Trends in Aggregate Employment, Hours Worked per Worker, and the Long-Run Labor Wedge*

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Abstract

We develop a novel representative agent Walrasian framework that disentangles household and firm decisions over the aggregate intensive and extensive margins of labor in the long run. Our objective is to advance the macroeconomic understanding of the driving forces behind trends in aggregate hours worked per population (the product of hours worked per worker and the employment-population ratio). The model features: endogenous search effort that households must expend in order to secure employment; and, motivated by a novel fact that we document—a relationship between capital taxes and employment—firm-side employment adjustment costs. A tax-inclusive version of the model can successfully explain the trend behavior of hours worked per population in both Europe and the US. In contrast, standard macroeconomic models yield stark counterfactual results for the US. Our framework can speak to the long-run neutrality of productivity on employment implied by traditional models of aggregate labor markets, and, more generally, is relevant for assessing the long run impact of host of factors beyond taxes, including labor market reforms, changes in productivity, and changes in demographics. As such, our model stresses the ongoing validity of representative agent Walrasian frameworks for analyzing aggregate labor markets.

Keywords: Business-cycle accounting; capital taxes; consumption taxes; employment-population ratio; extensive margin; hours worked per population; hours worked per worker; intensive margin; investment taxes; labor demand; labor supply; labor taxes

JEL Codes: E60, H20, J20.

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1 Introduction

We develop a novel representative-agent Walrasian framework that disentangles household and firm decisions over the aggregate intensive—hours worked per *worker*—and extensive—the employment-population ratio—margins of labor in the *long run*. Our objective is to advance the macroeconomic understanding of the driving forces behind *trends* in aggregate hours worked per *population* (the product of hours worked per *worker* and the employment-population ratio). Our motivation is three facts, which we discuss below and result in the following three specific questions. (1) What drives the U.S. tax puzzle (which we recap below)? (2) Does the extent to which standard macroeconomic models fail to explain the *trend* behavior of hours worked per population H make an effective case for building this variable up by explicitly modeling hours worked per worker h and the employment-population ratio e? (3) Can a novel relationship between capital taxes and H, which we document in this paper, inform how to best model the behavior of h and e for the purposes of trend *analysis*—regardless of whether taxes are a variable of interest or not?

While these three questions are specific, they guide us to arrive at a general modeling contribution whose applicability goes well beyond these specific questions. This is important given the enormous relevance of labor markets for aggregate economic activity. In particular, the generality and tractability of our model allows it to speak to the issue of long-run neutrality of productivity on employment implied by traditional models of aggregate labor markets (see, for instance, Layard, Nickell, and Jackman, 1991, Blanchard, 2007, Shimer, 2010, and Elsby and Shapiro, 2012). More generally, our model can be easily extended to address a wide range of issues beyond taxes, including the trend impact on the labor market of changes in productivity, changes in demographics, and labor market reforms, among others. All told, our model highlights the extent to which, in spite of major advances in macroeconomic modeling, ranging from search theory to heterogeneous agents and beyond, Walrasian representative agent frameworks continue to be an exceedingly reliable tool for analyzing aggregate labor markets.

Regarding our motivating facts, consider fact (1): As we review in the following section, standard macroeconomic models (which we refer to as being within the *combined labor margins—CLM—*category, that is, models that focus on hours worked per *population*, only, but do not disentangle the two margins of labor) fail to correctly account for the trend behavior of hours worked per population H in general and, specifically, in Europe and the US. This implies that results from CLM models are consistent with the existence of a prominent (long-run/trend) labor wedge—the extent to which the marginal product of labor differs from the marginal rate of consumption for leisure. A literature proposes that in the long run taxes are a natural candidate to close the long-run (trend labor) wedge. However, as we also review in the following section, a pervasive result of this literature is that taxes help close the long-run (trend) labor wedge in Europe, but *thoroughly exacerbate* the long-run labor wedge in the US (see, for instance, Shimer, 2009). What drives this "U.S. tax puzzle?" Our work suggests that this puzzle is artificial and stems from CLM models internally generating a long-run labor wedge by construction.

Fact (2): As we review in the following section, over the last several decades trends in U.S. hours worked per *population* were driven by trends in the employment-population ratio e. In contrast, on average, trends in H in Europe were driven by trends in hours worked per *worker* h (earlier literature highlights similar facts). Given facts (1) and (2) combined, is it the case that the U.S. puzzle is coming from an inability of CLM models to account for e, in which case to get at correct predictions of the long-run (trend) behavior of H it is necessary to build this variable up by explicitly modeling h and e? Our work suggests that the answer to both parts of this question is yes, since our work also suggests that after taxes are accounted for, much of what remains of the long-run labor wedge is actually employment. We argue immediately below that in a long-run context this result is highly surprising.

It is well known that CLM models face prominent limitations in accounting for the behavior of e at *business cycle frequency*. As such, for example, theories rooted in labor market frictions (see, for instance, Diamond, 1982, Mortensen and Pissarides, 1994, and Merz, 1995) have provided insights into the macroeconomic relevance of unemployment dynamics (and, by consequence, of employment) in closing the labor wedge at *business cycle frequency*. However, the fact that CLM models would face the same limitations in the long run and, especially, once taxes are accounted for, is extremely surprising. Indeed, within the long-run conceptual framework of full flexibility of labor markets, given trend changes in taxes it is intuitive to expect adjustments in trend equilibrium labor to occur largely on the extensive margin—case in point, the extent to which the labor force participation of secondary earners increased against the backdrop of the Reagan tax reforms (1981 through 1986). As such, once trend changes in taxes are accounted for, it is intuitive that results from CLM models would suggest that discrepancies between model predictions of trends in H and their empirical counterparts would stem from the model missing on the *intensive* margin of labor, *and not so much, if at all, on the extensive margin.*

Fact (3): As shown in the next section, we present what, to the best of our knowledge, is a new stylized fact: capital taxes are associated with the behavior of H, and this association traces back to e, only (capital taxes are not associated with the behavior of h).[1] That said, CLM models imply that capital taxes do not have direct impact on labor markets—only consumption and labor taxes matter. Can this novel empirical relationship between H and capital taxes that we document help guide a theory that furthers our understanding of the driving forces, *beyond taxes*, of the trend behavior of H, and,

[1] The importance of capital taxes that emerges from our analysis is of particular relevance given recent work on the evolution of wealth and income in advanced economies (see, most prominently, Piketty and Zucman, 2014, and and subsequent work).

more generally, can help address a broader set of issues related to aggregate labor markets, such as the long-run neutrality of productivity on employment implied by traditional models of aggregate labor markets? Our work suggests that the answer to this two-part question is yes, since a link between capital taxes and e can be intuitively driven by the employment demand side and, in particular, by firm-side employment adjustment costs. In addition, these costs are a straightforward way by which firm-level decisions on h and e can be disentangled, and, importantly, they can establish a positive link between trend productivity and trend employment (the broader relevance of adjustment costs is emphasized in a vast literature that includes, for instance, Cooper and Willis, 2004, Caballero and Engel, 2004, Cooper and Willis, 2008, and Mumtaz and Zanetti, 2014).

The first and main contribution of our work is the development of a novel model, which we refer to as the *Dual Labor Margins Model (DLM)*—a naming convention that we adopt to contrast our model with CLM frameworks. We operationalize a tax-inclusive version of our DLM model, but, more generally, given its generality and tractability our DLM model can easily speak to a host of issues well beyond taxes. For instance, our DLM model can speak to the issue of long-run neutrality of productivity on employment implied by traditional models of aggregate labor markets. In particular, our DLM model explicitly suggests that trend employment can be increasing in trend productivity as a result of employment adjustment costs, which, in turn, are a modeling element informed by the novel relationship between capital taxes and e that we highlight. More generally, our DLM model can be easily extended to address a wide range of issues beyond taxes, including the trend impact on the labor market of changes in productivity, changes in demographics, and labor market reforms, among others.

Our DLM model has two key ingredients: endogenous household search effort (the household must expend resources in order to secure employment); and firm-side employment adjustment costs. These two ingredients are sufficient to disentangle in highly tractable fashion both household and firm side optimal choices over trend h and e. We justify and ground with discipline the how and why of the inclusion of our DLM model's ingredients by initially developing our DLM model within a general equilibrium labor search context. However, our bottom-line DLM specification purges the model from several standard theoretical search characteristics, making this bottom-line specification quasi frictionless. The frictions that remain are an exogenous job destruction probability that is equal to 1 in every period (in the spirit of Arseneau and Chugh, 2012), and, given the presence of endogenous household side employment-securing search effort, an endogenous decision of the household to never set $e = 1$.

This "quasi frictionless" specification actually puts our bottom-line DLM model on par with CLM models from the point of view that our bottom-line DLM model is a representative agent framework where *there is no involuntary unemployment* (what does exist, though, is "nonemployment," but individuals in this state are indifferent between being employed or not) and, importantly in our DLM model *all markets are perfectly competitive, including the labor market*. All told, our DLM model is, in fact, an intuitive and tractable extension of CLM models. This is reflected in the fact that our DLM model is able to, from a theoretical perspective, exactly pinpoint why CLM models should, in general, not be expected to have full and concrete explanatory power over H.

Our DLM model yields two testable implications: a novel static equilibrium condition for h and a novel dynamic condition for equilibrium e. We bring a tax-inclusive version of the model to the data, in line with related research, using a business cycle accounting approach. We show that the model can explain very well the trend behavior of the two margins of labor in *both* Europe and the US, and, therefore, the trend behavior of H in *both* Europe and the US (in contrast to the stark counterfactual predictions for the US stemming from CLM models).

Turning to explicit results, our DLM model suggests that over the last several decades in both Europe and the US changes in labor taxes are associated with downward pressure on h of, respectively, about 18 and 8 percent. In the US over the last several decades the major force associated with higher e is robust gains in total factor productivity (TFP). These gains in TFP are associated with U.S. employment in 2014 being about 18 percent higher compared to what it would have been otherwise. As such, our DLM model is consistent with a positive relationship long-run relationship between productivity growth and equilibrium employment. Our DLM model suggests that U.S. consumption and capital taxes are each associated with U.S. employment being lower in 2014 by about 6 percent each compared to what it would have been otherwise, for a combined drag of 12 percent—*which is quantitatively important, since the 6 percent contribution of consumption and capital taxes, each, is a third of that of TFP, and the joint contribution of these taxes is two thirds of that of TFP*. Critically, our DLM model also suggests that in Europe the model-implied positive association between gains in TFP (which are empirically trend-wise roughly the same as in the US) and e are, for all purposes, fully and entirely offset by higher labor taxes—over the last several decades, there was a substantial increase Europe's labor taxes.

Our second contribution is a step forward in resolving the U.S. tax puzzle. Our DLM model's equation for equilibrium h is the same as the CLM model's equation for equilibrium H. The extent to which our DLM model is successful in tracking the cross-country behavior of h therefore suggests, in very explicit terms and with *sound theoretical justification*, that a CLM model's equation for H is not so—it is, instead, fully and explicitly an equilibrium condition for h. This is the fundamental explanation of the U.S. tax puzzle that our work sets forth. When the trend behavior of H is driven by h, as in Europe , CLM models can give the impression of being successful. In contrast, when trends in H are driven by trends in e, as in the US, CLM models fail. The implication is, therefore, that a substantial portion of the long-run labor wedge that remains in CLM models after accounting for taxes is employment itself. (Recall fact (1) as related to taxes, and fact (2) as related to cross-country heterogeneity in the relevance of h versus e in driving the trend behavior of H.)

Our third contribution speaks to the broad literature that examines the relationship between taxes and labor markets. Importantly, while the relationship between taxes and h and e is well understood from the macroeconomic perspective in both *business cycle* contexts (see, for instance, Krusell et al., 2008) and from *life-cycle* perspectives (see, for instance, Erosa et al., 2016, and Rogerson and Wallenius, 2009), an understanding of the relationship between *trends* in taxes and *trends* in h and e is generally lacking. We help bridge this gap. Moreover, earlier literature highlighted the relationship of trends in consumption and labor taxes with trends in H (see, for instance, Prescott 2004, and Ohanian et al., 2008). However, to the best of our knowledge, we are the first to document empirically that capital taxes can also matter for H via e, only, and our DLM model's dynamic equilibrium e condition speaks to this finding (while CLM models cannot).

This paper proceeds as follows. Section 2 discusses more extensively the key motivational factors of our research. Section 3 discusses related literature. Section 4 develops our DLM model. Section 5 describes how we operationalize our DLM model. Section 6 presents results from our DLM model and also from a version of the CLM model for the purposes of comparison. Section 7 concludes.

2 Details on Motivation

In this section we elaborate on the three motivating facts we discussed in the Introduction. Of note, all data used in this paper are at yearly frequency—*our research question and focus are entirely unrelated to detrended/cyclical components of the data.*[2] Regarding fact (1), standard macroeconomic frameworks that focus on modeling work hours per population, only (recall that we refer to such frameworks as being within the *combined labor margins—CLM—*category, since the two margins of labor are not modeled separately) generate a substantial long-run (trend) labor wedge. Of note, given its definition, this wedge can be calculated as the difference between the trend behavior of empirical hours and the trend behavior of model-predicted hours. For example, Figure 1 features Europe in its bottom panel and the US in the top panel.[3] This figure shows empirical hours worked per population (*H Empirical*) and also a

[2] The time span over which we present data is limited by the availability of extensive time series data on taxes across the countries in our sample.

[3] "Europe" is the simple average of the European countries in our sample, which are in line with related literature (motivated by the fact that there are extensive time series data on taxes for these countries as well as for the US): Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Spain, Sweden, Switzerland, and the UK. Results are for all purposes identical using alternative weighting schemes, including GDP weighted averages.

representative CLM model's predictions for this variable (*CLM without taxes*), which are flat.[4]

Also regarding fact (1), a literature proposes that, within the CLM framework, taxes—which, of course, are unaccounted for in benchmark modeling setups—are an intuitive candidate to help close the long-run (trend) labor wedge. In light of this, Figure 1 also plots a tax-inclusive CLM model's predictions for hours worked per population (*CLM with taxes*).[5] This model tracks the behavior of hours worked per population very well, on average, in Europe, but it generates stark counterfactual results for the US—this is a shared result of a host of tax-inclusive versions of the CLM model. Therefore, there is a substantial long-run labor wedge in the US that exists beyond taxes. In fact, note that relative to results from the CLM model without taxes, accounting for taxes makes the implied U.S. labor wedge worse. We refer to this as the "U.S. tax puzzle."

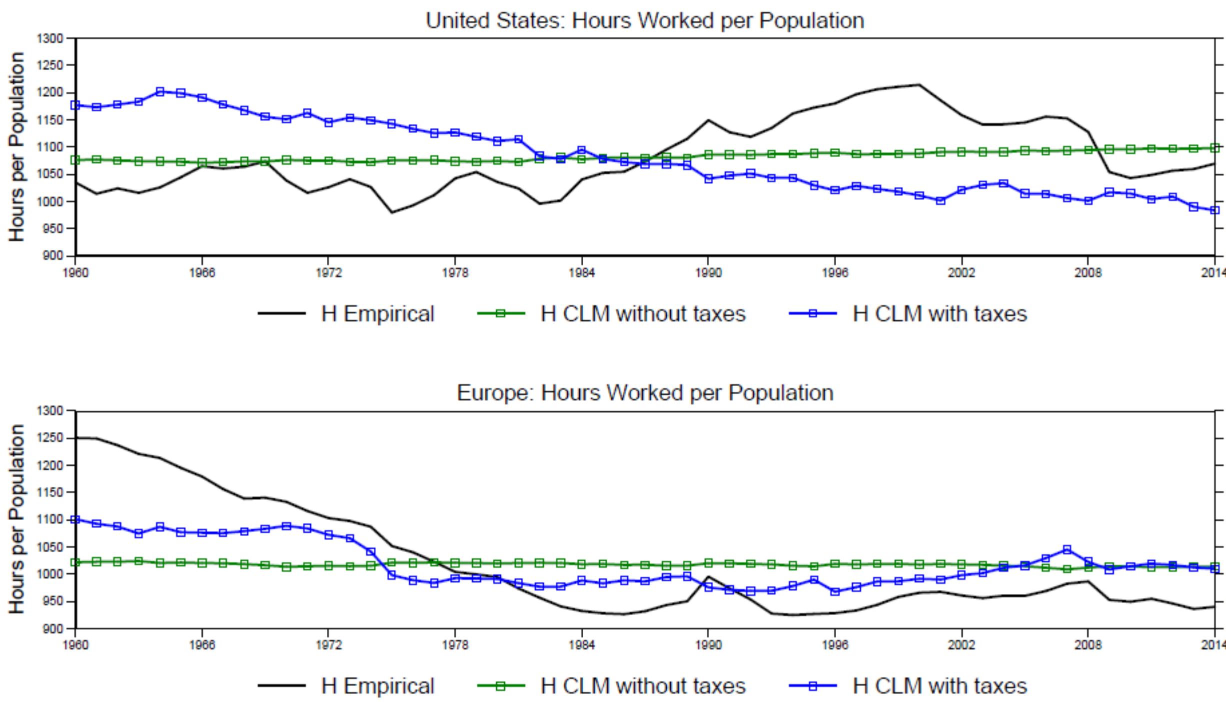

Figure 1: Empirical hours worked per population and predictions from CLM model with and without taxes for Europe (top panel) and the US (bottom panel).[6]

Fact (2) is addressed in Figure 2, were all data are at yearly frequency and in levels—*our research question and focus are entirely unrelated to detrended/cyclical components of the data.* As shown in the top panel of this figure, in the US hours worked per *population* (*H Empirical*) exhibit a shallow V-shape,

[4] This version of the CLM model is the one used in Shimer (2009), albeit without taxes.

[5] This version of the CLM model is exactly the one used in Shimer (2009).

[6] Notes: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, and population data are from the UN. CLM series are constructed using tax data are from McDaniel (2007) and consumption and output data from the Penn Word Tables.

decreasing through the early 1980s and then increasing—driven by trend gains in the employment-population ratio (denoted by *H fix h* and whose value is normalized at the 1960 value of *H Empirical*), while hours worked per *worker* decreased some (denoted by *H fix e* and whose value is normalized at the 1960 value of *H Empirical*). In contrast, as shown in the bottom panel of this figure Europe experienced a secular trend decline in hours per worked *population* driven by a trend decline in hours per *worker* against the backdrop of a flat employment-population.[7]

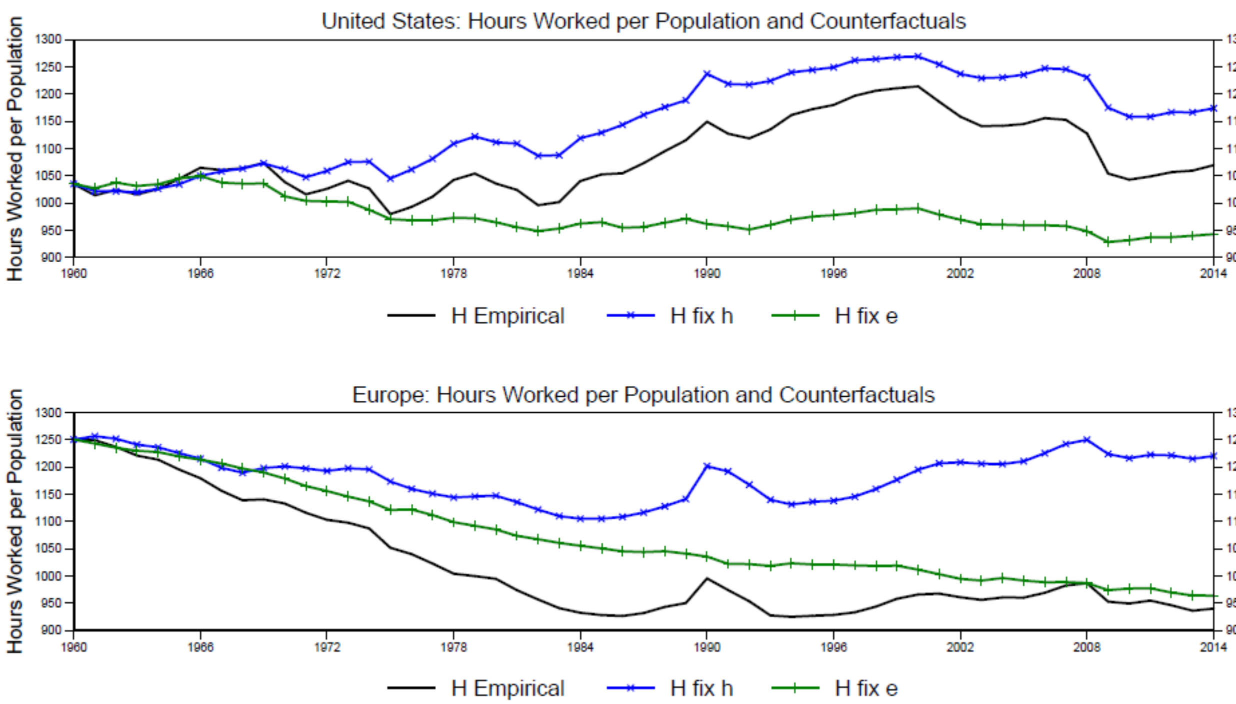

Figure 2: Hours worked per population, per worker, and the employment-population ratio in Europe (bottom panel) and the US (top panel).[8]

Related to fact (1), it is important to note that the equilibrium condition for hours worked per population in CLM models is static and, moreover, CLM frameworks are such that only consumption and labor taxes matter for the determination of hours worked per population. However, fact (3) is that a stylized empirical analysis, whose results are presented in Table 1, reveals that capital taxes are also associated with hours worked per population, and the channel for this is the extensive margin, only, as hours worked per worker are not significantly associated with capital taxes. To the best of our knowledge, this fact is novel.

[7] Earlier research, such as Rogerson (2006) and Blundell et al. (2011), highlights similar patterns.

[8] Note: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, and population data are from the UN.

Table 1 presents results from a panel regression of the twelve OECD countries in our sample (eleven European plus the US) in which all variables are in growth rates ($d\ln$—of course, even though our interest is not in the detrended/cyclical component of labor market data, running regressions on the level of time series data can lead to spurious correlations) the regressands are the growth rates ($d\ln$) the employment-population ratio, hours worked per worker, and hours worked per population, and the regressors are the growth rate of return -adjusted consumption $(1+\tau^c)$, labor $(1-\tau^l)$, capital $(1-\tau^k)$, and investment $(1+\tau^i)$ taxes. Each regression is run with and without a business cycle control (the growth rate of output per population Y).[9]

Table 1: panel regression of labor market variables on taxes[10]

Variable	$d\ln(e)$	$d\ln(e)$	$d\ln(h)$	$d\ln(h)$	$d\ln(H)$	$d\ln(H)$
$d\ln(1+\tau^c)$	0.02	-0.33	0.08	0.01	0.10	-0.32
	(0.61)	(0.34)	(0.13)	(0.08)	(0.71)	(0.37)
$d\ln(1-\tau^l)$	0.22**	0.16**	0.07*	0.05	0.29***	0.21***
	(0.09)	(0.07)	(0.03)	(0.03)	(0.07)	(0.06)
$d\ln(1-\tau^k)$	-0.18**	-0.14**	-0.05	-0.04	-0.23***	-0.17***
	(0.07)	(0.06)	(0.04)	(0.03)	(0.06)	(0.04)
$d\ln(1+\tau^i)$	0.01	0.59	-0.11	0.00	-0.10	0.59
	(1.09)	(0.53)	(0.22)	(0.11)	(1.29)	(0.62)
$d\ln(Y)$		0.29***		0.06***		0.35***
		(0.04)		(0.01)		(0.04)
Fixed effects	yes	yes	yes	yes	yes	yes
Obs.	648	648	648	648	648	648

Note that Table 1 implies a positive contemporaneous relationship between capital taxes, employment, and hours worked per population. As we show below, our DLM model can generate this relationship via the presence of firm-side employment adjustment costs. Importantly, our DLM model highlights that the contemporaneous positive relationship between capital taxes and employment shown in Table 1 is not causal. Instead, it is the result of optimal firm decisions from earlier periods. In fact, our DLM model implies that the firm's employment demand decision is forward looking and its contemporaneous employment demand is, in line with intuition, decreasing in capital taxes.

3 Related Literature

As noted in the Introduction, our focus on both margins of labor is partly motivated by the extent to which the employment margin can explain trends in long-run changes in total hours worked per *population*

[9] That data used for these regressions are the same as that used to generate the series shown in Figure 1 and, therefore and in particular, are at yearly frequency and span the years 1960 through 2014 (again, limited by the availability of extensive time series data for each country in our sample).

[10] Notes: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, population data are from the UN, output is from the Penn World Tables, and tax data are from McDaniel (2007).

across countries, which, among others, is also highlighted empirically by Rogerson (2006) and Blundell et al. (2011). Importantly, Blundell et al. (2011) stress that making a distinction between employment and hours worked per worker is widely accepted in microeconometric studies (Heckman, 1974, and Blundell and Macurdy, 1999, among others) as well as in a well-known public economics literature (Saez, 2002, Laroque, 2005, and Brewer et al, 2010, among others). Therefore, these studies highlight that the extent to which the employment margin can explain long-run changes in total work hours should also be considered a prominent question in the macroeconomics literature given the relevance of labor markets for macroeconomic outcomes. Addressing this question is, of course, a main objective of our paper. Of note, neither Rogerson (2006) nor Blundell et al. (2011) develop a theoretical framework that can explain differences in the *trend* behavior of employment and hours worked per worker. In contrast, putting forth such a framework is at the center of our paper, the result of which is our DLM model.

As noted in the Introduction, our DLM framework begins as a full-fledged general equilibrium labor search model. As such, we build on vast related literature, including, among others, Diamond (1982), Mortensen and Pissarides (1994), and Merz (1995). That said, because as we also note in the Introduction our bottom-line DLM specification is quasi-frictionless, which ultimately makes it a representative-agent Walrasian framework, then our work is also related to existing studies that consider the extensive margin of labor in such frameworks. This literature is vast and includes, among others, Hansen (1985), Rogerson (1987), Bils and Cho (1994), Cho and Cooley (1994), Mulligan (2001), Krusell et al. (2008), Llosa et al. (2014), and Erosa (2016). We contribute to these literatures by showing how accounting for endogenous household search effort and vacancy-posting adjustment costs are a means through which the extensive and intensive margins of labor can be easily disentangled in a representative-agent Walrasian framework.

It is important to note that in CLM models, hours worked per worker, h, and the employment-population ratio, e, are perfect substitutes in household labor disutility and the firm's production function. In other words, in CLM models the product of h and e, which equals hours worked per population H, is the relevant labor input variable that goes into labor disutility and the aggregate production function. As such, household and firm decisions over h and e cannot be disentangled. There are many ways to disentangle these decisions. The way we do so in our DLM model is entirely and completely guided by the principles that: (1.) the DLM model should be easily comparable to the CLM model; (2.) under certain straightforward assumptions our DLM model should deliver an equilibrium condition for H that is in line with that of the CLM model. In essence, what our modeling strategy aims for is tractability and the development of a model that is a straightforward extension of CLM models.

There is a large literature on the labor wedge—the extent to which, in equilibrium, the marginal product of labor differs from the marginal rate of substitution of consumption for leisure—which includes, among others, Hall (2009), Shimer (2009), Pescatori and Tasci (2011) , Karabarbounis (2014a and 2014b), Cheremukhin and Restrepo-Echavarria (2014), Cociuba and Ueberfeldt (2015), Gourio and Rudanko (2014), and Hou and Johri (2018). There is also an extensive macroeconomics literature examining the impact of taxes and fiscal policies on labor market outcomes using various modeling frameworks. Some recent examples include Krusell et al. (2008), Kaygusuz (2009), and Guner et al. (2012). Within this literature, Prescott (2004), Ohanian et al. (2008), Rogerson and Wallenius (2009), Berger and Heylen (2011), McDaniel (2011), and Bick et al. (2017) are examples of research focusing on the long-run behavior of hours worked per population.

Importantly, focusing on hours worked per population, Prescott (2004) highlights that taxes are a natural candidate to help explain the *long-run* labor wedge. Prescott (2004), Ohanian et al. (2008), and McDaniel (2011) are particular instances in which fairly standard tax-inclusive versions of the CLM model are used for the purposes of analysis, and in all cases the model yields counterfactual predictions for U.S. hours worked per population (to greater or lesser degree), but successful predictions for hours worked per population in European countries. This means that even after accounting for taxes, the *long-run* U.S. labor wedge remains. Importantly, Ohanian et al. (2008) note explicitly that the discrepancy between empirical hours worked per population in the US and hours predicted for the US by CLM models are so stark that it is a crucial question for future research in macroeconomics. Yet, in very broad terms, the literature that studies the relationship between trends in taxes and trends in hours worked per population ends with McDaniel (2011). This impasse in the literature is perhaps the result of it being the case that obvious resolutions to the U.S. tax puzzle are not evident within CLM modeling frameworks.

Neither Prescott (2004), Ohanian et al. (2008), nor McDaniel (2011) attempt to resolve the U.S. tax puzzle from a modeling perspective. In contrast to these papers, a main focus of our paper is to move the literature forward in resolving the U.S. tax puzzle, for which we argue that distinguishing between the relationship of trends in taxes with, separately, trends in employment and trends hours worked per worker, is critical. As such, given the extent to which our DLM model can correctly predict hours worked per population in *both* Europe *and* the US, we argue that our DLM model is an important step forward towards resolving the U.S. tax puzzle and, therefore, an important contribution to the labor wedge literature that focuses explicitly on taxes. Moreover, this U.S. tax puzzle contribution is an identity with our paper's contribution to the labor-wedge literature beyond taxes, which is showing that a large portion of the *long-run* labor wedge stems from deficiencies in the ability of standard macroeconomic models to account for the behavior of the extensive margin of labor. As noted in the Introduction, while this deficiency is well known at business cycle frequency, it is extremely surprising that it also is present in the long run and, in particular, once taxes are accounted for.

It is important to highlight that stylized empirical analysis that we perform reveals a fact that, to the best of our knowledge, is novel: capital taxes are associated with hours worked per population via employment (while capital taxes are unrelated to hours worked per worker). Related to this new stylized fact, in our DLM model the association of trends in capital taxes with trends in employment is driven by the presence of employment adjustment costs on the firm side. Introducing these costs is a modeling assumption based on the fact that they are an intuitive and tractable means through which a relationship between capital taxes and employment can arise (the broader relevance of adjustment costs is emphasized in a vast literature that includes, for instance, Cooper and Willis, 2004, Caballero and Engel, 2004, Cooper and Willis, 2008, and Mumtaz and Zanetti, 2014). More generally, while the relationship between taxes and the intensive and extensive margins of labor is well understood from the macroeconomic perspective in both business cycle contexts (see, for instance, Krusell et al., 2008) and from life-cycle perspectives (see, for instance, Erosa et al., 2016, and Rogerson and Wallenius, 2009), an understanding of the relationship between trends in taxes and trends in the different margin of labor is generally lacking. Our model's application to taxes helps move the literature forward in filling this gap, and, in doing so, highlights the interaction between capital taxes and employment at the trend level.

Returning to the labor wedge, it is important to note the extent to which heterogeneity can matter. For instance, Cociuba and Ueberfeldt (2015) examine the U.S. tax puzzle within the context of gender and marital status heterogeneity. Focusing only on the US, they argue that part of the labor wedge owes to this heterogeneity, which, of course, standard homogenous-agent models cannot account for. They further argue that accounting for this heterogeneity, which they do based on a theoretical model that they develop, can go a long way in resolving the U.S. tax puzzle. In contrast, our DLM model highlights that the U.S. tax puzzle can be well addressed within a representative-agent modeling framework, that is, *within the same context in which the U.S. tax puzzle is originally observed.* As such, another contribution of our paper is showing with DLM model that heterogeneity itself is not a critical factor for resolving the U.S. tax puzzle. Therefore, our work stresses the ongoing validity of representative-agent Walrasian frameworks for analyzing and understanding aggregate labor markets.

Finally, we highlight that a literature centers on the fact that, while a relationship between changes in productivity and equilibrium is intuitive, a definitive theory that links these two variables is lacking. For instance, Elsby and Shapiro (2012), note that in standard models of aggregate labor markets this link, in fact, does not exist. As such, the authors argue that the fact that, empirically, equilibrium employment indeed appears to respond to changes in productivity is an enduring macroeconomic puzzle. As related to this literature, see, in addition and for example,: Layard, Nickell, and Jackman (1991); Blanchard (2007), who surveys the literature on traditional models of aggregate labor markets and concludes that in these models there is long-run neutrality of unemployment to productivity growth; and Shimer (2010). That said, in addressing this neutrality result regarding the relationship between productivity and equilibrium employment, several explanations that go beyond standard models of aggregate labor markets have been considered, including Elsby and Shapiro (2012) proposing that the existence of returns to experience are a channel that can relate equilibrium employment and changes in productivity via changes in reservation wages. Complementing this literature, in the spirit of Pissarides (2000) our DLM model suggests that a direct link between changes in total factor productivity (TFP) and equilibrium employment can exist by TFP potentially affecting vacancy costs. While Pissarides (2000) models TFP as explicitly increasing vacancy costs, in our DLM model we are agnostic regarding the qualitative direction of the potential link between TFP and vacancy costs. Instead, when we bring our model to the data, we let the data, and results from stylized empirical estimation suggests that TFP lowers the costs of posting vacancies. We discuss the intuition behind this result later in the paper.

4 The DLM Model

Our DLM model begins as a full-fledged general equilibrium labor search model, which, among other benefits, provides a disciplined and well-grounded theoretical justification for the presence of household-side search effort. However, the ultimate specification of our DLM model is purged of certain elements of standard search theory. Amid this backdrop, in our DLM model all markets are competitive, including the labor market, which puts our DLM model on equal grounds as CLM models. The modeling decision to purge our bottom-line DLM specification of certain search elements is specifically guided by the principle of making our DLM model easily comparable to the CLM model and, moreover, making our DLM model a tractable and intuitive extension of the CLM model.

The population is (potentially) grouped in an aggregate household, and a household (*not social*) planner solves the household's optimization problem. In contrast to related literature, the economy's population is entirely selfish, atomistic, and autonomous. "Autonomous" in this paper means that each ("atomistic") household member has the power to renege on the household planner's solution if it is not incentive compatible. The resources of all economic agents within the household are pooled.

We assume that the household owns the economy's final-goods producing firm, and, without loss of generality, that the firm owns the economy's capital stock. Households can purchase corporate bonds and firms can issue debt. Corporate bonds and debt mature one period after being issued. Therefore, within any period the inflow of new bonds and debt is equivalent to the total stock of bonds and debt. The inclusion of debt guarantees that firms are able to pay any incurred vacancy-adjustment costs, which, as we show, ultimately link capital taxes to employment.

Mathematical details regarding all of the following can be found in the Appendix. Moving forward, all non-price variables are normalized by the aggregate population, which we assume has a unit mass, and all price variables are normalized by the price of consumption.

4.1 The Household

4.1.1 Evolution of Employment

From the household's point of view, employment evolves as follows:

$$e_t^S = (1-\rho)\, e_{t-1}^S + F(\chi_t)\, p_t s_t, \tag{4.1}$$

where e_t^S denotes employment supply, ρ is the time-t job destruction probability (which is exogenous to the model), and s_t is the endogenous mass of job searchers in period t. Moreover, $F(\chi_t)\, p_t$ is the household's effective job finding probability, where p is an exogenous component that the household takes as given, and χ is search effort, which the household controls and has an impact on its effective probability of finding a job. F has the following properties: $F' \geq 0$, $F'' \leq 0$, $F(0) = 0$, and $F_t \rightarrow 1$ as $\chi_t \rightarrow \infty$ (this guarantees that the household's effective job finding probability cannot be greater than p). *Of note, this equation of motion should be interpreted as saying that job search occurs at the beginning of a period, and jobs are destroyed at the end of a period.*[11] In words, equation (4.1) says that period t employment supply is equal to the sum of all individual who were employed last period and whose jobs were not destroyed ($(1-\rho)e_{t-1}^S$, per equation (4.1)) and the mass of successful contemporaneous searchers ($F(\chi_t)\, p_t s_t$, per equation (4.1)).

There is both theoretical and empirical background that justify our decision to incorporate search effort into our DLM model (see, for instance, Pissarides, 2000, and Chirinko, 1984). Given this background, we assume that there are diminishing returns to search effort. There are a host of real-world counterparts to search effort, which can include, among others, a person spending time searching for job opportunities, applying for jobs, practicing interviewing skills, and even simply improving how their CV is written up. Of course, if a person expends zero search effort—in our DLM model $\chi = 0$, which is the real-world analog of simply not applying for any job—then the effective probability of becoming employed is zero.

Following standard labor search theory, we assume that *in equilibrium* all individuals participate in

[11] However, We follow the timing convention in Arseneau and Chugh (20120), by which all employment matches become productive in the same period in which the match is formed. That said, Arseneau and Chugh (2012) do not include endogenous search effort.

the labor market. Therefore,

$$s_t = \left(1 - e_{t-1}^S\right) + \rho e_{t-1}^S. \tag{4.2}$$

In words: the mass of contemporaneous searchers is equal to all individuals who did not find a job yesterday, $1 - e_{t-1}^S$, plus the mass of all individuals whose jobs were destroyed at the end of the previous period, ρe_{t-1}^S. As such, in each period t there are three employment states: newly employed workers (those who flow into employment in period t, $F(\chi_t) p_t s_t$ per equation (4.1)), "old" employed workers (those whose jobs were not destroyed at the end of the previous period, $(1-\rho)e_{t-1}^S$, per equation (4.1)), and searchers who did not find jobs in this same period $(1 - F(\chi_t))p_t s_t$—*nonemployed individuals.*

It is important to note that the key feature of our DLM model to disentangle h from e is that household members need to devote search effort in order to secure employment, and this search effort comes at a utility cost. However, as noted above, we assume that only search effort tending to infinity could *potentially* result in a household member securing a job with probability 1 (if $p_t = 1$). However, this will always undesirable, since we will also assume that as search effort tends to infinity so does its associated disutility. Jointly, these assumptions guarantee that in our DLM model e supply will always be less than 1 as the result of an optimal and endogenous decision on behalf of the household. This decision therefore holds equilibrium e below 1 regardless of whether exogenously-perceived search frictions (job finding and filling probabilities) also contribute to this result.

4.1.2 Utility

The household's lifetime utility $\mathbf{U}_t$ is equal to the infinite sum of the weighted sum of the instantaneous utility of individuals in each employment state, where the weights are the mass of individuals in each employment state. In particular,

$$\mathbf{U}_t \equiv \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \begin{array}{c} e_s^{S,old} \overbrace{\left[u(C_s^{e,old}) - \gamma \frac{\varepsilon}{1+\varepsilon} (h_s^{S,old})^{\frac{1+\varepsilon}{\varepsilon}} \right]}^{\equiv v_s^{e,O}} \\ e_s^{S,new} \overbrace{\left[u(C_s^{e,Nnew}) - \gamma \frac{\varepsilon}{1+\varepsilon} (h_s^{S,new})^{\frac{1+\varepsilon}{\varepsilon}} - D(\chi_t) \right]}^{\equiv v_s^{e,N}} \\ + \left(1 - e_s^S\right) \overbrace{\left[u\left(C_s^n\right) - D\left(\chi_t\right) \right]}^{\equiv v_s^n} \end{array} \right\},$$

where β is the exogenous subjective discount factor, $e_s^{S,old}$ denotes the supply of old employed individuals, $e_s^{S,new}$ denotes the supply of newly employed individuals, and $1 - e_s^S$ is the mass of period-s unsuccessful searchers (nonemployed individuals). $v_s^{e,old}$, $v_s^{e,new}$, and v_s^n are, respectively, the instantaneous utilities of each of these individuals. Also, $h_s^{S,old}$ is the supply of hours worked per old employed individual and $h_s^{S,new}$ is the supply of hours worked per newly employed individual. $C_s^{e,old}$, $C_s^{e,new}$, and C_s^n is the consumption of, respectively, old employed individuals, newly employed individuals, and nonemployed individuals. We assume that these consumptions are, respectively, a fraction $\kappa_s^{e,old}$, $\kappa_s^{e,new}$, and $1 - \kappa_s^{e,old} - \kappa_s^{e,new}$ of aggregate consumption C_s. Finally, u is utility from consumption, whose properties are $u' > 0$ and $u'' < 0$, G is disutility from work hours whose properties are $G' > 0$ and $G'' > 0$, and D is disutility from search effort, whose properties are $D' > 0$, $D'' > 0$, and $D_t \to \infty$ as $\chi \to \infty$.

Note that the instantaneous utility of individuals in old employment does not include disutility from search effort: since their jobs were not destroyed in the previous period they did not have to search contemporaneously. However, the instantaneous utility of individuals in new employment does include disutility from search effort, since individuals in new employment started the period as searchers. Of course, because at the beginning of any period t both individuals in new employment and unsuccessful searched, then in equilibrium the search effort of both types of economic agents is the same per period: χ_t.

4.1.3 Constraints

As noted earlier, since the economy's population is atomistic, autonomous, and selfish, then the setup is such that the household planner faces a series of incentive compatibility constraints. First, $v_t^{e,old} \geq v_t^{e,new}$. Else, individuals in old employment have an incentive to quit at the beginning of a period and jump into the pool of searchers or jump into nonemployment after search has taken place. Second, $v_t^{e,new} \geq v_t^n$. Else, all individuals in new employment have an incentive to jump into nonemployment after search has taken place (note that the first and second constraints jointly imply that $v_t^{e,old} \geq v_t^n$, as should be the case). Third, $v_t^n \geq \bar{v}_t$, where $\bar{v}_t$ denotes the outside option of individuals from reneging on the household planner's decision and leaving the household. Else, all individuals have an incentive to exit the household—the outside option of the atomistic household members is positive since they can choose to participate (or not) in the labor market on their own and, therefore, as an atomistic household instead of as part of the aggregate household. An implication of them doing so is taking their portion of aggregate non-labor income with them (recall that all individuals who are effectively in the household pool their resources). Jointly, these constraints guarantee that individuals will accept the household planner's proposed solution.

The household's budget constraint is

$$(1+\tau_t^c)\,C_t + (b_t - b_{t-1}) \leq \left(1-\tau_t^l\right) w_t (h_t^{S,old} e_t^{S,old} + h_t^{S,new} e_t^{S,new})$$
$$+UBs_t + V_t + \left(1-\tau_t^k\right) r_{t-1} b_{t-1} + T_t, \tag{4.3}$$

where τ_t^c is the consumption tax, b_t denotes period-t bonds, τ_t^l is the labor tax, w_t is the real wage, UB denotes unemployment benefits, which are paid to every individual who searches in a period and is unsuccessful in finding a job, V_t denotes net-of-(capital)-tax dividends paid by the firm to the household (recall that the household owns the firm), τ_t^k is the capital tax, r_t is the period-t real interest rate, and T_t denotes government transfers. The household takes all taxes, prices, the unemployment benefit, dividends, and transfers as given.

4.1.4 Optimization

The household planner's choice variables are: C_t, $\kappa_t^{e,old}$, $\kappa_t^{e,new}$, κ_t^n, $h_t^{S,old}$, $h_t^{S,new}$, e_t^S, s_t, χ_t, and b_t. Of course, given that $e_t^{S,old} = (1-\rho)\,e_{t-1}^S$, then knowing s_t and e_t^S is sufficient to know the distribution of the entire population across employment states. As shown in the Appendix, the planner's choice of $\kappa_t^{e,old}$ and $\kappa_t^{e,new}$ is such that, in equilibrium, in each period t the instantaneous utility of individuals in each employment state is equalized, which is intuitive: while in the model this is a household-planning decision, this result would also be an endogenous sorting outcome in the absence of the existence of a household planner. This implies that, while individuals in different employment states consume different amounts, in the pooled household there is instantaneous utility risk sharing. As also shown in the Appendix, in equilibrium $h_t^{S,new} = h_t^{S,old}$, which is also intuitive and both of which we henceforth denote by h_t^S.

Importantly, note that because individuals are indifferent between being employed or nonemployed, then in the model there is, by definition of involuntary unemployment, *no involuntary unemployment whatsoever*. This is the case regardless of the presence of search frictions, and that is why, to make this clear, we refer to unsuccessful searchers as "nonemployed" instead of "unemployed." Moreover, note that because all incentive compatibility constraints bind, then maximizing the weighted sum of lifetime utility of individuals across employment states is exactly the same as maximizing the lifetime utility of an individual in any one employment state.

In what follows, for brevity, we present the household's optimality condition for the *bottom-line version of our DLM model, which is purged from certain elements of labor search theory* (optimality conditions from the full-fledged search model are in the Appendix). In particular, our purging assumes:

$p_t = 1$ in all periods t, $\rho = 1$ in all periods t, and $UB = 0$ in all periods t. These assumptions are broadly in line with those used in Arseneau and Chugh (2012) when the authors show how to collapse their general equilibrium labor search model to a standard real business cycle (RBC) model.

Note that the assumption that $\rho = 1$ in all periods implies that all jobs are destroyed at the end of a period. Given our assumption that all individuals participate in the labor force, then this means that at the beginning of a period the mass of searchers is equal to 1. Moreover, all employed individuals within a period are newly employed, only. Finally, note that equation (4.1) collapses to

$$e_t^S = F\left(\chi_t\right). \tag{4.4}$$

Amid this backdrop, it is clearly the case that there is no corner solution for equilibrium employment. With zero employment there is no production, and the household can clearly do better. Moreover, as noted earlier the corner solution with employment equal to 1 is not optimal either. Of course, $\rho = 1$ in all periods and the fact that optimally the household will never want $e^S = 1$ since this only occurs in the limit when search effort tends to infinity are indeed frictions. However, we refer to our model as being "quasi frictionless" because, in spite of these frictions, all markets are perfectly competitive, including the labor market, and as we discuss immediately below our DLM model maps in straightforward fashion into CLM frameworks.

Regarding a mapping between our DLM model and CLM frameworks, note that assuming that there are no search-effort costs, then in our DLM model the trivial decision of the household for employment supply is $e = 1$, in which case our DLM model's implications for the determination of equilibrium H are the same as those of CLM models (we address this in further detail later in the paper). Importantly, while our DLM model says that this is explicitly the case when $e = 1$, only, in CLM frameworks no such message is conveyed. In other words, CLM models do not speak to the composition of H. This is an important result, because it is one avenue by which our DLM model suggests that CLM models' equation for H is actually an equation for h, and therefore that CLM models should not be expected to have any explanatory power over the extensive margin of labor e.

The remaining optimality conditions of our (purged) DLM model are as follows. Using this representation of the planner's Lagrangian, the first order conditions are as follows. For aggregate consumption:

$$\frac{u'(\kappa_t^{e,new} C_t)\kappa_t^{e,new}}{(1+\tau_t^c)} = \lambda_t, \tag{4.5}$$

which is entirely standard in tax-inclusive frameworks save for the presence of $\kappa_t^{e,new}$ that nonetheless does not affect the intuition behind this equation: The time-t effective marginal utility (tax-inclusive) marginal utility of consumption is equal to the marginal value of real wealth, λ_t. For bonds:

$$1 = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[1 + \left(1 - \tau_{t+1}^k \right) r_t \right]. \tag{4.6}$$

This condition defines the stochastic discount factor, which for any period $t \geq s$ is given by $\Xi_{t|s} \equiv \beta^{t-s} \lambda_{t+1} / \lambda_s$.

Turning to labor market variables, the optimality condition for search effort is

$$D_t' = \psi_t^e F_t', \tag{4.7}$$

where ψ_t^e is the household's shadow value of a job, meaning that the marginal cost of search effort equals its marginal benefit. The optimality condition for hours worked per worker is

$$G_t' = \lambda_t \left(1 - \tau_t^l \right) w_t e_t^S, \tag{4.8}$$

meaning that the marginal cost of hours worked per worker equals its marginal benefit. Of course, this equation pins down the supply of hours worked per worker, and, importantly, the right-hand side of this equation implies that *the supply of hours worked per worker depends, among other things, on the supply of employment*. Finally, the optimality condition for employment implies that

$$\psi_t^e = \lambda_t \left(1 - \tau_t^l \right) w_t h_t^S. \tag{4.9}$$

This means that the shadow value of employment is equal to its marginal benefit, and that *the supply of employment depends, among other things, on the supply of hours worked per worker.*

Finally, note that combining the optimality conditions for χ_t and e_t^S implies that

$$D_t' = \left[\lambda_t \left(1 - \tau_t^l \right) w_t h_t^S \right] F_t'. \tag{4.10}$$

Given our purging of labor search components, then in our bottom-line DLM specification choosing χ_t is the same as choosing e_t^S. As such, equation (4.10) is the household's effective employment supply equation, which means that the marginal cost of employment (the right-hand side of this equation, which is χ-dependent) is equal to its marginal benefit (the equation's left-hand side, which is also χ-dependent).

4.2 The Firm

4.2.1 Environment

Aggregate output Y_t is given by the production function

$$Y_t = Y\left(Z_t, K_t, h_t^D e_t^D\right), \tag{4.11}$$

where: Z_t is exogenous total factor productivity; K_t denotes capital; and h_t^D and e_t^D denote, respectively, the demand of hours worked per worker and the demand of employment. In line with standard CLM literature we assume that Y_t is linear in Z_t and increasing and concave in K_t and the demand of total hours worked per population $H_t^D \equiv h_t^D e_t^D$. Therefore, our assumption of perfect substitutability of h and e in production is guided by the principle of making our DLM model a straightforward extension of CLM models.

As is standard, we assume that the firm posts vacancies in order to fill open positions. Therefore, the firm's objective function, net-of-(capital)-tax dividends, is given by

$$\mathbb{E}_t \sum_{s=t}^{\infty} \Xi_{s|t} \underbrace{\left\{ \overbrace{\left(1-\tau_s^k\right) \left[\begin{array}{c} Y\left(Z_t, K_t, h_t^D e_t^D\right) - w_s h_s^D e_s^D - I_s \\ -r_{s-1} d_{s-1} - \Lambda\left(Z_s\right)\left(\Phi v_s + \Omega(v_s/v_{s-1}, v_s w_s)\right) \end{array} \right]}^{\equiv \bar{V}_s} + \left(d_s - d_{s-1}\right) \right\}}_{\equiv V_s}. \tag{4.12}$$

Above, I_s is investment, d denotes debt, Φ is the exogenous flow cost of posting vacancies v_s, and we discuss the function Λ further below. Moreover, Ω is a standard adjustment cost function that is increasing and convex in the ratio v_s/v_{s-1} and also in the product $v_s w_s$ (we discuss the inclusion of this adjustment cost function further below). Moreover, Ω is equal to zero whenever v_s equals v_{s-1}, which, in particular, is the case in steady state. (The broader relevance of adjustment costs for labor markets is emphasized by Cooper and Willis, 2004, Caballero and Engel, 2004, Cooper and Willis, 2008, and Mumtaz and Zanetti, 2014, among others.) Of course, the change in the firm's debt position is not taxed, and, intuitively, we assume that $V_t \geq 0\ \forall t$. Moreover, we do not include investment taxes since, per the evidence in Table 1 they do not have an impact on labor market variables and, furthermore, in our DLM model, in line with CLM models, investment taxes do not have a theoretical impact on labor market variables either.

As an example of vacancy-posting adjustment costs, consider the following. Whether the firm is expanding or reducing their number of employees, the firm may need to reassess and reorganize some of its production techniques. This, for instance, can involve redistributing the amount of workers that are devoted to any one particular project, which in turn can involve workers having to learn new on-the-job skills, both of which can naturally be costly.

Regarding the firm's overall vacancy posting costs, following, for instance, Pissarides (2000), we assume that the firm's costs of posting vacancies are potentially a function of exogenous aggregate productivity, which is captured by the presence of $\Lambda(Z_t)$ in the firm's objective function. That said, we do not make any assumption on whether, if productivity is indeed related to vacancy posting costs, higher productivity makes it more costly or less costly for firms to post vacancies. Instead, as we will discuss below, we arrive at a conclusion regarding the potential relationship between productivity and vacancy posting empirically.

In terms of intuition, note that a relationship between productivity and vacancy posting costs can reflect a host of reduced form factors. For example, if higher productivity exacerbates the firm's costs of posting vacancies, among many real-world counterparts this can reflect the following. Given increasing productivity the firm decides to post more vacancies in order to boost employment and, therefore, boost output beyond the boost stemming from higher productivity. However, this trivially leads to more job applications, and with a larger pool of applicants these applications become harder to screen. In contrast, if higher productivity decreases the firm's costs of posting vacancies, then this can reflect, for instance, the following. Amid higher productivity, the firm in fact decreases screening as a result of it being the case that higher aggregate productivity more than offsets any concerns regarding dispersion of worker-side idiosyncratic productivity.

More generally, the relationship between aggregate productivity and vacancy costs incorporated in our DLM model establishes indirectly a link between productivity and employment. In our bottom-line/purged DLM specification this link becomes direct, and, anticipating empirical results, is positive. As such, our DLM framework speaks to the long-run neutrality of productivity on employment implied by traditional models of aggregate labor markets, which is problematic (see, for instance, Layard, Nickell, and Jackman, 1991, Blanchard, 2007, and Elsby and Shapiro, 2012). In broader terms, it follows that if aggregate productivity has an impact on the firm's costs of posting vacancies, then this can be interpreted as a driving force behind changes in matching efficiency. In that respect, Barnichon and Figura (2015) find that, empirically, matching efficiency is indeed increasing in productivity.

The firm faces two constraints. First, a standard equation of motion for the capital stock,

$$K_{t+1} = I_t + (1 - \delta) K_t, \tag{4.13}$$

where δ is the capital depreciation rate. Second, its perceived evolution of employment,

$$e_t^D = (1-\rho)\, e_{t-1}^D + q_t v_t, \tag{4.14}$$

where q_t is the job filling probability, which the firm takes as given. In words, this equation says that the firm's contemporaneous stock of employed individuals is equal to the sum of all of the previous periods how did not lose their job (the first term on the right-hand side) and all newly formed employment relationships, which are equal to all vacancies that are filled in a period (the second term on the right-hand side).[12]

4.2.2 Optimization

For brevity, here we present the firm's optimality condition for the bottom-line version of our DLM model, which, as noted earlier, is purged from certain elements of labor search theory (optimality conditions from the full-fledged search model are in the Appendix). In particular, as relevant for the firm, these purging assumptions are $q_t = 1$ in all periods t, $\rho = 1$ in all periods t, and $\Phi = 0$ in all periods t (recall the following: these assumptions are broadly in line with Arseneau and Chugh, 2012, when the authors show how to collapse their general equilibrium labor search model to a standard RBC model).

The firm's choice variables are K_{t+1} (i.e., I_t), d_t, h_t^D, e_t^D, and v_t. The first order condition for K_{t+1} is

$$\mathbb{E}_t \Xi_{t+1|t} \left(1-\tau_{t+1}^k\right) Y_{K_{t+1}} + \mathbb{E}_t \Xi_{t+1|t} \left(1-\tau_{t+1}^k\right)(1-\delta) - \left(1-\tau_t^k\right) = 0, \tag{4.15}$$

which is standard save for the fact that it is tax-inclusive. The first order condition for d_t is:

$$1 = \mathbb{E}_t \Xi_{t+1|t} \left[1 + \left(1-\tau_{t+1}^k\right) r_t\right]. \tag{4.16}$$

Note that equations (4.6) and (4.16) are identical and therefore always hold, meaning that bond/debt holdings are indeterminate. It would be straightforward to introduce convex costs of portfolio adjustments around a target leverage ratio for the firm, or similar modelling devices on the side of the household to resolve this indeterminacy. We choose not to do so, since the concern of this paper is not the capital structure of the firm, and the presence of bonds only serves to guarantee that firms make at least zero economic profits in the presence of vacancy-related costs.

[12] Interest payments on corporate debt $d_{t-1}r_{t-1}$, as well as depreciation of existing capital δK_t, are assumed to be deductible as expenses for the purpose of calculating the corporate tax bill. While this is the case in the United States, it is not uniformly applicable to all the countries that we use in our empirical analysis. We abstract from such differences in the model since our conclusions about the long term trends in employment and hours are robust to these assumptions about the tax treatment of interest payments and capital depreciation.

The first order condition for h_t^D implies that

$$Y_{h_t} = w_t e_t^D. \tag{4.17}$$

The first order condition for e_t^D implies that

$$\mathbf{J}_t = Y_{e_t} - w_t h_t^D, \tag{4.18}$$

where $\mathbf{J}_t$ is the firm's shadow value of a job.[13] *As such, the demand of hours worked per worker and employment depend on each other.* Finally, the first order condition for vacancies is:

$$\left(1-\tau_t^k\right)\left(\mathbf{J}_t - Z_t^{\zeta}\frac{\partial \Omega_t}{\partial v_t}\right) - \mathbb{E}_t \Xi_{t+1|t}\left(1-\tau_{t+1}^k\right) Z_{t+1}^{\zeta}\frac{\partial \Omega_{t+1}}{\partial v_t} = 0. \tag{4.19}$$

Alternatively, this last equation can be stated as

$$\mathbf{J}_t = \mathbb{E}_t \Xi_{t+1|t}\left(1-\tau_{t+1}^k\right) Z_{t+1}^{\zeta}\frac{\partial \Omega_{t+1}}{\partial v_t} + \left(1-\tau_t^k\right) Z_t^{\zeta}\frac{\partial \Omega_t}{\partial v_t}. \tag{4.20}$$

The intuition behind this equation is straightforward: the benefit of a filled and additional vacancy, $\mathbf{J}_t$, equals its expected marginal cost, which in turn equals the intertemporal cost of adjusting vacancies over the duration of a position remaining unfilled (recall that by assumption $q_t = 1$ in all periods). Moreover, note that, importantly, the firm's optimality condition for vacancies yields a novel link between vacancy posting and capital taxes. Because vacancy postings have a direct impact on employment, then *the optimality condition for vacancies also links employment to capital taxes (recall that, as shown in Table 1, there is an empirical statistically significant association between capital taxes and employment).*

Because adjustment costs are zero whenever v_t equals v_{t-1}, then in these periods this last equation implies that the firm's value of a job $\mathbf{J}_t$ is zero and, per equation (4.17), that in these periods $Y_{e_t} = w_t h_t^D$, which of course is also the case in steady state. That said, note that the fact that h and e are perfect substitutes in production implies that the wage is competitive. This result is clear from equation (4.17), which holds regardless of whether search frictions exist or not, *and therefore implies that whether there are search frictions or not the wage is competitive and the firm's shadow value of a job,* $\mathbf{J}$*, is in fact equal to zero in all periods.* Indeed, note that with h and e being perfect substitutes in production, then $Y_{h_t} h_t$ equals $Y_{e_t} e_t$. From equation (4.17) it follows that $Y_{h,t} h_t^D = w_t h_t^D e_t^D$, which given the fact that $Y_{h_t} h_t$ equals $Y_{e_t} e_t$ implies, *as a result*, that $Y_{e,t} e_t^D = w_t h_t^D e_t^D$. Multiplying through equation (4.17) therefore

[13] Importantly, note that given the presence of the vacancies adjustment cost function, then the firm's value of a job is analogous to Tobin's Q.

yields the result that $\mathbf{J}_t$ is equal to zero in all periods.

While this result regarding the wage may be surprising, the intuition behind it is as follows. When a worker and a firm meet, because hours worked per worker and employment are perfect substitutes in production, then in fact the firm and the worker are negotiating in a spot market: the spot market for hours worked per population. Indeed, note that if the firm rejects the match, its outside option is simply to adjust production via hours worked per worker, which it can do since it always has at least one employed individual. As such, for the firm rejecting the match is costless. Analogously, on the household side rejecting the match is costless as well because the household's outside option is simply to adjust earnings by adjusting the supply of hours worked per worker. All told, the fact that the wage is competitive in this search environment owes fully and entirely to the fact that h and e are perfect substitutes in production. (This competitive wage and its relationship to a noncompetitive wage determination protocol, Nash bargaining, is discussed in the Appendix.)

All told, the relevant optimality condition for vacancies is

$$\underbrace{Y_{e_t} - w_t h_t^D}_{=\,\mathbf{J}_t\,=\,0} - \Lambda\left(Z_t\right)\frac{\partial \Omega_t}{\partial v_t} = \mathbb{E}_t \Xi_{t+1|t} \frac{1-\tau_{t+1}^k}{1-\tau_t^k} \Lambda\left(Z_{t+1}\right)\frac{\partial \Omega_{t+1}}{\partial v_t}, \tag{4.21}$$

which can now be interpreted as firms finding it optimal to equate the post-tax marginal cost of changing vacancies today to the post-tax (discounted) marginal cost of changing vacancies tomorrow. Another way to interpret this equation is that *firms optimally decide on vacancy adjustments in order to smooth these costs over time (and, more specifically, smooth the consumption value of net-of-(capital)-tax dividends for the household)*. This effect is absent in models where vacancy costs do not have an intertemporal element. In particular, consider an example of how capital taxes affect vacancy postings. Suppose that at time t an expected increase in next period's capital tax rate τ_{t+1}^k lowers the ratio $\frac{1-\tau_{t+1}^k}{1-\tau_t^k}$. To restore equation (4.21) the firm will raise $\frac{\partial \Omega_t}{\partial v_t}$, which will require raising expected vacancy posting.

Finally, note that given our purging of specific search elements and, in particular, assuming that ρ and q_t are both equal to 1 in all periods, implies that equation (4.14) collapses to $e_t^D = v_t$. This clearly implies that equation (4.21) is, in our bottom-line DLM model, the firm's effective demand for employment. Clearly, then, in any period in which the firm adjusts employment demand, employment demand is pinned down by the following optimality condition:

$$-\Lambda\left(Z_t\right)\frac{\partial \Omega_t}{\partial e_t^D} = \mathbb{E}_t \Xi_{t+1|t} \frac{1-\tau_{t+1}^k}{1-\tau_t^k} \Lambda\left(Z_{t+1}\right)\frac{\partial \Omega_{t+1}}{\partial e_t^D}, \tag{4.22}$$

where we substituted out v_t by e_t^D. *This means that whenever the firm is adjusting employment, em-*

ployment demand is forward looking. However, in periods in which the firm does not adjust employment demand, adjustment costs are zero and equation (4.21) implies that employment demand is pinned down by the following optimality condition: $Y_{e_t} = w_t h_t^D$.

4.3 Closing the Model

In this section we focus on the equilibrium values of variables, so we drop all "demand" and "supply" superscripts. *Moreover, throughout the remainder of the paper we henceforth focus exclusively on the "purged" version of our DLM model.* In particular, recall that this purging involves assuming the following: ρ, p_t, and q_t are equal to 1 in all periods, and UB and Φ are equal to zero in all periods. To close the model (details regarding the full-fledged search version of our DLM model are in the Appendix) we assume that the government runs a balanced budget so that

$$T_t = \tau_t^c C_t + \tau_t^l w_t h_t e_t + \tau_t^k \left(r_{t-1} b_{t-1} + \bar{V}_t\right).$$

This implies that the aggregate resource constraint is given by

$$Y_t = C_t + I_t + \Omega\left(e_t^D / e_{t-1}^D, e_t^D w_t\right)$$

whenever e_t^D is not equal to e_{t-1}^D, and

$$Y_t = C_t + I_t$$

whenever the firm adjusts vacancies.

4.4 Equilibrium

In this version of our DLM model, an equilibrium is a vector

$$\{\kappa_t^{e,new},\ C_t,\ \chi_t,\ e_t,\ h_t,\ b_t,\ d_t,\ I_t,\ K_{t+1},\ w_t,\ r_t,\ \mathbf{J}_t\}$$

that, given the model's exogenous variables and parameters, satisfies the following equations (in this vector we only listed $\kappa_t^{e,new}$ since, given the fact that all jobs are destroyed in every period, then in each period there are only two effective employment estates given the equilibrium outcome that all individuals accept the planner's solution: newly employed; and nonemployed): $v_t^e \geq v_t^n$ (incentive compatibility constraint), (4.5), (4.7), (4.9), (4.8), (4.6), $b_t = d_t$ (bond market clearing), (4.13), (8.12), (4.15), (4.17), (4.16), and (4.18). *Of note, $\mathbf{J}_t$ should be interpreted as a shadow price—the shadow price of employment—akin to Tobin's Q.*

5 Operationalizing the Theory

Recall again that throughout the remainder of the paper we focus exclusively on the "purged" version of our DLM model.

5.1 Methodology

In related literature, the main approach used to assess the CLM model's fit regarding H is as follows. Take the implied theoretical equilibrium relationship between hours worked per population and aggregate prices/quantities to predict equilibrium H. Then, compare this prediction to empirical H and assess the extent to which each of the model-implied driving forces matter for the behavior of this variable. This approach falls within the "business cycle accounting" category (see for example, Prescott 2004 and Chari, Kehoe, and McGrattan, 2007, among others), and we follow this approach in order to make our DLM results directly comparable to this literature.

More specifically, the approach to business cycle accounting followed by related literature involves the following. Suppose a model predicts that some variable X_t is a function of the vector of variables $\mathbf{\Psi}_t$ such that $X_t = X(\mathbf{\Psi}_t)$. Then, the accounting methodology involves taking the equation $X_t = X(\mathbf{\Psi}_t)$ and feeding into it empirical data for $\mathbf{\Psi}_t$, which results in a theory-implied prediction for the behavior of X_t given the behavior of the empirical data $\mathbf{\Psi}_t$. Assessing the model's fit then involves comparing this theory-implied prediction for the behavior of X_t with its empirical behavior.

Of course, our DLM model's equilibrium condition for h (equation (5.1)) is static, so it is straightforward to test this condition using business cycle accounting. That said, because employment demand (equation (4.21)) is dynamic, then so is equilibrium employment. Therefore, to be conceptually in line with business cycle accounting, we will assess the fit of our DLM model's dynamic equilibrium employment equation from an ex-post vantage point. This allows us to proceed as described in the previous paragraph, in particular, feeding in (*realized*) empirical data into the model equation for employment in order to back out the model's predictions for employment. In line with this ex-post/realized business cycle accounting approach, we henceforth drop the expectation operators applicable to our DLM model's dynamic employment demand and equilibrium conditions.

5.2 DLM: Functional Forms

Our interest is in the equilibrium values labor market variables (therefore, unless otherwise noted we no longer implement a distinction for demand- and supply-side variables), which are henceforth our sole focus. As such, to operationalize the model we only need to make assumptions on a limited set of functions. Regarding production, in line with related literature, we assume a standard constant returns to scale production function $Y_t = Z_t K_t^{\alpha} (h_t e_t)^{1-\alpha}$ where $\alpha \in (0,\, 1)$ and therefore $Y_{h_t} h_t = (1-\alpha) Y_t$ and $Y_{e_t} e_t = (1-\alpha) Y_t$. Turning to employment demand, we assume that $\Lambda(Z_t) = Z_t^{\zeta}$, where, as discussed earlier, ζ will be estimated and its value could, in principle, be less than zero, greater than zero, or equal to zero.

We also assume the following cost function:

$$\Omega_t = \psi \mathbb{I}_t \left(\frac{e_t^D}{e_{t-1}^D} \right)^{\phi} e_t^D w_t,$$

where $\psi > 0$, $\phi > 1$, and $\mathbb{I}_t$ equals zero if e_t^D equals e_{t-1}^D and $\left(\frac{\beta\phi}{1+\phi}\right)^{t-1}$ otherwise. As such, $\mathbb{I}_t$ guarantees that adjustment costs are zero whenever the firm is not adjusting vacancies (which, in particular, is the case in steady state), and the adjustment-value of $\mathbb{I}_t$ guarantees that the growth rate of employment is zero in steady state. Regarding the fact that the product $e_t^D w_t$ enters the cost function linearly, in line related literature, this assumption reflects in a reduced form way the intuition that hiring costs can reflect expenditures on a human resources department, and therefore are a fraction of the wage bill.

Turning to the household, following the assumptions in Shimer (2009) as applicable to the present context we assume $G(h_t) = \gamma \frac{\varepsilon}{1+\varepsilon} (h_t)^{\frac{1+\varepsilon}{\varepsilon}}$, where: γ and ε are parameters that are strictly greater than zero. As such, in our DLM model ε is the Frisch elasticity of the supply of hours worked per *worker*. Also following Shimer (2009), let $u(\cdot) = \ln(\cdot)$. For expositional tractability we assume $F(\chi_t) = \varphi \chi_t^{\sigma}$, where $\sigma \in (0, 1)$ and $\varphi > 0$ (parameters are assumed to be in line with only infinite search effort being sufficient to approach a value of F_t equal to 1, even though our assumed functional form for F does not asymptote at 1), and also $D_t = \varrho \chi_t^{\tau}$, where $\tau > 1$ and $\varrho > 0$.

5.3 DLM: Testable Implications

The functional forms we assume imply the following. Note that since $u(\cdot) = \ln(\cdot)$, then in the household's objective function

$$\ln(\kappa_t C_t) = \ln(\kappa_t) + \ln(C_t),$$

which means that the first order condition for aggregate consumption is

$$\frac{1}{C_t (1 + \tau_t^c)} = \lambda_t.$$

Therefore, the optimal values of the κ variables are irrelevant for our analysis. Moreover, the household's stochastic discount factor is

$$\Xi_{s|t} \equiv \beta^{s-t} \frac{C_t}{C_s} \frac{1+\tau_t^c}{1+\tau_s^c},$$

where $s \geq t$, which means that higher consumption as well as higher consumption taxes (or, taken together, higher *consumption expenditures*) at time t lower the marginal value of consumption in that period.

5.3.1 Hours Worked per Worker

As shown in the Appendix, combining the demand and supply of hours worked per worker implies the following equilibrium condition, which is our model's first testable implication:

$$h_t = \left[\frac{1-\alpha}{\gamma} \frac{1-\tau_t^l}{1+\tau_t^c} \left(\frac{C_t}{Y_t} \right)^{-1} \right]^{\frac{\varepsilon}{1+\varepsilon}}. \tag{5.1}$$

Above, ε is the elasticity of hours worked per *worker* with respect to the wage. The intuition behind this equation is straightforward. Increases in labor taxes chip away at the value of the extra hour of work, putting downward pressure on work hours. Increases in consumption taxes raise the price of consumption, and therefore increase the opportunity cost of consumption in terms of leisure. Moreover, a higher consumption-output ratio is consistent with a lower marginal value of real wealth, which reduces the marginal value (in terms of consumption) of an additional hour of work.

To generate our DLM model's predictions of hours worked per worker, we proceed as follows. First, we run a panel regression of equation (5.1) in growth rates, i.e., a regression of

$$d \ln h_t = \left[\begin{array}{c} \frac{\varepsilon}{1+\varepsilon} d \ln \left(1-\tau_t^l\right) - \frac{\varepsilon}{1+\varepsilon} d \ln \left(1+\tau_t^c\right) \\ -\frac{\varepsilon}{1+\varepsilon} d \ln \left(C_t / Y_t\right) \end{array} \right], \tag{5.2}$$

with the coefficients constrained as implied by the theory, in order to arrive at a cross-country estimate of $\frac{\varepsilon}{1+\varepsilon}$. Then, for each country in our sample, we feed country-specific empirical time series data on consumption, consumption taxes, output, and labor taxes, into the right-hand side of equation (5.1) that, given our estimate of $\frac{\varepsilon}{1+\varepsilon}$, yield our DLM model's country-specific predictions for hours worked per worker (there is one for *each country* in our sample). Since our interest is in long run trends, in line with related literature, such as Shimer (2009), the value of $\frac{1-\alpha}{\gamma}$ is chosen so that the average of predicted hours worked per worker match the average of their empirical counterpart *on a country-by-country basis.*

5.3.2 Employment

Regarding employment, our assumed functional form yields the following "forward looking" condition for employment demand (which, as noted earlier, we focus on from an ex-post perspective in order to make business cycle accounting feasible)

$$d\ln e_t^D = \frac{1+\phi}{\phi} d\ln e_{t+1}^D + \frac{1}{\phi}\left[\begin{array}{c} d\ln w_{t+1} + \zeta d\ln Z_{t+1} \\ -d\ln\lambda_{t+1} + d\ln\left(1-\tau_{t+1}^k\right) \end{array}\right] \quad (5.3)$$

(see the Appendix for details). Regarding this expression, recall that in our purged DLM model firms optimally decide on employment adjustment in order to smooth adjustment costs over time. As such, *from the vantage point of the firm's period t decision making* equation (5.3) conveys the following optimal firm-side actions. Higher $d\ln w_{t+1}$, higher $d\ln e_{t+1}$, and *lower* $dln\lambda_{t+1}$ are associated with higher future output, given which the firm anticipates an expansion in future employment. In order to smooth adjustment costs, the firm frontloads some of this employment expansion, which puts upward pressure on $d\ln e_t$. For concreteness assume that $\zeta < 0$. As such, higher $d\ln Z_{t+1}$ means lower adjustment costs in the future. In smoothing these costs the firm postpones some contemporaneous adjustment, which puts downward pressure on $d\ln e_t$. Finally, higher capital taxes mean that future net-of-capital-tax dividends will be lower, given which the firm wants to adjust the least possible amount today in order to get as much net-of-tax-dividends today and, therefore, before the increase in capital taxes. This puts downward pressure on $d\ln e_t$.

Given the firm's optimal period-t actions in light of the behavior of future variables (so, the behavior of future variables *causes* the firm's contemporaneous actions), then, all else equal, in period $t+1$ the following should be observed as an *outcome* of the firm's previous period's optimal actions:

$$d\ln e_{t+1}^D = \frac{\phi}{1+\phi} d\ln e_t^D - \frac{1}{1+\phi}\left[\begin{array}{c} d\ln w_{t+1} + \zeta d\ln Z_{t+1} \\ -d\ln\lambda_{t+1} + d\ln\left(1-\tau_{t+1}^k\right) \end{array}\right]. \quad (5.4)$$

We arrive at this equation simply by solving equation (5.3) for $d\ln e_{t+1}^D$. Of course, relative to equation (5.3), in equation (5.4) signs on all variables except $d\ln e_t^D$ are flipped. But, again, notice that these flipped signs are period $t+1$ *outcomes* given the firm's optimal forward looking employment demand decision in period t. So, the period $t+1$ relationships between $d\ln e_{t+1}^D$, $d\ln w_{t+1}$, $\zeta d\ln Z_{t+1}$, $d\ln\lambda_{t+1}$,

$d\ln\left(1-\tau_{t+1}^{k}\right)$ and also, of course, with $d\ln e_t^D$, are correlations and do not imply causality.

Combining the dynamic equation for employment demand with a dynamic version of employment supply (see the Appendix for details) yields our DLM model's second testable implication, which is the following dynamic equation for equilibrium employment:

$$
\begin{aligned}
d\ln e_t = \; & \frac{\phi}{\phi+\left(\frac{\sigma}{\tau}\right)^{-1}} d\ln e_{t-1} \\
& + \frac{1}{\phi+\left(\frac{\sigma}{\tau}\right)^{-1}} \left[\begin{array}{c} \left(1+\frac{\varepsilon}{1+\varepsilon}\right) d\ln\left(1-\tau_t^l\right) - \frac{\varepsilon}{1+\varepsilon} d\ln\left(1+\tau_t^c\right) \\ -\frac{\varepsilon}{1+\varepsilon} d\ln\left(C_t/Y_t\right) - d\ln\left(1-\tau_t^k\right) - \zeta d\ln Z_t \end{array} \right].
\end{aligned} \tag{5.5}
$$

Intuitively, this is an autoregressive moving average process with exogenous variables (ARMAX) for equilibrium employment. Note that the coefficient on $d\ln e_{t-1}$is a number between 0 and 1, which means that this ARMAX process is not explosive in employment (similar comments are relevant for dynamic employment demand). Also, in this equation the variables scaled by coefficients that include εs trace back to employment supply, which, recall, is a function of hours worked per worker—hence the presence of the εs (see equation (5.1)). The presence of σ/τ also traces back to employment demand and reflects the relative degree of diminishing returns to search (recall that σ is the exponent on the job-attaining search-effort function F).

Recall that employment supply is static. Therefore, restated in growth rates (in order to combine employment supply and demand), unlike the firm's decision on employment demand, which is entirely forward looking, the household's employment supply decision is reflecting *contemporaneous* optimal actions. As such, while in equation (5.5) the terms that involve $d\ln e_{t-1}$, $d\ln\left(1-\tau_t^k\right)$, and $\zeta d\ln Z_t$ are correlations and do not reflect causality (these two terms come in from *forward-looking* employment demand, only), the terms that involve $\left(1+\frac{\varepsilon}{1+\varepsilon}\right) d\ln\left(1-\tau_t^l\right)$, $\frac{\varepsilon}{1+\varepsilon} d\ln\left(1+\tau_t^c\right)$, $\frac{\varepsilon}{1+\varepsilon} d\ln\left(C_t/Y_t\right)$ do indeed reflect contemporaneous *causal* relationships from labor taxes, consumption taxes, and the consumption-output ratio to employment supply. Given a higher τ^l or τ^c the marginal value of employment decreases for the household, since any given level of employment is worth less in consumption terms. This puts downward pressure on $d\ln\left(e_t\right)$. In addition, a higher $d\ln\left(C_t/Y_t\right)$ is consistent with a lower marginal value of real wealth. This also puts downward pressure on $d\ln\left(e_t\right)$ since a lower marginal value of real wealth means that, all else equal, the household places less value on additional labor income.

Before proceeding it is important to note the following. As highlighted earlier, in steady state there are no adjustment costs. Therefore in steady state employment demand is given by $Y_e = wh^D$. As such, it is straightforward to show that in steady state equilibrium employment is tax-wise only a function of labor taxes (decreasing) and consumption taxes (decreasing).

To operationalize equation (5.5) we begin by running a regression of this equation in order to obtain estimates of its coefficients. To be internally consistent, the regressors $\left(1+\frac{\varepsilon}{1+\varepsilon}\right) d\ln\left(1-\tau_t^l\right)$, $\frac{\varepsilon}{1+\varepsilon} d\ln\left(1+\tau_t^c\right)$, and $\frac{\varepsilon}{1+\varepsilon} d\ln\left(C_t/Y_t\right)$ are generated by using data on τ_t^l, τ_t^c, and C_t/Y_t and the estimates of ε obtained from running the regression of equation (5.2). As such, our (constrained, as implied by the theory) regression of equation (5.5) yields estimates of $\frac{\phi}{\phi+\frac{\tau}{\sigma}}$, $\frac{1}{\phi+\frac{\tau}{\sigma}}$, and $\frac{\zeta}{\phi+\frac{\tau}{\sigma}}$.

Recall that in estimating equation (5.2) we run a panel with all countries in our sample since with that equation we are ultimately estimating for ε, which is a preference parameter and therefore should not be assumed to be different across countries. However, in estimating equation (5.5) we are guided by the fact that there are well-known differences between the relative flexibility of European versus U.S. labor markets (see, for instance, Llosa et al., 2014). To example, consider the OECD's indicators of employment protection legislation (indices that go from 0, which implies least employment restrictions, to 6, which implies most employment restrictions)[14]. In our European sample, the average index for: protection of permanent workers against individual and collective dismissal is 2.47; the average protection of permanent workers against individual dismissal is 2.16; specific requirements for collective dismissal is 3.24; and regulation on temporary forms of employment is 1.91. In contrast, in the US these figures are, respectively: 1.17 (over 50 percent lower than in Europe); 0.49 (nearly 80 percent lower than in Europe); 2.88 (over 10 percent lower than in Europe); and 0.33 (over 80 percent lower than in Europe).

Importantly, note that in equation (5.5) all parameters to be estimated can be interpreted as reflecting, in a reduced form way, economic factors related to labor market rigidities. Indeed, a relatively higher value of ϕ means that it is more costly for firms to adjust employment, a relatively higher value of τ/σ means that relative returns to search decrease at a faster rate, and, assuming for concreteness that ζ is negative, a lower ζ means that higher productivity lessens employment adjustment costs by a relatively lower amount. In line with the evidence on labor market rigidities, there is no reason to expect that these parameters would be the same in Europe as in the US. As such, in estimating equation (5.5) we run a panel for Europe and a separate regression for the US.

Given the estimated parameter values from running the regressions of equation (5.5), we predict a level employment series for *each country* in our sample as follows. Let x_i denote empirical values for country i's variable x, $\hat{x}_i$ denote predicted values for country i of variable x, and $\hat{\beta}$ denote the applicable vector of parameter estimates from running the regressions of equation (5.5). First, we predict country i's employment growth between 1961 and 1962 (the first growth rate that we can predict since our data start in 1960 but a regressor is the growth rate of employment between 1960 and 1961) using the following

[14] https://www.oecd.org/employment/emp/oecdindicatorsofemploymentprotection.htm

equation:

$$d\widehat{\ln(e_{i,t})} = \hat{\beta}' \times \left[d\ln(e_{i,t-1}), d\ln\left(1-\tau^l_{i,t}\right), d\ln\left(1+\tau^c_{i,t}\right), d\ln\left(\frac{C_{i,t}}{Y_{i,t}}\right), d\ln\left(1-\tau^k_{i,t}\right), d\ln(Z_{i,t}) \right],$$

where $t = 1962$. Second, we use this predicted growth rate of employment for 1961-62 in the prediction of the growth rate of employment between 1962-1963, and so on recursively forward, so that:

$$d\widehat{\ln(e_{i,t})} = \hat{\beta}' \times \left[d\widehat{\ln(e_{i,t-1})}, d\ln\left(1-\tau^l_{i,t}\right), d\ln\left(1+\tau^c_{i,t}\right), d\ln\left(\frac{C_{i,t}}{Y_{i,t}}\right), d\ln\left(1-\tau^k_{i,t}\right), d\ln(Z_{i,t}) \right]$$

for all $t \geq 1962$. *Thus, in effect, the only empirical employment growth rate we use is the starting value for 1960-61: all growth rates that follow are entirely endogenous.* Third, once we have the full series of predicted employment growth rates from 1962 onwards, we use the empirical level of employment in 1961 along with the model-predicted growth rate between 1961-62 to arrive at the model's first predicted level value of employment: $\hat{e}_{i,1962} = e_{i,1961} \cdot (1 + d\widehat{\ln(e_{i,1962})})$. We generate the remaining level values of employment by using the previous period's predicted employment levels. In other words, for $t > 1962$, $\hat{e}_{i,t} = \hat{e}_{i,t-1} \cdot (1 + d\widehat{\ln(e_{i,t})})$.

Akin to our DLM model's hours worked per worker predictions, the (level) employment series predicted by our DLM model is rescaled to have the same mean as the empirical employment series on a country-by-country basis. Of note, we obtain our DLM model's predicted equilibrium employment series using its dynamic version since in our data no two adjacent employment figures are the same. In other words, in taking our model to the data the implication is that employment adjustment costs were endured in every period.

5.4 CLM Model

We operationalize a representative version of the CLM model, in particular, the one used in Shimer (2009), in order to compare results from this model regarding H to the results from our DLM model. This version of the CLM model yields tax-inclusive results that are representative of related literature: a successful prediction of hours worked per population in Europe and highly counterfactual results for the US.

In the CLM model the production function is the same as in our DLM model, and the household's instantaneous utility is

$$\ln(C_t) - \frac{\varepsilon}{1+\varepsilon} (H_t)^{\frac{\varepsilon}{1+\varepsilon}},$$

so in this case ε is the Frisch elasticity of the supply of hours worked per *population* (in contrast to the

DLM model, where ε is the Frisch elasticity of the supply of hours worked per *worker*). Of course, all taxes we consider in our DLM model remain the same in the CLM model, and in the CLM model the household's budget constraint is the same as in our DLM model.

All told, as shown in the Appendix, the following is the single equilibrium labor market condition that arises in the CLM model:

$$H_t = \left[\left(\frac{1-\alpha}{\gamma} \right) \left(\frac{1-\tau_t^l}{1+\tau_t^c} \right) \left(\frac{C_t}{Y_t} \right)^{-1} \right]^{\frac{\varepsilon}{1+\varepsilon}}. \tag{5.6}$$

(recall that the CLM model does not distinguish between the different margins of labor). Importantly, note that the right-hand side of equations and (5.6) are the same (therefore, the intuition behind equation (5.6) is entirely analogous to the intuition behind equation (5.1)). Of note, this means that per our DLM model the CLM model's equation for equilibrium H is, in fact, an equation for equilibrium h. Of course, in our DLM model equation (5.1) would only be an equation for H if $e = 1$.

In abstract terms it is straightforward to show that our DLM model's equilibrium condition for hours worked per worker is

$$h_t = G^{-1} \left(\lambda_t \left(1 - \tau_t^l \right) Y_{h,t} h_t \right)$$

and the CLM model's equilibrium condition for hours worked per population is

$$H_t = G^{-1} \left(\lambda_t \left(1 - \tau_t^l \right) Y_{H,t} H_t \right).$$

With perfect substitutability of e and h in production, then $Y_{h,t} h_t = Y_{H,t} H_t$. Therefore, the fact that our DLM model's equation for equilibrium h is the same as that of the CLM model for equilibrium H is not functional form dependent.

To operationalize equation (5.6) we first run a constrained panel regression of this equation in growth rates, i.e., of

$$d \ln H_t = \left[\begin{array}{c} \frac{\varepsilon}{1+\varepsilon} d \ln \left(1 - \tau_t^l \right) - \frac{\varepsilon}{1+\varepsilon} d \ln \left(1 + \tau_t^c \right) \\ - \frac{\varepsilon}{1+\varepsilon} d \ln \left(C_t / Y_t \right) \end{array} \right]. \tag{5.7}$$

This regression gives us a cross-country estimate of $\frac{\varepsilon}{1+\varepsilon}$ and, therefore, of ε, which is the CLM model's Frisch elasticity of hours worked per *population* (recall again that in our DLM model ε is the Frisch elasticity of hours worked per *worker*). Then, in operationalizing the CLM models equation for H we proceed in the same fashion, as described earlier, as we operationalize our DLM model's equation for h.

5.5 Data

Given our focus on the trend behavior of labor market variables, our analysis makes use of data at yearly frequency. The countries in our sample are those for which, in line with related literature, there is extensive time series data on taxes. These countries are the United States and the following eleven European countries: Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Spain, Sweden, Switzerland, and the United Kingdom. Our analysis spans the years 1960 - 2014, since at the time of writing this paper those are the years over which tax data are available for these twelve countries.

To operationalize our theory, we need data on consumption, output, the working-age population, work hours, employment, total factor productivity and, of course, taxes. Following related literature, we use data on consumption, output, and total factor productivity from the Penn World Tables (Feenstra et al., 2015), which are hosted by the Groningen Growth and Development Center.[15] Data on the working-age population (ages 15-64) come from the United Nations[16], while data on aggregate work hours and aggregate employment come from the Conference Board's Total Economy Database.[17]

In line with related literature, our main tax data are average taxes from McDaniel (2007). These data are at yearly frequency and publicly available from the author's website.[18] Of note, these tax series are calculated from national accounts data using procedures similar to Mendoza et al. (1994) and Carey and Rabesona (2002). In representative agent contexts, Mendoza, Razin, and Tesar (1994) suggest that average tax rates derived from national accounts are a useful proxy for marginal tax rates faced by a representative agent. Importantly, McDaniel (2007) compares her derived average taxes with marginal taxes and shows that these series track each other very well. As such, a number of studies have used the McDaniel (2007) tax data, including, among others, Karabarbounis (2014), Ragan (2013), and Epstein et al. (2016).

In brief, the McDaniel (2007) taxes are constructed as follows. The capital tax is the ratio of government capital-tax revenue to capital taxable income. Government capital-tax revenue is capital-tax revenue collected from households plus direct corporate taxes plus the fraction of production and import taxes that are property taxes paid by entities other than households. Taxable capital income is the gross domestic product minus taxes on production and import subsidies, weighted by the economy's capital share in production, minus gross operating surplus earned by the government. The consumption tax

[15] These data are available for download at https://www.rug.nl/ggdc/productivity/pwt/. Regarding TFP, these data feature, for each country, a TFP index, which is TFP relative to the United States in each period. Of course, then, U.S. TFP is normalized to 1 in all periods. Therefore, we construct U.S. TFP using a standard Solow residual appraoch and then use the Penn World Tables TFP indices to back out implied TFP levels for all other countries in our sample.

[16] These data are publicly available at https://esa.un.org/unpd/wpp/Download/Standard/Population/

[17] These data can be found at https://www.conference-board.org/data/economydatabase/ While these data are publicly available, at the time of writing this paper accessing the data requires creating an account, which is free of charge.

[18] http://www.caramcdaniel.com/

is the ratio of government tax revenue from consumption divided by total taxable consumption expenditures (household final consumption expenditure minus revenue collected from taxes on consumption expenditure).

Government revenue from investment comes from, among other sources, taxes on goods and services, excise taxes, import duties, and property taxes. The investment tax is, therefore, the ratio of tax revenue from investment to total investment expenditures. Government labor-income tax revenue comes from the household income tax and taxes on social security. Following Mendoza et al. (1994), McDaniel assumes that the tax rate on labor income is the same as the tax rate on capital income. Then, the labor tax is the ratio of government revenue from labor income to labor income.

In the Appendix, we present some additional results using data on male and female employment rates and male and female population for the United States. These data are publicly available from the Bureau of Labor Statistics (BLS)[19]. Also in the Appendix, as an alternative to the McDaniel U.S. labor tax series, we use data on labor taxes for the US from the National Bureau of Economic Research (NBER).[20] These tax series are available for federal taxes and federal plus state taxes, and are calculated from individual level income data using NBER TAXISM to obtain each individual's marginal tax rate on the next dollar of income earned.[21] These rates are then averaged at the state and federal levels to produce aggregate series.

As such, while the McDaniel (2007) taxes are average taxes, the NBER taxes are average marginal taxes. We highlight results using this alternative labor tax series since, while both the McDaniel and NBER taxes have similar trends, contour-wise the McDaniel tax series does not reflect the Reagan tax reforms (1981 through 1986) while the NBER tax series does. Of note, to the best of our knowledge, comparable average marginal income tax rates are not readily available for the European countries used in our sample. However, focusing on the US for comparison of the different labor tax series is still illustrative.

6 Results

6.1 Parameter Estimates

Table 2 shows results from running constrained panel regressions of equations (5.2) and (5.7) in columns 1 and 2, respectively. All parameter estimates are statistically significant. Moreover, for $d\ln(h)$, the implied estimate of ε is 1.44 (in this case the Frisch elasticity of the supply of hours worked per

[19] https://www.bls.gov/
[20] Tables 1 and 3 from http://users.nber.org/~taxsim/marginal-tax-rates/
[21] http://users.nber.org/~taxsim/

worker), and for $d\ln(H)$ the implied estimate of ε is 2.84 (in this case the Frisch elasticity of hours worked per population).[22] These results are qualitatively in line with, for instance, Chetty et al. (2011), where empirical evidence suggests that the Frisch elasticity of hours worked per population is greater than hours worked per worker, which is intuitive. *As such, results from Table 2 lend validity to our DLM model.*

Table 2: estimates of labor elasticities[23]

	$d\ln(h)$	$d\ln(H)$
$d\ln(C/Y)$	-0.59***	-0.74***
	(0.02)	(0.03)
$d\ln(1+\tau^c)$	-0.59***	-0.74***
	(0.02)	(0.03)
$d\ln(1-\tau^l)$	0.59***	-0.74***
	(0.02)	(0.03)
Fixed effects	yes	yes
Obs.	660	660
Implied ε	1.44	2.84

Table 3 shows results from running the constrained regression of equation (5.5) in panel form for Europe (first column) and individually for the US (second column). All parameter estimates are statistically significant. Moreover, the estimated coefficients imply that the value of $\phi + \tau/\sigma$ is 11.11 in Europe and 4.16 in the US. Also, the implied value ϕ is 5.56 in Europe and 1.78 in the US. This means that the elasticity of employment adjustment costs with respect to the relative size of adjustment is over twice as high in Europe compared to the US. The implied value of τ/σ in Europe is 5.55 and 2.38 in the US. Recall that a relatively higher value of τ/σ means that returns to search decrease at a faster rate, so our results suggest these returns decreasing at a rate over twice as high in Europe compared to the US. Finally, the implied value of ζ is -0.44 in Europe and -0.79 in the US. Since ζ is negative, this means that when productivity is higher, employment costs are lower, which is in line with our prior as discussed earlier. Moreover, the fact that the value of ζ in the US is less than its value in Europe implies that higher productivity decreases employment adjustment costs by less in Europe than the US.

All told, we speculated that parameters in our DLM model's dynamic equilibrium employment reflect underlying structural factors related to labor market rigidities. Results from Table 3 are indeed in line with this, as the implied rigidities from our results are consistent with these rigidities being substantially greater in Europe compared to the US, which is in line with empirical evidence that studies these rigidities

[22] Given our functional form assumptions, it is straightforward to show that in our DLM model the Frisch elasticity of employment supply is $\frac{\sigma}{\tau-\sigma}$, so that our model's Frisch elasticity of hours worked per population is $\varepsilon + \frac{\sigma}{\tau-\sigma}$ (of course, $\tau > \sigma$ by assumption).

[23] Notes: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, and population data are from the UN. Construction of model results use tax data from McDaniel (2007), and consumption and output data are from the Penn Word Tables.

explicitly. *Taken together, these results lend further validity to our DLM model.*

Table 3: estimates of employment equation[24]

	Europe	United States
	$d\ln(e)$	$d\ln(e)$
$d\ln(e_{-1})$	0.50***	0.43**
	(0.05)	(0.19)
$d\ln(TFP)$	0.04**	0.19**
	(0.02)	(0.08)
$d\ln(C/Y)$	-0.09**	-0.24**
	(0.03)	(0.09)
$d\ln(1+\tau^c)$	-0.09**	-0.24**
	(0.03)	(0.09)
$d\ln(1-\tau^k)$	-0.09**	-0.24**
	(0.03)	(0.09)
$d\ln(1+\tau^l)$	0.09**	0.24**
	(0.03)	(0.09)
Fixed effects	yes	
Obs.	583	53
Implied $\phi+\tau/\sigma$	11.11	4.16
Implied ϕ	5.56	1.78
Implied ζ	-0.44	-0.79

6.2 Cross-Country Relative Performance

In our DLM model, predicted hours worked per *population* for each country are constructed as the product of our DLM model's predictions of each country's hours worked per *worker* and employment-population ratio. In contrast, in the CLM framework hours worked per *population* are the only labor market variable delivered by the model. Of course, a key issue related to our research is understanding the extent to which our DLM model does relatively better or worse at predicting hours worked per *population* across countries compared to the CLM model.

To get at this assessment: let $\widehat{H}_{i,t}$ denote the growth rate of empirical hours worked per population between period $t-1$ and t for country i in our sample; let $\widehat{H}_{i,t}^{DLM}$ denote our DLM model's prediction for this variable; and let $\widehat{H}_{i,t}^{CLM}$, denote the CLM model's prediction for it. The metrics $\widehat{SSD}_i$ and $\widehat{SAD}_i$, defined below, then summarize the performance across time, in relative terms, of the two models for each country i. Specifically, $\widehat{SSD}_i$ and $\widehat{SAD}_i$ are defined as

$$\widehat{SSD}_i = \frac{\sum_t \left(\widehat{H}_{i,t}^{DLM} - \widehat{H}_{i,t}\right)^2}{\sum_t \left(\widehat{H}_{i,t}^{CLM} - \widehat{H}_{i,t}\right)^2}$$

[24] Notes: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, and population data are from the UN. Construction of model results use tax data from McDaniel (2007), and output, consumption and productivity data from the Penn Word Tables.

and

$$\widehat{SAD}_i = \frac{\sum_t \left| \widehat{H}^{DLM}_{i,t} - \widehat{H}_{i,t} \right|}{\sum_t \left| \widehat{H}^{CLM}_{i,t} - \widehat{H}_{i,t} \right|}$$

for each country i. As such, $\widehat{SSD}_i$ is the ratio of sum-of-squares of each model's prediction relative to empirical work hours, and $\widehat{SAD}_i$ shows the ratio of sum-of-absolute-deviations of each model's prediction relative to empirical work hours.

Then, for country i consider the numbers $100 \cdot (1 - \widehat{SSD}_i) \equiv \widetilde{SSD}_i$ and $100 \cdot (1 - \widehat{SAD}_i) \equiv \widetilde{SAD}_i$. This means that for $z_i \in \{\widetilde{SSD}_i, \widetilde{SAD}_i\}$ numbers $z_i > 0$ denote that our DLM does z_i percent better than the CLM model at predicting the growth rate of hours worked *per population*, while numbers $z_i < 0$ denote that our DLM does z_i percent worse than the CLM model at predicting the growth rate of hours worked *per population*. Of course, $z_i = 1$ implies that both models do equally well.

Table 4 shows $\widetilde{SSD}_i$ and $\widetilde{SAD}_i$ for all countries in our sample. Results show that, on net, our DLM performs much better compared to the CLM model at predicting the growth rates of empirical hours worked per population using both the $\widetilde{SSD}_i$ metric and the $\widetilde{SAD}_i$ metric, except for the case of France, in which case our DLM model does a tad worse using the $\widetilde{SAD}_i$ metric and about the same using the $\widetilde{SSD}_i$ metric. Per the $\widetilde{SSD}_i$ metric our DLM model's improvements relative to the CLM model range from 0.63 percent for France to 43 percent in the UK. Per the $\widetilde{SAD}_i$ metric our DLM model's improvements relative to the CLM model range from 4 percent in Austria to 28 percent in the UK. Importantly, our DLM model's improvements relative to the CLM model are 31 percent and 23 percent, respectively, using the $\widetilde{SSD}_i$ and $\widetilde{SAD}_i$ metrics. On average, on the basis of the $\widetilde{SSD}_i$ metric our DLM model predicts hours worked per population by about 20 percent better compared to the CLM model, and on the basis of the $\widetilde{SAD}_i$ metric our DLM model predicts hours worked per population about 12 percent better compared to the CLM model.

On the basis of these metrics, we conclude that, compared to the CLM framework, our DLM model is an important step forward in understanding and predicting the behavior of hours worked per population.

Table 4: Comparison of CLM and DLM model performance[25]

(in %)	Austria	Belgium	Finland	France	Germany	Italy
$\widetilde{SSD}_i$	16.26	32.64	23.64	0.63	25.26	11.91
$\widetilde{SAD}_i$	4.21	26.5	7.89	−2.43	11.50	9.03
(in %)	Netherlands	Spain	Sweden	Switzerland	UK	US
$\widetilde{SSD}_i$	18.24	11.43	10.72	12.64	43.11	31.49
$\widetilde{SAD}_i$	10.62	7.83	8.19	7.69	28.15	22.7

[25] Notes: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, and population data are from the UN. Construction of model results use tax data from McDaniel (2007), and output, consumption and productivity data from the Penn Word Tables.

6.3 Trend Analysis

For all cases that follow regarding the variable "Europe," all empirical data are simple averages of these data across the eleven European countries in our sample. In particular, regarding predictions of our DLM model, all hours worked per worker series are simple averages of our model's predictions of hours worked per worker for each European country in our sample, and similarly for employment. For our DLM model's predictions of hours worked per population, we obtain hours worked per population for each country in our sample by multiplying each country's predicted hours worked per worker and employment-population ratio. Then, the "Europe" series for hours worked per population is the simple average of our eleven European countries' country-specific hours worked per population. "Europe" hours worked per population are constructed in the same way in the case of the CLM model (again, predicting H separately for each country in our sample). Of note, using a host of alternatives to simple averaging, among others, GDP weighted averaging, yields European series that are for all purposes identical to those constructed using simple averaging.

6.3.1 DLM Model Hours Worked per Worker

The top panel of Figure 3 assesses our DLM model's results regarding the trend in U.S. hours worked per worker. In this figure, empirical hours worked per worker are denoted by *h Empirical* and hours worked per worker predicted by our DLM model are denoted by *h DLM.* Clearly, results from our DLM model track empirical hours worked per worker very well.

To get at our DLM model's implied driving forces behind the U.S. trend in hours worked per worker, the top panel of Figure 3 also shows our DLM model results by keeping each of the ingredients that go into our DLM model's equilibrium equation for hours worked per worker (see equation (5.1)) fixed one at a time. In particular, in the figure *h DLM Y/C fix* denotes our model's predictions holding constant the consumption-output ratio, *h DLM cons tax fix* shows results holding constant the consumption tax, and *h DLM lab tax fix* shows results holding constant the labor tax. The model suggests a minimum impact of consumption taxes, and that: changes in the consumption-output ratio are associated with hours worked per worker being about 8 percent lower in 2014 compared to what they would have been otherwise; and changes in labor taxes are associated with hours worked per worker being about 6 percent lower in 2014 compared to what they would have been otherwise. The combined drag of these associated tax changes on hours worked per worker is nontrivial: 14 percent.

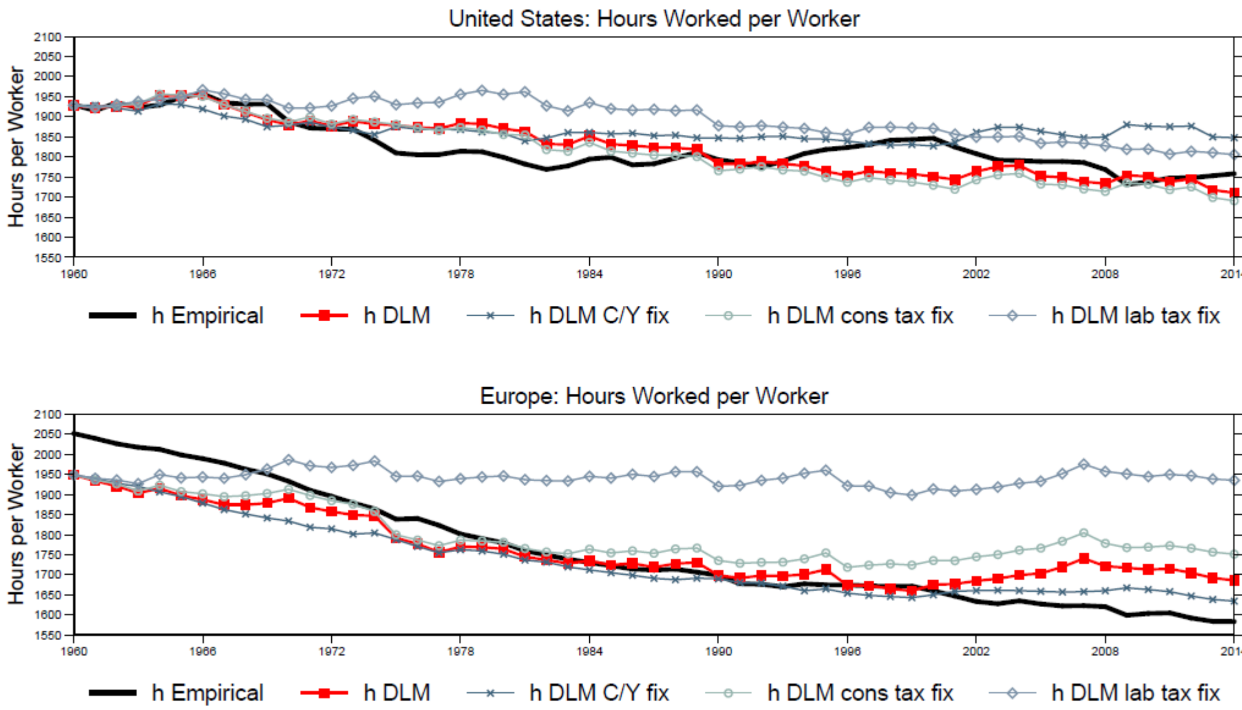

Figure 3: Empirical hours worked per population and DLM model predictions for Europe (bottom panel) and the US (top panel).[26]

These results reflect the behavior of aggregate U.S. empirical series shown in the top panel of Figure 4. This figure highlights, in particular, that relative to their (normalized) values in 1960: the U.S. consumption-output ratio (C/Y) trended up and was about 15 percent higher in 2014; the return-adjusted U.S. consumption tax rate ($1+\tau^C$, *1+C tax*) was unchanged; and the return-adjusted U.S. labor tax rate ($1-\tau^L$, *1-L tax*) was about 10 percent lower.

[26] Notes: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, and population data are from the UN. Construction of model results use tax data from McDaniel (2007), and output and consumption data from the Penn Word Tables.

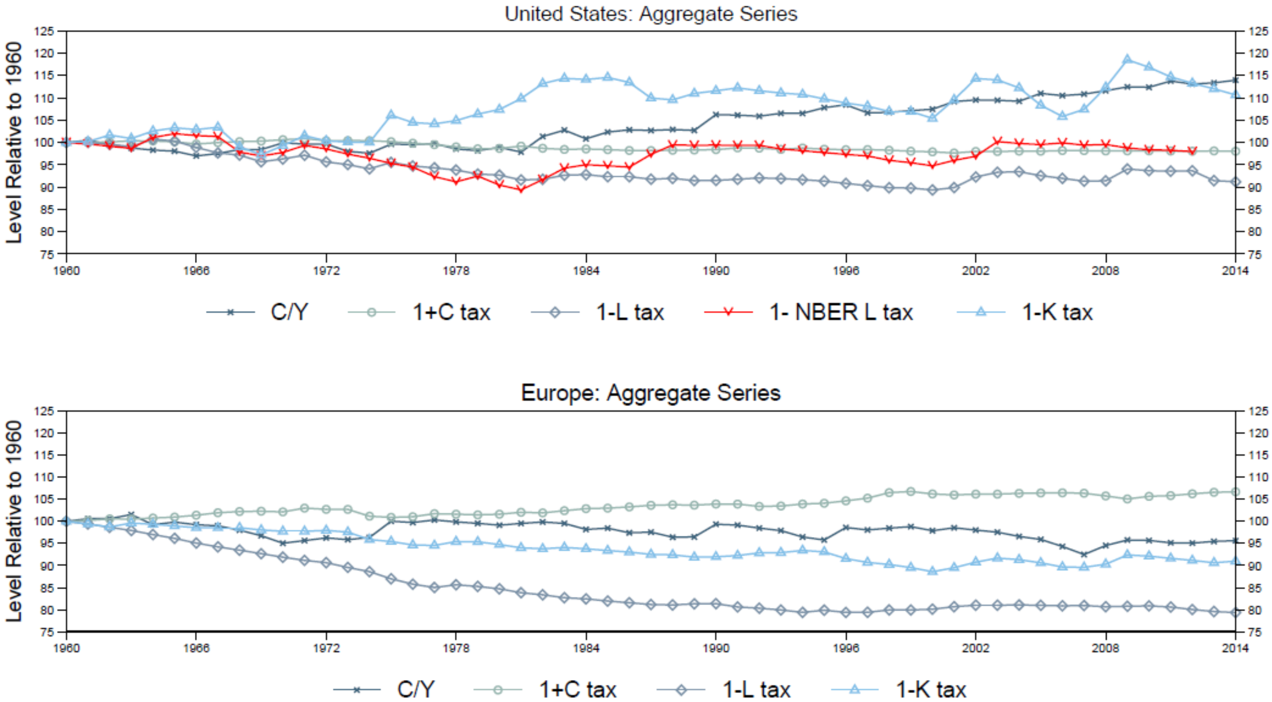

Figure 4: Empirical macroeconomic series for Europe (bottom panel) and the US (top panel).[27]

The bottom panel of Figure 3 assesses our DLM model's results regarding the trend in Europe of hours worked per worker. Again, our DLM model tracks empirical hours worked per worker very well. The decomposition of driving forces reveals that labor taxes are the major variable driving the downward trend in hours worked per worker: changes in labor taxes are associated with hours worked per worker being about 14 percent lower in 2014 compared to what they would have been otherwise. Secondary roles are played by changes in consumption taxes as of the late 1980s, which are associated with hours worked per worker being about 3 percent lower in 2014 compared to what they would have been otherwise, and changes in the consumption-output ratio as of the mid-1990s, which are associated with hours worked per worker being about 6 percent higher in 2014 compared to what they would have been otherwise. Note that, jointly, the drag of the associated consumption and labor changes on hours worked per worker is nearly 18 percent.

These results reflect the behavior of aggregate Europe empirical series shown in the bottom panel of Figure 4. This figure highlights, in particular, that relative to their (normalized) values in 1960: the Europe consumption-output ratio declined since the early 1990s and was about 5 percent lower in

[27] Notes: data on total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, population data are from the UN, tax data from McDaniel (2007), and output and consumption from the Penn Word Tables.

2014; the return-adjusted Europe consumption tax rate trended up since the early 1980s and was about 15 percent higher in 2014; and the return-adjusted Europe labor tax rate trended down through the mid-1990s and was had dropped by about 20 percent in 2014.

6.3.2 DLM Model Employment

The top panel of Figure 5 assesses our DLM model's results regarding the trend in the U.S. employment-population ratio. In this figure, the empirical employment-population ratio is denoted by *e Empirical* and the employment-population ratio predicted by our DLM model is denoted by *e DLM.* Our DLM model results track empirical employment very well, with a slight miss in the empirical hump between the early 1980s and the early 2000s. To get at our DLM model's implied driving forces behind the U.S. trend in employment the top panel of Figure 5 shows a similar exercise as we did earlier for hours worked per worker. In particular, this panel also shows our DLM model's results by keeping each of the ingredients that go into our DLM model's equilibrium equation for employment fixed one at a time (see equation (5.5)). All legends/series are the analog of those in the top panel of Figure 3, and *e DLM cap tax fix* holds constant the capital tax and *e DLM TFP fix* holds constant total factor productivity.

Clearly, the main factor associated with the trend in U.S. employment is TFP, and consumption taxes play no role. Changes in TFP are associated with employment in 2014 being about 18 percent higher compared to what it would have been otherwise. That said, the consumption-output ratio, labor taxes, and capital taxes share a fairly equal role in their individual relationships to employment. Indeed, changes in each of these variables are associated with employment in 2014 being lower by 6 percent compared to what it would have been otherwise. These results are nontrivial, since *the drag of associated consumption taxes and capital taxes on employment is each a third of the positive association of TFP with employment*, and their joint drag—12 percent—is a third of the positive association of TFP with employment. Of note, the importance of capital taxes that emerges from our analysis is of particular relevance given recent work on the evolution of wealth and income in advanced economies (see, most prominently, Piketty and Zucman, 2014, and subsequent work).

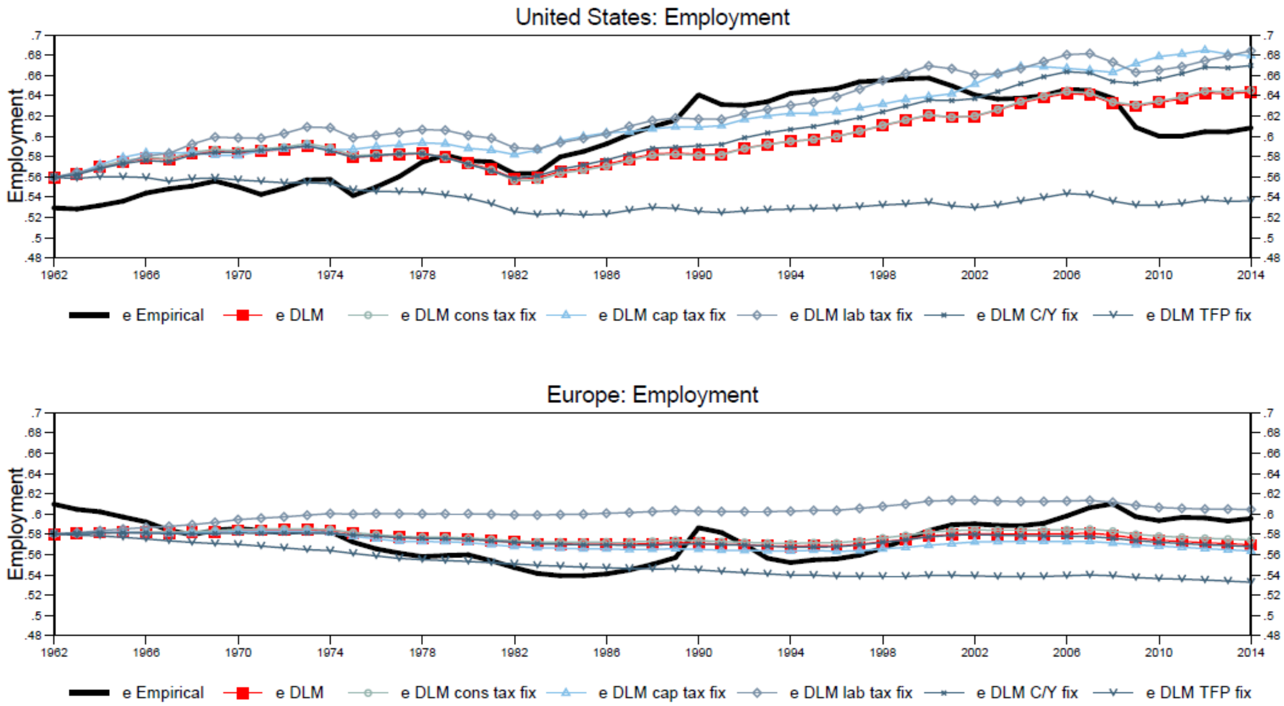

Figure 5: Empirical employment-population ratio and DLM model predictions for Europe (bottom panel) and the US (top panel).[28]

These results are in line with the aggregate U.S. empirical series shown in the top panel of Figure 4, where all series plotted were discussed earlier except for the capital tax ($1-\tau^k$, *1-K tax*), which as shown in the figure trended up throughout the sample and in 2014 were about 10 percent higher compared to 1960 (*recall that our DLM model suggests that the positive association between contemporaneous employment and capital taxes observed in the data and reproduced by the model is not causal; instead, it is a result of forward looking employment-demand decisions by which higher future capital taxes but downward pressure on employment growth*). Regarding TFP and its impact on employment, Figure 6 plots this series for both Europe and the US (the Europe series is relative to the US, so in 1960 TFP in Europe was about 30 percent lower than in the US). Relative to 1960, in 2014 U.S. TFP had nearly doubled.

[28]Notes: employment is from the OECD, and population data are from the United Nations. Construction of model results use tax data from McDaniel (2007), and output, consumption and productivity data from the Penn Word Tables.

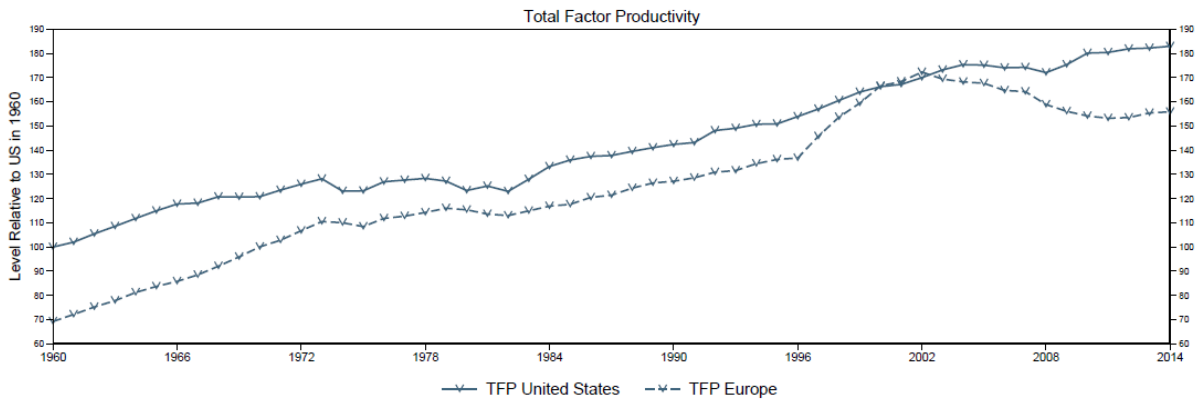

Figure 6: Total factor productivity in Europe and the US.[29]

Given that the rise in U.S. TFP is the main factor associated with the upward trend in U.S. employment then, in light of Figure 6, which shows that in Europe TFP increased on net throughout the sample period, it is natural to expect that our DLM model would also predict increasing employment in Europe in contrast to empirical employment being flat. However, results in the bottom panel of Figure 5 show that our DLM model tracks the behavior of employment in Europe very well. A decomposition of our DLM model's driving forces behind this result delivers a stark message. In Europe, the positive association of gains in TFP with employment have been entirely offset by the negative association of labor taxes with employment. In particular, changes in TFP are such that employment in Europe in 2014 was about 6 percent higher compared to what it would have been otherwise, and changes in labor taxes are associated with employment in Europe being over 5 percent lower in 2014 compared to what it would have been otherwise. These results, and the fact that all other variables have little association with the behavior of European employment, are in line with the empirical evidence in Figure 6 and the bottom panel of Figure 5.

6.3.3 Hours Worked per Population

Bringing the two margins of labor together, Figure 7 shows our DLM model's predictions for hours worked per population (the US in the top panel, and Europe in the bottom panel), which, again, are constructed on a country-by-country basis by predicting h and e for each country in our sample and then calculating their product. Our DLM model's predictions (H DLM) for Europe track the empirical series (H *empirical*) very well. In addition, our DLM model's predictions for the US successfully gets,

[29] Notes: data are from the Penn World tables.

contour-wise, at the flat V-shape of empirical U.S. hours worked per population. *As such, our DLM model is an important step forward in resolving the CLM framework's implied U.S. tax puzzle.* Given our model's fairly small miss on the employment side regarding the U.S. empirical hump in employment, it is not surprising that our DLM model misses part of the hump in U.S. empirical hours worked per population as well (early 1980s through early 2000s). That said, it is well known that a large portion of this hump is driven by trend increases in female labor force participation. Since our DLM model abstracts from demographics, we conclude that in predicting hours worked per population, our DLM model is extremely successful and validates the use of representative-agent Walrasian frameworks in predicting the behavior of this variable—*importantly, though, per our DLM model by first predicting the behavior of the two margins of labor separately.*

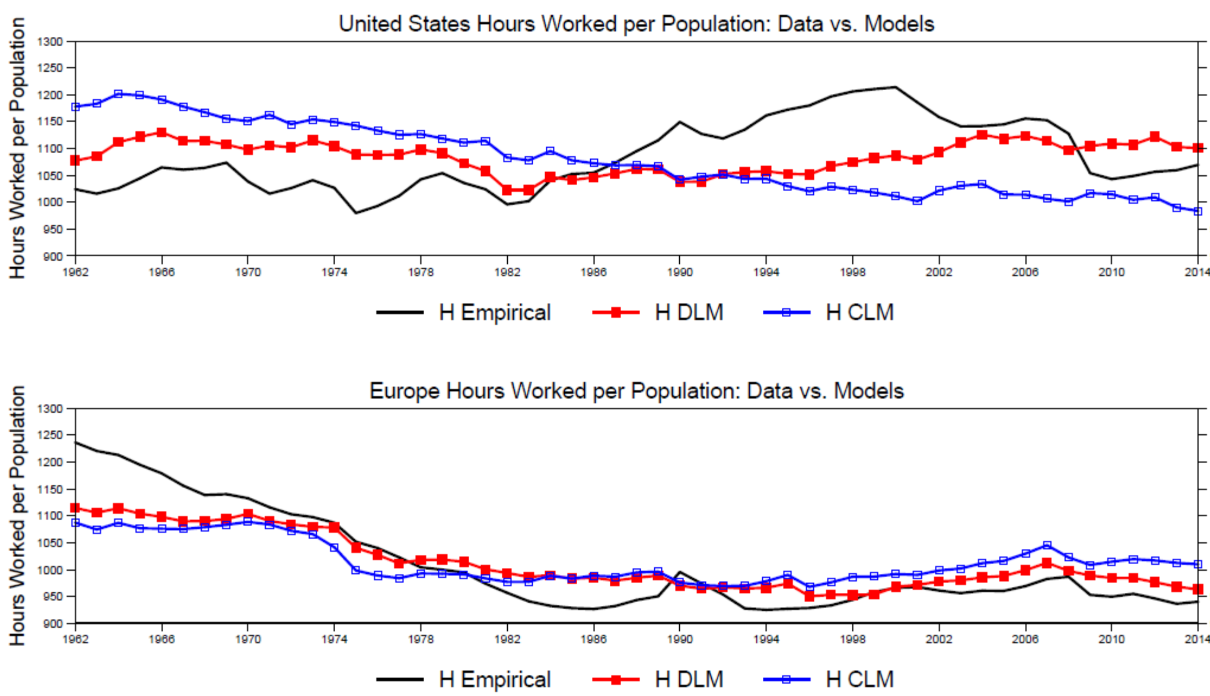

Figure 7: Empirical hours worked per population and DLM and CLM model predictions for Europe (bottom panel) and the US (top panel).[30]

The Appendix shows the extent to which demographics matter for the U.S. hump, and also that the U.S. labor-tax series from McDaniel (2007) does not reflect, contour wise, the Reagan tax reforms (1981 through 1986). Importantly, even though the McDaniel (2007) labor tax series and the NBER

[30]Notes: total work hours are from the Conference Board's Total Economy Database, employment is from the OECD, and population data are from the United Nations. Construction of model results use tax data from McDaniel (2007), and output, consumption and productivity data from the Penn Word Tables.

labor tax series (noted earlier in the data section) have similar trends, the NBER tax series indeed gets at the Reagan tax reforms. In the Appendix, we show that using the NBER tax series instead of the McDaniel tax series, our DLM model predictions for U.S. employment and hours worked per population continue to track their respective empirical counterparts in a much tighter way, *in particular getting at U.S. empirical humps in each of these variables much better and leaving a minimum gap between model predictions and data.* This further validates our DLM model.

In the Appendix, we also show that an extension of our DLM model that accounted for differences in trends in employment by gender would for all purposes close the small remaining gap (fairly small in our benchmark results, and much smaller using the NBER labor tax series instead of the McDaniel, 2007, labor tax series) that our DLM model predictions exhibit relative to the U.S. empirical humps. As noted in the Related Literature section, research such as Cociuba and Ueberfeldt (2015) develop models that account for gender and marital status heterogeneity, and shows that accounting for these factors can contribute to resolving the U.S. tax puzzle. That said, the extent to which our DLM successfully gets at the behavior of hours worked per population in the US implies that the U.S. tax puzzle can be well addressed within a representative-agent modeling framework, that is, *within the same context in which the U.S. tax puzzle is originally observed.* Again, it follows that our work stresses the ongoing validity of representative-agent Walrasian frameworks for analyzing and understanding aggregate labor markets.

Returning to Figure 7, this figure also recaps the CLM model's (tax-inclusive predictions) for hours worked per population ($H\ CLM$) that we showed in Figure 1. (Of course, we do not decompose the driving forces behind the CLM model's prediction for hours worked per population since, trivially, they are the same as the driving forces behind our DLM model's prediction for hours worked per worker—recall that the right-hand side of equations (5.6) and (5.1) are identical.) Comparing these results to those for hours worked per worker in Figure 3 strongly suggest that it is indeed the case that the CLM model is in fact predicting hours worked per *worker* instead of, as should be the case per the CLM theory, hours worked per *population. This suggests the following: the CLM model is never quite successful in predicting hours worked per population.* Indeed, when hours worked per population are driven by hours worked per worker, as in the case of Europe, the CLM model can give the impression of correctly predicting hours worked per population, since the CLM model is in fact prediction hours worked per worker. In contrast, when empirical hours worked per population are driven by employment, then the CLM model can give the impression of failing. In the Appendix we also use the U.S. NBER labor-tax series in the CLM model instead of the McDaniel (2007) labor taxes, *and we show that the model's counterfactual results for the US are not overturned.*

On net, these results suggest the following. First, that our DLM model is correct in explicitly delivering the result that the CLM model's equation for equilibrium hours worked per population is in fact an equilibrium hours worked per worker. Second, that *in the CLM model a majority of the labor wedge that remains after accounting for taxes is employment itself.*

7 Conclusions

The extent to which employment is a key driver of fluctuations in aggregate work hours at business cycle frequency is well known. In contrast, from a theoretical perspective, the extent to which disentangling the extensive and intensive margins of labor is important for making sense of the *long-run/trend* behavior of aggregate work hours is less understood. The objective of this paper is to help bridge this gap. To do so, we develop a representative agent Walrasian macroeconomic model whose key features are that the household must expend resources to secure employment and that the firm faces employment-adjustment costs. Our model is a tractable and intuitive extension of standard macroeconomic models (models that do not disentangle the two margins of labor, hours worked per *worker* and the employment-population ratio, and therefore only yield predictions for hours worked per *population*).

The analysis of our model is tax inclusive. This is motivated by the fact that (1.) the *long-run/trend* labor wedge (the extent to which the marginal rate of substitution of consumption for leisure differs from the marginal product of labor) that emerges from standard macroeconomics models' lack of explanatory power regarding the trend behavior of hours worked per *population* is substantial, and (2.) that taxes, which are a natural candidate for closing this wedge in standard macroeconomic models (*at the long-run/trend level*), indeed do so for European countries, on average, but not for the US. In fact, tax-inclusive standard macroeconomic models generate stark counterfactual predictions for the trend behavior of hours worked per population in the US. We refer to this result as the "U.S. tax puzzle."

Our model can successfully replicate the trend behavior of the two margins of labor and, therefore, of hours worked per population as well, in both Europe *and the US.* It follows that our work is a step forward in resolving the U.S. tax puzzle. This is one and the same with our model suggesting that a large fraction of the long-run labor wedge that remains after standard macroeconomic models incorporate taxes is, in fact, employment itself. Regarding this, of course it is well known that at business cycle frequency standard models fall short of explaining the behavior of hours worked per population given a profound lack of explanatory power of the behavior of employment. However, as we argue, *at the trend level and when taxes are accounted for, the fact that standard models cannot account for the behavior of employment is in fact extremely surprising.*

Importantly, as related to taxes, our work highlights what is, to the best of our knowledge, a new stylized fact: capital taxes are associated with the behavior of hours worked per population via their association with the behavior of employment. However, capital taxes are not associated with the behavior of hours worked per worker. Our model can successfully account for these relationship, which is also a guiding factor behind our modeling strategy.

More generally, given its generality and tractability, our model is relevant for assessing the long run impact of factors beyond taxes, such as labor market reforms, productivity changes, and demographics. Moreover, our model can also speak to a wider set of issues related to aggregate labor markets, such as the long-run neutrality of productivity on employment implied by traditional models of aggregate labor markets. All told, our model highlights the extent to which, in spite of major advances in macroeconomic modeling ranging from search theory to heterogeneous agents and beyond, Walrasian representative agent frameworks continue to be an exceedingly reliable tool for analyzing aggregate labor markets.

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