Monetary Independence and Rollover Crises *

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Abstract

This paper shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. We study a sovereign default model with self-fulfilling rollover crises, foreign currency debt, and nominal rigidities. When the government lacks monetary independence, lenders anticipate that the government would face a severe recession in the event of a liquidity crisis, and are therefore more prone to run on government bonds. In a quantitative application, we find that the lack of monetary autonomy played a central role in making Spain vulnerable to a rollover crisis during 2011-2012. Finally, we argue that a lender of last resort can go a long way towards reducing the costs of giving up monetary independence.

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JEL Codes: E4, E5, F34, G15

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1 Introduction

A prominent concern during the Eurozone crisis was the risk of a rollover crisis. Policymakers feared that an adverse shift in market expectations would restrict governments' ability to roll over their debt, creating liquidity problems that would feed back into investors' expectations and ultimately lead governments to default. At the same time, the premise was that the lack of monetary independence was aggravating sovereign debt problems in Southern Europe. In this context, the European Central Bank took unprecedented policy measures aimed at stabilizing financial markets and reducing the risks of a potential collapse of the monetary union.[1]

The goal of this paper is to investigate whether and how the lack of monetary independence affects the vulnerability to a rollover crisis. A central question we tackle is: Does a country become more vulnerable after joining a monetary union? We present a novel theory in which the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. The key insight is that lenders' pessimism can trigger a demand-driven recession, making the option to default more attractive and, in turn, validating the initial pessimism. A government that has monetary independence can alleviate the recession that results from a rollover crisis, making investors less prone to run in the first place. Quantitative simulations show that while an economy that possesses monetary independence is almost immune to a rollover crisis, it can become significantly vulnerable once it joins a monetary union. Moreover, we show that a lender of last resort can significantly mitigate the welfare costs from joining a monetary union.

The environment we consider is a version of the canonical model of sovereign default à la Eaton and Gersovitz (1981) that incorporates the possibility of rollover crises, as in Cole and Kehoe (2000). The government issues debt before deciding on whether to repay or default. When lenders expect the government to default, the government is shut-off from credit markets and is forced to repay the maturing debt exclusively out of its tax revenues. When the maturing debt is large enough, repayment becomes too costly for the government and lenders' pessimistic expectations are validated: a self-fulfilling rollover crisis arises. We depart from the standard endowment economy setup by considering nominal rigidities, which creates a scope for a stabilization role for monetary policy. External debt is denominated in real terms, or equivalently in foreign currency, eliminating the possibility of inflating away the debt. The model features tradable and non-tradable goods and downward nominal wage rigidity, as in Schmitt-Grohé and Uribe (2016). In this environment, a shock leading to a contraction in aggregate demand reduces the price of non-tradables in equilibrium, generating a decline in labor demand. When wages cannot fall sufficiently quickly to clear the labor market, involuntary un-

[1] On September 6, 2012, Mario Draghi, the president of the European Central Bank, expressed that "the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a 'bad equilibrium,' namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." Preceding these remarks, Draghi famously pledged to do "whatever it takes to preserve the euro."

employment arises and the economy goes through a recession. Following the classic principles from Friedman (1953), a government with an independent monetary policy can use the nominal exchange rate as a shock absorber, altering real wages, and reducing unemployment.

Our main theoretical result is that the lack of an independent monetary policy increases the vulnerability to a rollover crisis. To understand the mechanisms in the model, consider what happens when a government is trying to roll-over its debt and investors suddenly panic and refuse to lend to the government. As the government is shut-off from credit markets, it needs to raise tax revenues and cut down on expenditures in order to service the maturing debt. In the presence of nominal rigidities and constraints on monetary policy, this situation has macroeconomic implications. The fiscal contraction generates a decline in aggregate demand, which leads to involuntary unemployment and makes repayment less attractive for the government. If the increase in unemployment is sufficiently large, the government finds it optimal to default, which in turn validates the initial panic by investors and generates a self-fulfilling rollover crisis. Interestingly, for this pessimistic equilibrium to emerge, unemployment does not have to be realized in equilibrium. In fact, it is the off-equilibrium outcome of a large recession that pushes the government to default and triggers the rollover crisis.

On the other hand, if the government can use monetary policy, the fiscal contraction that results from being shut-off from credit markets does not have macroeconomic implications. In this scenario, the government's willingness to repay is relatively less affected by the lenders' pessimistic expectations. As a result, a panic is less likely to be triggered in the first place under monetary independence. We also show that our main theoretical insight applies along several extensions of the baseline model. Among other extensions, we show that the same results apply when the source of nominal rigidity is on prices rather than wages, when costs are associated with depreciating the exchange rate, when the government follows a fixed exchange rate regime, and finally, when debt is denominated in domestic currency.

We then proceed to conduct a quantitative investigation. We start by considering a calibration of the model under monetary independence, in particular a flexible exchange rate regime under which the government chooses the exchange rate optimally at each point in time. In this regime, the government finds it optimal to implement the full employment allocations by depreciating the currency, in line with the traditional argument for flexible exchange rates. (Notice, however, that the government cannot alter the value of the debt since it is denominated in foreign currency.) In a calibrated version of the model, we find that rollover crises play a modest role under a flexible exchange rate regime. In fact, less than 1% of default episodes in the simulations are driven by rollover crises. Almost all defaults occur because of fundamental factors.

We then examine the quantitative effects of giving up monetary independence. One can think of a small open economy that has a fixed exchange rate regime, or equivalently, a single small economy within a monetary union in which wages (and debt) are denominated in the currency of the union, and the conduct of monetary policy is exogenous to the single small economy. Keeping the same parameter values for the calibration of the flexible exchange rate regime, we find that the economy faces a significantly larger fraction of defaults due to rollover crises, which can reach about 20% compared with less than 1% in the flexible exchange rate regime. The large difference also remains if we recalibrate the fixed exchange rate regime to match the same targets for debt and spreads as for the flexible exchange rate regime. Our quantitative findings therefore suggest that joining a monetary union entails significant costs in terms of a higher exposure to rollover crises.

Using the calibrated model for the fixed exchange rate regime, we then simulate the path of the Spanish economy starting at the time of the adoption of the euro. We find that the economy hits the "crisis zone" around 2012, in line with the turmoil in sovereign debt markets that occurred at the time. As a counterfactual, we then consider what the outcome would have been if Spain had exited the Eurozone in 2012. According to our model, the government would have remained immune to a rollover crisis, thanks to the ability to use monetary policy for macroeconomic stabilization. The goal of this exercise, however, is not to argue that being part of a monetary union is undesirable, but only to point out a new cost from giving up monetary independence and help to shed light on effective policies. An important welfare consequence that emerges from our welfare analysis is that a lender of last resort can significantly reduce the costs of joining a monetary union.

Related literature. Our paper contributes to a vast literature on monetary unions, pioneered by the seminal work of Mundell (1961). The traditional view is that the benefits of joining a monetary union are given by larger international trade, fostered by lower transaction costs. A more modern view, stressed by Alesina and Barro (2002), has emphasized the benefits from a reduction in the inflationary bias generated by the time inconsistency problem of monetary policy from the seminal work of Barro and Gordon (1983). The main theme in the literature is that these benefits have to be traded-off against the losses from inefficient macroeconomic fluctuations due to nominal rigidities and the lack of monetary independence. A comprehensive discussion of these issues, which has taken center stage since the formation of the Eurozone, is provided in Alesina, Barro, and Tenreyro (2002), Santos Silva and Tenreyro (2010), and De Grauwe (2018). A related literature compares the performance of fixed versus flexible exchange rates. Of particular interest are studies that consider financial accelerator models and argue that fixed exchange rates may exacerbate financial distress (Céspedes, Chang, and Velasco, 2004; Gertler, Gilchrist, and Natalucci, 2007).[2]

Our paper adds a new dimension to the costs from giving up monetary independence: a higher exposure to rollover crises. Our welfare analysis shows that the new costs that we uncovered are substantial and suggests that an adequate evaluation of the overall net benefits should consider these costs. In this respect, our results shed some light on the Outright Monetary Transactions facility es-

[2] Also related in this regard is an active closed economy literature on how the interaction between household deleveraging and a zero lower bound can amplify demand shocks (Eggertsson and Krugman, 2012, Guerrieri and Lorenzoni, 2017).

tablished by the European Central Bank (ECB) to purchase government bonds of distressed countries, following Mario Draghi's July 2012 speech pledging to do "whatever it takes to preserve the euro." Indeed, the paper shows that a lender of last resort can substantially reduce the costs for a country to remain in a monetary union.

Our paper also belongs to the literature on rollover crises in sovereign debt markets, starting with Sachs (1984), Alesina, Prati, and Tabellini (1990), and Cole and Kehoe (2000). Our formulation follows Cole-Kehoe, which has become the workhorse model in the quantitative sovereign default literature in the tradition of Aguiar and Gopinath (2006) and Arellano (2008). Examples include Chatterjee and Eyigungor (2012), Aguiar, Chatterjee, Cole, and Stangebye (2016), Conesa and Kehoe (2017), Roch and Uhlig (2018), and Bocola and Dovis (2019). Different from these contributions, we consider an economy with production and nominal rigidities, and establish how the exchange rate regime is central to the risk of exposure to rollover crises. With a flexible exchange rate regime, we find the exposure to a rollover crisis to be minimal, which is in line with Chatterjee and Eyigungor (2012), who showed that in a canonical endowment economy model with long-term debt calibrated to the data, the presence of rollover crises has a negligible effect on debt and spreads. By contrast, we show that with a fixed exchange rate regime, the multiplicity region expands significantly, and the government is heavily exposed to a rollover crisis.[3]

The paper that is perhaps most closely related to ours is Aguiar, Amador, Farhi, and Gopinath (2013), who address the question of whether the government's ability to inflate away its debt reduces its exposure to rollover crises, an argument notably raised by De Grauwe (2013) and Krugman (2011), who made the observation that Spain and Portugal had higher levels of sovereign spreads compared to the UK, despite having lower levels of debt. Aguiar et al. consider an endowment economy with domestic currency debt and show that when commitment to low inflation is weak, an independent monetary policy can actually increase the vulnerability to a rollover crisis, contrary to De Grauwe and Krugman's argument. Our paper also studies how monetary policy matters for the exposure to a rollover crisis but considers instead a model with nominal rigidities and foreign currency debt. Our results show that the lack of monetary autonomy does increase vulnerability to a rollover crisis and provides a new perspective that ascribes a role for monetary policy to deal with rollover crises, even

[3]With one-year maturity, as in Cole and Kehoe (1996, 2000), the exposure to a rollover crisis is typically large because the government has to roll over a large amount of debt relative to output every period. While they were motivated by the 1994 Mexican crisis with maturity of less than a year, the typical maturity for sovereign bonds, is much larger, averaging around six years for the Eurozone. With debt duration calibrated to the Eurozone, Conesa and Kehoe (2017) and Bocola and Dovis (2019) achieve a somewhat more significant role for rollover crises but rely implicitly on a minimum subsistence level for consumption, which they set to about 70% of income, and require debt levels of around 100% of GDP for a typical rollover crisis. Overall, Bocola and Dovis (2019) still finds that non-fundamental risk played a limited role during the Italian debt crisis. Our model, in the context of a monetary union, does attribute a significant role to rollover crises in Spain, despite lower observed external debt levels of about 30%. Notably, the model predicts that Spain was vulnerable to a rollover crisis in 2012, precisely around the time during which the ECB's promise to buy government bonds effectively led to a sharp reduction in sovereign spreads.

when debt is denominated in foreign currency.[4]

A related literature studies sovereign debt crises, but in the tradition of Calvo (1988), in which the government lacks commitment to debt issuances. If investors expect high inflation, the government borrows at a high rate and finds it optimal to inflate ex post, validating the initial expectations. In this line of work, the fact that debt is denominated in domestic currency and that the government can inflate away the debt is at the core of the fragility problem.[5] We consider a baseline model with debt in real terms, which allows us to abstract from the use of inflation to reduce the real value of the debt (and the multiplicity issues associated) to highlight a new channel by which monetary policy can actually help to reduce a fragility problem that arises from rollover crises.

Our paper is also related to an emerging literature that integrates nominal rigidities into the workhorse sovereign default model. Na, Schmitt-Grohé, Uribe, and Yue (2018) study a sovereign default model with downward nominal wage rigidity and show that it can account for the joint occurrence of large nominal devaluations and defaults, a phenomenon known as the "twin Ds." Bianchi, Ottonello, and Presno (2019) analyze a trade-off between the expansionary effects of government spending and the increase in sovereign risk and show how it can generate the observed fiscal procyclicality. Other recent papers include Arellano, Mihalache, and Bai (2019) who study the comovements of sovereign spreads with domestic nominal rates and inflation, and Bianchi and Sosa-Padilla (2019) who study the accumulation of international reserves as a macroeconomic stabilization tool. In contrast to this literature, we consider the possibility of rollover crises, which allows us to provide the first analysis of how nominal rigidities and monetary policy affect vulnerability to rollover crises. Another contribution of our paper is to provide an analytical characterization of how nominal rigidities affect the incentives to default.

Layout. Section 2 presents the model, and Section 3 presents the theoretical analysis. Section 4 presents the quantitative analysis, and Section 6 concludes. The proofs are listed in Appendix A.

[4]Aguiar, Amador, Farhi, and Gopinath (2015) consider a setup similar to Aguiar, Amador, Farhi, and Gopinath (2013) but with multiple countries and a union-wide monetary policy. They show that for a country with a high level of debt, it is preferable to join a monetary union with a mix of high and low debt countries as a way to balance the costs from inflationary bias and the reduction in the vulnerability to rollover crises by inflating away the debt ex post. Although we focus on a single country, a likely implication of our analysis is that an optimal currency area would feature countries with similar debt positions, more in line with Mundell's criteria for an optimal currency area. Other recent papers addressing issues of debt crises with a focus on the Eurozone are Broner, Erce, Martin, and Ventura (2014), Gourinchas, Martin, and Messer (2017), De Ferra and Romei (2018).

[5]A large literature on multiple equilibria follows this tradition: Corsetti and Dedola (2016) study the role of central bank backstop policies; Farhi and Maggiori (2017) study the implications for the international monetary system; Bacchetta, Perazzi, and Van Wincoop (2018) study conventional and unconventional monetary policy, building on a dynamic version of Calvo by Lorenzoni and Werning (2013). The role of inflation as a partial default also plays a key role in recent work by Araujo, Leon, and Santos (2013), Du and Schreger (2016), Bassetto and Galli (2019), Nuño and Thomas (2017), Camous and Cooper (2018), and Hur, Kondo, and Perri (2018).

2 Model

We study a small open economy (SOE) model of endogenous sovereign default subject to rollover crises. The SOE is populated by households, firms, and a government. In the international financial markets, risk-neutral lenders buy the long-term government bonds of the SOE denominated in foreign currency. A single tradable good can be traded without any frictions, and as a result, the law of one price holds. A non-tradable good in the SOE is produced using labor, and downward nominal wage rigidity creates the possibility of involuntary unemployment. We next describe the decision problems of households, firms, lenders, and the government.

2.1 Households

There is a unit measure of households with preferences over consumption given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U(c_t), \tag{1}$$

with

$$c_t = C(c_t^T, c_t^N) = [\omega(c_t^T)^{-\mu} + (1-\omega)(c_t^N)^{-\mu}]^{-1/\mu}, \quad \omega \in (0,1), \; \mu > -1.$$

The utility function $U(c)$ satisfies the standard properties $u' > 0$, $u'' < 0$, and Inada conditions, where c is a composite of tradable (c^T) and non-tradable goods (c^N), with constant elasticity of substitution (CES) equal to $1/(1+\mu)$.[6]

Each period, households receive y_t^T units of tradable endowment, which is stochastic and follows a stationary first-order Markov process. We assume a constant unit price of tradable goods in terms of foreign currency. The value of the tradable endowment in domestic currency is therefore given by $e_t y^T$, where e_t denotes the exchange rate measured as domestic currency per foreign currency (an increase in e_t denotes a depreciation of the domestic currency). Households also receive firms' profits, which we denote by ϕ_t^N, and labor income, $W_t h_t$, where W is the wage expressed in domestic currency and h is the amount of hours worked. Households inelastically supply $\overline{h}$ hours of work to the labor markets, but because of the presence of downward wage rigidity, they will work a strictly lower amount of hours when wage rigidity is binding. As we will discuss below, when wage rigidity is binding, the actual hours worked will be determined by firms' labor demand given prices and wages.

As is standard in the sovereign debt literature, we assume that households do not have direct access to external credit markets, although the government can borrow abroad and distribute the net proceedings to the households using lump-sum taxes or transfers. The households' budget constraint,

[6] The assumption of CES is only done to simplify some of the expressions. For our theoretical results, any homothetic preferences over tradables and non-tradables, or even normality, would suffice.

expressed in domestic currency, is therefore given by

$$e_t c_t^T + P_t^N c_t^N = e_t y_t^T + \phi_t^N + W_t h_t - T_t, \tag{2}$$

where P_t^N denotes the price of non-tradables in domestic currency, and T_t denotes lump-sum taxes/transfers in units of domestic currency.

The households' problem consists of choosing c_t^T and c_t^N to maximize (1) given the sequence of prices for non-tradables $\{P_t^N\}$, labor income $\{W_t h_t\}$, profits $\{\phi_t^N\}$, and taxes $\{T_t\}$. The static optimality condition equates the relative price of non-tradables to the marginal rate of substitution between tradables and non-tradables:

$$\frac{P_t^N}{e_t} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu}. \tag{3}$$

Thanks to homotheticity, the relative demand of tradable and non-tradable consumption goods is only a function of the relative price.

2.2 Firms

Firms operate a production function $y^N = F(h)$ where y_t^N denotes the output of non-tradable goods, and h_t denotes employment, the sole input. The production function $F(\cdot)$ is a differentiable, increasing, and concave function. In particular, we will consider a homogeneous production function $F(h) = h^\alpha$ where $\alpha \in (0, 1]$.

Firms operate in perfectly competitive markets, and each period they maximize profits that are given by

$$\phi_t^N = \max_{h_t} P_t^N F(h_t) - W_t h_t. \tag{4}$$

The optimal choice of labor employment h_t equates the value of the marginal product of labor to the nominal wage:

$$P_t^N F'(h_t) = W_t. \tag{5}$$

Given the price of non-tradables, a higher wage leads to lower employment. Likewise, given the wage, a lower price of non-tradables leads to lower employment. As we will see below, how the price of non-tradables reacts in general equilibrium will have important implications for debt crises.

2.2.1 Downward Nominal Wage Rigidity

We model downward nominal wage rigidity, following Schmitt-Grohé and Uribe (2016). For an economy that is outside a currency union, we assume, in particular, that wages in domestic currency cannot fall below $\overline{W}$:[7]

$$W_t \geq \overline{W} \tag{6}$$

for all t.

The parameter $\overline{W}$ determines the severity of the wage rigidity.[8] There are two cases. If the nominal wage that clears the labor market is higher than $\overline{W}$, the economy is at full employment and (6) is not binding. If, however, the nominal wage that would clear the market is below $\overline{W}$, the economy experiences involuntary unemployment. In this case, the amount of employment in equilibrium is determined by the amount of labor demand (5), and households work strictly less than their endowment of hours. Formally, wages and employment need to satisfy the following slackness condition:

$$\left(W_t - \overline{W}\right)\left(\overline{h} - h_t\right) = 0. \tag{7}$$

For an economy within a currency union, wages are set in foreign currency (the currency of the union), and therefore the lower bound is also in foreign currency. As we will see, in a fixed exchange rate regime, the wage also becomes effectively rigid in foreign currency.

2.3 Government

The government can issue a non-contingent long-term bond and can default at any point in time. As in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), a bond issued in period t promises an infinite stream of coupons that decrease at an exogenous constant rate $1-\delta$.[9] In particular, a bond issued in period t promises to pay $\delta(1-\delta)^{j-1}$ units of foreign currency in period $t+j$, for all $j \geq 1$. Given the assumption of a constant unitary price of the tradable good in terms of foreign currency, it is equivalent to specify the bond in terms of the tradable good. Debt dynamics can be represented by the following law of motion:

$$b_{t+1} = (1-\delta)b_t + i_t, \tag{8}$$

where b_t is the stock of bonds due at the beginning of period t, and i_t is the stock of new bonds issued in period t.

[7]There is a large amount of empirical evidence on downward wage rigidity. In particular, a recent literature has used micro-level data to highlight the important role that this friction played in the European crisis (e.g. Faia and Pezone, 2018; Ronchi and Di Mauro, 2017).

[8]In Schmitt-Grohé and Uribe (2016), $\overline{W}$ depends on the previous period wage and a parameter that controls the speed of wage adjustment. For numerical tractability, we take $\overline{W}$ as an exogenous (constant) value, as in Bianchi et al. (2019). Notice also that allowing for indexation to CPI inflation would not affect our theoretical mechanism because a nominal exchange rate depreciation in a state with unemployment would lead to a real exchange rate depreciation, and the price of non-tradables in domestic currency would rise by more than the increase in wages due to indexation.

[9]We take maturity as a primitive. There is an active literature studying maturity choices in sovereign default models (Arellano and Ramanarayanan, 2012; Bocola and Dovis, 2019; Sanchez, Sapriza, and Yurdagul, 2018).

Debt contracts cannot be enforced. If the government chooses to default, it faces two punishments. First, the government switches to financial autarky and cannot borrow for a stochastic number of periods. Second, there is a utility loss $\kappa(y_t^T)$, assumed to be increasing in tradable income. We think of this utility loss as capturing various default costs related to reputation, sanctions, or the misallocation of resources.[10]

The government's budget constraint in a period starting with good credit standing is

$$\delta e_t b_t (1 - d_t) = T_t + e_t q_t i_t (1 - d_t), \tag{9}$$

where $d_t = 0(1)$ if the government repays (defaults) and $q(\cdot)$ denotes the price schedule, which we will characterize below. The budget constraint indicates that repayment of outstanding debt obligations is made by collecting lump-sum taxes and issuing new debt. [11]

The timing within each period follows Cole and Kehoe (2000). At the beginning of each period, the government has outstanding debt liabilities b_t and could be in good or bad credit standing. If the government is in good credit standing, it chooses new debt issuances at the price schedule offered by investors. At the end of each period, the government decides whether to default or repay the initial debt outstanding. The difference with respect to Eaton and Gersovitz (1981) that will give rise to multiplicity is that here the government does not have the ability to commit to repaying within the period.[12] As we will see, negative beliefs about the decision of the government to default can become self-fulfilling.

Monetary regimes. Regarding the policy for exchange rates, we will consider two regimes: a flexible exchange rate and a fixed exchange rate. In the flexible exchange rate regime, the government

[10] Utility losses from default in sovereign debt models are also used by Bianchi, Hatchondo, and Martinez (2018) and Roch and Uhlig (2018), among others. An alternative often used is an output cost. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications. In any case, as will become clear below, what will be most important for our mechanism is the difference in the values of repayment for the government when investors are willing to lend and when they refuse to lend, but not the explicit form of the default cost.

[11] As is well-understood, allowing for specific taxes on consumption or payroll subsidies can mimic a nominal depreciation, as studied, for example, in Farhi, Gopinath, and Itskhoki (2013) and Schmitt-Grohé and Uribe (2016) (see also Correia, Nicolini, and Teles, 2008). As long as there are some limitations on the use of these policies (either political or economic), there remains a role for explicit nominal depreciations. Quantitatively, we will calibrate wage rigidity to match the observed increase in unemployment, and so implicitly this captures that the government could in practice be using these policies to some extent. From a normative standpoint, the importance of the exchange rate regime that we will uncover applies to the role of fiscal devaluation policies.

[12] A different source of multiplicity following Calvo (1988) arises if the government has to issue a fixed amount of debt revenues. In this case, the fact that bond prices decrease with debt generates a Laffer curve, which leads, directly through the budget constraint, to a high debt/spreads equilibrium and a low debt/spreads equilibrium. Lorenzoni and Werning (2013) explore this kind of multiplicity in a dynamic context with fiscal rules and long-term maturity and show how this gives rise to "slow moving debt crises" (see also Ayres, Navarro, Nicolini, and Teles, 2016).

will choose the optimal exchange rate at all dates without commitment.[13] In the fixed exchange rate regime, we assume that the government fixes the exchange rate to an exogenous level $e = \overline{e}$ at all times. Equivalently, one can also interpret the fixed exchange rate regime as the policy of a single economy that enters a monetary union and gives up its currency, in which case wages would be directly denominated in the foreign currency. In the former, monetary policy is set by the country to which the currency is pegged. In the latter, monetary policy is set at the level of the monetary union.[14] The important point is that in both cases, the government lacks monetary independence.

2.4 International Lenders

Sovereign bonds are traded with atomistic, risk-neutral foreign lenders. In addition to investing through the defaultable bonds, lenders have access to a one-period risk-free security paying a net interest rate r. By a no-arbitrage condition, equilibrium bond prices when the government repays are then given by

$$q_t = \frac{1}{1+r}\mathbb{E}_t[(1-d_{t+1})(\delta + (1-\delta)q_{t+1})]. \tag{10}$$

Equation (10) indicates that in equilibrium, an investor has to be indifferent between investing in a risk-free security and buying a government bond at price q_t, bearing the risk of default. In case of repayment next period, the payoff is given by the coupon δ plus the market value q_{t+1} of the nonmaturing fraction of the bonds. Since we assume no recovery, the bond price is zero in the event of default.

2.5 Equilibrium

In equilibrium, the market for non-tradable goods clears:

$$c_t^N = F(h_t). \tag{11}$$

In addition, using the households' and government budget constraint (2) and the definition of the firms' profits and market clearing condition (11), we obtain the resource constraint for tradable goods in the economy:

$$c_t^T = y_t^T + (1-d_t)[\delta b_t - q_t(b_{t+1} - (1-\delta)b_t)]. \tag{12}$$

[13] Given the path for nominal exchange rates that the government chooses, there is a domestic nominal interest rate that would implement this path. This interest rate can be obtained, via an interest parity condition, by introducing nominal bonds in domestic and foreign currency traded among domestic households.

[14] One could also allow some degree of correlation between the monetary policy conducted at the union level or by the target country by allowing P^* to follow a stochastic process correlated with the small open economy. Theoretically, all our results would remain unchanged.

Before proceeding to study a Markov equilibrium in which the government chooses policies optimally without commitment, let us examine the equilibrium for given government policies. A competitive equilibrium given government policies in our economy is defined as follows:

Definition 1 (Competitive Equilibrium)**.** Given an initial debt b_0, an initial credit standing, government policies $\{T_t, b_{t+1}, d_t, e_t\}_{t=0}^{\infty}$, and an exogenous process for the tradable endowment $\{y_t^T\}_{t=0}^{\infty}$ and for reentry after default, a *competitive equilibrium* is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$ such that:

1. Households and firms solve their optimization problems.
2. Government policies satisfy the government budget constraint (9).
3. The bond pricing equation (10) holds.
4. The market for non-tradable goods clears (11), and the resource constraint for tradables (12) holds.
5. The labor market satisfies conditions (6), (7), and $h \leq \overline{h}$.

Employment, Consumption, and Wages Using market clearing for non-tradable goods (11), together with the optimality conditions for households (3) and firms (5), we can obtain a useful (partial) characterization of equilibrium in a system of these three static equations and three variables (c_t^T, h_t, w_t), where $w_t \equiv W_t/e_t$ denotes the wage denominated in tradable goods. Using this system of equations, we can then derive in every period a real equilibrium wage solely as a function of (c_t^T, h_t).

Lemma 1. *In any equilibrium, the real wage in terms of tradable goods is a function of tradable consumption and employment,*

$$\mathcal{W}(c_t^T, h_t) \equiv \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\mu} F'(h_t). \tag{13}$$

Moreover, $\mathcal{W}(c_t^T, h_t)$ is increasing with respect to c_t^T and decreasing with respect to h_t.

One implication of Lemma 1, which will be important once we turn to the determination of the entire dynamic equilibrium, is that a decrease in the amount of tradable consumption is associated with a lower equilibrium wage. This occurs because lower tradable consumption is associated in equilibrium with a lower relative price of non-tradables (via household optimality), which, in turn, leads to a lower demand for labor and a decrease in the real wage, for a given level of employment. In addition, for a given level of tradable consumption, a decrease in employment is associated in equilibrium with a higher price of non-tradables (via market clearing and household optimality), which, in turn, requires a higher real wage to be consistent with firms' labor demand.

In equilibrium, we then have that downward nominal wage rigidity can be expressed as

$$\mathcal{W}(c_t^T, h_t)e_t \geq \overline{W}. \tag{14}$$

According to this lemma, we then have that if (14) is binding, a reduction in the amount of tradable consumption is associated with low employment in equilibrium. This result has important implications for the general equilibrium effects in the full dynamic system. If a shock reduces the demand for total consumption, we must have that for a given level of non-tradable output, the price of non-tradables needs to decline so that households switch consumption from tradables toward non-tradables and the market for non-tradable goods clears. Absent wage rigidity, we would have that the wage falls, and the only implication for the real economy is the reduction in tradable consumption. However, if wages are downwardly rigid, the decline in the relative price of non-tradables will lead to a decline in employment and non-tradable consumption.

Based on Lemma 1, we can also analogously construct an equilibrium employment that is a function of c_t^T and $\overline{w_t} \equiv \overline{W}/e_t$.

Lemma 2. *In any equilibrium, employment is given by*

$$\mathcal{H}(c_t^T, \overline{w_t}) = \begin{cases} \left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\overline{w_t}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}} & \text{if } c_t^T \leq \overline{c}_{\overline{w_t}}^T, \\ \overline{h} & \text{if } c_t^T > \overline{c}_{\overline{w_t}}^T \end{cases}, \tag{15}$$

where

$$\overline{c}_{\overline{w_t}}^T = \left[\frac{\omega}{1-\omega}\left(\frac{\overline{w_t}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \left(\overline{h}\right)^{\frac{1+\alpha\mu}{1+\mu}}.$$

This condition implies that when the wage rigidity is binding and there is unemployment, the government will realize that repaying debt and cutting back on consumption will create more unemployment. We will see below how the implied increase in the cost of repayment affects incentives to default and vulnerability to a rollover crisis.

2.6 Recursive Government Problem

We consider the optimal policy of a benevolent government with no commitment, which chooses consumption and external borrowing to maximize households' welfare, subject to the implementability conditions. We focus on the Markov equilibria.

Every period in which the government enters with access to financial markets, it evaluates the lifetime utility of households if debt contracts are honored against the lifetime utility of households if they are repudiated. We use $\mathbf{s} = (y^T, \zeta)$ to denote the vector of exogenous states in every period. The variable ζ is a sunspot variable to index for the possibility of multiplicity of equilibria, as in Cole and Kehoe (2000), which we will describe below. Different from the equilibrium according to the timing in Eaton and Gersovitz (1981), the possibility of a rollover crisis implies that the bond price is a function of the initial debt position and the sunspot, in addition to the debt choice and current income shock.

Regarding the policy for exchange rates, we will start with the case in which the government is under a fixed exchange rate regime. That is, the exchange rate is fixed at an exogenous level $e = \overline{e}$ for every period. We can define a *real wage rigidity* constraint as $w \geq \overline{w}$ where $\overline{w} \equiv \overline{W}/\overline{e}$ and $w \equiv W/e$. We can then rewrite (14) as $\mathcal{W}(c^T, h) \geq \overline{w}$. Later on, we will study the case in which we allow the government to depreciate its currency. As should be clear from (14), an exchange rate depreciation will be able to undo the wage rigidity, and this will be the optimal policy for the government.

The government problem with access to financial markets can be formulated in recursive form as follows:

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \{(1-d)V_R(b, \mathbf{s}) + dV_D(y^T)\}, \tag{16}$$

where $V_R(b, \mathbf{s})$ and $V_D(y^T)$ denote, respectively, the values of repayment and default.

The value of repayment is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \overline{h}} \left\{u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}')\right\} \tag{17}$$

subject to

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

where $q(b', b, \mathbf{s})$ denotes the debt price schedule, taken as given by the government, and $\mathcal{W}$ is defined in (13).[15] Meanwhile, the value of default is given by

$$V_D(y^T) = \max_{c^T, h \leq \overline{h}} \left\{u\left(c^T, F(h)\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi)V_D(y^{T\prime})\right]\right\} \tag{18}$$

subject to

$$c^T = y^T$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

where $\psi \in [0, 1]$ is the probability of reentering financial markets after a default.

A Markov-perfect equilibrium is then defined as follows.

[15] An equivalent representation uses equilibrium employment (15) rather than the explicit wage rigidity constraint.

Definition 2 (Markov-perfect equilibrium)**.** A Markov-perfect equilibrium is defined by value functions $\{V(b, \mathbf{s}), V_R(b, \mathbf{s}), V_D(y^T)\}$, policy functions $\{\hat{d}(b, \mathbf{s}), \hat{c}^T(b, \mathbf{s}), \hat{b}(b, \mathbf{s}), \hat{h}(b, \mathbf{s})\}$, and a bond price schedule $q(b', b, \mathbf{s})$ such that

1. Given the bond price schedule, policy functions solve problems (16), (17), and (18).
2. The debt price schedule satisfies

$$q(b', b, \mathbf{s}) = \begin{cases} \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathbf{s}'))] & \text{if } \hat{d}(b, \mathbf{s}) = 0, \\ 0 & \text{if } \hat{d}(b, \mathbf{s}) = 1 \end{cases},$$

where

$$\begin{aligned} b'' &= \hat{b}(b', \mathbf{s}') \\ d' &= d(b', \mathbf{s}'). \end{aligned}$$

For the economy with a flexible exchange rate, the only difference is that the government also chooses e, in addition to the prices and allocations that are chosen under the fixed exchange rate regime subject to the implementability conditions. It is also straightforward to expand the definition for an arbitrary Markov exchange rate policy.

2.7 Multiplicity of Equilibrium

As in Cole and Kehoe (2000), the government is subject to self-fulfilling rollover crises. Let us define the debt price schedule, assuming there will be no default and the break-even condition of lenders is satisfied. We will call this the *fundamental* debt price schedule:

$$\tilde{q}(b', y^T) \equiv \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathbf{s}'))], \tag{19}$$

where $b'' = \hat{b}(b', \mathbf{s}')$ and $d' = d(b', \mathbf{s}')$. This debt price schedule does not depend on the sunspot nor on the current amount of debt held by the government. Using this price schedule, we can construct the value of repayment when international lenders believe that the government will honor its debt commitments at the end of the period and therefore extend credit to the government. This value is as follows:

$$V_R^+(b, y^T) = \max_{b', c^T, h \leq \overline{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}') \right\} \tag{20}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + \tilde{q}(b', y^T)[b' - (1-\delta)b]$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

Denote by $\hat{b}^+(b, y^T)$ the solution to the previous problem. Divide the state space where the government finds it optimal to issue strictly positive amounts of debt:

$$\mathcal{B} \equiv \left\{ (b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad \hat{b}^+(b, y^T) > (1-\delta)b \right\}.$$

Consider now the case in which investors are unwilling to lend to the government. Denote by V_R^- the value function in this case, when the government decides to repay. If $(b, y^T) \notin \mathcal{B}$, we have that $V_R^-(b, y^T) = V_R^+(b, y^T)$, as the government is not issuing debt even when investors are willing to lend to the government. Then, if $(b, y^T) \in \mathcal{B}$, the value is given by

$$V_R^-(b, y^T) = \max_{c^T, h \leq \overline{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V((1-\delta)b, \mathbf{s}') \right\} \tag{21}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

Lemma 3 states that the value of repayment when lenders refuse to roll over government bonds is never greater than the value when lenders are willing to roll over. This must be the case since the government can always choose not to borrow when lenders are willing to roll over.[16]

Lemma 3. *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b \in \mathbb{R}$, we have that $V_R^+(b, y^T) \geq V_R^-(b, y^T)$.*

Because tradable consumption is lower when the government does not have access to borrowing , the wage rigidity constraint will become binding for lower levels of debt, in line with Lemma 1. As a result, the presence of wage rigidity will have a stronger effect on V_R^- than on V_R^+ and lead to an increase in the gap between these two values. As we will see below, this will have important implications for the occurrence of self-fulfilling rollover crises.

[16] One element implicit here is that if the government were to try to repurchase debt when investors are unwilling to lend, the price of bonds would rise to the fundamental price, and hence the budget constraint when $(b, y^T) \in \mathcal{B}$ would be $c^T = y^T - \delta b$, as reflected in (21). See Aguiar and Amador (2013) and Bocola and Dovis (2019) for an elaboration of this point.

Three zones. Let us separate the state space (b, y^T) into three zones: the safe zone, default zone, and crisis zone. The *safe zone* will denote those states in which the government finds it optimal to repay its debt even if international lenders are not willing to issue more debt to the government. That is,

$$\mathcal{S} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-(b, y^T)\right\}.$$

The *default zone* defines those states in which the government finds it optimal to default even if international lenders are willing to lend at the fundamental debt price schedule. That is,

$$\mathcal{D} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+(b, y^T)\right\}.$$

Finally, the *crisis zone* will correspond to those states in which the government finds it optimal to repay if investors are willing to lend at the fundamental debt price schedule, but finds it optimal to default if investors are not willing to lend. That is,

$$\mathcal{C} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_R^+(b, y^T) > V_D(y^T) > V_R^-(b, y^T)\right\}.$$

In the crisis zone, the outcome is undetermined and depends on investors' beliefs. If investors believe the government will repay, the government will find it optimal to repay, whereas if they believe that the government will default, the government will default. To select an equilibrium, we will use a sunspot $\zeta \in \{0, 1\}$. If $\zeta = 0$, we will say there is a "good sunspot," in which case the equilibrium with repayment is selected. If $\zeta = 1$, we will say there is a "bad sunspot," in which case the equilibrium with default is selected. We assume that ζ follows an *i.i.d.* process with probability π of selecting the bad sunspot.

Following these definitions, the optimal binary default decision and the optimal debt price schedule will satisfy

$$d(b, \mathbf{s}) = \begin{cases} 1 & \text{if } (b, y^T) \in \mathcal{D} \\ 1 & \text{if } (b, y^T) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ 0 & \text{if } (b, y^T) \in \mathcal{C} \quad \& \quad \zeta = 0 \\ 0 & \text{if } (b, y^T) \in \mathcal{S} \end{cases} \tag{22}$$

$$q(b', b, \mathbf{s}) = \begin{cases} 0 & \text{if } (b, y^T) \in \mathcal{D} \\ 0 & \text{if } (b, y^T) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}. \tag{23}$$

3 Theoretical Analysis

In this section, we provide an analytical characterization of how monetary policy and downward nominal wage rigidity affect the government's incentives to default. The central point we will establish is that fixing the exchange rate leaves a government more vulnerable to a rollover crisis. In other words, we will show that the crisis zone will be larger for an economy with a fixed exchange rate.

3.1 Flexible Exchange Rate Regime

The flexible exchange rate regime allows the government to choose any nominal exchange rate every period. The following proposition characterizes the optimal exchange rate policy.

Proposition 1 (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states.*

This proposition establishes that the government finds it optimal to choose an exchange rate that delivers full employment, a result that can be seen from the value functions (17) and (18). If there was unemployment in the economy, the government could always relax the wage constraint by sufficiently depreciating the nominal exchange rate without bearing any other costs. This basic result is of course in line with the traditional benefit of having a flexible exchange rate in the presence of nominal rigidities, going back to Friedman (1953) and Mundell (1961). One difference here is that to ensure full employment, the government needs to depreciate the currency not only on-equilibrium but also off-equilibrium.

We would like to highlight two points. First, while the only role of monetary policy is to help stabilize unemployment through real wages, this is usually seen as a central channel of monetary policy in open economies. Milton Friedman, for example, highlighted the dangers of Europe eliminating the exchange rate adjustment because of possible misalignments in real wages.[17]

Second, it is worth pointing out that while we do not explicitly model why the government would fix the exchange rate or join a monetary (and therefore depart from the optimal exchange rate policy), doing so, in practice, offers a number of well-studied benefits. The gains could arise, for example, from lower inflationary bias (Alesina and Barro, 2002, Barro and Gordon, 1983) or from improvements in trade due to lower volatility and transaction costs (Mundell, 1961, Frankel and Rose, 2002). Following a

[17] As expressed by Milton Friedman in "Why Europe Can't Afford the Euro," *Times* (London), November 19, 1997.

> If one country is affected by negative shocks that call for, say, lower wages relative to other countries, that can be achieved by a change in one price, the exchange rate, rather than by requiring changes in thousands on thousands of separate wage rates, or the emigration of labour. The hardships imposed on France by its "franc fort" policy illustrate the cost of a politically inspired determination not to use the exchange rate to adjust to the impact of German unification. Britain's economic growth after it abandoned the exchange-rate mechanism a few years ago to refloat the pound illustrates the effectiveness of the exchange rate as an adjustment mechanism.

large literature on monetary unions, we do not model explicitly these gains. We will show, however, that the main theoretical results to be presented below will continue to hold in the presence of an exogenous specified costs from current or future nominal exchange rate fluctuations.[18]

3.2 Uncovering the Role of Nominal Rigidities and Monetary Policy

In this section, we study how wage rigidity and the exchange rate regime shape default decisions and the exposure to a rollover crisis. As a starting point, we consider the impact of a temporary change in the degree of wage rigidity. In particular, we examine a one-period tightening of wage rigidity. Since $\overline{w} = \frac{\overline{W}}{e}$, the tightening can arise because of an increase in $\overline{W}$, a more appreciated exchange rate, or some combination of the two.[19] The advantage of assuming that the change in wage rigidity (or the exchange rate) is only for one period is that the *fundamental* price schedule remains the same. This is because continuation values do not change, and hence future default decisions also remain unchanged. Notice that it is equivalent to consider a stochastic *i.i.d.* process for $\overline{w}$. In this case, a shock to $\overline{w}$ will also leave the fundamental price schedule unchanged and the theoretical propositions that we will derive, would apply in the same way. We will later study the consequences of permanent changes in the exchange rate regime, but studying a temporary change in wage rigidity is useful because it allows us to isolate current changes in monetary policy while leaving future policies constant.

Denote the current value functions with the one-period change in wage rigidity $\overline{w}$ as $\tilde{V}_D(y^T;\overline{w})$, $\tilde{V}_R^+(b, y^T;\overline{w})$, and $\tilde{V}_R^-(b, y^T;\overline{w})$. We maintain the same notation for continuation values V and the bond price q which correspond to a Markov equilbrium with an arbitrary exchange rate policy.[20] The problem the government faces when there is a temporary change in wage rigidity can then be expressed as

$$\tilde{V}_D(y^T;\overline{w}) = \max_{c^T, h\leq\overline{h}} \left\{u\left(c^T, F(h)\right) - \kappa(y^T) + \beta\mathbb{E}\left[\psi V(0,\mathbf{s}') + (1-\psi)V_D(y^{T\prime})\right]\right\} \tag{24}$$

subject to

$$c^T = y^T,$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

[18]It remains a topic for future research to provide a more comprehensive analysis that integrates the gains from joining a monetary union studied in the literature and the new cost identified in this paper.

[19]Alternatively, one can think of a decrease in the price of tradables in foreign currency P^*, which we have assumed constant and normalized to one for simplicity. In this case, by the law of one price we have $\overline{w} = \frac{\overline{W}}{eP^*}$, and a reduction in P^* would have the same tightening effects.

[20]To fix ideas, one can think about these continuation values being associated with the economy under flexible wages. What is important in this comparative static exercise is that we change the current wage rigidity ($\overline{w_t}$) leaving the $t+1, t+2....$ rigidity constant, and so our results apply for any arbitrary continuation Markov equilibrium, including the one under the flexible exchange rate.

$$\tilde{V}_R^+(b, y^T; \overline{w}) = \max_{b', c^T, h \leq \overline{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}') \right\} \tag{25}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + \tilde{q}(b', y^T)(b' - (1-\delta)b),$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

and

$$\tilde{V}_R^-(b, y^T; \overline{w}) = \max_{c^T, h \leq \overline{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V((1-\delta)b, \mathbf{s}') \right\} \tag{26}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b,$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

To show how the three zones (safe, default, and crisis) are affected by nominal rigidity, we first present some useful properties regarding the thresholds at which the government is indifferent between repaying and defaulting.

Lemma 4 (Debt Thresholds). *For every $y^T \in \mathbb{R}_+$, there exist debt thresholds $\bar{b}^+(\overline{w}), \bar{b}^-(\overline{w}) \in \mathbb{R}_+$ such that $\tilde{V}_D(y^T; \overline{w}) = V_R^+\left(b, y^T; \overline{w}\right) \forall b \geq \bar{b}^+(\overline{w})$ and $\tilde{V}_D(y^T; \overline{w}) \geq V_R^-\left(b, y^T; \overline{w}\right) \forall b \geq \bar{b}^-(\overline{w})$. Moreover, $\tilde{V}_D(y^T; \overline{w}) = V_R^+\left(\bar{b}^+(\overline{w}), y^T; \overline{w}\right)$ and $\tilde{V}_D(y^T; \overline{w}) = V_R^-\left(\bar{b}^-(\overline{w}), y^T; \overline{w}\right)$. Finally, we have $\bar{b}^+(\overline{w}) \geq \bar{b}^-(\overline{w})$.*

(Notice that to streamline notation, we omit the dependence of these thresholds on y^T.)

Using that the repayment value functions are strictly decreasing with respect to current debt and that the value of default is independent of debt, Lemma 4 presents the debt intervals that characterize the repayment/default decision. The lemma also establishes that the amount of debt that makes the government indifferent between repaying and defaulting is higher when investors are willing to lend.

Using these results, we can construct a safe region, a default region, and a crisis region for every level of wage rigidity:[21]

$$\tilde{S}(\overline{w}) \equiv \left(-\infty, \bar{b}^-(\overline{w})\right], \quad \tilde{C}(\overline{w}) \equiv \left(\bar{b}^-(\overline{w}), \bar{b}^+(\overline{w})\right], \quad \text{and} \quad \tilde{D}(\overline{w}) \equiv \left(\bar{b}^+(\overline{w}), \infty\right).$$

Figure 1 shows how the debt space can be separated into these distinct regions by the thresholds defined in Lemma 4 for every possible y^T.

[21]Different from the "zones" constructed above, which are in the (b, y^T) space, the "regions" fix the level of y^T and $\overline{w}$. Recall that the dependence of the debt threshold y^T is made implicit to avoid clutter. We will often refer to the thresholds, in short, as b^+ or b^-.

Figure 1: Safe, crisis, and default regions and debt thresholds

Comparative static results. We now study how these regions expand or contract with the degree of wage rigidity. Notice that to characterize the three regions, it is sufficient to determine the threshold values of $\bar{b}^{-}$ and $\bar{b}^{+}$. A property that will be useful in the characterization, established in Lemma 7 in the appendix, is the existence of wage rigidity thresholds $\overline{w}_D$, $\overline{w}_R^{+}$, and $\overline{w}_R^{-}$. When $\overline{w}$ is below these thresholds, the value functions are identical to their corresponding flexible wage counterparts, and when $\overline{w}$ is above these thresholds, the value functions become strictly inferior to their corresponding flexible wage counterparts. We will be using this property to show how $\bar{b}^{-}$ and $\bar{b}^{+}$ change with $\overline{w}$.

We first establish that the safe region contracts with higher wage rigidity.

Proposition 2 (Safe Region Threshold). *For every y^T and taking a pair of wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*

$i)$ *If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^{-}(\overline{w}_2) \le \bar{b}^{-}(\overline{w}_1)$, or equivalently, $\mathcal{S}(\overline{w}_2) \subseteq \mathcal{S}(\overline{w}_1)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^{-}, \overline{w}_D\right]$, then $\bar{b}^{-}(\overline{w}_2) < \bar{b}^{-}(\overline{w}_1)$. Equivalently, $\mathcal{S}(\overline{w}_2) \subset \mathcal{S}(\overline{w}_1)$.*

$ii)$ *Consider $F(h) = h$, $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^{-}, \infty\right)$, then $\bar{b}^{-}(\overline{w}_2) \le \bar{b}^{-}(\overline{w}_1)$, or equivalently, $\mathcal{S}(\overline{w}_2) \subseteq \mathcal{S}(\overline{w}_1)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^{-}, \overline{w}_D\right)$, then $\bar{b}^{-}(\overline{w}_2) < \bar{b}^{-}(\overline{w}_1)$, or equivalently, $\mathcal{S}(\overline{w}_2) \subset \mathcal{S}(\overline{w}_1)$.*

An implication of $\bar{b}^{-}$ decreasing with wage rigidity is that a higher level of rigidity can push the government away from the safe zone. Key for Proposition 2 is that, relative to being in default, wage rigidity is more binding under a state in which the government cannot roll over the debt. The first part of this proposition analyzes a range of $\overline{w}$ of low wage rigidity in the sense that there is unemployment only when the government repays while being unable to roll over the debt. Within this range, the value of repayment is decreasing in $\overline{w}$ whereas the value of default stays constant. The result is that a higher $\overline{w}$ always leads to a contraction of the safe zone. Once $\overline{w}$ is high enough, both the value of default and the value of repayment are affected, and the strong monotonicity cannot be ensured for general conditions. With linear production and log utility, we can establish that the changes in these value functions once the wage rigidity becomes binding under both default and repayment are of the same magnitude.[22] Hence, the safe region for high wage rigidity such that unemployment occurs under default is strictly higher than the safe region for any lower wage rigidity such that

[22] The reason is that under these assumptions, the non-tradable consumption becomes linear in tradable consumption and with log utility, and one can then separate the component associated with tradable consumption from the wage rigidity component.

unemployment occurs only when the government cannot roll over the debt. Furthermore, we are able to show in Proposition 5 in the appendix that for any utility function that satisfies separability between tradables and non-tradables, a fixed exchange rate always displays a smaller safe zone compared with a flexible exchange rate regime.

One implication of the fact that the safe zone contracts with a higher $\overline{w}$ is that the maximum default region that includes both the crisis region and the (fundamental) default region expand. Moreover, the crisis region expands to the left, meaning that the government is vulnerable to a rollover crisis for lower levels of debt. To examine the overall change in the crisis region and how the default region change, we need to examine the other debt threshold $\bar{b}^{-}$.

We show next that the effect of wage rigidity on $\bar{b}^{-}$ depends crucially on the trade balance in the flexible exchange rate economy when the government repays while being able to roll over the debt. Let $TB_R^{flex,+} = y^T - \hat{c}_R^+(\bar{b}^+(0), y^T; 0)$ denote such a trade balance. Proposition 3 shows that $\bar{b}^+(\overline{w})$ is increasing in $\overline{w}$ if $TB_R^{flex,+} \leq 0$. Conversely, $\bar{b}^+(\overline{w})$ is decreasing in $\overline{w}$ if $TB_R^{flex,+} \geq 0$.

Proposition 3 (Default Region Threshold)**.** *For every y^T and taking a pair of wage rigidities $\overline{w}_1 < \overline{w}_2$, we have that:*

If $TB_R^{flex,+} \leq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_R^+]$, then $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$, or equivalently, $\mathcal{D}(\overline{w}_2) \subseteq \mathcal{D}(\overline{w}_1)$. Moreover, if $\overline{w}_2 \in (\overline{w}_D, \overline{w}_R^+]$, then $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$, or equivalently, $\mathcal{D}(\overline{w}_2) \subset \mathcal{D}(\overline{w}_1)$.

ii) Consider $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$, then $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$, or equivalently, $\mathcal{D}(\overline{w}_2) \subseteq \mathcal{D}(\overline{w}_1)$. Moreover, if $\overline{w}_1 \in [\overline{w}_D, \overline{w}_R^+)$, then $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$, or equivalently, $\mathcal{D}(\overline{w}_2) \subseteq \mathcal{D}(\overline{w}_1)$.

If $TB_R^{flex,+} \geq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$, or equivalently, $\mathcal{D}(\overline{w}_1) \subseteq \mathcal{D}(\overline{w}_2)$. Moreover, if $\overline{w}_2 \in (\overline{w}_R^+, \overline{w}_D]$, then $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. Equivalently, $\mathcal{D}(\overline{w}_1) \subset \mathcal{D}(\overline{w}_2)$.

ii) Consider $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in [\overline{w}_R^+, \infty)$, then $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. Equivalently, $\mathcal{D}(\overline{w}_1) \subseteq \mathcal{D}(\overline{w}_2)$. Moreover, if $\overline{w}_1 \in [\overline{w}_R^+, \overline{w}_D)$, then $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$, or equivalently, $\mathcal{D}(\overline{w}_1) \subset \mathcal{D}(\overline{w}_2)$.

This proposition tells us that depending on the sign of the trade balance, the default zone will contract or expand. If $TB_R^{flex,+} \geq 0$, we have that a higher wage rigidity can push the government to the default region. On the other hand, if $TB_R^{flex,+} \leq 0$, a higher wage rigidity can push the government out of the default region.

The intuition for this proposition is that the trade balance determines (together with y^T) the amount of tradable resources available for consumption and affects how binding wage rigidity is.

If the trade balance is positive when the government repays, this implies that there are fewer tradable resources available under repayment than under default (since the trade balance is zero in this case).[23] Starting from a low $\overline{w}$, this implies that an increase in $\overline{w}$ generates unemployment first under repayment, and only after a sufficiently large increase does unemployment emerge under default too. As a result, starting at an indifferent point between repayment and defaulting under flexible wages, there is a range of $\overline{w}$ such that the value of repayment is decreasing in $\overline{w}$ while the value of default is unchanged. In this range, the default region expands with $\overline{w}$.[24] If the trade balance is negative, these results revert, since now more resources are available for consumption under repayment than under default. Hence, a tighter wage rigidity first affects the value of default, and only after a sufficiently large increase is the value of default affected. The result is a default region that contracts with $\overline{w}$.

Putting together Propositions 2 and 3 that establish how b^+ and b^- change with $\overline{w}$, we can now examine what happens with the crisis zone. Since the safe region contracts with $\overline{w}$, we know that the crisis region expands to the left (i.e., the government becomes vulnerable for lower values of debt). We also know that if $TB_R^{Flex,+} < 0$, the crisis region expands to the right as well. If $TB_R^{Flex,+} > 0$, we can also show that the crisis region expands, in general, for moderate degrees of rigidity. Furthermore, under stricter conditions, we can show that it always expands for any degree of rigidity. Proposition 4 establishes this central result.

Proposition 4 (Crisis Region Expansion)**.** *For every y^T and taking a pair of wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*

$i)$ *If $\overline{w}_1, \overline{w}_2 \in \left[0, \min\left\{\overline{w}_R^+, \overline{w}_D\right\}\right]$, then $\tilde{C}\left(\overline{w}_1\right) \subseteq \tilde{C}\left(\overline{w}_2\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \min\left\{\overline{w}_R^+, \overline{w}_D\right\}\right]$, then $\tilde{C}\left(\overline{w}_1\right) \subset \tilde{C}\left(\overline{w}_2\right)$.*

$ii)$ *Under $TB_R^{flex,+} \leq 0$, $F(h) = h$, and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\tilde{C}\left(\overline{w}_1\right) \subseteq \tilde{C}\left(\overline{w}_2\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_R^+\right)$, then $\tilde{C}\left(\overline{w}_1\right) \subset \tilde{C}\left(\overline{w}_2\right)$.*

Key for this result is that starting from full employment, a small increase in wage rigidity first affects the safe zone, thereby increasing the crisis region, and only after a sufficiently large increase does the default region start to increase (and the crisis region starts to contract to the right). When $TB_R^{flex,+} \leq 0$, and under log utility and linear production, the monotonicity can be extended further. In general, for any value of $\overline{w}$, we can obtain the change in the length of the crisis zone as follows: [25]

$$\frac{\partial[\bar{b}^+(\overline{w}) - \bar{b}^-(\overline{w})]}{\partial \overline{w}} = \frac{\frac{\partial \tilde{V}_R^+(\bar{b}^+, y^T; \overline{w})}{\partial \overline{w}} - \frac{\partial \tilde{V}_D(\overline{w})}{\partial \overline{w}}}{\frac{\partial \tilde{V}_R^+(\bar{b}^+, y^T; \overline{w})}{\partial b}} - \frac{\frac{\partial V_R^-(\bar{b}^-, y^T; \overline{w})}{\partial \overline{w}} - \frac{\partial \tilde{V}_D(\overline{w})}{\partial \overline{w}}}{\frac{\partial \tilde{V}_R^-(\bar{b}^-, y^T; \overline{w})}{\partial b}}. \tag{27}$$

[23] Assuming a loss in tradable output under default would lead to lower tradable resources available compared with the case under repayment with a zero trade balance. Considering the same percentage loss in non-tradable output, however, leads to an increase in the relative price of non-tradables, which offsets the losses in tradable output and generates the same amount of unemployment, leaving $\overline{w}^-$ and the debt threshold unchanged.

[24] Notice that when investors are pessimistic and the government repays, $b' = (1 - \delta)b$, and since $\delta \leq 1$, this implies that the trade balance is always positive, giving rise to a safe region that contracts with wage rigidity.

[25] To see this, apply the implicit function theorem in $V_R^+(\bar{b}^+, y^T; \overline{w}) = V^D(\overline{w})$ and $V_R^-(\bar{b}^-, y^T; \overline{w}) = V^D(\overline{w})$.

This shows that the higher the decrease in V_R^- is relative to V_R^+ when there is a tightening of wage rigidity, the higher the increase tends to be in the crisis region.

3.3 Graphical Illustration

Following the theoretical analysis above, this section provides a graphical illustration of how wage rigidity affects incentives to default and, in particular, raises the vulnerability to a rollover crisis. To construct the following figures, we use the calibrated version of our model, which we will explain in the quantitative section. In addition, as we vary the current level of rigidity, we fix the continuation values from the economy with flexible wages.

In Figure 2 we present the values $\tilde{V}_D, \tilde{V}_R^+, \tilde{V}_R^-$ for different levels of debt. We fix the tradable endowment to the average value in default episodes in our simulation exercise for the flexible exchange rate regime (technically, the element in the grid that is closest to this point). This level is 4.3% below average. To facilitate the reading of the figures, we normalize debt by average GDP. Unless we specify otherwise, all numbers reported will be expressed in this way. Notice that in Figure 2, the actual equilibrium value function is given by the upper envelope of $\tilde{V}_D$ and $\tilde{V}_R^+$ in the case of the good sunspot and by the upper envelope of $\tilde{V}_D$ and $\tilde{V}_R^-$ in the case of the bad sunspot. Panel (a) presents the values for the flexible exchange rate regime. It should be understood that when we refer to the flexible exchange rate, we mean the exchange rate policy that delivers the full employment case in all states. For the case of a fixed exchange rate, it will be useful to consider two values for $\overline{w}$. Panel (b) corresponds to the fixed exchange rate regime with "low" wage rigidity and panel (c) to a fixed exchange rate regime with "high" wage rigidity.

Using these value functions, it is straightforward to graphically represent the safe region, crisis region, and default region in Figure 2. The crisis region (i.e., the levels of debt at which a default would occur if investors turn pessimistic) appears shaded in the figure. The safe region (i.e., the levels of debt at which the government repays regardless of lenders' beliefs) is to the left of the crisis region. The default region (i.e., the levels of debt at which the government defaults regardless of lenders' beliefs) is to the right. It is apparent from these figures that vulnerability to a rollover crisis is higher in a fixed exchange rate regime than in a flexible one for both degrees of wage rigidity. A difference between the low and high rigidity is that under the former, only the safe region and the crisis region change, whereas the default region remains the same.

Crisis region for flexible exchange rate. Let us now describe how we arrive at the crisis region in the flexible exchange regime in panel (a) of Figure 2. The value of default $\tilde{V}_D$ is a constant because it does not depend on the amount of debt the government owes.[26] The values of repayment $\tilde{V}_R^+$, $\tilde{V}_R^-$ are

[26] In case of the flexible exchange rate regime, we have $\tilde{V} = V$. However, we keep the notation with $\tilde{V}$ to make it more uniform with the fixed exchange rate regime.

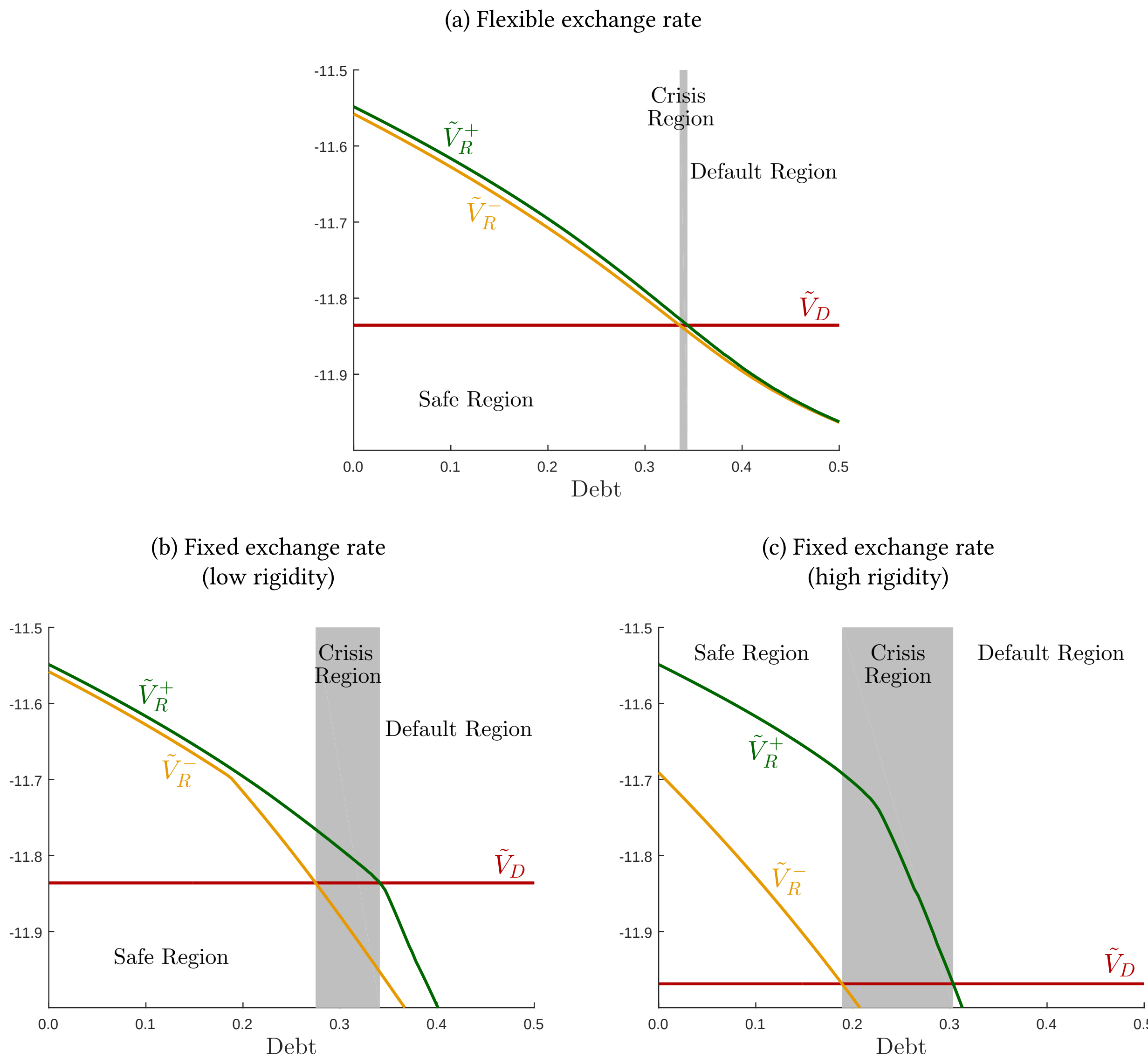

Figure 2: Value Functions and Crisis Regions

Notes: The income shock in the three panels is set to -4.3% below the mean, which is the average income shock before a default episode in the flexible exchange rate regime. Panel (a) uses parameter values from the calibrated flexible exchange rate economy. Panels (b) and (c) use the same parameters with the exception of the current level of wage rigidity $\overline{w}$. In panel (a), $\overline{w}$ is set to its highest value where full employment is achieved under a good sunspot. This is 1.33 times the real wage in the flexible exchange rate regime. Panel (c) increases the wage rigidity to 1.66 times higher than the wage in the flexible exchange rate regime.

decreasing in debt in both cases because the resource constraint becomes tighter. The value function $\tilde{V}_R^+$ is uniformly above $\tilde{V}_R^-$. Moreover, the difference between these two values is higher for low levels of debt (when the government wants to issue more debt), and the values become identical for very high levels of debt (when the government does not issue debt even when it has access to financial markets).

At the debt level in which the curves $\tilde{V}_R^+$ and $\tilde{V}_D$ intersect, the government is indifferent between repaying when having access to credit markets and defaulting. For debt positions higher than this level, the government defaults regardless of the international lenders' beliefs. This is what we define as the default region. On the other hand, at the debt level in which the curves $\tilde{V}_R^-$ and $\tilde{V}_D$ intersect, the government is indifferent between repaying when unable to roll over the debt and defaulting. For debt positions lower than this level, the value of repayment is higher than the value of default, and the government repays its debt. This is what we define as the safe region: levels of debt at which the government repays even if investors are pessimistic. In between these two regions, there is an interval of debt positions in which the government will default only if international lenders are unwilling to roll over the debt. This is what we define as the crisis region. This region, which appears shaded in panel (a) of Figure 2, is less than 1% of debt in terms of average GDP: the range is between 33.5% and 34.4%. The region in which the government is vulnerable to a rollover crisis is small for a flexible exchange rate regime.

Crisis region for fixed exchange rate. Panels (b) and (c) of Figure 2 consider the one-period fixed exchange rate regime. As described above, we consider a situation in which there is a fixed exchange rate regime for only the current period and the flexible regime prevails from next period onward. The impact of the fixed exchange rate regime depends, of course, on the level of nominal wages and the level of the exchange rate—in particular, a sufficient variable is $\overline{w}$, the lower bound on wages in foreign currency. We consider two values for this real wage rigidity $\overline{w}$. In panel (b), we consider the highest value of $\overline{w}$ such that only b^- changes while b^+ remains the same. This case allows us to have a situation in which the default region remains unchanged relative to the flexible wage (as characterized in the first item of Proposition 4). One can also see that $\tilde{V}_D$ is at exactly the same level as in the flexible regime because the wage rigidity constraint is not binding for this income shock. In panel (c) we consider a higher degree of wage rigidity, in which case we see a decline in b^- and an increase in the default region.[27] Notice that here we also see a reduction in $\tilde{V}_D$ because the wage rigidity is also binding under default.

Panels (b) and (c) reveal that there is a bigger gap between $\tilde{V}_R^+$ and $\tilde{V}_R^-$ with a fixed exchange rate regime compared with the flexible exchange rate regime. In other words, both values drop, but $\tilde{V}_R^-$ is reduced by much more than V_R^+. Key for this result is the behavior of unemployment, as we will see below. The consequence of the increase in the gap between these two curves is the increase in vulnerability to a rollover crisis, in line with Proposition 4. In panel (b), the range of the crisis region is about 7% of GDP and goes from 27.1% to 34.4%. In panel (c), the crisis region increases to more than 12 percentage points of GDP and represents more than a third of the average debt to GDP. Moreover, the economy enters a rollover crisis with a level of debt that is 14 percentage points of GDP lower than the level it takes under a flexible exchange rate regime.

[27] In this case, we have $\overline{w}^D > \overline{w}^+$ since $TB_R^{Flex,+} > 0$, in line with Proposition 3.

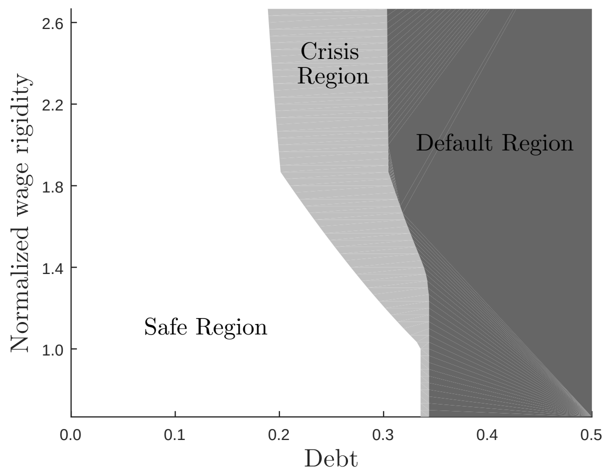

Figure 3: Safe, Crisis, and Default Regions under Different Wage Rigidities

Crisis regions for range of $\overline{w}$ and y^T. So far we have illustrated how the exchange rate regime shapes the crisis region for two values of $\overline{w}$. In Figure 3 we show how the safe, crisis, and default regions change for a whole range of $\overline{w}$, keeping the income level the same as before. The value of $\overline{w}$ is normalized by the highest $\overline{w}$ that is consistent with no changes in the three zones.[28] In this way, a value lower than unity in Figure 3 will correspond effectively to the flexible exchange rate regime. As soon as $\overline{w}$ rises above one, given the normalization, wage rigidity becomes binding and the safe region contracts. For low values of wage rigidity, the intersection between $\tilde{V}_D$ and $\tilde{V}_R^+$ remains unaffected, and hence the crisis region expands at the expense of the safe region without changes in the default region. Once $\overline{w}$ reaches 1.33, which is the value used in panel (b) of Figure 2, the value of default starts to increase, which in turn leads to an expansion of the default region at the expense of the crisis region.[29] However, we can see in Figure 3 that the crisis region continues to expand significantly because the safe region contracts by an amount greater than the default region expansion.

These regions were also constructed for a given level of the tradable endowment. To have a more complete picture, we show in Figure 4 the three zones in the (b, y^T) state space. For any given level of debt, the economy is in the default zone for a sufficiently low level of tradable endowment. As we increase the tradable endowment, the economy arrives in the crisis zone at some point. Finally, increasing it even further makes the economy reach the safe zone. Again, we can clearly see how vulnerability to a rollover crisis is lower in a flexible exchange rate regime compared with a fixed exchange rate regime, and this occurs for all income levels.[30]

[28]This level can be computed by first obtaining $\hat{b}^-_{y^T}$ such that $\tilde{V}_R^-(\hat{b}^-_{y^T}(\overline{w}), y^T) = \tilde{V}_D(y^T; \overline{w})$ and then finding $\overline{w}$ such that $\tilde{V}_R^+(\hat{b}^+_{y^T}(\overline{w}), y^T; \overline{w}) = \tilde{V}_D(y^T; \overline{w})$.

[29]If we were to have $TB_R^{Flex,+} < 0$, we would also have an expansion in the crisis region to the right, in line with Proposition 3.

[30]Along the horizontal y-axis with $y^T = 0.957$ for all panels of Figure 4, we recover exactly the same thresholds that separate the three regions in Figure 2. Notice also that for any income level different from $y^T = 0.957$, the default region will change in panel (b).

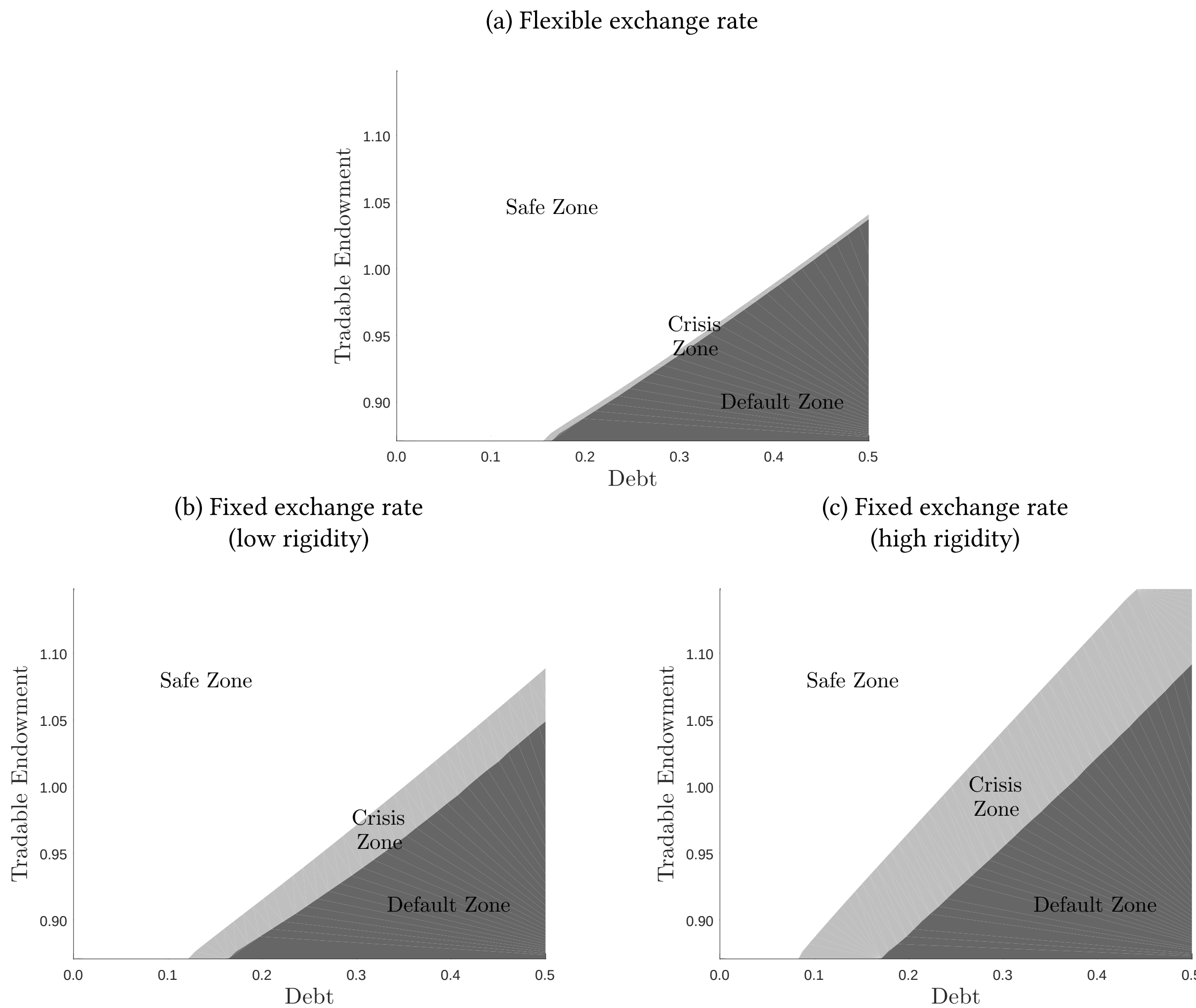

Figure 4: Safe, Crisis, and Default Zones under Different Wage Rigidities

3.4 Inspecting the Mechanism

In this section, we delve deeper into the differences in the incentives to default under a fixed and flexible exchange rate regime. We highlight the role of unemployment in generating a higher vulnerability to a rollover crisis under a fixed exchange rate regime.

On and off equilibrium unemployment. Figure 5 shows the behavior of unemployment under a fixed exchange rate for the two levels of $\overline{w}$ considered earlier. For each panel, there are three lines: u_D denotes the unemployment rate if the government chooses to default: u_R^+ is the unemployment rate if the government chooses to repay when investors are willing to roll over, and u_R^- is the unemployment rate if the government chooses to repay when investors refuse to roll over.

When the government repays, unemployment is increasing in the current amount of debt both when the government can access the debt market and when it cannot. This is because a higher debt level reduces aggregate demand, which in turn generates a decline in the price of non-tradables.[31] Under a fixed exchange rate, the wage rigidity in terms of domestic currency becomes a wage rigidity in foreign currency. Because of the downward rigidity in wages, the decline in the price of non-tradables leads to a rise in unemployment. A crucial feature of the model is that when investors refuse to roll over, unemployment starts rising strictly for lower levels of debt, and it is always higher than when investors are willing to roll over the government bonds. The reason is that when the government is forced to raise tax revenues to repay the maturing debt, this generates a more severe contraction in aggregate demand, causing larger unemployment.

The unemployment level that is realized on equilibrium depends on the initial debt level and possibly on investors' beliefs. In the safe region, the on-equilibrium unemployment rate is u_R^+, whereas in the default region, it is u_D. In both cases, it is determined. In the crisis region, unemployment rate can be either u_D or u_R^+, depending on the realization of the sunspot. It is interesting to realize that in the low rigidity case of panel (a) in Figure 5, no unemployment equilibrium arises *on* the equilibrium path. For debt levels such that $u_R^+ > 0$, the government finds it optimal to default, and given the value of $\overline{w}$, unemployment is zero in this case. For debt levels such that $u_R^- > 0$ and the government finds it optimal to repay, investors do *not* refuse to roll over on the equilibrium path. As a result, unemployment is given by u_R^+, which is zero in this case. Finally, for debt levels such that $u_R^- > 0$ and the government finds optimal to default, investors do run and the government defaults on the equilibrium path. As a result, we arrive again at $u_D = 0$. The takeaway is that what leads the government to default in a rollover crisis (and investors to run) is not the realization of unemployment per se but the desire to avoid the large unemployment that would emerge if the government were to repay while being unable to borrow.[32]

This increase in unemployment that emerges from fluctuations in labor demand from the non-tradable sector is at the heart of the mechanism to generate a larger exposure to a rollover crisis. It is useful to point out that having production in the tradable sector would not affect the differences in employment when investors lend vis-à-vis when investors refuse to lend. The level of the exchange rate would affect employment in the tradable sector, but this would be independent of investors' beliefs. The key idea is that for tradable goods, the relevant demand is the international one. On the other hand, in the non-tradable sector, the availability of domestic resources is critical in determining the domestic price of tradables and firms' labor demand.

[31] A decline in aggregate demand requires a decline in the relative price of non-tradables to clear the market for non-tradables. Since the price of tradables in terms of domestic currency is constant (because of the fixed exchange rate and the assumption of zero foreign inflation), it is the price of non-tradables that falls (in both domestic and foreign currency).

[32] In panel (b), because the wage rigidity is tighter, we do observe unemployment on the equilibrium path. For the level of rigidity considered, there is positive unemployment when the government defaults, either in the crisis region or in the default region, but not in the safe region. An even larger degree of rigidity would lead to unemployment in the safe region too. In these cases, the level of $u_R^- > 0$ would be even larger, but this level of unemployment would not be observed in equilibrium.

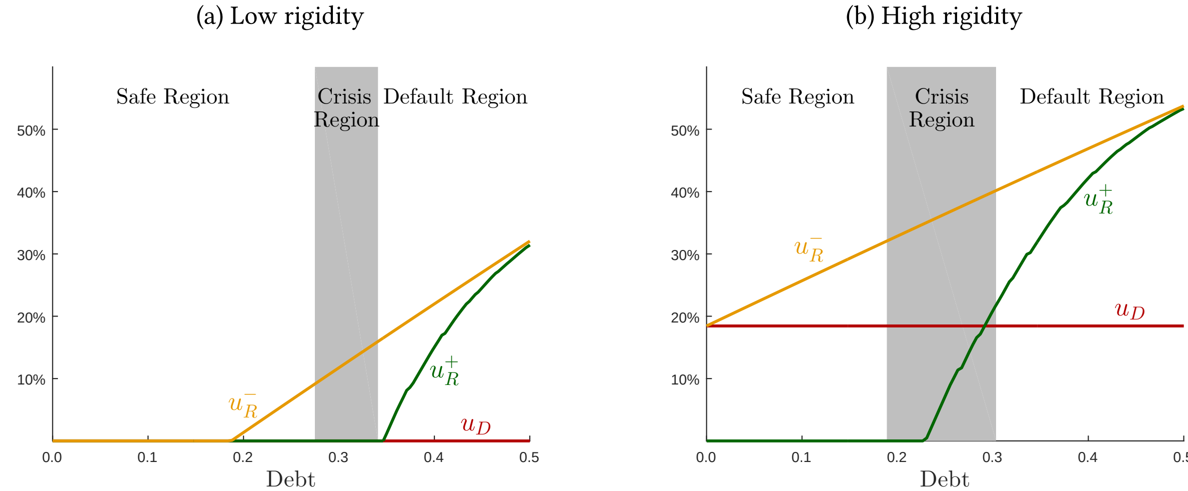

Figure 5: Unemployment Rates with Fixed Exchange Rate Regime

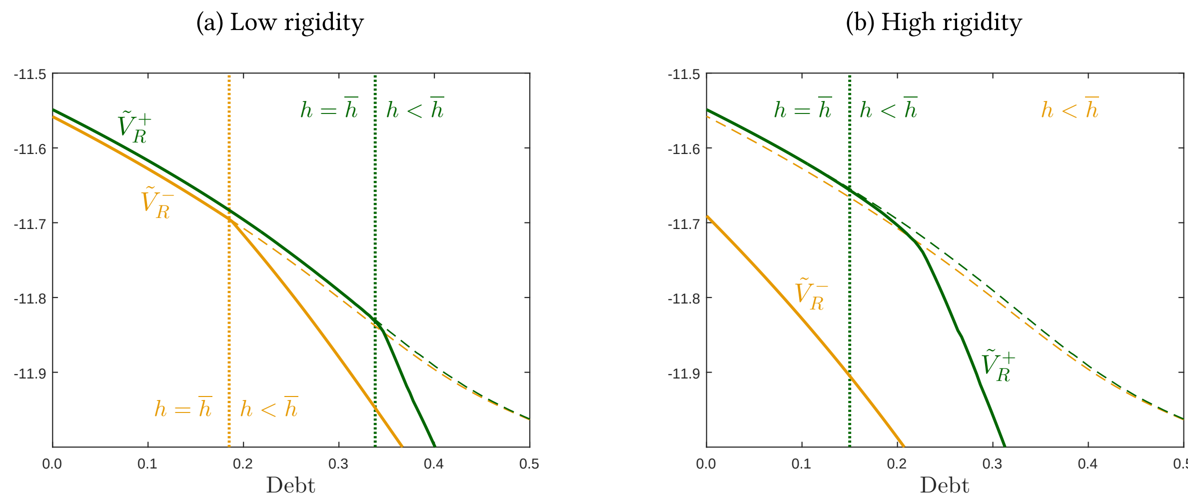

Figure 6: Values of Repayment in Flexible vs. Fixed Exchange Rate Regime.

Notes: Dashed lines correspond to the flexible exchange rate regime and straight lines correspond to the fixed exchange rate regime. Green (dark) lines correspond to $\tilde{V}_R^+$, and yellow (light) lines correspond to $\tilde{V}_R^-$.

Unemployment and value functions. These differences in unemployment that arise depending on whether investors are willing to lend or not translate into differences in the value functions. Figure 6 shows how $\tilde{V}_R^+$ and $\tilde{V}_R^-$ change when we introduce rigidities. These are the same value functions from Figure 2, but now we put them together to better appreciate the differences and mark the thresholds at which unemployment emerges. Consider first the repayment value functions under a flexible exchange rate regime, which are denoted with dashed lines. We can see that the gap between the two is very small: there is zero unemployment regardless of whether investors lend or not. Moreover, the gap is relatively wider at very low levels of debt (because the government wants to issue more debt). However, at those levels of debt, the government has a value of repayment that is far larger than the value of default, and hence this gap between $\tilde{V}^+$ and $\tilde{V}^-$ is innocuous. As debt increases and we approach the value of default, the gap becomes smaller (because the government does not want to issue as much debt). The outcome is a narrow crisis region.

Figure 6 shows that when the exchange rate is fixed, all value functions drop relative to the flexible case, and there is a strict decline at the debt threshold in which unemployment emerges. Most importantly, however, is that $\tilde{V}^-$ is reduced by more than $\tilde{V}^+$, and hence there is a bigger gap between the two compared with the flexible exchange rate. This arises because of the substantially different unemployment levels that arise depending on whether investors lend or not. Moreover, the widening of the gap occurs precisely at debt levels at which lenders' beliefs matter for the repayment decision. The outcome is a wide crisis region.

3.5 Extensions, Generalizations and a Simple Example

The main theoretical results that we have presented so far can be extended and generalized in a relatively straightforward manner. While the full details are presented in the Online Addendum, we discuss here the main elements of each of these extensions.[33] In addition, we also present in Section 3.5.1 a simple example with deterministic income.

The same results can be obtained in a model with sticky prices instead of sticky wages. Consider a situation in which investors become pessimistic and the government raises taxes to service the maturing debt. With sticky wages, we showed that the resulting decline in aggregate demand leads to a decline in the price of non-tradables, which generates a decline in employment and makes repayment more costly. With sticky prices, firms respond to the cut in demand by reducing production, which in turn generates lower labor demand, lower wages, and lower employment. In both cases, repayment becomes very costly when investors turn pessimistic, and this precipitates a rollover crisis. Appendix A shows how all the propositions extend to the case of price stickiness and also shows that with linear production functions, results are identical.

The same results can also be obtained when there are costs from exchange rate fluctuations, so that a fixed exchange rate regime is not necessarily a dominated regime. In our baseline model, a higher exchange rate unambiguously increase the utility flow at any particular state given that it reduces unemployment and does not involve any cost (see Proposition 1). We consider two specifications of costs from exchange rate fluctuations (see Appendix B.1). In one specification, we consider a quadratic cost of departing from a target exchange rate $\bar{e}$. These costs could come from redistributive effects or monetary distortions, but we prefer not to take a stance on the source of these costs. In this situation, the government will tradeoff the benefits from higher employment with the costs of exchange rate fluctuations. The higher the costs are from depreciating, the more similar the economies will be under flexible and fixed exchange rate regimes. Regardless of how large the costs are, however, an

[33]We leave the case with an arbitrary maturity structure and elastic labor supply entirely for the appendix.

economy under flexible exchange rate regime displays a smaller crisis zone, and all our theoretical results continue to hold.

In the second specification, we consider a version of the model in which the costs from exchange rate fluctuations arise from the expectation of future depreciations, rather than from the current one. Lacking commitment to an exchange rate policy, the government always finds it optimal to depreciate the currency enough to deliver full employment, generating an inflationary bias. An economy that fixes the exchange rate or enters a monetary union is able to avoid this inflationary bias, and doing so could be desirable if these costs are sufficiently large. Still, the economy under a flexible exchange rate will feature a lower exposure to rollover crises, as in our baseline model. These two specifications are useful because they highlight that our main result is not altered by the fact that our baseline model abstracts from modeling the reasons why the government implements a fixed exchange rate regime.

We also consider an inflation-targeting regime. In particular, we focus on a regime in which the government keeps constant the price of the composite consumption good. When investors turn pessimistic, the government can depreciate the currency to alleviate unemployment, but there is a limit given by achieving the inflation target. A negative shock to aggregate demand in this economy leads to deflationary pressures and a reduction in the price of non-tradables. As the government depreciates the domestic currency, real wages fall, which stimulates labor demand. The price of tradable goods rises at the same time, thereby placing a limit on the government's ability to stabilize employment while fulfilling the inflation target. Appendix C shows how the same propositions as in our baseline model hold under inflation targeting. The general lesson here is that the presence of monetary policy constraints in the form of a fixed exchange rate or an inflation targeting regime, can make an economy more vulnerable to rollover crises.

We also consider a model with debt denominated in domestic currency. In the baseline model, the only difference between a flexible exchange rate regime and a fixed exchange rate regime is that in the former, the government can use monetary policy to stabilize macroeconomic fluctuations. We made this assumption partly to better highlight the new channel regarding the role of monetary policy in reducing the vulnerability to rollover crises. In principle, however, an economy that is outside a monetary union can also issue debt in domestic currency, which opens the possibility to inflating away the debt. We argue that the main insight of the paper remains when we allow for this possibility. In Appendix F, we consider a version of the model in which a nominal depreciation allows for simultaneously affecting the real value of the debt as well as the level of employment.In this economy, depreciating the currency allows for an increase in the amount of consumption by effectively diluting the real value of foreign lenders' debt. Importantly, this allows for an increase in aggregate demand and, through the mechanism highlighted above, also reduces unemployment and makes repayment less costly in the event of investors's panic. Therefore, the possibility of depreciating the currency, again reduces the investors' incentives to run, thereby reducing the exposure to a rollover crisis.

Finally, another point worth considering is whether other differences between a fixed and a flexible exchange rate regime could alter our conclusions. Specifically, some observers have argued that defaulting while being in a monetary union might be more costly, perhaps because punishments are easier to enforce. Interestingly, an increase in the default cost has the direct implication of always reducing the fundamental default region, but the crisis region may expand. Key for the results is that crisis region depend mainly on the gap between $\tilde{V}_R^+$ and $\tilde{V}_R^-$. Specifically, the change in the size of the crisis region upon a change in the value of default can be calculated, via the implicit function theorem, as

$$\frac{\partial\left(\bar{b}^+(\overline{w}) - \bar{b}^-(\overline{w})\right)}{\partial \tilde{V}_D} = \frac{1}{\frac{\partial \tilde{V}_R^+}{\partial b}} - \frac{1}{\frac{\partial \tilde{V}_R^-}{\partial b}}. \tag{28}$$

Notice that the derivatives of these value functions are negative. If the absolute value of the derivative of $\tilde{V}_R^-$ is higher than the one of $\tilde{V}_R^+$ evaluated respectively at $\bar{b}^-$ and $\bar{b}^+$, the distance between $\bar{b}^-$ and $\bar{b}^+$ goes up. In other words, if $\tilde{V}_R^-$ varies relatively more than $\tilde{V}_R^+$ with the level of debt, the crisis region expands when default costs increase.[34]

Beyond these specific extensions, our main result is quite general in the sense that it hinges on only two key robust elements: (i) A sudden panic by investors triggers capital outflows, if the government chooses to repay; (ii) The costs of sudden capital outflows are more severe under a fixed exchange rate (because the government is unable to mitigate the contraction in aggregate demand). The combination of these two elements implies that the government is more tempted to default during a panic under a fixed exchange rate regime, and, hence, investors are more prone to run.

3.5.1 Simple Example

In this section, we consider a simple version of the model in which (i) the tradable endowment is deterministic $y_t^T = y^T$, (ii) $\beta(1+r) = 1$, (iii) the exclusion after default is permanent, and (iv) debt is one period $\delta = 1$. With these assumptions, the dynamics are simplified significantly, following Cole and Kehoe (2000). In particular, when the economy is in the crisis zone, the government is motivated to reduce the debt until it exits the crisis zone. The key additional insight that we are able to show is that a government under a fixed exchange rate regime has incentives to exit the crisis zone more *slowly*. In other words, a country within a monetary union is more vulnerable to a rollover crisis not only is because the crisis zone is larger but also because it exits more slowly.

In contrast with the theoretical analysis presented above, we now allow for a permanent change in wage rigidity. That is, rather than changing only the current wage rigidity keeping future rigidities constant, we change $\overline{w}$ over all periods. Following the same steps as above, we proceed to analyze how the default thresholds $\bar{b}^+$ and $\bar{b}^-$ change with the rigidity. Thanks to the simplifying assumptions in this example, characterizing the thresholds is now straightforward. In particular, given that there

[34] This point can be seen by a simple inspection of the introduction of a parallel shift in the value of default in Figure 6.

is permanent exclusion, the value of default is entirely determined by exogenous parameters:

$$V_D(\overline{w}) = \frac{1}{1-\beta}[u\left(y^T, F(\mathcal{H}\left(y^T, \overline{w}\right))\right) - \kappa]$$

with $\mathcal{H}$ defined as in Lemma 2. The value of V_R^- also simplifies because once the government pays the entire stock of debt ($\delta = 1$), from tomorrow onward, it consumes a constant amount given by the annuity value of the income, a result that follows from the fact that $\beta R = 1$ and income being deterministic.

$$V_R^-(b;\overline{w}) = u\left(y^T - b, F(\mathcal{H}\left(y^T - b\right), \overline{w})\right) + \frac{\beta}{1-\beta}u(y^T, F(\mathcal{H}\left(y^T, \overline{w}\right))).$$

The value of V_R^+ depends on whether the government is in the safe zone or the crisis zone. If the economy is in the safe zone, the government simply consumes every period the annuity value of the income minus the interest payments. On the other hand, if the economy is not in the safe zone, the optimal policy for the government is to gradually reduce the debt until it is able to exit the crisis zone. The value function can be written as

$$V_R^+(b;\overline{w}) = \begin{cases} \frac{1}{1-\beta}u\left(y^T - \frac{r}{1+r}b, F(\mathcal{H}\left(y^T - \frac{r}{1+r}b, \overline{w}\right)\right), & \forall b \leq \bar{b}^+(\overline{w}) \\ \max_{b'} u\left(y^T - b + qb', F\left(\mathcal{H}\left(y^T - b + qb', \overline{w}\right)\right)\right) & \\ \qquad +\beta\left[\pi V_D(\overline{w}) + (1-\pi)V_R^+(b';\overline{w})\right] & \forall b > \bar{b}^+(\overline{w}) \end{cases}$$

The government finds it optimal to save its way out of the crisis zone because it wants to avoid the default costs that carry the realization of a bad sunspot while in the crisis zone. A key question is how much the government should save, or equivalently, how fast the government should exit, depending on the exchange rate regime. We address this question next.

Figure 7 compares the incentives to save for the fixed and flexible exchange rate economies. Panels (a) and (b) show the policy functions for debt: the top panels present the flexible and fixed case, respectively, and the dashed line denotes the 45^o line. The crisis zone is again larger for the fixed exchange rate economy although now both economies appear larger because debt has one-period maturity in this simple example, which exacerbates the liquidity problems.[35] In particular, following the same logic as before, we can see that $\bar{b}^-$ moves to the left, and hence the government is exposed to a rollover crisis for smaller levels of debt. In addition, the default region now always expands, for two reasons. First, in this simple example, that the government reduces the path of debt while in the crisis zone implies that it is running a trade surplus, and, in line with Proposition 3, tighter wage

[35] The advantage of modelling one-period debt is that the government moves immediately to the safe zone after repaying the debt when it is unable to roll over the debt. This result simplifies the computation of the debt thresholds. Regardless of the maturity, however, the results we emphasize regarding the difference between the flexible and fixed regimes would be essentially the same.

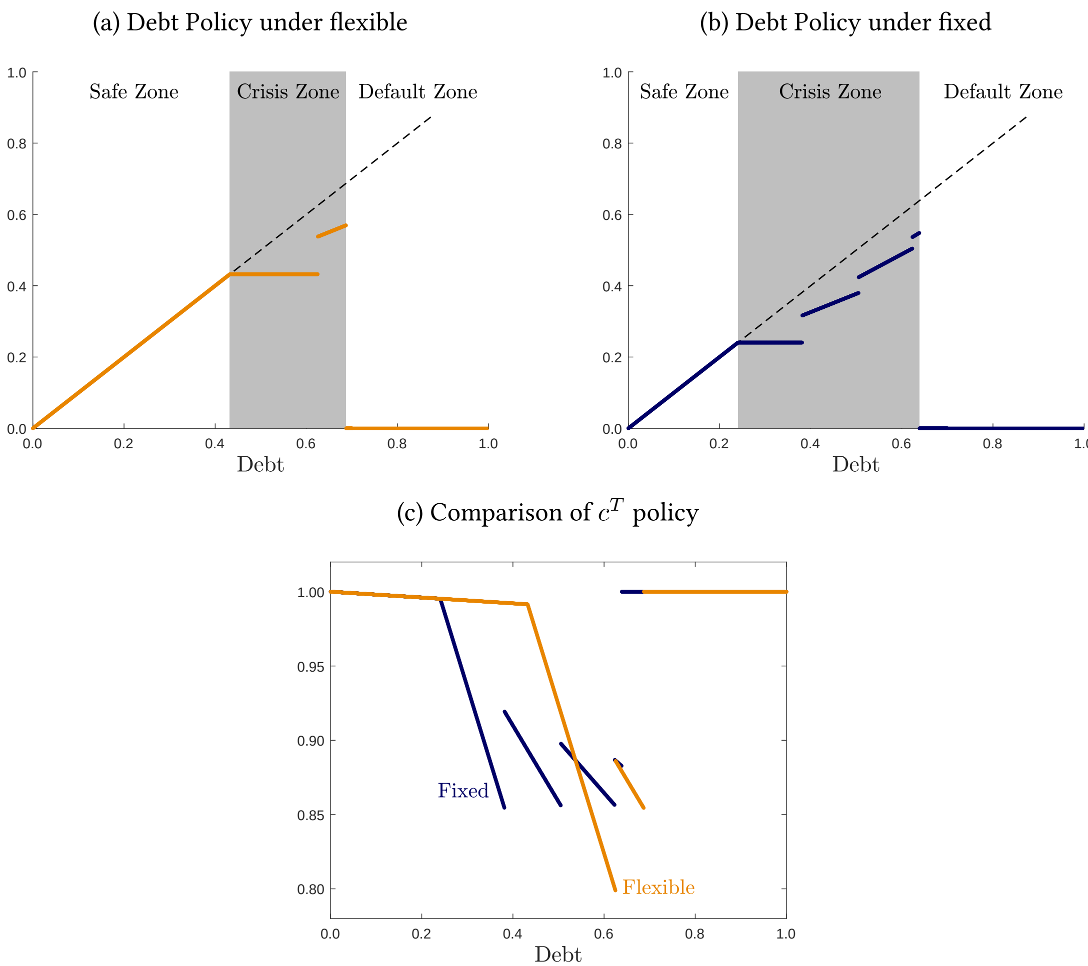

Figure 7: Policy Functions for Debt and Consumption

Note: Parameter values follow the calibration set in the following section, with the exception of the maturity parameter $\delta = 1$, the discount factor $\beta = \frac{1}{1+r}$, permanent autarky default penalty $\psi = 0$, and the utility penalty for defaulting is a constant level $\kappa = 0.150$.

rigidity has more of an effect on the value of repayment than default. Second, that the change in wage rigidity is permanent implies that continuation values under repayment fall. Even if wage rigidity is small enough so that unemployment is triggered only when the government is shut-off from credit markets, that $\bar{b}^{-}$ falls implies that the government needs to save more to exit the crisis zone, thereby reducing the value from repayment.

Government savings differ markedly between the safe zone and the crisis zone. When the economy is in the safe zone, debt is kept constant. Because $\beta(1+r) = 1$, the government finds it optimal to keep debt and consumption constant over time, as long as a rollover crisis is not possible. When the economy is in the crisis zone, the government reduces b', as illustrated by the policies being below the 45^o line. Essentially, the government chooses a constant consumption profile while in the crisis zone.[36] The level of consumption it chooses guides the speed at which it exits the crisis zone. The debt policy functions are therefore piecewise flat with discontinuities at the points where the government decides to take one further period to exit the crisis zone. The further away from the crisis zone, the longer it takes to exit.

A comparison of panels (a) and (b) of Figure 7 shows that the government policy function features more jumps under a fixed exchange rate regime, which implies, importantly, that the government exits the crisis zone more slowly. One could argue that this is a natural consequence because a larger crisis zone implies that the government should take longer to exit, particularly if it were to follow the same savings policy as in the flexible exchange rate regime. However, this is only one part of the story. Under a fixed exchange rate regime, the government realizes that if it were to save more today to speed up the exit from the crisis zone, it would generate a recession today. This force pushes the government to reduce borrowing at a slower pace instead of saving more to speed up the exit. Panel (c) of Figure 7 shows that for high levels of debt, the government actually consumes more under a fixed exchange rate regime. That is, when the government is deep in the crisis zone, it saves *less* under a fixed exchange rate regime. In an attempt to avoid a current recession, the government gambles for redemption, hoping that investors remain optimistic and a default does not occur.

In Figure 8, we further examine how the speed at which the government exits the crisis zone changes with wage rigidity. Fixing the initial amount of debt as the highest level of debt in the safe zone under a flexible exchange rate regime, we show how the number of periods it takes to exit the crisis zone increases with wage rigidity. One can see that the tighter wage rigidity is, the longer it takes to exit.

To conclude, this simple example shows that a government within a monetary union is more vulnerable to a rollover crisis, both because the crisis zone is larger and because it slows down the increase in savings and the exit to the safe zone. In the next section, we will generate simulated data from the general model of Section 2 to investigate how often the government defaults because of rollover crises when the government is in a monetary union and how does this compare when the economy is outside a monetary union.

[36]The reason for this result is that throughout the crisis zone, the probability of default is fixed at π, and hence a local change in debt does not affect this probability. Right after jumping to the safe zone, when the probability of default drops to zero, the government increases consumption and again keeps it constant.

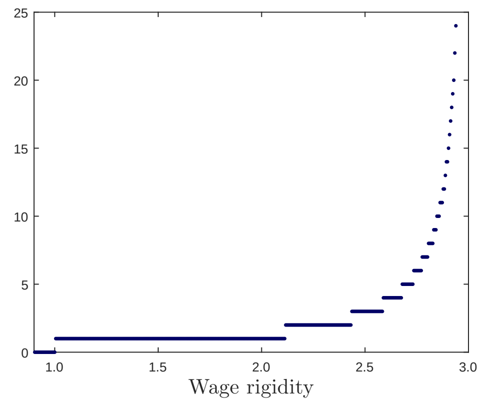

Figure 8: Time to Safety

Note: The figure depicts the number of periods it takes for the government to exit the crisis zone if no bad sunspot is triggered. Wage rigidity is normalized by the level of $\overline{w}$ at which the solution corresponds to the flexible exchange rate regime. The initial debt level corresponds to the largest debt level in the safe zone in the flexible exchange rate regime.

4 Quantitative Analysis

This section presents the quantitative analysis and has three goals. First, we conduct model simulations to quantitatively assess how often an economy is exposed to a rollover crises and examine how this exposure depends on the exchange rate regime. Second, we perform welfare computations to determine how significant the costs from monetary independence are, and, we additionally assess the potential gains from a lender of last resort depending on the exchange rate regime. Third, we perform a counterfactual experiment applied to the recent crisis in Spain to shed light on whether the crisis was triggered by fundamentals or self-fulfilling beliefs.

4.1 Calibration

We calibrate the model at an annual frequency using Spain as a case study.[37]

Functional forms. We use a CRRA utility function,

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \text{with} \quad \sigma > 0.$$

We parameterize the default utility cost as follows:

$$\kappa(y^T) = \max\left\{0, \kappa_0 + \kappa_1 \ln\left(y^T\right)\right\}.$$

[37]The model is solved numerically using value function iteration with interpolation. Linear interpolation is used for the endowment and debt levels. We use 25 grid-points for the tradable endowment grid and 99 grid-points for debt. To compute expectations, we use 105 quadrature points for the endowment realizations.

As shown in Arellano (2008) and Chatterjee and Eyigungor (2012), a non-linear specification of the cost of default is important to allow the model to match the levels of debt and spreads in the data. In particular, we follow Bianchi et al. (2018) in specifying this default cost function in terms of utility.

The tradable endowment process follows a lognormal AR(1) process,

$$\ln(y_t^T) = \rho \ln(y_{t-1}^T) + \sigma_y \varepsilon_t,$$

where $|\rho| < 1$ and the shock ε_t is $i.i.d.$ and normally distributed, $\varepsilon \sim N(0, 1)$. To estimate the tradable endowment stochastic process, we use the value-added series in the manufacturing and agricultural sectors in Spain. After we log-quadratically detrend the series, we estimate a persistence parameter of $\rho = 0.777$ and a standard deviation of $\sigma_y = 2.9\%$.

Model Parameters. Table 1 shows all the baseline calibration values for the parameters of the model. A first subset of parameters is specified directly. These are parameters that can be calibrated straight from the data or are relatively standard in the literature. We then choose a second subset of parameters to match key moments in the data under two different regimes: flexible exchange rates and fixed exchange rates.

Table 1: Parameter Values

Parameter	Value		Description
$\overline{h}$	1.000		Normalization
σ	2.000		Standard risk aversion
ω	0.197		Share of tradables
μ	1.000		Unitary elasticity of substitution between T-NT
ρ	0.777		Output persistence
σ_y	0.029		Standard deviation of tradable output shock
α	0.750		Labor share in non-tradable sector
r	0.020		German 6-year government bond yield
δ	0.141		Spanish bond maturity 6 years
ψ	0.240		Reentry to financial markets probability
π	0.030		Sunspot probability
Calibration	Flexible	Fixed	Target
β	0.914	0.908	Average external debt-GDP ratio 29.05%
κ_0	0.101	0.315	Average spread 2.01%
κ_1	0.759	3.273	Standard deviation interest rate spread 1.42%
$\overline{w}$	-	2.493	Δ unemployment rate 2.00%

We start with the first subset of parameters. First, we specify the parameters governing preferences and technology, which will take standard values in the literature. The coefficient of risk aversion will be set to $\sigma = 2$. Meanwhile, the elasticity of substitution between tradable and non-tradable goods is set to $\frac{1}{1+\mu} = 0.5$, which is in the range of empirical estimates. The share of tradable goods in the consumption aggregator is set to $\omega = 0.197$, so it matches the ratio between tradable output and total output, which averages around 20% for Spain in the period considered.[38] Regarding the labor share in non-tradable production, we set $\alpha = 0.75$, an estimate from Uribe (1997) for the non-tradable sector. Last, we normalize the inelastic labor supply of households to $\overline{h} = 1$.

Next, we set the parameters from financial markets. We set the international risk-free interest rate to $r = 2\%$, which is the average annual gross yield on German 6-year government bonds over the period 2000 to 2015. We calculate a maturity parameter of $\delta = 0.141$ to reproduce an average bond duration of 6 years, in line with Spanish data.[39] We set the reentry to financial markets probability after default to $\psi = 0.24$ to capture an average autarky spell of 4 years, in line with Gelos, Sahay, and Sandleris (2011). Finally, we need to set the sunspot probability, which is a more difficult parameter to calibrate. In the literature, the probability of drawing a bad sunspot is usually set to a relatively low value (e.g., Chatterjee and Eyigungor, 2012, study a range between [0,0.10]). Our baseline value is 3%, but we examine a wide range as well.

For the second subset of parameters $\{\beta, \kappa_0, \kappa_1, \overline{w}\}$, we will set these parameters so that the moments in the model match the counterparts in the data. Since we have two different exchange rate regimes, we have two sets of parameters. The difference in the two calibrations is that $\overline{w}$ is set to zero for the flexible exchange rate regime, whereas this value has to be calibrated for the fixed exchange rate regime. In particular, we calibrate $\overline{w}$ in the fixed exchange rate regime to be consistent with the increase in unemployment during episodes of high sovereign spreads. In the data for Spain, the increase in unemployment relative to the HP-filtered trend was 2% in 2011, the year prior to the EU and ECB's intervention.[40] With a value of $\overline{w} = 2.493$ and given the rest of the calibrated parameters, the average increase in unemployment in the year prior to default is 2% in the model, matching the empirical counterpart.[41]

For both regimes, we calibrate the parameters β, κ_0, and κ_1 to match three moments from the data, and we follow Hatchondo, Martinez, and Sosa-Padilla (2016) in considering the moments in the

[38] In a non-stochastic version of the model with a mean value of debt $\bar{b}$ and average employment $\bar{h}$, the value of ω can be pinned down from $\frac{y^T}{y^T + \frac{1-\omega}{\omega}\left(\frac{y^T + r\bar{b}}{F(\bar{h})}\right)^{\mu+1}} = 20\%$.

[39] The Macaulay duration of a bond with price q and our coupon structure is given by

$$D = \sum_{t=1}^{\infty} t \frac{\delta}{q} \left(\frac{1-\delta}{1+i_b} \right)^t = \frac{1+i_b}{\delta + i_b},$$

where the constant per-period yield i_b is determined by $q = \sum_{t=1}^{\infty} \delta (\frac{1-\delta}{1+i_b})^t$.

[40] We use a smoothing parameter of 100 for the HP filtering. If we use a log-quadratic filter, we obtain a value closer to 3%.

[41] As we mentioned in footnote 11, governments have available to them fiscal instruments such as payroll subsidies to stimulate employment. In terms of our model, this would imply that the wage rigidity would be governed by $\overline{w}$ net of these subsidies. Our approach to calibrating $\overline{w}$, therefore, incorporates these effects implicitly.

years following 2008 to concentrate on the period around the crisis. The three moments targeted are the average debt-GDP ratio, and the average and standard deviation of spreads. For the average debt-GDP ratio, we target an average external debt of 29%. For the average and the volatility of spreads, we target 2.0 and 1.4, respectively.[42] The resulting values for these parameters appear in Table 1.

4.2 Simulation Results: Exposure to Rollover Crises

We conduct simulations to investigate how the exchange rate regime determines which type of default—fundamental or rollover crisis is more likely.

Degree of wage rigidity. We start from the flexible exchange rate economy. In this economy, out of 100 default episodes, the share of defaults due to a rollover crisis is only 0.92. In line with this result, only 0.77% of the time, the economy is in the crisis zone and therefore vulnerable to a rollover crisis. To examine how the degree of wage rigidity matters for the exposure to a rollover crisis, we vary the wage rigidity parameter $\overline{w}$ while keeping the same calibrated parameters for the flexible exchange rate economy.[43] In Figure 9 (panel (a)), we present the fraction of defaults that are explained by rollover crises as a function of $\overline{w}$. We can see here that the tighter wage rigidity is, the larger the fraction of defaults that are explained by non-fundamentals. In line with this result, panel (b) of Figure 9 also shows that the fraction of time the economy spends in the crisis region becomes larger with the degree of rigidity.

It is worth highlighting that in these simulations, we also obtain that the average debt level falls with $\overline{w}$ (panel (c)). Two reasons explain this. First, the government faces borrowing terms that are more adverse, given that incentives to default in the future are higher. Second, the government also attempts to stay further away from the crisis zone by reducing debt. Despite this attempt, the fact that the crisis region expands significantly implies that the government ends up being more heavily exposed to a rollover crisis.

When we vary the degree of wage rigidity, the long-run moments to which we calibrate the flexible exchange rate economy also change. In particular, as mentioned above, the economy under a fixed exchange rate ends up borrowing less than the economy under flexible exchange rate. To take these changes into account, we complement the results by recalibrating the economy to hit the same targets: mean debt, mean spreads, and volatility of spreads, while calibrating the value of $\overline{w}$ to match the increase in unemployment, as described in Section 4.1. We present these results in the first two

[42]The debt level in the model is computed as the present value of future payment obligations discounted at the risk-free rate r. Given our coupon structure, we thus have that the debt level is $\frac{\delta}{1-(1-\delta)/(1+r)} b_t$.

[43]Different from our analysis in the comparative statics exercise of Section 3, the change in $\overline{w}$ is now permanent, and therefore the bond price schedule is affected.

columns of Table 2, which show that the differences in vulnerability remain significant.[44]

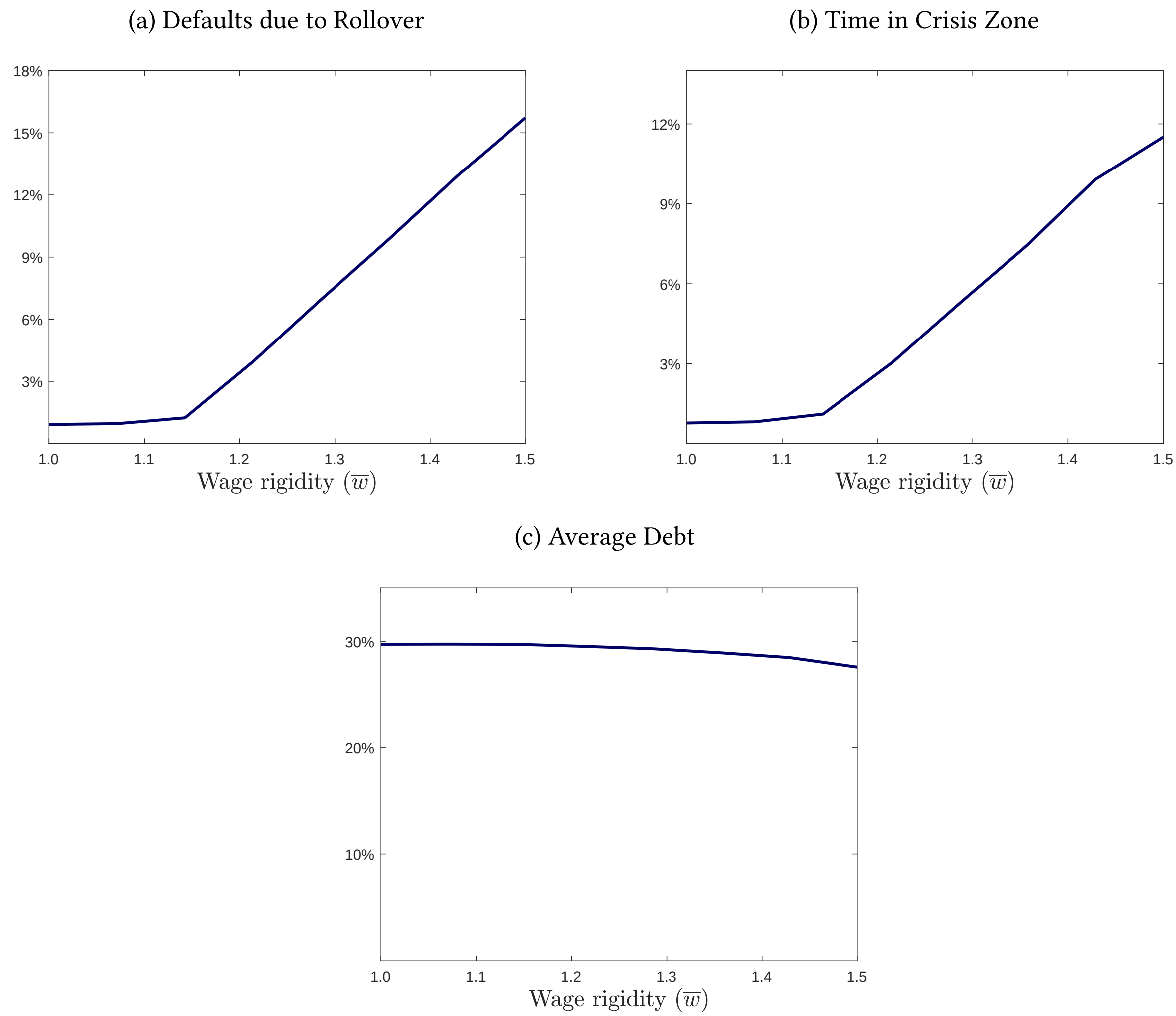

Figure 9: Role of Wage Rigidity

Sunspot Probability. The fraction of defaults that are the outcome of a rollover crisis depends on two factors. One factor is the probability of a bad sunspot (i.e., the probability of selecting the bad equilibrium whenever the economy is in the crisis zone). The second factor is the probability of ending up in the crisis zone in the first place, which is an endogenous outcome that depends critically on borrowing decisions and on the monetary policy regime. Next, we increase the probability of selecting the bad equilibrium while keeping the rest of the parameter values for fixed and flexible exchange rate regimes at their respective baseline values.

[44] Because of the highly non-linear nature of the model, we have been unable to perfectly match the moments that we target. In particular, spreads in the flexible (fixed) exchange rate regime are higher (lower) than the target. See Aguiar et al. (2016) for a discussion of some of the challenges in the calibration of sovereign default models.

Table 2: Sensitivity to Sunspot Probability

Sunspot probability (percentage %)	$\pi = 3\%$ Flexible	Fixed	$\pi = 10\%$ Flexible	Fixed	$\pi = 20\%$ Flexible	Fixed
Average spread	2.46	1.43	2.45	1.47	2.46	1.53
Average debt-income	29.73	31.33	29.58	29.29	29.37	28.53
Spread volatility	1.33	1.60	1.30	1.72	1.31	1.75
Unemployment increase	0.00	1.83	0.00	1.80	0.00	1.35
Fraction of time in crisis region	0.77	2.59	0.68	1.93	0.58	1.41
Fraction of defaults due to rollover crisis	0.92	6.53	3.70	11.80	6.20	19.80

Notes: All parameter values correspond to the benchmark calibrations for fixed and flexible exchange rate regimes. The benchmark calibration uses $\pi = 3\%$.

Table 2 shows how increasing the likelihood of a bad sunspot increases the fraction of defaults due to a rollover crisis for the two economies, and particularly for the economy under a fixed exchange rate regime. Specifically, when the probability of a bad sunspot is 20%, up from 3% in the baseline, about one-fifth of all defaults are for non-fundamental reasons. Moreover, one can see that the fraction of time spent in the crisis region decreases as the government reduces its exposure, but this duration is not enough to offset the higher likelihood of a bad sunspot.

4.3 Welfare Consequences

Here we tackle two important welfare considerations: (i) What is the welfare cost of the lack of monetary independence? (ii) What are the welfare costs of rollover crises?

Our first result is that the possibility of a rollover crisis substantially increases the welfare costs of giving up monetary independence. We examine, for all initial states, how much household are willing to give up of the composite consumption good to move to a flexible exchange rate for *one period.* Two steps are involved. The first step is to compute the value to the government of being able to vary the exchange rate today. We denote this value by $V_{0,Flex}(b, s)$. In a state in which the government is participating in financial markets, we have

$$V_{0,Flex}(b, s) = \max[V^D_{0,Flex}(s), V^R_{0,Flex}(b, s)] \tag{29}$$

where

$$\begin{aligned} V^R_{0,Flex}(b, \mathbf{s}) &= \max_{c^T, b'} u(c^T, \bar{h}^\alpha) + \beta \mathbb{E} V(b', s) \\ c + b &= y^T + q(b', b, \mathbf{s})(b' - (1 - \delta)b) \end{aligned} \tag{30}$$

and

$$V^D_{0,Flex}(s) = u(y^T, F(\bar{h})) + \beta \left[\psi V(0, s') + (1-\psi) V^D(y^{T\prime}\right]) \tag{31}$$

The second step is to compare the value $V_{0,Flex}(b, \mathrm{s})$ with the value to the government in a fixed exchange rate regime from the Markov equilibrium. Using these two values, we compute the welfare gain $\theta_0^{Flex}(b, \mathrm{s})$, as given by

$$\begin{aligned} V_{0,Flex}(b, \mathrm{s}) = (1 - \hat{d}(b, \mathrm{s}))[(1 + \theta_0^{flex}(b, \mathrm{s}))^{1-\sigma} u(\hat{c}^T(b, \mathrm{s}), \hat{c}^N(b, \mathrm{s})) + \beta \mathbb{E} V(\hat{b}'(b, \mathrm{s}), \mathrm{s})] + \\ \hat{d}(b, \mathrm{s})[(1 + \theta_0^{Flex}(b, \mathrm{s}))^{1-\sigma} u(\hat{c}^T(b, \mathrm{s}), \hat{c}^N(b, \mathrm{s})) + \beta \left[\psi V(0, \mathrm{s}') + (1-\psi) V^D(y^{T\prime})\right] \end{aligned} \tag{32}$$

where $\hat{d}, \hat{c}^T, \hat{c}^N, \hat{b}'$ correspond to the optimal policies from the Markov equilibrium under fixed exchange rate that solve (16)-(18). When the government under a fixed exchange rate finds it optimal to repay, the welfare gain from having a flexible exchange rate can be obtained by equating $V_0(b, \mathrm{s})$ with the first line in eq. (32). The second line in eq. (32) allows for the same computation when the government finds it optimal to default.

Figure 10 shows the welfare cost of belonging to a monetary union in the current period for a range values of debt and for a given endowment shock. For reference, we show the safe region, crisis region, and default region for the economy under a fixed exchange rate, and the welfare gains are presented for the good sunspot, $\zeta = 0$, and the bad sunspot, $\zeta = 1$. Starting from the left, we see that if debt is very low, there is no unemployment and no cost from having a fixed exchange rate.[45] As debt approaches 0.2, unemployment emerges, and there is a positive welfare cost. Under the good sunspot, the welfare cost increases continuously until debt reaches about 0.3, at which point the government chooses to default under a fixed exchange rate and this helps to mitigate the effects from the wage rigidities. Here, the welfare costs from a fixed exchange rate become decreasing in the level of debt because the value function is independent of debt under a fixed exchange rate but it is still decreasing under flexible exchange rate. Importantly, while the economy under a fixed exchange rate features no unemployment, there is still a welfare cost from a fixed exchange rate because it is precisely the lack of flexibility that triggers the government default, and the economy suffers from the default costs. For debt levels higher than 0.35, the government under a flexible exchange rate also chooses to default and there are no costs from rigidity. Under the bad sunspot, the welfare costs increase discretely once the debt enters the crisis zone. This occurs because the lack of exchange rate flexibility prompts the government to default if investors refuse to rollover the government bonds.

The next welfare consideration that we tackle is on the welfare cost of rollover crises. We interpret these costs as the potential gains from having a lender of last resort from the perspective of the small open economy. As is well understood, a third party with deep pockets can eliminate the coordination

[45] If we were to consider the permanent costs from belonging to a monetary union, there would be strictly positive costs in this range of debt arising from the discounted future costs. Moreover, the higher exposure to future rollover crisis would have negative effects on spreads today.

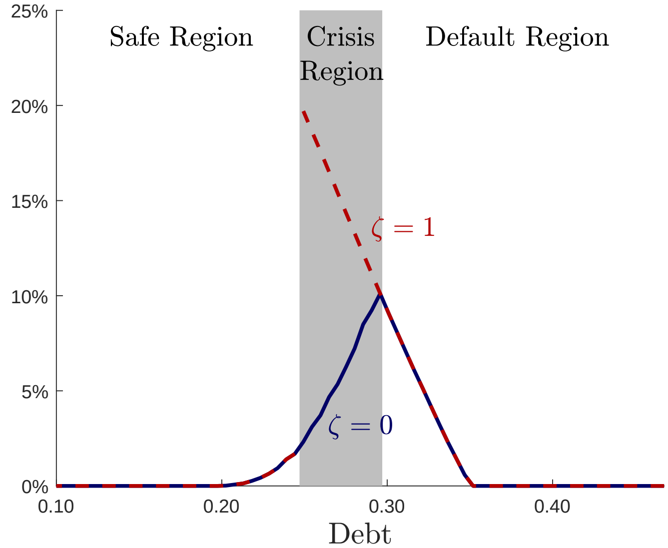

Figure 10: Welfare gains from one-period flexible exchange rate

The regions highlighted in the future correspond to the economy with fixed exchange rate.

problem behind a rollover crisis. The basic argument is that by purchasing a sufficiently large amount of government bonds, in either the primary or the secondary market, this can induce the government to repay and therefore make investors willing to lend to the government.[46]

We ask how much households would be willing to pay in terms of consumption to permanently eliminate the possibility of a rollover crisis. To compute these welfare costs, we take the fixed and flexible exchange rate economies with their respective calibrations, and solve for the Markov equilibrium after setting the sunspot probability to zero. For each economy, we compute the welfare gains in terms of consumption equivalence as

$$\theta^{roll-over}(b, \mathbf{s}) = \left(\frac{V(b, \mathbf{s})^{NoSunspot}}{V(b, \mathbf{s})} \right)^{1/(1-\sigma)} - 1 \tag{33}$$

for every initial state.[47] Under a fixed exchange rate regime, the gains from having a lender of last resort can reach about 1.5% of permanent consumption and average 0.5%. Having access to a lender of last resort allows for both an improvement in the borrowing terms and a reduction in default costs. For the flexible exchange rate, however, the unconditional welfare gains from having a lender of last resort are negligible, in line with the minimal exposure to roll over crises.

It is worth highlighting that a successful implementation of lender of last resort hinges on the ability to correctly identify whether a default is being driven by fundamentals or by self-fulfilling beliefs. Moral hazard concerns would naturally emerge when the government and investors expect

[46]See Roch and Uhlig (2018) and Bocola and Dovis (2019) for an analysis of a lender of last resort in the context of the Outright Monetary Transactions (OMT) program by the ECB.

[47]Equation (33) uses homotheticity of the utility function and transforms default costs into consumption equivalence.

interventions in defaults driven by fundamentals.[48] Therefore, in a scenario in which the lender of last resort does not observe the source of the default, a trade-off is likely to emerge between the benefits from offsetting the coordination problem and the moral hazard effects.[49] Our analysis shows that while economies that lack monetary independence are likely to strongly benefit from a lender of last resort, this is less valuable for a flexible exchange rate regime, since defaults are almost exclusively driven by fundamental reasons.

Overall, this welfare analysis provides several important lessons. First, in the presence of rollover crises, the lack of monetary independence can become very costly. In particular, governments can become severely exposed to a rollover crisis and costly defaults because of the lack of monetary independence. Second, a lender of last resort can help to ease the costs of an economy giving up monetary independence.

4.4 The Path to Spain's 2012 Rollover Crisis

In this section, we use the model to shed light on the Spanish experience after giving up the peseta and adopting the euro. Two main points we wish to emphasize here. First, the model predicts that Spain is in the crisis zone in 2012 whereas exiting the Eurozone would make the economy safe from a rollover crisis. Second, the bulk of the welfare losses from lacking monetary independence can be mitigated by access to a lender of last resort.

The exercise we conduct is as follows. Starting at Spain's external debt-GDP ratio in 2000, we feed the sequence of shocks to tradable output and simulate the model under a fixed exchange rate regime. From 2000 until 2011, we find that the economy remains in the safe zone (and hence the sunspot realization is irrelevant). As it turns out, the model predicts that the economy is in the crisis zone in 2012, and a negative sunspot would trigger a rollover crisis and a default. Spain did not actually default in 2012, but a €100 billion assistance package by the European Union was channeled through the European Financial Stability Fund and the European Stability Mechanism. In addition, the announcement of the ECB's OMT bond purchasing program following the "whatever it takes" speech by Mario Draghi, led to a drastic collapse in spreads, which appeared to have dissipated concerns over the emergence of a rollover crisis.

Figure 11 summarizes the results of the exercise. Panel (a) shows the tradable output we feed into the model and the one-period-ahead probability of falling into the crisis zone. To compute this probability, we use the end-of-period level of debt and compute the probability of receiving an income shock in the following period that would push the economy into the crisis zone. Panels (b) and (c) of

[48] As argued by Aguiar et al. (2015), an alternative to a lender of last resort would be some form of fiscal union in which the government receives transfers from other countries, but this is more likely to be plagued by moral hazard and other problems.

[49] See Bianchi (2016) for a quantitative analysis of the trade-off between the moral hazard effects from bailouts and the stabilization benefits in the context of firms' borrowing.

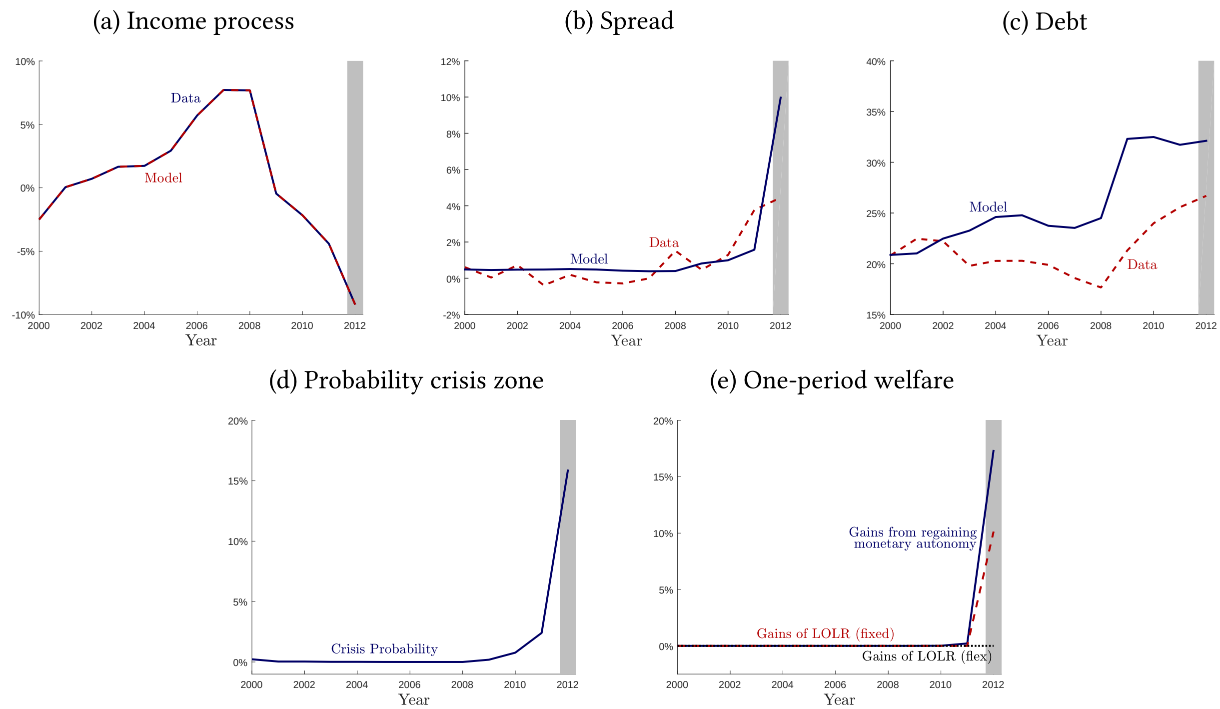

Figure 11: Path to Spain's 2012 Rollover Crisis

Notes: Welfare gains in panel (e) correspond to policies that are in place for one period, reverting to the baseline Markov equilibrium. Crisis probability denotes the probability that the economy would be in a crisis region in the following period given the current choices of debt and initial states. The tradable endowment shock was obtained by applying a log-quadratic filter to the Spanish tradable output from 1995 to 2017. Debt levels in the data correspond to Spain's external debt-GDP ratio. The shaded region denotes that the economy is in the crisis zone.

Figure 11 show the dynamics of debt and spread in these simulations. In early 2000, the government increases its debt, and this is driven by the low initial debt (recall that the calibrated mean external debt is close to 30%) and by relatively good income shocks that allow for favorable borrowing terms. These dynamics are fairly similar to those in the data, except that the model overpredicts the initial increase. One can also see that the model is able to replicate the low and stable spreads before 2008 in the data. Finally, the evolution of the probability of being in the crisis zone in panel (d) of Figure 11 reveals interesting dynamics. After the debt accumulation that occurs initially and the negative income shocks that pile up after 2008, the economy's probability of a rollover crisis becomes more significant. By 2012, the year in which the ECB intervened, the economy becomes significantly exposed to a rollover crisis, with a 20% probability of being in the crisis zone.

The final block of the exercise is a series of policy counterfactuals. Building on the analysis of Section 4.3, we first consider what the costs are from the lack of monetary independence. More precisely, throughout the simulations, we ask how much households in Spain are willing to pay in terms of current consumption to recover monetary independence for one period. As the blue line in panel (e) of Figure 11 shows, the gains are about zero until 2011 when a modest increase takes place, and strikingly the welfare cost reaches about 17 percent in 2012. As it turns out, close to 60% of these costs are due to the presence of rollover crises. That is, if there were no possibility of a rollover crisis, the increase in welfare from regaining monetary independence would be reduced by 60%; the remaining 40% would be a direct reduction in unemployment. To illustrate this, we compute the gains from eliminating a rollover crisis throughout the simulation for the fixed exchange rate regime (see the red line in panel (e)). As the figure shows, the gains are zero from 2000 to 2011 since the economy is in the safe zone, but these gains reach 10% in 2012 as the economy arrives in the crisis zone. For comparison, one can see in the figure that the gains from eliminating a rollover crisis would remain zero if the government were to regain monetary autonomy. In other words, the bulk of the welfare losses from the lack of monetary independence arise because it exposes the government to a rollover crisis and costly default. By the same token, a lender of last resort would help to significantly ease the costs involved in giving up monetary autonomy.

According to this experiment, if Spain had exited the monetary union in 2012, it would not have been subject to a rollover crisis.[50] Two remarks about this counterfactual experiment are in order. First, we are keeping everything else constant when we analyze the implications of exiting the Eurozone. We are therefore abstracting from any possible structural changes that Spain could experience upon exiting a monetary union. Notwithstanding, to the extent that these structural changes would symmetrically affect ${V_R}^+$ and ${V_R}^-$, we expect that the large gap between these two values that arise because of the inability to depreciate the currency would remain intact, and hence these structural changes should not significantly alter the crisis region. Second, we do not suggest that Spain would have been better off by exiting the monetary union. Being in a monetary union indeed has many benefits from that we are not modeling. Our goal is to point out an additional cost of remaining in a monetary union, which arises from the higher exposure to rollover crises.

5 Conclusion

This paper shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis and points to a new cost from joining a monetary union. When a government lacks monetary autonomy, a run on government bonds can lead to a large recession in the presence of nominal rigidities. In turn, anticipating that the government will find it more costly to repay, investors become more prone to run and the crisis becomes self-fulfilling. In a calibrated version of the model, we have found that an economy with a flexible exchange rate is relatively immune to a rollover crisis. On the other hand, a substantial defaults under a fixed exchange rate regime are driven by rollover crises.

[50] While the welfare results of Figure 11 correspond to a situation in which Spain regains monetary autonomy for one period, the same result would hold if there were a permanent exit from the Eurozone. In both cases, we continue to assume that debt remains denominated in foreign currency, a natural assumption since a currency redenomination would be akin to a default. While it is quite likely that Spain would start issuing debt in domestic currency after exiting, this would apply only to new issuances of debt, not the existing stock, which is to a large extent the most relevant in understanding the incentives to default and how they change if the government remains in or exits the monetary union.

Our analysis provides a new perspective on discussions about whether the lack of monetary autonomy in Southern European countries made them more vulnerable to a rollover crisis. According to a popular view, the fact that their debt was not denominated in domestic currency contributed to their vulnerability by preventing them from inflating away the debt. We argue instead that monetary policy has a role in preventing rollover crises that goes beyond the ability to inflate away the debt. Our analysis also suggests that a lender of last resort contributes to easing the costs from giving up monetary independence and could be highly beneficial for the stability of a monetary union.

Extending beyond our current analysis, several avenues remain for future work. In terms of debt management, our model suggests that economies with more rigid labor markets or a less flexible monetary policy should seek longer debt maturities. Another interesting avenue is to provide a more explicit modeling of the benefits from joining a monetary union and quantify the relevant trade-offs involved. Finally, one could also extend the analysis to consider spillovers within a monetary union.

6 Conclusion

This paper shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis and points to a new cost from joining a monetary union. When a government lacks monetary autonomy, a run on government bonds can lead to a large recession in the presence of nominal rigidities. In turn, anticipating that the government will find it more costly to repay, investors become more prone to run and the crisis becomes self-fulfilling. In a calibrated version of the model, we have found that an economy with a flexible exchange rate is relatively immune to a rollover crisis. On the other hand, a substantial defaults under a fixed exchange rate regime are driven by rollover crises.

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A Proofs

Proof of Lemma 1

In any equilibrium, the real wage in terms of tradable goods is a function of tradable consumption and employment:

$$\mathcal{W}(c_t^T, h_t) \equiv \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\mu} F'(h_t).$$

Moreover, $\mathcal{W}(c_t^T, h_t)$ is increasing with respect to c_t^T and decreasing with respect to h_t.

Proof. Using the firm's first order condition (5) and the equilibrium relative price, the equilibrium real wages in terms of tradable goods can be defined as a function of tradable consumption goods and employment:

$$\mathcal{W}(c_t^T, h_t) = p_t^N F'(h_t) = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\mu} F'(h_t).$$

Using this, we can find that

$$\frac{\partial \mathcal{W}_t}{\partial c_t^T} = \frac{(1+\mu)p_t^N F'(h_t)}{c_t^T} \qquad \text{and}$$
$$\frac{\partial \mathcal{W}_t}{\partial h_t} = -(1+\mu)p_t^N F'(h_t)\left(\frac{F'(h_t)}{F(h_t)} + \left(\frac{1}{1+\mu}\right)\frac{-F''(h_t)}{F'(h_t)}\right).$$

Because $F(\cdot)$ is a non-negative, strictly increasing, and decreasing returns to scale function, we know that F, $F' > 0$, and $F'' < 0$. Therefore, we can conclude that $\frac{\partial \mathcal{W}_t}{\partial c_t^T} > 0$ and $\frac{\partial \mathcal{W}_t}{\partial h_t} < 0$. □

Proof of Lemma 2

Under a fixed exchange rate regime, the employment function is a piecewise linear function:

$$\mathcal{H}(c^T) = \begin{cases} \left[\left(\frac{1-\omega}{\omega}\right)\left(\frac{\alpha}{\overline{w}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c^T\right)^{\frac{1+\mu}{1+\alpha\mu}} & \textit{if } c^T \leq \overline{c}^T_{\overline{w}} \\ \overline{h} & \textit{if } c^T > \overline{c}^T_{\overline{w}} \end{cases},$$

where

$$\overline{c}^T_{\overline{w}} = \left[\left(\frac{\omega}{1-\omega}\right)\left(\frac{\overline{w}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \overline{h}^{\frac{1+\alpha\mu}{1+\mu}}.$$

Proof. When the real wage rigidity is binding,

$$\overline{w} = \mathcal{W}(c^T, h) = \frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu} F'(h) = \frac{1-\omega}{\omega}\left(\frac{\left(c^T\right)^{1+\mu}}{F(h)^{\mu}}\right)\left(\frac{F'(h)}{F(h)}\right).$$

Using the property that $F(\cdot)$ is a homogeneous function of degree $\alpha \in [0,1]$, we then know that $F'(h)$ is homogeneous of degree $\alpha - 1$. Moreover, because it is a unidimensional function, we can also assert that $F(h) = h^{\alpha}$ and $F'(h) = h^{\alpha-1}$. Finally, we can also say that $hF'(h) = \alpha F(h)$. Hence, we can say that

$$\overline{w} = \frac{1-\omega}{\omega}\left(\frac{\alpha\left(c^T\right)^{1+\mu}}{h^{1+\alpha\mu}}\right).$$

Hence, solving for h, we get

$$h_{\overline{w}}(c^T) = \left[\left(\frac{1-\omega}{\omega}\right)\left(\frac{\alpha}{\overline{w}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c^T\right)^{\frac{1+\mu}{1+\alpha\mu}}.$$

Moreover, this labor function is increasing with respect to the consumption of non-tradables. Knowing that labor cannot go above the household's labor endowment of $\overline{h}$, we can compute the consumption threshold for tradables at which employment reaches this cap. Hence, we solve for this level:

$$\overline{c}^T_{\overline{w}} = \left[\left(\frac{\omega}{1-\omega}\right)\left(\frac{\overline{w}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \overline{h}^{\frac{1+\alpha\mu}{1+\mu}}.$$

For levels of tradable consumption above this threshold, the supply of labor in the economy will be in full employment. □

Proof of Lemma 3

For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b \in \mathbb{R}$, we have that $V_R^+(b, y^T) \geq V_R^-(b, y^T)$.

Proof. Realize that problem (21) is a particular case of (20). That is,

$$\begin{aligned} V_R^+(b, y^T) &= \max_{b', h \leq \overline{h}} \left\{ u(y^T - \delta b + \tilde{q}(b', y^T)\left(b' - (1-\delta)b\right), h) + \beta\mathbb{E}\left[V(b', \mathbf{s}')\right]\right\} \\ &\geq \max_{h \leq \overline{h}} \left\{ u(y^T - \delta b, h) + \beta\mathbb{E}\left[V((1-\delta)b, \mathbf{s}')\right]\right\} \\ &= V_R^-(b, y^T), \end{aligned}$$

where both problems satisfy the same labor and wage constraints. □

Proof of Proposition 1

Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states.

Proof. The value of repayment when the government can choose the exchange rate is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \bar{h}, e} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}') \right\} \tag{34}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$\mathcal{W}(c^T, h)e \geq \overline{W}.$$

Meanwhile, the value of default when the government can choose the exchange rate is given by the following Bellman equation:

$$V_D(y^T) = \max_{c^T, h \leq \bar{h}, e} \left\{ u\left(c^T, F(h)\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D(y^{T\prime})\right] \right\} \tag{35}$$

$$\text{subject to}$$

$$c^T = y^T$$

$$\mathcal{W}(c^T, h)e \geq \overline{W}.$$

It is immediate from (34) and (35) that an increase in e relaxes the wage rigidity constraint without tightening any other constraint. Fully relaxing the wage rigidity constraint allows the government to achieve full employment.

□

Proof of Lemma 4

For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exist levels of debt $\bar{b}^+, \bar{b}^- \in \mathbb{R}$ such that $\tilde{V}_D(y^T) = V_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D(y^T) = V_R^-\left(\bar{b}^-, y^T\right)$. Furthermore, it also satisfies $\bar{b}^+ \geq \bar{b}^-$.

Proof. First, realize that for every level of tradable endowment $y^T \in \mathbb{R}_+$, if $b = 0$ then $V_D(y^T) \leq \tilde{V}_R^-(0, y^T) \leq \tilde{V}_R^+(0, y^T)$. Because V is strictly decreasing in b, we can choose a level of debt sufficiently high $b >> 0$ such that $\tilde{V}_D(y^T) > \tilde{V}_R^+(b, y^T) \geq \tilde{V}_R^-(b, y^T)$. Because $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are continuous functions, by the intermediate value theorem there exist levels of debt $\bar{b}^+, \bar{b}^- \in \mathbb{R}$ such that $\tilde{V}_D(y^T) =$

$\tilde{V}_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D(y^T) = \tilde{V}_R^-\left(\bar{b}^-, y^T\right)$. Acknowledge that for every level of endowment $y^T \in \mathbb{R}_+$,

$$\tilde{V}_R^-\left(\bar{b}^-, y^T\right) = \tilde{V}_D(y^T) = \tilde{V}_R^+\left(\bar{b}^+, y^T\right) \geq \tilde{V}_R^-\left(\bar{b}^+, y^T\right).$$

Using that V_R^- is decreasing, we can conclude that $\bar{b}^+ \geq \bar{b}^-$. Furthermore, we have $\tilde{V}_D(y^T;\overline{w}) = V_R^+\left(b, y^T;\overline{w}\right) \forall b \geq \bar{b}^+\left(\overline{w}\right)$ and $\tilde{V}_D(y^T;\overline{w}) \geq V_R^-\left(b, y^T;\overline{w}\right) \forall b \geq \bar{b}^-\left(\overline{w}\right)$ □

Proof of Proposition 2

For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:

$i)$ *If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^-\left(\overline{w}_2\right) \leq \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}\left(\overline{w}_2\right) \subseteq \mathcal{S}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_D\right]$, then $\bar{b}^-\left(\overline{w}_2\right) < \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}\left(\overline{w}_2\right) \subset \mathcal{S}\left(\overline{w}_1\right)$.*

$ii)$ *Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\bar{b}^-\left(\overline{w}_2\right) \leq \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}\left(\overline{w}_2\right) \subseteq \mathcal{S}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_D\right)$, then $\bar{b}^-\left(\overline{w}_2\right) < \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}\left(\overline{w}_2\right) \subset \mathcal{S}\left(\overline{w}_1\right)$.*

Proof. Choose an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$ and two arbitrary wage rigidities $\overline{w}_1, \overline{w}_2 \in \mathbb{R}_+$ such that $\overline{w}_1 < \overline{w}_2$. Let us now proceed by cases:

$i)$ First, let the arbitrary wage rigidities be $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$. Using Lemma 4, call the debt thresholds that limit the safe region $\bar{b}^-\left(\overline{w}_1\right)$ and $\bar{b}^-\left(\overline{w}_2\right)$. Abusing notation, rename the previous thresholds as $\bar{b}_1^- \equiv \bar{b}^-\left(\overline{w}_1\right)$ and $\bar{b}_2^- \equiv \bar{b}^-\left(\overline{w}_2\right)$. Acknowledging that a higher real wage rigidity imposes a tighter constraint on the government, we have $\tilde{V}_R^-\left(b, y^T;\overline{w}_1\right) \geq \tilde{V}_R^-\left(b, y^T;\overline{w}_2\right)$ for any amount of debt $b \in \mathbb{R}$. Thus,

$$\tilde{V}_R^-\left(\bar{b}_1^-, y^T;\overline{w}_1\right) = \tilde{V}_D\left(y^T;\overline{w}_1\right) = \tilde{V}_D\left(y^T;\overline{w}_2\right) = \tilde{V}_R^-\left(\bar{b}_2^-, y^T;\overline{w}_2\right) \leq \tilde{V}_R^-\left(\bar{b}_2^-, y^T;\overline{w}_1\right).$$

Using Lemma 5, it follows that $\bar{b}_2^- \leq \bar{b}_1^-$. This implies that $\tilde{S}\left(\overline{w}_2\right) \subseteq \tilde{S}\left(\overline{w}_1\right)$. Moreover, assume that $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_D\right]$. Call $\hat{h}_R^{1,-}, \hat{h}_R^{2,-} \in \mathbb{R}_+$ the labor in the economy under the wage rigidities $\overline{w}_1$ and $\overline{w}_2$ in repayment with no borrowing, respectively. Because $\overline{w}_2 > \overline{w}_R^-$, under this wage rigidity the wage constraint binds, ensuring strictly positive unemployment in repayment with no borrowing. Using Lemma 1, it follows that $\hat{h}_R^{2,-} < \hat{h}_R^{1,-} \leq \overline{h}$. Hence, it follows that $\tilde{V}_R^-\left(b, y^T;\overline{w}_1\right) > \tilde{V}_R^-\left(b, y^T;\overline{w}_2\right)$ for any amount of debt $b \in \mathbb{R}$. Thus,

$$\tilde{V}_R^-\left(\bar{b}_1^-, y^T;\overline{w}_1\right) = \tilde{V}_D\left(y^T;\overline{w}_1\right) = \tilde{V}_D\left(y^T;\overline{w}_2\right) = \tilde{V}_R^-\left(\bar{b}_2^-, y^T;\overline{w}_2\right) < \tilde{V}_R^-\left(\bar{b}_2^-, y^T;\overline{w}_1\right).$$

Using Lemma 5, it follows that $\bar{b}_2^- < \bar{b}_1^-$. This implies that $\tilde{S}\left(\overline{w}_2\right) \subset \tilde{S}\left(\overline{w}_1\right)$.

$ii)$ Let the arbitrary wage rigidities be $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$. Using Lemma 4, call the debt thresholds that limit the safe region $\bar{b}^-\left(\overline{w}_1\right)$ and $\bar{b}^-\left(\overline{w}_2\right)$. Abusing notation, rename the previous thresholds as $\bar{b}_1^- \equiv \bar{b}^-(\overline{w}_1)$ and $\bar{b}_2^- \equiv \bar{b}^-(\overline{w}_2)$. Using the definition of the debt threshold and the implicit function theorem, we know that

$$\frac{d\bar{b}^-}{d\overline{w}} = \frac{\frac{\partial \tilde{V}_R^-}{\partial \overline{w}} - \frac{\partial \tilde{V}_D}{\partial \overline{w}}}{-\frac{\partial \tilde{V}_R^-}{\partial b}}.$$

Under linear production $F(h) = h$ and log-utility in final consumption $u(c) = \ln(c)$ and using Lemma 6 we reach that $\frac{d\bar{b}^-}{d\overline{w}} = 0$ because $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$. In other words, $\bar{b}_2^- = \bar{b}_1^-$, implying that $\tilde{S}(\overline{w}_2) = \tilde{S}(\overline{w}_1)$. Joining this with part i), we can conclude that, if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\bar{b}_2^- \leq \bar{b}_1^-$ and $\tilde{S}(\overline{w}_2) \subseteq \tilde{S}(\overline{w}_1)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_D\right)$, then $\bar{b}_2^- < \bar{b}_1^-$ and $\tilde{S}(\overline{w}_2) \subset \tilde{S}(\overline{w}_1)$.

□

Proof of Proposition 3

For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:
If $TB_R^+ \leq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, then $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$. Equivalently, $\tilde{D}(\overline{w}_2) \subseteq \tilde{D}(\overline{w}_1)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_D, \overline{w}_R^+\right]$, then $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$. Equivalently, $\tilde{D}(\overline{w}_2) \subset \tilde{D}(\overline{w}_1)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$, then $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$. Equivalently, $\tilde{D}(\overline{w}_2) \subseteq \tilde{D}(\overline{w}_1)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_D, \overline{w}_R^+\right)$, then $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$. Equivalently, $\tilde{D}(\overline{w}_2) \subset \tilde{D}(\overline{w}_1)$.

If $TB_R^+ \geq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. Equivalently, $\tilde{D}(\overline{w}_1) \subseteq \tilde{D}(\overline{w}_2)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^+, \overline{w}_D\right]$, then $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. Equivalently, $\tilde{D}(\overline{w}_1) \subset \tilde{D}(\overline{w}_2)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$, then $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. Equivalently, $\tilde{D}(\overline{w}_1) \subseteq \tilde{D}(\overline{w}_2)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^+, \overline{w}_D\right)$, then $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. Equivalently, $\tilde{D}(\overline{w}_1) \subset \tilde{D}(\overline{w}_2)$.

Proof. Choose an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$ and two arbitrary wage rigidities $\overline{w}_1, \overline{w}_2 \in \mathbb{R}_+$ such that $\overline{w}_1 < \overline{w}_2$. Let us now proceed by cases:

i) First, let the arbitrary wage rigidities be $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$. Using Lemma 4, call the debt thresholds that limit the default region $\bar{b}^+(\overline{w}_1)$ and $\bar{b}^+(\overline{w}_2)$. Abusing notation, rename the previous thresholds as $\bar{b}_1^+ \equiv \bar{b}^+(\overline{w}_1)$ and $\bar{b}_2^+ \equiv \bar{b}^+(\overline{w}_2)$. Acknowledging that a higher real wage rigidity in the relevant domain makes the value of default more likely to bind and may result in a higher unemployment, we know that $\tilde{V}_D\left(y^T;\overline{w}_1\right) \geq \tilde{V}_D\left(y^T;\overline{w}_2\right)$ for any amount of debt $b \in \mathbb{R}$. Thus,

$$\tilde{V}_R^+\left(\bar{b}_1^+, y^T;\overline{w}_1\right) = \tilde{V}_D\left(y^T;\overline{w}_1\right) \geq \tilde{V}_D\left(y^T;\overline{w}_2\right) = \tilde{V}_R^+\left(\bar{b}_2^+, y^T;\overline{w}_2\right) = \tilde{V}_R^+\left(\bar{b}_2^+, y^T;\overline{w}_1\right).$$

Using Lemma 5, it follows that $\bar{b}_2^- \geq \bar{b}_1^-$. This implies that $\tilde{D}\left(\overline{w}_2\right) \subseteq \tilde{D}\left(\overline{w}_1\right)$. Moreover, assume that $\overline{w}_2 \in \left(\overline{w}_D, \overline{w}_R^+\right]$. Call $\hat{h}_D^1, \hat{h}_D^2 \in \mathbb{R}_+$ the labor in the economy under the wage rigidities $\overline{w}_1$ and $\overline{w}_2$ in default, respectively. Because $\overline{w}_2 > \overline{w}_R^-$, under this wage rigidity the wage constraint binds, ensuring strictly positive unemployment in default. Using Lemma 1, it follows that $\hat{h}_D^2 < \hat{h}_D^1 \leq \overline{h}$. Hence, it follows that $\tilde{V}_D\left(y^T;\overline{w}_1\right) > \tilde{V}_D\left(y^T;\overline{w}_2\right)$. Thus,

$$\tilde{V}_R^+\left(\bar{b}_1^+, y^T;\overline{w}_1\right) = \tilde{V}_D\left(y^T;\overline{w}_1\right) > \tilde{V}_D\left(y^T;\overline{w}_2\right) = \tilde{V}_R^+\left(\bar{b}_2^+, y^T;\overline{w}_2\right) = \tilde{V}_R^+\left(\bar{b}_2^+, y^T;\overline{w}_1\right).$$

Using Lemma 5, it follows that $\bar{b}_2^- > \bar{b}_1^-$. This implies that $\tilde{D}\left(\overline{w}_2\right) \subset \tilde{D}\left(\overline{w}_1\right)$.

$ii)$ Let the arbitrary wage rigidities be $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$. Using Lemma 4, call the debt thresholds that limit the default region $\bar{b}^+\left(\overline{w}_1\right)$ and $\bar{b}^+\left(\overline{w}_2\right)$. Abusing notation, rename the previous thresholds as $\bar{b}_1^+ \equiv \bar{b}^+\left(\overline{w}_1\right)$ and $\bar{b}_2^+ \equiv \bar{b}^+\left(\overline{w}_2\right)$. Using the definition of the debt threshold and the implicit function theorem, we know that

$$\frac{d\bar{b}^+}{d\overline{w}} = \frac{\frac{\partial \tilde{V}_R^+}{\partial \overline{w}} - \frac{\partial \tilde{V}_D}{\partial \overline{w}}}{-\frac{\partial \tilde{V}_R^+}{\partial b}}.$$

Under linear production $F(h) = h$ and log-utility in final consumption $u(c) = \ln(c)$ and using Lemma 6, we reach that $\frac{d\bar{b}^+}{d\overline{w}} = 0$ because $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$. In other words, $\bar{b}_2^+ = \bar{b}_1^+$, implying that $\tilde{D}\left(\overline{w}_2\right) = \tilde{D}\left(\overline{w}_1\right)$. Joining this with part $i)$, we can conclude that, if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_D, \infty\right)$, then $\tilde{D}\left(\overline{w}_2\right) \subseteq \tilde{D}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_D, \overline{w}_R^+\right)$, then $\tilde{D}\left(\overline{w}_2\right) \subset \tilde{D}\left(\overline{w}_1\right)$.

□

Proof of Proposition 4

For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:

$i)$ *If $\overline{w}_1, \overline{w}_2 \in \left[0, \min\left\{\overline{w}_R^+, \overline{w}_D\right\}\right]$, then $\tilde{C}\left(\overline{w}_1\right) \subseteq \tilde{C}\left(\overline{w}_2\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \min\left\{\overline{w}_R^+, \overline{w}_D\right\}\right]$, then $\tilde{C}\left(\overline{w}_1\right) \subset \tilde{C}\left(\overline{w}_2\right)$.*

$ii)$ *Under $TB_R^+ \leq 0$, $F(h) = h$, and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\tilde{C}\left(\overline{w}_1\right) \subseteq \tilde{C}\left(\overline{w}_2\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_R^+\right)$, then $\tilde{C}\left(\overline{w}_1\right) \subset \tilde{C}\left(\overline{w}_2\right)$.*

Proof. Choose an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$ and two arbitrary wage rigidities $\overline{w}_1, \overline{w}_2 \in \mathbb{R}_+$ such that $\overline{w}_1 < \overline{w}_2$. Let us now proceed by cases:

i) First, let the arbitrary wage rigidities be $\overline{w}_1, \overline{w}_2 \in \left[0, \min\left\{\overline{w}_R^+, \overline{w}_D\right\}\right]$. Using Lemma 4, call the debt thresholds that limit the safe region $\bar{b}^-\left(\overline{w}_1\right)$ and $\bar{b}^-\left(\overline{w}_2\right)$. Abusing notation, rename the previous thresholds as $\bar{b}_1^- \equiv \bar{b}^-\left(\overline{w}_1\right)$ and $\bar{b}_2^- \equiv \bar{b}^-\left(\overline{w}_2\right)$. Using Proposition 2, it follows that $\bar{b}_2^- \leq \bar{b}_1^-$. In addition, using Lemma 4 again, call the debt thresholds that limit the default region $\bar{b}^+\left(\overline{w}_1\right)$ and $\bar{b}^+\left(\overline{w}_2\right)$. Abusing notation, rename the previous thresholds as $\bar{b}_1^+ \equiv \bar{b}^+\left(\overline{w}_1\right)$ and $\bar{b}_2^+ \equiv \bar{b}^+\left(\overline{w}_2\right)$. Using Proposition 3, it follows that $\bar{b}_2^+ = \bar{b}_1^+$. By construction, the crisis region under the different wage rigidities satisfy $\tilde{C}\left(\overline{w}_1\right) \subseteq \tilde{C}\left(\overline{w}_2\right)$. Moreover, assume that $\overline{w}_2\left(\overline{w}_R^-, \min\left\{\overline{w}_R^+, \overline{w}_D\right\}\right]$. Using Proposition 2, it follows that $\bar{b}_2^- < \bar{b}_1^-$. Hence, it follows that the crisis region satisfies $\tilde{C}\left(\overline{w}_1\right) \subset \tilde{C}\left(\overline{w}_2\right)$.

ii) Let the arbitrary wage rigidities $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$. Firstly, realize that when $TB_R^+ \leq 0$, then $\overline{w}_R^+ \geq \overline{w}_D$. Using Lemma 4, call the debt thresholds that limit the safe region $\bar{b}^-\left(\overline{w}_1\right)$ and $\bar{b}^-\left(\overline{w}_2\right)$. Abusing notation, recall the previous thresholds as $\bar{b}_1^- \equiv \bar{b}^-\left(\overline{w}_1\right)$ and $\bar{b}_2^- \equiv \bar{b}^-\left(\overline{w}_2\right)$. Using Proposition 2, it follows that $\bar{b}_2^- = \bar{b}_1^-$. In addition, using Lemma 4 again, call the debt thresholds that limit the default region $\bar{b}^+\left(\overline{w}_1\right)$ and $\bar{b}^+\left(\overline{w}_2\right)$. Abusing notation, rename the previous thresholds as $\bar{b}_1^+ \equiv \bar{b}^+\left(\overline{w}_1\right)$ and $\bar{b}_2^+ \equiv \bar{b}^+\left(\overline{w}_2\right)$. Using Proposition 3, it follows that $\bar{b}_2^+ = \bar{b}_1^+$. Joining this with part i) and using Proposition 2 and Proposition 3, we can conclude that if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$ then $\bar{b}_2^- \leq \bar{b}_1^-$ and $\bar{b}_2^+ \geq \bar{b}_1^+$. In other words, by construction of the crisis region $\tilde{C}\left(\overline{w}_1\right) \subseteq \tilde{C}\left(\overline{w}_2\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_R^+\right)$, then $\bar{b}_2^- < \bar{b}_1^-$ and $\bar{b}_2^+ \geq \bar{b}_1^+$. Hence, it follows that the crisis region satisfies $\tilde{C}\left(\overline{w}_1\right) \subset \tilde{C}\left(\overline{w}_2\right)$.

□

Lemma 5. *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to the debt b.*

Proof. Consider two different debt values $b_1, b_2 \in \mathbb{R}$ such that $b_2 > b_1$. Let (c_2^T, b_2') be the optimal policies associated with b_2 when the government can borrow. Under b_1, it is feasible for the government to choose $(c_1^T, b_1') = (c_2^T + \delta(b_2 - b_1) + \tilde{q}(b_2', y^T)(1-\delta)(b_2 - b_1), b_2')$, and this choice delivers higher utility.

Because $b_1' = b_2'$, continuation values are the same. To show that $(c_2^T + (b_2' - b_1), b_2')$ is feasible and delivers higher utility, it is then sufficient to show that the resource constraint is satisfied and that non-tradable consumption is higher:

$$
\begin{aligned}
c_2^T + \delta(b_2 - b_1) + \tilde{q}(b_2', y^T)(1-\delta)(b_2 - b_1) &\leq y^T - \delta b_1 + \tilde{q}(b_2', y^T)(b_2' - (1-\delta)b_1) \\
c_2^T &\leq y^T - \delta b_1 + \tilde{q}(b_2', y^T)(b_2' - (1-\delta)b_1) \\
&\quad - \delta(b_2 - b_1) - \tilde{q}(b_2', y^T)(1-\delta)(b_2 - b_1) \\
c_2^T &\leq y^T - \delta b_2 + \tilde{q}(b_2', y^T)(b_2' - (1-\delta)b_2),
\end{aligned}
$$

which holds since (c_2^T, b_2') is feasible under b_2. To see that non-tradable consumption is also higher, recall from Lemma 1 that $\mathcal{W}(c^T, h)$ is increasing in c^T and decreasing in h. Since $c_1^T > c_2^T$, it follows that c^N is higher under b_1. Therefore, we can conclude that $\tilde{V}_R^+(b_2) < \tilde{V}_R^+(b_1)$.

The proof that $\tilde{V}_R^-$ is decreasing in b can be obtained by differentiating the value function and, as is the case for $\tilde{V}_R^+$, it follows directly from the fact that a decrease in debt relaxes both the resource constraint and the wage rigidity constraint. □

Lemma 6 (Wage Constraint Lagrange Multiplier)**.** *When the wage constraint binds, the change in any of the value functions with respect to wage rigidity is strictly negative. Moreover, under linear production $F(h) = h$ and log-utility in final consumption $u(c) = \ln(c)$, the change in all the value functions is the same constant contingent on wage rigidity.*

Proof. Using the envelope theorem and by definition of the Lagrange multiplier associated with the wage constraint when binding $\eta > 0$, we know that

$$\frac{\partial V_D}{\partial \overline{w}} = -\eta_D < 0, \qquad \frac{\partial V_R^-}{\partial \overline{w}} = -\eta_R^- < 0, \qquad \text{and} \qquad \frac{\partial V_R^+}{\partial \overline{w}} = -\eta_R^+ < 0,$$

where the Lagrange multipliers are evaluated in the optimal solutions of each of their environments. Now, let us take the first derivative of the utility function and the real wage function:

$$F'(h)\frac{\partial u}{\partial c^N} = U'(C)\frac{\partial C}{\partial c^T}\mathcal{W}\left(c^T, h\right) \qquad \text{and} \qquad \frac{\partial W}{\partial h} = -\mathcal{W}\left(c^T, h\right)\left(\frac{1+\mu}{F(h)} + \frac{-F''(h)}{F'(h)}\right).$$

By definition, the Lagrange multiplier associated with the wage constraint is

$$\eta = -\frac{F'(h) \cdot \partial u / \partial c^N}{\partial \mathcal{W} / \partial h} = U'(C)\frac{\partial C}{\partial c^T}\left(\frac{1+\mu}{F(h)} + \frac{-F''(h)}{F'(h)}\right)^{-1},$$

by the properties of the utility function, the consumption aggregator, and the production function. Moreover, using linear production $F(h) = h$ and log-utility in final consumption $u(c) = \ln(c)$, it can be rewritten as

$$\eta = \frac{1}{1+\mu}\left(1 + \left(\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{1+\mu}} \overline{w}^{\frac{\mu}{1+\mu}}\right)^{-1}\right)^{-1}\frac{1}{\overline{w}}.$$

Realize that the Lagrange muliplier η is a constant contingent on wage rigidity in all of the maximization problems when the wage constraint binds. In other words, we have that $\eta = \eta_D = \eta_R^+ = \eta_R^-$. For simplicity, call $\overline{\eta}$ the Lagrange multiplier when the wage constraint binds and using linear production $F(h) = h$ and log-utility in final consumption $u(c) = \ln(c)$. □

Lemma 7 (Real Wage Rigidity Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, the following claims hold:*

i) There exists a real wage rigidity threshold $\overline{w}_D \in \mathbb{R}_+$ such that if, for any wage rigidity $\overline{w} \leq \overline{w}_D$, then $\tilde{V}_D\left(y^T; \overline{w}\right) = \tilde{V}_D\left(y^T; 0\right)$. Moreover, if, for any wage rigidity $\overline{w} > \overline{w}_D$, then $\tilde{V}_D\left(y^T; \overline{w}\right) < \tilde{V}_D\left(y^T; 0\right)$.

ii) *There exists a real wage rigidity threshold* $\overline{w}_R^+ \in \mathbb{R}_+$ *such that if, for any wage rigidity* $\overline{w} \leq \overline{w}_R^+$, *then* $\tilde{V}_R^+ \left(\bar{b}^+(0), y^T; \overline{w}\right) = \tilde{V}_R^+ \left(\bar{b}^+(0), y^T; 0\right)$. *Moreover, if, for any wage rigidity* $\overline{w} > \overline{w}_R^+$, *then* $\tilde{V}_R^+ \left(\bar{b}^+(0), y^T; \overline{w}\right) < \tilde{V}_R^+ \left(\bar{b}^+(0), y^T; 0\right)$.

iii) *There exists a real wage rigidity threshold* $\overline{w}_R^- \in \mathbb{R}_+$ *such that if, for any wage rigidity* $\overline{w} \leq \overline{w}_R^-$, *then* $\tilde{V}_R^- \left(\bar{b}^-(0), y^T; \overline{w}\right) = \tilde{V}_R^- \left(\bar{b}^-(0), y^T; 0\right)$. *Moreover, if, for any wage rigidity* $\overline{w} > \overline{w}_R^-$, *then* $\tilde{V}_R^- \left(\bar{b}^-(0), y^T; \overline{w}\right) < \tilde{V}_R^- \left(\bar{b}^-(0), y^T; 0\right)$.

In addition, define the equilibrium trade balance in the flexible exchange rate regime in repayment when borrowing is allowed as $TB_R^+ = y^T - \delta\bar{b}^+(0) + \tilde{q}^{flex}\left(\hat{b}_R^+\left(\bar{b}^+(0), y^T; 0\right) - (1-\delta)\bar{b}^+(0)\right)$, *where* $\tilde{q}^{flex} = \tilde{q}\left(\hat{b}\left(\bar{b}^+(0), y^T; 0\right), y^T; 0\right)$. *The following order of the thresholds is satisfied:*

i) *If* $TB_R^+ \leq 0$, *then* $\overline{w}_R^- \leq \overline{w}_D \leq \overline{w}_R^+$.

ii) *If* $TB_R^+ \geq 0$, *then* $\overline{w}_R^- \leq \overline{w}_R^+ \leq \overline{w}_D$.

Proof. Choose an arbitrary tradable endowment $y^T \in \mathbb{R}_+$ and let us proceed by cases:

i) Consider the following real wage rigidity threshold:

$$\overline{w}_D \equiv \mathcal{W}\left(y^T, \overline{h}\right) = \frac{1-\omega}{\omega}\left(\frac{y^T}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right),$$

Now choose an arbitrary real wage rigidity $\overline{w} \leq \overline{w}_D$ and realize that by construction, $\mathcal{W}\left(y^T, \overline{h}\right) \geq \overline{w}$. Therefore, the real wage constraint is not binding, achieving the same optimal allocations reached in the flexible exchange rate regime, $\tilde{V}_D\left(y^T; \overline{w}\right) = \tilde{V}_D\left(y^T; 0\right)$. On the other hand, choose an arbitrary real wage rigidity $\overline{w} > \overline{w}_D$ and realize that by construction, $\mathcal{W}\left(y^T, \overline{h}\right) < \overline{w}$. In this case, the wage constraint is binding, lowering labor in equilibrium $h < \overline{h}$ and yielding $\tilde{V}_D\left(y^T; \overline{w}\right) < \tilde{V}_D\left(y^T; 0\right)$.

ii) Abusing notation, let $\bar{b}^+ = \bar{b}^+(0)$, $\hat{b}_R^+ = \hat{b}_R^+\left(\bar{b}^+, y^T; 0\right)$, and $\tilde{q} = \tilde{q}^{flex}\left(\hat{b}_R^+, y^T\right)$. Consider the following real wage rigidity threshold:

$$\begin{aligned}\overline{w}_R^+ &\equiv \mathcal{W}\left(y^T - \delta\bar{b}^+ + \tilde{q}\left(\hat{b}_R^+ - (1-\delta)\bar{b}^+\right), \overline{h}\right) \\ &= \frac{1-\omega}{\omega}\left(\frac{y^T - \delta\bar{b}^+ + \tilde{q}\left(\hat{b}_R^+ - (1-\delta)\bar{b}^+\right)}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right).\end{aligned}$$

Choose an arbitrary real wage rigidity $\overline{w} \leq \overline{w}_R^+$ and realize that by construction, $\mathcal{W}\left(y^T - \delta\bar{b}^+ + \tilde{q}\left(\hat{b}_R^+ - (1-\delta)\bar{b}^+\right), \overline{h}\right) \geq \overline{w}$. Therefore, the real wage constraint is not binding, achieving the same optimal allocations reached in the flexible exchange rate regime, $\hat{b}_R^+\left(\bar{b}^+, y^T; \overline{w}\right) = \hat{b}_R^+\left(\bar{b}^+, y^T; 0\right)$ and $\tilde{V}_R^+\left(\bar{b}^+, y^T; \overline{w}\right) = \tilde{V}_R^+\left(\bar{b}^+, y^T; 0\right)$. On the other hand, choose an arbitrary real wage rigidity $\overline{w} > \overline{w}_R^+$ and realize that by construction, $\mathcal{W}\left(y^T - \delta\bar{b}^+ + \tilde{q}\left(\hat{b}_R^+ - (1-\delta)\bar{b}^+\right), \overline{h}\right) < \overline{w}$. In this case, the wage constraint is binding, lowering labor in equilibrium $h < \overline{h}$ and yielding $\tilde{V}_R^+\left(\bar{b}^+, y^T; \overline{w}\right) < \tilde{V}_R^+\left(\bar{b}^+, y^T; 0\right)$.

iii) Abusing notation, let $\bar{b}^+ = \bar{b}^+(0)$ and $\bar{b}^- = \bar{b}^-(0)$. Consider first the case in which the solution to the flexible exchange rate regime yields that it is optimal to buy off debt $\hat{b}_+^R\left(\bar{b}^+, y^T; 0\right) < \delta\bar{b}^+$. In this case, set $\overline{w}_R^- \equiv \overline{w}_R^+$ because by construction $\tilde{V}_R^- = \tilde{V}_R^+$ and follow case *ii*). On the other hand, let $\hat{b}_+^R\left(\bar{b}^+, y^T; 0\right) \geq \delta\bar{b}^+$. Consider the following real wage rigidity threshold:

$$\overline{w}_R^- \equiv \mathcal{W}\left(y^T - \delta\bar{b}^-, \overline{h}\right) = \frac{1-\omega}{\omega}\left(\frac{y^T - \delta\bar{b}^-}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right).$$

Choose an arbitrary real wage rigidity $\overline{w} \leq \overline{w}_R^-$ and realize that by construction, $\mathcal{W}\left(y^T - \delta\bar{b}^-, \overline{h}\right) \geq \overline{w}$. Therefore, the real wage constraint is not binding, achieving the same optimal allocations reached in the flexible exchange rate regime, $\tilde{V}_R^-\left(\bar{b}^-, y^T; \overline{w}\right) = \tilde{V}_R^-\left(\bar{b}^-, y^T; 0\right)$. On the other hand, choose an arbitrary real wage rigidity $\overline{w} > \overline{w}_R^-$ and realize that by construction, $\mathcal{W}\left(y^T - \delta\bar{b}^-, \overline{h}\right) < \overline{w}$. In this case, the wage constraint is binding, lowering labor in equilibrium $h < \overline{h}$ and yielding $\tilde{V}_R^-\left(\bar{b}^-, y^T; \overline{w}\right) < \tilde{V}_R^-\left(\bar{b}^-, y^T; 0\right)$.

Finally, by construction and by definition of TB_R^+, the following order is satisfied:

i) If $TB_R^+ \leq 0$, then $\delta\hat{b}^+ \leq \tilde{q}\left(\bar{b}_R^+ - (1-\delta)\bar{b}^+\right)$. This implies that

$$\begin{aligned}
\overline{w_D} &= \frac{1-\omega}{\omega}\left(\frac{y^T}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right) \\
&\leq \frac{1-\omega}{\omega}\left(\frac{y^T - TB_R^+}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right) \\
&= \frac{1-\omega}{\omega}\left(\frac{y^T - \delta\hat{b}^+ + \tilde{q}\left(\bar{b}_R^+ - (1-\delta)\bar{b}^+\right)}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right) \\
&= \overline{w}_R^+.
\end{aligned}$$

Also, because $\bar{b}^-(0) \leq 0$, then $\overline{w}_R^- \leq \overline{w}_D$. Finally, we arrive at the order $\overline{w}_R^- \leq \overline{w}_D \leq \overline{w}_R^+$.

$ii)$ If $TB_R^+ \geq 0$, then $\delta\hat{b}^+ \geq \tilde{q}\left(\bar{b}_R^+ - (1-\delta)\bar{b}^+\right)$. This implies that

$$\begin{aligned}
\overline{w_D} &= \frac{1-\omega}{\omega}\left(\frac{y^T}{F\left(\bar{h}\right)}\right)^{1+\mu} F'\left(\bar{h}\right) \\
&\geq \frac{1-\omega}{\omega}\left(\frac{y^T - TB_R^+}{F\left(\bar{h}\right)}\right)^{1+\mu} F'\left(\bar{h}\right) \\
&= \frac{1-\omega}{\omega}\left(\frac{y^T - \delta\hat{b}^+ + \tilde{q}\left(\bar{b}_R^+ - (1-\delta)\bar{b}^+\right)}{F\left(\bar{h}\right)}\right)^{1+\mu} F'\left(\bar{h}\right) \\
&= \overline{w}_R^+.
\end{aligned}$$

Also, by Lemma 3, we know that $\overline{w}_R^- \leq \overline{w}_R^+$. Finally, we arrive to the order $\overline{w}_R^- \leq \overline{w}_R^+ \leq \overline{w}_D$.

□

Proposition 5 (Smaller safe region under fixed)**.** Assume that $U(c(c^T, c^N)) = u(c^T) + u(c^N)$. For every y^T, the following claims hold: $\bar{b}^-\left(\overline{w}\right) \leq \bar{b}^-\left(0\right) \forall \overline{w}$. Moreover, if $\overline{w} > \overline{w}_R^-$, we have $\bar{b}^-\left(\overline{w}\right) < \bar{b}^-\left(0\right)$. Equivalently, we have $\mathcal{S}\left(\overline{w}\right) \subseteq \mathcal{S}\left(0\right) \forall \overline{w}$ and if $\overline{w} > \overline{w}_R^-$, $\bar{b}^-\left(\overline{w}\right) < \bar{b}^-\left(0\right)$.

Proof. By definition of $\bar{b}^-$,

$$\tilde{V}_D(y^T; 0) = \tilde{V}_R^-(\bar{b}^-\left(0\right), y^T; 0)$$

$$\begin{aligned}
&U(y^T - \delta\bar{b}^-\left(0\right), F(\bar{h})) + \beta\mathbb{E}V((1-\delta)\bar{b}^-\left(0\right), \mathbf{s}') = \\
&\qquad U\left((y^T, F(\bar{h})\right) - \kappa(y^T) + \beta\mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi)V_D(y^{T\prime})\right].
\end{aligned}$$

Using that preferences are separable,

$$u(y^T - \delta\bar{b}^-\left(0\right)) + \beta\mathbb{E}V((1-\delta)\bar{b}^-\left(0\right), \mathbf{s}') = u(y^T) - \kappa(y^T) + \beta\mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi)V_D(y^{T\prime})\right].$$

We also know that since $\mathcal{H}$ is increasing in c^T and combining the last two expressions, we arrive at

$$\begin{aligned}
u(y^T - \delta\bar{b}^-\left(0\right)) + u(F(\mathcal{H}(y^T - \delta\bar{b}^-\left(0\right), \bar{w}))) + \beta\mathbb{E}V((1-\delta)b, \mathbf{s}') &\leq u(y^T) + u(F(\mathcal{H}(y^T, \bar{w}))) \\
&\quad - \kappa(y^T) + \beta\mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi)V_D(y^{T\prime})\right],
\end{aligned}$$

which implies $\tilde{V}_D(y^T; \overline{w}) \geq \tilde{V}_R^-(\bar{b}^-\left(0\right), y^T; \overline{w})$. By definition, $\tilde{V}_D(y^T; \overline{w}) = \tilde{V}_R^-(\bar{b}^-\left(\overline{w}\right), y^T; \overline{w})$. Since $\tilde{V}_R^-$ is decreasing in debt, this implies $\bar{b}^-\left(0\right) \geq \bar{b}^-\left(\overline{w}\right)$. When $\overline{w} > \overline{w}_R^-$, the previous equation holds with strict inequality, and we arrive at $\bar{b}^-\left(0\right) > \bar{b}^-\left(\overline{w}\right)$.

□

Online Addendum to "Monetary Independence and Rollover Crises"

By Javier Bianchi and Jorge Mondragon

In this addendum, we show that our main theoretical results hold in various extensions from our baseline model. In Section A, we consider sticky prices. In Section B, we consider a version of the model in which there are costs from exchange rate fluctuations, providing a rationale for adopting a fixed exchange rate or joining a monetary union. In Section C, we consider an inflation-targeting regime, which provides an "intermediate" regime between a flexible exchange rate regime that seeks to achieve full employment at every period and a fixed exchange rate regime. In Section D, we show how the results can be extended in a model with a rich maturity structure. In Section E, we consider an elastic labor supply.

A Sticky Prices

In this section, we explore price rigidity as an alternative to wage rigidity. We assume that wages and the price of tradables are flexible and that the nominal non-tradable price in the economy cannot go lower than a threshold $\overline{P} > 0$.[51] Using the disequilibrium formulation of Barro and Grossman (1971), we have that firms supply $h^s = F^{-1}(h)$ whenever $p^N > \overline{P}$.

The constraint in the economy can be described as $P^N \geq \overline{P}$. Using the optimality condition of the household and the non-tradable market clearing condition, we can construct the real non-tradable price function as

$$\mathcal{P}(c^T, h) \equiv \frac{P^N}{e} = \frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu}.$$

Lemma A1. *The real non-tradable price function is increasing in consumption of tradables and decreasing in labor.*

Proof. Taking the first derivatives with respect to the consumption of tradables and labor, we have that

$$\frac{\partial \mathcal{P}}{\partial c^T} = (1+\mu)\frac{\mathcal{P}(c^T,h)}{c^T} > 0 \qquad \text{and} \qquad \frac{\partial \mathcal{P}}{\partial h} = -(1+\mu)F'(h)\frac{\mathcal{P}(c^T,h)}{F(h)} < 0.$$

Hence, the real non-tradable price function is increasing in the consumption of tradables and decreasing in labor. □

We will first define the government problem and the bond pricing under this new environment. The problem of the government to either default or repay debt can be described as

$$V(b,\mathbf{s}) = \max_{d\in\{0,1\}} \left\{ dV_D(y^T) + (1-d)V_R(b,\mathbf{s}) \right\}.$$

[51] We could also assume that price rigidity goes in both directions, but we model the asymmetry to have a more direct comparison with the model with downward wage rigidity. The approach of downward price rigidity is commonly attributed to "social norms" and is followed, for example, by Caballero and Farhi (2017).

In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{e,c^T,h\leq\overline{h}} & \left\{u\left(c^T, F\left(h\right)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{P} \leq e\mathcal{P}(c^T, h). \end{aligned}$$

The value of repayment transforms to

$$\begin{aligned} V_R\left(b, \mathbf{s}\right) = \max_{e,b',c^T,h\leq\overline{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T = y^T - \delta b + q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) \\ & \overline{P} \leq e\mathcal{P}(c^T, h). \end{aligned}$$

Proposition A1 (Nominal Rigidities Equivalence)**.** *If the production function $F(h)$ is linear, then if $\left\{V, V_D, V_R, q, \hat{b}\right\}$ is the solution to the Markov recursive equilibrium from Definition 2 with downward nominal wage rigidity $\overline{W} \in \mathbb{R}_+$, then it also is an equilibrium for the environment in this section when the downward nominal non-tradable price rigidity satisfies $\overline{P} \equiv \overline{W}/F'(h)$.*

Proof. By assuming that the production function is linear, then $F'(h) = z$ is a constant that does not depend on labor. The only difference between the models is the downward nominal rigidities, so inspecting the downward nominal wage rigidity, we have that

$$\overline{W} \leq e\mathcal{W}(c^T, h) = e\frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu} F'(h) = e\mathcal{P}(c^T, h)z.$$

By defining the downward nominal non-tradable price rigidity $\overline{P} \equiv \overline{W}/z$, we conclude that both rigidities are the same. Hence, the solutions are the same. □

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition on the part of international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$\begin{aligned} V_R^+\left(b, y^T\right) = \max_{e,b',c^T,h\leq\overline{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\ & \overline{P} \leq e\mathcal{P}(c^T, h). \end{aligned} \tag{A.1}$$

Call $\hat{b}_R^+\left(b, y^T\right)$ the optimal solution that solves the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} = \left\{\left(b, y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \hat{b}_R^+\left(b, y^T\right) > (1-\delta)b\right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b, y^T\right) \notin \mathcal{B}$, the government finds if optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b, y^T\right) = V_R^+\left(b, y^T\right)$ because the government is buying back its debt. Nevertheless, if $\left(b, y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$, representing their reluctance to issue new debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^-\left(b, y^T\right) = \max_{e, c^T, h \leq \bar{h}} \left\{u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \tag{A.2}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{P} \leq e\mathcal{P}(c^T, h).$$

The following lemma follows the same steps as the one stated before, following the fact that V_R^- is a particular case of a V_R^+ maximization problem.

Lemma A2. *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level b, we have that $V_R^+\left(b, y^T\right) \geq V_R^-\left(b, y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone as

$$\begin{aligned}
\mathcal{S} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b, y^T\right)\right\} \\
\mathcal{D} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b, y^T\right)\right\} \\
\mathcal{C} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b, y^T\right) \quad \text{and} \quad V_D(y^T) > V_R^-\left(b, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the bond pricing, following the no-arbitrage condition for each maturity structure, can be represented by the following recursion:

$$\tilde{q}(b', y^T) = \frac{1}{1+r}\mathbb{E}\left[\left(1 - d(b', \mathbf{s}')\right)\left(\delta + (1-\delta) q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right)\right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}.$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}$$

The following proposition follows the same steps as the one stated before.

Proposition A2 (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states.*

Let us now focus on the flexible exchange rate regime and solve the model. Call the flexible exchange rate regime solutions $\left\{V^{flex}, V_D^{flex}, \tilde{q}^{flex}\right\}$, and let us study the one-period fixed exchange rate regime shocks. To do this, let us define the downward real non-tradable price rigidity $\overline{p} \equiv \overline{P}/\overline{e}$ under a fixed exchange rate regime. The value of default will transform to

$$\tilde{V}_D\left(y^T;\overline{w}\right)=\max_{c^T,h\leq\overline{h}}\left\{u\left(c^T,F\left(h\right)\right)-\kappa\left(y^T\right)+\beta\mathbb{E}\left[\psi V^{flex}\left(0,\mathbf{s}'\right)+(1-\psi)V_D^{flex}\left(y^{T\prime}\right)\right]\right\}$$
$$\text{s.t. } c^T=y^T$$
$$\overline{p}\leq\mathcal{P}(c^T,h).$$

Also, the value of repayment when rollover debt is allowed is

$$\tilde{V}_R^+\left(b,y^T;\overline{w}\right)=\max_{b',c^T,h\leq\overline{h}}\left\{u\left(c^T,F\left(h\right)\right)+\beta\mathbb{E}\left[V^{flex}\left(b',\mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T-\tilde{q}\left(b',y^T\right)\left(b'-(1-\delta)b\right)=y^T-\delta b$$
$$\overline{p}\leq\mathcal{P}(c^T,h).$$

Finally, the value of repayment when new debt contracts of any maturity are forbidden is

$$\tilde{V}_R^-\left(b,y^T;\overline{w}\right)=\max_{c^T,h\leq\overline{h}}\left\{u\left(c^T,F\left(h\right)\right)+\beta\mathbb{E}\left[V^{flex}\left((1-\delta)b,\mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T=y^T-\delta b$$
$$\overline{p}\leq\mathcal{P}(c^T,h).$$

The following lemmas and propositions follow the same steps stated in the previous section.

Lemma A3. *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to the debt b.*

Lemma A4 (Debt Thresholds). *For every level of tradable endowment $y^T\in\mathbb{R}$, there exist levels of debt $\overline{b}^+\left(\overline{p}\right),\overline{b}^-\left(\overline{p}\right)\in\mathbb{R}$, such that $\tilde{V}_D\left(y^T;\overline{p}\right)=\tilde{V}_R^+\left(\overline{b}^+\left(\overline{p}\right),y^T;\overline{p}\right)$ and $\tilde{V}_D\left(y^T;\overline{p}\right)=\tilde{V}_R^-\left(\overline{b}^-\left(\overline{p}\right),y^T;\overline{p}\right)$. Furthermore, $\overline{b}^+\left(\overline{p}\right)\geq\overline{b}^-\left(\overline{p}\right)$.*
(To avoid clutter, we omit the dependence of these thresholds on y^T, but it should be understood throughout that the thresholds depend on y^T.)

Now call the regions

$$\tilde{S}_{y^T}\left(\overline{p}\right)\equiv\left(-\infty,\overline{b}^-\left(\overline{p}\right)\right],\qquad\tilde{C}_{y^T}\left(\overline{p}\right)\equiv\left(\overline{b}^-\left(\overline{p}\right),\overline{b}^+\left(\overline{p}\right)\right],\qquad\text{and}\qquad\tilde{D}_{y^T}\left(\overline{p}\right)\equiv\left(\overline{b}^+\left(\overline{p}\right),\infty\right).$$

The following propositions follow the same steps stated in the paper for Proposition 2, Proposition 3, and Proposition 4. The price rigidity thresholds follow the same narrative as in Lemma 7.

Proposition A3 (Safe Region Threshold). *For every y^T and taking arbitrary wage rigidities $\overline{p}_1<\overline{p}_2$, the following claims hold:*

i) If $\overline{p}_1,\overline{p}_2\in[0,\overline{p}_D]$, then $\overline{b}^-\left(\overline{p}_2\right)\leq\overline{b}^-\left(\overline{p}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{p}_2\right)\subseteq\mathcal{S}_{y^T}\left(\overline{p}_1\right)$. Moreover, if $\overline{p}_2\in\left(\overline{p}_R^-,\overline{p}_D\right]$, then $\overline{b}^-\left(\overline{p}_2\right)<\overline{b}^-\left(\overline{p}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{p}_2\right)\subset\mathcal{S}_{y^T}\left(\overline{p}_1\right)$.

ii) Under $F(h)=h$ and $u(c)=\ln(c)$; if $\overline{p}_1,\overline{p}_2\in\left[\overline{p}_R^-,\infty\right)$, then $\overline{b}^-\left(\overline{p}_2\right)\leq\overline{b}^-\left(\overline{p}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{p}_2\right)\subseteq\mathcal{S}_{y^T}\left(\overline{p}_1\right)$. Moreover, if $\overline{p}_1\in\left[\overline{p}_R^-,\overline{p}_D\right)$, then $\overline{b}^-\left(\overline{p}_2\right)<\overline{b}^-\left(\overline{p}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{p}_2\right)\subset\mathcal{S}_{y^T}\left(\overline{p}_1\right)$.

Proposition A4 (Default Region Threshold). *For every y^T and taking arbitrary wage rigidities $\overline{p}_1<\overline{p}_2$, the following claims hold:*
If $TB_R^{flex,+}\leq 0$:

i) *If* $\overline{p}_1, \overline{p}_2 \in \left[0, \overline{p}_R^+\right]$*, then* $\bar{b}^+(\overline{p}_1) \leq \bar{b}^+(\overline{p}_2)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_2) \subseteq \mathcal{D}_{y^T}(\overline{p}_1)$*. Moreover, if* $\overline{p}_2 \in \left(\overline{p}_D, \overline{p}_R^+\right]$*, then* $\bar{b}^+(\overline{p}_1) < \bar{b}^+(\overline{p}_2)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_2) \subset \mathcal{D}_{y^T}(\overline{p}_1)$*.*

ii) *Under* $F(h) = h$ *and* $u(c) = \ln(c)$*; if* $\overline{p}_1, \overline{p}_2 \in [\overline{p}_D, \infty)$*, then* $\bar{b}^+(\overline{p}_1) \leq \bar{b}^+(\overline{p}_2)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_2) \subseteq \mathcal{D}_{y^T}(\overline{p}_1)$*. Moreover, if* $\overline{p}_1 \in \left[\overline{p}_D, \overline{p}_R^+\right)$*, then* $\bar{b}^+(\overline{p}_1) < \bar{b}^+(\overline{p}_2)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_2) \subseteq \mathcal{D}_{y^T}(\overline{p}_1)$*.*

If $TB_R^{flex,+} \geq 0$*:*

i) *If* $\overline{p}_1, \overline{p}_2 \in [0, \overline{p}_D]$*, then* $\bar{b}^+(\overline{p}_2) \leq \bar{b}^+(\overline{p}_1)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_1) \subseteq \mathcal{D}_{y^T}(\overline{p}_2)$*. Moreover, if* $\overline{p}_2 \in \left(\overline{p}_R^+, \overline{p}_D\right]$*, then* $\bar{b}^+(\overline{p}_2) < \bar{b}^+(\overline{p}_1)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_1) \subset \mathcal{D}_{y^T}(\overline{p}_2)$*.*

ii) *Under* $F(h) = h$ *and* $u(c) = \ln(c)$*; if* $\overline{p}_1, \overline{p}_2 \in \left[\overline{p}_R^+, \infty\right)$*, then* $\bar{b}^+(\overline{p}_2) \leq \bar{b}^+(\overline{p}_1)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_1) \subseteq \mathcal{D}_{y^T}(\overline{p}_2)$*. Moreover, if* $\overline{p}_1 \in \left[\overline{p}_R^+, \overline{p}_D\right)$*, then* $\bar{b}^+(\overline{p}_2) < \bar{b}^+(\overline{p}_1)$*. Equivalently,* $\mathcal{D}_{y^T}(\overline{p}_1) \subset \mathcal{D}_{y^T}(\overline{p}_2)$*.*

Proposition A5 (Crisis Region Expansion)**.** *For every* y^T *and taking arbitrary wage rigidities* $\overline{p}_1 < \overline{p}_2$*, the following claims hold:*

i) *If* $\overline{p}_1, \overline{p}_2 \in \left[0, \overline{p}_R^+\right]$*, then* $\tilde{C}_{y^T}(\overline{p}_1) \subseteq \tilde{C}_{y^T}(\overline{p}_2)$*. Moreover, if* $\overline{p}_2 \in \left(\overline{p}_R^-, \overline{p}_R^+\right]$*, then* $\tilde{C}_{y^T}(\overline{p}_1) \subset \tilde{C}_{y^T}(\overline{p}_2)$*.*

ii) *Under* $TB_R^{flex,+} \leq 0$*,* $F(h) = h$*, and* $u(c) = \ln(c)$*; if* $\overline{w}_1, \overline{w}_2 \in \left[\overline{p}_R^-, \infty\right)$*, then* $\tilde{C}_{y^T}(\overline{p}_1) \subseteq \tilde{C}_{y^T}(\overline{p}_2)$*. Moreover, if* $\overline{p}_1 \in \left[\overline{p}_R^-, \overline{p}_R^+\right)$*, then* $\tilde{C}_{y^T}(\overline{p}_1) \subset \tilde{C}_{y^T}(\overline{p}_2)$*.*

Figure 12 compares the crisis region under rigid wages and rigid prices, and shows that these two forms of rigidities yield very similar implications.

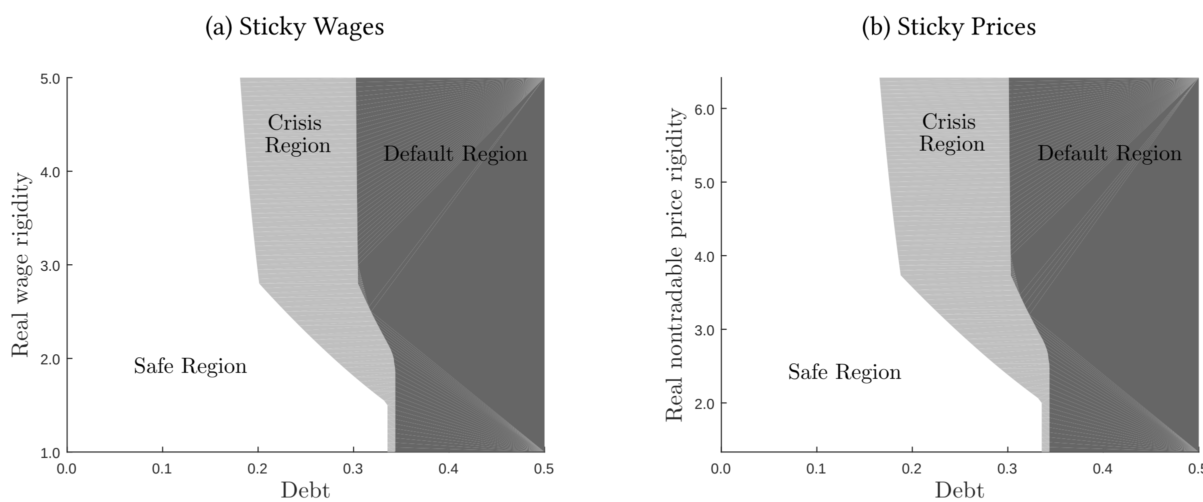

Figure 12: Regions change under different downward nominal rigidities

Notes: The tradable endowment is fixed at its long-run level. The grid for the real non-tradable price rigidity is set to its level in default corresponding to the real wage rigidity grid.

B Devaluation Costs

In this section, we explore a version of the model in which the government can choose the exchange rate every period, but a cost is associated with exchange rate fluctuations. We consider two variants: in one version are costs from current depreciations, and in another version are costs from future expected depreciations. Both versions provide a rationale for joining a monetary union or fixing the exchange rate.

B.1 Costs from Current Depreciations

We assume that exchange rate devaluations above a "natural" level $\overline{e} > 0$ incur a penalty $\Phi\left(e - \overline{e}\right) \geq 0$ that satisfies $\Phi(0) = \Phi'(0) = 0$ and $\Phi'(\cdot) > 0$. Using the downward nominal rigidity constraint and the properties of the devaluation utility cost, the optimal exchange rate can be expressed as

$$e(c^T) = \overline{e} \cdot \max\left\{\frac{\overline{w}}{\mathcal{W}\left(c^T, \overline{h}\right)}, 1\right\}. \tag{B.1}$$

We will first define the government problem and the bond pricing under this new environment. The problem of the government to either default or repay debt can be described as

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{dV_D(y^T) + (1-d)V_R\left(b, \mathbf{s}\right)\right\}.$$

In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{e, c^T} & \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\} \\ & \text{s.t. } c^T = y^T. \end{aligned}$$

The value of repayment transforms to

$$\begin{aligned} V_R\left(b, \mathbf{s}\right) = \max_{e, b', c^T} & \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \kappa\left(y^T\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ & \text{s.t. } c^T = y^T - \delta b + q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right). \end{aligned}$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition on the part of international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$\begin{aligned} V_R^+\left(b, y^T\right) = \max_{e, b', c^T} & \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ & \text{s.t. } c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b. \end{aligned} \tag{B.2}$$

Call $\hat{b}_R^+\left(b, y^T\right)$ the optimal solution that solves the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} = \left\{\left(b, y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \hat{b}_R^+\left(b, y^T\right) > (1-\delta)b\right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b, y^T\right) \notin \mathcal{B}$, the government finds it optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b, y^T\right) = V_R^+\left(b, y^T\right)$ because the government is buying back its debt. Nevertheless, if $\left(b, y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$, representing their reluctance to issue new debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^-\left(b, y^T\right) = \max_{e, c^T}\left\{u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \tag{B.3}$$
$$\text{s.t. } c^T = y^T - \delta b.$$

The following lemma follows the same steps as the one stated before, following the fact that V_R^- is a particular case of the V_R^+ maximization problem.

Lemma B5. *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exist levels of debt $\overline{b}_{y^T}^+\left(\overline{w}\right), \overline{b}_{y^T}^-\left(\overline{w}\right) \in \mathbb{R}_+$ such that $\tilde{V}_D(y^T; \overline{w}) = V_R^+\left(\overline{b}_{y^T}^+\left(\overline{w}\right), y^T; \overline{w}\right)$ and $\tilde{V}_D(y^T; \overline{w}) = V_R^-\left(\overline{b}_{y^T}^-\left(\overline{w}\right), y^T; \overline{w}\right)$. Furthermore, $\overline{b}_{y^T}^+\left(\overline{w}\right) \geq \overline{b}_{y^T}^-\left(\overline{w}\right)$.*

Now, let us define the safe zone, default zone, and repayment zone as

$$\begin{aligned}
\mathcal{S} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b, y^T\right)\right\} \\
\mathcal{D} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b, y^T\right)\right\} \\
\mathcal{C} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b, y^T\right) \quad \text{and} \quad V_D(y^T) > V_R^-\left(b, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the bond pricing, following the no-arbitrage condition, can be represented by the following recursion:

$$\tilde{q}(b', y^T) = \frac{1}{1+r}\mathbb{E}\left[\left(1 - d(b', \mathbf{s}')\right)\left(\delta + (1-\delta)q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right)\right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}.$$

The following proposition relies that under a flexible exchange rate, the downward nominal rigidity can be ignored. In addition, when there are no devaluation costs, then the objective functions coincide in both environments. Hence the maximization problems are the same.

Proposition B6 (No Devaluation Costs). *If $\left\{V^{flex}, V_D^{flex}, q^{flex}, \hat{b}^{flex}\right\}$ is a recursive equilibrium under a flex-*

ible exchange rate regime from Section 3.2, then it is also a recursive equilibrium in the environment when there are no devaluation costs $\Phi(\cdot) = 0$.

Let us now focus on the no devaluation costs environment. Call the no devaluation costs environment solutions $\left\{V^{flex}, V_D^{flex}, \tilde{q}^{flex}\right\}$, and let us study the one-period devaluation cost shock. The value of default will transform to

$$\tilde{V}_D\left(y^T\right) = \max_{c^T} \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \kappa\left(y^T\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta\mathbb{E}\left[\psi V^{flex}\left(0, \mathbf{s}'\right) + (1-\psi)V_D^{flex}\left(y^{T\prime}\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T.$$

Also, the value of repayment when rollover debt is allowed is

$$\tilde{V}_R^+\left(b, y^T\right) = \max_{b', c^T} \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta\mathbb{E}\left[V^{flex}\left(b', \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b.$$

Finally, the value of repayment when new debt contracts are forbidden is

$$\tilde{V}_R^-\left(b, y^T\right) = \max_{c^T} \left\{u\left(c^T, F\left(\overline{h}\right)\right) - \Phi\left(e\left(c^T\right) - \overline{e}\right) + \beta\mathbb{E}\left[V^{flex}\left((1-\delta)b, \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T - \delta b.$$

The following lemmas and propositions follow the same steps stated in the previous section.

Lemma B6. *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to the debt b.*

Lemma B7 (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists levels of debt that currently matures $\overline{b}^+, \overline{b}^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\overline{b}^+, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\overline{b}^-, y^T\right)$. Furthermore, $\overline{b}^+ \geq \overline{b}^-$.*
(To avoid clutter, we omit the dependence of these thresholds on y^T, but it should be understood throughout that the thresholds depend on y^T.)

Now call the regions

$$\tilde{S}_{y^T}\left(\overline{w}\right) \equiv \left(-\infty, \bar{b}^-\left(\overline{w}\right)\right], \quad \tilde{C}_{y^T}\left(\overline{w}\right) \equiv \left(\bar{b}^-\left(\overline{w}\right), \bar{b}^+\left(\overline{w}\right)\right], \quad \text{and} \quad \tilde{D}_{y^T}\left(\overline{w}\right) \equiv \left(\bar{b}^+\left(\overline{w}\right), \infty\right).$$

Lemma B8 (Devaluation Costs Ordering)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$ and level of debt $b \in \mathbb{R}$, the devaluation and its penalty needed when borrowing is not allowed is at least as high as when rollover is allowed.*

Proof. Because $\tilde{V}_R^-$ is a particular problem of $\tilde{V}_R^+$ and full employment is achieved in both, it follows that $\mathcal{W}\left(\hat{c}_R^+, \overline{h}\right) \geq \mathcal{W}\left(\hat{c}_R^-, \overline{h}\right)$. This implies by (B.1) that $e\left(\hat{c}_R^-\right) \geq e\left(\hat{c}_R^+\right)$. Moreover, because of the properties of the devaluation penalty, $\Phi_R^- \geq \Phi_R^+$. □

The following propositions follow the same steps stated in the paper for Proposition 2, Proposition 3, and Proposition 4. The wage rigidity thresholds follow the same narrative as in Lemma 7. We use Lemma B8 to argue that the devaluation incurred in the no borrowing scenario is deeper than the one in which a rollover is allowed. Using the properties of the devaluation utility cost, a deeper devaluation implies a higher utility loss.

Proposition B7 (Safe Region Threshold)**.** *For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^-(\overline{w}_2) \leq \bar{b}^-(\overline{w}_1)$. Equivalently, $\mathcal{S}_{y^T}(\overline{w}_2) \subseteq \mathcal{S}_{y^T}(\overline{w}_1)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_D\right]$, then $\bar{b}^-(\overline{w}_2) < \bar{b}^-(\overline{w}_1)$. Equivalently, $\mathcal{S}_{y^T}(\overline{w}_2) \subset \mathcal{S}_{y^T}(\overline{w}_1)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\bar{b}^-(\overline{w}_2) \leq \bar{b}^-(\overline{w}_1)$. Equivalently, $\mathcal{S}_{y^T}(\overline{w}_2) \subseteq \mathcal{S}_{y^T}(\overline{w}_1)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_D\right)$, then $\bar{b}^-(\overline{w}_2) < \bar{b}^-(\overline{w}_1)$. Equivalently, $\mathcal{S}_{y^T}(\overline{w}_2) \subset \mathcal{S}_{y^T}(\overline{w}_1)$.

Proposition B8 (Default Region Threshold)**.** *For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*
If $TB_R^{flex,+} \leq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, then $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_2) \subseteq \mathcal{D}_{y^T}(\overline{w}_1)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_D, \overline{w}_R^+\right]$, then $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_2) \subset \mathcal{D}_{y^T}(\overline{w}_1)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$, then $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_2) \subseteq \mathcal{D}_{y^T}(\overline{w}_1)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_D, \overline{w}_R^+\right)$, then $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_2) \subseteq \mathcal{D}_{y^T}(\overline{w}_1)$.

If $TB_R^{flex,+} \geq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}(\overline{w}_2)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^+, \overline{w}_D\right]$, then $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_1) \subset \mathcal{D}_{y^T}(\overline{w}_2)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$, then $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}(\overline{w}_2)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^+, \overline{w}_D\right)$, then $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. Equivalently, $\mathcal{D}_{y^T}(\overline{w}_1) \subset \mathcal{D}_{y^T}(\overline{w}_2)$.

Proposition B9 (Crisis Region Expansion)**.** *For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*

i) If $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, then $\tilde{C}_{y^T}(\overline{w}_1) \subseteq \tilde{C}_{y^T}(\overline{w}_2)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_R^+\right]$, then $\tilde{C}_{y^T}(\overline{w}_1) \subset \tilde{C}_{y^T}(\overline{w}_2)$.

ii) Under $TB_R^{flex,+} \leq 0$, $F(h) = h$, and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\tilde{C}_{y^T}(\overline{w}_1) \subseteq \tilde{C}_{y^T}(\overline{w}_2)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_R^+\right)$, then $\tilde{C}_{y^T}(\overline{w}_1) \subset \tilde{C}_{y^T}(\overline{w}_2)$.

B.2 Costs from Future Depreciations

In this section, we consider a version of the model in which the costs from depreciating the exchange rate arise from the expectation of *future* depreciations, rather than from the current one. We assume that every period, there is a cost incurred today whenever there is positive expected depreciation. Following the section above, assume an exogenous long-run level of exchange rate $\overline{e} > 0$ and that there is penalty $\Phi = (\mathbb{E}e_{t+1} - \overline{e}) \geq 0$ associated with an expected exchange rate above this level. We assume the following increasing convex function as the expected devaluation penalty: $\Phi(\mathbb{E}e_{t+1} - \overline{e}) = \phi(\mathbb{E}e_{t+1} - \overline{e})^2$.

Using the downward nominal wage rigidity constraint, the equilibrium exchange rate policy can be described as a function of tradable consumption and labor:

$$e(c^T, h) = \max\left\{\frac{\overline{W}}{\mathcal{W}\left(c^T, h\right)}, \overline{e}\right\}.$$

In this setup, under a discretionary optimal policy, the government will choose to deliver the devaluation that is necessary to achieve full employment ex post. This devaluation will be excessive from an ex ante point of view, in the spirit of Barro and Gordon (1983). A government would like to promise a lower depreciation in the future, but ex post, the optimal policy is always to achieve full employment by depreciating the exchange rate. In a Markov equilibrium, the government will take into consideration how its current choices affect future exchange rate policies because this will affect the current devaluation costs.

Let us denote the equilibrium devaluation penalty in repayment and default scenarios as

$$\Phi_R\left(b, \mathbf{s}\right) \equiv \phi\left(e\left(\hat{c}\left(b, \mathbf{s}\right), \overline{h}\right) - \overline{e}\right)^2 \qquad \text{and} \qquad \Phi_D\left(y^T\right) \equiv \phi\left(e\left(y^T, \overline{h}\right) - \overline{e}\right)^2,$$

where the equilibrium consumption in repayment is defined as

$$\hat{c}\left(b, \mathbf{s}\right) = y^T - \delta b + q\left(\hat{b}\left(b, \mathbf{s}\right), b, \mathbf{s}\right)\left(\hat{b}\left(b, \mathbf{s}\right) - (1-\delta)b\right).$$

First, the maximization problem of the government between defaulting or repaying debt remains unchanged as

$$V\left(b, \mathbf{s}\right) = \max\left\{V_R\left(b, \mathbf{s}\right), V_D\left(y^T\right)\right\}.$$

Nevertheless, the value of default and repayment will incorporate the costs of devaluating in future periods. The maximization problem in default can be written as

$$\begin{aligned} V_D\left(y^T\right) = \max_{c^T} & \left\{u\left(c^T, F(\overline{h})\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi\left(V\left(0, \mathbf{s}'\right) - \Phi_R\left(0, \mathbf{s}'\right)\right) + (1-\psi)\left(V_D\left(y^{T\prime}\right) - \Phi_D\left(y^{T\prime}\right)\right)\right]\right\} \\ \text{s.t.} \quad & c^T = y^T. \end{aligned}$$

Meanwhile the value of repayment can be written as

$$\begin{aligned} V_R\left(b, \mathbf{s}\right) = \max_{b', c^T} & \left\{u\left(c^T, F(\overline{h})\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right) - \Phi_R\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t.} \quad & c^T - q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \end{aligned}$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition on the part of international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$\begin{aligned} V_R^+\left(b, y^T\right) = \max_{b', c^T} & \left\{u\left(c^T, F\left(\overline{h}\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right) - \Phi_R\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t.}\ & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b. \end{aligned}$$

Call $\hat{b}_R^+\left(b, y^T\right)$ the optimal solution that solves the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} \equiv \left\{ \left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \qquad \hat{b}_R^+(b, y^T) > (1-\delta)b \right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $(b, y^T) \notin \mathcal{B}$, the government finds it optimal to reduce debt issuances. In this case, we can say that $V_R^-(b, y^T) = V_R^+(b, y^T)$ because the government is buying back its debt. Nevertheless, if $(b, y^T) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set the price of debt to zero representing their reluctance to issue new debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^-\left(b, y^T\right) = \max_{c^T} \left\{ u\left(c^T, F\left(\overline{h}\right)\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right) - \Phi_R\left((1-\delta)b, \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T - \delta b.$$

Table 3 shows how an economy with a flexible exchange rate but which incurs these expected devaluation costs remains relatively immune to a rollover crisis, as in the baseline model.

Table 3: Sensitivity to Expected Devaluation Costs

Devaluation penalty ϕ	Benchmark	0.20	0.40	0.60	0.80
Average spread	2.40	2.01	1.37	1.51	1.43
Average debt-income	29.61	28.52	25.58	26.14	25.04
Spread volatility	1.29	1.11	0.95	1.00	1.09
Unemployment increase	0.00	0.00	0.00	0.00	0.00
Fraction of time in crisis region	0.80	0.62	0.45	0.46	0.37
Fraction of defaults due to rollover crisis	0.96	1.28	1.16	0.68	0.92

Notes: All parameter values correspond to the benchmark calibrations for flexible exchange rate regimes. The benchmark calibration uses $\phi = 0.0$; that is, there are no penalties from expected depreciations.

C Inflation Targeting

In this section, we present a version of the model in which the government adopts an inflation-targeting regime. The goal is to consider a regime that falls in the middle between a fully flexible exchange rate regime that achieves full employment at every period and a fixed exchange rate regime that fully stabilizes the nominal exchange rate. In this intermediate regime, the economy has monetary autonomy, yet inflation-targeting acts as a monetary policy constraint that limits the ability to achieve full employment.

In line with the inflation targeting regime, we assume there is a long-run aggregate consumption price level $\overline{P} > 0$. Using the final consumption aggregator, we can construct the final consumption price aggregator as

$$P\left(P^T, P^N\right) \equiv \left(\omega^{\frac{1}{1+\mu}}\left(P^T\right)^{\frac{\mu}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}}\left(P^N\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}}.$$

Hence, the inflation-targeting condition that must be satisfied can be expressed as $P\left(P^T, P^N\right) = \overline{P}$. Define the real aggregate price function as

$$\mathcal{P}\left(c^T, h\right) \equiv \frac{1}{\omega}\left(\frac{c^T}{c\left(c^T, F(h)\right)}\right)^{1+\mu}.$$

Lemma C9. *The inflation-targeting condition yields an exchange rate policy* $e = \overline{P}/\mathcal{P}\left(c^T, h\right)$.

Proof. The real aggregate price function is increasing in the consumption of tradable goods and decreasing in labor. Furthermore, realize that the final consumption price aggregator with the law of one price and the optimality conditions from the households and firms can be rewritten as

$$\begin{aligned}
P\left(e, P^N\right) &= e\left(\omega^{\frac{1}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}}\left(\frac{P^N}{e}\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}} \\
&= e\left(\omega^{\frac{1}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}}\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu}\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}} \\
&= \frac{e}{\omega}\left(\frac{c^T}{c\left(c^T, F(h)\right)}\right)^{1+\mu} \\
&= e\mathcal{P}\left(c^T, h\right).
\end{aligned}$$

Hence, the exchange rate policy follows $e = \overline{P}/\mathcal{P}\left(c^T, h\right)$. □

Define the aggregate price real wage function as $\mathcal{F}\left(c^T, h\right) \equiv \mathcal{W}\left(c^T, h\right)/\mathcal{P}\left(c^T, h\right)$. In this way, the downward nominal wage rigidity using the exchange rate that follows the inflation-targeting condition transforms to $\overline{W} \leq \overline{P}\mathcal{F}\left(c^T, h\right)$.

Lemma C10. *The aggregate price real wage function is increasing in the consumption of tradables and decreasing in labor.*

Proof. Realise that the aggregate price real wage function can be rewritten as

$$\mathcal{F}\left(c^T, h\right) = \frac{\mathcal{W}\left(c^T, h\right)}{\mathcal{P}\left(c^T, h\right)} = (1-\omega)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu} F'(h).$$

Taking the partial derivatives for the aggregate price real wage function, we find that

$$\frac{\partial \mathcal{F}}{\partial c^T} = \omega(1-\omega)(1+\mu)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu}\left(\frac{c\left(c^T, F(h)\right)}{c^T}\right)^{\mu}\left(\frac{F'(h)}{c^T}\right) > 0 \qquad \text{and}$$

$$\frac{\partial \mathcal{F}}{\partial h} = \omega(1-\omega)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu} F'(h)\left[\frac{F''(h)}{F'(h)} - (1+\mu)\left(\frac{c\left(c^T, F(h)\right)}{c^T}\right)^{\mu}\frac{F'(h)}{F(h)}\right] < 0,$$

because $F(\cdot)$ is concave, and thus $F''(\cdot) < 0$. In other words, the aggregate price real wage function is increasing in the consumption of tradables and decreasing in labor. □

We will first define the government problem and the bond pricing under this new environment. The problem of the government to either default or repay debt can be described as

$$V(b, \mathbf{s}) = \max_{d\in\{0,1\}} \left\{ dV_D(y^T) + (1-d)V_R(b, \mathbf{s}) \right\}.$$

In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{c^T, h\leq \bar{h}} & \left\{u\left(c^T, F\left(\bar{h}\right)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{W} \leq \overline{P}\mathcal{F}\left(c^T, h\right). \end{aligned}$$

The value of repayment transforms to

$$\begin{aligned} V_R(b, \mathbf{s}) = \max_{b', c^T, h\leq \bar{h}} & \left\{u\left(c^T, F\left(\bar{h}\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T = y^T - \delta b + q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) \\ & \overline{W} \leq \overline{P}\mathcal{F}\left(c^T, h\right). \end{aligned}$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition on the part of international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$\begin{aligned} V_R^+\left(b, y^T\right) = \max_{b', c^T, h\leq \bar{h}} & \left\{u\left(c^T, F\left(h\right)\right) + \beta\mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\ & \overline{W} \leq \overline{P}\mathcal{F}\left(c^T, h\right). \end{aligned} \tag{C.1}$$

Call $\hat{b}_R^+\left(b, y^T\right)$ the optimal solution that solves the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} = \left\{\left(b, y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \hat{b}_R^+\left(b, y^T\right) > (1-\delta)b\right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b, y^T\right) \notin \mathcal{B}$, the government finds it optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b, y^T\right) = V_R^+\left(b, y^T\right)$ because the government is buying back its debt. Nevertheless, if $\left(b, y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$, representing their reluctance to issue new debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^- \left(b, y^T\right) = \max_{c^T, h \leq \overline{h}} \left\{u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \tag{C.2}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{W} \leq \overline{P}\mathcal{F}\left(c^T, h\right).$$

The following lemma follows the same steps as the one stated before, following the fact that V_R^- is a particular case of the V_R^+ maximization problem.

Lemma C11. *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level b, we have that $V_R^+\left(b, y^T\right) \geq V_R^-\left(b, y^T\right)$.*

Now let us define the safe zone, default zone, and repayment zone

$$\begin{aligned}
\mathcal{S} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b, y^T\right)\right\} \\
\mathcal{D} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b, y^T\right)\right\} \\
\mathcal{C} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b, y^T\right) \quad \text{and} \quad V_D(y^T) > V_R^-\left(b, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the bond pricing following the no-arbitrage condition can be represented by the following recursion

$$\tilde{q}(b', y^T) = \frac{1}{1+r}\mathbb{E}\left[\left(1 - d(b', \mathbf{s}')\right)\left(\delta + (1-\delta)q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right)\right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}.$$

Now define the real aggregate wage rigidity as $\overline{w} \equiv \overline{W}/\overline{P}$ and let us now focus on the wage rigidity environment $\overline{w} = 0$. Call the solutions to it $\left\{V^{flex}, V_D^{flex}, \tilde{q}^{flex}\right\}$, and let us study the one-period rigidity shock. The value of default will transform to

$$\tilde{V}_D\left(y^T\right) = \max_{c^T, h \leq \overline{h}} \left\{u\left(c^T, F\left(h\right)\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V^{flex}\left(0, \mathbf{s}'\right) + (1-\psi)V_D^{flex}\left(y^{T\prime}\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T$$
$$\overline{w} \leq \mathcal{F}(c^T, h).$$

Also, the value of repayment when rollover debt is allowed is

$$\tilde{V}_R^+\left(b, y^T\right) = \max_{b', c^T, h \leq \bar{h}} \left\{ u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left(b', \mathbf{s}'\right)\right] \right\}$$
$$\text{s.t. } c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b$$
$$\overline{w} \leq \mathcal{F}(c^T, h).$$

Finally, the value of repayment when new debt contracts are forbidden is

$$\tilde{V}_R^-\left(b, y^T\right) = \max_{c^T, h \leq \bar{h}} \left\{ u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left((1-\delta)b, \mathbf{s}'\right)\right] \right\}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{w} \leq \mathcal{F}(c^T, h)$$

The following lemmas and propositions follow the same steps stated in the previous section.

Lemma C12. *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to the debt b*

Lemma C13 (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists levels of debt that currently matures $\bar{b}^+, \bar{b}^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\bar{b}^-, y^T\right)$. Furthermore, $\bar{b}^+ \geq \bar{b}^-$.*
(To avoid clutter, we omit the dependence of these thresholds on y^T, but it should be understood throughout that the thresholds depend on y^T.)

Now, call the regions

$$\tilde{S}_{y^T}\left(\overline{w}\right) \equiv \left(-\infty, \bar{b}^-\left(\overline{w}\right)\right], \quad \tilde{C}_{y^T}\left(\overline{w}\right) \equiv \left(\bar{b}^-\left(\overline{w}\right), \bar{b}^+\left(\overline{w}\right)\right], \quad \text{and} \quad \tilde{D}_{y^T}\left(\overline{w}\right) \equiv \left(\bar{b}^+\left(\overline{w}\right), \infty\right).$$

The following propositions follow the same steps stated in the paper for Proposition 2, Proposition 3, and Proposition 4. The wage rigidity thresholds follow the same narrative as in Lemma 7.

Proposition C10 (Safe Region Threshold)**.** *For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^-\left(\overline{w}_2\right) \leq \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subseteq \mathcal{S}_{y^T}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_D\right]$, then $\bar{b}^-\left(\overline{w}_2\right) < \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subset \mathcal{S}_{y^T}\left(\overline{w}_1\right)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\bar{b}^-\left(\overline{w}_2\right) \leq \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subseteq \mathcal{S}_{y^T}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_D\right)$, then $\bar{b}^-\left(\overline{w}_2\right) < \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subset \mathcal{S}_{y^T}\left(\overline{w}_1\right)$.

Proposition C11 (Default Region Threshold)**.** *For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*
If $TB_R^{flex,+} \leq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, then $\bar{b}^+\left(\overline{w}_1\right) \leq \bar{b}^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}\left(\overline{w}_2\right) \subseteq \mathcal{D}_{y^T}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_D, \overline{w}_R^+\right]$, then $\bar{b}^+\left(\overline{w}_1\right) < \bar{b}^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}\left(\overline{w}_2\right) \subset \mathcal{D}_{y^T}\left(\overline{w}_1\right)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$, then $\bar{b}^+\left(\overline{w}_1\right) \leq \bar{b}^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}\left(\overline{w}_2\right) \subseteq \mathcal{D}_{y^T}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_D, \overline{w}_R^+\right)$, then $\bar{b}^+\left(\overline{w}_1\right) < \bar{b}^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}\left(\overline{w}_2\right) \subseteq \mathcal{D}_{y^T}\left(\overline{w}_1\right)$.

If $TB_R^{flex,+} \geq 0$*:*

i) *If* $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, *then* $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_2 \in \left(\overline{w}_R^+, \overline{w}_D\right]$, *then* $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subset \mathcal{D}_{y^T}(\overline{w}_2)$.

ii) *Under* $F(h) = h$ *and* $u(c) = \ln(c)$*; if* $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$, *then* $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_1 \in \left[\overline{w}_R^+, \overline{w}_D\right)$, *then* $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subset \mathcal{D}_{y^T}(\overline{w}_2)$.

Proposition C12 (Crisis Region Expansion)**.** *For every* y^T *and taking arbitrary wage rigidities* $\overline{w}_1 < \overline{w}_2$, *the following claims hold:*

i) *If* $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subseteq \tilde{C}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_R^+\right]$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subset \tilde{C}_{y^T}(\overline{w}_2)$.

ii) *Under* $TB_R^{flex,+} \leq 0$, $F(h) = h$, *and* $u(c) = \ln(c)$*; if* $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subseteq \tilde{C}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_R^+\right)$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subset \tilde{C}_{y^T}(\overline{w}_2)$.

D Maturity Choice

In this section we expand our baseline model to show that our theoretical results hold when the government chooses a portfolio of bonds with different maturities.

Define the set of different debt maturities in the beginning of the period t as $\mathbf{b}_t \equiv \{b_{t,n}\}_{n=0}^{\infty}$, where $b_{t,n}$ represents the amount of the government's debt due n periods ahead. In this sense, when the government has access to international markets, it chooses a new portfolio of debt with different maturities $\mathbf{b}_{t+1}$. Then, the budget constraint of the government in period t can be described as

$$c_t^T = y_t^T - b_t + \sum_{n=1}^{\infty} q_{t,n} \left(b_{t+1,n-1} - b_{t,n}\right),$$

where $q_{t,n}$ is the price of the bond in period t that matures n periods ahead.

We will first define the government problem and the bond pricing under this new environment. The problem of the government to either default or repay debt can be described as

$$V(\mathbf{b}, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ dV_D(y^T) + (1-d) V_R\left(\mathbf{b}, \mathbf{s}\right) \right\}.$$

We will assume that if the government defaults, then it will default on all of its portfolios of different debt maturities. In addition, we will assume that when the government reenters international financial markets, then its portfolio resets to zero for all of the different debt maturities. In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{e, c^T, h \leq \bar{h}} & \left\{ u\left(c^T, h\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V\left(\mathbf{0}, \mathbf{s}'\right) + (1-\psi) V_D\left(y^{T\prime}\right)\right] \right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{W} \leq e \mathcal{W}\left(c^T, h\right), \end{aligned}$$

where $\mathbf{0}$ is an infinite vector with zeros for all entries. The value of repayment transforms to

$$\begin{aligned} V_R\left(\mathbf{b}, \mathbf{s}\right) = \max_{e, \mathbf{b}', c^T, h \leq \bar{h}} & \left\{ u\left(c^T, h\right) + \beta \mathbb{E}\left[V\left(\mathbf{b}', \mathbf{s}'\right)\right] \right\} \\ \text{s.t. } c^T &= y^T - b_0 + \sum_{n=1}^{\infty} q_n\left(\mathbf{b}', \mathbf{b}, \mathbf{s}\right)\left(b'_{n-1} - b_n\right) \\ \overline{W} &\leq e \mathcal{W}\left(c^T, h\right), \end{aligned}$$

where $q_n\left(\mathbf{b}', \mathbf{b}, \mathbf{s}\right)$ is the bond pricing that is contingent not only on the future portfolio of debt maturities $\mathbf{b}'$ but also on the current portfolio of debt maturities $\mathbf{b}$ and the sunspot ζ.

The value of repayment can be studied under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition on the part of international lenders is applied. As before, call this fundamental bond pricing $q_n\left(\mathbf{b}', y^T\right)$ for every single bond with maturity n. Then, the value of repayment when rollover is allowed transforms to

$$V_R^+\left(\mathbf{b}, y^T\right) = \max_{e,\mathbf{b}',c^T,h\le\overline{h}} \left\{u\left(c^T,h\right) + \beta\mathbb{E}\left[V\left(\mathbf{b}',\mathbf{s}'\right)\right]\right\} \tag{D.1}$$
$$\text{s.t. } c^T = y^T - b_0 + \sum_{n=1}^{\infty} \tilde{q}_n\left(\mathbf{b}', y^T\right)\left(b'_{n-1} - b_n\right)$$
$$\overline{W} \le e\mathcal{W}\left(c^T, h\right).$$

Call $\hat{\mathbf{b}}_R^+\left(\mathbf{b}, y^T\right)$ the optimal solution to new portfolio debt maturities that solve the previous problem. Call the state space in which it is optimal for the government to create new debt contracts that incur in a positive flux of resources from international lenders as

$$\mathcal{B} = \left\{\left(\mathbf{b}, y^T\right) \in \mathbb{R}^\infty \times \mathbb{R}_+ : \quad \sum_{n=1}^{\infty} \tilde{q}_n\left(\hat{\mathbf{b}}\left(\mathbf{b}, y^T\right), y^T\right)\left(\hat{b}^+_{R,n-1}\left(\mathbf{b}, y^T\right) - b_n\right) > 0\right\}.$$

If $\left(\mathbf{b}, y^T\right) \in \mathcal{B}$, we can say that the government finds it optimal in that state to overall incur more debt overall with international lenders. In other words, the government increases its net debt issuances.

The value of repayment when rollover is not allowed can be divided into two cases. When $\left(\mathbf{b}, y^T\right) \notin \mathcal{B}$, the government finds it optimal to reduce net debt issuances, changing its portfolio of different debt maturities. In this case, we can say that $V_R^-\left(\mathbf{b}, y^T\right) = V_R^+\left(\mathbf{b}, y^T\right)$ because the international lenders are being repaid instead of being asked for more net debt. Nevertheless, if $\left(\mathbf{b}, y^T\right) \in \mathcal{B}$, then the government wants to increase its net debt issuances. International lenders set a price of $\tilde{q}_n = 0$ for all different maturities n, representing their reluctance to issue new debt. In this way, the value of repayment when new debt contracts of any maturity are forbidden can be expressed as

$$V_R^-\left(\mathbf{b}, y^T\right) = \max_{e,c^T,h\le\overline{h}} \left\{u\left(c^T,h\right) + \beta\mathbb{E}\left[V\left(\left\{b_n\right\}_{n=1}^{\infty}, \mathbf{s}'\right)\right]\right\} \tag{D.2}$$
$$\text{s.t. } c^T = y^T - b_0$$
$$\overline{W} \le e\mathcal{W}\left(c^T, h\right).$$

The following lemma follows the same steps as the one stated before, with the difference that now the option of choice is not a single debt level but a portfolio of different debt maturities.

Lemma D14. *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt portfolio $\boldsymbol{b} = \{b_n\}_{n=0}^{\infty} \in \mathbb{R}^\infty$, we have that $V_R^+\left(\boldsymbol{b}, y^T\right) \ge V_R^-\left(\boldsymbol{b}, y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone contingent to the portfolio of different debt maturities as

$$\begin{aligned}
\mathcal{S}(\mathbf{b}) &\equiv \left\{y^T \in \mathbb{R}_+ : \quad V_D(y^T) \le V_R^-\left(\mathbf{b}, y^T\right)\right\} \\
\mathcal{D}(\mathbf{b}) &\equiv \left\{y^T \in \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(\mathbf{b}, y^T\right)\right\} \\
\mathcal{C}(\mathbf{b}) &\equiv \left\{y^T \in \mathbb{R}_+ : \quad V_D(y^T) \le V_R^+\left(\mathbf{b}, y^T\right) \quad \text{and} \quad V_D(y^T) > V_R^-\left(\mathbf{b}, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the probability of defaulting for each forward period using the optimal portfolio of different debt maturities can be defined as

$$p_n(\mathbf{b}', y^T) \equiv (1-\pi)\left[\int_{\mathcal{S}(\mathbf{b}')\cup\mathcal{C}(\mathbf{b}')} p_{n-1}\left(\hat{\mathbf{b}}\left(\mathbf{b}', y^{T\prime}\right), y^{T\prime}\right) dF(\mathbf{b}'', y^{T\prime}) dF(y^{T\prime}|y^T)\right] + \pi\left[\int_{\mathcal{S}(\mathbf{b}')} p_{n-1}\left(\hat{\mathbf{b}}\left(\mathbf{b}', y^{T\prime}\right), y^{T\prime}\right) dF(y^{T\prime}|y^T)\right],$$

for every forward period $n \in \mathbb{N}$ and where $p_0 = 1$. Taking these probabilities, the bond pricing following the no-arbitrage condition for each maturity structure can be represented by the following recursion:

$$\tilde{q}_n(\mathbf{b}', y^T) = \left(\frac{1}{1+r}\right)^n p_n\left(\mathbf{b}', y^T\right), \qquad \text{for every } n \in \mathbb{N}.$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q_n\left(\mathbf{b}', \mathbf{b}, \mathbf{s}\right) = \begin{cases} 0 & \text{if } y^T \in \mathcal{D}\left(\mathbf{b}\right) \\ 0 & \text{if } y^T \in \mathcal{C}\left(\mathbf{b}\right) \quad \text{and} \quad \zeta = 1 \\ \tilde{q}_n(\mathbf{b}', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(\mathbf{b}, \mathbf{s}\right) = \begin{cases} 1 & \text{if } y^T \in \mathcal{D}\left(\mathbf{b}\right) \\ 0 & \text{if } y^T \in \mathcal{C}\left(\mathbf{b}\right) \quad \text{and} \quad \zeta = 0 \\ 1 & \text{if } y^T \in \mathcal{C}\left(\mathbf{b}\right) \quad \text{and} \quad \zeta = 1 \\ 0 & \text{if } y^T \in \mathcal{S}\left(\mathbf{b}\right) \end{cases}$$

The following proposition follows the same steps as the previous one.

Proposition D13 (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states*

Let us now focus on the flexible exchange rate regime and solve the model. Call the flexible exchange rate regime solutions $\left\{V^{flex}, V_D^{flex}, \left\{\tilde{q}_n^{flex}\right\}_{n=1}^{\infty}\right\}$, and let us study the one-period fixed exchange rate regime shocks. The value of default will transform to

$$\begin{aligned} \tilde{V}_D\left(y^T\right) = \max_{c^T, h \leq \overline{h}} & \left\{u\left(c^T, F(h)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V^{flex}\left(\mathbf{0}, \mathbf{s}'\right) + (1-\psi)V_D^{flex}\left(y^{T\prime}\right)\right]\right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{w} \leq \mathcal{W}\left(c^T, h\right). \end{aligned}$$

Also, the value of repayment when rollover debt is allowed is

$$\begin{aligned} \tilde{V}_R^+\left(\mathbf{b}, y^T\right) = \max_{\mathbf{b}', c^T, h \leq \overline{h}} & \left\{u\left(c^T, F(h)\right) + \beta\mathbb{E}\left[V^{flex}\left(\mathbf{b}', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T = y^T - b_0 + \sum_{n=1}^{\infty} \tilde{q}_n^{flex}\left(\mathbf{b}', y^T\right)\left(b'_{n-1} - b_n\right) \\ & \overline{w} \leq \mathcal{W}(c^T, h). \end{aligned}$$

Finally, the value of repayment when new debt contracts of any maturity are forbidden is then

$$\tilde{V}_R^-\left(\mathbf{b}, y^T\right) = \max_{\mathbf{b}', c^T, h \leq \bar{h}} \left\{ u\left(c^T, F(h)\right) + \beta \mathbb{E}\left[V^{flex}\left(\{b_n\}_{n=1}^{\infty}, \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T - b_0$$
$$\overline{w} \leq \mathcal{W}(c^T, h).$$

For convenience, define the portfolio of current, not yet matured debt as $\mathbf{b}_{-0} = \{b_n\}_{n=1}^{\infty}$. The following lemmas and propositions follow the same steps stated in the previous section, with the difference that besides fixing a tradable endowment $y^T \in \mathbb{R}_+$, we also fix a portfolio of current, not yet matured debt $\mathbf{b}_{-0} \in \mathbb{R}^{\infty}$. The important level of debt to study is the current debt matured in the period $b_0 \in \mathbb{R}$. We focus on this amount of debt and define the regions studied before.

Lemma D15. *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to debt that currently matures b_0*

Lemma D16 (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$ and portfolio of current, not yet matured debt $\boldsymbol{b}_{-0} \in \mathbb{R}^{\infty}$, there exists levels of debt that currently mature $\bar{b}_0^+, \bar{b}_0^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\left\{\bar{b}_0^+, \boldsymbol{b}_{-0}\right\}, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\left\{\bar{b}_0^-, \boldsymbol{b}_{-0}\right\}, y^T\right)$. Furthermore, $\bar{b}_0^+ \geq \bar{b}_0^-$.*
(To avoid clutter, we omit the dependence of these thresholds on y^T, but it should be understood throughout that the thresholds depend on y^T.)

Now call the regions in the space of the debt maturing in the current period as

$$\tilde{S}_{y^T}^{\mathbf{b}_{-0}}\left(\overline{w}\right) \equiv \left(-\infty, \bar{b}_0^-\left(\overline{w}\right)\right], \qquad \tilde{C}_{y^T}^{\mathbf{b}_{-0}}\left(\overline{w}\right) \equiv \left(\bar{b}_0^-\left(\overline{w}\right), \bar{b}_0^+\left(\overline{w}\right)\right], \qquad \text{and} \qquad \tilde{D}_{y^T}^{\mathbf{b}_{-0}}\left(\overline{w}\right) \equiv \left(\bar{b}_0^+\left(\overline{w}\right), \infty\right).$$

The following propositions follow the same steps stated in the paper for Proposition 2, Proposition 3, and Proposition 4 by adding a given arbitrary distribution of debt not maturing in the current period $\mathbf{b}_{-0}$. The wage rigidity thresholds follow the same narrative as in Lemma 7.

Proposition D14 (Safe Region Threshold)**.** *For every y^T, $\boldsymbol{b}_{-0}$; and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}_0^-\left(\overline{w}_2\right) \leq \bar{b}_0^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subseteq \mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_D\right]$, then $\bar{b}_0^-\left(\overline{w}_2\right) < \bar{b}_0^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subset \mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\bar{b}_0^-\left(\overline{w}_2\right) \leq \bar{b}_0^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subseteq \mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_D\right)$, then $\bar{b}_0^-\left(\overline{w}_2\right) < \bar{b}_0^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subset \mathcal{S}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$.

Proposition D15 (Default Region Threshold)**.** *For every y^T, $\boldsymbol{b}_{-0}$; and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*
If $TB_R^{flex,+} \leq 0$:

i) If $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, then $\bar{b}_0^+\left(\overline{w}_1\right) \leq \bar{b}_0^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subseteq \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_D, \overline{w}_R^+\right]$, then $\bar{b}_0^+\left(\overline{w}_1\right) < \bar{b}_0^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subset \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$, then $\bar{b}_0^+\left(\overline{w}_1\right) \leq \bar{b}_0^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subseteq \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_D, \overline{w}_R^+\right)$, then $\bar{b}_0^+\left(\overline{w}_1\right) < \bar{b}_0^+\left(\overline{w}_2\right)$. Equivalently, $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_2\right) \subseteq \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}\left(\overline{w}_1\right)$.

If $TB_R^{flex,+} \geq 0$*:*

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$*, then* $\bar{b}_0^+(\overline{w}_2) \leq \bar{b}_0^+(\overline{w}_1)$*. Equivalently,* $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*. Moreover, if* $\overline{w}_2 \in \left(\overline{w}_R^+, \overline{w}_D\right]$*, then* $\bar{b}_0^+(\overline{w}_2) < \bar{b}_0^+(\overline{w}_1)$*. Equivalently,* $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subset \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*.*

ii) Under $F(h) = h$ *and* $u(c) = \ln(c)$*; if* $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$*, then* $\bar{b}_0^+(\overline{w}_2) \leq \bar{b}_0^+(\overline{w}_1)$*. Equivalently,* $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*. Moreover, if* $\overline{w}_1 \in \left[\overline{w}_R^+, \overline{w}_D\right)$*, then* $\bar{b}_0^+(\overline{w}_2) < \bar{b}_0^+(\overline{w}_1)$*. Equivalently,* $\mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subset \mathcal{D}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*.*

Proposition D16 (Crisis Region Expansion)**.** *For every* y^T *and taking arbitrary wage rigidities* $\overline{w}_1 < \overline{w}_2$*, the following claims hold:*

i) If $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$*, then* $\tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subseteq \tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*. Moreover, if* $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_R^+\right]$*, then* $\tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subset \tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*.*

ii) Under $TB_R^{flex,+} \leq 0$*,* $F(h) = h$*, and* $u(c) = \ln(c)$*; if* $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$*, then* $\tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subseteq \tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*. Moreover, if* $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_R^+\right)$*, then* $\tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_1) \subset \tilde{C}_{y^T}^{\boldsymbol{b}_{-0}}(\overline{w}_2)$*.*

E Elastic Labor Supply

In this section, we expand the baseline model to allow for an elastic supply of labor. While the amount of hours will continue to be demand determined when wage rigidity is binding, the amount of hours will not be fixed at $\bar{w}$ when wage rigidity is slack because households will adjust their labor supply. In this setup, the household problem is to solve

$$\max_{h_t, c_t^N, c_t^T} \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \right] \right\}$$
$$\text{s.t. } P_t^T c_t^T + P_t^N c_t^N = P_t^T y_t^T + W_t h_t + \phi_t + T_t$$
$$c_t = \left(\omega \left(c_t^T \right)^{-\mu} + (1-\omega) \left(c_t^N \right)^{-\mu} \right)^{-\frac{1}{\mu}}.$$

The first-order conditions are

$$\omega \left(\frac{c_t}{c_t^T} \right)^{1+\mu} U_c(t) = \lambda_t P_t^T \qquad (1-\omega) \left(\frac{c_t}{c_t^N} \right)^{1+\mu} U_c(t) = \lambda_t P_t^N \qquad -U_h(t) = \lambda_t W_t.$$

Let us drop the time subscript and define the real wages and nontradable prices as $w = W/P^T$ and $p^N = P^N/P^T$. Joining the first-order conditions, we have

$$\frac{1-\omega}{\omega} \left(\frac{c^T}{c^N} \right)^{1+\mu} = p^N \qquad \& \qquad \frac{w}{p^N} = \frac{1}{1-\omega} \left(\frac{-U_h}{U_c} \right) \left(\frac{c^N}{c} \right)^{1+\mu}$$

Recall the first-order condition of the firm and the market clearing of non-tradable goods $p^N F'(h) = w$ and $c_N = F(h)$, respectively. Hence,

$$1 = (1-\omega) F'(h) \left(\frac{U_c}{-U_h} \right) \left(\frac{c}{F(h)} \right)^{1+\mu}.$$

Assumption E1. *The production function and utility functions can be described respectively as*

$$F(h) = h^{\alpha} \qquad \textit{and} \qquad U(c,h) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{h^{1+\nu}}{1+\nu}.$$

It will be useful to establish the following lemma, which is analogous to Lemma 1 in the main text.

Lemma E17. *Under a flexible exchange rate regime, the real wage function is increasing with respect to the consumption of tradables, and optimal labor supply is increasing in the consumption of tradables,* $\frac{dh}{dc^T} > 0$ *and* $\frac{\partial \mathcal{W}}{\partial c^T} > 0$*, respectively.*

Proof. Define the function $\mathcal{F}(c_T, h)$ from joining the first-order conditions from the households and firms as

$$\mathcal{F}(c^T, h) \equiv h^{\alpha(1-\sigma)-(1+\nu)} \left(\omega \left(\frac{c^T}{h^{\alpha}} \right)^{-\mu} + (1-\omega) \right)^{\frac{1+\mu-\sigma}{-\mu}} = \frac{\chi}{\alpha(1-\omega)}.$$

Therefore,

$$\frac{dh}{dc^T} = \frac{\partial \mathcal{F}/\partial c^T}{-\partial \mathcal{F}/\partial h} = \frac{h}{c^T}\left(\frac{1+\mu}{\alpha(1+\mu) + ((1+\nu) - \alpha(1-\sigma))\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}\right).$$

In other words,

$$\frac{c^T}{h}\frac{dh}{dc^T} = \frac{1+\mu}{\alpha(1+\mu) + ((1+\nu) - \alpha(1-\sigma))\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}.$$

Because $\left(\frac{1-\omega}{\omega}\right)\left(\frac{c_T}{h^\alpha}\right)^\mu > 0$, then

$$\begin{aligned}0 &< \alpha(\sigma+\mu) + (1+\nu)\\ &= \alpha(1+\mu) + (1+\nu) - \alpha(1-\sigma)\\ &< \alpha(1+\mu) + ((1+\nu) - \alpha(1-\sigma))\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right).\end{aligned}$$

In other words, $\frac{dh}{dc^T} > 0$.

Now, also realize that

$$\mathcal{W}(c^T, h) = \alpha\frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu} F'(h) = \left(\frac{1-\omega}{\omega}\right)\frac{(c^T)^{1+\mu}}{h^{1+\alpha\mu}}.$$

So,

$$\frac{\partial \mathcal{W}}{\partial c^T} = \frac{\alpha(1+\mu)}{h}\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)\left(1 - \left(\frac{1+\alpha\mu}{1+\mu}\right)\frac{c^T}{h}\frac{dh}{dc^T}\right).$$

Therefore,

$$\begin{aligned}1 - \left(\frac{1+\alpha\mu}{1+\mu}\right)\frac{c^T}{h}\frac{dh}{dc^T} &= 1 - \frac{1+\alpha\mu}{\alpha(1+\mu) + ((1+\nu) - \alpha(1-\sigma))\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}\\ &= \frac{((1+\nu) - \alpha(1-\sigma))\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right) - (1-\alpha)}{\alpha(1+\mu) + ((1+\nu) - \alpha(1-\sigma))\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right)}.\end{aligned}$$

Because $\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu > 0$, then

$$\begin{aligned}0 &< \alpha\sigma + \nu\\ &= \sigma + \nu - (1-\alpha)\sigma\\ &= \sigma + \nu + (1-\alpha)(1-\sigma) - (1-\alpha)\\ &= ((1+\nu) - \alpha(1-\sigma)) - (1-\alpha)\\ &< ((1+\nu) - \alpha(1-\sigma))\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{h^\alpha}\right)^\mu\right) - (1-\alpha).\end{aligned}$$

In other words, $\frac{\partial \mathcal{W}}{\partial c^T} > 0$.

Finally, we can conclude that when downward nominal wage rigidity is not binding, then the real wage function is increasing with respect to the consumption of tradables, and optimal labor is increasing in the consumption of tradables. □

To ease the formulation, define the equilibrium labor function in terms of the consumption of tradables as $\hat{h}\left(c^T\right)$, which satisfies the following first-order condition:

$$1 = \left(\alpha + \frac{((1+\nu) - \alpha(1-\sigma))}{1+\mu}\right)\left(1 + \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{\hat{h}\left(c^T\right)^{\alpha}}\right)^{\mu}\right)\left(\frac{\hat{h}'\left(c^T\right)}{\hat{h}\left(c^T\right)}\right)c^T.$$

Acknowledge that from Lemma E17, we know that this function satisfies $\hat{h}'(\cdot) > 0$.

Now, we will define the government problem and the bond pricing under this new environment. The problem of the government to either default or repay debt can be described as

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ dV_D(y^T) + (1-d)V_R(b, \mathbf{s}) \right\}.$$

In this way, the maximization problem in default can be described as

$$\begin{aligned} V_D\left(y^T\right) = \max_{e, c^T, h \leq \hat{h}(c^T)} & \left\{u\left(c^T, F(h)\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\} \\ \text{s.t. } & c^T = y^T \\ & \overline{W} \leq e\mathcal{W}(c^T, h). \end{aligned}$$

The value of repayment transforms to

$$\begin{aligned} V_R(b, \mathbf{s}) = \max_{e, b', c^T, h \leq \hat{h}(c^T)} & \left\{u\left(c^T, F(h)\right) + \beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T = y^T - \delta b + q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) \\ & \overline{W} \leq e\mathcal{W}(c^T, h). \end{aligned}$$

The value of repayment can be studied as before under two different scenarios: when rollover debt is allowed and when it is not. Let us start by analyzing the problem when new debt contracts can be issued and hence rollover debt is allowed. Under this scenario, the bond pricing that satisfies the no-arbitrage condition on the part of international lenders is applied. Then, the value of repayment when rollover is allowed transforms to

$$\begin{aligned} V_R^+\left(b, y^T\right) = \max_{e, b', c^T, h \leq \hat{h}(c^T)} & \left\{u\left(c^T, F(h)\right) + \beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } & c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b \\ & \overline{W} \leq e\mathcal{W}(c^T, h). \end{aligned} \tag{E.1}$$

Call $\hat{b}_R^+\left(b, y^T\right)$ the optimal solution to the previous problem. As before, call the state space in which it is optimal for the government to increase debt issuances as

$$\mathcal{B} = \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad \hat{b}_R^+\left(b, y^T\right) > (1-\delta)b\right\}.$$

As before, the value of repayment when rollover is not allowed can be divided into two cases. When $\left(b, y^T\right) \notin$

$\mathcal{B}$, the government finds it optimal to reduce debt issuances. In this case, we can say that $V_R^-\left(b, y^T\right) = V_R^+\left(b, y^T\right)$ because the government is buying back its debt. Nevertheless, if $\left(b, y^T\right) \in \mathcal{B}$, then the government wants to increase its debt issuances. International lenders set a price of $\tilde{q} = 0$, representing their reluctance to issue new debt. In this way, the value of repayment when new debt contracts are forbidden can be expressed as

$$V_R^-\left(b, y^T\right) = \max_{e, c^T, h \leq \hat{h}(c^T)} \left\{u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V\left((1-\delta)b, \mathbf{s}'\right)\right]\right\} \tag{E.2}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{W} \leq e\mathcal{W}(c^T, h).$$

The following lemma follows the same steps as the one stated before, following the fact that V_R^- is a particular case of the V_R^+ maximization problem.

Lemma E18. *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level b, we have that $V_R^+\left(b, y^T\right) \geq V_R^-\left(b, y^T\right)$.*

Now, let us define the safe zone, default zone, and repayment zone as

$$\begin{aligned}
\mathcal{S} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-\left(b, y^T\right)\right\} \\
\mathcal{D} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+\left(b, y^T\right)\right\} \\
\mathcal{C} &\equiv \left\{\left(b, y^T\right) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+\left(b, y^T\right) \quad \text{and} \quad V_D(y^T) > V_R^-\left(b, y^T\right)\right\}.
\end{aligned}$$

Using these zones, the bond pricing following the no-arbitrage condition can be represented with the following recursion

$$\tilde{q}(b', y^T) = \frac{1}{1+r}\mathbb{E}\left[\left(1 - d(b', \mathbf{s}')\right)\left(\delta + (1-\delta)q\left(\hat{b}\left(b', \mathbf{s}'\right), b', \mathbf{s}'\right)\right)\right].$$

Finally, using the zones and the multiplicity of equilibria, the overall bond pricing can be described as

$$q\left(b', b, \mathbf{s}\right) = \begin{cases} 0 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ \tilde{q}(b', y^T) & \textit{in every other case} \end{cases}$$

and the optimal default decision as

$$d\left(b, \mathbf{s}\right) = \begin{cases} 1 & \text{if } \left(b, y^T\right) \in \mathcal{D} \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 0 \\ 1 & \text{if } \left(b, y^T\right) \in \mathcal{C} \quad \text{and} \quad \zeta = 1 \\ 0 & \text{if } \left(b, y^T\right) \in \mathcal{S} \end{cases}.$$

Let us now focus on the flexible exchange rate regime and solve the model. Call the flexible exchange rate regime solutions $\left\{V^{flex}, V_D^{flex}, \tilde{q}^{flex}\right\}$ and let us study the one-period fixed exchange rate regime shocks. The value of default will transform to

$$\tilde{V}_D\left(y^T\right) = \max_{c^T, h \leq \hat{h}(c^T)} \left\{ u\left(c^T, F\left(h\right)\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V^{flex}\left(0, \mathbf{s}'\right) + (1-\psi) V_D^{flex}\left(y^{T\prime}\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T$$
$$\overline{w} \leq \mathcal{W}(c^T, h).$$

Also, the value of repayment when rollover debt is allowed

$$\tilde{V}_R^+\left(b, y^T\right) = \max_{b', c^T, h \leq \hat{h}(c^T)} \left\{ u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left(b', \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T - \tilde{q}\left(b', y^T\right)\left(b' - (1-\delta)b\right) = y^T - \delta b$$
$$\overline{w} \leq \mathcal{W}(c^T, h).$$

Finally, the value of repayment when new debt contracts are forbidden is

$$\tilde{V}_R^-\left(b, y^T\right) = \max_{c^T, h \leq \hat{h}(c^T)} \left\{ u\left(c^T, F\left(h\right)\right) + \beta \mathbb{E}\left[V^{flex}\left((1-\delta)b, \mathbf{s}'\right)\right]\right\}$$
$$\text{s.t. } c^T = y^T - \delta b$$
$$\overline{w} \leq \mathcal{W}(c^T, h).$$

The following lemmas and propositions follow the same steps stated in the previous section.

Lemma E19. *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to the debt b*

Lemma E20 (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}$, there exists levels of debt that currently matures $\bar{b}^+, \bar{b}^- \in \mathbb{R}$, such that $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D\left(y^T\right) = \tilde{V}_R^-\left(\bar{b}^-, y^T\right)$. Furthermore, $\bar{b}^+ \geq \bar{b}^-$.*
(To avoid clutter, we omit the dependence of these thresholds on y^T, but it should be understood throughout that the thresholds depend on y^T.)

Now, call the regions

$$\tilde{S}_{y^T}\left(\overline{w}\right) \equiv \left(-\infty, \bar{b}^-\left(\overline{w}\right)\right], \quad \tilde{C}_{y^T}\left(\overline{w}\right) \equiv \left(\bar{b}^-\left(\overline{w}\right), \bar{b}^+\left(\overline{w}\right)\right], \quad \text{and} \quad \tilde{D}_{y^T}\left(\overline{w}\right) \equiv \left(\bar{b}^+\left(\overline{w}\right), \infty\right).$$

The following propositions follow the same steps stated in the paper for Proposition 2, Proposition 3, and Proposition 4. The wage rigidity thresholds follow the same narrative as in Lemma 7.

Proposition E17 (Safe Region Threshold)**.** *For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*

i) If $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, then $\bar{b}^-\left(\overline{w}_2\right) \leq \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subseteq \mathcal{S}_{y^T}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_D\right]$, then $\bar{b}^-\left(\overline{w}_2\right) < \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subset \mathcal{S}_{y^T}\left(\overline{w}_1\right)$.

ii) Under $F(h) = h$ and $u(c) = \ln(c)$; if $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, then $\bar{b}^-\left(\overline{w}_2\right) \leq \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subseteq \mathcal{S}_{y^T}\left(\overline{w}_1\right)$. Moreover, if $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_D\right)$, then $\bar{b}^-\left(\overline{w}_2\right) < \bar{b}^-\left(\overline{w}_1\right)$. Equivalently, $\mathcal{S}_{y^T}\left(\overline{w}_2\right) \subset \mathcal{S}_{y^T}\left(\overline{w}_1\right)$.

Proposition E18 (Default Region Threshold)**.** *For every y^T and taking arbitrary wage rigidities $\overline{w}_1 < \overline{w}_2$, the following claims hold:*
If $TB_R^{flex,+} \leq 0$:

i) *If* $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, *then* $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_2) \subseteq \mathcal{D}_{y^T}(\overline{w}_1)$. *Moreover, if* $\overline{w}_2 \in \left(\overline{w}_D, \overline{w}_R^+\right]$, *then* $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_2) \subset \mathcal{D}_{y^T}(\overline{w}_1)$.

ii) *Under* $F(h) = h$ *and* $u(c) = \ln(c)$; *if* $\overline{w}_1, \overline{w}_2 \in [\overline{w}_D, \infty)$, *then* $\bar{b}^+(\overline{w}_1) \leq \bar{b}^+(\overline{w}_2)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_2) \subseteq \mathcal{D}_{y^T}(\overline{w}_1)$. *Moreover, if* $\overline{w}_1 \in \left[\overline{w}_D, \overline{w}_R^+\right)$, *then* $\bar{b}^+(\overline{w}_1) < \bar{b}^+(\overline{w}_2)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_2) \subseteq \mathcal{D}_{y^T}(\overline{w}_1)$.

If $TB_R^{flex,+} \geq 0$:

i) *If* $\overline{w}_1, \overline{w}_2 \in [0, \overline{w}_D]$, *then* $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_2 \in \left(\overline{w}_R^+, \overline{w}_D\right]$, *then* $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subset \mathcal{D}_{y^T}(\overline{w}_2)$.

ii) *Under* $F(h) = h$ *and* $u(c) = \ln(c)$; *if* $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^+, \infty\right)$, *then* $\bar{b}^+(\overline{w}_2) \leq \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subseteq \mathcal{D}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_1 \in \left[\overline{w}_R^+, \overline{w}_D\right)$, *then* $\bar{b}^+(\overline{w}_2) < \bar{b}^+(\overline{w}_1)$. *Equivalently,* $\mathcal{D}_{y^T}(\overline{w}_1) \subset \mathcal{D}_{y^T}(\overline{w}_2)$.

Proposition E19 (Crisis Region Expansion). *For every* y^T *and taking arbitrary wage rigidities* $\overline{w}_1 < \overline{w}_2$, *the following claims hold:*

i) *If* $\overline{w}_1, \overline{w}_2 \in \left[0, \overline{w}_R^+\right]$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subseteq \tilde{C}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_2 \in \left(\overline{w}_R^-, \overline{w}_R^+\right]$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subset \tilde{C}_{y^T}(\overline{w}_2)$.

ii) *Under* $TB_R^{flex,+} \leq 0$, $F(h) = h$, *and* $u(c) = \ln(c)$; *if* $\overline{w}_1, \overline{w}_2 \in \left[\overline{w}_R^-, \infty\right)$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subseteq \tilde{C}_{y^T}(\overline{w}_2)$. *Moreover, if* $\overline{w}_1 \in \left[\overline{w}_R^-, \overline{w}_R^+\right)$, *then* $\tilde{C}_{y^T}(\overline{w}_1) \subset \tilde{C}_{y^T}(\overline{w}_2)$.

F Nominal Debt

In this section, we start from the simplified version of the model from Section 3.5.1 that has deterministic income, $\beta R = 1$, permanent exclusion after default and one-period debt. In addition, we consider a cost from depreciating the currency as in section B. The resource constraint for tradables is given by

$$c^T = y^T - \frac{b}{e} + q\frac{b'}{e} \tag{F.1}$$

Notice in this equation how an increase in e reduces real payments to foreigners and increase tradable consumption.

Denote $\mathcal{E}(b)$ the optimal exchange rate as a function of the level of debt $e' = \mathcal{E}(b')$. From investors' side, notice that arbitrage implies that the fundamental bond price must satisfy

$$q(b')(1+r) = \frac{e}{\mathcal{E}(b')}.$$

If investors are pessimistic, the value of repayment for the government is

$$V_R^-(b) = \max_{e,b'} u\left(y^T - \frac{b}{e}, \mathcal{H}\left(y^T - \frac{b}{e}, \frac{W}{e}\right)\right) - \psi(e/\bar{e}) + \beta V_R^+(0). \tag{F.2}$$

The value of default is

$$V^D = \max_e \frac{u(y^T, \mathcal{H}\left(y^T, \frac{W}{e}\right)) - \kappa - \psi(e/\bar{e})}{1-\beta} \tag{F.3}$$

Inspecting (F.2), one can observe how the increase in e not only raises employment through the reduction in the real wage rigidity constraint but also through the increase in tradable consumption (by effectively reducing debt repayment to foreigners). The effect of a devaluation is therefore stronger under repayment than under default, and hence contribute to reduce the vulnerability to a debt crisis.