Fiscal Multipliers: a Tale from the Labor Market^{*}

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Abstract

This paper uncovers the role of firms' hiring decisions as a novel source of state dependence in the fiscal spending multiplier. Hiring is a costly activity as it requires firms to temporarily divert employees from production to recruitment and training of the new hires. Thus, a firm that hires faces a tradeoff between current and future production. A fiscal stimulus carried out when the hiring rate is already high induces firms to hire more when it is costlier, resulting in a weaker response to the increased aggregate demand. Differently from previous studies, I provide reduced form evidence that expansionary spending multipliers depend on the hiring rate of firms, but not on aggregate labor market conditions. I show that they are lower when the hiring rate is higher. I then develop a general equilibrium model with hiring frictions in the labor market to study the propagation of government spending shocks across labor market states. In line with the reduced form evidence, the model shows that output is less responsive to expansionary fiscal policy when the hiring rate is higher.

Keywords: fiscal policy, state dependence, labor market, hiring frictions JEL Classification: E62, E22, E24

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1 Introduction

A key question in macroeconomics is to understand whether and to what extent government spending stimulates the economy. While earlier studies have investigated this issue independently from the state of the economy, the more recent literature started to explore whether the effect of fiscal spending depends on the phase of the business cycle. The underlying intuition rests upon the argument that a fiscal expansion carried out in slack times is less likely to crowd out private consumption and investment and hence stimulates output more.

As part of the ongoing debate on whether fiscal policy transmission depends on the state of the economy, various studies have used the labor market condition to proxy for the phase of the business cycle. The theoretical literature on labor markets proposes multiple summary statistics to measure the state of the labor market. Some are related to the aggregate labor market. The most common ones are tightness -the ratio between posted vacancies and unemployment– and the unemployment rate. The fact that the efficacy of fiscal stimulus may depend on the aggregate labor market condition stems from an idea of labor supply constraint: a fiscal stimulus carried out when the labor market is tight, increases tightness even further and makes it harder for firms to fill their vacancies. Along with these measures of aggregate labor market, the macro labor literature has proposed an alternative summary statistic: the hiring rate –see Merz and Yashiv (2007). This variable, which is the ratio between new hires and total employees, captures hiring decisions internal to firms. The dependence of fiscal policy transmission on the hiring rate is related to an idea of labor demand constraint. Hiring is a costly activity for firms, which have to temporarily divert some of their internal resources from production to recruiting and training the new hires. If a fiscal stimulus is implemented when firms are facing an already high hiring rate, it becomes more costly for them to further expand hiring.

I contribute to the discussion on how fiscal policy depends on the slack of the economy by showing that the transmission of government spending stimuli is affected by the hiring rate of firms, but not by the aggregate labor market conditions. I start by empirically testing which measures of the labor market affect the transmission of fiscal spending shocks. I use local projections to compute state and sign dependent impulse responses to the fiscal spending shocks. When using aggregate measures of labor market slack, I do not find any significant dependence. Rather, I do find a stark dependence associated to the hiring rate of firms. I show that expansionary fiscal policy depends on whether the hiring rate is high or low and that fiscal spending stimuli are less effective when the hiring rate is higher. I compute the cumulative multiplier for a fiscal spending expansion over a five-year horizon to be as big as 3.5 when the hiring rate is below trend, and not significantly different from zero when the hiring rate is above trend.

The empirical results that the transmission of government spending stimuli is affected by the hiring rate of firms, but not by the aggregate labor market conditions, have impli-





Note: The hiring rate is taken from the Current Population Survey and is defined as the ratio between the number of newly hired employees from unemployment or non-employment and the total number of employees. Tightness is defined as vacancies over unemployed. Vacancies are proxied by the help wanted index of Barnichon (2010). The dashed lines are trends obtained by running a Hodrick-Prescott filter with smoothing parameter 10000. Grey bars indicate NBER recessions.

cations for the dynamics of the fiscal multiplier over the business cycle. Figure 1 displays the hiring rate and tightness along with NBER recessions.

Recessions are periods in which tightness moves from its peak to its trough. To the contrary, due to anticipatory effects, firms start hiring well before recessions are over. Hence, the hiring rate moves from low to high values over a recessionary period. As I found expansionary spending multipliers to be lower when the hiring rate is higher, this indicates that the end of a recession is not a good time to do a fiscal stimulus. A better time would be towards the end of an expansion when the hiring rate is below trend and the multiplier is higher. This implication for the timing of fiscal expansions is different from the usual idea that fiscal policy is more effective during recessions and from what fluctuations in tightness would indicate if I had found the spending multiplier to depend on them.

To shed light on the mechanism driving the reduced-form evidence, I build a general equilibrium model with hiring frictions in the labor market and exogenous government expenditure. The dependence of fiscal policy on firms' hiring rate originates from firms' hiring costs being modelled as a function of the hiring rate. This formulation is supported by micro evidence from the literature showing that the biggest component of the hiring cost is not related to vacancy posting, but to training.¹ This assumption further translates into modelling hiring costs as forgone output, which is also supported by micro estimates.²

¹See Manning (2011) for a review of the empirical evidence on hiring costs, and Silva and Toledo (2009) and Faccini and Yashiv (2019) for micro estimates. The functional form is structurally estimated by Yashiv (2000), Merz and Yashiv (2007), and Christiano et al. (2011) and used by Gertler et al. (2008), Gertler and Trigari (2009).

²See Bartel et al. (2014), Cooper et al. (2015), and Faccini and Yashiv (2019) for micro evidence on the disruption caused by hiring.

Diversion of resources from production to training generates a tradeoff between current and future production. When the value of production is low, it is a good time to hire: diverting employees from production to training is relatively less expensive. On the other hand, when the value of production is high, it is not a good time to hire: the tradeoff between current and future production becomes starker. In short, fluctuations in the value of output generate variations in the marginal cost of hiring. A fiscal stimulus that is implemented when the hiring rate is already significant, induces firms to expand hiring when it is costlier, as it makes the tradeoff between current and future production more severe. This costly diversion of internal resources temporarily reduces the production efficiency of firms, and results in a lower output response to a fiscal expansion.

Solving the model non-linearly allows me to study the propagation of fiscal shocks across different levels of the hiring rate. I show that when an increase in government spending is simulated from a state when the hiring rate is high, it generates a wider rise in the value of output. As hiring costs are denominated in terms of the value of output, this increase results in more significant hiring costs to be faced by firms. Firms choose to raise hiring less, which in turn produces a smaller output increase. These theoretical responses mirror the empirical responses estimated with the local projections, showing that my modelling framework is able to generate the asymmetries found in the reduced form evidence.

To check what happens when the state of the labor market is captured by an aggregate measure, I extend the hiring cost function to allow for vacancy posting costs. Following Sala et al. (2013), I assume that the hiring cost is not only a function of the hiring rate, which is a firm-specific object, but also of the vacancy filling rate, which instead reflects the aggregate labor market conditions. This specification reintroduces the more traditional component of vacancy posting costs. With this extended hiring cost function, which allows for both vacancy posting and training costs, I repeat the exercise carried out before to study the propagation of fiscal expansions. However, I now identify the state of the labor market by looking at the level of labor market tightness instead of the hiring rate. I show that allowing for vacancy posting costs results in a much smaller state dependence in the transmission of fiscal expansions. This is in line with my reduced form evidence, which found no dependence of fiscal policy transmission on the aggregate state of the labor market.

This paper is related to three main streams of literature. First, it is connected to the expanding reduced form literature on state dependent fiscal multipliers. There is an unsettled debate on whether the efficacy of fiscal policy varies across different phases of the business cycle. Barro and Redlick (2011) produce estimates that are not precise enough to conclusively establish whether multipliers have cyclical variation. Owyang et al. (2013) find mixed evidence on the state dependence of fiscal multipliers. They show that multipliers are higher in periods of slack for Canada, but not for the US. Auerbach and Gorodnichenko (2012, 2013) argue that fiscal policy is more effective during recessionary periods. Fazzari et al. (2015)'s empirical findings also support state dependent effects of fiscal policy. Using military spending news, Ramey and Zubairy (2018) contend that this is not the case. Barnichon and Matthes (2019) try to reconcile the two views by arguing that the difference of results lies in the sign dependence of the fiscal shocks and that a contractionary shock generates a multiplier which is much higher than an expansionary shock. Caggiano et al. (2015) show that while the effectiveness of fiscal stimuli does not vary across different phases of the business cycles, it does differ when these phases are extreme in their intensity. Riera-Crichton et al. (2015) analyse state and sign dependent fiscal policy across recessions and expansions and show that multipliers are asymmetric depending on both the sign of the fiscal shock and the state of the business cycle.³ Often in these studies, the phase of the business cycle is identified with the amount of aggregate slack in the economy. My paper contributes to this stream of literature by showing that the empirically relevant variable affecting the transmission of fiscal policy is not closely related to the business cycle. In particular, I show that there is no dependence directly deriving from the state of the business cycle or the conditions of the aggregate labor market, which are often used as proxies for the state of the business cycle. Rather, my analysis shows that what affects the transmission of fiscal stimuli is the tradeoff in production that hiring generates.

This paper is also related to the growing literature that examines the non-linear effects of fiscal policy using structural models. Various papers study the effects of fiscal expansions over the business cycle, and provide theories for when fiscal policy is more effective. Brinca et al. (2019) show that the fiscal multiplier of government purchases is increasing in the size of the spending shock and argue that this empirical fact can be explained by the response of labor supply across the wealth distribution in a heterogeneous agent model. Hagedorn et al. (2019) carry out a thorough quantitative exercise aiming at gauging the fiscal multiplier in a heterogenous agent context encompassing nominal rigidities. Faria-e Castro (2018) focusses on the US fiscal policy response during the Great Recession, highlighting different channels of transmission of fiscal stimulus in a recessionary period. Sims and Wolff (2018a,b) study the state dependent effects of respectively tax shocks and government spending shocks by showing that fiscal shocks vary across different phases of the business cycle, but the mechanism for why this happens is not fully explored. Canzoneri et al. (2016) propose a model that features costly financial intermediation and countercyclical financial frictions and is able to generate state dependent fiscal multipliers across the business cycle. In this paper, I focus my analysis on the transmission of expansionary fiscal policy depending on the conditions of the labor market. These do not exactly overlap with the states of the business cycle, as often during recessions unemployment goes from its lowest to its highest level. More specifically, I provide a mechanism explaining why the

³Other papers focus on different types of state dependence. Examples are Navarro and Ferriere (2018), who study tax regime state dependence, Ilzetzki et al. (2013), who look at different country characteristics, and Roulleau-Pasdeloup (2016) and Ramey and Zubairy (2018) who analyse the effects of government spending at the zero lower bound.

level of the hiring rate is an important factor affecting the transmission of a fiscal stimulus. To my knowledge, this is the first paper that uncovers hiring decisions as a source of state dependence in the transmission of fiscal policy. Another paper that studies how labor market tightness affects the transmission of fiscal policy is Michaillat (2014). He studies the interaction between the size of the public sector and the labor market. In his case, an increase in the size of the public sector lowers the number of unemployed people in the labor market, for a given labor force participation. This decrease in unemployment increases labor market tightness for private firms, which have now more difficulty in filling their vacancies. This public sector channel could be an alternative channel through which fiscal stimuli and labor market interact. I show that, in the data, I do not find state dependence of fiscal expansions related to the aggregate labor market tightness. This evidence points towards my hiring rate friction being the empirically relevant congestion affecting fiscal policy transmission. Another paper studying the interaction between fiscal policy and labor markets is Gomes (2015). The latter focuses on how public wages should be determined optimally to achieve an efficient allocation of jobs between the private and the public sector. Differently from it, my paper focuses on the effect that government purchases have on the rest of the economy considering hiring frictions in the private sector. Further, Cacciatore et al. (2019) study how employment protection legislation affects the size of fiscal multipliers. While they analyse labor market frictions in the form of firing costs, my main focus is on hiring costs. Our results are consistent with each other and indicate that labor frictions internal to firms lower the size of multipliers.

The third stream of literature this paper is related to is the one modelling hiring frictions. The standard Diamond-Mortensen-Pissarides (DMP) framework models hiring frictions as a function of the vacancy posting cost denominated in pecuniary terms. Micro evidence shows that the biggest share of the hiring costs borne by firms is related to the effort of bringing the productivity of the new hires to the level of the already experienced employees – see for example Silva and Toledo (2009), Manning (2011), Cooper et al. (2015) and Faccini and Yashiv (2019). These costs are mainly related to training and are a function of the new hires as a share of the existing employees. They are firm-specific and are denominated in terms of foregone output. I contribute to this line of studies by analysing how various types of hiring frictions affect the propagation of fiscal shocks differently. In particular, I show that the hiring frictions generating state dependence in the transmission of fiscal policy are those related to training costs. To the contrary, hiring frictions associated to vacancy posting costs are less prominent in affecting the transmission of fiscal spending stimuli.

This paper is organised as follows. Section 2 shows the reduced form evidence on expansionary fiscal shocks. Section 3 discusses the modelling assumptions of hiring frictions. Section 4 describes the model and the theoretical results. Section 5 analyses asymmetries in the impulse responses. Section 6 studies an extension of the hiring cost function. Section 7 concludes.

2 Reduced Form Evidence

In this section I estimate fiscal multipliers that are dependent on the state of the labor market and the sign of the fiscal shock. I begin by discussing identification and how I define the labor market.

2.1 Fiscal Shocks

To study fiscal shocks, I focus on government purchases (consumption and investment) in the US. Following Auerbach and Gorodnichenko (2012, 2013), I identify unanticipated government spending shocks as the forecast error FE_t for the growth rate of government purchases. The forecast error is defined as the difference between the forecast made at time t - 1 for government spending growth at t and the actual, first-release government spending growth rate at time t. The underlying idea is that the component of government spending growth, which is not forecasted one period ahead, can be considered as unanticipated by economic agents. Forecasts for government spending are available from the Survey of Professional Forecasters (SPF) since 1982.⁴ For the period before that, Greenbook forecasts prepared by the Federal Reserve Board for the Federal Open Market Committee meetings are available from 1966 to 2004. By splicing the Greenbook and the SPF forecasts, Auerbach and Gorodnichenko (2012) compute a forecast error series running from 1966Q3 to 2010Q2.

Other strategies are also used in the literature to identify fiscal shocks. A leading alternative is the narrative series of military spending news constructed by Ramey (2011). What prevents me from using this series is a data restriction. Over the sample period when the hiring rate is available, there are only 27 military spending news, which is not enough to meaningfully compute multipliers in four states, that is the combination of expansionary/contractionary fiscal shocks and high/low hiring rate. Ramey (2011) shows that the forecast error computed by using the SPF forecast has an R-squared of 60 percent for government spending growth for the sample 1968Q4 - 2008Q4, which makes a potentially more powerful indicator of news than the military spending over the sample considered.⁵

2.2 The State of the Labor Market

The theoretical literature on labor market frictions identifies two main measures to describe the state of the labor market: one captures the aggregate labor market conditions, while the other is related to firms' specific conditions. The first is usually referred to as labor

⁴The Survey of Professional Forecasters has a quarterly frequency. The forecasters are asked to provide quarterly projections of various macro variables for five quarters (the quarter when the survey takes place and the four following quarters) and annual projections for the current year and the following year. More information can be found at https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/

 $^{^5\}mathrm{From}$ 1968Q4 to 1981Q2 the SPF predicts nominal defence spending, while from 1981Q3 the SPF predicts total federal spending.

market tightness and is the ratio between posted vacancies and unemployment. The second is the hiring rate of firms and is defined as the ratio between new hires and employees. Figure 1 shows both the labor market tightness and the hiring rate. Labor market tightness is computed using the help wanted index⁶ from Barnichon (2010) and the unemployment series from the FRED Database. The hiring rate is calculated using the series of hires and employment from the Current Population Survey (CPS). Ideally, one would want the flows of new hires to come from both non-employment and employment. Unfortunately, though, the CPS series of hires from employment is only available from 1994. As I need a longer time span to estimate state dependent multipliers, in my baseline analysis I use the series of new hires from non-employment, which is available at quarterly frequency since 1976Q1.⁷ The hiring rate series shows a decreasing trend from the seventies to the early ninities. This secular decline is discussed thoroughly in Davis and Haltiwanger (2014), who impute it to a decline in the US labor market fluidity.⁸ My analysis does not aim at explaining the secular decline in the trend, rather it focusses on the business cycle component. Labor market tightness and the hiring rate are only mildly correlated. Their correlation is 0.15. This indicates that the aggregate and the firms-specific conditions capture different features of the labor market.

2.3 Local Projections

To compute fiscal multipliers, I use local projections, which has been proposed by Jordà (2005) as a more flexible alternative to structural vector autoregressions. Local projections allow for a direct estimation of impulse response functions without imposing any dynamic restriction. Moreover, they are particularly suited to study state dependence as they provide a flexible environment to introduce non-linearities. In its linear version, the local projection that I am interested in is:

$$x_{t+h} = \gamma_h + \phi_h(L)z_t + \beta_h F E_t + \phi_h \operatorname{trend}_t + \epsilon_{t+h} \qquad h \ge 0, \tag{1}$$

where FE_t is the forecast error of government spending growth, x_{t+h} is the logarithm of the variable of which I want to compute the impulse response, z_t is a vector of controls, $\phi_h(L)$ is a lag polynomial of degree L, γ_h is a constant term, and trend_t is a linear time trend. The estimated coefficient β_h is the impulse response of x_{t+h} to an unanticipated

⁶The help wanted index is an 'index that captures the behavior of total – "print" and "online"– help-wanted advertising, by combining the print Help-Wanted Index with the online Help-Wanted Index published by the Conference Board since 2005.', Barnichon (2010).

⁷The Job Openings and Labor Turnover Survey from the Bureau of Labor Statistics provides data on new hires from both non-employment and employment and is available from 2000M12. In Section 2.5.3, I conduct robustness checks using an extended version of this series.

⁸They observe the following: 'An ageing workforce and a secular shift away from younger and smaller employers partly account for the long-term decline in labor market fluidity. These forces are not the main story, however. Instead, we find large declines in the rate at which workers reallocate across employers within cells defined by gender and age and by gender and education. Likewise, there are large declines in the rate at which jobs reallocate across employers within cells defined by industry, employer size and employer age.'

government spending shock at t as captured by FE_t . To study sign and state dependence, I need a non-linear version of Equation (1). Specifically, I introduce the following dummy variables: $\mathbb{1}_t^+ = 1$ when $FE_t > 0$ and $\mathbb{1}_t^+ = 0$ when $FE_t \leq 0$; $\mathbb{1}_t^{Slack} = 1$ when the labor market variable (tightness or the hiring rate) is below trend and $\mathbb{1}_t^{Slack} = 0$ when it is above trend. The non-linear local projection is as follows:

$$\begin{aligned} x_{t+h} &= \mathbb{1}_{t}^{+} \mathbb{1}_{t-1}^{Slack} \left[\gamma_{PS,h} + \phi_{PS,h}(L) z_{t} + \beta_{PS,h} F E_{t} \right] \\ &+ \mathbb{1}_{t}^{+} (1 - \mathbb{1}_{t-1}^{Slack}) \left[\gamma_{PT,h} + \phi_{PT,h}(L) z_{t} + \beta_{PT,h} F E_{t} \right] \\ &+ (1 - \mathbb{1}_{t}^{+}) \mathbb{1}_{t-1}^{Slack} \left[\gamma_{NS,h} + \phi_{NS,h}(L) z_{t} + \beta_{NS,h} F E_{t} \right] \\ &+ (1 - \mathbb{1}_{t}^{+}) (1 - \mathbb{1}_{t-1}^{Slack}) \left[\gamma_{NT,h} + \phi_{NT,h}(L) z_{t} + \beta_{NT,h} F E_{t} \right] \\ &+ \phi_{h} \operatorname{trend}_{t} + \epsilon_{t+h}, \quad h \geq 0. \end{aligned}$$

This regression allows me to distinguish among four states: positive fiscal shock in a slack labor market (PS), positive fiscal shock in a tight labor market (PT), negative fiscal shock in a slack labor market (NS), and negative fiscal shock in a tight labor market (NT). Notice that the dummy variable for the sign of the shock is indexed at time t, while the dummy variable for the state of the labor market is indexed at time t-1. This lag is introduced to avoid contemporaneous feedback from policy action into the state of the economy. The coefficients $\beta_{PS,h}$, $\beta_{PT,h}$, $\beta_{NS,h}$, and $\beta_{NT,h}$ are the coefficients of interest and are the impulse responses of variable x_{t+h} at horizon t+h to an unanticipated government spending shock at t. The control variables which I include are: log real output, log real government expenditure, debt-to-GDP, and unemployment rate. The lag L is equal to one. By controlling for lagged output and lagged government expenditure, the forecast error is purified from any component that could have been predicted by professional forecasters the previous period. I control for debt-to-GDP so as to make sure that the state dependence is not imputable to different levels of deficit.⁹ In addition, including unemployment rate allows me to control for the phase of the business cycle. The regression is estimated by ordinary least squares. Since standard errors are serially correlated when employing this methodology, I use Newey-West robust standard errors.

Given the impulse responses obtained from Equation (2), it is possible to compute cumulative fiscal multipliers.¹⁰ This consists of a three-step procedure as follows: first, run Equation (2) for x_{t+h} equal to log real GDP for h = 0, ..., H and sum all the β_h ; second, run Equation (2) for x_{t+h} equal to log real government expenditure for h = 0, ..., H and sum all the β_h ; third, divide the first sum by the second sum to get the elasticity of

 $^{^{9}}$ Gali et al. (2007) show that the fiscal multiplier critically depends on how the fiscal expansion is financed. In particular, they argue that it is increasing with the extent of deficit financing.

¹⁰As argued by Ramey and Zubairy (2018), the policy relevant concept of fiscal multiplier is the *cumulative multiplier*. This is defined as the cumulative GDP response relative to the cumulative government spending response over a given period, as proposed by Mountford and Uhlig (2008), Uhlig (2010), and Fisher and Peters (2010). The cumulative multiplier differs from the concept of *peak multiplier*, defined as the peak of the output response to the initial government spending shock, or the *average multiplier*, defined as the ratio between the average response of output over the horizon of interest and the initial government shock.

output to exogenous increases in government expenditure. This three-step procedure is equivalent to using an instrumental variable local projection approach as proposed by Ramey and Zubairy (2018). This method allows me to directly compute the cumulative fiscal multipliers and the associated standard errors. In its linear formulation the regression to estimate is:

$$\sum_{j=0}^{h} y_{t+j} = \gamma_h + \phi_h(L) z_t + m_h \sum_{j=0}^{h} g_{t+j} + \phi_h \text{ trend}_t + \epsilon_{t+h},$$
(3)

where $\sum_{j=0}^{h} y_{t+j}$ is the sum of log real output, and $\sum_{j=0}^{h} g_{t+j}$ is the sum of log real government expenditure instrumented by the fiscal shock FE_t . Since g_{t+j} and y_{t+j} are in logs, the coefficient m_h is an elasticity. To convert it to dollar terms, I multiply m_h by the average ratio between real output and real government expenditure over the sample period.¹¹ Standard errors are also converted by the relevant factor by using the delta method. The non-linear version of Equation (3) is the following:

$$\sum_{j=0}^{h} y_{t+j} = \mathbb{1}_{t}^{+} \mathbb{1}_{t-1}^{Slack} \left[\gamma_{PS,h} + \phi_{PS,h}(L)z_{t} + m_{PS,h} \sum_{j=0}^{h} g_{t+j} \right] \\ + \mathbb{1}_{t}^{+} (1 - \mathbb{1}_{t-1}^{Slack}) \left[\gamma_{PT,h} + \psi_{PT,h}(L)z_{t} + m_{PT,h} \sum_{j=0}^{h} g_{t+j} \right] \\ + (1 - \mathbb{1}_{t}^{+}) \mathbb{1}_{t-1}^{Slack} \left[\gamma_{NS,h} + \psi_{NS,h}(L)z_{t} + m_{NS,h} \sum_{j=0}^{h} g_{t+j} \right] \\ + (1 - \mathbb{1}_{t}^{+}) (1 - \mathbb{1}_{t-1}^{Slack}) \left[\gamma_{NT,h} + \psi_{NT,h}(L)z_{t} + m_{NT,h} \sum_{j=0}^{h} g_{t+j} \right] \\ + \phi_{h} \operatorname{trend}_{t} + \epsilon_{t+h}, \qquad h \ge 0,$$

where again $\sum_{j=0}^{h} g_{t+j}$ is instrumented with the fiscal shock FE_t .

2.4 Fiscal Multipliers

Impulse Responses. Figure 2 and Figure 3 show the impulse response functions of government spending, output, consumption and investment to a positive fiscal shock. Figure 2 displays the responses when the hiring rate is used to identify the state of the labor market, while Figure 3 exhibits the responses when tightness is used instead.¹² The blue solid line depicts the responses when the series identifying the labor market state is

¹¹The concern raised by Ramey and Zubairy (2018) towards this conversion procedure as responsible for inflating or deflating multipliers does not apply here, as the ratio between output and government expenditure is not very volatile over my sample.

¹²For completeness, Appendix B.1 shows the impulse responses in absence of any state dependence, that is in the linear case.



Figure 2 – **Impulse responses to a positive spending shock depending on hiring rate** Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate.

below trend: it corresponds to coefficients $\beta_{PS,h}$ for h = 1, ..., 20 in Equation (2). The red dashed line depicts the responses when the series identifying the labor market state is above trend: it corresponds to the coefficients $\beta_{PT,h}$ for h = 1, ..., 20. As can be observed from Figure 2, when the hiring rate is used to identify the labor market state, responses to an expansionary shock are very much state dependent. All variables are more responsive when the hiring rate is below than when it is above trend. Moreover, an expansionary shock when the hiring rate is above trend slightly crowds out private consumption and investment, generating a mild drop in output. To the contrary, the picture is much less clear when tightness is used in place of the hiring rate. As shown by Figure 3, responses of output, consumption, and investment are not statistically different in the two states.

Cumulative Multipliers. From Figure 2, it is immediately evident that the peak multiplier is higher when the hiring rate is below trend.¹³ However, since the more policy relevant measure of the fiscal multiplier is the cumulative multiplier defined as the cumulative output response relative to the cumulative government spending response over a given horizon, Table 1 reports the cumulative multipliers for a horizon of 20 quarters. These multipliers correspond to the estimated coefficients $m_{PS,h}$ and $m_{PT,h}$ in Equation (3). To be precise, since these coefficients are elasticities of output to fiscal shocks, they are

¹³The peak multiplier is the ratio between the peak of the output response and the initial government spending shock.



Figure 3 – Impulse responses to a positive spending shock depending on tightness Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) tightness.

Table 1 – Cumulative multipliers for a positive shock						
Labor Market Variable	Horizon	Tight	Slack	p-value		
Hiring Rate	5Y	-0.81	3.58	0.005		
		(0.59)	(0.31)			
Tightness	5Y	2.52	2.51	0.981		
		(0.45)	(0.44)			

Table 1 – Cumulative	$\mathbf{multiplier}$	s for a	positive shock
Labor Market Variable	Uonizon	Tight	Slady p value

converted to dollar equivalent by multiplying them by the average output-to-government spending ratio over the sample period. When the hiring rate is used to identify the labor market state, multipliers to an expansionary fiscal shock are as big as 3.58 when the hiring rate is below trend, but they are not significantly different from zero when the hiring rate is above trend. Moreover, the p-value testing whether the multiplier estimates differ across states indicates that the multipliers are statistically different in the two states. On the other hand, when tightness is used to identify the labor market state, the five-year cumulative multipliers to an expansionary fiscal shock are not statistically different in the two states of tightness above and below trend.

Note: Newey-West standard errors in parenthesis. 'p-value' indicates the p-value for the test that the multiplier estimates are different across states. It is based on heteroscedastic- and autocorrelation-consistent standard errors.

2.5 Robustness Checks

2.5.1 Alternative Measures of Labor Market Tightness

Other measures are used in the literature as proxies for the aggregate state of the labor market. Appendix B.2 shows responses when NAIRU or the unemployment rate trend are used as threshold to identify the state of the aggregate labor market. In the former case, the labor market is defined as tight when the unemployment rate is below the NAIRU and slack otherwise. In the latter case, the labor market is defined as tight when the unemployment rate is below its trend and slack otherwise. In both cases, no state dependence of the impulse responses to an expansionary fiscal shock is evident.

2.5.2 Robustness Checks for the CPS Hiring Rate Series

Appendix B.3 conducts some robustness checks for the local projection results where the labor market state is identified by using the CPS hiring rate series. The first set of robustness checks concerns the specification of the regression model. Results are robust to eliminating the time trend, adding two lags, and changing the control variables to taxes and log employment.

The second set of robustness checks concerns the method applied to detrend the hiring rate. As a matter of fact, the state dependence may be affected by the way the hiring rate series is detrended. Results still hold when changing the HP filter smoothing parameter, as well as when a polynomial trend or a moving average are employed to detrend the series.

2.5.3 An Alternative Measure of the Hiring Rate: JOLTS Extended Series

As an additional robustness check, I use an alternative measure of the hiring rate, which, differently from the CPS series, also includes the new hires from employment. This is the hiring rate series from the Job Openings and Labor Turnover Survey. This series, which starts in 2001, is extended back to 1990 by Davis et al. (2012).¹⁴ To study state dependence, it is important for me to have a long time series. For this reason, I further extend the hiring rate series by Davis et al. (2012) as follows. Over the period 1990-2018, I run the regression:

$$\frac{H_t}{N_t} = \beta_0 + \beta_1 u_t + \beta_2 \Delta \log N_t + \beta_3 t + \epsilon_t,$$
(5)

where H_t/N_t is the hiring rate series of Davis et al. (2012), u_t is the unemployment rate, $\Delta \log N_t$ is the first difference of log employment, and t is a linear time trend. I can then use the estimated coefficients $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, the unemployment rate, and the employment series to get the fitted values of the hiring rate prior to 1990. Using the fitted hiring rate,

¹⁴I thank Jason Faberman for sharing this hiring rate series with me.

I rerun Equation (2) and Equation (4). Appendix B.4 shows that results are robust to using this extended hiring rate series to identify the labor market state.

2.5.4 Measures of Recessions vs Expansions

The literature has so far focussed on fiscal spending multipliers dependent on the business cycle. Appendix B.5 illustrates that defining the state of the economy according to the hiring rate is not the same as following the standard definitions of business cycle. In particular, the Appendix shows that expansionary fiscal multipliers are not state dependent when the NBER recessions or Auerbach and Gorodnichenko (2012)'s smooth transition threshold are used to define the state of the economy.

3 Modelling Hiring Frictions

In light of the reduced-form evidence provided in Section 2, this section is going to discuss how the literature has approached the modelling of hiring frictions and how I am going to tackle it. In the standard DMP framework hiring frictions have two salient features: i) they are modelled as costs of posting new vacancies; ii) they are denominated in terms of the final composite good, i.e. as pecuniary. The first feature implies that the main cost of hiring a worker is borne before the match is created. In particular, it assumes that, once a new worker is hired by a firm, he can start working with the same productivity as the one of experienced workers. The second feature requires the cost of hiring to enter the resource constraint of the economy, thus constituting a share of aggregate demand. Micro-evidence shows that these features are not fully supported by empirical evidence. For this reason, I am modelling hiring frictions as i) mainly post-match and ii) expressed in terms of foregone output. I am going to discuss these two assumptions more in details in the following subsections.¹⁵

3.1 Pre-match versus Post-match Hiring Costs

When hiring a new worker, employers face two types of hiring costs: pre-match costs and post-match costs. Pre-match costs are those related to posting vacancies, head hunting, and interviewing. Post-match costs are those referring to training and all the activities that raise the productivity of a newly employed worker to the level of an already experienced worker. Pre-match costs are also referred to as external costs as they depend on the conditions of the aggregate labor market, which are external to the firm. Postmatch costs are also known as internal costs as they depend on the internal conditions of the firm, meaning that they depend on the share of new hires to the already employed workers. While most of the macro literature has focussed on pre-match costs, new micro

 $^{^{15}}$ For an overview on hiring frictions and their theoretical and empirical relevance see Manning (2011). Estimates on the magnitude of hiring frictions vary by country, sector, and skill in a range between 2.4% and 11.2% of the wage bill.

evidence is showing that the biggest share of the hiring costs borne by firms is actually post-match rather than pre-match. Using German survey data, Faccini and Yashiv (2019) compute that 82.3% of hiring costs are post-match.¹⁶ In the Handbook of Labor Economics, Manning (2011) highlights that the bulk of hiring costs is related to training rather than recruitment. Similarly, Silva and Toledo (2009) review the literature on hiring costs and report that, based on US data, training costs are much more significant than recruiting costs. Muchlemann and Leiser (2018) decompose the cost of hiring into search costs, adaptation costs, and disruption costs and use Swiss administrative establishmentlevel survey data for 2000, 2004, and 2009 to estimate the cost components. They find that 'The search costs only accounted for 21 percent of the costs incurred to fill a vacancy and most of a firm's hiring expenses occurred after the signing of a contract. Adaptation costs (i.e., training costs and the initially low productivity of a new hire) accounted for 53 percent and disruption costs (i.e., productivity losses because other workers could not perform their regular tasks while providing informal training to new hires) accounted for 26 percent of the total hiring costs.' In parallel to the growing micro evidence, macroeconomic studies started modelling hiring frictions both as pre-match and post-match. Yashiv (2000), Christiano et al. (2011), Furlanetto and Groshenny (2016), and Faccini and Melosi (2019) have specifications of hiring costs which allow for both pre-match and post-match costs. They estimate their models using different datasets¹⁷ and they all find that the biggest share of costs is the post-match one. Given all this micro and macro evidence, in my baseline specification I model hiring costs as post-match following Gertler et al. (2008), Gertler and Trigari (2009), and Faccini and Yashiv (2019). In particular, the hiring cost function $\tilde{g}_{i,t}$ of firm *i* that I assume is as follows:

$$\tilde{g}_{i,t} = \frac{e}{2} \left(\frac{H_{i,t}}{N_{i,t}}\right)^2,\tag{6}$$

where $H_{i,t}/N_{i,t}$ is the ratio between the new hires and the workforce of firm *i*, and *e* is a scale parameter. This formulation assumes that hiring costs are quadratic in the hiring rate. This means that the costs get increasingly more significant with the rise in the hiring rate.¹⁸ Using US data, Merz and Yashiv (2007) structurally estimate the functional form of firms' adjustment costs and find strong evidence in favour of their convexity. Since the quadratic specification is the most commonly adopted, I follow the literature in specifying my costs as quadratic. Notice nonetheless that what matters for my results is the convexity of the hiring cost function, not the specific degree of convexity.

¹⁶Namely, they use a survey conducted by the Federal Institute for Vocational Education Training over the years 2012-2013.

¹⁷Yashiv (2000) uses Israeli data, Christiano et al. (2011) use Swedish data, and Furlanetto and Groshenny (2016) and Faccini and Melosi (2019) use US data.

¹⁸There could be other forces at play such as economies of scales that would push towards a concave specification. However, structural estimation indicates that these costs are convex in the hiring rate.

3.2 Non-pecuniary versus Pecuniary Hiring Costs

In the standard DMP framework, hiring costs are denominated in units of the final good, which is usually the economy numeraire. Yet, micro evidence shows that the most burdensome aspect of hiring a new employee is not the pecuniary costs actually faced. Rather it is the disruption that a new hire causes along two dimensions. First, it takes some time to train the new hires and bring their productivity to the level of already trained employees.¹⁹ Second, unless the training activity is outsourced, internal training generates a disruption in production as some employees have to be diverted from production to the training of the newly hired workers.²⁰ This cost is quantifiable as foregone production. Faccini and Yashiv (2019) bring micro evidence that these costs represent around 80% of the total hiring cost. Along the same lines, Bartel et al. (2014) find that the arrival of a newly hired nurse in a hospital lowers the productivity of the team when the nurse is hired externally, while Cooper et al. (2015) show that labor adjustment disrupts the production process of manufacturing plants. Given this evidence, I follow Faccini and Yashiv (2019) and Faccini and Melosi (2019) in modelling hiring costs as non-pecuniary. Let's assume that $f_{i,t}$ is the production function of firm *i*. Having non-pecuniary costs implies forgoing some of the production to actually hire. In analytical terms this concept translates into the following equation:

$$Y_{i,t} = f_{i,t}(1 - \tilde{g}_{i,t}).$$
(7)

In other words, to match demand $Y_{i,t}$ firm *i* has to produce $f_{i,t}(1 - \tilde{g}_{i,t})$.

4 Model

To study the effects of an expansionary fiscal shock across different states of the labor market, I develop a general equilibrium model with hiring frictions and nominal rigidities. I assume that there is a government, which spends wastefully following an exogenous process. Nominal rigidities are needed as the mechanism through which hiring costs affect the propagation of fiscal shocks hinges upon the interaction between these two types of frictions.

The economy is populated by a continuum of households fully sharing risk. Households consume, work, choose whether to join the labor force, invest in capital and one-period bonds. Firms are of two types. Final good firms aggregate intermediate differentiated

¹⁹Also anecdotal evidence points towards this being the case. Fastcompany, an American business magazine, writes: 'It can take as long as long as eight months for an employee to become fully productive'. Hundred5, a modern skilled-based hiring platform, notes: 'When you hire someone new, they most likely won't be fully productive their first day of work. In fact, it can take up a few months for them to get comfortable in their new role. As confirmed by research, it will take 8 to 26 weeks for an employee to achieve full productivity. Before this time runs out, you are essentially losing money – the new employee costs more than they are earning for the company. After this period, they provide positive return on investment for your company, i.e. this is the break-even point.'

 $^{^{20}}$ Harvard Business Review declares that hiring 'disrupts the culture and burdens peers who must help new hires figure out how things work.'

goods into a homogeneous product. This homogenous good is sold to the households and the government in a perfectly competitive market. Intermediate good firms produce their differentiated good by renting capital and hiring labor from households. They face a quadratic cost of hiring and a quadratic cost of adjusting prices. Labor markets are frictional and wages are set according to Nash bargaining. Real wages are adjusted with some inertia. Monetary policy follows a standard Taylor rule where the interest rate reacts to deviations of inflation and output from their steady state. Wasteful government spending is funded by levying lump-sum taxes on households and issuing one-period bonds. The economy is subject to exogenous shocks in preferences, technology, marginal efficiency of investment, monetary policy and fiscal policy.

4.1 Labor Market

There are three different employment states: employed, unemployed but actively looking for a job, and unemployed but inactive. N_t is the number of employed, while U_t is the number of unemployed actively looking for a job. The fraction of people supplying labor, and therefore actively participating in the labor market, is

$$L_t = N_t + U_t. ag{8}$$

The labor market is subject to search and matching frictions. People who are unemployed at the beginning of the period, indicated by $U_{0,t}$, actively look for a job, while firms post vacancies V_t . New hires H_t are created according to a standard Cobb-Douglas function:

$$H_t = m U_{0,t}^l V_t^{1-l}, (9)$$

where m is a parameter controlling the matching efficiency and l is the elasticity of hires to beginning-of-period job seekers. The vacancy filling rate and the job finding rate are respectively:

$$q_t = \frac{H_t}{V_t}$$
 and $x_t = \frac{H_t}{U_{0,t}}$. (10)

Aggregate employment evolves according to the following law of motion:

$$N_t = (1 - \delta_N)N_{t-1} + H_t, \tag{11}$$

where $\delta_N \in (0, 1)$ is the exogenous separation rate between firms and workers.

4.2 Household

There is a representative household who gets utility from consuming C_t and get disutility from supplying labor L_t . The period utility function of the household is:

$$\mathcal{U}_t(C_t, L_t) = \frac{\left(\eta_t^p C_t - \frac{\chi}{1+\phi} L_t^{1+\phi}\right)^{1-\sigma}}{1-\sigma},\tag{12}$$

where χ is the parameter that controls the disutility of supplying labor, ϕ is the inverse Frisch elasticity of labor supply, and σ is the intertemporal elasticity of substitution. η_t^p is an exogenous AR(1) process with Gaussian shocks, which will be referred to as preference shocks.

The household can choose to save by investing in capital K_{t+1} or buying zero-coupon government bonds at discounted value $\frac{B_{t+1}}{R_t}$, where R_t is the nominal interest rate set by the monetary policy authority. Capital evolves according to the following law of motion:

$$K_{t+1} = (1 - \delta_K)K_t + \eta_t^{Inv} \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t,$$
(13)

where I_t are investments, $\delta_K \in (0,1)$ is the capital depreciation rate, and $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2$ are quadratic investment adjustment costs. As in Justiniano et al. (2011), η_t^{Inv} follows an AR(1) process and affects the marginal efficiency of investment.

The household gets a return R_t^K on capital rented to firms and earns nominal wage W_t from working. Moreover, he gets dividends Θ_t from owning firms and pays lump-sum taxes Θ_t . The budget constraint of the household is:

$$P_t C_t + P_t I_t + \frac{B_{t+1}}{R_t} = R_t^K K_t + W_t N_t + B_t + \Theta_t - T_t,$$
(14)

where P_t is the price of the final composite good in which both consumption and investment are denominated.

Given the initial value of bonds B_0 and the discount factor β , the household chooses state-contingent sequences $\{C_t, L_t, N_t, K_{t+1}, I_t, B_{t+1}\}_{t=0}^{\infty}$ to maximise the discounted present value of utility:

$$\max_{\{C_t, L_t, N_t, K_{t+1}, I_t, B_{t+1}\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t(C_t, L_t)$$
(15)

subject to the labor supply constraint (8), the employment law of motion (11), the capital law of motion (13), and the budget constraint (14).

4.3 Firms

There are two types of firms of measure one: perfectly competitive final good producers and monopolistically competitive intermediate good producers.

4.4 Final Good Producers

Final good producers buy differentiated goods from intermediate producers, aggregate them into a final homogeneous good, and sell it to the households and the government in a perfectly competitive market. To aggregate intermediate goods, final producers use a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}},\tag{16}$$

where ϵ indicates the elasticity of substitution between different varieties of intermediate good. Cost minimisation gives the demand function for the intermediate good:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t,\tag{17}$$

where $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the price index for the final good.

4.5 Intermediate Good Producers

Each intermediate good producer *i* produces a differentiated good by using capital $K_{i,t}$, and labor $N_{i,t}$. These factors of production are combined into $f_{i,t}$ according to the Cobb-Douglas function:

$$f_{i,t} = (A_t N_{i,t})^{\alpha} K_{i,t}^{1-\alpha},$$
(18)

where α is the labor share in the inputs of production and A_t is a labor-augmenting technology, which follows an AR(1) process. Intermediate good producers rent capital from households at nominal rate R_t^K and hire labor in a frictional labor market.

4.5.1 Hiring frictions

Hiring costs as training. When hiring workers, intermediate good firms face hiring frictions. As discussed in Section 3.1, these costs are modelled as post-match and mainly refer to training and all the activities required to bring the productivity of a newly hired worker to the level of an already experienced one. Following Gertler et al. (2008), Gertler and Trigari (2009), and Faccini and Yashiv (2019), the hiring cost function $\tilde{g}_{i,t}$ of firm *i* is:

$$\tilde{g}_{i,t} = \frac{e}{2} \left(\frac{H_{i,t}}{N_{i,t}}\right)^2,\tag{19}$$

where $H_{i,t}/N_{i,t}$ is the ratio between the new hires and the workforce of firm *i*, and *e* is a scale parameter.²¹

Hiring costs as foregone output. As documented in Faccini and Yashiv (2019) by using micro data, the biggest component of hiring costs is not borne in pecuniary terms, but in terms of disruption to production. Therefore, as discussed in Section 3.2 and following Faccini and Yashiv (2019), I choose to model hiring costs as a fraction of foregone output. This modelling assumption captures the idea that hiring is internally costly as it requires firms to divert part of their workforce from production to hiring activities. The net output of intermediate firm i is then given by:²²

$$Y_{i,t} = f_{i,t}(1 - \tilde{g}_{i,t}).$$
(20)

For notational convenience, I define $g_{i,t} \equiv \tilde{g}_{i,t} f_{i,t}$.

4.5.2 Problem of the Intermediate Good Producers

Intermediate good producers set their price monopolistically and are subject to quadratic price adjustment costs à la Rotemberg (1982).

They solve the following problem:

$$\max_{\{P_{i,t}H_{i,t},N_{i,t},K_{i,t}\}} \mathbb{E}_{t} \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left\{ \frac{P_{i,t}}{P_{t}} Y_{i,t} - \frac{W_{t}}{P_{t}} N_{i,t} - \frac{R_{t}^{K}}{P_{t}} K_{i,t} - \frac{\zeta}{2} \left(\frac{P_{it}}{P_{i,t-1}\bar{\Pi}} - 1 \right)^{2} Y_{t} \right\}$$
(21)

subject to the employment law of motion (11), the demand function (17), and the constraint requiring demand to be fully satisfied by production net of hiring costs (20). ζ is the parameter that controls price rigidities, $\overline{\Pi}$ is the steady-state gross inflation rate, while $\Lambda_{t,t+1}$ is the household's stochastic discount factor.

4.6 Wage Bargaining

Once a match is created between a firm and a worker, wage is bargained according to a standard Nash bargaining process. The Nash wage W_t^{NASH}/P_t maximises the geometric average of the household's and firm's surplus weighted by the the parameter γ , indicating the bargaining power of the household:

$$\frac{W_t^{NASH}}{P_t} = \arg\max\left\{\left(\mathcal{V}_t^N\right)^{\gamma} \left(Q_t^N\right)^{1-\gamma}\right\}.$$
(22)

²¹In this model, workers' skills are homogeneous. Therefore, there is no heterogeneity in hiring costs arising from workers' skill heterogeneity. In fact, it could be that hiring more skilled workers is more costly. This margin is left to further explorations.

²²Alternatively, I could assume that the hiring cost is not directly proportional to output, but only indirectly through hiring. This specification would impy: $Y_{i,t} = f_{i,t} - \tilde{g}_{i,t}$. I have run the model with this alternative formulation and verified that it gives rise to the same mechanism and model dynamics. Results are available upon request.

 \mathcal{V}_t^N and Q_t^N are the marginal values of a job for the household and the intermediate firm i and correspond to the Lagrange multipliers of the labor law of motions in the respective maximisation problems.

I assume that there is real wage inertia as in Hall (2005):

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\omega} \left(\frac{W_t}{P_t}\right)^{1-\omega},$$

where ω controls the degree of wage rigidity.

4.7 Fiscal and Monetary Policies and Market Clearing

Fiscal authority. The government spends G_t wastefully.²³ This expenditure is financed through a lump-sum tax T_t on the household and a zero-coupon bond issued at the discounted value $\frac{B_{t+1}}{R_t}$. The government's budget constraint is:

$$P_t G_t - T_t = \frac{B_{t+1}}{R_t} - B_t.$$
 (23)

Government spending obeys the following AR(1) process:

$$\ln G_t = (1 - \rho_G) \ln \bar{G} + \rho_G G_{t-1} + \epsilon_{G,t}, \qquad (24)$$

where \overline{G} is the steady state value of government spending, ρ_G is the autoregressive parameter, and $\epsilon_{G,t}$ is an i.i.d. shock.

Monetary authority. Monetary policy is set according to a standard Taylor rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{r_y} \epsilon_t^R,\tag{25}$$

where barred variables indicate steady state values, r_{π} and r_y control the response of monetary policy to inflation and output deviations from their steady state values, and ϵ_t^R is an i.i.d. monetary policy shock.

Market Clearing. Consolidating the household's and government's budget constraint, imposing a symmetric equilibrium across firms, and substituting for the intermediate firms' profits yields the following resource constraint:

$$\left[1 - \frac{\zeta}{2} \left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)^2\right] Y_t = C_t + I_t + G_t, \tag{26}$$

 $^{^{23}}$ I could alternatively assume that government spending is not wasteful. For example, I could suppose that it enters the utility function of the household as a complement to the private consumption good. In this case, I would get an even stronger state dependence of expansionary fiscal shocks than in the case of wasteful expenditure. Results are available upon request.

where $Y_t = f_t(1 - \tilde{g}_t)$, that is Y_t indicates aggregate output net of aggregate hiring costs, and $\bar{\Pi}$ is the steady state inflation.

4.8 The Employment and Hiring Decisions

The employment decision. The marginal value of a job to the household is given by the first order condition (FOC) of the household's problem with respect to N_t :

$$\mathcal{V}_t^N = \frac{\Phi_t}{\lambda_t P_t} + \frac{W_t}{P_t} + (1 - \delta_N) E_t \Lambda_{t,t+1} \mathcal{V}_{t+1}^N, \qquad (27)$$

where \mathcal{V}_t^N is the Lagrange multiplier associated with the employment law of motion and Φ_t is the Lagrange multiplier associated to the labor supply constraint.²⁴ In particular, $\frac{\Phi_t}{\lambda_t P_t}$ is the marginal rate of substitution between consumption and leisure. Hence, the marginal value of a job is equal to the sum of the marginal rate of substitution between consumption and leisure, the real wage, and the continuation value.

Imposing symmetry across firms, the marginal value of a job to firms is given by the FOC of the firm's problem with respect to N_t :

$$Q_t^N = \xi_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t} + (1 - \delta_N) E_t \Lambda_{t,t+1} Q_{t+1}^N,$$
(28)

where \mathcal{Q}_t^N is the Lagrange multiplier associated with the employment law of motion (11), ξ_t is the Lagrange multiplier associated with the demand function (17), while $f_{N,t}$ and $g_{N,t}$ are the derivatives of the production and the hiring cost function with respect to N_t . In particular, the marginal value of a job for the firm is equal to the marginal revenue obtained with an additional employee net of the real wage plus a continuation value.

The hiring decision. The FOC of the firm with respect to H_t is:

$$Q_t^N = \xi_t g_{H,t}.$$
(29)

This condition equates the marginal value of a job to the marginal cost of a hire. It is important to notice that the marginal cost of a hire depends on ξ_t , which can be interpreted as the shadow value of output. Fluctuations in the shadow value of output generate fluctuations in the marginal value of a job and, hence, in the hiring decisions of firms. This is because hiring costs are modelled as non pecuniary in nature, meaning that hiring activities require diverting employees from production to recruitment activities. This feature plays an important role in the propagation of a fiscal shock as is going to be explained in Section 4.12.

4.9 Exploring the Main Mechanism

Intuition: Because of search and matching frictions in the labor market, hiring can be

 $^{^{24}\}mathrm{See}$ Appendix D for the Lagrangian of the household's problem and the FOCs.

thought of as an investment activity in workers - as a matter of fact, employment N is a state variable. The decision of investing in workers, i.e. hiring, is therefore going to depend on the future value of workers. When the hiring rate is high today, the value of workers is already high and it is not expected to further increase very significantly. A smaller expected change in the value of workers implies a higher output shadow value today. As hiring costs are denominated in terms of output shadow value, this results in a costlier hiring activity. On the contrary, if the hiring rate is not as high today, the value of workers is expected to increase much more. The expected increase in the value of workers is going to lower the output shadow value, thus making investment in employment more valuable.

Analytically: To better understand the hiring decision of the firm, it is useful to rearrange Equation (28) as follows:

$$\xi_t = \frac{\frac{W_t}{P_t}}{f_{N,t} - g_{N,t}} + \frac{\mathcal{Q}_t^N - (1 - \delta_N) E_t \Lambda_{t,t+1} \mathcal{Q}_{t+1}^N}{f_{N,t} - g_{N,t}},\tag{30}$$

which allows me to decompose the output shadow value ξ_t in two components. The first term is the ratio of the real wage to the marginal productivity of labor and represents the real unit labor cost. The second term arises in the presence of hiring frictions and is a correction for the marginal value of employment relative to the *expected* marginal value of employment, that is the expected change in the value of employment. Differently from a standard search and matching model with pecuniary costs of posting vacancies, in this model the marginal value of a job Q_t^N is a function of the marginal hiring cost $g_{H,t}$ evaluated at the output shadow value ξ_t , as can be seen from Equation (29). Fluctuations in the output shadow value are going to affect firms' hiring decisions. Hence, to better identify the determinants of the output shadow value, I substitute Equation (29) into Equation (30) to get:

$$\xi_t = \frac{\frac{W_t}{P_t} - (1 - \delta_N) E_t \Lambda_{t,t+1} \xi_{t+1} g_{H,t+1}}{f_{N,t} - g_{N,t} - g_{H,t}}.$$
(31)

Equation (31) shows that the current output shadow value is a function of multiple objects: the difference between the real wage and the continuation value of a job; the difference between the marginal productivity of labor and the marginal hiring cost. Being a general equilibrium model, all these variables are endogenous. Nonetheless, the following observations can be made from Equation (31). Because of the convexity of the hiring cost function g_t , when the hiring rate increases the marginal hiring cost $g_{H,t}$ rises, resulting in a higher output shadow value. As hiring costs are denominated in terms of output shadow value, an increase in the latter raises the cost of hiring. As for the numerator of Equation (31), it depends on the expected value of next period output shadow value ξ_{t+1} and next period marginal hiring cost $g_{H,t+1}$. The former implies that when the output shadow value is expected to increase in the next period, it gets relatively cheaper for firms to front-load hiring today. The latter has the following implication. A higher expected hiring rate increases the expected marginal hiring cost $g_{H,t+1}$. This makes it relatively more convenient for firms to hire today as it lowers the current output shadow value. If a fiscal expansion hits the economy in a state of already high hiring rate, the expectation that this hiring rate is going to increase even further, thus increasing the expected value of workers, is much lower than if the fiscal expansion hits the economy in a normal labor market condition.

These reasonings carry two implications. First, the output shadow value is going to be higher if the fiscal expansion happens when the hiring rate is already high. Second, the output shadow value can decrease following an expansionary fiscal shock. In fact, differently from models with competitive labor markets where the output shadow value is simply given by the real unit labor cost, in the presence of hiring frictions there is an additional adjustment term. This term captures the fluctuations in the expected value of employment. If the value of employment is expected to increase enough to compensate the increase in the real unit labor cost, the output shadow value will decrease following an expansionary fiscal shock. As hiring costs in this model are denominated in terms of output, when the shadow value of output decreases hiring costs also decline.

4.10 Solution Method and Calibration

To study state dependence, the model is solved via a third order perturbation method around the steady state. This allows me to take non-linear effects into account.²⁵

Table 2 reports the parameters of the model. The discount factor is calibrated to 0.99 to target a quarterly interest rate of 1%, while the inverse Frisch elasticity of labor supply is set to the standard value of 2. The intertemporal elasticity of substitution is set to 2.7, which is within the range entertained by the literature. The capital depreciation rate targets a quarterly investment rate of 2.5%, while the investment adjustment cost is set to 5. The Rotemberg adjustment cost parameter maps into a five-quarter price stickiness with Calvo type price rigidities. The elasticity of substitution between varieties is calibrated to 11, implying a steady state price markup of 10%. The elasticity of output to labor is set to the standard 0.66.

Moving to parameters related to the labor market, the unemployment rate is calibrated to the US average of 0.06 over the period 1976Q1-2019Q2. Following Faccini and Melosi (2019), who target an average quarterly hiring rate of 12.76%, the separation rate is calibrated to 0.126. Also the steady state labor supply, the vacancy filling rates, and the elasticity of hires to the beginning of period unemployment are set following Faccini and Melosi (2019). Wage stickiness is calibrated to 0.87 to match the persistence of the US wages as in Faccini and Yashiv (2019). The scale parameter e in the hiring cost function

 $^{^{25}}$ To analyse state dependence, an alternative strategy would be to simulate the model under perfect foresight as in Michaillat (2014). Third-order perturbation does not force me to assume that agents have perfect foresight.

Parameter	Description	Value	Target/Source				
Standard Parameters							
β	Discount factor	0.99	quarterly interest rate of 1%				
φ	Inverse Frisch elasticity	2	standard				
σ	Intertemporal elasticity of substitution	2.5	in standard range				
δ_K	Capital depreciation rate	0.024	quarterly investment rate of 2.5%				
ϕ	Investment adjustment cost	5	in standard range				
ζ	Rotemberg adj. cost parameter	140	reset prices every five quarters				
ϵ	Elasticity of substitution	11	steady state mark up of 10%				
α	Elasticity of output to labor	0.66	standard				
	Labor Market F	Parameter	s				
Urate	Unemployment rate	0.06	US average				
δ_N	Separation rate	0.126	average quarterly hiring rate 12.76%				
\bar{L}	Steady state labor supply	0.65	US average				
l	Elasticity of hires to U_0	0.597	Faccini and Melosi (2019)				
q	Vacancy filling rate	0.7	Faccini and Melosi (2019)				
ω	Wage stickiness	0.87	match persistence of the US real wage				
e	Hiring friction parameter	4.17	Faccini and Melosi (2019)				
γ	Household's bargaining power	0.305	match u_{rate}				
χ	Disutility of labor supply	3.709	match \bar{L}				
<i>m</i>	Matching efficiency	0.704	match q				
Policy parameters							
r_{π}	Taylor rule coefficient on inflation	2	standard				
r_y	Taylor rule coefficient on output	0.125	standard				
s_G	Output share of public spending	0.2	US average				
	Exogenous P	rocesses					
ρ_G	Persistence of G	0.90	Faccini and Melosi (2019)				
σ_G	Volatility of G shock	0.009	Faccini and Melosi (2019)				
$ ho_A$	Persistence of TFP	0.98	Faccini and Melosi (2019)				
σ_A	Volatility of TFP	0.003	Faccini and Melosi (2019)				
$ ho_p$	Persistence of preference shock	0.45	Faccini and Melosi (2019)				
σ_p	Volatility of preference shock	0.004	Faccini and Melosi (2019)				
$ ho_I$	Persistence of investment shock	0.81	Faccini and Melosi (2019)				
σ_I	Volatility of investment shock	0.008	Faccini and Melosi (2019)				

Table 2 – Parameter values

is set to the value estimated by Faccini and Melosi (2019).

The Taylor rule coefficients on inflation and output are set to the standard 2 and 0.125, while the output share of public spending is calibrated to the US average of 20%. Since three steady state variables are calibrated (unemployment rate, vacancy filling rate and labor supply), this leaves three parameters to be determined in steady state: the workers' bargaining power γ , the matching efficiency parameter m, and the disutility of supplying labor χ .

Persistence and standard deviations of the exogenous processes are parametrised following the estimated values of Faccini and Melosi (2019).

4.11 State dependent responses

Impulse responses in non-linear models. There are two main features that differentiate impulse responses of non-linear models from those of linear (or linearised) ones. First, while in linear models the path of variables in response to a shock does not depend on the initial state of the simulation, in non-linear models impulse responses do depend on the initial state. Second, in linear models there is a natural benchmark against which to compare impulse responses. This benchmark is the behavior of the model absent the shock, that is the deterministic steady state. In non-linear models, instead, identifying this benchmark is less straightforward. As a matter of fact, the model reacts differently depending on the initial state and the sequence of shocks, so even in absence of a contemporaneous shock, the model may behave differently due to a different history of shocks. In the coming analysis, I will be specific on the initial state of the simulation and the benchmark against which to compare impulse responses.²⁶

My exercise. My goal is to study how differently an expansionary fiscal shock propagates throughout the economy depending on the hiring rate level when the shock hits. In brief, I first obtain a distribution for the hiring rate. Then, I study the responses to an expansionary fiscal shock simulated from an economy with different hiring rate levels. More specifically, I proceed as follows. First, I compute the stochastic steady state, that is the steady state to which the model converges when it is not subject to shocks. Second, starting from the stochastic steady state, I simulate the model for one period and repeat this simulation for 100 000 times.²⁷ By doing so, I obtain a distribution for the hiring rate. I take the simulation in which the hiring rate corresponds to the 90^{th} percentile of its distribution. Then, I simulate an expansionary fiscal shock starting from the state corresponding to the 90^{th} percentile of the hiring rate distribution and from the stochastic steady state. As benchmark against which to compare the model subject to the fiscal shock, I take the model absent the fiscal shock. Therefore, I subtract the path of the variables when the model receives a fiscal shock at time $t = 0, Z_t^{\text{shock}}$, with the path of the same variables when the model does not receive any fiscal shock at time t = 0, $Z_t^{\text{no shock}}$. Finally, since the path of the variables converges to the stochastic steady state, I standardise the responses by the latter, $Z^{\text{stoch. SS}}$. In short, for each variable Z_t and

$$s_{t} = \Phi_{s,0} + As_{t-1} + B\epsilon_{t} + \frac{1}{2}\Phi_{s,1}(s_{t-1}\otimes s_{t-1}) + \frac{1}{2}\Phi_{s,2}(\epsilon_{t}\otimes \epsilon_{t}) + \Phi_{s,3}(s_{t-1}\otimes \epsilon_{t}) + h.o.t.$$
$$x_{t} = \Phi_{x,0} + Cs_{t-1} + D\epsilon_{t} + \frac{1}{2}\Phi_{x,1}(s_{t-1}\otimes s_{t-1}) + \frac{1}{2}\Phi_{x,2}(\epsilon_{t}\otimes \epsilon_{t}) + \Phi_{x,3}(s_{t-1}\otimes \epsilon_{t}) + h.o.t.$$

 $^{^{26}\}mathrm{A}$ non-linear state space representation of the model is as follows:

where s_t and x_t are the vectors of state and non-state variables respectively, ϵ_t is the vector of shocks, capital letters indicate matrices of coefficients, and *h.o.t.* refers to higher order terms. From this representation it is clear that, since the vector of shocks interacts with the state vector, the response of the system depends on the part of the state space where the economy is when the shock hits.

 $^{^{27}}$ Another possibility is to simulate the model for 100 000 times starting from the stochastic steady state, and then compute the distribution of labor market tightness. Results hold also with this type of simulation.

time t = 0, ..., T, the impulse response that I plot is:

$$IRF(Z_t|s_0) = \frac{(Z_t^{\text{shock}}|s_0) - (Z_t^{\text{no shock}}|s_0)}{Z^{\text{stoch. SS}}},$$
(32)

where s_0 is the initial state of the simulation. Because of higher order approximation, $IRF(Z_t|s_0)$ will depend on s_0 . In particular, $IRF(Z_t|s_0)$ will be different when the initial state s_0 corresponds to the 90th percentile of the hiring rate distribution or to the stochastic steady state.

4.12 Fiscal Shocks

Figure 4 plots the impulse responses to a one standard deviation increase in public spending. The blue solid line shows $IRF(Z_t|s_0 = \text{stochastic SS})$, the response of variable Z_t at quarterly horizon t = 0, ..., 10 when the shock is simulated from the stochastic steady state. The red dashed line displays $IRF(Z_t|s_0 = 90^{th} \text{ percentile})$, the response of variable Z_t at quarterly horizon t = 0, ..., 10 when the shock is simulated from the state corresponding to the 90^{th} percentile of the hiring rate distribution.

Because of price rigidities, a rise in the fiscal expenditure makes the shadow value of output ξ_t fluctuate. In particular, while the shadow value of output increases on impact following a fiscal shock simulated from a high hiring rate (90th percentile of its distribution), it decreases when starting from the stochastic steady state.²⁸ The different response in the shadow value of output triggers a different behaviour of the marginal value of a job, as given by Equation 29. Hiring costs are non-pecuniary and hiring is an activity that implies forgoing current production. Hence, the higher the value of output gets, the costlier it is to engage in hiring as this activity requires diverting some resources from production to recruiting and training. As a consequence, hires rise less when the fiscal expansion is simulated from the state corresponding to the 90th percentile of the hiring rate distribution than from the stochastic steady state. Given that the separation rate is exogenous and constant, a lower increase in hires translates into a lower rise in employment.

Moving to output and its subcomponents, consumption and investment drop on impact following the fiscal shock simulated from the 90^{th} percentile of the hiring rate distribution. After the initial drop, they start rising again. To the contrary, both consumption and investment increase when the fiscal expansion is simulated starting from the stochastic steady state. Because of the combined effect of the fiscal expansion on employment, consumption, and investment, output increases more when the fiscal expansion is simulated from the stochastic steady state. Notice that these theoretical impulse responses are broadly in line with the empirical impulse responses computed through the local projections and displayed in Figure 2.

²⁸As explained in Section 4.9, the shadow value of output can decrease following a fiscal expansion. This happens when the expected value of employment increases more than the real unit labor cost.



Figure 4 – Responses to a one standard deviation expansionary shock in gov. spending Note: Responses are in deviation from their stochastic steady state. The figure shows responses to an expansionary fiscal shock simulated from two states: the stochastic steady state (solid blue line) and the state corresponding to the 90^{th} percentile of the hiring rate distribution (red dashed line).

5 Asymmetries

Figure 5 shows a quadratic hiring cost function. The convexity of the function creates asymmetries in the strength of the friction. In particular, it shows that the hiring cost is increasingly bigger in the hiring rate. This has at least two implications. First, a positive government spending shock carried out when the hiring rate is higher is less expansionary than when the hiring rate is lower because the hiring cost gets increasingly more significant.²⁹ Second, the effects of positive and negative spending shocks are more asymmetric when carried out from a higher hiring rate. These points are better illustrated by Figure 6 and Figure 7. They compare responses to a positive and negative spending shock of the same size carried out from the states corresponding to the 10^{th} and the 90^{th} percentile of the hiring rate distribution (for the sake of comparison, responses to a contractionary shock are multiplied by -1). When the positive and negative shocks are implemented from the 90^{th} percentile, the difference between the responses is more marked than when implemented from the 10^{th} percentile. This is because at the 90^{th} percentile, the hiring friction is more severe: a positive shock is much less expansionary on output as the hiring friction gets increasingly bigger. Instead, at the 10^{th} percentile, the asymmetry

²⁹As indicated by Equation 29, the convexity of the hiring cost function is not the only reason why hiring costs are increasing in the hiring rate. Fluctuations in the output shadow value also play a key role in determining how hiring costs depend on the hiring rate.



Note: The figure shows that the hiring cost is increasingly bigger in the hiring rate. 10^{th} and 90^{th} indicate respectively the 10^{th} and 90^{th} percentiles of the hiring rate distribution.

between a positive and a negative shock is less severe as the convexity is milder.

The state dependence of responses is due to two features: the convexity of the hiring cost function and the behaviour of the output shadow value. Figure 6 and Figure 7 illustrate how much of the state dependence has to be imputed to the convexity of the hiring cost function. The remaining difference between the positive and the negative response is to be attributed to the state dependent dynamics of the output shadow value.

6 Allowing for Vacancy Costs

In Section 4.5.1 I have modelled hiring costs $\tilde{g}_{i,t}$ as dependent only on the hiring rate of firms. However, as highlighted in Section 3, the more traditional, though minor component of the hiring frictions is related to pre-match costs. In this Section, I allow the hiring costs to depend on the vacancy filling rate q_t . The latter is meant to capture the conditions of the aggregate labor market and be a proxy for the magnitude of the pre-match costs. The specification of this hiring cost function follows Sala et al. (2013), Faccini and Yashiv (2019), and Faccini and Melosi (2019):

$$\tilde{g}_{i,t} = \frac{e}{2} q_t^{-\eta} \left(\frac{H_{i,t}}{N_{i,t}}\right)^2,\tag{33}$$

where e > 0 is a scale parameter. With this specification, hiring costs depend on two factors: i) external conditions of the aggregate labor market as captured by the vacancy filling rate q_t ; ii) firm-level conditions as captured by the hiring rate $H_{i,t}/N_{i,t}$, that is the ratio of new hires to the workforce of the firm. The parameter $\eta \in [0, 2]$ controls the



Figure 6 – Impulse response functions to a positive and negative fiscal shock

Note: The figure compares impulse responses to a positive and negative fiscal shock of the same magnitude (one standard deviation) simulated from the state corresponding to the 10^{th} percentile of the hiring rate distribution. For the sake of comparison, impulse responses to a negative shock are multiplied by -1.

relative share of external (pre-match) vs internal (post-match) costs of hiring. When $\eta = 0$, only internal costs are present, as in Section 4.5.1. When $\eta = 2$, because $H_{i,t} = q_t V_{i,t}$, the function simplifies to $\tilde{g}_{i,t} = \frac{e}{2} \left(\frac{V_{i,t}}{N_{i,t}}\right)^2$, thus capturing only vacancy posting costs. For every value $\eta \in (0, 2)$, both internal and external hiring costs are present. I calibrate the parameter η following Sala et al. (2013), who estimate a value of 0.49.

With this extended specification of hiring costs, I carry out a similar exercise to the one in Section 4.11. Yet, instead of using the hiring rate to identify the state of the labor market, I now use labor market tightness V_t/U_t . In particular, I simulate the fiscal shock starting from two different states: the stochastic steady state and the state in which labor market tightness is at its 90th percentile. Figure 8 shows the responses to an expansionary fiscal shock starting from these two states. As can be observed comparing Figure 8 with Figure 4, state dependence is still present, but it is much milder. Output shadow value still increases more for the fiscal expansion simulated from a tight labor market, but the difference with the response from the stochastic steady state is much smaller than in the case of Figure 4. Hence, also the increase in hires is barely state dependent.

6.1 Denominating Hiring Costs in Pecuniary Terms

In line with micro evidence, the baseline model of Section 4 assumes that hiring frictions are denominated in units of forgone intermediate output. The standard DMP framework,



Figure 7 – Impulse response functions to a positive and negative fiscal shock Note: The figure compares impulse responses to a positive and negative fiscal shock of the same magnitude (one standard deviation) simulated from the state corresponding to the 90^{th} percentile of the hiring rate distribution. For the sake of comparison, impulse responses to a negative shock are multiplied by -1.

though, denominates hiring frictions in units of the final good, which is the economy numeraire. To model hiring frictions in pecuniary terms, I would need to change the following equilibrium conditions: the intermediate good output, the resource constraint and the FOCs with respect to employment and hires. In particular, Equations (20), (26), (28), and (29) would be substituted with conditions:

$$Y_{i,t} = f_{i,t} \tag{34}$$

$$\left[1 - \frac{\zeta}{2} \left(\frac{\Pi_t}{\overline{\Pi}} - 1\right)^2\right] Y_t = C_t + I_t + G_t + g_t, \tag{35}$$

$$Q_t^N = \xi_t f_{N,t} - g_{N,t} - \frac{W_t}{P_t} + (1 - \delta_N) E_t \Lambda_{t,t+1} Q_{t+1}^N,$$
(36)

$$Q_t^N = g_{H,t}. (37)$$

As shown by Equation (34), hiring costs are no longer deducted from the intermediate good output. Instead, being denominated in units of final rather than intermediate good, they enter the resource constraint (35). Equation (37) shows that the hiring decision does no longer depend on the shadow value of output ξ_t , but only on the marginal hiring cost $g_{H,t}$. Nonetheless, because of the convexity in the hiring cost, higher hiring rates still result in more significant hiring costs.



Figure 8 – Responses to a one standard deviation expansionary shock in gov. spending Note: Responses are in deviation from their stochastic steady state. The figure shows responses to an expansionary fiscal shock simulated from two states: the stochastic steady state (solid blue line) and the state corresponding to the 90^{th} percentile of the tightness distribution (red dashed line). Both pre-match and post-match components are included in the hiring cost function.

On a side, as already noticed by Faccini and Yashiv (2019), this model with pecuniary hiring costs is prone to indeterminacy when hiring and price frictions are high. The main reason behind this lies in the fact that both the price adjustment cost and the hiring cost enter the resource constraint. An increase in demand following an expansionary fiscal shock pushes firms to increase hires. As hiring costs add to the resource constraint, this further stimulates aggregate demand triggering self-fulfilling expectations of higher demand.

7 Conclusion

This paper has conjectured that fiscal policy transmission is dependent on labor market slackness. It has then shown that while no evidence of state dependence is found when using aggregate measures of labor market slackness, a stark state dependence is associated to the hiring rate of firms. In particular, it has brought reduced-form evidence that fiscal expansions are less effective when the hiring rate is higher. It has developed a theory to explain this empirical evidence. It has built a general equilibrium model with hiring frictions to study the propagation of expansionary fiscal shocks for different levels of the hiring rate. Hiring frictions have been modelled as training costs that are disruptive of firms' production. Hiring entails a temporary loss of firm-level production efficiency, as internal resources are temporarily diverted from production to recruiting and training the new hires. If a fiscal stimulus takes place when the hiring rate is already high, firms' ability to further expand hiring is limited, and their response to the increased aggregate demand is weaker. Due to this mechanism, the model is able to replicate the asymmetries obtained with the empirical estimation of impulse responses. In particular, the responses of output, consumption, and investment to an expansionary fiscal shock are weaker if implemented when the hiring rate of firms is higher. This result suggests that governments should time their fiscal expansions by taking the hiring rate of firms into consideration.

In this paper I have studied how hiring frictions affect the propagation of fiscal shocks. The initial trigger for the model dynamics is a rise in aggregate demand generated by an expansion in wasteful government purchases. This increase in aggregate demand could be generated by other types of shocks such as a monetary expansion. I leave the exploration of how the level of hiring frictions affects the propagation of other demand shocks to future studies.

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A Additional Figures

Figure 9 shows the distribution of forecast errors from Auerbach and Gorodnichenko (2012). Forecast errors are computed as the difference between the professional forecast made at time t - 1 for government spending growth at t and the actual, first-release government spending growth rate at time t. Auerbach and Gorodnichenko (2012) splice the Greenbook forecasts, prepared by the Federal Reserve Board for the Federal Open Market Committee meetings and available from 1966 to 1981, and the forecast done by the Survey of Professional Forecasters (SPF), which is available from 1982 onwards. Forecast errors have both negative and positive sign, with mean -0.15, standard deviation 4.46, and skewness 0.42. I use this series as my identified unanticipated fiscal shocks.



Figure 9 – Distribution of the forecast errors of government spending growth Note: Forecast errors are computed by Auerbach and Gorodnichenko (2012) as the difference between the realised and the forecasted growth rate of government purchases (consumption and investment) at quarterly frequency. They are both negative and positive, with mean -0.15, standard deviation 4.46, and skewness 0.42.

B Robustness Checks for the Local Projection

This Appendix shows additional robustness checks for the local projection. Appendix B.1 reports impulse response functions to a positive fiscal shock (identified as the quarterly forecast error of government purchases growth) in the linear specification. Appendix B.2 displays impulse response functions for the state dependence specification where alternative series to tightness are used to capture the aggregate labor market condition. Appendix B.3 exhibits robustness checks concerning the specification of the local projection and the detrending method used for the hiring rate series. Appendix B.4 reports robustness where the CPS hiring rate series is substituted with an extended version of the hiring rate series from JOLTS.

B.1 Linear Local Projection

This Appendix reports impulse response functions to a positive fiscal shock in the linear local projection. The five-year cumulative multiplier is 1.44, with SE=0.401.



Figure 10 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors.

B.2 Robustness Checks for Tightness

This Appendix reports impulse response functions to a positive fiscal shock where alternative measures to tightness are used to identify the aggregate labor market state. Figure 11 shows state dependent responses where the labor market state is defined as tight if the unemployment rate is below the NAIRU and slack otherwise. In Figure 12 the labor market state is identified by using an HP filter trend as a threshold for the unemployment rate. Namely, labor market is defined as tight when the unemployment rate is above trend and slack otherwise. In both Figures, the state dependent impulse responses are not statistically different from each other, showing that expansionary fiscal policy is not dependent on the state of the aggregate labor market.



Figure 11 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of tight (unemployment rate below NAIRU) and slack (unemployment rate above NAIRU) labor market.





Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of tight (unemployment rate below trend) and slack (unemployment rate above trend) labor market. The uneployment rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 10000.

B.3 Robustness Checks for the CPS Hiring Rate

This Appendix shows the robustness checks for the state dependent local projection when the CPS hiring rate series is used to identify the state of the labor market. The first set of robustness checks concerns the specification of the regression model. Figure 13, Figure 14, and Figure 15 display the impulse responses where the following changes are made to the specification of the local projection: no time trend, two lags, and taxes and log employment as control variables in place of unemployment rate and debt-to-GDP. Results are robust to these changes.





Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 10000. No trend is included in the local projection.



Figure 14 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 10000. Two lags are included for the control variables in the local projection.



Figure 15 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 10000. Taxes and logged employment are included as controls in place of debt-to-gdp and unemployment rate.

The second set of robustness checks concerns the detrending method, as the state dependence may indeed be affected by the way the hiring rate series is filtered. Figure 16

and Figure 17 show the impulse responses for the case in which the smoothing parameter of the HP filter is lower (5000) or higher (16000) than the one in the baseline regression (10000). Figure 18 displays responses to a state dependent local projection where the threshold to identify the labor market state is defined by fitting a polynomial trend of degree four to the hiring rate series. Figure 19 shows responses when the threshold is defined by a moving average with fifteen lags and fifteen leads. Results still hold to these variations.



Figure 16 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 5000.



Figure 17 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 16000.





Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by fitting a polynomial of degree four.



Figure 19 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by using a moving average with 15 leads and 15 lags.

Table 3 displays the cumulative multipliers corresponding to all the different specifications mentioned above. Results are in line with the cumulative multipliers in the baseline local projection.

Robustness	Horizon	High	Low	p-value
no trend	5Y	-0.81 (0.59)	$3.58 \\ (0.31)$	0.005
two lags	5Y	1.47 (0.28)	$3.11 \\ (0.34)$	0.092
smoothing parameter $= 5000$	5Y	-1.12 (0.57)	$3.68 \\ (0.37)$	0.002
smoothing parameter $= 16000$	5Y	-1.09 (0.53)	3.67 (0.32)	0.000
controls: log empl and taxes	5Y	-1.04 (0.51)	3.97 (0.43)	0.001
polynomial trend	5Y	-0.69 (0.46)	$3.72 \\ (0.34)$	0.000
MA(15,1,15)	5Y	0.63 (0.40)	$3.85 \\ (0.33)$	0.013

Table 3 – Cumulative multipliers for a positive fiscal shock

Note: The table shows fiscal multipliers computed with an instrumental variable-local projection approach. Columns 'High' and 'Low' report the expansionary cumulative multipliers when the hiring rate is above and below trend respectively.

B.4 Robustness Checks Using the Fitted Series From Davis et al. (2012)

Figure 20 shows the hiring rate series from Davis et al. (2012) as well as the fitted hiring rate computed as described in Section 2.5.3. The regression has an $R^2 = 0.93$.

I now use the fitted hiring rate series to define my labor market state. To make sure that results are robust to various filtering methods, I use different filters to detrend the series: Hodrick-Prescott, Butterworth, and Christiano and Fitzgerard filter.

Using the fitted hiring rate series, I rerun Equation (2) and Equation (4). Figure 21, Figure 22, and Figure 23 show the responses using the different filters to detrend the hiring rate series. Table 4 displays the corresponding cumulative multipliers to expansionary shocks. Responses are still state dependent, even though the state dependence is not always as stark as when using the CPS series. The reason could be that while the CPS series only contains hires from non-employment, the hiring rate series from Davis et al. (2012) includes hires both from non-employment and from employment. This may weaken the strength of the hiring rate friction – a new hire coming from employment may be faster at reaching the productivity of the other workers than a new hire coming from unemployment or inactivity.



Figure 20 – Fitted hiring rate and hiring rate from Davis et al. (2012) Note: The fitted values are obtained by estimating the following regression: $\frac{H_t}{N_t} = \beta_0 + \beta_1 u_t + \beta_2 \Delta \log N_t + \beta_3 t + \epsilon_t$, where u_t is the unemployment rate, N_t the employment and t is a time trend. $R^2 = 0.93$.

B.4.1 Hodrick-Prescott Filter



Figure 21 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The hiring rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 5000. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate.

B.4.2 Butterworth filter



Figure 22 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The hiring rate series is detrended by using a Butterworth filter of order 1 with maximum period 32. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate.

B.4.3 Christiano and Fitzgerard filter



Figure 23 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The hiring rate series is detrended by using a Christiano and Fitzgerard filter with minumum number of periods 2 and maximum 32. The figure compares impulse responses to a positive fiscal shock in the two states of high (above trend) and low (below trend) hiring rate.

Table 4 – Cumulative multipliers for a positive shock						
Robustness	Horizon	High	Low	p-value		
Hodrick-Prescott filter	5Y	-1.52 (0.91)	$3.30 \\ (0.55)$	0.036		
Butterworth filter	5Y	-0.51 (0.66)	3.72 (0.64)	0.075		
Christiano-Fitzgerard filter	5Y	-3.31 (1.42)	2.57 (0.38)	0.111		

Note: The table shows fiscal multipliers computed with an instrumental variable-local projection approach. Columns 'High' and 'Low' report the expansionary cumulative multipliers when the hiring rate is above and below trend respectively.

B.5 Measures of Recessions vs Expansions

This Appendix shows that identifying the state of the labor market according to the hiring rate is different from using measures of business cycles that have been used in the literature. In particular, Figure 24 shows responses to a positive fiscal shock where the economy is defined to be in an expansion or a recession according the NBER timing of

recession. Responses in the two states of expansion and recession are not statistically different from each other.



Figure 24 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. Recessionary periods (blue solid line) correspond to NBER recessions.

Figure 25 displays responses where the state of the economy follows the definition of Auerbach and Gorodnichenko (2012). Accordingly, the state of the economy is defined with a smooth transition threshold based on a seven-quarter moving average of output growth, s_t . The transition function used is $F(s_t) = \frac{exp(-\gamma s_t)}{1+exp(-\gamma s_t)}$, where the parameter γ is calibrated by Auerbach and Gorodnichenko (2012) to 1.5 implying that the economy spends one fifth of of time in a recessionary period. Responses in the two states look different. However, both government purchases and output follow a similar pattern, being much more reactive and persistent in the recessionary period. When I compute the fiveyear cumulative spending multipliers I find that the two states are not statistically different from each other. This is shown by Table 5.



Figure 25 – Impulse response functions to a positive fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. Recessionary (blue solid line) and expansionary (red dashed line) periods are defined according to smoothing transition of Auerbach and Gorodnichenko (2012).

Lä	Table 5 – Cumulative multipliers for a positive shoc								
	State Variable	Horizon	Tight	Slack	p-value				
	AG (2012) state	5Y	2.31	3.50	0.378				
			(0.64)	(0.42)					

 \mathbf{k}

Note: Newey-West standard errors in parenthesis. 'p-value' indicates the p-value for the test that the multiplier estimates are different across states. It is based on heteroscedastic- and autocorrelation-consistent standard errors.

Table 6 reports the correlations between different measures of tightness. The Table highlights that periods in which the labor market is slack are not equivalent to the recessionary periods as measured by the NBER recessions or the Auerbach and Gorodnichenko (2012) business cycle measure. In particular, while the latter two measures are positively and highly correlated with each other (0.57), their correlations with the measures of labor market slackness in very low.

State	Hiring Rate	Tightness	Un. Rate (trend)	Un. Rate (NAIRU)	NBER rec.	AG (2012)
Hiring Rate	1					
Tightness	-0.26	1				
Un. Rate (trend)	-0.38	0.86	1			
Un. Rate (NAIRU)	-0.30	0.64	0.59	1		
NBER recessions	0.14	0.24	0.12	0.24	1	
AG (2012)	0.14	0.11	0.01	0.06	0.57	1

Table 6 – Correlation between different measures of state in the local projection

Note: For 'Hiring Rate' and 'Tightness' the state is defined as tight when they are above their trend, and slack otherwise. For 'Un. Rate (trend)' and 'Un. Rate (NAIRU)', the state is defined as tight when they are below trend or below the NAIRU respectively, and slack otherwise. For 'NBER recessions', the state is defined as slack when there is an NBER recession and tight otherwise. 'AG (2012)' indicate the case in which the state is defined according to the smooth transition threshold of Auerbach and Gorodnichenko (2012).

C Responses to a Contractionary Shock

Figure 26 and Figure 27 display the impulse response functions of government spending, output, consumption and investment to a negative fiscal shock when respectively the hiring rate and the tightness are below (blue solid line) or above (red dashed line) trend. They correspond to the series of coefficients $\beta_{NS,h}$ and $\beta_{NT,h}$ for h = 1, ..., 20 in Equation (2). The state dependence of the responses is not marked, especially in the case of labor market tightness.





Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a negative fiscal shock in the two states of high (above trend) and low (below trend) hiring rate. The hiring rate series is detrended by using a Hodrick-Prescott filter with smoothing parameter 10000.



Figure 27 – Impulse response functions to a negative fiscal shock

Note: Impulse responses are estimated by local projection. Shaded areas are the 68% confidence intervals calculated using Newey-West standard errors. The figure compares impulse responses to a negative fiscal shock in the two states of high (above trend) and low (below trend) tightness. Tightness is detrended by using a Hodrick-Prescott filter with smoothing parameter 10000.

D Model Equilibrium Conditions

This Appendix reports the Lagrangian for the household's and for intermediate firms' maximisation problem, as well as the model equilibrium conditions.

D.1 Household's problem

$$\max_{\{C_{t},L_{t},U_{t},N_{t},K_{t},I_{t},B_{t+1}\}} \mathbb{E}_{t} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{\left[\eta_{t}^{p}C_{t} - \frac{\chi}{1+\phi}L^{1+\phi} \right]^{1-\sigma}}{1-\sigma} + \Phi_{t} \left[N_{t} + U_{t} - L_{t} \right] - \lambda_{t}P_{t}\mathcal{V}_{t} \left[N_{t} - (1-\delta_{N})N_{t-1} - H_{t} \right] - \lambda_{t}P_{t}\mathcal{V}_{t} \left[N_{t} - (1-\delta_{N})N_{t-1} - H_{t} \right] - \lambda_{t} \left[P_{t}C_{t} + P_{t}I_{t} + \frac{B_{t+1}}{R_{t}} - R_{t}^{K}K_{t-1} - W_{t}N_{t} - B_{t} - \Theta_{t} + T_{t} \right] - \lambda_{t}Q_{t}^{K}P_{t} \left[K_{t} - (1-\delta_{K})K_{t-1} - \eta_{t}^{I} \left[1 - S\left(\frac{I_{t}}{I_{t-1}}\right) \right] I_{t} \right] \right\}$$

D.2 Intermediate firms' problem

$$\max_{\{P_{i,t}H_{i,t},N_{i,t},K_{i,t}\}} \mathbb{E}_{t} \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left\{ \frac{P_{i,t}}{P_{t}} \left(\frac{P_{i,t}}{P_{t}} \right)^{-\epsilon} Y_{t} - \frac{W_{t}}{P_{t}} N_{i,t} - \frac{R_{t}^{k}}{P_{t}} K_{i,t} - \frac{\zeta}{2} \left(\frac{P_{i,t}}{P_{i,t-1}\overline{\Pi}} - 1 \right)^{2} Y_{t} - Q_{t}^{N} \left[N_{t} - (1 - \delta_{N}) N_{t-1} - H_{t} \right] + \xi_{t} \left[f(K_{i,t}, N_{i,t}) - g_{i,t}(H_{i,t}, K_{i,t}, N_{i,t}) - \left(\frac{P_{i,t}}{P_{t}} \right)^{-\epsilon} Y_{t} \right] \right\}$$

D.3 First order conditions

Households

$$[C] \qquad \left[\eta_t^p C_t - \frac{\chi}{1+\phi} L_t^{1+\phi}\right]^{-\sigma} \eta_t^p = \lambda_t P_t \tag{1}$$

$$[L] \qquad \left[\eta_t^p C_t - \frac{\chi}{1+\phi} L^{1+\phi}\right]^{-\sigma} \left(-\chi L_t^\phi\right) = \Phi_t \tag{2}$$

$$[U] \qquad \Phi_t = -\frac{1}{\bar{\omega}} \lambda_t P_t \mathcal{V}_t \frac{x_t}{1 - x_t} \tag{3}$$

$$[N] \qquad \mathcal{V}_t = \frac{\Phi_t}{\lambda_t P_t} + \frac{W_t}{P_t} + E_t \Lambda_{t,t+1} \mathcal{V}_{t+1} (1 - \delta_N) \tag{4}$$

$$[K] \qquad Q_t^K = E_t \Lambda_{t,t+1} \left[\frac{R_{t+1}^K}{P_{t+1}} + (1 - \delta_K) Q_{t+1}^K \right]$$
(5)

$$[I] \qquad Q_t^K = \frac{1 - E_t \Lambda_{t,t+1} Q_{t+1}^K \eta_{t+1}^I S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2}{\eta_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \left(\frac{I_t}{I_{t-1}}\right)\right]} \tag{6}$$

$$[B] \qquad \frac{1}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \tag{7}$$

Firms

$$[K] \qquad \frac{R_t^K}{P_t} = \xi_t (f_{K,t} - g_{K,t}) \tag{8}$$

$$[H] \qquad Q_t^N = \xi_t g_{H,t} \tag{9}$$

$$[N] \qquad Q_t^N = \xi_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t} + (1 - \delta_N) E_t \Lambda_{t,t+1} Q_{t+1}^N \tag{10}$$

$$\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right) \left(\frac{\Pi_t}{\bar{\Pi}}\right) = \frac{1}{\zeta} \left(1 - \epsilon\right) + \xi_t \frac{\epsilon}{\zeta} + E_t \Lambda_{t,t+1} \left(\frac{\Pi_{t+1}}{\bar{\Pi}} - 1\right) \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right) \tag{11}$$

Wage

$$\frac{W_t}{P_t}^{NASH} = \gamma \xi_t \left[f_{N,t} - g_{N,t} \right] - (1 - \gamma) \left[\frac{\Phi_t}{\lambda_t P_t} \right]$$
(12)

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\omega} \left(\frac{W_t}{P_t}^{NASH}\right)^{1-\omega}$$
(13)

Output and hiring cost functions

$$Y_t = f_t - g_t \tag{14}$$

$$f_t = (A_t N_t)^{\alpha} K_{t-1}^{1-\alpha}$$
(15)

$$g_t = \frac{e}{2} q_t^{-\eta} \left(\frac{H_t}{N_t}\right)^2 f_t \tag{16}$$

$$f_{N,t} = \alpha \frac{f_t}{N_t} \tag{17}$$

$$f_{K,t} = (1 - \alpha) \frac{f_t}{K_{t-1}}$$
(18)

$$g_{H,t} = eq_t^{-\eta} \frac{H_t}{N_t^2} f_t \tag{19}$$

$$g_{K,t} = (1 - \alpha) \frac{g_t}{K_{t-1}}$$
(20)

$$g_{N,t} = (\alpha - 2) \frac{e}{2} q_t^{-\eta} H_t^2 A_t^{\alpha} K_{t-1}^{1-\alpha} N_t^{\alpha-3}$$
(21)

Resource constraint

$$\left[1 - \frac{\zeta}{2} \left(\frac{\Pi_t}{\overline{\Pi}} - 1\right)^2\right] Y_t = C_t + I_t + G_t, \tag{22}$$

Taylor rule

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi^*}\right)^{r_\pi} \left(\frac{\tilde{Y}_t}{Y^*}\right)^{r_y} \right]^{1-\rho_R} \eta_t^R$$
(23)

Capital law of motion

$$K_t = (1 - \delta_K)K_{t-1} + \eta_t^I \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t$$
(24)

Government purchases

$$\ln G_t = (1 - \rho_G) \ln \bar{G} + \rho_G G_{t-1} + \epsilon_{G,t},$$
(25)

Labor force

$$L = N + U \tag{26}$$

Employment law of motion

$$N_t = (1 - \delta_N)N_{t-1} + H_t \tag{27}$$

Hires

$$H_t = x_t \frac{U_t}{1 - x_t} \tag{28}$$

Vacancy filling rate

$$q_t = m \left(\frac{x_t}{m}\right)^{-\frac{l}{1-l}} \tag{29}$$

Stochastic discount factor

$$\Lambda_{t,t+1} = \beta E_t \frac{\lambda_{t+1} P_{t+1}}{\lambda_t P_t} \tag{30}$$