factor productivity A , capital level K_c and labor L_c , such that: $Y = F(K_c, L_c) = AK_c^\alpha L_c^{1-\alpha}$. Capital depreciates at rate $\delta = 0.07$ in both sectors.

Adjustment costs and liquidation value When investing in an incumbent business or when founding a new business, owners of privately held businesses are assumed to pay a cost ϕ_u on each unit of positive investment, such that $\mathcal{C}(k, k', \phi) = \phi_u(k' - k)$. The parameter ϕ_u over the adjustment cost is calibrated endogenously such as to reproduce the aggregate fraction of private businesses investing a positive capital amount, which is 60% in the SBO.³¹

When liquidating the business, the entrepreneur is assumed to recover a fraction $(1 - \phi_d)$ of the value of the private business. We set the parameter ϕ_d to 30% (corresponding to a recovery rate of the business capital value of 70%). We perform sensitive analysis on the effect of these parameters in the Appendix.

Matching probabilities The parameters μ_b and μ_s are set to 0.1 of the price, meaning that brokers charge a commission of 10%, consistent with what is reported on *Bizbuysell.com*. The parameters $A_s = \kappa_s^{\frac{\gamma-1}{\gamma}}$, $A_b = \kappa_b^{\frac{\gamma-1}{\gamma}}$ are determined jointly to match the ratio of sellers to exiting entrepreneurs and the ratio of buyers to new entrants. γ is estimated by OLS from the data using the following specification:

$$\log(\text{Proba}_t^{\text{sell}}) = \alpha_s + \frac{\gamma}{1-\gamma} \log(q_t) + \epsilon_t \quad (23)$$

where $\log(\text{Proba}_t^{\text{sell}})$ is computed as the probability to sell the business during a quarter (i.e. as proxied by 91 days over the time needed to sell the business) and q_t is the price index.

In order to estimate the price index at date t , we first regress the ratio of the sale price over the EBITDA of the business on the business characteristics (age, number of employees,

³¹This value is also consistent with [Cooper and Haltiwanger \(2006\)](#) who takes into account firms in the manufacturing sector. They find that around 80% of that sector are investing each year.

inventories, business category, reasons to sell, etc.). Such that:

$$\frac{Price_{i,t}}{EBITDA_{i,t}} = \tilde{q}_t + \beta X_{i,t} + u_{i,t} \quad (24)$$

where \tilde{q}_t is the time fixed effect capturing the average price index at date t . We normalized the price level as the difference between the price index at date t , q_t , with the price index at date $t = 0$ (corresponding to 2014:Q1), such that $q_t = \tilde{q}_t - \tilde{q}_0$. The resulting estimate for the elasticity of the probability to sell the business and the price level, γ , is 0.20 for our sample.

Other parameters The estate taxation is set to 30%, consistent with the statutory tax rate in the US and the value used in [Cagetti and De Nardi \(2009\)](#). Taxes on the sale of a private business is set to 15%, which sits in the lowest area of the statutory tax rate on capital gains when selling a business that has a typical range between 15% and 20%. Taxes associated to the purchase of an existing business are set to 5% in the benchmark economy. Finally, we calibrate the transition probability of the entrepreneurial ability process ι : we endogenously determine $p_\iota = P(\iota' = 1 | \iota = 0)$ to match the share of self-employed in the economy while we fix $P(\iota' = 0 | \iota = 1) = \chi(0)$.

4.2 Non-Stationary Calibration and Joint Parameterization

Recent evidence show the large demographic effects on the decision to become an entrepreneur (see, among others, [Liang et al. \(2014\)](#) and [Pugsley et al. \(2016\)](#)). As most of our data counterpart are observations after 2000 and are taken at different periods (mostly in 2007 and 2016), those moments are largely influenced by the demographic structure of the economy. [Table 3](#) shows that the proportion of entrepreneurs close to the retirement age has largely increased between 1989 and 2016. The changing demographic structure of the US economy has important consequences on the stock of business assets in the economy and is related to the *aging of entrepreneurs*. In [Table 4](#), we show that it also quite substantially affects entrepreneurial exit. Between 2007 and 2016, the fraction of entrepreneurs who declare having exited due to retirement or/and because they sold their businesses drastically increased. In the next section, we detail the non-negligible impact of the demographic changes on price dynamics, and in turn on individual incentives. Finally, our calibration strategy, by adding a dynamic dimension to the model, lets us compare both the cross-sectional and dynamic effects.

Table 3. Share of business assets and entrepreneurs' age

Entrepreneur with	% of entrepreneur			% business assets		
	1989	2004	2016	1989	2004	2016
age 55 and over	26.1	33.7	47.1	48.6	45.1	63.9
age 60 and over	17.3	22.2	31.7	36.5	30.0	45.4

Source: owner's computation using SCF, waves 1989, 2004 and 2016.

Table 4. Entrepreneurs' reasons for ceasing their activity ^a

	SBO (2007)	ASE (2016)		
	All	All	< 2 years	> 5 years
% of business owners ceasing per year (%)	5.5	6.4	8.6	5.5
business conditions (%)	44.0	22.7	35.1	16.5
owner(s) retired (%)	9.9	18.9	3.5	27.3
owner(s) sold the business (%)	11	17.9	11.0	21.0
other reasons ^b (%)	35.1	34.2	50.4	35.2

^a As the question related to the ceasing reasons allow for multiple choices. We normalize the number in terms of the total number of answers.

^b Contains reasons linked to illness, injury or died, started another business and another reasons.

The specific exercise we perform is the following. We choose a set of parameters corresponding to the observed stationary distributions of occupations, age structure and wealth as the initial condition. We then shock the economy by increasing the number of new young individuals on the labor market following the baby-boom. This demographic shock propagates along the transitional years. In 2007, we compute the distance between the model generated moments and the data.³² This experiment is repeated until finding the set of parameters that minimizes this distance. Formally, for a vector $\{\mathbf{X}_t\}_{t_0}^T$ of structural variables changing over time from t_0 to T and Ξ a set of structural parameters of the economy, the calibration strategy seeks to minimize the distance between the M generated moments indexed k , denoted $\tilde{m}_{k,t_r}(\Xi, \{\mathbf{X}_t\}_{t_0}^T)$, and the observed moments m_{k,t_r} :

$$\hat{\Xi} = \underset{\Xi}{\operatorname{argmin}} \sum_{k=1}^M |m_{k,t_r} - \tilde{m}_{k,t_r}(\Xi, \{\mathbf{X}_t\}_{t_0}^T)|$$

The reference period is thus $t_r = 2007$. The set of structural parameters include the demographic change $\mathbf{X}_t = \varkappa_{d,t}$ with $\varkappa_{d,t}$ the growth rate of the population of 25 years of age in period t . We choose $\varkappa_{d,t}, \forall t$ between $t = 1970$ and $t = 1980$ such as to reproduce a growth rate of the

³²We choose this specific year because all fundamental observables are available. Upon data availability, we could have chosen an alternative year or tracked a path of moments over several years while letting the model be over-identified.

population in the US relative to the average growth rate as shown in figure 4.³³ We assume that the economy recovers its stationary property (with a higher mass of individuals) in $T = 2170$.

Our calibration leaves us with 9 parameters, $\Xi = \{\beta, \nu, \kappa_s, \kappa_b, \zeta, u_R, \tau_u, \theta, p_i\}$, that are picked to match data moments. From the CPS, we target a share of self-employed individuals relative to the working age population between 8% and 10%, a self-employment exit rate (per year) of 25% and a share of worker becoming self-employed of about 2.5%. From the SCF (averaged over the 2001, 2004 and 2007 waves), we target a fraction of entrepreneurs relative to the population in the last age brackets (corresponding to retirement) of 2.0% and a share of early-stage entrepreneurs buying their business of 25% (excluding the inheritance and gift options). From the SBO, we target a fraction of private businesses expanding of about 60% and a share of business sold relative to exiting entrepreneurs of 10%. The target for the share of production coming from the SMEs is of about 48%.³⁴ Finally we target a capital-to-output ratio of 3.0. Our model is exactly identified, with 9 parameters used to pin down 9 moments. The resulting parameter values are reported in table 5.

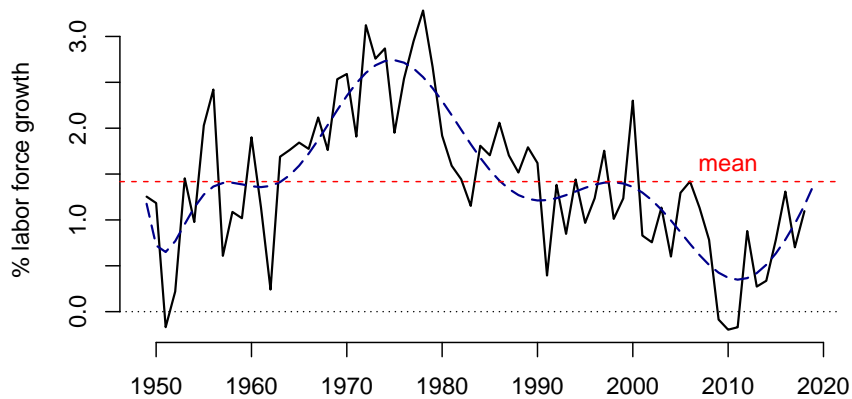


Figure 4. Civilian growth rate (16 years and more). Note: the dashed blue line is the polynomial approximation.

5 Results

In this section we establish that our model behavior matches our empirical findings concerning business transfers and assess the effect of various key parameters. We also quantify the aggregate and cross-sectional importance on the US economy of the aging economy.

³³We assume that the economy displays a normal population growth rate of 1.5%. We therefore let the population grows between 1% to 2% between 1970 and 1980.

³⁴This share is taken from the Small Business Administration report *Small Business GDP: Update 2002-2010*.

Table 5. Model Parameterization

DESCRIPTION	SYMBOL	VALUE	TARGET	MODEL	SOURCE/MAIN MOMENT*
A. External Parameters					
Risk aversion	σ	1.5			Standard value
Degree of altruism	Λ	0			Benchmark normalization
Depreciation of capital	δ	0.07			Standard value
Returns to scale in corp.	α	0.33			Standard value
Proba of aging/die	$P(j' j)$	<i>see text</i>			5y age bracket, 11y retirement
Persistency/SD of ability y	ρ_y, σ_y	0.96, 0.2			Cagetti and De Nardi (2006)
Life-cycle earnings $h(j)$	$\alpha_1^j, \alpha_2^j, \alpha_3^j$	<i>see text</i>			Earnings Gini 0.38 (PSID)
Replacement rate	ρ	40%			Shimer (2005)
Wedge mature/immature	v_s, v_m	2%, 1.6%			SSBF (2003)
Prob. switch mature	P_m	0.2			5 years in operation
Probability to fail	$\chi(m)$	{18%, 6%}			ASE (2016), ceasing reasons
Matching technology	γ	0.2			BBS data
Broker's commission	$\mu_s = \mu_b$	10%			<i>Bizbuysell</i>
Discount of business liqui.	ϕ_d	30%			60-80% of business value
Tax on selling/purch. a bus.	τ_s, τ_b	15%, 5%			IRS capital gains
Estate taxation	τ_a	30%			IRS statutory tax rates
B. Jointly Determined Parameters					
Discount factor	β	0.911	3.0	2.8	K/Y
Returns to scale priv. bus.	ν	0.813	48	48	% production priv. bus. (<i>see text</i>)
Buyer/seller's side broker cost	κ_b	0.1	25	31	% purchasing existing bus. (SCF)
Seller's side broker cost	κ_s	0.9	10	11	% selling their bus. (SBO)
Prob. to fail for exo reasons	ζ	0.17	25	25	% exiting self-employed (CPS)
Disutility of working (retired)	u_R	0.6	2.0	1.9	% retired entrepreneurs (SCF)
Adjustment cost	ϕ_u	0.1	60	55	% of SME expanding (SBO)
Borrowing constraint	θ	0.4	8-10	8.9	% self-employed (CPS)
Probability entrep. ability	p_i	0.047	2.5	2.3	% of worker \rightarrow entrep. (CPS)

* The main moments are indicative. Changing one endogenous parameter affects the whole equilibrium.

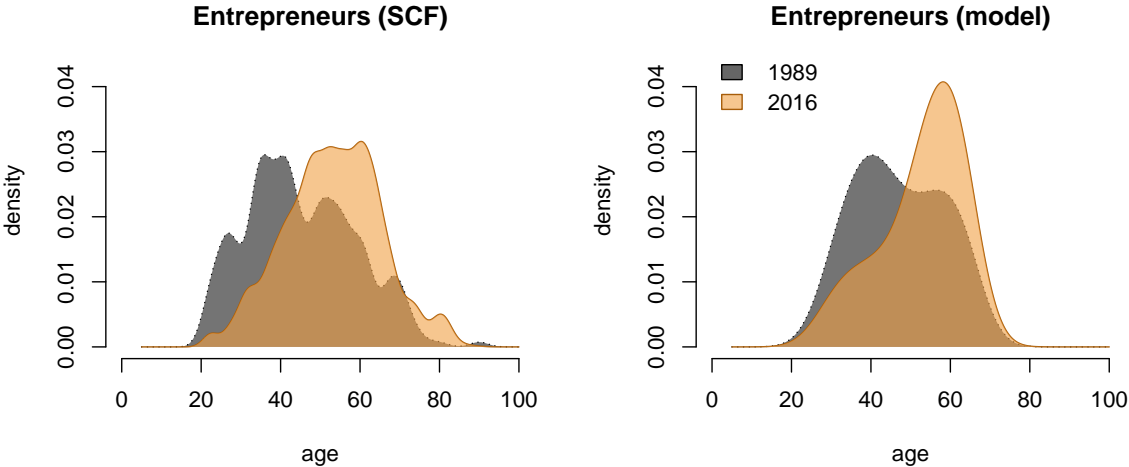
5.1 Cross Sectional Distribution and Aggregate Statistics

We start by briefly describing the cross-sectional outcomes of the model. While not the primary focus of the paper, the resulting wealth inequality implies a wealth Gini of 0.75 in 2007, close to the estimates in the US data of 0.8. Additionally, the share of wealth held by the top 10% is 26% and the median wealth ratio of self-employed individuals to that of workers is about 5 in the model, against respectively 30-40% and 6 in the SCF.

Concerning the distribution of occupations over the life-cycle at different periods of time, we display on [Figure 5](#) the distribution of entrepreneurs in 1989 and in 2016 in the model and in the SCF. Following the demographic change, the cross-sectional distribution of entrepreneurs at these two different periods is very similar to the patterns found in the data. The mass of

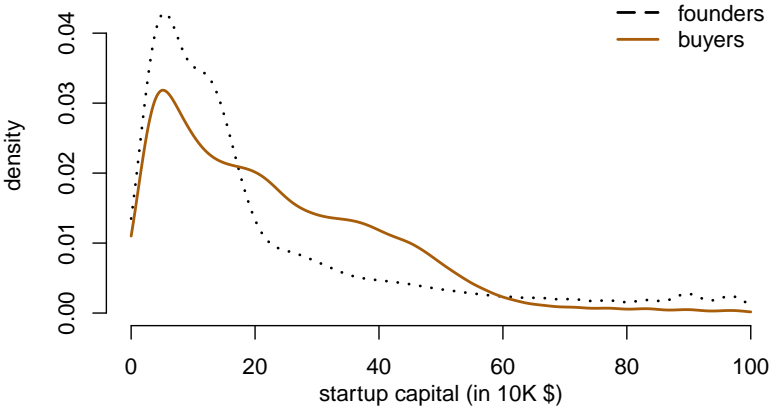
entrepreneurs peak around age 35-45 in 1989, while it peaks around 60 in 2016, which, as we will show later, impose supply pressures on the SME for sale market.

Figure 5. Distribution of entrepreneurs in 1989 and 2016, model *versus* data.



On Figure 6, we display the distribution of buyers and founders by startup capital. As in the data, the distribution of buyers is shifted to the right. The buyers of existing businesses tend to buy larger businesses as compared to founders of new businesses. Comparing the median startup capital in the two cases, we find that buyers possess 46% more capital at the start as compared to founders. Buyers tend to be older and richer, consistently with the data.

Figure 6. Distribution of buyers and founders in the model.



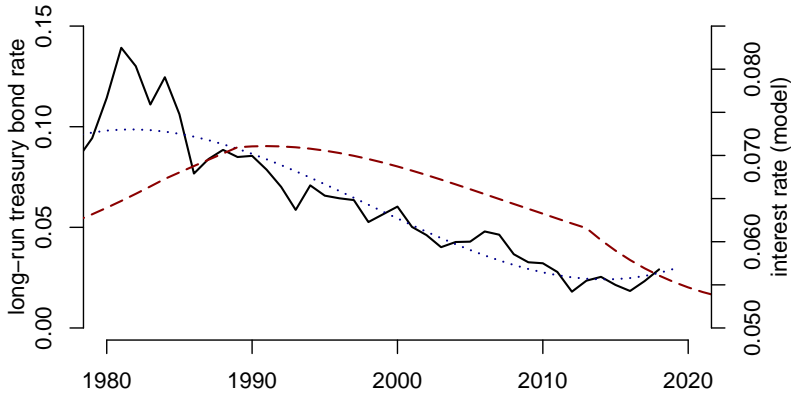
5.2 The Dynamic of Entrepreneurship and the SME for Sale Market

We now describe the results of our baseline calibration when we take into account the changing demographic structure since 1970 and evaluate the model properties in 2007 along the dynamics. We show that there are substantial effects due to the entry of the baby-boom generation in the labor market, highlighting the importance of taking into account the *over the time dynamics* when evaluating cross-sectional effects, in particular for questions related to entrepreneurship,

a sector that has been shown to be on the decline since the 80s (see, for instance [Pugsley et al. \(2016\)](#)).

[Figure 7](#) displays the path of the equilibrium interest rate and emphasizes the importance of the baby-boom effect on equilibrium prices. Concomitant with the entry of the baby-boomers, the equilibrium interest rate increases. A similar picture is displayed in the data, when taking the long-run treasury bond at all maturities. After 1985-1990, the interest rate falls, a feature which is well reproduced in the model. The explanation is straightforward and is due to a composition effects: since individuals in the baby-boomers cohort are becoming older, they tend to save more and hold more wealth, lowering the equilibrium interest rate.

Figure 7. Dynamic of the interest rate in the model (dashed red line) and in the data (black line).

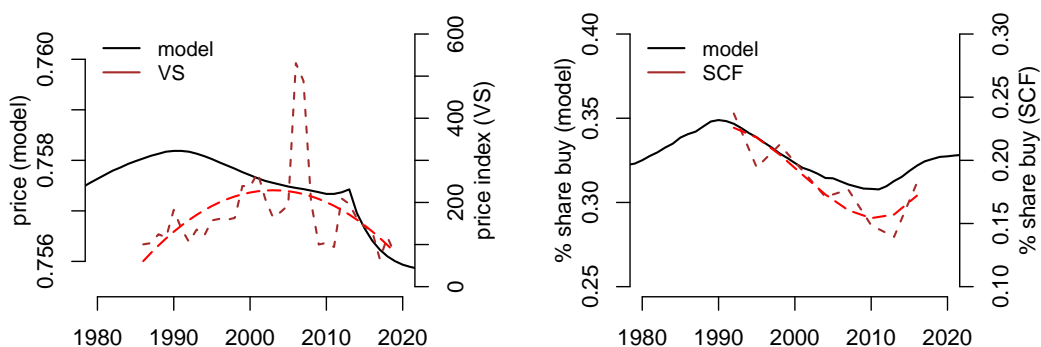


Note: The data report the long-run treasury bond at constant maturity. The dashed blue line is the polynomial approximation of the data trend.

[Figure 8](#) (left) displays the prices on the SME for sale market. Again the model is able to reproduce the global hump shape of the price evolution, with the rise of business price levels accompanying the arrival of the baby boomers and the declining trend since the 2000’s. On the SME for sale market, people between 30-40 years of age are the most likely to buy businesses. As a consequence, the share of people who tend to buy increase until 1990, as shown on [Figure 8](#) (right). During the 2000’s, the price level fall, following the decrease in the number of new buyers, and the rise of entrepreneurs who want to sell their businesses. Indeed, from around 2010, individuals in the baby-boom cohort start to retire and old entrepreneurs start to sell their company. This is particularly important since, as shown on [Figure 9](#), old entrepreneurs (those of age 65 and over) are the most likely to sell their businesses (in the model and in the PSID), essentially due to a retirement motive. As a result, the price level falls and the share of buyer starts increasing again as buying an existing business becomes relatively cheaper as compared to founding. Those patterns are surprisingly consistent to what can be seen in the SCF and the ValueSource (VS) data and confirm the validity of the model dynamics along the transitional

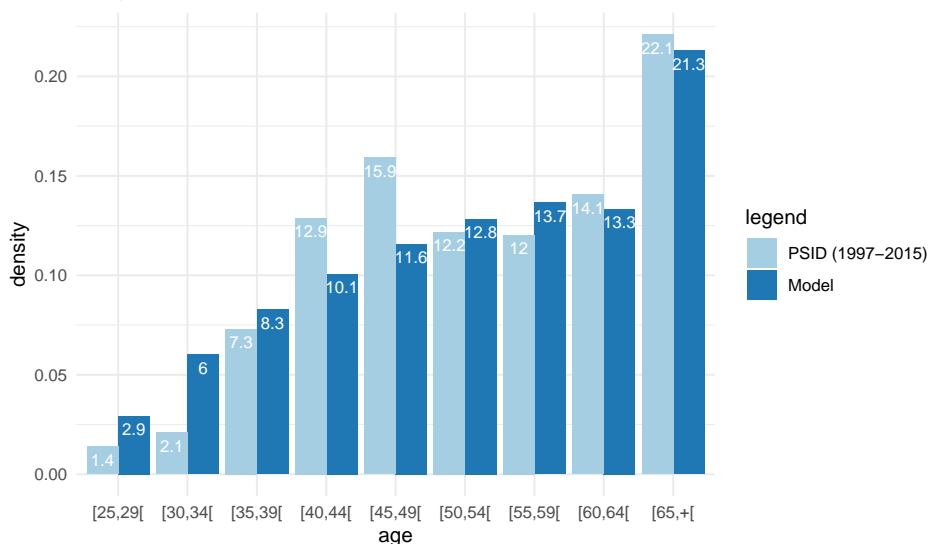
path.³⁵

Figure 8. Dynamic of the Business Sale Market in the data and the model.



Note: VS is for ValuSource.

Figure 9. Distribution of sellers in the model and the data (PSID).



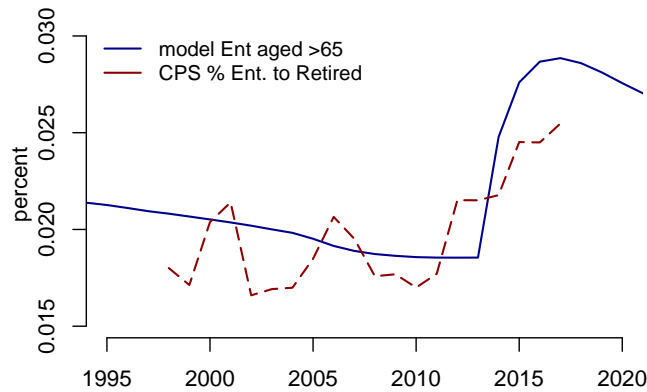
To get a sense of the magnitude of the retirement of old entrepreneurs starting in the 2010s, we display on [Figure 10](#) the share of entrepreneurs of age 65 and over in the model compared to the fraction of entrepreneurs switching to retirement in the CPS. Between 2010 and 2017, the probability to switch from entrepreneurship to retirement in the CPS has increased by 60%. Consistently, in the model, starting in the 2010s, there is an important flow of entrepreneurs of age 65 and more who are retiring.³⁶

Finally, the model is also consistent with the recent debate on the declining start-up rate

³⁵ValuSource is a dataset containing business transaction observations from 1990 onwards that complement our BBS dataset. In the data the increase of the buying share after 2010 could also be reinforced by the era of exceptionally accommodating interest rates in the wake of the financial crisis.

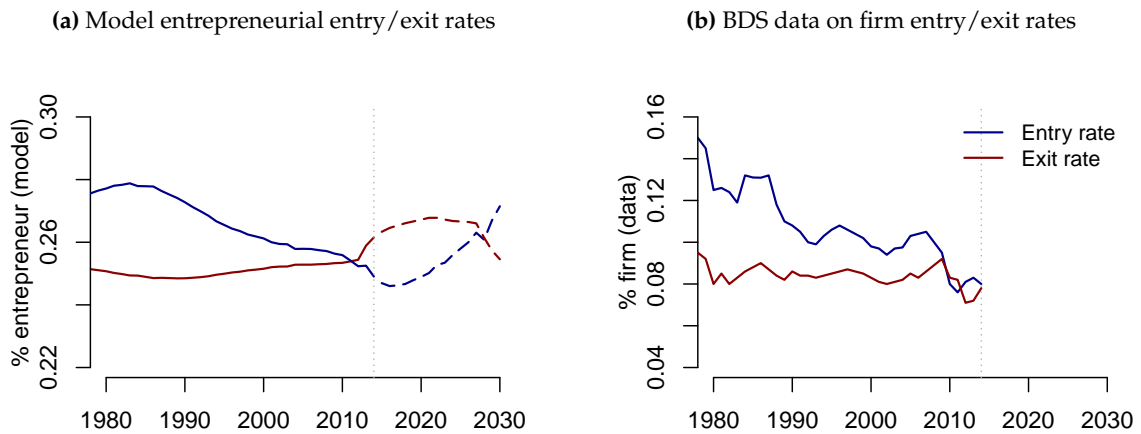
³⁶Recall that in the model, entrepreneurs still in activity after the retirement age are imposed a fixed utility cost u_R encouraging the poorest of them (those with the lowest business returns) into retirement.

Figure 10. Dynamic of retirement in the model and the data (CPS).



(and to a broader extent, with the debate on the US declining business dynamism) highlighted by [Decker et al. \(2016\)](#) or [Pugsley et al. \(2016\)](#) among many others. In particular, the firm exit rate was shown to be fairly constant over time, while the firm entry rate declines starting in the 1980s as displayed on ??, panel (b). Consistently, as shown in panel (a), the model displays similar properties. The average entrepreneurial exit rate, defined as the total number of exits over the total number of entrepreneurs in the economy, is almost constant over time. On the contrary, the entry rate, defined as the number of new entrepreneurs over the total number of entrepreneurs does respond to the demographic shock, as more individuals enter entrepreneurship between 1980-1990, and then decreases afterwards.

Figure 11. Entry and exit rate in the model and the data.



We therefore see the model as a good representation of the evolution of the economy. In the cross-section, the model realistically reproduce both the wealth distribution and the distribution of buyers and sellers in terms of startup capital while correctly matching their life-cycle properties. Importantly, the model is also consistent with the dynamics of the economy be-

tween 1970 and 2015.

6 Quantitative Results: the Importance of the SME for Sale Market

In this section, we evaluate the importance of the SME for sale market on the US economy. To this end, we conduct a series of experiments. First, we compute a counterfactual experiment in which we reduce sellers and buyers matching in order to reduce the number of business transfers. We then highlight the role of key parameters in the model that drive our results. Finally, we investigate the role of taxation: we show that lowering selling and buying taxes have a significantly favorable effect on business transfers. These experiments are conducted in alternative economies in which we use the exact same baseline structural parameters. We then compute their dynamics from 1970 to 2170 to take into account the demographic change.

6.1 Restricting access to the SMEs for sale market

To assess the importance of the SME for sale market, we run the model under the alternative specification where the matching technology, characterized by $A_s = \kappa_s^{\frac{\gamma-1}{\gamma}}$ and $A_b = \kappa_b^{\frac{\gamma-1}{\gamma}}$, are altered to significantly reduce or cut the probability to find an existing business on the buyer and the seller's side. [Table 6](#) displays our results.

Table 6. Model results under alternative SME for sale markets

Model	Nb	Matching par.		Moments ^a				
		A_s	A_b	Y_{SME}	K	r	% Ent.	% $m = 1$
Benchmark	(1)	0.9	0.1	1.52	12.0	5.3	8.10	55.3
High proba.	(2)	0.9	0.2	2.07	12.3	5.2	9.55	72.2
Low proba.	(3)	0.225	0.025	1.34	9.97	6.0	7.88	32.7
No SME	(4)	0.0	0.0	0.81	7.14	7.6	7.42	16.9

^a The values reported correspond to the date $t = 2007$ for the benchmark and the alternative economies with lowered probability to find a seller / buyer.

This experiment, that is similar to either a partial (experiment (3)) or a full (experiment (4)) shutdown of the SMEs for sale market, leads to a significant reduction of the production in the SME sector, Y_{SME} . Aggregate capital is also significantly dampened and the interest rate increases in this economy with respect to the benchmark economy. This comes from the fact that the transfer of reputable businesses among households happens less often or is prevented. The ratio of mature businesses in the economy reduces significantly as owners of those ventures are forced to liquidate. Mature businesses are less likely to fail, and have access to lower interest rates, allowing their owners to accumulate capital and expand their activity. As a consequence, the more developed the SME for sale market is, the higher is the share of mature businesses

in the economy and the higher is the GDP from the SME sector. Models with only capital adjustment costs as the only means of exit are unable to capture this aspect of SMEs dynamics. Moreover, in terms of occupational choice, there are large consequences. When the purchase of an existing business is easier, a larger fraction of individuals turns to entrepreneurship and a lower fraction of incumbents fail, increasing the fraction of entrepreneurs.

6.2 Decomposition: The Role of Business Maturity

Business maturity plays a key role in this model by generating a persistent wedge between immature businesses and mature businesses. In particular, mature businesses pay lower interest rate (i.e. $r_b(1) < r_b(0)$) and display a lower exogenous exit probability due to business conditions (i.e. $\chi(1) < \chi(0)$). Moreover, the adjustment cost has a particular role in the model, by making the buying of existing businesses, which implies no adjustment cost, more attractive as compared to founding a new business. We assess in this section the relative importance of each of these components in generating a wedge between buying an existing business *versus* founding a new business. In a first decomposition, we assume that $r_b(0) = r_b(1) = r + v_m$; that is, immature businesses are charged the exact same interest rate on their debt. In the second experiment, we assume $\chi(1) = 10\%$. Therefore, mature businesses are now more likely to fail, and the probability to fail for immature business is now 1.8 higher than the one of mature businesses (against 3 in the baseline economy). Finally, we assess the model when lowering the adjustment cost to $\phi_u = 1\%$. [Table 7](#) summarizes the results.³⁷

Table. 7. Decomposition: the role of the interest rate wedge and exit rate

Model	Nb	Parameter		Moments ^a				
		Symbol	Value	Y_{SME}	K	r	% Ent.	% $m = 1$
Benchmark	(1)	0.9	0.1	1.52	12.0	5.3	8.10	55.3
No wedge	(5)	v_s	0.0	1.59	10.2	5.9	8.26	54.5
High mature exit	(6)	$\chi(1)$	0.1	1.17	8.70	6.6	7.77	47.1
Low adj. cost	(7)	ϕ_u	0.01	4.38	20.7	3.6	12.41	44.6

^a The values reported correspond to the date $t = 2007$ for the benchmark and the alternative economies with lowered probability to find a seller / buyer.

The three margins considered have important implications for the tradeoff between founding a new business versus buying an existing business. In particular, the results show that in the no wedge case (row (5)), the model displays a lower fraction of mature businesses, while generating a higher production in the SME sector. Because the interest rate wedge on im-

³⁷It is worth noting the an existing business is not more likely to be more productive. Therefore, we view our results as a lower bound since early-stage businesses could be significantly less productive than older one, especially because they are in a process of acquiring market shares.

mature businesses is removed, it becomes more attractive to found a business relative to the benchmark, lowering the relative number of mature businesses. Moreover, because the wedge is removed, operating costs are lowered, making it easier to expand a small business. As a consequence, Y_{SME} increases.

When the probability to exit is higher for mature businesses, we also find significant effects. First, the number of mature businesses fall for two reasons: (i) the exit rate is higher for mature businesses, (ii) a lower number of entrants decide to purchase an existing business. This lower number of mature business, the higher failure rate and the lower share of entrepreneurs reduce the production of the SME sector.

Finally, the adjustment cost play a crucial role by directly acting on the options of founding a new business versus purchasing an existing one. When lowering substantially the adjustment cost, the share of mature business drastically fall due to the increased number of new immature startups. Production in the SME sector increase significantly when lowering adjustment costs.

6.3 The Role of Capital Taxes on Business Transfers

In this section, we investigate the role of capital taxation when selling and buying an existing business. In the US, significant taxes are levied when an entrepreneur decides to sell part of or the entirety of a business. Typically, the IRS requires the seller to report capital gains following the sale of each bundle of assets (intangible and tangible) separately. Then the seller pays taxes on this bundle at specific or identical rates. On the buyer side, local and state sales taxes must be paid whenever tangible assets are purchased. We test the implications of lowering selling and liquidating capital income taxes in [Table 8](#).

Table 8. Lower capital income taxes

Model	Nb	Parameter		Moments ^a				
		Symbol	Value	Y_{SME}	K	r	% Ent.	% $m = 1$
Benchmark	(1)	0.9	0.1	1.52	12.0	5.3	8.10	55.3
Low tax	(8)	$\tau_s = \tau_l$	0.05	2.04	12.5	5.2	8.23	55.1

^a The values reported correspond to the date $t = 2007$ for the benchmark and the alternative economies with lowered probability to find a seller / buyer.

Lowering capital income taxes significantly improves the production in the SME sector. This is due to two reasons: (i) liquidating a business is less costly now and as a result entrepreneurs are more likely to accumulate, (ii) individuals are more likely to start/enter the SME market, since now selling and liquidating a business is less costly. The relative value between founding an existing business or starting a new business is however relatively unchanged as seen with the fraction of mature businesses, which stays almost constant after the tax reform.

7 Conclusion

In this paper, we develop an incomplete markets heterogeneous agents general equilibrium model with stylized life-cycle dynamics and occupational choices. Importantly, our model lets incumbent entrepreneurs sell their businesses and prospective workers buy or found businesses, subject to an endogenous business price and adjustments costs. This framework is able to take into account the underlying mechanisms of entrepreneurial entry and exit. We build a new dataset using data from a major online business selling platform and provide evidences that transferring a business is subject to significant frictions that our theoretical setup is able to convey. We show that illiquid business assets, frictions on the business for sale market and the life-cycle components of entrepreneurship are key in reproducing our empirical findings. We show that our model is tractable and can be used to simulate the demographic changes appearing on the labor and entrepreneurial markets due to the aging of the entrepreneurial population.

Appendix

A Data complement

A.1 Summary statistics

Table. 9. Summary statistics: BBS data

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)	Max
number_employees	23,597	11.219	235.073	2,000	5,000	9,000	35,000,000
price	52,922	645,640.100	2,376,567.000	119,500	250,000	570,000	200,000,000
cashflow	33,736	199,519.500	420,001.900	70,840	124,279.5	222,833.8	50,000,000
gross_revenue	40,210	1,346,217.000	22,530,589.000	261,741.8	532,813	1,100,000	2,591,205,000
ebitda	5,416	342,890.100	5,933,854.000	62,000	118,294	255,112	435,000,000
ffe	18,520	155,886.200	473,154.200	25,000	65,000	150,000	22,000,000
inventory	10,719	82,916.840	440,765.500	3,000	10,000	40,000	23,798,180

A.2 Further evidence on mismatch: IBBA and European comparison.

A substantial number of transactions are terminated without a successful deal according to the International Business Broker Association (IBBA), a private source aggregating transaction information from business brokers. In their estimates, between 18% to 30% of businesses for

sale were successfully transferred to a new owner. The remaining was terminated.³⁸

In Europe, 690000 of the 1.7 millions EU-27 SMEs closing every year attempt transfer, of which 60% fail to transfer. Among those business transfer failures, 25% failed for reasons unrelated to the performance but due to inefficiencies in business transfers, representing 9% of total business closures (European Commission (2011)).

B Financing conditions by acquisition type and maturity

We use the 2003 SSBF data in order to estimate the effect of the type of acquisition (purchased versus founded) on the interest rate charged, with the idea that purchased businesses are, on average, well established since many years. We then run the following regression on the sample of early stage firms (acquired in the last 5 years) and the full sample of businesses:

$$int.rate_i = \alpha_i + \beta_i D(purchased)_i + X_i + u_i \quad (25)$$

where $D(purchased)_i$ is a dummy variable indicating whether a business as been purchased by the current owner or founded. X_i is a vector of controls for both the business (employment and capital size, sector etc.) and the owners (net worth, age, education, sex, entrepreneurial experience, past default history etc.) and u_i is an error term. Table 10 summarizes the results.

Table. 10. Interest rate, credit limit and type of acquisition.

	Interest rate (%)	
	All firms (1)	Young firms (<5y) (2)
Purchased business	-0.58*** (0.13)	-1.59*** (0.28)
<i>N</i>	8,919	1,811
<i>R</i> ²	0.12	0.31

Notes: p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.

We find a significative relationship between the type of acquisition and the interest rate. Recent owners that purchased an existing business are charged, on average, an interest rate 1.59% smaller than founders. Taking the sample of all firms, we consistently find that purchasers are charged an interest rate smaller than founders, but the effect is lower than for young firms. We interpret these findings in the following way: financial intermediaries lower the premium charged on their loans when they are able to infer business characteristics over a longer period.

³⁸This survey is available here: <https://www.ibba.org/resource-center/industry-research/> and corresponds to the 2018Q3 report.

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