Should monetary policy care about redistribution? Should fiscal policy care about inflation? Optimal monetary policy with heterogeneous agents

François Le Grand   Alaïs Martin-Baillon   Xavier Ragot*

Abstract

We derive optimal monetary and fiscal policies in heterogeneous-agent economy with nominal frictions and aggregate shocks. This enables us to investigate the redistributive role of optimal monetary policy. We determine the optimal dynamics of nominal interest rate, capital and labor taxes, and public debt. The role of monetary policy is shown to crucially depend on the fiscal tools that are available. When linear taxes on capital and labor are available, then there is no redistributive role for monetary policy. When fiscal tools are incomplete, we identify three new objectives for monetary policy which generate deviations from price stability, and only due to agents heterogeneity. The first objective is an information channel when time-varying capital taxes are not available. The second one is a real-wage objective : Inflation is used to affect the real wage over the business cycle for redistributive purposes. This real-wage channel generates empirically consistent dynamics of public debt. The third objective is a public finance channel, related to the provision of public debt for liquidity needs. We provide analytical and numerical results thanks to an extensive use of the Lagrangian approach to derive optimal policies. This approach is well-suited for monetary policy with heterogeneous agents.

Keywords: Heterogeneous agents, optimal policies, monetary policy.

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1 Introduction

Monetary policy generates redistributive effects through various channels that have been studied in a vast empirical and theoretical literature, reviewed below. However, it is not clear how these channels *should* change the conduct of monetary policy. An option is to consider that monetary policy should take into account these effects to improve welfare, and thus participate in a function usually devoted to fiscal policy. An opposite position is to claim that monetary policy should only focus on monetary goals and let fiscal tools either dampen or strengthen the redistributive effects of monetary policy. However, in both cases, studying the normative implications of monetary policy requires to understand the interactions between monetary and fiscal policies.

The goal of this paper is to study optimal monetary policy with heterogeneous agents, with a relevant set of fiscal instruments. Obviously, in such a setting, redistribution is key aspect of public policies. Following the Bewley literature, we assume incomplete insurance markets for idiosyncratic risks to be the main source of agents heterogeneity. This framework is known to be general enough to generate realistic consumption and wealth distributions. In this setup, we add nominal frictions, modeled as costly price adjustments. This environment has been named HANK following the seminal paper of Kaplan, Moll, and Violante (2018). Thanks to some methodological contribution explained below, we can derive optimal monetary and fiscal policies, with three fiscal instruments: a linear tax on capital income, a linear tax on labor income and issuance of riskless public debt.

First and foremost, we show that an economy, in which all these instruments are available, constitutes a meaningful benchmark. Indeed, when the government can levy resources through both capital and labor taxes, the redistributive effects of monetary policy are shown to be inexistent, after both a technology and a public spending shock. In this case, monetary policy solely aims at ensuring price stability in each period – as in any representative agent economy – and to let fiscal policy alone deal with redistribution. In this sense, there is a perfect dichotomy between the objectives of monetary and fiscal policies. The redistributive role of monetary policy only stems from missing fiscal instruments or, more precisely, from non-optimally time-varying fiscal instruments. Considering various assumptions about the availability of fiscal instruments, we identify three channels through which optimal monetary policy should deviate from price stability for redistributive purposes. Comparing model outcomes to the data could enable us to identify the most relevant framework to think about optimal public policies along the business cycle.

In a first economy, we characterize the optimal redistributive effect of monetary policy when capital taxes are optimal at the steady state, but not time-varying. We find that inflation is volatile only at the impact of an aggregate shock in order to affect the real interest rate. This interest rate channel is only active for one period, because the nominal interest rate is set in advance. Changes in inflation at the date of an aggregate shock is a way to achieve redistribution through the real interest rate, when capital taxes cannot be adjusted. For this reason, we label this effect the *information channel.* It is worth mentioning that the availability of labor-income taxes allows the government to separate the redistributive effect of real wages, coming from *post-tax* wages, and the dynamics of inflation, stemming from *before-tax* wages through the Phillips curve. As a consequence, the government has no incentive to change inflation expectations for redistribution purposes through real wages and inflation expectations remain approximately constant and equal to zero.

This is not the case anymore in the second setup, where we derive optimal monetary policy when labor tax is constant and when the government uses the capital-income tax to balance its budget. As labor taxes do not adjust, monetary policy now induces redistribution through real wages by influencing inflation expectations. Inflation now deviates from price stability.

In our third setup, we derive optimal monetary policy when labor and capital income taxes are constant. Importantly, public debt is a well-defined concept in our economy because markets are incomplete and because only distorting taxes are available (see Aiyagari and McGrattan 1998). Hence, there is a positive amount of public debt which is issued by government for agents to be able to self-insure themselves. When the government uses the inflation rate to affect the real interest rate, this also directly affects the governmental budget. We call this last effect the *public finance effect*.

These effects are identified at a theoretical level, thanks to two methodological contributions. The first one is the use of a truncated representation of incomplete insurance market economies that we apply here to a monetary economy. This theory of the truncation is presented in LeGrand and Ragot (2017). The basic idea of the theory is to design a partial insurance mechanism guaranteeing that heterogeneity only depends on a finite but arbitrarily large number, denoted N, of past consecutive realizations of idiosyncratic risk. As a theoretical outcome, agents having the same idiosyncratic risk history for the previous N periods choose the same consumption and hold the same wealth. The full-fledged Bewley economy corresponds to the case where $N = \infty$ – which means no partial insurance mechanism. The representative agent is also mapped into this setup and corresponds to the case where $N = 0$ – where there is full insurance among agents. The gain of the truncated representation is that the equilibrium features a finite – though possibly arbitrarily large – number of heterogeneous agents. This allows us to use the same tools as in representative agent models. Second, we show that the Lagrangian approach, used in Marcet and Marimon (2011), is particularly well-suited for monetary economies. This allows us to derive first-order conditions in non-linear cases and obtain simple intuition about optimal monetary and fiscal policies.

Finally, we simulate optimal public policies after a public spending shock, with different assumptions about available fiscal instruments. The only empirically relevant case is the one where both monetary tool and labor tax are used to smooth shocks, but not the capital tax. In this case, both public debt and inflation increase after a public spending shock due to the information and real wage channels. The public finance channel plays a minor role in the adjustment.

Related literature. This paper is related to the literature on monetary policy with nominal frictions and heterogeneous agents. This is a vast literature (including the seminal work of Bewley 1980 and 1983). The more recent literature, in which our work is embedded, focuses on sticky prices as the main friction. For instance, Gornemann, Kuester, and Nakajima (2017) McKay, Nakamura, and Steinsson (2016), Kaplan, Moll, and Violante (2018) study the transmission channels of monetary policy in this setup. McKay and Reis (2016) investigate the interaction between monetary and fiscal policies. Auclert (2017) also analyzes the transmission channels of monetary policy with heterogeneous agents. He identifies three transmission channels, which are close but different from the ones we identify in our paper. Indeed, as we focus on optimal policies, we investigate the reasons explaining why monetary policy should handle redistribution.

Regarding normative issues, Bhandari, Evans, Golosov, and Sargent derive optimal monetary policy when the government has access to non-distorting taxes, labor-income tax and public debt. However, their results crucially depend on the presence of lump-sum taxes, which makes public debt irrelevant, and simplifies the computation of optimal policies in their setup. Our truncated approach allows us to dispense with this assumption and to consider a framework where public debt is well-defined. Nuño and Moll (2017) use a continuous-time approach and mean-field games to characterize optimal monetary policies for economies without aggregate shocks. They do not consider additional fiscal tools (or public debt neither). As a consequence, their results can be consider as an upper bound of the redistributive objective of monetary policy.

2 The environment

Time is discrete, indexed by $t \geqslant 0$. The economy is populated by a continuum of agents of size 1, distributed on a segment J following a non-atomic measure ℓ: $J(\ell) = 1$.

2.1 Risk

The only aggregate shock in the model affects technology level in the economy. We denote this risk by $(z_t)_{t \geq 0}$ and we assume that is follows an AR(1) process: $z_t = \rho_z z_{t-1} + u_t^z$ with $\rho_z > 0$ the persistence parameter and the shock u_t^z being a white noise with a normal distribution $\mathcal{N}(0, \sigma_z^2)$. The economy-wide productivity, denoted $(Z_t)_{t\geq 0}$ is assumed to relate to z_t through the following functional form: $Z_t = e^{z_t}$.

In addition of this aggregate shock, agents face an uninsurable idiosyncratic labor productivity shock $e_t \in \{0, 1\}$ that can take two different values. We interpret these two states as employment and unemployment. Employed agents in state $e_t = 1$ have a constant labor productivity equal to 1 and they can freely adjust their supply. Unemployed agents in state $e_t = 0$ have a zero labor productivity. Such agents have no choice about their labor supply and they must supply a fixed quantity of labor δ for home production to obtain δ units of final goods. The individual productivity process $(e_t)_{t\geq 0}$ follows a discrete first-order Markov process where the probability to stay employed is denoted α_t and the probability to remain unemployed is ρ_t. With this notation, the job separation rate is equal $1 - \alpha_t$, while the job finding rate is $1 - \rho_t$. The history of idiosyncratic shocks up to date t is denoted by $e^t = \{e_0, \ldots, e_t\} \in \mathcal{E}^{t+1}$.

2.2 Preferences

In each period, the economy has two goods: a consumption good and labor. Households are expected utility maximizers that rank streams of consumption $(c_t)_{t\geq 0}$ and of labor $(l_t)_{t\geq 0}$ according to a time-separable intertemporal utility function equal to $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$, where $\beta \in (0, 1)$ is a constant discount factor and $U(c, l)$ is an instantaneous utility function. As is standard in this class of models, we focus on the case where U is a Greenwood-Hercowitz-Huffman (GHH) utility function, exhibiting no wealth effect for the labor supply. For any consumption c and labor supply l, the instantaneous utility $U(c, l)$ can be expressed as:

$$U(c, l) = u\left(c - \chi^{-1} \frac{l^{1+1/\varphi}}{1 + 1/\varphi}\right),$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously derivable, increasing, and concave, with $u'(0) = \infty$.

2.3 Production

The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y_t(j))_j$ from different sectors indexed by $j \in [0, 1]$ using a standard Dixit-Stiglitz aggregator. Denoting by ε the elasticity of substitution for the goods belonging to the different sectors, we obtain that the production Y_t can be expressed using a CES aggregation of individual productions:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

For any intermediate good $j \in [0,1]$, the production $y_t(j)$ is realized by a monopolistic firm. The profit maximization for the firm producing the final good implies that its demand for the intermediate good is:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t,$$

where P_t is the price of the consumption good. The zero profit condition of the firm producing the final good implies that the price P_t can be expressed as:

$$P_t = \left(\int_0^1 p_t(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}.$$

Intermediary firms are endowed with a linear production technology and use labor as a sole production factor, such that every unit of labor is transformed into Z_t units of intermediate good. Denoting the amount of labor used in sector j by $\tilde{l}_t(j)$, we obtain that the production of goods j amounts to $Z_t\tilde{l}_t(j)$. At the equilibrium, this production will exactly cover the demand $y_t(j)$ for the good j, that will sold with the real price $p_t(j)/P_t$. The sole production cost will be labor cost. Denoting as $\tilde{w}_t$ the real before-tax wage – that is identical for all firms – the real labor cost for firm j producing $y_t(j) = Z_t\tilde{l}_t(j)$ will be $\tilde{w}_t\tilde{l}_t(j)$ or $\frac{\tilde{w}_t(1-\tau^Y)}{Z_t}y_t(j)$. As is standard, the tax τ^Y will be a labor subsidy to compensate for steady-state distortions.

In addition to the labor cost, intermediate firms face a quadratic price adjustment cost *à la* Rotemberg (1982) when setting their price in the period. The price adjustment cost is proportional to the magnitude of the firm's relative price change, which is in other words the magnitude of the inflation in firm's price. More formally, the adjustment cost can be expressed as $\frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t$, where $\kappa \geq 0$ is a scaling factor. We can thus deduce the real profit, denoted $\Omega_t(j)$, at date t of firm j:

$$\Omega_t(j) = \left(\frac{p_t(j)}{P_t} - \frac{\tilde{w}_t(1-\tau^Y)}{Z_t}\right)\left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t - \frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t - t_t^Y.$$

where t_t^Y is a lump-sum tax financing the subsidy τ^Y. Computing the firm j's intertemporal profit requires to define the firm's pricing kernel. In a heterogeneous agent economy, there is no obvious choice for the pricing kernel. We discuss the reasons and the several options below. For the moment, we assume that the firm's j pricing kernel is independent of its type and we denote the pricing kernel at date t by $\frac{M_t}{M_0}$. With this notation, the firm j's program consisting in choosing the price schedule $(p_t(j))_{t\geq 0}$ maximizing the intertemporal profit at date 0, can be expressed as follows:

$$\max_{(p_t(j))_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty} \frac{M_t}{M_0}\left(\left(\frac{p_t(j)}{P_t} - \frac{\tilde{w}_t(1-\tau_t^Y)}{Z_t}\right)\left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t - \frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t - t_t^Y\right)\right]. \quad (1)$$

Observing that the program (1) is independent of the firm type j, we deduce that in the symmetric equilibrium, all firms will charge the same price: $p_t(j) = P_t$ for all $j \in [0,1]$. Denoting the gross inflation rate as $\Pi_t = \frac{P_{t+1}}{P_t}$, we deduce the following first-order condition for the firm's program:

$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon}{\kappa}(e^{-z_t}\tilde{w}_t(1-\tau_t^Y) - \frac{\varepsilon - 1}{\varepsilon}) + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1)\frac{Y_{t+1}}{Y_t}\frac{\Gamma_{t+1}}{\Gamma_t}, \tag{2}$$

which the equation characterizing the Phillips curve in our environment. We set $\tau^Y = \frac{1}{\varepsilon}$ to obtain an efficient steady-state. It gives:

$$\Omega_t = \left(1 - e^{-z_t}\tilde{w}_t - \frac{\kappa}{2}\pi_t^2\right) Y_t.$$

Choosing the pricing kernel. As explained above, in a heterogeneous agent economy, there is no straightforward choice for the firm's pricing kernel. In a representative agent economy, the unique agent's is necessarily the firm's owner and there is no possible dispute about the firm's pricing kernel, which has to be the representative agent's pricing kernel. The choice that we make is to assume that the firm pricing kernel is defined based on the average marginal utility among agents. We provide a formulation in equation (17) below in the truncated economy. Quantitatively, the choice of the pricing kernel seems to have a small effect.

2.4 Assets

Agents have the possibility to trade two assets. The first one is public debt, denoted by B. Public debt is issued by the government. The second asset is private debt, which is issued by households. Both assets are assumed to be perfect substitutes. In particular, we assume the existence of an enforcement technology that prevents agents from defaulting on their debt. Both private and public are assumed to be exempt of default risk. We denote by i_{t-1} the nominal interest rate, common to both assets, prevailing between period $t-1$ and period t. Both assets being substitute, there is no actual portfolio choice and we will denote by $a_{t,i}$ the agent's total asset holdings.

Agents face borrowing constraints, and their total asset holdings must be higher than $-\bar{a} \leq 0$. Alternatively, this constraint states that agents cannot borrow more than the amount $\bar{a}$. In the rest of the paper, we will focus on the case where the credit limit is above the steady-state natural borrowing limit.[1]

[1] Aiyagari (1994) discusses the relevant values of $\bar{a}$, called the natural borrowing limit in an economy without aggregate shocks. Shin (2006) provides a similar discussion in presence of aggregate shocks. A standard value in the literature is $\bar{a} = 0$, which ensures that consumption remains positive in all states of the world.

2.5 Government, fiscal tools and resource constraints

In each period t, the government has to finance an exogenous and possibly stochastic public good expenditure $G_t \equiv G_t(z_t)$. The government has several tools for financing the expenditure. First, the government can levy two distorting taxes. A first tax τ_t^K is based on payoffs of all interest-rate bearing assets. The second tax τ_t^L concerns labor income. Second, in addition to these distorting taxes on households, the government can also tax the profits of firms. Finally, besides taxation, the government can also issue a one-period public bond, that is assumed to be riskless.

The real after tax wage w_t, as well as the real after-tax interest rate from periods $t-1$ to t, denoted R_t, can therefore be expressed as follows:

$$w_t = (1 - \tau_t^L)\tilde{w}_t, \tag{3}$$

$$R_t = (1 - \tau_t^K)\frac{1 + i_{t-1}}{\Pi_t}. \tag{4}$$

Regarding firm taxation, we assume that the government fully taxes the profit of all firms. The main advantage of this solution is that it greatly simplifies the question of the distribution of firm profits among the population of heterogeneous agents. We can now express the governmental budget constraints at date t

$$G_t + \frac{1 + i_{t-1}}{\Pi_t}B_{t-1} + \left(\tau_t^Y - t_t^Y\right) L_t\tilde{w}_t =$$
$$\tau_t^L L_t\tilde{w}_t + \tau_t^K\frac{1 + i_{t-1}}{\Pi_t}B_{t-1} + \left(1 - \frac{\tilde{w}_t}{Z_t}(1 - \tau_t^Y + t_t^Y) - \frac{\kappa}{2}\pi_t^2\right) Y_t + B_t,$$

that simply states that government spending, consisting of public good expenditure, public debt repayment, and labor tax subsidies are equal to governmental resources, made of household taxes on labor and capital, firm profits taxes and public debt issuance. Using the fact that $Y_t = Z_tL_t$, the governmental budget constraint can be simplified into:

$$G_t + R_tB_{t-1} + w_tL_t = B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right) Z_tL_t. \tag{5}$$

3 The truncated economy

3.1 Presentation

We now propose a description of our truncated economy that follows the main lines of LeGrand and Ragot (2017). In this environment, the choices of each agent, and in particular their savings, do not depend on the whole history of their idiosyncratic shocks, but only on a fixed number of consecutive past periods. Consequently, the population features a finite, though possibly large, heterogeneity. This finite-state property contrasts with standard equilibrium representation in heterogeneous agent models. Indeed, in general, wealth levels present a growing heterogeneity in these models and the equilibrium representation implies a time-varying distribution of wealth levels, whose support is infinite. This raises considerable theoretical and computational challenges, that we address here using a new approach based on a finite-state representation.

The length of the truncation for idiosyncratic histories is denoted by $N \geq 0$. For each agent, its whole history is truncated in a history covering the previous N periods. This N-period history is represented by a vector $\tilde{e}_N \in \{0,1\}^N$. The vector $\tilde{e}_N$ can also be decomposed into a set of coordinates $(\tilde{e}_{N-1}, \ldots, \tilde{e}_0)$, with $\tilde{e}_k \in \{0,1\}$ for any k. The element $\tilde{e}_0$ is the current beginning-of-period idiosyncratic state, while for any $k \geq 1$, $\tilde{e}_k$ is the beginning-of-period idiosyncratic state that occurred k periods ago.

In each period t, every agent is therefore endowed with a given history denoted as $\tilde{e}^N = (\tilde{e}^N_{-N+1}, \ldots, \tilde{e}^N_0)$. In the next period, the agent experiences a new realization of the idiosyncratic shock e_0 and becomes endowed with a new N-period history $e^N = (e^N_{-N+1}, \ldots, e^N_0)$. By construction, the history e^N must be a possible continuation of the previous period history $\tilde{e}^N$, that we denote as $e^N \succeq \tilde{e}^N$. Formally, with our notation, this means that for all $k = 1, \ldots, N-1$, we have $e^N_k = \tilde{e}^N_{k-1}$. In words, this means that the shock that occurred $k-1$ periods ago in the previous period now dates from k periods from the current date perspective. The probability to switch from history $\tilde{e}^N$ in period t to history e^N in period $t+1$ will be denoted $H_{t,\tilde{e}^N,e^N}$. With our notation, this probability is equal to the probability to transit from state $\tilde{e}^N_0$ at date t to state e^N_0 at date $t+1$, provided that history e^N is a possible continuation of history $\tilde{e}^N$. Formally, the probability can be expressed as $H_{t,\tilde{e}^N,e^N} = 1_{e^N \succeq \tilde{e}^N} M_{\tilde{e}^N_0, e^N_0}(s_t)$, where $1_{e^N \succeq \tilde{e}^N} = 1$ if e^N is a possible continuation of history $\tilde{e}^N$ and 0 otherwise. From these transition probabilities, we can deduce the recursion characterizing the size of the agents' population with history e^N in each period denoted $(S_{t,e^N})_{t \geq 0, e^N \in \mathcal{E}^N}$:

$$S_{t+1,e^N} = \sum_{\hat{e}^N \in \{0,1\}^N} S_{t,\tilde{e}^N} H_{t,\tilde{e}^N,e^N}, \tag{6}$$

where the initial population distribution $(S_{-1,e^N})_{e^N \in \mathcal{E}^N}$, with $\sum_{e^N \in \mathcal{E}^N} S_{-1,e^N} = 1$, is given. The law of motion (6) of $(S_{t,e^N})_{t \geq 0, e^N \in \mathcal{E}^N}$ is thus valid from period 0 onwards.

The trick to reduce heterogeneity is to assume that each agent receives a lump-sum transfer, which depends on her $N+1$-history. This set of specific lump-sum transfers, denoted $\left(\Gamma^*_{t,N+1}(e^{N+1})\right)_{e^{N+1} \in \{0,1\}^{N+1}}$ at date t, is fully balanced in each period, such that we have $\sum_{e^{N+1} \in \{0,1\}^{N+1}} \Gamma^*_{t,N+1}(e^{N+1}) = 0$. A consequence of this well-chosen transfer is that all agents with history e^N hold, in the beginning of period t, the *same* wealth $\tilde{a}_{t,e^N}$, which is equal to:

$$\tilde{a}_{t,e^N} = \sum_{\hat{e}^N \in \{0,1\}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} H_{t-1,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}, \tag{7}$$

where $a_{t-1,\hat{e}^N}$ is the end-of-period asset holding of agents with history $\hat{e}^N$ at date $t-1$. Since all agents with the same N-period history have the same beginning-of-period wealth, they will also choose the same consumption, the same savings and the same labor supply.

What is the expression of lump-sum transfers $\left(\Gamma^*_{t,N+1}(e^{N+1})\right)_{e^{N+1}\in\{0,1\}^{N+1}}$? Consider an agent with the $N+1$-history e^{N+1}. This history can first be written as $(\tilde{e}^N, e)$, where e is the date-t idiosyncratic state following the N-period history $\tilde{e}^N$ starting from date $t-1$. The history can also be written as $(\tilde{e}, e^N)$, where the N-period history e^N starting at date t is preceded by state $\tilde{e}$, $N+1$ periods ago. The transfer $\Gamma^*_{t,N+1}(e^{N+1})$ can then be defined as:

$$\Gamma_{t,N+1}(e^{N+1}) = R_t(\tilde{a}_{t,e^N} - a_{t-1,\tilde{e}^N}), \tag{8}$$

which swaps, at date t, the beginning-of-period wealth $R_t a_{t-1,\tilde{e}^N}$ with the wealth $R_t\tilde{a}_{t,e^N}$. Importantly, this lump-sum transfer does not affect agents' choices. Indeed, since each agent is atomistic in the continuum with mass $S_{\tilde{e}^N}$ of agents with history $\tilde{e}^N$, all agents take transfers $\left(\Gamma^*_{t,N+1}(e^{N+1})\right)_{e^{N+1}\in\{0,1\}^{N+1}}$ as given.

3.2 Agents' program

Agents are expected-utility maximizers taking fiscal policy as given. At each date, every agent chooses her consumption level c, her labor effort l and her savings a. Because of our limited-heterogeneity equilibrium, every agent is characterized by two state variables only: her beginning-of-period wealth a and her $N+1$-period history. Agents' program can be written recursively as follows:[2]

$$V_{N+1}(a, e^{N+1}) = \max_{a',c,l} \; U(c,l) + \beta\mathbb{E}\left[\sum_{(e^{N+1})'\succeq e^{N+1}} H_{e^{N+1},(e^{N+1})'} V_{N+1}(a', \left(e^{N+1}\right)')\right], \tag{9}$$

$$a' + c = w\theta_{e_0^N} l + \delta 1_{e_0^N=0} + Ra + \Gamma_{N+1}(e^{N+1}), \tag{10}$$

$$c,\, l \geq 0,\;\; a' \geq -\bar{a}, \tag{11}$$

where condition (10) is the budget constraint of an agent with history e^{N+1}, and inequalities in (11) are positivity constraints on consumption and labor choices as well as the credit constraint.

An interesting feature of the equilibrium is that despite the value function depends on the $N+1$-period history of agents, their choices only depend on their N- period history. Consequently, for sake of conciseness, every agent-dependent variable x will be denoted as x_{t,e^N} for agent e^N at date t. If we denote by ν_{t,e^N} the Lagrange multiplier of the credit constraint $a' \geq -\bar{a}$ for an agent with history e^N at date t, the first-order conditions of

[2] In line with the literature, we denote the savings choice in the current period by a'. The value a is thus the beginning-of-period wealth.

the agent's program (9)–(11) can be expressed as follows:

$$U_c(c_{t,e^N}, l_{t,e^N}) = \beta \mathbb{E}_t \left[\sum_{\tilde{e}^N \succeq e^N} H_{t,e^N,\tilde{e}^N} U_c(c_{t+1,\tilde{e}^N}, l_{t+1,\tilde{e}^N}) R_{t+1} \right] + \nu_{t,e^N}, \tag{12}$$

$$l_{t,e^N} = \left(w_t \theta_{e_0^N} \right)^{\varphi} + \delta 1_{e_0^N = 0}, \tag{13}$$

$$\nu_{t,e^N}(a_{t,e^N} + \bar{a}) = 0 \text{ and } \nu_{t,e^N} \geq 0. \tag{14}$$

Equation (12) is the Euler equation for consumption of an agent with history e^N. Similarly, equation (13) is the Euler equation for labor. The very simple expression is a direct consequence from the GHH utility function. Finally, equation (14) is the complementary slackness condition for asset holdings, stating that either the credit constraint is binding or the Lagrange multiplier is null.

3.3 Market clearing and pricing kernel

Using the limited-heterogeneity notation, we can express equilibrium condition on the labor and asset market. For the labor market, start noticing that for any history e^N, a population of agents of size S_{t,e^N} supplies the same labor quantity amounting to l_{t,e^N}, with the productivity $\theta_{e_0^N}$. Consequently, the labor supply in efficient units for history e^N amounts to $\theta_{e_0^N} S_{t,e^N} l_{t,e^N}$. The aggregation over all possible histories of $\{0,1\}^N$ implies that the total labor supply L_t in efficient units can be expressed as:

$$L_t = \sum_{e^N \in \mathcal{E}^N} \theta_{e_0^N} S_{t,e^N} l_{t,e^N}. \tag{15}$$

Regarding the financial market clearing, the total demand of private assets sums up to the public debt supply B_t. Indeed, private and public claims are perfectly fungible and private savings are issued in net zero supply. In consequence, the aggregate supply of private and public savings is equal to the amount of public debt. Consequently, financial market clearing at date t implies the following equality:

$$B_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} a_{t,e^N}. \tag{16}$$

Finally, we can provide a formal expression for the firms pricing kernel. This pricing kernel that equals M_t/M_0 is assumed to depend on the a sum of the agents' marginal utility for consumption. Formally, the expression of M_t is:

$$M_t = \beta^t \sum_{e^N \in \mathcal{E}^N} S_{e^N} U_c \left(c_{e^N,t}, l_{e^N,t} \right). \tag{17}$$

3.4 Sequential equilibrium

We can finally formalize the expression of a sequential equilibrium.

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual allocations* $\left(c_{t,e^N}, l_{t,e^N}, \tilde{a}_{t,e^N}, a_{t,e^N}\right)_{t\geq 0, e^N\in\mathcal{E}^N}$*, of aggregate quantities* $(L_t, B_t)_{t\geq 0}$*, of population sizes* $\left(S_{t,e^N}\right)_{t\geq 0,e^N\in\mathcal{E}^N}$*, of price processes* $(w_t, R_t, \tilde{R}_t, \tilde{w}_t)_{t\geq 0}$*, and of a fiscal policy* $(\tau^K_{t+1}, \tau^L_t, B_t)_{t\geq 0}$*, and of gross inflation rate* $(\Pi_t)_{t\geq 0}$ *such that, for an initial distribution of population and wealth* $\left(S_{-1,e^N}, a_{-1,e^N}\right)_{e^N\in\mathcal{E}^N}$*, and for initial values of public debt* B_{-1}*, of capital tax* τ_0*, and of the initial aggregate shock* z_{-1}*, we have:*

1. *given prices and fiscal policies, individual strategies* $\left(a_{t,e^N}, c_{t,e^N}, l_{t,e^N}\right)_{t\geq 0,e^N\in\mathcal{E}^N}$ *solve the agents' optimization program in equations (9)–(11);*

2. *number of agents with history* e^N*,* $\left(S_{t,e^N}\right)_{t\geq 0,e^N\in\mathcal{E}^N}$*, and beginning-of-period individual wealth,* $\left(\tilde{a}_{t,e^N}\right)_{t\geq 0,e^N\in\mathcal{E}^N}$*, are consistent with the laws of motion (7) and (6);*

3. *labor and financial markets clear at all dates: for any* $t \geq 0$*, equations (15)–(16) hold;*

4. *the government budget constraint (5) holds at any date;*

5. *factor prices* $(w_t, R_t, \tilde{R}_t, \tilde{w}_t)_{t\geq 0}$ *are consistent with (3) and (4);*

6. *the inflation path* $(\Pi_t)_{t\geq 0}$ *is consistent with the dynamics of the Phillips curve: at any date* $t \geq 0$*, equation (2) holds.*

4 Optimal fiscal policy

4.1 The Ramsey problem

We now solve the Ramsey problem in our incomplete-market economy with aggregate shocks. The Ramsey problem requires the government to choose a fiscal policy and an inflation path that maximize aggregate welfare. This fiscal policy consists of a path for labor and capital taxes, as well as a path of public debt issuances. For a given truncation length $N > 0$, this aggregate welfare, computed using a utilitarian criterion, can simply be expressed as:

$$\sum_{t=0}^{\infty}\beta^t \sum_{e^N\in\varepsilon^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N}). \tag{18}$$

In other words, the government has to select the competitive equilibrium associated to the highest welfare subject to a constraint of balanced governmental budget.

We can formalize the Ramsey program as follows:

$$\max_{\left(R_t,w_t,B_t,\Pi_t,(a_{t,e^N},c_{t,e^N},l_{t,e^N})_{e^N\in\{0,1\}^N}\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}U(c_{t,e^N},l_{t,e^N})\right], \tag{19}$$

$$G_t + R_tB_{t-1} + w_tL_t = B_t + \left(1-\frac{\kappa}{2}\pi_t^2\right)Z_tL_t. \tag{20}$$

for all $e^N \in \{0,1\}^N$:

$$a_{t,e^N} + c_{t,e^N} \leq w_t\theta_{e_t^N}l_{t,e^N}1_{e_0^N\neq 0} + \delta 1_{e_0^N=0} + R_t\tilde{a}_{t,e^N}, \tag{21}$$

$$U_c\left(c_{t,e^N},l_{t,e^N}\right) - \nu_{t,e^N} = \beta\mathbb{E}_t\left[\sum_{\tilde{e}^N\in\mathcal{E}^N}H_{t+1,e^N,\tilde{e}^N}U_c\left(c_{t+1,\tilde{e}^N},l_{t+1,\tilde{e}^N}\right)R_{t+1}\right], \tag{22}$$

$$l_{t,e^N} \geq \left(w_t\theta_{e_t^N}\right)^{\varphi} + \delta 1_{e_0^N=0}, \tag{23}$$

$$\Pi_t(\Pi_t-1) = \frac{\varepsilon}{\kappa}(e^{-z_t}\tilde{w}_t(1-\tau_t^Y) - \frac{\varepsilon-1}{\varepsilon}) \tag{24}$$

$$+\ \beta\mathbb{E}_t\Pi_{t+1}(\Pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}, \tag{25}$$

$$\nu_{t,e^N}(a_{t,e^N}+\overline{a}) = 0, \tag{26}$$

$$B_t = \sum_{e^N\in\mathcal{E}^N}S_{t,e^N}a_{t,e^N},\ L_t = \sum_{e^N\in\mathcal{E}^N}S_{t,e^N}\theta_{e_t^N}l_{t,e^N}, \tag{27}$$

and additionally subject to the law of motion (6) of $(S_{t,e^N})_{t\geq 0,e^N\in\mathcal{E}^N}$, the definition (7) of $(\tilde{a}_{t,e^N})_{t\geq 0,e^N\in\mathcal{E}^N}$, and the positivity of labor and consumption choices. All constraints in the Ramsey program have already been interpreted. Equations (20) and (21) are governmental and individual budget constraints. Equations (22) and (23) are Euler equations for consumption and labor, while equation (24) characterizes the Phillips curve. Finally, (26) is the complementary slackness condition, while (27) collect aggregation equations. An detailed formulation of the program can be found in Appendix A.

The Ramsey program can be reformulated by integrating in the objective function the individual Euler equations (22) for consumption as well as the equation for the Phillips curve. Following the same lines as LeGrand and Ragot (2017), we denote by $\beta^tm^t(s^t)S_{t,e^N}\lambda_{t,e^N}$ the Lagrange multiplier of the Euler equation of agent e^N in state s^t. We also define for all $e^N \in \mathcal{E}^N$:

$$\Lambda_{t,e^N} = \frac{\sum_{\hat{e}^N\in\mathcal{E}^N}S_{t-1,\hat{e}^N}\lambda_{t-1,\hat{e}^N}\Pi_{t,\hat{e}^N,e^N}}{S_{t,e^N}}. \tag{28}$$

The quantity Λ_{t,e^N} can be interpreted as the average of previous period Lagrange multipliers for the Euler equation. Similarly, we denote as $\beta^tm^t(s^t)\alpha_t$ the Lagrange multiplier of the equation (24) of the Phillips curve. With this notation, the objective of the Ramsey program (19) becomes (see Appendix B for further detail about the analytical derivation):

$$
\begin{aligned}
J =&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) \\
&+ U_c(c_{t,e^N}, l_{t,e^N}) \left(\Lambda_{t,e^N} R_t - \lambda_{t,e^N} \right), \\
&+ U_c(c_{t,e^N}, l_{t,e^N}) \left(\alpha_t \frac{\varepsilon - 1}{\kappa} \left(e^{-z_t} \tilde{w}_t - 1 \right) - (\alpha_t - \alpha_{t-1}) \Pi_t (\Pi_t - 1) \right) e^{z_t} L_t \Big).
\end{aligned} \tag{29}
$$

With this notation, the Ramsey program (19)–(27) can now be expressed as:

$$
\max_{\left(\Pi_t, R_t, w_t, B_t, (a_{t,e^N}, c_{t,e^N}, l_{t,e^N})_{e^N \in \mathcal{E}^N}\right)_{t \geq 0}} J
$$

subject to constraints (20), (21), (23), and (27), as well as subject to the law of motion (6) of $(S_{t,e^N})_{t \geq 0, e^N \in \mathcal{E}^N}$, and the definition (7) of $(\tilde{a}_{t,e^N})_{t \geq 0, e^N \in \mathcal{E}^N}$.

Marcet and Marimon (2011) rely on a similar transformation for individual Euler equations. We here use the same methodology to cope with the Phillips curve. The main idea of this representation is that minimizing the cost of the constraints is now an objective.

Second, it is worth mentioning a new difference between real and monetary frameworks. As is standard in optimal fiscal policy literature and following Chamley (1986), the Ramsey problem is written in post-tax prices R_t and w_t. To derive fiscal policy, the literature usually solves for the the post-tax allocation and then derive the value of the taxes comparing post-tax prices and marginal productivities. This methodology cannot be applied when the Phillips curve is a binding constraint (i.e., $\alpha_t \neq 0$), because the before-tax price now enters into the objective of the planner. This difference is the basis of the real-wage effect described below. As for the real economy, we will solve for the allocation in the monetary policy and we then derive the value of the instruments.

Finally, in all expressions below, it is useful to introduce a new intuitive concept, that we call the *social valuation of liquidity for agents* e^N and that we denote by ψ_{t,e^N}. It is formally defined as:

$$
\psi_{t,e^N} \equiv U_c(c_{t,e^N}, l_{t,e^N}) - U_{cc}(c_{t,e^N}, l_{t,e^N}) \left(\lambda_{t,e^N} - (1 + r_t) \Lambda_{t,e^N} \right). \tag{30}
$$

The valuation ψ_{t,e^N} differs from the marginal utility of consumption $U_c(c_{t,e^N}, l_{t,e^N})$ (which can be seen as the private valuation of liquidity for agents e^N) since ψ_{t,e^N} takes into consideration the saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$ and thus makes her Euler equation more "binding". This more "binding"constraint reduces the utility by the algebraic quantity $U_{cc}(c_{t,e^N}, l_{t,e^N})\lambda_{t,e^N}$, where λ_{t,e^N} is the Lagrange multiplier of the agent's Euler equation at date t. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity Λ_{t,e^N}. Finally, the last key Lagrange multiplier is denoted μ_t and is applied to the budget constraint of the government. Hence, for the government, the marginal cost at period t of transferring resources to households is μ_t.

4.2 Understanding the role of monetary policy: A decomposition

The understanding of the rationale for deviation from price stability, we study five different economies:

1. the real economy with both time-varying capital and labor taxes, without monetary frictions;
2. the monetary economy, with nominal frictions and time-varying capital and labor taxes;
3. the economy without time-varying capital taxes and with only time-varying labor taxes, allowing to identify the information channel;
4. the economy without time-varying labor taxes and with only time-varying capital taxes, allowing to identify the real wage channel;
5. the economy without time-varying capital and optimal labor taxes.

The roadmap, is the following. The first two economies will prove that the real economy (economy 1) generates the constrained efficient allocation. This allocation can be reached with the full set of instruments (economy 2). The following two economies (economy 3 and 4) allow identifying the *real interest* and the *real wage* channels, respectively. Both effects are combined in economy 5.

In all these economies, we allow for time-varying public debt, as it is an obvious time-varying tool in the business cycle.[3] We study the effect of an ad-hoc debt dynamics in economy 5, to discuss the *public finance* channel.

4.2.1 The benchmark: The real economy case

To understand the impact of nominal frictions on our results, we compare a monetary economy with frictions to a frictionless economy. We define the real-economy allocation as a flexible-price economy, where the government can choose in each period capital and labor taxes, and public debt, so as to optimize the aggregate welfare. More formally, the

[3]Results with constant public debt would be very easy to derive, with no further economic insights.

real economy allocation is the solution of the following program:

$$\max_{\left(R_t,w_t,B_t,(a_{t,e^N},c_{t,e^N},l_{t,e^N})_{e^N\in\mathcal{E}^N}\right)_{t\geq 0}} J,$$

with $\kappa = 0$ and $\alpha_t = 0$ for all t (real economy) and subject to equations (5), (7), (10), (13), (16), and (17). We recall that expression of the objective J can be found in equation (29). We formally derive the first-order conditions in Section C of the Appendix. We will use these conditions to explain the new role for monetary policy.

4.2.2 An irrelevance result with incomplete markets: The monetary economy with a full set of fiscal tools

We first solve for the optimal monetary and fiscal policies when the government has access to a full set of fiscal tools. This program can be written as:

$$\max_{\left(\Pi_t,R_t,w_t,B_t,(a_{t,e^N},c_{t,e^N},l_{t,e^N})_{e^N\in\mathcal{E}^N}\right)_{t\geq 0}} J,$$

subject to equations (5), (7), (10), (13), (16), and (17). Observing the constraints, there is an obvious choice of inflation. Because of the presence of both capital and labor taxes, the inflation is redundant and only destroys resources because of the price-adjustment cost. Consequently, the government sets $\Pi_t = 1$. The Phillips curve then implies $\tilde{w}_t = e^{z_t}$ and $\alpha_t = 0$. The before-tax wage is therefore equal to the marginal product of labor and the Phillips curve is not a binding constraint. The Ramsey program is therefore exactly the same as the one in the real economy. The optimal allocations of both economies are therefore identical. Any redistributive effect that inflation could generate is achieved more efficiently by fiscal policy. We summarize this first result in the following proposition.

Proposition 1 (An irrevelance result) *When both labor and capital taxes are available, the government reproduces the real-economy allocation.*

This allocation can be reproduced with several combinations of instruments. Nominal interest rate and capital taxes are not uniquely pinned down in this allocation. Indeed, the allocation pins down the post tax real interest rate, which is when $\Pi_t = 1$:

$$R_t \;=\; (1-\tau_t^K)\,(1+i_{t-1})\,.$$

As a consequence, any pair of τ_t^K and i_{t-1} satisfying the previous equality generates the optimal allocation. A simple policy is to set:

$$i_{t-1} = \mathbb{E}_t R_{t+1} + \phi^{\Pi}\,(\Pi_t - 1)\,,$$

with $\phi^{\Pi} > 1$ to insure price determinacy. In this case, $\mathbb{E}_t\tau_{t+1}^K = 0$ and the capital tax in the period only adjusts to the new information set.

4.2.3 The economy without time-varying capital taxes

We now consider the economy where the planner has only access to time-varying labor taxes. The capital tax is constant and fixed at its optimal steady-state τ_{SS}^K to avoid steady-state distortions. The problem of the planner can now be written as:

$$\max_{\left(\Pi_t, R_t, w_t, B_t, (a_{t,e^N}, c_{t,e^N}, l_{t,e^N})_{e^N \in \mathcal{E}^N}\right)_{t \geq 0}} J,$$

subject to equations (5), (7), (10), (13), (16), and (17), as well as with the new additional constraint $R_t = (1 - \tau_{SS}^K)\frac{1+i_{t-1}^b}{\Pi_t}$, reflecting the new steady state value of the capital tax.

We derive the first-order conditions in Appendix D. We only here discuss the relevant ones that matter for understanding the optimal allocation. First, the optimal choice of the nominal interest rate implies the following equation:

$$\underbrace{\mathbb{E}_t\left[\sum_{e^N \in \mathcal{E}^N} S_{t+1,e^N}\left[\left(\psi_{t+1,e^N}\tilde{a}_{t+1,e^N} + U_c(c_{t+1,e^N}, l_{t+1,e^N})\Lambda_{t+1,e^N}\right)\right]\right]}_{\text{expected redistributed gains}} = \underbrace{B_t \mathbb{E}_t \mu_{t+1}.}_{\text{expected public fin. cost}} \quad (31)$$

The planner equalizes the expected redistributive gains of a change in the real interest rate to the expected public financial cost. The reason is that the expected inflation will be roughly 0, as shown in the next paragraph. The redistributive gain of a change in the real interest rate is the sum of two terms. The first one channels through the tax base made of asset holdings $\tilde{a}_{t+1,e^N}$ and is priced by the liquidity value ψ_{t+1,e^N}. The second term involves Λ_{t+1,e^N} and captures the benefits in the saving incentives for a marginal increase in the real interest rate. This gain is equalized to the expected cost for public finance of an increase in the real interest rate. This cost is related to higher interest payment on the public debt and is scaled by the amount of outstanding debt B_t. This cost is priced by the expected governmental liquidity $\mathbb{E}_t \mu_{t+1}$.

Second, the inflation dynamics satisfy the following equation:

$$0 = \mathbb{E}_t\left[\kappa \mu_{t+1} \Pi_{t+1}(\Pi_{t+1} - 1) L_{t+1} e^{z_{t+1}}\right]. \quad (32)$$

This constraint is provided in the general non-linear case. It implies that inflation expectation is roughly 0. Indeed, at the first order the previous equality implies $\mathbb{E}_t \pi_{t+1} = 0$. This is not exactly true in the non-linear case as the government values differently increase and decrease in inflation.

This last property helps understand monetary policy. The planner commits to a monetary policy such that the inflation is (approximately) null in expectations, which minimizes price adjustment costs. The nominal interest rate is then set to balance the gains and costs of the variations in the real interest rate. However, this does not imply that inflation is constant in every period. Indeed, the inflation is set in every period such that the costs and the gains of inflation changes are balanced. This is given by the following first-order condition for inflation:

$$\underbrace{\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left[\psi_{t,e^N} \tilde{a}_{t,e^N} + U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N}\right]}_{\text{distributive gain of inflation}} = \underbrace{\mu_t \left(\kappa \frac{\Pi_t(1-\Pi_t)}{R_t} e^{z_t} L_t + B_{t-1}\right)}_{\text{public financial cost}}. \quad (33)$$

The left hand side is the welfare gain (possibly negative) of an increase in inflation on welfare going through a change in the real interest rate. This term is therefore identical to the left hand side of equation (31). The right hand side the cost for public finances. It includes the effect on public debt interest rate, scaled by the outstanding public debt amount B_{t-1}. The total cost embeds a second term, which reflects the resource destruction implied by the price adjustment cost – and the deviation from price stability. Because of this price adjustment term, inflation is an imperfect substitute to a change in the capital tax within the period, that does not generate any waste of resources.

How can the planner reach this allocation and manipulate inflation? The answer is that the Phillips curve generates a relationship between the before-tax real wage and inflation (when inflation expectations are 0). The government can thus choose the labor tax to create a wedge between the after-tax real wage, which determines the labor supply and the before-tax real wage which affects inflation. Actually, it appears that the Phillips curve is not a constraint in this allocation ($\alpha_t = 0$), but it pins down the labor tax with the inflation target.

We summarize this finding in the next proposition.

Proposition 2 *In the economy with constant capital taxes,*

1. *net inflation expectations are* 0 *at the first order;*

2. *inflation adjusts for only one period after the shock to adjust to the new information;*

3. *the labor tax adjusts in the period to affect the real wage; the Phillips curve is thus not a constraint (*$\alpha_t = 0$*).*

As the inflation adjusts only to take into account the information which was not available when nominal interest rate is set, we call this channel the *information channel* for optimal monetary policy. Inflation is an imperfect substitute for time-varying capital taxes.

4.2.4 The economy without time-varying labor taxes

We now consider an economy where time-varying labor taxes are missing, but where time-varying capital taxes are available. The government has an additional constraint which is $\tau_t^L = \tau_{SS}^L$, where τ_{SS}^L is the optimal labor tax at the steady state. This economy exhibits possibly a larger inflation persistence than in the previous case. In this economy, we indeed have $w_t = \left(1-\tau_{SS}^L\right)\tilde{w}_t$. As a consequence, the before-tax real wage affects both inflation dynamics through the Phillips curve, and the income of agents through its direct effect on the after-tax real wage. Conversely, inflation dynamics will have redistributive effects because it affects the after-tax real wage through the Phillips curve. In this economy, the Phillips curve will thus be a constraint and $\alpha_t \neq 0$. Again, we derive the full set of first-order conditions in Appendix E and focus here on the main expression.

First, the government will set the inflation rate for the real wage to satisfy the following expression:

$$\underbrace{\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \frac{\theta_{e^N} l_{t,e^N}}{e^{z_t} L_t} \psi_{t,e^N}}_{\text{distrib. effect real wage}} + \underbrace{\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \left[\alpha_t F^1_{t,e^N} + \alpha_{t-1} F^2_{t,e^N}\right]}_{\text{effect on the Phillips curve}}$$
$$= \underbrace{\mu_t \left[\frac{1+\varphi}{\varphi}(1-\tau^L)\frac{\tilde{w}_t}{e^{z_t}} + \frac{\kappa}{2}\left(\Pi_t - 1\right)^2 - 1\right] \frac{\varphi}{\tilde{w}_t(1-\tau^L)}}_{\text{effect on the government budget}}, \quad (34)$$

The first term at the left hand side captures the welfare effect of an increase in the real wage for households. For the planner, it is the valuation of liquidity for agents of each history multiplied by the tax base $\frac{\theta_{e^N} l_{t,e^N}}{e^{z_t} L_t}$. The second term at the left hand side is more involved. It captures the change in the pricing kernel of the firm due to the change in consumption, which is itself the outcome of the change in the real wage. The expressions of F^1_{t,e^N} and F^2_{t,e^N} are given in Appendix E. This term can be expected to be close to zero at the first order for aggregate shocks. The last term captures the effect of an increase in before-tax real wage on the budget of the government: tax increases, the labor supply decreases, output decreases, and so does the price adjustment cost. The optimal inflation rate satisfies the following equality:

$$\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N})(1-2\Pi_t)(\alpha_t - \alpha_{t-1}) = \mu_t \kappa(\Pi_t - 1). \quad (35)$$

The left hand side is the cost of a change in inflation on the ability of government to implement the optimal allocation though the Phillips curve. The right-hand side captures the cost of a change in inflation for the government.

The third equation characterizing the optimal capital tax has now a familiar expression, which is:

$$\underbrace{\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left(\psi_{t,e^N} \tilde{a}_{t,e^N} + \Lambda_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N})\right)}_{\text{distrib. effect of real int. rate}} = \underbrace{\mu_t B_{t-1}.}_{\text{effect on gov. bud. cons.}} \quad (36)$$

It equalizes the gain of an increase in the after-tax real interest rate with the public finance cost. As in the economy with both time-varying taxes, presented in Section 4.2.2, the nominal and real interest rates are not determined. Only the product of the two are determined: $R_t = (1-\tau_t^K)(1+i_{t-1})$. The absence of labor taxes does not change this result. The three equations (34), (35), and (36) determine jointly inflation capital taxes and public debt, with the government budget constraint. It is difficult to provide more analytical insights with general utility functions. We rely below on simulations to investigate the relative importance of these various effects.

4.2.5 The economy without time-varying labor taxes and without time-varying capital taxes : Fiscal dominance

We finally treat the case where no fiscal tool is available and only monetary policy intervenes in the business cycle. Obviously, this case is a mix of the two previous ones and all the effects are present. To study interesting cases and the effect of public debt dynamics, we introduce a fiscal rule (which is purposely not optimal), in order to investigate the effect of public debt. Following Bohn (1998) and many others, we assume that the labor tax τ_t^L follows a rule depending on the deviation of public debt:

$$\tau_t^L = \tau_{SS}^L + \phi^G(G_t - G_{SS}).$$

We will play later on the parameter ϕ^G. The main choice concerns the inflation dynamics. It is determined by the following first-order condition:

$$\begin{aligned}\underbrace{\sum_{e^N\in\mathcal{E}^N} S_{t,e^N}\left[\psi_{t,e^N}\tilde{a}_{t,e^N} + U_c(c_{t,e^N}, l_{t,e^N})\Lambda_{t,e^N}\right]}_{\text{distributive gain of inflation}} &+ \underbrace{\sum_{e^N\in\mathcal{E}^N} S_{t,e^N} PC^{\Pi}_{t,e^N}}_{\text{adj. of Phillips curve}} \\ &= \underbrace{\mu_t\left(\kappa\frac{\Pi_t(1-\Pi_t)}{R_t}e^{z_t}L_t + B_{t-1}\right)}_{\text{public financial cost}}.\end{aligned} \tag{37}$$

This equation is very close to (33) derived in the case where only capital taxes were missing. When labor taxes are missing, the Phillips curve is a constraind and the planner has to consider the cost of affecting the real wage though the Phillips curve. This is summarized by the term PC^{Π}_{t,e^N}, the expression of which is given in Appendix. This last term is complex, as it embeds the change in the pricing kernel due to the cohange in the consumption of all agents after a change in inflation.

The condition for the nominal interest rate choice is

$$\begin{aligned}0 &= \mathbb{E}_t\left[\kappa\mu_{t+1}\Pi_{t+1}(\Pi_{t+1}-1)e^{z_{t+1}}L_{t+1}\right] \\ &+ \underbrace{\mathbb{E}_t\left[(\alpha_{t+1}-\alpha_t)\Pi_{t+1}(2\Pi_{t+1}-1)e^{z_{t+1}}L_{t+1}\sum_{e^N\in\mathcal{E}^N} S_{t+1,e^N}U_c(c_{t+1,e^N}, l_{t+1,e^N})\right]}_{\text{Phillips curve adj.}}\end{aligned}$$

This expression is close to the expression (32), it includes an additional term for the adjustment of the Phillips curve. The first-order condition for the before-tax real wage is the same as (34). Again, we rely on simulations in Section 5 to investigate the property of optimal monetary policy.

5 Quantitative properties of the optimal tax system

We now provide a quantitative investigation of the optimal monetary policy after a public spending shock. We derive the joint optimal monetary and fiscal policies that maximize the aggregate welfare.

5.1 Calibration and simulation

Parameter calibration. The period is a quarter and the discount factor is $\beta = 0.99$. The utility function is $u\left(c - \chi^{-1}\frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) = \log\left(c - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$, with a Frisch elasticity of labor supply set to $\varphi = 1$, which is the standard value taken in monetary economics. Public spending is assumed to follow an AR(1) process $G_t = \rho_g G_{t-1} + \varepsilon_t^g$, where $(\varepsilon_t^g)_{t\geq 0}$ is a white-noise process with a distribution $\mathcal{N}(0, \sigma_g^2)$. With this specification the steady-state public spending is 0, as is standard in New-Keynesian models. We made this choice for comparison purposes with the New-Keynesian literature, but the model could perfectly handle a non-zero average public spending. We set the persistence of the public spending to $\rho_g = 0.97$ and the standard deviation to $\sigma_g = 7\%$, following Chari, Christiano, and Kehoe (1994) and Farhi (2010).

As in Imrohoroğlu (1992) or Krusell and Smith (1998) among others, the idiosyncratic risk in our paper is to assumed to be an unemployment risk. We derive transition probabilities using a calibration based on the strategy of Shimer (2003). The quarterly transition matrix is:

$$M = \begin{bmatrix} 0.21 & 0.79 \\ 0.05 & 0.95 \end{bmatrix}.$$

This matrix implies that the quarterly job finding rate is 80%, while the quarterly job separation rate is 5%. The home production parameter δ is set such that home production amounts to 50% of market income in real terms. Regarding the credit constraint, we set the credit limit to $\bar{a} = 0.1$. Note that this quantity has a smaller impact on economy outcomes than in an economy with capital.[4]

Finally, concerning the monetary aspect of the model, we follow the standard calibration practice for the Phillips curve. We namely choose $\kappa = 100$ and $\varepsilon = 6$.

Table 1 summarizes our calibration for standard parameters.

[4] Indeed, as shown in LeGrand and Ragot (2017), in an economy with capital, the government aims at holding the whole capital stock to minimize distortions.

β	ϕ	$\delta/(wl)$	ρ_g	σ_g	$\bar{a}$	κ	ε
0.99	1	50%	0.97	0.07	0.01	100	6

Table 1: Parameter values

Choosing N. We consider the case where $N = 4$. As a consequence, there are $2^4 = 16$ different households in our economy. We checked that credit constraints are binding in equilibrium and our results do not drastically depend on the choice of N.

Numerical methods. To solve the model, we first compute the steady state. This is not a difficult task, as the above equations define an almost linear system. Second, we write a code that writes the set of dynamic equations in Dynare for an arbitrary N. This allows us to use the Dynare solver to double-check our steady-state computations and to simulate the model. We check that the standard stability and credit-constraint conditions are fulfilled and that all variables converge back to their steady state value after the shocks. Simulating the model takes a couple of seconds once the steady-state is found.

5.2 Steady-state tax system

We simulate the model to determine the optimal fiscal and monetary policy at the steady state. We report in Table 2 the results for the complete-market economy ("CM" economy for which $N = 0$) and the incomplete-market economy ("IM" economy with $N = 4$). The government is assumed to have in hand three fiscal tools: capital and labor taxes, and public debt, as well as one monetary tool, the gross inflation rate.

First, as the steady-state public spending level is null, $G = 0$, the fiscal system in the CM economy is trivial: $\tau^{K,CM} = \tau^{L,CM} = B^{CM} = 0$. Regarding the monetary policy, the gross inflation rate is $\Pi^{CM} = 1$, implying a zero net inflation rate $\pi^{CM} \equiv \Pi^{CM} - 1 = 0$. This choice also avoids any price adjustment cost. Finally, the gross real interest rate is $R^{CM} = 1/\beta$, because in presence of complete markets, there is no self-insurance motives. Overall, the CM economy features no distortion at the steady state and the CM allocation is identical to the first-best allocation, which is also the benchmark steady-state allocation in New-Keynesian economies.

In the IM economy, though close for some aspects, the allocation is different along other aspects. First, the optimal net inflation rate is also zero to avoid price adjustment costs. Second, the nominal interest rate and the capital tax are jointly indeterminate. Indeed, the government determines the optimal after-tax real interest rate, which is by construction equal to $R^{IM}_{SS} = \left(1 - \tau^{K,IM}_{SS}\right)\left(1 + i^{IM}_{SS}\right)$. Every couple $\left(\tau^{K,IM}_{SS}, i^{IM}_{SS}\right)$ satisfying the previous equality for a given optimal R^{IM} are admissible choices. So as to avoid indetermination, we impose as a normalization a zero capital tax at the steady-state:

$\tau_{SS}^{K,IM} = 0$. This will also simplify comparisons with the CM economy. As a result, we have $1 + i_{SS}^{IM} = R_{SS}^{IM}$. Furthermore, the real interest rate is lower in the IM economy than in the CM one. We therefore have $\beta R_{SS}^{IM} < 1$, even if βR_{SS}^{IM} remains quantitatively close to 1. This outcome is standard in Bewley-type economies. The less standard outcome of the IM economy steady-state concerns the labor tax and the public debt. First, the public debt is not zero and we find that the quarterly public debt-to-GDP ratio amounts to 42%. As a consequence, the provision of liquidity for self-insurance motives, resulting from market incompleteness for the unemployment risk, has a strong impact on household liquidity demand and ultimately on public debt. This amount is found to be close to the results of LeGrand and Ragot (2017) in an economy with capital. The labor tax needs to adjust for financing public debt interest debt payments. The labor tax remains however small and equal to 0.4%. This small amount, compared to LeGrand and Ragot (2017), stems from the choice of a zero public spending in steady state. Contrary to New-Keynesian standard calibration, LeGrand and Ragot (2017) follow the calibration of the public finance literature and consider a public debt-to-GDP ratio (on an annual basis) amounting to 33%, that matches US data.

Finally, our truncated economy features a partial risk sharing arrangement through the pooling transfers $\left(\Gamma_{N+1}^*(e^{N+1})\right)_{e^{N+1}\in\{0,1\}^{N+1}}$. These pooling transfers being constant and equal to 0 is a Bewley economy, we proxy the magnitude of the risk sharing in the truncated economy by the standard deviation, across agents, of the transfers $\left(\Gamma_{N+1}^*(e^{N+1})\right)_{e^{N+1}\in\{0,1\}^{N+1}}$, normalized by the total income $Inc_{e^{N+1}}$ of agents with history e^{N+1} ($Inc_{e^{N+1}} = w\theta_{e_0^{N+1}} l_{e^{N+1}} + \delta 1_{e_0^{N+1}=0} + (1+r)a_{e^{N+1}}$). We denote by sd_Γ this standard deviation. In the IM economy, with our calibration, this value is found to be equal to $sd_\Gamma = 3.8\%$. The standard deviation of pooling transfers relative to agents' own *individual* income is 3.8%, which is a low value.

	τ^K (%)	τ^L (%)	π	$1-\beta R$	B/Y	sd_Γ (%)
CM	0.0	0.0	0.0	0	0	–
IM	0.0	0.4	0.0	10^{-6}	42%	3.8

Table 2: Steady-state optimal fiscal system

In the exercises we conduct below, in which we close successively the labor and the capital tax, we always start from the same steady state in IM economies. Furthermore, if the capital and/or the labor tax is not available, we impose that the corresponding tax rates is constant and equal to their steady-state value. Formally, we impose $\tau_t^{L,IM} = \tau_{SS}^{L,IM}$ (for all t) if labor tax is closed or $\tau_t^{K,IM} = \tau_{SS}^{K,IM}$ (for all t) if capital tax is closed.

5.3 The role of monetary policy in the business cycle

To understand the role of monetary policy, and consistently with the theoretical analysis, we first various economies after a public spending shock:

1. The "first-best economy" with complete markets and in which no distorting fiscal tool are used.
2. The real economy with both time-varying capital and labor taxes, without monetary frictions. This is the constrained efficient allocation.
3. The economy without time-varying capital tax and with only time-varying labor tax, allowing to identify the information channel.
4. The economy without time-varying labor tax and with only time-varying capital tax, allowing to identify the real wage channel.
5. The economy without time-varying capital nor optimal labor taxes.

For the sake of the homogeneity of results, we consistently plot IRFs for key variables after a public spending shock of 1% of GDP. For IRFs, we always focus on 9 key variables that are plotted on the same 9 panels, as can be seen in Figures 1–4. The first panel is the public spending shock, plotting the same 1% shock. The second panel plots aggregate consumption, Ctot. The third panel plots consumption inequality measures, sd(C), that is proxied by the standard deviation of consumption levels across agents. The fourth panel is the post-tax real wage, w. The fifth panel is the labor tax, taul. The sixth panel is public debt, B. The seventh panel is the post-tax real gross interest rate, R. The eighth panel is the net nominal interest rate, i. The ninth and last panel is the net inflation rate, pi. All variables are plotted in percentage deviation from their steady-state value, except tax rates and inflation rate, which are in level deviations from steady state values.

Comparing the first-best economy and the real economy. We plot the results for the first-best economy (hereafter FB) in Figure 1 and we compare them to the outcomes of the constrained-efficient economy. Blue dashed lines are for FB results and black lines are for constrained-efficient economies. The main lesson of the comparison is that both economies yield very similar allocations concerning aggregate consumption, and labor supply. However, in the IM economy, this allocation is obtained with a sharp decrease in the real interest rate for one period (and thus a sharp increase in the capital tax rate). This standard outcome comes from the fact that a change in capital tax for one period is not distorting, which generates a front loading of the negative wealth shock on the government budget constraint. This fall in capital tax finances a fall in the public debt-to-GDP ratio of 10% at the quarterly level. The labor tax barely moves in the business

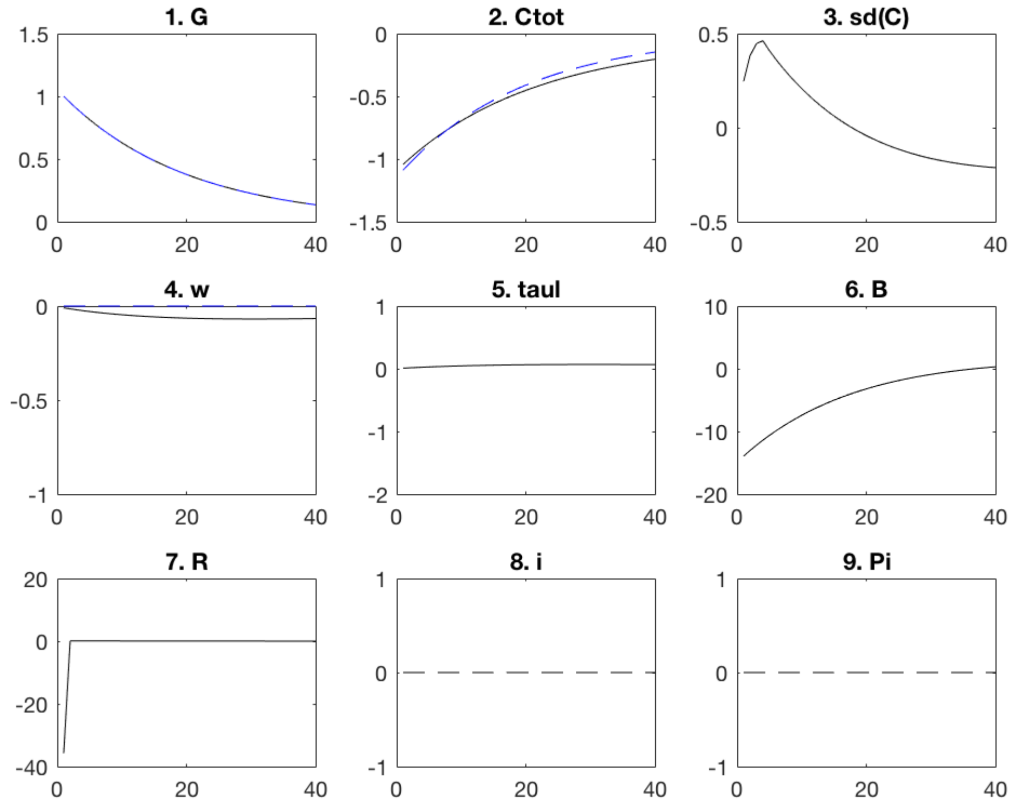

Figure 1: Comparison between the first-best economy (blue dashed line) and the real economy (black solid line)

cycle. Finally, inequality increases (panel 3) because of the lower level of public debt that implies a smaller supply of assets available for self-insurance. This smaller public debt supply ultimately increases equilibrium consumption dispersion.

Despite the absence of capital, these effects are highly similar to those described in LeGrand and Ragot (2017).

Comparing the real economy and the economy without capital taxes: the information channel. We plot the results for the labor-tax-only economy in Figure 2 (blue dashed line), and they are compared to those of the constrained-efficient economy (black solid line).

The main difference is that, in absence of labor tax, the real interest rate does not move at the impact (blue dashed line in panel 7). Consequently, the public debt does not feature any front loading after a public spending shock. Furthermore, the public debt is now countercyclical, and gradually increases, instead of decreasing at the impact in the constrained-efficient economy. Labor tax increases to finance the higher public debt repayments. This higher distorting tax reduces labor supply and aggregate consumption.

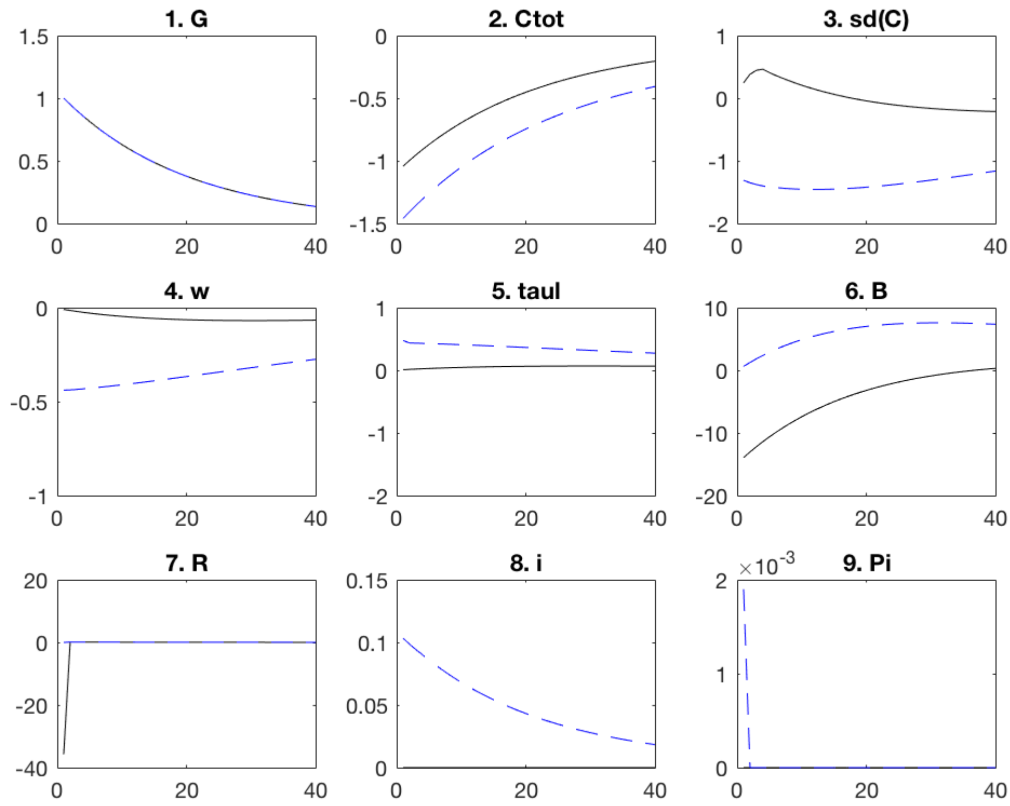

Figure 2: Comparison between the real economy and the economy without capital taxes, and with labor taxes (blue dashed line)

Inequality now decreases (panel 3) because the labor supply of high-income agents decreases relatively more than those of low-income agents. Finally, the government mimics a fall in the real interest rate on impact by increasing the inflation rate for one period. This is what we call the *information channel*. But as this increase in inflation is costly and destroys resources, the inflation raise remains quantitatively small tiny, and amounts to $2 \times 10^{-3}\%$ – note that the scale of panel 9 is in percentage points.

Comparing the real economy and the economy without labor tax: the real wage channel.

We now plot the results for the capital-tax-only economy in Figure 3 (blue dashed line), and we compared these results to those of the constrained-efficient economy (black solid line).

When only time-varying capital tax is available, we also observe a sharp decrease in the real interest rate and a front-loading by a decrease in public debt. In absence of time-varying labor tax, the inflation now slightly decreases for redistributive purpose. We refer to this as the *real wage channel*.

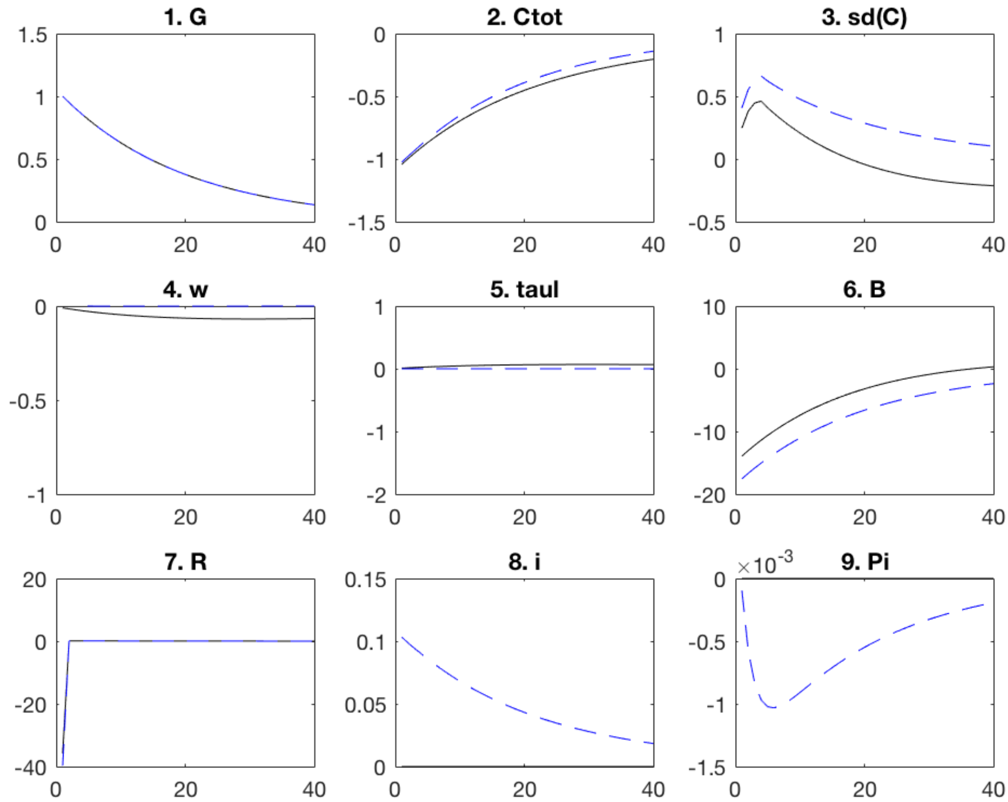

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Figure 3: Comparison between the real economy and the economy without labor taxes, but with capital taxes (blue dashed line)

Comparing the "constrained-efficient IM economy" and the "debt-only IM economy": public finance channel.

Finally, we now plot the results for the debt-only economy in Figure 4 (blue and red dashed lines), while we plot in black solid line the results to those of the constrained-efficient economy.

In the public debt-only economy, fiscal policy is exogenous and non-optimal, while monetary policy only is optimal. Regarding the labor tax, setting a constant amount yields an exploding public debt path and for the stability of the public debt path, we have to impose a fiscal rule for the labor tax, that takes the debt dynamics into account. More precisely, we choose:

$$\tau_t^L = \tau_{SS}^L + \phi^G (G_t - G_{SS}),$$

where the parameter $\phi^G > 0$ drives how sensitive to public debt evolution the fiscal rule is. Our results are reported for two values of ϕ^G, namely 1.7 (dashed red line) and 1.5 (dashed blue line). The higher the taxes the higher public debt and inflation.

This last experiments is a case of fiscal dominance, as inflation is used to balance the budget of the gouvernment. Importantly, this occurs by two channels. First, inflation is

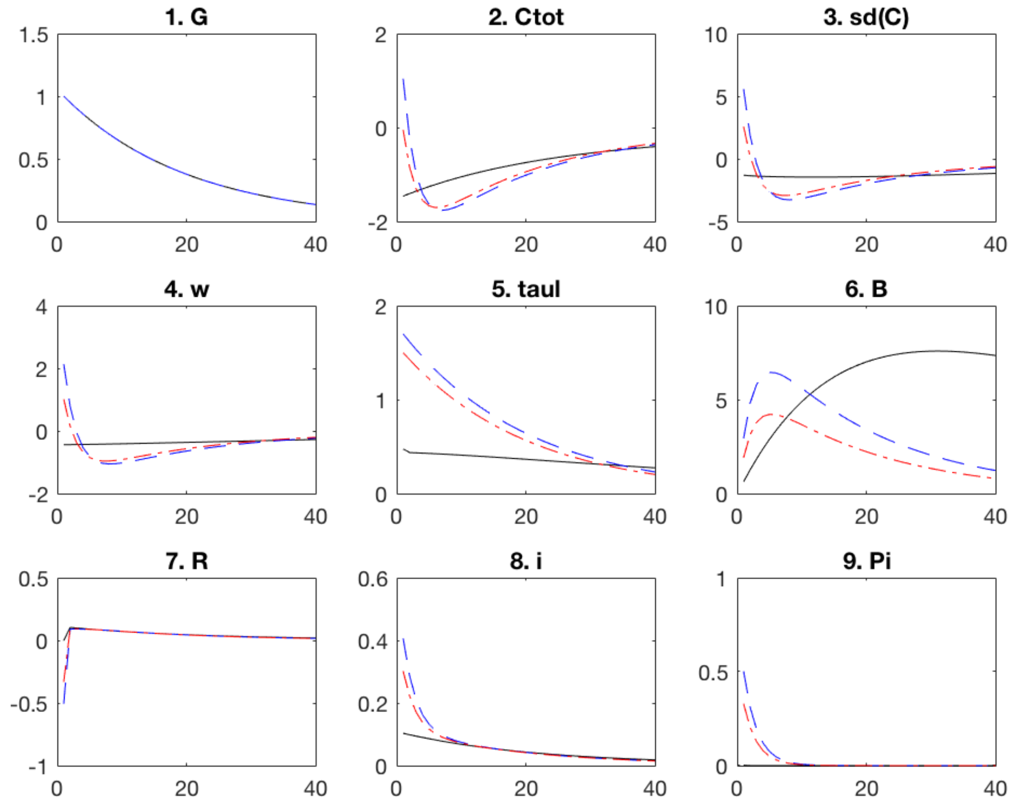

Figure 4: Comparison between the labor-tax economy (black solid line) and economies with exogenous tax policy (dashed lines)

used to change the ex-post real interest rate on public debt, through the Fisher effect. This occurs only at the moment of the shock, as the future path of inflation is forecasted and included in prices (as can be seen from Pannel 7). Second and more importantly, unflation is used to affect the before-tax real wage $\tilde{w}_t$ through the Phillips curve. This affect the labor tax income to balance the budget of the government $\tau_t^L \tilde{w}_t$.

6 Conclusion

We derive optimal monetary policy in an economy with incomplete insurance markets and nominal frictions. Optimal monetary policy crucially depends on assumptions about fiscal policy, notably the availability of capital and labor taxes. We identify three reasons why optimal monetary policy should depart from price stability, that holds in complete markets. When capital taxes are not time-varying, the inflation is a costly substitute to the redistribution of wealth across agents, at the impact of the shock. We call this channel the information channel, because inflation only moves due to new information. Second, when labor taxes are not time-varying, deviation from price stability is used to affect the real wage, and to generate redistribution through the real wage. This second effect is called the real interest rate channel. Finally, in the two previous cases, the inflation affects the budget of the government as public debt is positive to provide liquidity. This last channel is called the public finance channel. The analysis of the outcomes in various cases lets conclude that the most relevant case is the economy without time-varying capital taxes, but with time-varying labor taxes. In this case, public debt increases after a positive shock to public spending. The possibility to derive our analytical results is based on two contributions. First, we use a truncated approach to limit heterogeneity, such that we study a finite (and arbitrarily high) number of agents. Second, we show that the Lagrangian approach is particularly well-suited for monetary economics.

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A Ramsey program

We provide below a detailed expression of the Ramsey program.

$$\max_{\left(\Pi_t,B_t^b,w_t,(a_{t,e^N},c_{t,e^N},l_{t,e^N})_{e^N\in\mathcal{E}^N}\right)} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\sum_{e^N\in\varepsilon^N}\left[S_{t,e^N}U(c_{t,e^N},l_{t,e^N})\right]\right],$$

subject to equations:

$$\begin{aligned}
a_{t,e^N}+c_{t,e^N} &= \theta_{e^N}l_{t,e^N}w_t+\delta 1_{e_0=0}+R_t\tilde{a}_{t,e^N}\\
a_{t,e^N} &\geq -\bar{a}\\
\xi_{e^N}U_c(c_{t,e^N},l_{t,e^N}) &= \beta\mathbb{E}_t\left[\sum_{\hat{e}^N\succeq e^N}H_{t,e^N,\hat{e}^N}U_c(c_{t+1,\hat{e}^N},l_{t+1,\hat{e}^N})R_{t+1}\right]+\nu_{t,e^N},\\
w_t\theta_{e^N}U_c(c_{t,e^N},l_{t,e^N}), &= -U_l(c_{t,e^N},l_{t,e^N})\\
\nu_{t,e^N}(a_{t,e^N}+\bar{a}) &= 0 \text{ and } \nu_{t,e^N}\geq 0\\
l_{t,e_0} &= \delta \text{ if } e_0=0\\
\tilde{a}_{t,e^N} &= \sum_{\hat{e}^N\in\mathcal{E}^N}\frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}}H_{t-1,\hat{e}^N,e^N}a_{t-1,\hat{e}^N}\\
\Pi_t\left(\Pi_t-1\right) &= \frac{\epsilon-1}{\kappa}\left(e^{-z_t}\tilde{w}_t-1\right)+\beta\mathbb{E}_t\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{L_{t+1}}{L_t}\frac{M_{t+1}}{M_t}\frac{e^{z+1}}{e^{z_t}}\\
G_t+R_tB_{t-1}+w_tL_t &= B_t+\left(1-\frac{\kappa}{2}\left(\Pi_{t+1}-1\right)^2\right)e^{z_t}L_t\\
B_t &= \sum_{e^N\in\mathcal{E}^N}S_{t,e^N}a_{t,e^N}\\
w_t &= (1-\tau_t^L)\tilde{w}_t\\
R_t &= (1-\tau_t^K)\frac{1+i_{t-1}}{\Pi_t}\\
c_{t,e^N},\ l_{t,e^N}\geq 0,\ ,a_{t,e^N} &\geq -\bar{a}, \text{for all } e^N\in\mathcal{E}^N\\
(S_{-1,e^N})_{e^N\in\mathcal{E}^N} &\text{ and } (a_{-1,e^N})_{e^N\in\mathcal{E}^N} \text{ are given}
\end{aligned}$$

B Transforming the Ramsey program

Denote $\beta^t m^t(s^t)S_{t,e^N}\lambda_{t,e^N}$ the Lagrange multiplier of the Euler equation for island e^N at date t. Denote $\beta^t m^t(s^t)\alpha_{t,e^N}$ the Lagrange multiplier of the Phillips curve at date t.

The objective of the Ramsey program can be rewritten as:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N})$$
$$-\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \lambda_{t,e^N} \left(U_c(c_{t,e^N}, l_{t,e^N}) - \nu_{t,e^N} - \beta \mathbb{E}_t \left[\sum_{\hat{e}^N \in \mathcal{E}^N} H_{t+1,e^N,\hat{e}^N} U_c(c_{t+1,\hat{e}^N}, l_{t+1,\hat{e}^N}) R_{t+1} \right] \right)$$
$$-\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \alpha_t \left(\Pi_t (\Pi_t - 1) e^{z_t} L_t M_t - \frac{\epsilon - 1}{\kappa} \left(e^{-z_t} \tilde{w}_t - 1 \right) e^{z_t} L_t M_t + \beta \mathbb{E}_t \left[\Pi_{t+1} (\Pi_{t+1} - 1) e^{z_{t+1}} L_{t+1} M_{t+1} \right] \right).$$

With $\lambda_{t,e^N} \nu_{t,e^N} = 0$ and the definition of Λ_{t,e^N}, (28) and of Γ_t(17), we obtain after some manipulations the following expression for the objective of the Ramsey program:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N})$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \lambda_{t,e^N} \left(U_c(c_{t,e^N}, l_{t,e^N}) \right)$$
$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \lambda_{t,e^N} \mathbb{E}_t \left[\sum_{\hat{e}^N \in \mathcal{E}^N} H_{t+1,e^N,\hat{e}^N} U_c(c_{t+1,\hat{e}^N}, l_{t+1,\hat{e}^N}) R_{t+1} \right]$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \alpha_t \left(\Pi_t (\Pi_t - 1) e^{z_t} L_t M_t \right)$$
$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \alpha_t \mathbb{E}_t \left[\Pi_{t+1} (\Pi_{t+1} - 1) e^{z_{t+1}} L_{t+1} M_{t+1} \right]$$
$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} \alpha_{t,e^N} \frac{\epsilon - 1}{\kappa} \left(e^{-z_t} \tilde{w}_t - 1 \right) e^{z_t} L_t M_t.$$

Finally:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(\xi_{e^N} U(c_{t,e^N}, l_{t,e^N})$$
$$+\xi_{e^N} U_c(c_{t,e^N}, l_{t,e^N}) \left(\Lambda_{t,e^N} R_t - \lambda_{t,e^N} + \alpha_t \frac{\epsilon - 1}{\kappa} \left(e^{-z_t} \tilde{w}_t - 1 \right) e^{z_t} L_t - (\alpha_t - \alpha_{t-1}) \Pi_t (\Pi_t - 1) e^{z_t} L_t \right) \Big).$$

C First-order conditions for the real economy

Derivative with respect to w_t: the labor tax. We define the efficient labor share of households with history e^N as $\omega^L_{t,e^N} \equiv \frac{S_{t,e^N} l_{t,e^N} \theta_{e^N_t}}{L_t}$, which represents the share of workers with history e^N in the labor-tax base at date t. Note that $\sum_{e^N \in \mathcal{E}^N} \omega^H_{t,e^N} = 1$. We have:

$$\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} l_{t,e^N} \theta_{e^N} \psi_{t,e^N} = \mu_t L_t \left(1 - \varphi \frac{e^{z_t} - w_t}{w_t} \right),$$

or finally:

$$\varphi\frac{e^{z_t}-w_t}{w_t}=1-\frac{\sum_{e^N\in\mathcal{E}^N}\omega^L_{t,e^N}\psi_{t,e^N}}{\mu_t} \tag{38}$$

The distortions induced by the labor tax are equalized to the equilibrium gain of transferring resources from all households to the government.

The left-hand side is a measure of the marginal cost of raising resources with the labor tax, taking into account distortions, which are an increasing function of labor elasticity φ. The right-hand side is a measure of the marginal gain. If the government and households value liquidity identically, i.e. if $\sum_{e^N\in\mathcal{E}^N}\omega^L_{t,e^N}\psi_{t,e^N}=\mu_t$, then the right-hand side is null, and so is the labor tax τ^L. There is no use for a distorting tool. Conversely, when the government has a greater liquidity need than that of households: $\mu_t^> \sum_{e^N\in\mathcal{E}^N}\omega^L_{t,e^N}\psi_{t,e^N}$, the labor tax becomes positive.

The household valuation of liquidity is an average across households, which can be written as $\sum_{e^N\in\mathcal{E}^N}\omega^L_{t,e^N}\psi_{t,e^N}=\sum_{e^N\in\mathcal{E}^N}\psi_{t,e^N}+cov_{e^N}(\omega^L_{t,e^N},\psi_{t,e^N})$. The covariance term (across histories) highlights an additional net cost of using the labor tax, stemming from its redistributive effect. If the covariance is negative (as in the case in the quantitative investigation below), households with a high labor income have a low liquidity need. In this case, the labor tax tends to be progressive and has a small redistributive cost. As a result, when the covariance becomes increasingly negative, the labor tax will rise, all other things being constant. The reverse holds when the covariance is positive.

Derivative with respect to R_t: the capital tax. We have:

$$\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}\Lambda_{t,e^N}U_c(c_{t,e^N},l_{t,e^N})+\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}\psi_{t,e^N}\tilde{a}_{t,e^N}=\mu_tB^b_{t-1} \tag{39}$$

The distortions induced by the capital tax are equalized to the equilibrium gain of transferring resources from all households to the government.

Defining $\omega^K_{t,e^N}\equiv\frac{S_{t,e^N}\tilde{a}_{t,e^N}}{A_{t-1}}$ one can rewrite the expression as:

$$\frac{\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}\Lambda_{t,e^N}U_c(c_{t,e^N},l_{t,e^N})}{B_{t-1}}+\sum_{e^N\in\mathcal{E}^N}\omega^K_{t,e^N}\psi_{t,e^N}=\mu_t \tag{40}$$

The left-hand side is a measure of the marginal cost of raising resources with the capital tax. It is the sum of the intertemporal distortion and the redistributive effects generated by this tax. The intertemporal distortion generated by the capital tax is a decreasing function of the capital stock, as one additional unit of resources is generated by a smaller marginal increase in the capital tax when the capital tax base, B_{t-1}, is higher. The cost of levying resources depends on the term $\sum_{e^N\in\mathcal{E}^N}\omega^K_t\psi_{t,e^N}=\sum_{e^N\in\mathcal{E}^N}\psi_{t,e^N}+cov_{e^N}(\omega^K_{t,e^N},\psi_{t,e^N})$, where the covariance term again captures the redistributive effect of the capital tax. The more negative the covariance term, the less costly it becomes to levy resources with capital tax. The right hand side is the government valuation of one unit of liquidity raised using the capital tax.

Derivative with respect to B_t: the public debt. We obtain:

$$\mu_t = \beta \mathbb{E}_t R_{t+1} \mu_{t+1} + \gamma_t. \tag{41}$$

This expression equalizes the marginal benefit of issuing one additional unit of debt at time t to the marginal the cost of this additional unit of debt that is equal to the cost of satisfying the financial market equilibrium in time t plus its cost in time $t+1$ using the after tax return R_{t+1} to value the next period.

Derivative with respect to a_{t,e^N}: the net saving of consumers . For all $e^N \in \mathcal{E}^N \setminus \mathcal{C}_t$, this yields:

$$\psi_{t,e^N} = \beta \mathbb{E}_t \sum_{e^N \in \mathcal{E}^N} R_{t+1} H_{t,\hat{e}^N,e^N} \psi_{t+1,e^N} + \gamma_t. \tag{42}$$

This equation states that this marginal cost of saving in additional unit of asset at date t is equal to benefit of relaxing the financial asset constraint in date t (to increase its insurance toward shock) plus the marginal benefit of this unit of asset at date $t+1$.

D First-order conditions for the economy without capital taxes

Derivative with respect to Π_t .

$$\begin{aligned} &\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \frac{1}{\Pi_t} \left[\psi_{t,e^N} \tilde{a}_{t,e^N} + U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N} \right] \\ &= \mu_t \left(\kappa \frac{(1-\Pi_t)}{(1-\tau^K)\frac{(1+i_{t-1})}{\Pi_t}} e^{z_t} L_t + \frac{B_{t-1}}{\Pi_t} \right). \end{aligned} \tag{43}$$

The left-hand side is a measure of the marginal cost of raising inflation. It is the sum of the intertemporal distortion and the redistributive effects generated by the variation of the level of price. The right hand side is the net government's benefit of raising inflation. It equal to the marginal benefit for the govenrment of raising inflation, that is proportional to the real public debt issued in time $t-1$ (because it decreases the real cost of this debt) minus the loss associated with the profit of firms that have to support the cost of price adjustment that is proportional to the level of output and to the degree of nominal rigidities (recall that the government taxes the entire profit of firms).

Derivative with respect to i_{t-1}.

$$\mathbb{E}_t\left[\sum_{e^N\in\mathcal{E}^N} S_{t+1,e^N}\left(\psi_{t+1,e^N}\tilde{a}_{t+1,e^N}+U_c(c_{t+1,e^N},l_{t+1,e^N})\Lambda_{t+1,e^N}\right)\right]=B_t\mathbb{E}_t\left[\mu_{t+1}\right].$$

This equation states that in date t the cost of raising the nominal interest rate is equal to the expected cost of generate intertemporal distortion and to the cost of redistributive effects in time $t+1$ and that this cost is equal to the benefit of an increase in the nominal interest rate in date t that is proportional to the stock of debt issued at this date using the expected valuation of government liquidity in date $t+1$ to value it. It implies that:

$$\mathbb{E}_t\left[\mu_{t+1}\Pi_{t+1}(1-\Pi_{t+1})L_{t+1}e^{z_{t+1}}\right]=0.$$

At date t, the expected level of inflation at the next period is approximately equal to 1.

The first order conditions with respect to the after-tax wage, the public debt and the net saving of agents are similar to the ones in the real economy and are then omitted.

E First-order conditions for the economy without labor taxes

Define:

$$D_{t,e^N}=\theta_{e^N}l_{t,e^N}\frac{U_{cc}(c_{t,e^N},l_{t,e^N})}{U_c(c_{t,e^N},l_{t,e^N})}+\frac{\varphi}{(1-\tau^L)\tilde{w}_t},$$

and:

$$F^1_{t,e^N}=\frac{\varepsilon-1}{\kappa}\left(\left(D_{t,e^N}\left(\tilde{w}_te^{-z_t}-1\right)+\frac{e^{-z_t}}{1-\tau^L}\right)-\frac{\kappa}{\varepsilon-1}D_{t,e^N}\Pi_t\left(\Pi_t-1\right)\right),$$
$$F^2_{t,e^N}=D_{t,e^N}\Pi_t\left(\Pi_t-1\right).$$

With these notations the first-order conditions are the following.

Derivative with respect to $\tilde{w}_t$: the before-tax real wage.

$$\sum_{e^N\in\mathcal{E}^N} S_{t,e^N}\frac{\theta_{e^N}l_{t,e^N}}{e^{z_t}L_t}\psi_{t,e^N}+\sum_{e^N\in\mathcal{E}^N} S_{t,e^N}U_c(c_{t,e^N},l_{t,e^N})\left[\alpha_tF^1_{t,e^N}+\alpha_{t-1}F^2_{t,e^N}\right]$$
$$=\mu_t\left[\frac{1+\varphi}{\varphi}(1-\tau^L)\frac{\tilde{w}_t}{e^{z_t}}+\frac{\kappa}{2}\left(\Pi_t-1\right)^2-1\right]\frac{\varphi}{\tilde{w}_t(1-\tau^L)}.$$

The equation states that the cost of the redistributive effect generated by increasing the real wage (the first expression of the first line) plus the cost of satisfying the Phillips curve (the second expression of the first ligne) is equal to the benefit of changing the real wage on the budget of the government.

Derivative with respect to Π_t.

$$\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N})(2\Pi_t - 1)(\alpha_t - \alpha_{t-1}) = -\mu_t \kappa (\Pi_t - 1).$$

The other first-order conditions are the same as in the previous sections and we then omitted them here. Here Π_t will be different from 1 and the interest rate is indeterminate.

F First-order conditions for the economy without capital and labor taxes

Define

$$F_t^3 = \left(\alpha_t \frac{\epsilon - 1}{\kappa}(e^{-z_t}\tilde{w}_t - 1) - (\alpha_t - \alpha_{t-1})\,\Pi_t\,(\Pi_t - 1)\right) e^{z_t} L_t$$

Then the correction for the Phillips curve in equation (37) is

$$PC^{\Pi}_{t,e^N} = \tilde{a}_{t,e^N} U_{cc}(c_{t,e^N}, l_{t,e^N}) F_t^3 + U_c(c_{t,e^N}, l_{t,e^N}) \frac{\Pi_t}{R_t}\,(\alpha_t - \alpha_{t-1})\,(2\Pi_t - 1)\, e^{z_t} L_t$$