Forward guidance with life-cycle motives

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November 2019 - Preliminary, please do not circulate

Abstract

We propose a tractable New Keynesian model with an overlapping generations structure that offers a resolution to the forward guidance puzzle. Two elements contribute to making the economy less sensitive to future interest rate cuts. First, since households have to save for retirement, a low equilibrium real interest rate is consistent with a higher rate of time preference than in standard New Keynesian models, thereby weakening the strength of the intertemporal substitution channel. Second, and more importantly, forward guidance shocks benefit disproportionately to young generations, notably those who are about to enter the labor market at the time of the announcement. These heterogeneous effects of forward guidance reduce the strength of general equilibrium forces which, in standard models, amplify the initial response of the economy to the shock.

Keywords: Forward Guidance, Saving for Retirement

JEL Codes: E31, E32, E52

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1 Introduction

Forward guidance has become an increasingly important tool of monetary policy in the years following the Great Recession. Given the secular decline in interest rates observed in advanced economies, the effective lower bound on nominal interest rates is likely to constrain the efforts of central banks at stimulating the economy more often in the future. The extent to which policymakers can undo this constraint, and provide the economy with proper accommodation, depends on the effectiveness of unconventional monetary policies, among which forward guidance. Such a policy has been shown to have unrealistically powerful effects in New Keynesian models. In Eggertsson and Woodford (2003), a recession can be entirely avoided if the central bank commits to keeping interest rates at the zero lower bound for an additional few quarters beyond what is justified by contemporaneous economic conditions. In Calstrom, Fuerst, and Paustian (2015), explosive dynamics arise as the duration of forward guidance approaches and exceeds some critical value. Del Negro, Giannoni, and Patterson (2015) show that this outsized impact of forward guidance on the economy in macroeconomic models is at odds with empirical evidence, a phenomenon they call the "forward guidance puzzle".

In this paper, we propose a resolution of the forward guidance puzzle based on generational turnover and life-cycle motives. We consider a New Keynesian model with an overlapping generations structure, as in Blanchard's (1985) and Yaari's (1965) perpetual youth model, that also incorporates a "savings for retirement" motive and age-specific labor income volatilities. On the firm side, infinitely-lived monopolistic firms use labor as their only input in the production process and set prices while facing price adjustment costs. This gives rise to a Phillips Curve relating markups to inflation. On the household side, a new generation is born in every period without any financial wealth while existing generations die with positive probability. As in Blanchard (1985), the labor productivity of individuals declines as they age. As a result, individuals save and accumulate financial wealth while young in anticipation that their labor income whill shrink later in life. This "savings for retirement" motive allows us to obtain a low equilibrium real interest rate without having to assume that the household discount factor is close to one. Moreover, consistent with the evidence in Jaimovich, Pruitt, and Siu (2013), we assume that the cyclicality of labor income differs across cohorts. New generations negotiate their wages flexibly when first entering the labor market, while the wages of existing generations respond only sluggishly to current macroeconomic conditions. Thus, the lifetime earnings of an individual depend heavily on the state of the economy when he or she first entered the labor market.

We consider an announcement by the central bank that the real interest rate will be cut by 25 basis point during 4 quarters at an horizon of 5 years. The real interest rate is assumed to stay constant in all other periods. In the standard New Keynesian model, and under the assumption of log preferences, consumption would immediately jump upwards by about one percent in all periods preceding the forward guidance shock. That is, the response of consumption would be the same regardless of how far in the future the interest rate cut is. In contrast, in our model, the response of consumption to the shock is approximately 50 percent lower at the time of the annoucement of the shock than at the time of its realization. In order to better understand these results, we decompose the consumption response in a direct effect, which captures how the future interest cut modifies the household optimal consumption path for a given stream of income, and an indirect effect, which captures how current and future income streams are affected through general equilibrium. In the standard New Keynesian model, both effects contribute positively to the consumption response, with the importance of the direct (indirect) effect decreasing (increasing) with the horizon of the shock. In our model, the direct effect contributes positively to the consumption response, albeit to a lesser extent than in the New Keynesian model, while the indirect effect is negative.

What are the drivers of these differences? First, a key factor contributing to household's willingness to shift consumption intertemporally is the rate of time preference. In the New Keynesian model, a low steady-state real interest rate implies a low rate of time preference. In our model, another determinant of the real interest rate is household's desire to save for retirement. For a given steady-state real interest rate, the rate of time preference is therefore higher than in the New Keynesian model, which reduces the strength of the intertemporal substitution channel. Second, and more importantly, the age-specific responses of labor income to the announcement induce strong redistributive effects between generations. Since the wages of generations alive at the time of the shock adjust only sluggishly, their permanent labor income does not increase much following the forward guidance annoucement and the ensuing boost in demand. However, generations entering the labor market at a later date will be able to negotiate their wages flexibly and therefore benefit from the more favorable economic conditions. This results in a fall in future profits and dividends, which limits the boost to stock market valuations arising from lower interest rates and the initial procyclical response of dividends. These positive, but limited, responses of permanent labor income and financial wealth for existing generations explain the initial muted response of consumption.

Our paper is related to other attempts at solving the forward guidance puzzle. Some studies have emphasized the role of unemployment risk and precautionary savings (McKay, Nakamura and Steinsson 2016), while others have proposed explanations based on departures from the rational expectations hypothesis (Gabaix 2016, Fahri and Werning 2018, Andrade,Gaballo, Mengus and Mojon 2019). Del Negro, Giannoni, and Patterson (2015) also propose an explanation based on an overlapping generations model. The calibration of their model, however, raises several concerns, and is not compatible with a purely demographic explanation. The birth and the death rate are set at 3 percent per quarter. Taken literally, this means a life expectancy of 8 year. The *quarterly* steady state real interest rate is equal to 1.5 percent, largely above standard estimates of the natural interest rate. Once a version of their model is recalibrated to match a more realistic value for the steady-state real interest rate, the forward guidance puzzle reappears.

Moreover, our modeling of the labor market is connected to several strands of literature. First, as noted above, Jaimovich, Pruitt, and Siu (2013) document that the cyclical volatilities of hours and wages are larger for workers aged 15-29 than for prime-aged workers. Second, a substantial body of literature has documented persistent effects of recessions on lifetime earnings for new entrants on the labor market. Orepoulos et al. (2012) show that individuals graduating in recessions get lower wages than their peers graduating in expansions. This relative decline lasts ten years and is particularly large and persistent for low-skilled workers. Similar results have been found in the Japanese case by Genda et al. (2010). The recent literature on earnings dynamics at the individual level has confirmed the importance of the early stages of a career for lifetime earnings. Using microdata from administrative sources, Ozkan et al. (2015) show that, until the 90th percentile of lifetime earnings, most of earnings growth occurs between the ages of 25 and 35. Studying the cyclicality of earnings risks, Guvenen et al. (2014) find that expansions differ from recessions in that they are characterized by large earnings growth at the bottom of the earnings distribution, whereas the middle of the distribution experiences roughly the same earnings growth across the business cycle. Third, the literature on wage cyclicality also brings some evidence in favor of our mechanism. Bils (1985) shows that wage cyclicality is larger for new hires than for incumbents. Using longitudinal data from the PSID survey, Beaudry and Di Nardo (1991) document some history dependence in labor market outcomes at the individual level. To rationalize their findings, they propose a model with implicit contracts. The implict contracts model received some recent empirical backing on the firm side by Kudlyak (2014) and Basu and House (2016).

The paper is organized as follows. Section 2 describes the model. Section 3 details the forward guidance experiment and the main results. Secton 4 provides some intuition for our results by decomposing the consumption response in direct and indirect effects. Section 5 proposes a comparison to Del Negro et al. (2015).

2 Model

We consider a simple and tractable overlapping generations model of the perpetual youth type (Yaari 1965, Blanchard 1985). This approach has two advantages. First, it allows us to characterize analytically how announcements about the future course of monetary policy affect the economy. Second, it facilitates comparison with the standard New Keynesian model, which is nested in our framework when the horizon of agents goes to infinity. In introducing a life-cyle or "saving for retirement" motive in the standard perpetual youth model, we follow Blanchard (1985) and assume that individuals' productivity declines exponentially as they age.

2.1 Households

In every period j, a new cohort is born with mass γ and each cohort has a constant probability of dying γ. The horizon of any individual is therefore constant and equal to $1/\gamma$ regardless of age. Each household is endowed with one unit of labor when born. The budget constraint for members of cohort j is given by

$$C_{j,t}+\int_0^1 Q_t(i)S_{j,t+1}(i)di+E_t\left\{\mathcal{F}_{t,t+1}B_{j,t+1}\right\} \leq \frac{1}{1-\gamma}\left[B_{j,t}+\int_0^1 \left(Q_t(i)+D_t(i)\right)S_{j,t}(i)di\right]+w_{j,t}L_{j,t}-T_t \tag{1}$$

where $S_{j,t}(i)$ is the number of shares issued by firm i and bought by individuals of cohort j. These shares offer (real) dividends $D_t(i)$, and their real price is $Q_t(i)$. Complete markets for state-contingent bonds are assumed, with $B_{j,t+1}$ denoting the stochastic payoff (expressed in units of the consumption index) generated by a set of contingent claims purchased in period t and with value $E_t\left\{\mathcal{F}_{t,t+1}B_{j,t+1}\right\}$, where $\mathcal{F}_{t,t+1}$ is the stochastic discount factor for one-period ahead stochastic real payoffs. Financial wealth held by cohort j at the beginning of period t is given by $\Omega_{j,t}=\frac{1}{1-\gamma}\left(B_{j,t}+\int_0^1 \left(Q_t(i)+D_t(i)\right)S_{j,t}(i)di\right)$. As in Blanchard (1985), households enter an annuity contract in which the fraction γ of cohort members dying in each period leaves its wealth to those remaining alive. This explains the presence of the term $\frac{1}{1-\gamma}$, which captures the extra return on wealth resulting from the annuity contract. Consumption $C_{j,t} \equiv \left[\int_0^1 C_{j,t}(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ is a Dixit-Stiglitz aggregator of the different varieties of goods produced by firms and θ is the elasticity of substitution between goods. The optimal allocation of income on each variety is $C_{j,t}(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} C_{j,t}$, where $P_t = \left[\int_0^1 P_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ is the aggregate price index and $P_t(i)$ is the price of a good of variety i. $L_{j,t}$, and $w_{j,t}$ are labor supply and real wages for cohort j at time t. Lump-sum taxes T_t are common to all cohorts. Each generation has a different productivity level $Z_{j,t}$ related to aggregate productivity according to

$$Z_{j,t} = \delta Z_t \left(1-\alpha\right)^{t-j} \tag{2}$$

where $0 < \alpha < 1$. Productivity $Z_{j,t}$ is a decreasing function of age $t-j$. This is meant as an approximation for the loss of labor income upon retirement, as in Blanchard (1985), and is used to create a "saving for retirement" motive. In period t, the lifetime utility for an individual of cohort j is given by the recursive equation

$$V_{j,t} = \left[\left(C_{j,t} - \overline{C}\right)^{\frac{\sigma-1}{\sigma}} + \beta(1-\gamma) E_t \left(V_{j,t+1}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where β is the subjective discount factor of households and $\bar{C}$ is a subsistence point in consumption. Individuals discount the future at the rate $\beta(1-\gamma)$ to take into account the probability of dying. The first-order conditions for an optimum include the budget constraint (1) holding with equality as well as intertemporal conditions for the two assets

$$Q_t(i) = E_t \mathcal{F}_{t,t+1} \left(Q_{t+1}(i) + D_{t+1}(i)\right) \tag{4}$$

$$\mathcal{F}_{t,t+1} = \beta \left(\frac{C_{j,t+1} - \overline{C}}{C_{j,t} - \overline{C}}\right)^{-\frac{1}{\sigma}} \tag{5}$$

Complete markets imply that the stochastic discount factor is common to all (surviving) individuals. The nominal gross return $1+R_t$ on a risk-free one-period bond paying one unit of currency in period $t+1$ in all states of the world is defined by the no-arbitrage condition

$$E \mathcal{F}_{t,t+1} \frac{1+R_t}{\Pi_{t+1}} = 1 \tag{6}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the gross inflation rate between periods t and $t+1$. Following Gertler (1999), one can derive from these equations a policy function for consumption,

$$C_{j,t} = \overline{C} + \rho_t \left[\Omega_{j,t} + E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} (1-\gamma)^s (w_{j,t+s} L_{j,t+s} - T_{t+s} - \overline{C}) \right] \tag{7}$$

where the marginal propensity to consume ρ_t is defined by the recursive equation

$$\rho_t = 1 - \beta^{\sigma} \left(E_t \mathcal{F}_{t,t+1} \right)^{1-\sigma} (1-\gamma) \frac{\rho_t}{\rho_{t+1}} \tag{8}$$

Households consume out of financial and human wealth with propensity ρ_t , where human wealth – the second term in brackets – is equal to the expected discounted sum of future labor income adjusted for the presence of subsistence points.

2.2 Labor Market

Households supply inelastically one unit of labor. However, the labor market does not clear and households face some rationing in hours worked. The rationing scheme implies that every cohort works the same number of hours in period t. The mass of households being equal to one, hours worked by a household of cohort j is equal to aggregate hours worked.

$$L_t^j = L_t \tag{9}$$

Newborn households negotiate a real wage per unit of efficient hours and this initial bargained wage has persistent effects on the whole sequence of wages that they can expect to earn during their lifetime. More precisely, the wage of members of cohort j in period t, $w_{j,t}$, depends on three components. The first component is cohort-specific and depends on the wage negociated upon entry in the labor market, $w_{j,j}$. The second component depends on macroeconomic conditions in period t. The third component evolves linearly with cohort $j's$ time t productivity level $Z_{j,r}$. The wage of a household of cohort j in period t is thus equal to

$$w_{j,t} = w_{j,j} \tilde{W}_t Z_{j,t} \tag{10}$$

We do not model explicitly the bargaining process between workers and firms or the matching process (or, in other words, the rationing scheme) on the labor market. We simply assume that the negotiated wage, $w_{j,j}$, is an increasing function of aggregate employment and that labor demand by firms is satisfied. In other words, rationing occurs only on workers' side. The equation for $w_{j,j}$ is

$$w_{j,j} = \chi_1 L_j^{\lambda} \tag{11}$$

The equation for the second component, $\tilde{W}_t$, is very similar

$$\tilde{W}_t = \chi_2 L_t^{\varphi} \tag{12}$$

We use this specification of the wage process for two reasons. First, as emphasized by Ascari and Rankin (2007), one limitation of the perpetual youth model is that labor supply becomes negative for some (old) agents if leisure is a normal good. Our specification circumvents this issue by getting rid of the wealth effect on labor supply. Second, and more importantly, using this specification helps us make clear that, in this framework, the effects of forward guidance policies depend crucially on the cyclicality of wages and its distribution across cohorts, which are controled by the parameters φ and λ.

Workers from different cohorts are pooled in a composite labor unit, and then sold to firms. Thus, even if workers are actually perfect substitutes, firms cannot replace a worker from a high wage cohort by a low wage cohort worker.

2.3 Aggregation

Consider any variable $X_{j,t}$ for a cohort born at time j and define the aggregate variable

$$X_t \equiv \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t-j} X_{j,t} \tag{13}$$

X_t is a weighted average of the individual $X_{j,t}$, where the weights are given by the mass of each cohort $\gamma(1-\gamma)^{t-j}$. One exception to this notation applies to time t bonds and shares. Although they are indexed with a time t subscript, they are acquired by households at time $t-1$ and must be aggregated using time $t-1$ averages. Aggregation of (2) pins down the parameter δ

$$\delta = \frac{\alpha}{\gamma} + 1 - \alpha \approx \frac{\alpha+\gamma}{\gamma} \tag{14}$$

where the last step follows from the assumption that α and γ are both close to 0. Aggregation of (4) gives

$$Q_t = E_t \frac{1+R_t}{\Pi_{t+1}} \left(Q_{t+1} + D_{t+1}\right) \tag{15}$$

where $Q_t = \int_0^1 Q_t(i)di$. In order to facilitate the aggregation of individual consumption functions (7), we split human wealth in two subcomponents

$$H_{1,j,t} \equiv w_{j,t} L_{j,t} + E_t \mathcal{F}_{t,t+1} (1-\gamma) H_{1,j,t+1} \tag{16}$$

$$H_{2,t} = T_t + \bar{C} + E_t \mathcal{F}_{t,t+1} \left(1-\gamma\right) H_{2,t+1} \tag{17}$$

In Appendix 1, we show that $H_{1,t} = \overline{W}_t N_t$, where $\overline{W}_t$ and N_t are defined by the two recursive equations

$$\overline{W}_t = (\gamma+\alpha)\chi_1 L_t^{\lambda} + (1+\alpha+\gamma)\overline{W}_{t-1} \tag{18}$$

$$N_t = \chi_2 Z_t L_t^{1+\varphi} + E_t (1-\gamma)(1-\alpha)\mathcal{F}_{t,t+1} N_{t+1} \tag{19}$$

Therefore, aggregation of individual consumption functions gives

$$C_t = \overline{C} + \rho_t \left[\Omega_t + \overline{W_t} N_t - H_{2,t}\right] \tag{20}$$

The aggregate wage W_t is given by (see Appendix 1)

$$W_t = \chi_2 L_t^{\varphi} Z_t \overline{W_t} \tag{21}$$

2.4 Firms

A continuum of monopolistic firms, indexed by i, produce differentiated goods according to the linear technology $Y_t(i) = \sum_{j=-\infty}^{t} \gamma (1-\gamma)^{t-j} Z_{j,t} L_{j,t}(i)$ and face quadratic price adjustment costs $\Phi_t(i) = \frac{\phi^p}{2}(\frac{P_t(i)}{P_{t-1}(i)} - 1)^2 Y_t$. These costs have the same composition as the aggregate consumption basket and are proportional to aggregate output. Firms choose $P_{t+s}(i)$, $Y_{t+s}(i)$, and $L_{j,t+s}(i)$, to maximize the expected discounted sum of future profits

$$E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} \left[\frac{P_{t+s}(i)}{P_{t+s}} Y_{t+s}(i) - \sum_{j=-\infty}^{t+s} \gamma(1-\gamma)^{t+s-j} w_{j,t+s} L_{j,t+s}(i) - \frac{\phi^p}{2} \left(\frac{P_{t+s}(i)}{P_{t+s-1}(i)} - 1 \right)^2 Y_{t+s} \right]$$

subject to the production function $Y_{t+s}(i) = \sum_{j=-\infty}^{t+s} \gamma(1-\gamma)^{t+s-j} Z_{j,t+s} L_{j,t+s}(i)$ and the demand for goods $Y_{t+s}(i) = \left(\frac{P_{t+s}(i)}{P_{t+s}}\right)^{-\theta} Y^d_{t+s}$, where Y^d is aggregate demand. In equilibrium, all firms face a similar problem and choose the same price, which implies that $Y_t = \int_0^1 Y_t(i)di = Y_t^d$. We obtain

$$\mu_t = \frac{Z_t}{W_t} \tag{22}$$

$$1 - \theta + \frac{\theta}{\mu_t} - \phi^p \Pi_t \left(\Pi_t - 1\right) + E_t \mathcal{F}_{t,t+1} \phi^p \Pi_{t+1} \left(\Pi_{t+1} - 1\right) \frac{Y_{t+1}}{Y_t} = 0 \tag{23}$$

where equation (23) is a nonlinear Phillips Curve relating markups to inflation. Aggregating production functions across firms leads to a production function for aggregate output

$$Y_t = Z_t L_t \tag{24}$$

2.5 Government and resource constraint

The government budget contraint is

$$G_t + \frac{b_t}{\Pi_t} = T_t + \frac{b_{t+1}}{1 + R_t} \tag{25}$$

where $b_t = \frac{B_t}{P_{t-1}}$ and (wasteful) government spending G_t is allocated among differentiated consumption goods in the same manner as individual consumption, $G_t = \left[\int_0^1 G_t(j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$. For given G_t, and assuming that the tranversality condition $lim_{s\to\infty} E_t \mathcal{F}_{t,t+1} \frac{B_{t+s}}{P_{t+s}} = 0$ holds, lump-sum taxes T_t must adjust to satisfy (25). Since the model is non-Ricardian, the tax policy will matter for equilibrium outcomes. In every period, firms redistribute profits to shareholders in the form of dividends. We normalize the number of shares to one. Aggregate dividends are equal to

$$D_t = \int_0^1 D_t(i) S_{t-1}(i) di = Y_t - \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t-j} w_{j,t} L_{j,t} - \frac{\phi^p}{2} (\Pi_t - 1)^2 Y_t \tag{26}$$

Aggregating the budget constraint of households, equation (1), and substituting (25) and (26) gives the aggregate resource constraint

$$C_t + G_t = Y_t \left(1 - \frac{\phi^p}{2}(\Pi_t - 1)^2\right) \tag{27}$$

2.6 Equilibrium conditions

A competitive equilibrium is a set of plans $\left\{C_t, Y_t, W_t, \overline{W_t}, L_t, N_t, H_{2,t}, b_t, \Pi_t, \mu_t, \mathcal{F}_{t,t+1}, Q_t, D_t, \Omega_t, \rho_t, R_t\right\}_{t=0}^{\infty}$ satisfying equations (6), (8), (15), (17), (18), (19), (20), (21), (22), (23), (24), (25), (26), and (27) as well as the equation for aggregate financial wealth $\Omega_t = Q_t + D_t + b_t$, given specifications for monetary policy $\{R_t\}_{t=0}^{\infty}$, fiscal policy $\{G_t, T_t\}_{t=0}^{\infty}$, the exogenous productivity process $\{Z_t\}_{t=0}^{\infty}$. Equilibrium conditions are summarized in Appendix 2.

3 A forward guidance experiment

We focus on a simple version of the model where real government spending G_t is equal to zero and government bonds are in zero net supply. Section 3.1 calibrates the model. Section 3.2 performs a forward guidance experiment.

3.1 Calibration

We first assign values to parameters that are commonly found in the New Keynesian literature. The elasticity of substitution between goods is assumed to be equal to $\theta = 6$, which corresponds to a markup of 20% in the zero-inflation steady state. The price adjustment cost parameter ϕ^p is chosen according to the following logic. The linearized Phillips curve of the model is observationally equivalent to the one derived under Calvo pricing. Assuming an average contract duration of four quarters, the slope of the linearized Phillips curve under Calvo pricing would be equal to 0.0858. In the above model with Rotemberg pricing, this slope is given by $\frac{\theta-1}{\phi^p}$. Matching coefficients implies that $\phi^p = 58$. Finally, $\chi_1$and χ_2 are set to target respectively a steady-state output equal to one and a steady-state current component for wages equal to one.

We now turn to parameters that are specific to the overlapping generations environment. Assuming that individuals are "born" at the age of twenty and can expect to live, on average, until the age of eighty implies that $\gamma = 0.0042$. Using actuarial life tables from the Social Security Administration, Del Negro et al. (2015) find a similar value when γ is strictly interpreted as a death probability. The rate at which individual productivity declines with age, α, is set to 0.0063 in order to approximate a 2.5 percent annual transition rate from workforce to retirement consistent with a forty years average career duration. The parameter β is then fixed to match a steady-state real interest rate of 1.5% (see Appendix 3). In our baseline experiment, we fix the elasticity of intertemporal susbtitution at 1 (equivalent to a logarithmic specification) and the subsistence consumption level $\overline{C}$ to half of steady-state consumption.

Finally, we have to calibrate the contemporaneous wage elasticity φ and the elasticity of newly-negotiated wages with respect to aggregate employment λ. In our experiment, we set φ to 0 and λ to 5. This implies that wage cyclicality only comes from newly-negotiated wages. For a given cohort, wages become fully rigid after entry on the labor market. A one percent increase in employment in a given quarter leads to a five percent rise in newly-negotiated wages. As a new cohort represents one percent of the total workforce once we adjust for productivity, this implies that aggregate wages increase by about 0.05 percent.

3.2 A forward guidance experiment

We consider a "pure" forward guidance experiment. We assume that the central bank announces that the real interest rate falls by 25 basis points during one year at a horizon of 20 quarters in the future, whereas real interest rates remain unchanged for all other quarters. The path of the real interest rate is plotted in Figure 2. In Figure 1, we plot the response of output to the announcement. In the standard New Keynesian model, output responds proportionally with changes in real interest rates, regardless of the horizon of the shock. In contrast, in this framework, the output response declines with the horizon of the shock, and significantly so. Just after the announcement, output jumps by about 0.2 percent whereas the increase at the time of the realization of the first shock is slightly lower than 0.5 percent.

How does the response of output to forward guidance shocks differ from that in the standard perpetual youth model? In order to answer to this question, we shut down the "savings for retirement" motive by setting α to zero and assume that all wages are renegotiated in every period. We calibrate the elasticity of wages with respect to employment at 0.25, a plausible value given the small procyclicality of real wages. All other parameters are calibrated as in our baseline model. The response of output in the standard perpetual youth model is also displayed in Figure 1. There is practically no difference

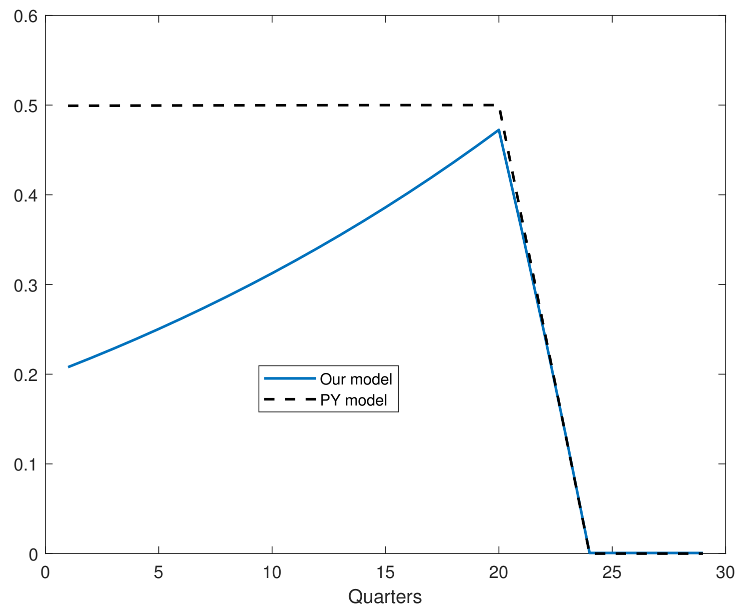


Figure 1: Log deviation of output from steady-state in response to a 25 basis point cut in real interest rates for 4 quarters at a horizon of 20 quarters, holding real interest rates fixed in other periods.

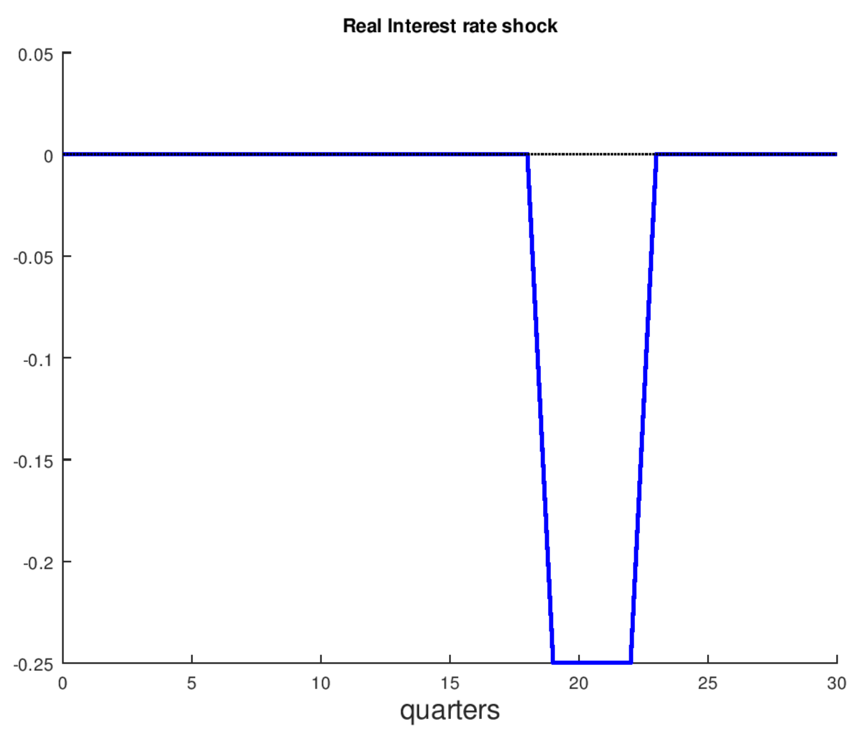


Figure 2: Assumed real interest rate path

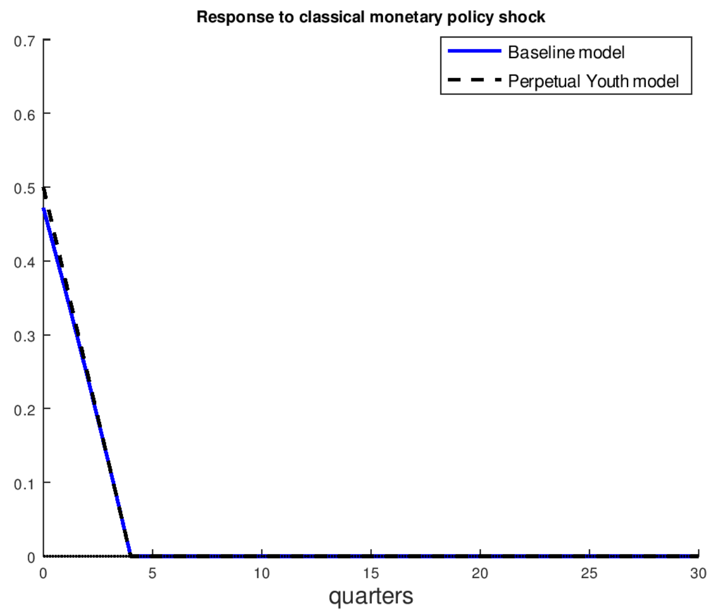


Figure 3: Response of output to a contemporaneous monetary policy shock in our model and in the standard perpetual youth model

between the response of output at the announcement and at the realization of the shock.

It would be possible that our model dampens not only forward guidance shocks but also contemporaneous monetary policy shock. To see that, we compare how our baseline model and the standard perpetual youth model respond after a surprise four-quarters increase in the real interest rate. Figure 3 shows that the response of output to such a shock in our model is quite plausible and is actually very similar to the response in the perpetual youth model.

4 Decomposing the response to forward guidance shocks

4.1 Direct and indirect effects

Forward guidance shocks, like contemporaneous monetary policy shocks, affect individual and aggregate consumption directly, by modifying the optimal consumption path of households for a given stream of income, and indirectly by changing the income stream through general equilibrium effects. In order to better understand how forward guidance shocks are transmitted to real activity in our framework, we reformulate equation (20), and decompose the response of aggregate consumption between direct and indirect effects. Equation (20) rewrites

$$C_t = \overline{C} + \rho_t E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} \left[\tilde{Y}_{t+s} - \sum_{j=-\infty}^{t+s} \gamma (1-\gamma)^{t+s-j} w_{j,t+s} L_{t+s} + \sum_{j=-\infty}^{t} \gamma (1-\gamma)^{t+s-j} w_{j,t+s} L_{t+s} - (1-\gamma)^s (\overline{C} + T_{t+s}) \right] \tag{28}$$

where $\tilde{Y}_t$ is output net of adjustment costs $\tilde{Y}_t = Y_t \left(1 - \frac{\phi^p}{2}(\Pi_t - 1)^2\right)$. Consumption in period t depends on: 1) the expected discounted sum of future dividends, which is itself equal to the expected discounted sum of future output net of adjustment costs minus future labor income paid out *to all generations*, as captured by the first two terms in brackets; 2) future labor income paid out to *generations currently alive*, as captured by the third term in brackets; 3) future taxes and subsistence points, as captured by the last term in brackets. Equation (28) simplifies to

$$C_t = \overline{C} + \rho_t E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} \left[\tilde{Y}_{t+s} - \sum_{j=t+1}^{t+s} \gamma (1-\gamma)^{t+s-j} w_{j,t+s} L_{t+s} - (1-\gamma)^s (\overline{C} + T_{t+s}) \right] \tag{29}$$

We use this last equation to perform our decomposition between direct and indirect effects.

Proposition 1. *At first order, the reponse of aggregate consumption to a t+n real interest rate shock can be decomposed between direct and indirect effects.*

$$\frac{\partial C_t}{\partial RR_{t+n}} = Direct_t + Indirect_t \tag{30}$$

where

$$Direct_t = \rho \sum_{s=0}^{n} \frac{\partial \mathcal{F}_{t,t+s}}{\partial RR_{t+n}} \left(\tilde{Y}_{t+s} - \sum_{j=t+1}^{t+s} \gamma (1-\gamma)^{t+s-j} w_{j,t+s} L_{t+s} - (1-\gamma)^s (\overline{C} + T_{t+s}) \right) \tag{31}$$

$$Indirect_t = \rho \sum_{s=0}^{n} \mathcal{F}_{t,t+s} \left(\frac{\partial \tilde{Y}_{t+s}}{\partial RR_{t+n}} - \sum_{j=t+1}^{t+s} \gamma (1-\gamma)^{t+s-j} \frac{\partial \left(w_{j,t+s} L_{t+s} \right)}{\partial RR_{t+n}} \right) \tag{32}$$

This formula is derived under the assumption of an intertemporal elasticity of substitution equal to one, which implies that the marginal propensity to consume ρ is constant[1]. The term on the first line captures the direct, or partial equilibrium, effect, which is equal to the sum of income and substitution

[1]We make this assumption in order to keep the formula simple and understandable. Similar results obtain when ρ is endogenous.

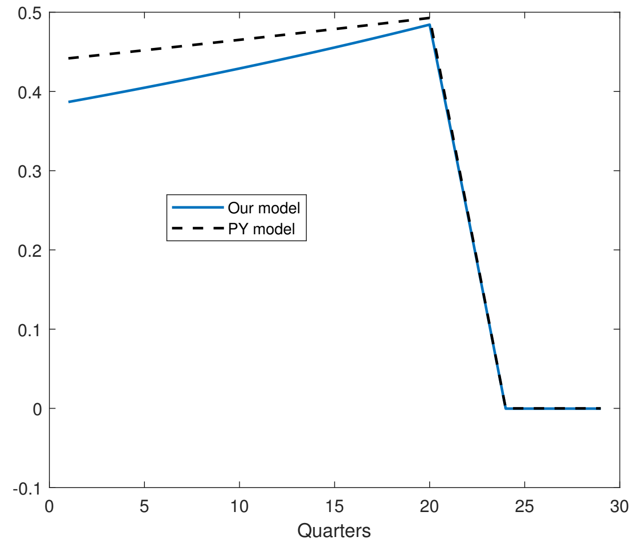


Figure 4: Direct effect of forward guidance - Log deviation of output from steady-state in response to a 25 basis point cut in real interest rates for 4 quarters at a horizon of 20 quarters, holding real interest rates fixed in other periods.

effects. The term on the second line captures the indirect, or general equilibrium, effect.

Figures 4 and 5 display the response of output due to direct and indirect effects for both our model and the standard perpetual youth model. The direct effect is slightly weaker in our framework than in the perpetual youth model. However, most of the difference in the power of forward guidance between the two models is accounted for by the indirect effect. In our model, the strength of this effect declines with the horizon of forward guidance. In the perpetual youth model, the strength of general equilibrium forces increases with the horizon of forward guidance. The next two sections provide intuition for these results by analyzing the drivers of direct and indirect effects in greater details.

4.2 The direct effect: income, substitution, and reevaluation effects

The direct effect of forward guidance on consumption operates through standard substitution and income effects, as well as a financial reevaluation effect.

Proposition 2. *At first order, the direct reponse of aggregate consumption to a $t+n$ real interest rate shock can be decomposed in a substitution effect, an income effect, and a financial reevaluation effect.*

$$Direct_t = Substitution_t + Income_t + Reevaluation_t \tag{33}$$

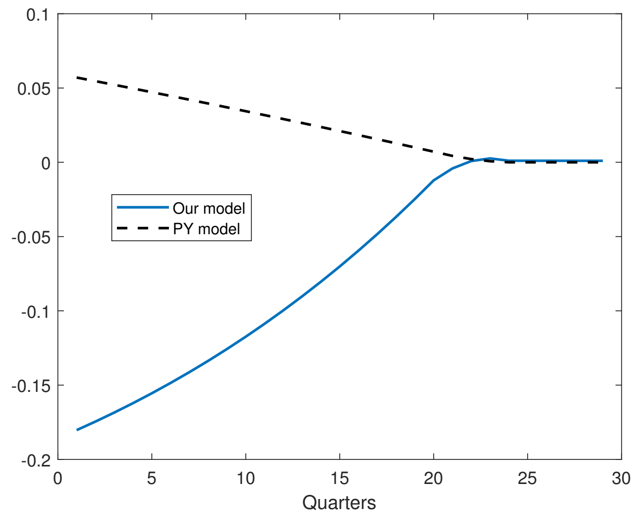


Figure 5: Indirect effect of forward guidance - Log deviation of output from steady-state in response to a 25 basis point cut in real interest rates for 4 quarters at a horizon of 20 quarters, holding real interest rates fixed in other periods.

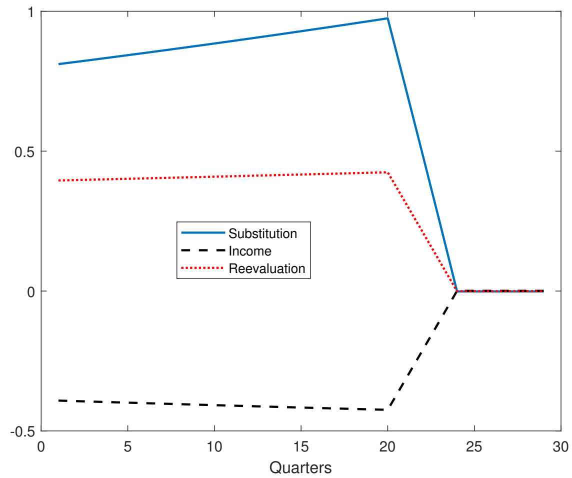
where

$$Substitution_t = \rho \sum_{s=0}^{\infty} E_t (1-\gamma)^s \frac{\partial \mathcal{F}_{t,t+s}}{\partial RR_{t+n}} \left(\sum_{j=-\infty}^{t} \gamma \left(1-\gamma\right)^{t-j} C_{j,t+s} - \bar{C} \right) \tag{34}$$

$$Income_t = \rho E_t \sum_{s=0}^{\infty} \frac{\partial \mathcal{F}_{t,t+s}}{\partial RR_{t+n}} (1-\gamma)^s \left(\sum_{j=-\infty}^{t} \gamma \left(1-\gamma\right)^{t-j} w_{j,t+s} L_{j,t+s} - T_{t+s} - \sum_{j=-\infty}^{t} \gamma \left(1-\gamma\right)^{t-j} C_{j,t+s} \right) \tag{35}$$

$$Reevaluation_t = \rho E_t \sum_{s=0}^{\infty} \frac{\partial \mathcal{F}_{t,t+s}}{\partial RR_{t+n}} D_{t+s} \tag{36}$$

For clarity, we still assume a value of σ equal to one. The three effects are plotted in Figure 6. As before, we also report the results from a standard perpetual youth model. In both models, the financial reevaluation effect is positive, the income effect is negative, and they broadly cancel one another. For given dividends, a decrease in real interest rates n periods ahead raises the expected discounted value of dividends today, thereby increasing stock prices and leading to an upward reevaluation of financial wealth which boosts consumption. At the same time, the decrease in future real interest rates makes households poorer intertemporally as they are, on average, net savers. In face of this negative income effect, households reduce consumption.



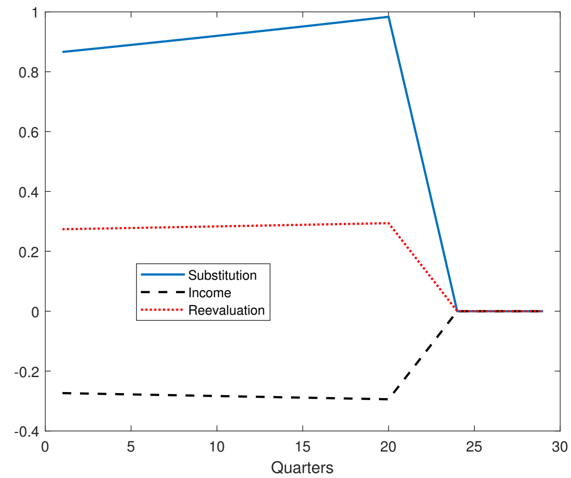


Figure 6: Decomposition of the direct effect of forward guidance - Left panel: our model. Right panel: standard perpetual youth model.

Most of the differences in the direct effects of forward guidance between our model and the standard perpetual youth model stem from the substitution effect. In order to understand this result, it is useful to reformulate equation (34)

$$Substitution_t = \left(C_t - \bar{C}\right) \left(\beta(1-\gamma)\right)^{n+1} \frac{dRR_{t+n}}{1+RR_{t+n}} \tag{37}$$

The speed at which the substitution effect decays with the horizon of forward guidance is controled by the parameters β and γ. The death rate γ is the same in the two models. The discount factor β is, however, lower in our model than in the perpetual youth model. This is linked to the presence of a "savings for retirement" motive in our model, which pushes down the equilibrium real interest rate. Thus, matching a similar equilibrium real interest rate implies that the preference for the present of households has to differ in the two models. In the perpetual youth model, β is equal to 0.9975. In our model, β is equal to 0.9794.

4.3 The indirect effect: wage cyclicality and generational turnover

Equations (28) and (29) make clear that current consumption depends negatively on labor income paid out to all generations working in future periods, and positively on future labor income paid out to generations currently alive. Put otherwise, consumption depends negatively on future labor income paid out to generations that are yet to be born, as this leads to a decrease in future dividends without

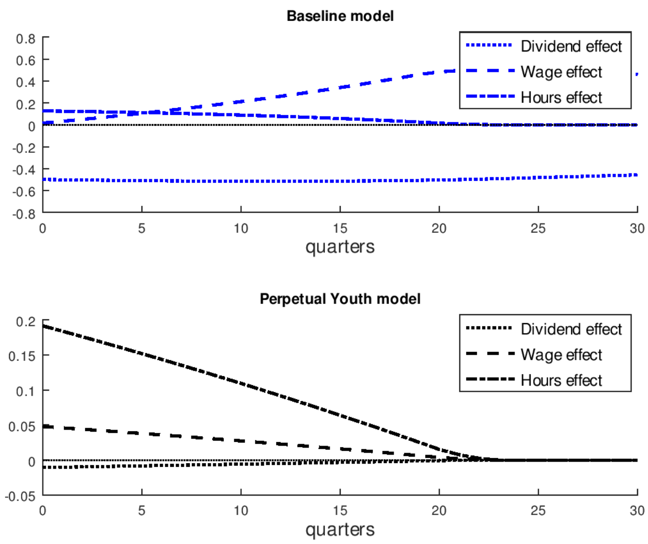


Figure 7: Decomposition of the indirect effect

a commensurate increase in the future labor income of generations currently alive. If, as in our model, the labor income of the young is responsive to economic conditions while that of the old is not, the effects of forward guidance on consumption can be substantially reduced. Indeed, in that case, forward guidance implies a limited increase in permanent labor income and a significant decrease in permanent capital income for current consumers.

In order to illustrate this, we decompose the indirect effect by distinguishing between a dividend effect, a wage effect, and an hours effect, as shown in Figure 7. We also contrast our results with those obtained in the standard perpetual youth model. In the perpetual youth model, the wage effect becomes larger as the horizon of forward guidance grows. In our model, the opposite happens. In each period, new cohorts enter in the labor market with a higer wage and thus a higher permanent income than the previous generation, leading to them to consume more. Progressively, the share of these new cohorts in aggregate consumption increases and so does aggregate consumption itself. We now turn to the dividend effect. This effect is small in the perpetual youth model but deeply negative in our model and drives the small initial response of consumption. But, and this is the crucial point, that does not mean that dividends are countercyclical. Figure 8 displays the behavior of dividends in the two models. Dividends decrease after the forward guidance announcement in the perpetual youth model but *increase* in our model. The key factor behind our result is that, despite this initial increase, dividends will fall a lot between the announcement and the realization of the shocks, as new

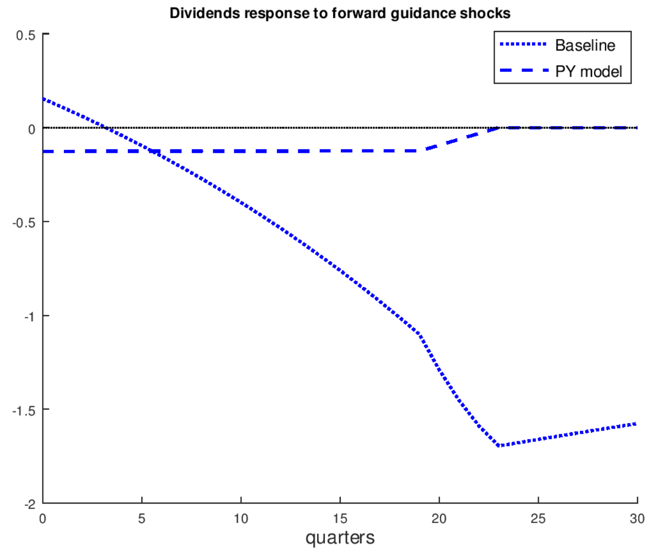


Figure 8: Response of dividends

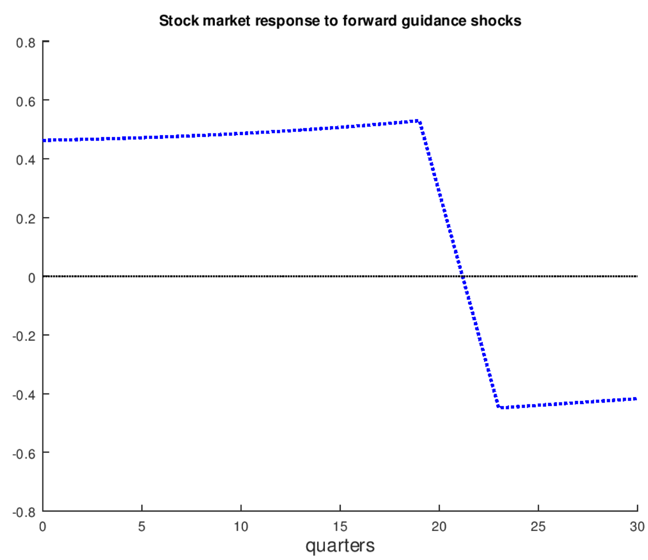


Figure 9: Stock market response

generations with higher wages enter the labor market and start accounting for a growing share of the workforce. Hence, while dividends are procyclical conditional to contemporaneous shocks in our model, the discounted sum of dividends is countercyclical conditional to a forward guidance shock.

Does the response of dividends imply a strong negative reaction of the stock market to expansionary forward guidance announcement ? Surprisingly, the actual response of share prices displayed in Figure 9 is actually positive. The price of stocks increase by about 0.5 percent at the time of the announcement and is stable until the realization of the shock. Indeed, the fall in the expected discounted value of dividends is compensated for by the decrease in real interest rates. After the realization of the shock, the price of shares decreases to -0.5 percent as the boost stemming from lower interest rates disappears. Share prices then recover over a long period, in line with the persistent behavior of dividends.

5 Relation to Del Negro. et al (2015)

Del Negro et al. (2015) (DNGP thereafter) show that introducing a perpetual youth structure in a standard macroeconomic model can help solve the forward guidance puzzle. In this section, we argue that perpetual youth alone needs three questionable features to be able to produce a significant discounting of forward guidance shocks. More precisely, a simple New Keynesian model with a perpetual youth structure needs: (i) an implausibly high steady state real interest rate; (ii) an implausibly high death rate (if we interpret death literally); (iii) countercyclical dividends. These points have already been made separately. DNGP acknowledge that their baseline death rate cannot be interpreted as a true death rate. The importance of dividends in DNGP has been suggested by Werning (2016). The contribution of this section is to show that the *three* assumptions are important for DNGP's result.

We consider a simple New Keynesian model with a perpetual youth structure, without subsistence points, declining labor income, and cohort-specific wage component. We compare the calibration of our model, that we label the "demographic calibration", with a calibration close to DNGP. In the demographic calibration, the quarterly steady-state real interest rate is set at 0.4 percent. The quarterly death rate is also set at 0.4 percent. We also assume a small elasticity of wages with respect to employment (φ is set at 0.25). This value generates acyclical dividends. The baseline calibration in DNGP implies a quarterly death rate of 3 percent and a quarterly real rate of 2 percent. Wages are supposed to be more flexible in order to produce countercyclical dividends. φ is set at 2. We plot

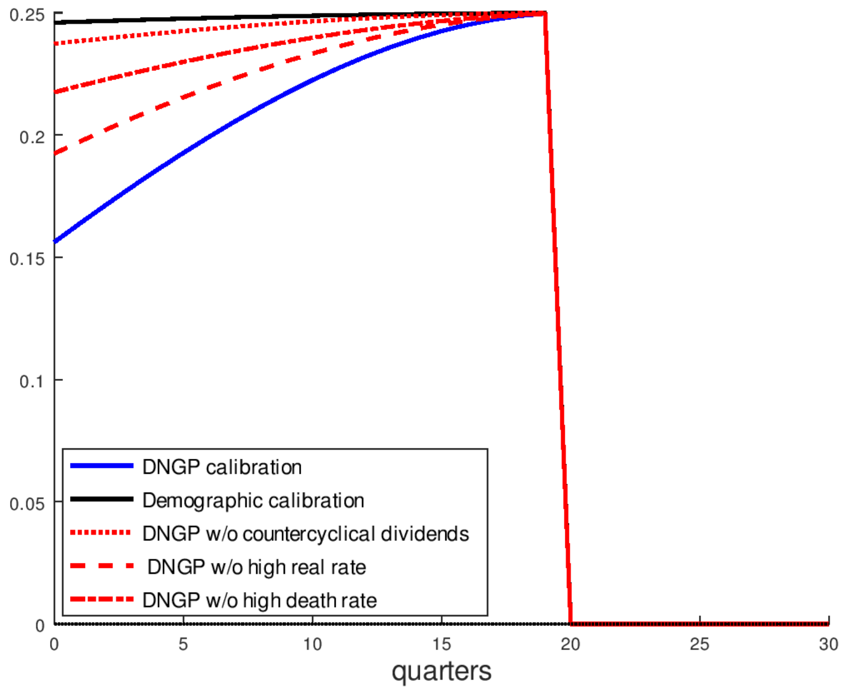


Figure 10: Sensitivity of output response to forward guidance in standard perpetual youth model

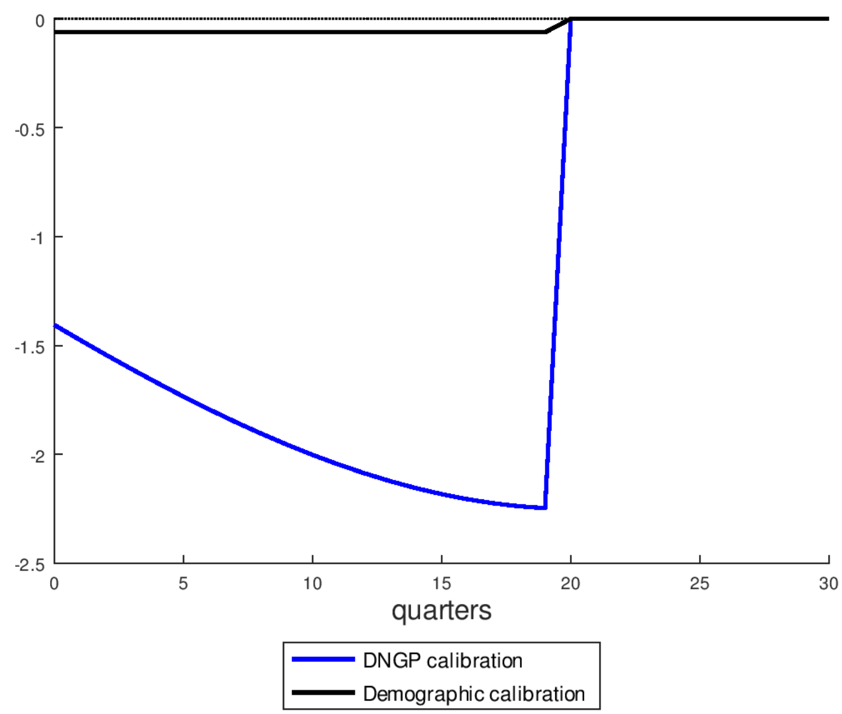


Figure 11: Dividends response in simple perpetual youth model

the result for the two calibrations in Figure 10. The calibration close to DNGP allows us to obtain a significant discounting, as the blue line shows. By contrast, the demographic calibration generates a very small one. We then study the relative importance of the three aforementioned assumptions by performing the following exercise. Starting from DNGP'scalibration, we successively consider a lower value for the real interest rate, a lower value for the death rate, and a lower wage elasticity. For each experiment, we set the changing parameter at its value in the demographic calibration and we keep other parameters at their DNGP value. It appears that all three assumptions are important. A standard value for the steady-state real rate reduces the discounting by nearly half. A death rate compatible with demographic data has even more effects but the more important assumption is relative to the behavior of dividends. Even with the same real rate and the same death rate as in DNGP, the discounting disappears if real wages are sticky enough to generate acyclical dividends.

Figure 11 shows the response of dividends under the DNGP calibration. Dividends falls by 1.5 percent on impact (ten times the magnitude of the output response) and by more than 2 percent when the shock occurs. By contrast, dividends are nearly acyclical with the demographic calibration, as the black line shows.

6 Conclusion

To be added.

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7 Appendix

7.1 Human Wealth and wage Aggregation

We compute the first component of human wealth.

$H_{1,t} = \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t-j} E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s}(1-\gamma)^s w_{j,t+s} L_{j,t+s}$ $\quad H_{1,t} = \sum_{j=-\infty}^{t} \gamma\delta(1-\gamma)^{t-j} E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s}(1-\gamma)^s (1-\alpha)^{t+s-j} w_{j,j} \tilde{W}_{t+s} Z_{t+s} L_{t+s}$

$H_{1,t} = \sum_{j=-\infty}^{t} \gamma\delta(1-\gamma)^{t-j} (1-\alpha)^{t-j} w_{j,j} E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s}(1-\gamma)^s (1-\alpha)^s \tilde{W}_{t+s} Z_{t+s} L_{t+s}$

$H_{1,t} = \left(\sum_{j=-\infty}^{t} (\alpha+\gamma)(1-\gamma)^{t-j} (1-\alpha)^{t-j} w_{j,j} \right) \left(E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s}(1-\gamma)^s (1-\alpha)^s \chi_2 Z_{t+s} L_{t+s}^{1+\varphi} \right)$ $\quad H_{1,t} = \overline{W}_t N_t$

where

$\overline{W}_t \equiv \sum_{j=-\infty}^{t} (\alpha+\gamma)(1-\gamma)^{t-j} (1-\alpha)^{t-j} w_{j,j} = (\alpha+\gamma) w_{t,t} + (1-\alpha-\gamma)\overline{W}_{t-1}$

$N_t = \chi_2 Z_t L_t^{1+\varphi} + E_t \mathcal{F}_{t,t+1}(1-\gamma)(1-\alpha) N_{t+1}$

The aggregate wage is given by

$W_t = \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t-j} w_{j,t}$

$W_t = \sum_{j=-\infty}^{t} \gamma\delta(1-\gamma)^{t-j}(1-\alpha)^{t-j} w_{j,j} \tilde{W}_t Z_t$

$W_t = \tilde{W}_t Z_t \sum_{j=-\infty}^{t} (\alpha+\gamma)(1-\gamma)^{t-j}(1-\alpha)^{t-j} w_{j,j}$

$W_t = \tilde{W}_t Z_t \overline{W}_t$

7.2 Consumption function aggregation

The consumption function at the individual level is given by equation (6)

$$C_{j,t} = \overline{C} + \rho_t \left[\Omega_{j,t} + E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s}(1-\gamma)^s (w_{j,t+s} L_{j,t+s} - T_{t+s} - \overline{C}) \right]$$

By assumption, we have that$L_{j,t+s} = L_{t+s}$. Thus, aggregating gives

$$C_t = \overline{C}+\rho_t \left[E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} D_{t+s} + \sum_{j=-\infty}^{t} E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} \gamma(1-\gamma)^{t+s-j} w_{j,t+s} L_{t+s} - E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} (1-\gamma)^s \left(T_{t+s} + \overline{C} \right) \right]$$

Dividends are equal to

$$D_t = Y_t \left(1 - \frac{\phi^p}{2} \left(\Pi_t - 1 \right)^2 \right) - \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t-j} w_{j,t} L_t$$

Substituting back in the previous expression, we obtain equation (28)

$$C_t = \overline{C}+\rho_t E_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} \left[\tilde{Y}_{t+s} - \sum_{j=-\infty}^{t+s} \gamma(1-\gamma)^{t+s-j} w_{j,t+s} L_{t+s} + \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t+s-j} w_{j,t+s} L_{t+s} - (1-\gamma)^s \left(T_{t+s} + \overline{C} \right) \right]$$

7.3 Equilibrium Conditions

Equilibrium conditions are the production function, the market clearing condition for goods, the aggregate consumption function, the second component of human wealth definition, the persistent component of wages, the forward looking part of labor income human wealth, the law of motion for public debt, the aggregate wage equation, the Philips curve, the stock price equation, the financial wealth equation, the dividend equation, the markup equation, and the stochastic disocunt factor. In addition of these equation, processes for spendings and taxes and the monetary policy rule have to be defined.

$$Y_t = Z_t L_t$$

$$C_t + G_t = Y_t \left(1 - \tfrac{\phi^p}{2} (\Pi_t - 1)^2 \right)$$

$$C_t = \overline{C} + \rho_t \left[\Omega_t + \overline{W}_t N_t - H_{2,t} \right]$$

$$H_{2,t} = T_t + \bar{C} + E_t \mathcal{F}_{t,t+1} \left(1 - \gamma \right) H_{2,t+1}$$

$$\overline{W}_t = (\gamma + \alpha)\chi_1 L_t^{\lambda} + (1 - \alpha - \gamma)\overline{W}_{t-1}$$

$$N_t = \chi_2 Z_t L_t^{1+\varphi} + E_t \mathcal{F}_{t,t+1} (1-\gamma)(1-\alpha) N_{t+1}$$

$$G_t + \tfrac{b_t}{\Pi_t} = T_t + \tfrac{b_{t+1}}{1+R_t}$$

$W_t = \chi_2 L_t^{\varphi} Z_t \overline{W_t}$

$1 - \theta + \frac{\theta}{\mu_t} - \phi^p \Pi_t \left(\Pi_t - 1\right) + E_t \mathcal{F}_{t,t+1} \phi^p \Pi_{t+1} \left(\Pi_{t+1} - 1\right) \frac{Y_{t+1}}{Y_t} = 0$

$E_t \frac{1+R_{t+1}}{\Pi_{t+1}} Q_t = E_t \left(Q_{t+\mathbf{1}} + D_{t+1}\right)$

$\Omega_t = Q_t + D_t + b_t$

$D_t = Y_t \left(1 - \frac{\phi^p}{2} (\Pi_t - 1)^2\right) - W_t L_t$

$\mu_t = \frac{Z_t}{W_t}$

$E_t \mathcal{F}_{t,t+1} \frac{1+R_t}{\Pi_{t+1}} = 1$

$\rho_t = 1 - \mathcal{F}_{t,t+1}^{1-\sigma} \beta^{\sigma} (1 - \gamma) \frac{\rho_t}{\rho_{t+1}}$

7.4 Zero-inflation steady state

The challenge is mainly to compute the wealth consumption ratio and the real interest rate. It is characterized by the following relationships, where the absence of a time subscript indicates a steady-state value

- $Q = \frac{1}{R} D$
- $D = Y \left(1 - \frac{1}{\mu}\right)$
- $\Omega = Q + D = \frac{1+R}{R} \frac{1}{\theta} Y$
- $C = \rho(\Omega + H_1 - H_2)$
- $H_1 = \frac{1+R+\gamma+\alpha}{R+\gamma+\alpha} WL = \frac{1+R+\gamma+\alpha}{R+\gamma+\alpha} \frac{\theta-1}{\theta} Y$
- $H_2 = \frac{1+R+\gamma}{R+\gamma} \overline{C}$
- $\rho = 1 - \beta^{\sigma}(1 - \gamma)(1 + R)^{\sigma-1}$
- $\mu = \frac{\theta}{\theta-1}$
- $C = Y$
- $H = W = \frac{\theta-1}{\theta} Z$

- $L = \left(\frac{\theta-1}{\theta}\frac{1}{\chi}\right)^{1/\lambda}$

- $\tilde{W} = 1$

- $W = \overline{W}Z$

- $Y = Z\left(\frac{\theta-1}{\theta}\frac{1}{\chi}\right)^{1/\lambda}$

We can now solve for the steady-state real interest rate,

$$(1-\beta^{\sigma}(1-\gamma)(1+R)^{\sigma-1})\left(\frac{1+R}{R}\frac{1}{\theta}+\frac{1+R+\gamma+\alpha}{R+\gamma+\alpha}\frac{\theta-1}{\theta}-\frac{1+R+\gamma}{R+\gamma}\frac{\overline{C}}{Y}\right)=1$$

We choose to express the β parameter with respect to other parameters.

$$\beta = \left[\left(1-\frac{1}{\left(\frac{1+R}{R}\frac{1}{\theta}+\frac{1+R+\gamma+\alpha}{R+\gamma+\alpha}\frac{\theta-1}{\theta}-\frac{1+R+\gamma}{R+\gamma}\frac{\overline{C}}{Y}\right)}\right)\frac{1}{1-\gamma}(1+R)^{1-\sigma}\right]^{1/\sigma}$$

7.5 Log-linearized equilibrium conditions

$y_t = z_t + l_t$

$\frac{C}{Y}c_t + (1-\frac{C}{Y})g_t = y_t$

$c_t = \frac{\rho(\Omega+\overline{W}N-H_2)}{C}\hat{\rho}_t + \frac{\rho\Omega}{C}\omega_t + \frac{\rho\overline{W}N}{C}(\overline{w}_t + n_t) - \frac{\rho H_2}{C}h_{2,t}$

$h_{2,t} = (R+\gamma)\frac{T}{T+\overline{C}}t_t + \frac{1-\gamma}{1+R}(h_{2,t+1} + E_t\hat{f}_{t,t+1})$

$\overline{w_t} = (\gamma+\alpha)\widehat{w_{t,t}} + (1-\alpha-\gamma)\overline{w_{t-1}}$

$n_t = \frac{(1-\gamma)(1-\alpha)}{1+R}(n_{t+1} + E_t\hat{f}_{t,t+1}) + \frac{R+\gamma+\alpha}{1+R}(z_t + (1+\varphi)l_t)$

$b_{t+1} = (1+R)(b_t + r_t - \pi_t) + g_t - t_t$

$w_t = z_t + \tilde{w}_t + \overline{w_t}$

$\pi_t = \frac{1}{1+R}E_t\pi_{t+1} - \frac{\theta-1}{\phi^p}\hat{\mu}_t$

$q_t = E_t\hat{f}_{t,t+1} + E_t\omega_{t+1}$

$\omega_t = \frac{1}{1+R} q_t + \frac{R}{1+R} d_t$

$d_t = y_t + (\theta - 1)\, \hat{\mu}_t$

$\tilde{w_t} = \varphi l_t$

$\hat{w_{t,t}} = \lambda l_t$

$w_t - z_t = -\hat{\mu_t}$

$E_t \hat{f}_{t,t+1} = E_t \pi_{t+1} - r_t$

$\hat{\rho_t} = -\frac{\beta^\sigma (1-\gamma)}{(1+R)^{1-\sigma} - \beta^\sigma (1-\gamma)} \left((1-\sigma) \hat{f}_{t,t+1} - \rho \hat{_{t+1}} + \hat{\rho_t} \right)$

7.5.1 Measuring direct and indirect effects of forward guidance

The evolution of wealth can be rewritten

$$\omega_t = \frac{R}{1+R} d_t + \frac{1}{1+R} E_t \hat{\mathcal{F}}_{t,t+1} + \frac{1}{1+R} E_t \omega_{t+1}$$

The direct effect of real interest changes is given by the change in marginal propensity to consume plus the wealth effect (through financial and human wealth) of the real interest rate change.

$$c_{direct,t} = \frac{\rho(\Omega + \overline{W}N - H_2)}{C} \hat{\rho}_t + \frac{\rho\Omega}{C} \tilde{\omega}_t + \frac{\rho \overline{W} N}{C} \tilde{h_{1,t}} - \frac{\rho H_2}{C} \tilde{h_{2,t}}$$

where

$$\tilde{\omega_t} = \frac{1}{1+R} \left(E_t \hat{f}_{t,t+1} + \tilde{\omega}_{t+1} \right)$$

$$\tilde{h_{1,t}} = \frac{(1-\gamma)(1-\alpha)}{1+R} \left(E_t \hat{f}_{t,t+1} + \tilde{h_{t+1}} \right)$$

$$\tilde{h_{2,t}} = h_{2,t} = \frac{1-\gamma}{1+R} E_t \left(E_t \hat{f}_{t,t+1} + h_{2,t+1} \right)$$

The indirect effect is

$$c_{indirect,t} = \frac{\rho\Omega}{C} \mathring{\omega}_t + \frac{\rho \overline{W} N}{C} \mathring{h_{1,t}} + \frac{\rho \overline{W} N}{C} \overline{w_t}$$

where

$$\mathring{\omega}_t = \frac{R}{1+R} d_t + \frac{1}{1+R} E_t \omega_{t+1}^{\circ}$$

$$h_{1,t}^{\circ} = \frac{(1-\gamma)(1-\alpha)}{1+R} h_{1,t}^{\circ} + \frac{R+\gamma+\alpha}{1+R} (z_t + (1+\varphi) l_t)$$

where $\omega_t = \tilde{\omega_t} + \mathring{\omega}_t$ and $h_{1,t} = \tilde{h_{1,t}} + h_{1,t}^{\circ}$.